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### THE APPLICATION OF HF CHANNEL DATA

### TO THE DEVELOPMENT OF BLOCK ERROR CORRECTORS

#### SEPTEMBER 1966

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Prepared for

DEPUTY FOR ENGINEERING AND TECHNOLOGY RANGE SYSTEMS DIVISION ELECTRONIC SYSTEMS DIVISION AIR FORCE SYSTEMS COMMAND UNITED STATES AIR FORCE L. G. Hanscom Field, Bedford, Massachusetts



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#### ABSTRACT

Error patterns of high-speed digital data transmission over an operational long-haul HF data link have been obtained for extensive time periods and used to develop statistical descriptions. Distribution functions have been calculated for the cases of consecutive errors, error-free intervals, and bursts and their associated intervals, and processed to obtain maximum error correction with minimum-size interleaving.

#### REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Phill

OTIS R. HILL Colonel, USAF Director of Aerospace Instrumentation



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#### SECTION I

#### INTRODUCTION

#### STATEMENT OF THE PROBLEM

The HF medium (2 mc to 30 mc) offers the potential for efficient long – range communications at a significantly lower cost than any other method available today where a modest number of voice channels are required.\* The superior tactical flexibility of military HF communications systems for over the horizon communications requirements remains unchallenged today and will probably remain so for some years to come. Specifically, the importance of digital data transmission using the HF medium has increased rapidly. This development is due in part to the fact that the majority of non-voice information transferred is in binary format. In addition, the digitization of voice transmission has become an operational objective for the great majority of military communication links. If the advantages of HF communications systems are to be realized, the problem of reliable transmission of digital data using this medium must be solved.

<sup>\*</sup> For example, a full-duplex four-channel (voice) HF terminal (including a receiver, single sideband exciter and l0-kw power amplifier, synthesizer, and a pair of log periodic antennas) can be purchased today for less than \$100,000. The initial cost of the antenna (30-foot diameter dish) for a ground satellite terminal is well in excess of \$100,000. In addition, a satellite repeater vehicle is a multimillion-dollar investment. In order to make economical use of satellite terminals, a large number of channels must be utilized so that the high initial investment can be spread out, resulting in a competitive cost per channel. In addition to the high initial cost of satellite ground terminals, there is a relatively high degree of technical support required for their installation, as well as a relatively high level of logistics support necessary, compared to an HF terminal.

The basic mechanism of HF propagation is fairly well understood; however, the problem of knowing what the medium is doing when one wants to use it, and of knowing what to do about it, is another matter. The principal difficulty in using the HF medium efficiently stems from the extreme time variability of propagation conditions of any specific path. Several new developments in recent years have improved this situation. For example, it is now a relatively straightforward matter to determine the optimum operating frequency for any selected path and time using ionospheric sounders. Since this knowledge is not unique to any single user, however, other stations using the same path tend to cluster in the optimum frequency range and interfere with one another. Other constraints, such as restricted frequency allocations, impose additional problems on the user of the HF medium. In order to appreciate the gross effects of HF data transmission channels on digital data transmission, the following analysis is made.

When human voice is transmitted over an HF channel, the limitations of the transmission medium are well disguised by the high redundancy of voice . nd the corresponding low information rate. For example, the normal speech rate is somewhat less than 300 words per minute for an average conversation. This amounts to an information rate of 150 bits\* per second. This information rate is equivalent to the transmission rate of two standard teletype channels. The transmission of analog voice signals requires a system with a 3-kc bandwidth because of the very high degree of redundancy inherent in speech. Now consider what takes place with a digital data transmission over the same HF system. A typical system user will attempt to transmit at 2400 bits per second through a 3-kc channel. This is an increase in data rate of more than one order of magnitude compared to the information transmission rate of voice.

<sup>\*</sup> An average of six characters per word and five bits per character is assumed.

The high digital data transmission rate is achieved by the reduction of redundancy compared to normal voice transmission in the same bandwidth. This data transmission is, however, much more vulnerable to errors, so steps must be taken to realize the accurate transfer of data.

#### PROPOSED SOLUTION

The problems of data transfer are rapidly compounded because the information transfer rate is normally much higher when digital data is transmitted over a typical HF transmission facility than in the case of normal voice. Several remedies are available which must be used in conjunction with forward error detection and correction. These include such classical remedies as frequency diversity transmission and reception, space diversity reception, and possibly ARQ (Automatic Retransmission Request), where this is permissible. The present state-of-the-art requires that a bit error rate no greater than  $10^{-2}$  exist on the channel before error correction can be effectively applied. One solution, then, is to make the best of what is available in the HF spectrum by taking advantage of more sophisticated signal processing techniques, and by including error detection and correction as an additional technique.

The ARQ error detection and retransmission technique has been used successfully for many years on HF teletype channels. This technique is not useful for quasi-real time data transmission, however, because of the synchronous data transmission requirement.

A method which is rapidly emerging from the theoretical stage to practical implementation is forward error detection and correction. Analysis of the observed error patterns introduced by a typical high speed (l200 and 2400 bits per second) HF data transmission link shows that error patterns intro-

duced by an HF transmission path can be separated into three general classes, which can be described and defined mathematically.

