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AN INVESTIGATION OF SOME OF THE EFFECTS OF THE IONOSPHERE ON ELECTROMAGNETIC WAVE PROPOGATION

John C. Mester

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Project No. TB3-0538C of the Research and Development Division, Ordnance Corps

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30 July 1951

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ABSTRACT

A study is made of the magneto-ionic splitting and reflection of electro-magnetic waves incident obliquely on a stratified ionosphere. Certain simplifying assumptions are applied to the theory. Under these assumptions the opinion is offered that for sufficiently high frequencies the loss of signal strength due to splitting and reflection may be neglected.

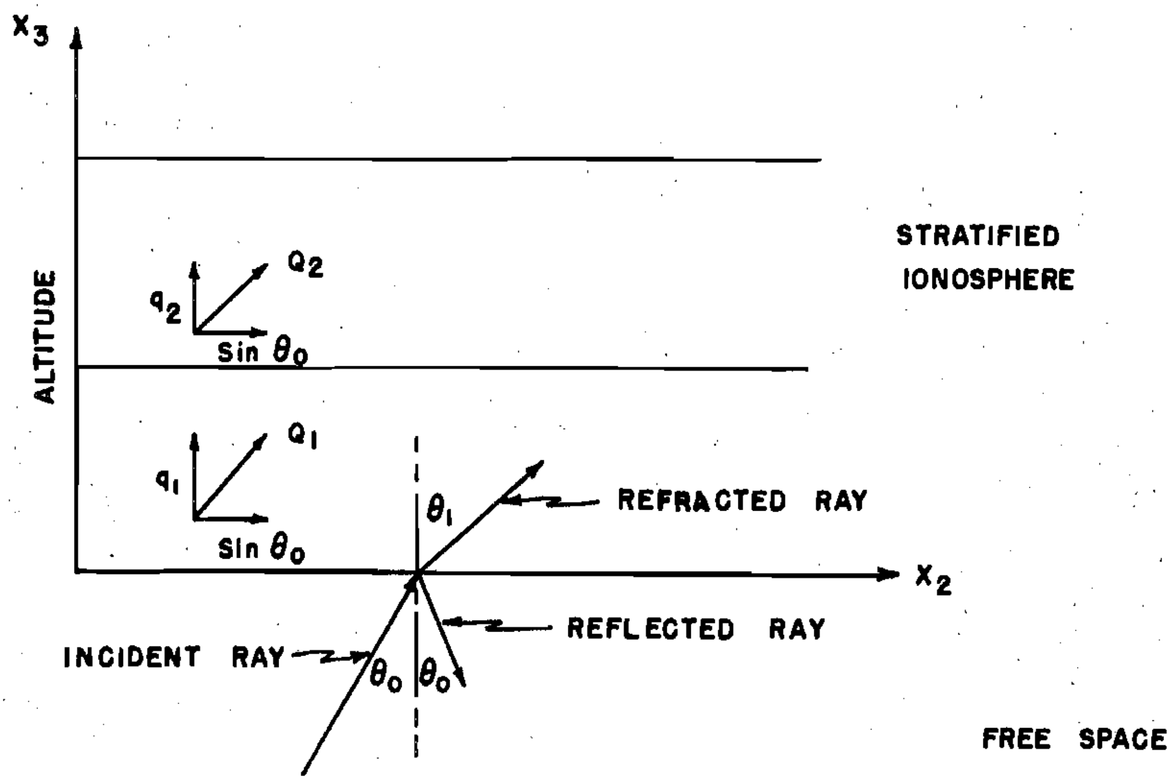
X_3
ALTITUDE
STRATIFIED
IONOSPHERE
Q_2
q_2
Sin θ_0
Q_1
q_1
θ_1
REFRACTED RAY
Sin θ_0
X_2
REFLECTED RAY
INCIDENT RAY
θ_0
θ_0
FREE SPACE

FIGURE 1

INTRODUCTION

It appears advisable, in view of the long range missile program to determine the reduction in signal strength of the DOVAP[1] system for waves propagated in the ionosphere. At long ranges and hence large angles of incidence, the reflection, refraction and magneto-ionic splitting of waves propagated in the ionosphere might possibly cause sufficient loss of signal strength as to effect the tracking distance of the system. Using the magneto-ionic theory in the form given by Appleton[2] and others and as generalized by Booker[3] an attempt will be made to determine quantitatively to a first approximation the reduction in field strength due to these causes.

MAGNETO-IONIC WAVE SPLITTING

Following Booker, we take a cartesion coordinate system (X_1, X_2, X_3) where X_3 is altitude and consider the ionosphere to be horizontally stratified, dependent only on X_3. (See Figure 1) It has been shown[4] that under these conditions the propagation of phase in the neighborhood of a point in the ionosphere may be described by a vector $(0, \sin\Theta, q)$ where the wave is incident at an angle Θ in the (2,3) plane. The magnitude of this vector (designated Q) is the index of refraction of the medium, that is, the ratio of the velocity of light in free space to the velocity of phase propagation in the medium. For an ionized medium Q is less than one. The horizontal component of this vector depends only on the angle of incidence and does not vary from point to point in the layer; the vertical component (q) is a somewhat complicated function of frequency, angle of incidence, the earth's magnetic field and ion density. Booker has shown, under the assumption that only free electrons in the ionosphere affect propagation, that q may be expressed by means of a quartic equation of the form

$$a_4 q^4 + a_3 q^3 + a_2 q^2 + a_1 q + a_0 = 0 \qquad (1.1)$$

(1) Doppler Velocity And Position: a system for determining velocity and position of a missile in flight by phase comparison of two radio signals transmitted between two ground stations - one signal directly, the other by way of the missile. The investigation, however, is applicable to most radio position determination systems.