The characteristic properties of these three classes of error patterns form the basis for a general method of effectively applying forward error detection and correction using block coding methods. The proposed method of attack on the problem of effectively applying forward error correction and detection to an HF data transmission link is based on the strategy of transforming channel error patterns into randomly distributed errors. Thus, bursterror patterns which are difficult to correct in their existing form are transformed into easily correctable random-error patterns. The net result of the randomization process performed by interleaving is the nearly equal distribution of channel errors over the words in a message. Significant improvement of error rate is demonstrated for actual data with average uncorrected error rates as low as  $10^{-2}$ , using realizable block codes and coding delays of less than 5 seconds. The first step necessary in the application of these procedures is to obtain error patterns from the HF link.

#### REAL-TIME ERROR DATA MEASUREMENTS

One phase of the Range Digital Data Transmission Improvement Program being conducted at The MITRE Corporation for the Electronics System Division and the National Range Division is concerned with the significant reduction of data transmission errors introduced by operational real-time HF data transmission. The method of attack that has been developed makes use of forward error correction techniques. In order to achieve this objective, the first step taken was to determine the nature of the data transmission errors introduced by the HF medium.

A test program was developed and implemented using the Eastern Test Range operational HF data link between Antigua Island and Ascension Island.

The objective was to gather error pattern data over a wide range of conditions on a typical operational HF data link. The test was conducted using the normal operating equipment and circuits on a non-interference basis for a period of six weeks during the fall of 1965. The test path and a block diagram of the test implementation are shown in Figure 1. A 52-bit digital test message was repetitively transmitted from the Antigua Transmitter Facility to the Receiver Facility at Ascension Island, where the message was detected and retransmitted to Antigua. This received test message was compared with the original test message suitably delayed to match the total transmission time of the test link. The comparison was made using a modulo-two adder which summed the original test message (delayed) and the received test message. The output of the modulo-two adder indicated a binary 'one' state whenever the two input signals were not the same. The output signal of the modulo-two adder was recorded in real-time on magnetic tape at Antigua Island. The test transmissions were divided into test periods called runs, which were each approximately ten minutes in duration. This was done so that error patterns can be easily associated with the specific channel conditions and data rates of any individual run. The test message was transmitted at data rates of 1200 bits per second and 2400 bits per second.

The data modem used for these tests was the Collins Radio Company Model TE-216 unit, which is designed to operate at high data transmission rates (up to 3,600 bits per second) over HF channels of voice bandwidth. This modem uses 4-level phase modulation of each of 16 frequency-multiplexed audio tones for a transmission rate of 2400 bits per second. This was accomplished using time-differential phase-shift keying. Only eight tones were used for 1200 bits per second data transmission rate. The modulation technique used in the modem permits a data rate of 150 bits per second per tone. Coherent time-differential phase-shift detection was used to detect the phase-



Figure 1. Typical Operational HF Data Link

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#### (b) functional block diagram, error data test implementation

modulated signal for each incoming tone. The method of modulation used for these tests is important because specific error pattern characteristics can be identified as systematic-type errors attributable to the modulation technique used. Specific methods for eliminating these errors will be shown later on.

After the field tests were completed, the magnetic tape recordings of the error patterns introduced by the transmission medium were sent back to The MITRE Corporation facilities in Bedford, Massachusetts, for processing and analysis. The field recordings of error patterns were translated into IBMcompatible tapes so that the data were compatible with the IBM 7030 computing facility used by MITRE. The next step was to process and analyze the error pattern data and extract significant characteristics that will be useful in establishing performance and design criteria for the implementation of forward error detection and correction.

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#### SECTION II

#### SUMMARY

General characteristics of the observed data error patterns introduced in an HF digital data transmission system will be derived and shown to be useful in the evaluation and design of forward error detection and correction using interleaving and block error correction codes. This technique is applicable to digital data transmission links which require near real-time synchronous operation and low error rates (less than  $10^{-5}$  bit error rate). Error correction capability is obtained at the price of transmission delay time introduced by error correction implementation and the addition of redundant data bits to provide the necessary coding structure for each data word transmitted. It is a fundamental requirement that if <u>a priori</u> knowledge of the message information transmitted is not available at the receive terminal of the transmission link, redundant data bits must be added to the information transmitted to achieve error detection and correction capability.

Error pattern data gathered from an operational HF data transmission link are shown to have characteristics which permit the direct classification of the observed error patterns into predominantly burst, random, and periodic error categories. Random-error pattern data is characterized by the relatively uniform distribution of errors throughout the observed data. On the other hand, burst-error pattern data is characterized by the appearance of clusters of errors in the data stream. Periodic errors are manifested by a high error rate on one of the modem tone channels relative to the error rate on the other tone channels.

The distribution of observed data is tabulated in Figure 2 in terms of random, burst, and periodic error runs. Data runs with error rates greater



(a) in terms of random and burst errors



## Figure 2. Distribution of Observed Data

than  $10^{-3}$  were predominantly (94 percent) burst-error type runs. Over 72 percent of the data runs with error rates less than  $10^{-4}$  were random-error type runs. The random-error data runs that had error rates between  $10^{-3}$  and  $10^{-4}$  were approximately equal to the burst-error data runs of the same error rate class. The occurrence of periodic errors increased with increasing error rate. For example, 83 percent of the runs in the  $10^{-2}$  error-rate decade contained periodic errors, whereas only 4.5 percent of the runs in the  $10^{-5}$  decade contained periodic errors.

The measure of randomness of observed errors is established by comparison with distributions of known random-error data derived from a computer program that was designed to generate random-error pattern data using a random-number generator program. The magnitude of the area between the observed error data distribution and the corresponding known random-error data run was used to evaluate the degree of randomness of observed data errors and interleaved data errors.

Six typical data runs are selected from a total of 151 observed data runs. These data runs are then analyzed using the postulated characteristic distribution functions of the error patterns. Three of the data runs are selected as random-error data runs and the other three are selected as burst-error data runs. Error correction without interleaving is evaluated for the random data runs, and it is found that the best performance is achieved using powerful random-error-correcting codes such as Bose-Chandhuri (255, 123, 19) code.

The three burst-error runs are used to evaluate performance of error correction with and without interleaving. Interleaving used with a modified Golay error-correcting code gives results superior to those obtained using this code without interleaving. Care must be exercised, however, to select interleavers that do not magnify the effects of periodic errors.

A valid method is established which permits the quantitative evaluation of interleaver performance in terms of effective randomization of burst-error patterns. Criteria are also established for proper design of the interleaver to minimize the effect of periodic errors.



#### SECTION III

#### ERROR PROCESSING METHODS

#### ERROR DATA CLASSIFICATION

The problem in the design of forward error detection and correction equipment is to determine which characteristics of the test data error patterns are significant, i.e., useful in the design of forward error detection and correction equipment. The selected classifications (and corresponding characteristics) must be independent of message structure. The following classes and their corresponding characteristics were tentatively selected as meeting this requirement:

- (a) Random-error patterns (randomness)
- (b) Burst-error patterns (burstiness)
- (c) Systematic-error patterns (periodicity)

These three classes of error patterns were selected for the following reasons: (1) they can be easily identified from the error location printout of observed test data, (2) they are amenable to a mathematical definition, and (3) they can be used to describe the observed error patterns. An intuitive definition for each of these classes will be given first; an explicit definition will be presented in later sections of this paper.

The random-error pattern is characterized by the occurrence of errors that are uncorrelated; that is, each error occurrence is independent of the immediate past history of error patterns in a message. Burst-error patterns display a clustering effect in some region of the message data stream, showing a dependence on the past history of errors; these burst regions are separated by relatively error-free interval regions. Systematic errors occur in periodic patterns. Each of these types of error patterns is illustrated in Figure 3. These examples are taken from the actual test data used for this report. The computer printout for error patterns shows the location of errors relative to the previous error when read from left to right, starting with the top row. Periodic errors occur when one tone has an exceptional error rate due to frequency selective fading or interference. This is shown in Figure 3c.

#### FUNCTIONAL ERROR CORRECTION IMPLEMENTATION

The basic method of error detection and correction discussed in this paper relies on effective randomization of error patterns introduced by the transmission medium. Once these error patterns are reduced to a single class, i.e., random errors, a powerful random-error correction code can be used to correct these errors. The functional implementation of this technique is shown in Figure 4. The data stream from a data source is coded and interleaved before transmission. The interleaver unit essentially performs a systematic time ordering of the coded data so that burst-error patterns introduced by the transmission medium are spread out in time. At the receive terminal, the interleaved data stream is then reassembled into the original coded data format and is checked for errors which are then corrected (within the limitations of the code capability).

The key to the successful correction of systematic and burst-error patterns lies in the basic design of the data interleaver unit. In this paper, a method is developed which permits a systematic and quantitative evaluation of the degree of burst-error randomization achieved for any realizable interleaver design. The results and analysis presented here are based on computer modeling of interleaver design, and the application of actual error patterns from an operational HF data link as the interleaver input signal.

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(a) random data sample

RUNNING	COUNT	0F TI	HE NUMBER	0F G000 81	ITS BEFORE	ERRDR	(INCLUDING	THE ERROR)				
236	151		74	115	81	32	107	155	10	73	254	44
82	63		23	112	70	56	273	314	80	29	196	119
295	77		154	208	33	60	62	29	73	65	21	218
250	85		50	101	16	1	1.16	10	231	135	160	46
10	63		6	132	1	143	18	141	43	239	23	23
21	275		l	20	194	42	56	22	72	16	163	7
86	58		34	35	114	74	141	36	158	379	103	387
215	45		153	31	54	100	25	51	67	251	70	6
67	4		66	85	27	45	23	266	80	32	69	54
151	200		21	12	265	10	87	55	133	67	б	38
53	18		111	59	64	66	64	40	001	23	141	67
19	89		141	33	47	22	115	157	158	67	112	282
40	48		117	66	48	181	127	171	01	96	88	21
80	83		78	31	199	21	4 8	37	163	180	12	33
ŝ	15		65	156	31	64	25	82	58	204	124	299
85	58		96	33	174	27	89	18	239	19	53	138
27	56		88	284	1	116	061	103	10	62	100	17
23	160		100	621	26	374	127	65		64	13	340
23	87		234	78	2	15	224	62	86	69	147	83
46	282		237	162	241	186	130	252	139	324	27	22
72	15		1	48	137	69	11	125	247	60	ŝ	87
175	79		96	6	72	383	172	60	114	35	18	159
58	174		173	66	141	68	160	6	307	87	228	73
28	24		6	127	55	13	119	212	22	140	264	53
7	156		106	211	96	28	20	23	26	36	25	13
142	10		35	153	34	1	186	11	327	16	185	16
156	185		69	130	32	83	6	25	103	11	34	20

AUNNING COUNT OF THE NUMBER OF GOOD BITS BEFORE ERROR (INCLUDING THE ERROR)

31326	37215	1	3522	2	1	1	1974	3667	9	1	1	2	1	e	1	39	4062	1	5	122	1	20	1	1	38	15
13	72939	13641	21271	2	1	8630	1	270293	4	3	1	1	1	1	11	24	17	2	2	11	158	1	1	39	ŝ	4
e	27317	2	1	7316	2510	1	1	e.	637302	1	1	1	9	2	1	1	1	2	*	Ŷ	1	361	276	16	112	9
1	2	1	4014	1	1	1	8488	85040	3	2	1	5	1	1	1	1	3	2	1	36	103	2	24	23	1	16
ē	10137	5851	1	15253	1	1	1	216209	3998	1	2	2	4	1	1	1	156558	9	2	103	45	116	138	100	7	Ŷ
2	1	60216	2	3	1	37286	1	3946	6	1	1	2	٠	11	1	14	327028	3	1	1	52	Ŷ	29	1	14	1
2	2	65	1	25812	13760	2	8717	2	9234	5	4	2	2	1	2	1	234052	1	1	39	1	34	1	2	186	3
e	2	516820	5186	4	2	2645	1	2	9	2	2	1	1	1	1	2	17115	3	2	615	107	125	2	1	41	11
ē	2	491265	2	12675	4316	1	8032	4309	1	2	2	1	2	9	4	1	44554	*	1	196	15	42	114	180	19	9
1	16	54952	51582	1	2	11876	9	1	10945	2	2	ŝ	1	*	0	ē	240825	1	Ŷ	2	04	55	1	115	16	40
ŝ	13441	14	2	1	1748	1	8383	1	2	1	1	*	ŝ	9	¢	*	31216	*	Э.	808135	5	274	9	221	3	56
295313	2228	272129	5659	1	61161	3678	1	1	1	1	2	9	1	1	Ţ	12	17	2	1	22985	1	21	4	121	17	1

(b) burst data sample

4

.....

TEST RUN NUMBER = 309 MODEM = KINEPLEX INUMBER OF TONES =16

#### TONE NUMBERS IN REVERSE SEQUENCE

#### ERROR RATE FOR EACH OF THE 16 DATA TONES

TONE	NO. OF BITS	NO. OF ERRORS	ERROR RATE
1	89970	2343.	.02604201
2	89970	2974.	.03305546
3	89970	318.	.00353451
4	89970	104.	.00115594
5	89970	101.	.00112260
6	89970	106.	.00117817
7	89970	112.	.00124486
8	89970	47.	•00052240
9	89970	29.	.00032235
10	89970	30.	.00033344
11	89970	19.	.00021118
12	89970	14.	.00015561
13	89970	11.	.00012226
14	89970	14.	.00015561
15	89970	22.	·J0024453
16	89970	8.	.00008892

(c) tone error rates

The basic measure of error pattern randomness presented herein is determined by comparing the cumulative distribution function of observed error bursts with the cumulative distribution function of error bursts for a synthetic random-error run\* which has the same average error rate and is known to have a random-error distribution. As an additional check, the gap and interval distribution functions of experimental data are also compared with the corresponding distribution functions of the same synthetic random-error runs. It is found that good agreement exists among all three methods of comparison.





<sup>\*</sup> Appendix A

#### SECTION IV

#### DESCRIPTIVE STATISTICS

The following statistical parameters of observed error patterns will be shown here to have the capability of discriminating predominantly randomerror data from predominantly burst-error data:

- (a) Error-free gap cumulative distribution function
- (b) Burst cumulative distribution function
- (c) Interval cumulative distribution function

The gap distributions were calculated for a large number of experimental data runs. However, the computation time required for the same number of experimental data runs to determine the burst and interval distributions would be prohibitive. In this case, six runs were selected which represent the randomand burst-type error patterns that are observed on a typical HF data link. These runs are analyzed in detail in the following paragraphs.

#### GAP DISTRIBUTIONS

An error-free gap is defined as that region of the observed data stream which begins with a correct bit that is immediately preceded by an error and ends with the last consecutive correct bit that is immediately followed by an error. For example, ecccce... is an error-free gap of size four.

The method of processing the recorded error pattern data to determine the gap distribution is as follows. The data runs (151) were first sorted into decades of average error rate:  $A \ge 10^{-Z}$  where  $1 < A \le 10$  and Z = (2, ...5). Then the data were separated into 1200 and 2400 bits per second subclasses for each decade of error rate. The reason for proceeding in this manner was to reduce the total bulk of output data while still obtaining meaningful results.

Instead of 151 individual gap distribution functions, only 8 gap distribution functions are presented. Each gap distribution is really the mean cumulative distribution function of gaps for error runs within a selected subclass of error rate decade and data rate.

The procedure of generating mean distribution functions in this manner is permissible since the combination of density functions (each of which sums to unity) and their corresponding means is identical to the sum of the means of the subpoints which generate each distribution. The gap distribution for each class of data is shown in Figures 5a and 5b. The abscissa shown for each gap size is the actual gap length.

There are two important characteristics of the data which are shown by the gap distribution curves. These are the occurrence of periodic errors and the relative randomness of the error-data patterns. The periodic errors result in discrete jumps in the cumulative distribution curve. For example, the occurrence of periodic errors of 16- and 32-bit periods is shown in Figure 6. These periodic errors are related to the design of the modem used for this test and the relationship of the incoming serial bit stream bits to the modem tones. For example, at an input data rate of 2400 bits per second, 16 tones are used by the modem. Every 32nd bit of the incoming bit stream will appear on the same tone of the modem, so that any interference or degradation of this tone at the receive terminal will result in a periodic error pattern of 32 bits (2 bits/baud) in the received data stream.

The degree of randomness of the errors of the specific data run under observation is determined by comparing the resulting cumulative gap distribution curve with the gap distribution of a synthetic random run of the same average error rate. Using the intuitive definition of random and burst error patterns, two data runs were selected. Run 117 was selected as a randomtype run and run 264 was selected as a burst run. A comparison of these two



(a) data rate = 1200 b/s









experimental runs with the corresponding gap distribution of the synthetic run of the same average error rate is made in Figure 7. It should be noted that run 117 more closely resembles the synthetic data run. Thus, the degree of randomness of the specific run under observation can be determined by measuring the disparity between the cumulative gap distribution of the run under observation and the reference distribution.





#### BURST DISTRIBUTIONS

A burst is defined as that region of the data stream within which a minimum of two errors exists. The region begins with an error bit that is immediately preceded by a correct bit. A specified minimum density of errors,  $\Delta$ , must exist in the region, where  $\Delta$  is defined as the ratio of error bits to total bits in the burst region for maximum burst length in the region. A burst always ends with an error bit that is immediately followed by a correct bit. The following example is a burst where the selected minimum error density is 0.5.. ceecececcc. The burst length in this exar ple is six bits.

Cumulative burst distributions for six typical data runs, 111, 117, 121, 264, 309, and 339, are shown in Figure 8 (a) through (f) respectively. The burst distribution is shown for two values of minimum error density (0.1 and 0.3) for each run. The corresponding synthetic random-data cumulative distributions are also shown with these curves, for comparison. One characteristic of random-error-pattern data to be expected is the frequent occurrence of short bursts and relatively long interval regions.

Examination of error locations in runs 111, 117, and 121, and the burst distributions for these runs, indicates that these runs are predominantly random in nature. The burst distribution for each of these runs shows that over 84 percent of the bursts are less than 20 bits long. Comparison with the burst distributions for each of these corresponding synthetic data runs shows that over 84 percent of the bursts in these runs are less than 18 bits long. This is an excellent agreement with the observed error data runs, which are thus interpreted to be predominantly random.

Examination of error locations in run 264 and the corresponding burst distribution reveals that over 20 percent of the bursts are greater than 20 bits long and 5 percent of the bursts are greater than 50 bits long ( $\Delta = 0.1$ ). On the other hand, the corresponding synthetic data run has no bursts greater than 20 bits in length. The evidence indicates that this run (264) is predominantly bursty rather than random. Similarly, runs 309 and 339 are classified as burst-error runs. A more dramatic indication of the burstiness of this run will be shown in the following discussion on interval distributions.





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Figure 8. Frequency Distribution on Lengths of Bursts







(d) Run No. 264



(e) Run No. 309



(f) Run No. 339

#### INTERVAL DISTRIBUTIONS

Interval regions are those sequences of bits which lie between bursts. Every interval region begins with a correct bit which is immediately preceded by an error bit, and ends with a correct bit which is immediately followed by an error bit. The density of errors in the interval region is always less than  $\Delta$ , the minimum error density selected for burst regions. The error-free gap can be considered as a limiting case of intervals when  $\Delta = 1$  is used as an input in burst calculations.

The interval distributions for runs 111, 117, 121, 264, 309, and 339 are shown in Figure 9 (a) through (f) respectively. It was previously established that runs 111, 117, and 121 were predominantly random, based on the characteristics of their burst distributions. Now a comparison of the interval distributions for these runs will be made with the corresponding interval distributions of the synthetic random data runs. Examination of the interval distribution for run 121 indicates that 90 percent of its intervals are greater than 100 bits in length. Over 99 percent of the intervals for the corresponding synthetic random data run are greater than 100 bits in length. This is in excellent agreement with the observed data, and reinforces the previous classification based on the burst characteristics of this run. The errors in this run are predominantly random.

Now let us examine run 264, which was previously established as having predominantly burst-type error characteristics. The interval distribution for this run shows that only 50 percent of the intervals are greater than 100 bits long. Another characteristic of this run is the relatively frequent occurrence of short intervals. In fact, the density distribution of intervals for this run is heavily weighted at the short interval lengths. Since every burst is always followed by an interval, the bursts in run 264 are closely spaced.



(a) Run No. 111



(b) Run No. 117

Figure 9. Frequency Distribution on Lengths of Intervals



(c) Run No. 121



(d) Run No. 264



(e) Run No. 309



(f) Run No. 339

The distinguishing characteristics for random and burst-type error pattern runs have been established. The next step to be taken is the application of this knowledge to implement error correction using interleavers and powerful random error correction codes.

#### SECTION V

#### ERROR CORRECTION PERFORMANCE

Forward error-correcting codes are used when the data that is transmitted must be corrected synchronously (i.e., real-time relationship must be maintained in the received information). Thus, the error-correcting code used must be constrained to meet this requirement. A short fixed delay introduced by the coding implementation into the information data stream is permissible providing the delay is not excessive relative to the real-time data transmission requirement. In general, the more powerful the error correction, the longer the fixed delay that is introduced by the error correction implementation. Clearly, the objective is to achieve the maximum error correction within the limits of the permissible fixed delay requirement, channel bandwidth, and required information transfer rate.

The implementation of error correction of data with predominantly bursty error patterns requires the effective randomization of this data and then error correction. The criteria for measurement of the degree of data randomization have already been established in the previous section. These criteria are now used to establish the effectiveness of interleavers (burst randomizers), using experimentally obtained data that has a high degree of error bursts. It is shown here that the combination of interleaving and error correction provides significantly better performance in error correction than error correction coding alone. In addition, the degree of improvement is shown to be directly related to the effective randomization achieved by the interleaver.

#### ERROR CORRECTION WITHOUT INTERLEAVING

The encoding process for error correction block codes generally takes the form of examining each block of n data bits and calculating k bits of parity to form an (n + k)-bit code word, which is transmitted as digital data. The code parameters are identified as (n + k, n, e) where e is the number of correctable errors in an (n + k)-bit code word. When this code word is received at the other end of the link, it is processed in accordance with the mathematical rules from which the original parity bits are derived. The code will correct up to e errors in this code word where e is a function of n, k, and the mathematical rule for decoding<sup>[1]</sup>. If the errors are uniformly distributed in the received message so that the occurrence of more than e errors in a code word is rare, the probability is quite high that all the errors will be corrected in the message. However, if the errors (same number) are distributed in such a manner that a large number of code words have more than e errors, then the number of errors after decoding will still be significant. This latter situation is the case when the error patterns in the message are bursty rather than random in distribution.

The performance of seven error-correcting codes was evaluated, using the six experimental data runs described in Section III. This evaluation was performed using a computer simulation of the system in Figure 4, with no interleaving. These codes included the Hamming (7, 4, 1) code (which corrects one error in seven bits, using three parity bits), the modified Golay code (24, 12, 3), and five near half rate Bose-Chandhuri codes of length 2  $^{P}$ -1 (where 4 ). The performance of these codes is tabulated in Table I. The average bit error rate and total number of errors are given for each run togetherwith the percent of errors corrected by each code for each of the six runs. Theimprovement is also tabulated for each code and data run, and is defined as theratio of number of input errors to the uncorrected errors in the output.

Examination of Table I provides further verification that the classification of the six data runs as established in Section III is correct. For example, the best performance of error correction for a burst-error run (run 264) was with

Table I

Error Corrections for Seven Codes

	-		Code (Bose-(	Chandhuri excel	pt as indicated)		
Run	(7, 4, 1)	(15, 7, 2)	(24, 12, 3)*	(31, 16, 3)	(63, 30, 6)	(127, 64, 10)	(255, 123, 19)
No. 264 BER: 2.9 x 10 <sup>-3</sup> No. of Frrons: 4193							
Improvement	4.8	12.1	24.0	17.3	20.7	17.9	14.5
% Errors Corrected	79.1	91.7	95.8	94.2	95.1	94.4	93.1
No. 309 BER: 4.3 x 10 <sup>-3</sup>							
Improvement	4.7	17.4	38.8	37.2	38.1	30.0	29.7
% Errors Corrected	78.7	94.2	97.4	97.3	97.3	96.6	96.6
No. 339 BER: 1.2 x 10 <sup>-3</sup> No. of Errors <sup>,</sup> 18576							
Improvement	2.7	4.5	8.7	6.6	7.5	5.4	5.4
% Errors Corrected	62.9	77.7	88.5	84.8	86, 6	81.4	81.4
No. 111 BER: 1.7 x 10 <sup>-3</sup> No. of Errors: 652							
Improvement	8.2	65.2	54.3	38.3	72.4	8	8
% Errors Corrected	87.8	98.4	98.1	97.3	98.6	100.0	100.0
No. 117 BER: 1.2 x 10 <sup>-3</sup> No. of Errors: 924							
Improvement	6.4	23.1	26.4	18.4	28.8	61.6	8
% Errors Corrected	84.3	95.6	96.2	94.5	96.5	98.3	100.0
No. 121 BER: 2.5 x 10 <sup>-3</sup> No. of Ferrore, 1805							
Improvement	7.4	42.9	62.2	58.2	8	Ŕ	ý
% Errors Corrected	86.4	97.6	98.3	98.2	100.0	100.0	100.0

\* Modified Golay

-

the modified Golay code. This code is the most powerful burst-error correcting code used here. The long random-error correction codes such as the (127, 64, 10) Bose-Chandhuri code performed best on the predominantly random-error type runs such as 111, 117, and 121.

Since the baseline has now been established, the improvement introduced by a combination of interleaving and coding (using one of the codes in Table I) can now be measured. These measurements are carried out using the computer simulation with interleaving inserted.

#### INTERLEAVER CHARACTERISTICS

Consider the following situation which is not uncommon for HF digital data transmission systems. Dense error bursts occur such that a large number of received code words of length (n + k) contain either more than e errors or no errors at all. It is postulated that the code selected can correct up to e errors in an (n + k)-length word. It is clear that the forward error correction code alone is practically useless in this situation. If a method of interspersing these error bursts into the interval regions is used, reducing the number of blocks with more than e errors (while decreasing the number of blocks with no errors), the error-correcting code would provide a significant reduction in received-data error rate. The device which performs this dispersal of errors is an interleaver unit. This unit is divided into two parts, one at the transmit terminal which performs the data time spreading, and the conjugate unit at the receive terminal which restores the time-dispersed message to its original form before error correction is performed.

#### INTERLEAVER DESCRIPTION

A typical interleaver configuration is shown in Figure 10. The interleaver consists of a rectangular array of digital storage elements of m code words,



### Figure 10. General Interleaver Configuration

each (n+k) bits long. Two identically configured interleavers are used in a data transmission link, one at the transmit terminal and the other at the receive terminal. The transmit interleaver is filled on a row-by-row basis from the encoder. When the unit is filled, the stored data are then transmitted on a column-by-column basis. Thus, adjacent bits in the same code word are separated in transmission by (m - 1) bits. At the receive interleaver, the data are read on a column-by-column basis so that the original configuration of the code words is established by reading out of the receive terminal interleaver on a row-by-row basis into the decoder unit. If the error-correcting code is capable of correcting three errors in a code word, a burst of three columns in length would be completely correctable if there are no other errors in the block, since no more than three errors would have been introduced into any one code word. An additional dispersal of errors can be achieved by transmitting the

first [m (n + k)/2] bits of the present interleaved block with the last [m (n + k)/2] bits of the previous interleaved block.

The detailed operation of the interleaver model used for this evaluation is shown in Figure 11. Each bit position in the interleaver is specified by the expression  $i_{jh}^{r}$ , where j identifies the row, h identifies the column, and r identifies the block. The following transmission sequence is read from right to left, so the transmission sequence would be expressed by:

$$i_{2, (\frac{n+k}{2}+1)}^{r-1}$$
  $i_{21}^{r}$  ...  $i_{3, (\frac{n+k}{2}+1)}^{r-1}$   $i_{31}^{r}$   $i_{1, (\frac{n+k}{2}+1)}^{r-1}$   $i_{11}^{r}$   $i_{m, (\frac{n+k}{2}+1)}^{r-1}$   $i_{m}^{r+1}$ 

This interleaver is much more complicated than the simplified model previously discussed. The first bit transmitted in this two-block interleaved pattern is located at  $i_{m1}^{r}$ , followed by the bit  $i_{m[(n+k)/(2)+1]}^{r-1}$ , which is located in



Figure 11. Interleaver Model

the previous block. All the odd-row bits in columns 1 and  $\left[\frac{n+k}{2}+1\right]$  are then transmitted, followed by the even-row bits in these two columns. The same procedure is followed for the next column in blocks r and r-l, and repeated until all the data have been transmitted. At the receive interleaver the data are replaced in the same locations as they were at the transmit interleaver, so that the original data are reconstructed. The length of the interleaver unit is selected to be some prime number, so that periodic errors introduced into the data stream will be non-periodic when taken out of the receive interleaver unit. If p is the length of a periodicity expected in the data stream, the value of m is selected such that the magnitude  $(m \pm np)$  is maximum (for m a prime number and np the multiple of the error period closest to m). It is not always possible to select an interleaver length optimally; however, experience indicates that a reasonable distance from multiples of expected periodicities in the data in interleaver length will practically eliminate periodic error patterns at the input to the decoder unit. On the other hand, when the interleaver length is inadvertently selected as some multiple of periodic errors, the error correction performance can be drastically reduced.

#### INTERLEAVER PERFORMANCE

In order to demonstrate the randomization effect of interleaving that was described above, a burst-error pattern run was selected and interleaved, using the portion of the computer simulation between the input to the source interleaver and the output of the sink interleaver. The resultant error locations and the burst and interval distributions were analyzed. It was found that the interval distribution after interleaving (shown in Figure 12) demonstrated that a significant degree of randomization was introduced by interleaver implementation. Before interleaving of run 264, less than 18 percent of its intervals were longer than 1500 bits (see Figure 9d; error density = 0.3); after interleaving, over 40 percent of its intervals were greater than 1500 bits in length.



Figure 12. Frequency Distribution on Lengths of Intervals, Run No. 264, Interleaved

This is a reliable indication that a significant amount of randomization was achieved. The interleaver length selected was m = 199. Comparison of the randomized distribution for a synthetic random data run shows that considerably more randomization can be achieved using longer interleaver lengths.

The behavior of consecutive error occurrences before and after interleaving is presented in Table II. The occurrence of triple errors was eliminated, as were all consecutive errors of higher order. The number of double errors was reduced by a factor of approximately six. The occurrence of all orders of multiple consecutive errors was reduced by a factor of eight by interleaving. The gap distribution for run 264 is shown in Figure 13 for the cases of non-interleaving and interleaving, together with the theoretical gap distribution function for a synthetic random run. Prior to interleaving, 78 percent of the gaps were shorter than 100 bits; after interleaving, 49 percent of the gaps were shorter than 100 bits. The appearance of periodic errors is indicated by

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	Frequency of C	Occurrence
Number of	Before	After
Consecutive Errors	Interleaving	Interleaving
1	3487	4085
2	307	54
3	23	
4	3	
5	1	
6	1	
Total Errors	4193	4193

### Effects of Interleaving on Consecutive Errors (Run 264)



Figure 13. Effects of Interleaving on Error Patterns for a 100% Correction Case

the sharp rise in the gap distribution function for a 32-bit-length gap. The interleaver has removed this periodic error effect, as shown by the resultant distribution after interleaving.

In addition to the results obtained so far in analyzing the performance of interleaving on burst-error-type data, the most significant result is the improvement in the error correction with interleaving.

#### ERROR CORRECTION WITH INTERLEAVING

The performance of error correction, as implemented by computer simulation, using a combination of selected interleaver configurations and the modified Golay code, is examined here. The performance is measured in terms of improvement relative to performance without coding. The effects of interleaver lengths are also examined. The modified Golay code was selected on the basis of the high degree of flexibility permissible in modeling variable interleaver lengths.

The burst-error pattern runs (264, 309 and 339) were processed through selected interleaver lengths which are near optimal in terms of the periodic error constraint discussed in the previous section. In each case, the error correction improvement increased monotonically as the interleaver length was increased to the next optimal length. This is shown in Figure 14. In every case, the improvement was superior to that achieved using the modified Golay code alone without interleaving. An improvement in errors corrected was observed when the transition from no interleaver to a short interleaver length was made, indicating that even a little interleaving was superior to none at all for situations which cannot permit longer interleaver delays in data transmission.



Figure 14. Improvement Factor

The improvement factor as a function of interleaver length shown in Figure 14 for runs 264, 309, and 339 gives a conservative picture of the improvement achieved. For example, on the basis of word error rate, the improvement factor would be 330 and the percentage of words correctly transmitted would be 99.79 based on 24-bit words for run 339, with an interleaver length of m = 211 words.<sup>\*</sup> The percentage of errors corrected for these three runs as a function of interleaver length, shown in Figure 15, is based on bit error-rate improvement.

The performance of interleaving with error correction coding for randomerror pattern runs was also examined. It was found that all the errors in runs 111, 117, and 121 are corrected with an interleaver of length m = 9, using the modified Golay code. The prime interleaver length restriction was relaxed, since there were no periodic errors. The percentage of errors corrected by the modified Golay code alone (Table I) was 98.1, 96.2, and 98.3, respectively.

2

The performance of the interleaver/error-correcting code combination was examined for the case of periodic errors. The data were observed to have periodic errors that were multiples of 32 (twice the number of tones used by the modem). The performance for an interleaver of length m = 96, 97, and 89 is shown in Table III. It is demonstrated that an interleaver of length 96 was the poorest choice, and a non-optimal prime length resulted in significantly poorer performance than the nearest optimal prime length interleaver (i. e., m = 89).

<sup>100</sup> percent correction was achieved with an interleaver length m = 23 on run No. 339 with a 0.25 rate code.



Figure 15. Percent of Errors Corrected

18,60

Interlegence	In	nprovement Facto	r
Interleaver	Run No. 339	Run No. 309	Run No. 264
96	2.6	2.9	4.6
97	17.4	183.8	174.7

### Table III

### Effects of Non-Optimal Interleaver Selection

#### SECTION VI

#### CONCLUSIONS

Error-rate improvement of burst-type error pattern data can be improved significantly using a combination of optimum interleaving and error correction, such that the interleaving maximizes the randomness of errors. Interleaver design must include the capability to randomize periodic errors, otherwise the performance can be considerably poorer than error correction without interleaving.

The degree of error location randomization can be measured quantitatively using burst and interval distributions of the input and output error patterns of the interleaver unit.

Error rate improvement performance expressed in terms of bit error rate improvement is a very conservative measure of performance of error correction. When ARQ techniques are used in addition to error detection and correction, the symbol, character, or word error rate is more significant. Word or block retransmission requests are significantly reduced using forward error correction, thus preventing the ARQ from temporary breakdown (i. e. continually requesting retransmission of a block under high error conditions). For example, the bit error rate improvement factor of 39 is equivalent to a word error rate improvement factor (using 24-bit word) of 330, for run 339.

The observed error data indicates that the probability is high that error patterns are burst-type in cases where the average error rate is greater than  $10^{-3}$ , and are predominantly random in cases where the error rate is less than  $10^{-4}$ . The occurrence of periodic errors is more frequent for data with average error rates greater than  $10^{-4}$ .



#### APPENDIX A

#### SYNTHETIC DATA GENERATION AND VALIDATION

In order to make comparisons of experimental error data runs with data known to be random, synthetic data runs were generated using a computer-programmed, uniformly distributed, random-number generator [2, 3]. Random data were generated using this computer program by setting the desired probability of error as an input parameter\*. Each time the random-generator output number was less than or equal to the specified probability of error, an error was entered into the synthetic data stream. This procedure was repeated until the desired length of run was generated. If the errors generated by this process are random, then the cumulative probability distribution of a sequence of n correct bits followed by an error is given by the following expression.

$$\Pr\left\{c^{n}e\left|e\right\} = p\sum_{k=0}^{n}(1-p)^{k}$$

n = number of consecutive correct bits

where:

c = correct bit

e = error bit

p = probability of error

The cumulative distribution of the synthetic data runs was calculated for the occurrence of  $c^n e$ . A comparison between this cumulative distribution function and the random distribution function shows excellent agreement, as do all other statistics of the synthetic run (see Figure 16), indicating that the assumption of randomness for the synthetic data runs is valid<sup>\*</sup>.

\* Uniform on (0, 1) with  $\chi^2$  at level 0.05.



Figure 16. Comparison of Theoretical and Actual Distributions

The next step was to derive the burst and interval distribution functions of these synthetic runs by processing through the computer program which activates the burst definition. The reference distributions for synthetic random error data are now established for comparison with the corresponding distribution functions of the actual data.

à

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