(2) Appleton; Journ. of Inst. of Elec. Engr. Vol. 71 P-642 (1932)

(3) Booker; Phil. Trans. Roy. Soc. Vol. 237 P-411 (1938)

(4) Booker; Proc. of Conference on Ionospheric Research Vol. 1 (D) (1949)

It is further assumed that damping by collision has little practical affect on phase propagation. Then the coefficients in (1.1) may be written as

$$a_4 = 1 - \left(\frac{e\,H}{2\pi f m v_0}\right)^2 - \frac{4\pi e^2}{4\pi^2\, m f^2}\, N \left\{1 - \left(\frac{e\,H_3}{2\pi m v_0 f}\right)^2\right\}$$

$$a_3 = 2S\left(\frac{4\pi e^2 N}{4\pi^2 f^2 m}\right)\left(\frac{e\,H_2}{2\pi f m v_0}\right)\left(\frac{e\,H_3}{2\pi f m v_0}\right)$$

$$a_2 = -2\left\{\left(C^2 - \frac{4\pi e^2 N}{4\pi^2 f^2 m}\right)\left(1 - \frac{4\pi e^2 N}{4\pi^2 f^2 m}\right) - \left(\frac{e\,C\,H}{2\pi f m v_0}\right)^2\right\} + \frac{4\pi e^2 N}{4\pi^2 f^2 m}$$

$$\left\{\left(\frac{e\,S\,H_2}{2\pi f m v_0}\right)^2 - \left(\frac{e C H_3}{2\pi f m v_0}\right)^2 - \left(\frac{e\,H}{2\pi f m v_0}\right)^2\right\}$$

$$a_1 = -\,2\,C^2\,S\left(\frac{4\pi e^2 N}{4\pi^2 f^2 m}\right)\left(\frac{e\,H_2}{2\pi f m v_0}\right)\left(\frac{e\,H_3}{2\pi f m v_0}\right)$$

$$a_0 = \left(C^2 - \frac{4\pi e^2 N}{4\pi^2 f^2 m}\right)\left\{\left(C^2 - \frac{4\pi e^2 N}{4\pi^2 f^2 m}\right)\left(1 - \frac{4\pi e^2 N}{4\pi^2 f^2 m}\right) - \left(\frac{e\,C\,H}{2\pi f m v_0}\right)^2\right\}$$

$$-\left(\frac{C\,S\,H_2 e}{2\pi f m v_0}\right)^2\left(\frac{4\pi e^2 N}{4\pi^2 f^2 m}\right)$$

where

e, m = charge and mass of electron in esu and grams respectively

N = equivalent electron density, number of electrons per cc

$\vec{H} = (H_1\ H_2\ H_3)$ earths magnetic field in gauss $|\vec{H}| = H$

v_0 = velocity of light in free space (cm/sec)

f = wave frequency (cps)

θ = angle of incidence, $S = \sin\theta$, $C = \cos\theta$

q = upward vertical component of phase propagation vector.

The general solution of this equation for q gives rise to some difficulty and will in general involve complex values of q. However under the transformation $\bar{a}_i = f^4 a_i$, one can rewrite the coefficients as

$$\bar{a}_4 = f^2 \left\{ f^2 - \left(\frac{e}{2\pi m v_o}\right)^2 H_2^{\,2} \right\} - \frac{e^2}{\pi m} \left\{ f^2 - \left(\frac{e}{2\pi m v_o}\right)^2 H_3^{\,2} \right\} N$$

$$\bar{a}_3 = \frac{e^4}{2\pi^3 m^3 v_o^{\,2}} \left\{ S\, H_2\, H_3\, N \right\}$$

$$\bar{a}_2 = -2 \left\{ (f^2\, C^2 - \frac{e^2}{\pi m} N)(f^2 - \frac{e^2}{\pi m} N) - \left(\frac{e}{2\pi m v_o}\right)^2 C^2\, H^2\, f^2 \right\} +$$

$$\frac{e^4 N}{4\pi^3 m^3 v_o^{\,2}} \left\{ S^2\, H_2^{\,2} - C^2\, H_3^{\,2} - H^2 \right\}$$

$$\bar{a}_1 = \frac{-e^4}{2\pi^3 m^3 v_o^{\,2}} \left\{ S\, C^2\, H_2\, H_3\, N \right\}$$

$$\bar{a}_o = \frac{1}{f^2} (f^2\, C^2 - \frac{e^2}{\pi m} N) \left\{ (f^2\, C^2 - \frac{e^2}{\pi m} N)(f^2 - \frac{e^2}{\pi m} N) - \right.$$

$$\left. \left(\frac{e}{2\pi m v_o}\right)^2 C^2\, H^2\, f^2 \right\} - \frac{e^4}{4\pi^3 m^3 v_o^{\,2}} \left\{ C^2\, S^2\, H_2^{\,2}\, N \right\}$$

It can be seen from this that the coefficients of the odd powers of q are independent of frequency while the coefficients of the even powers and the constant term involve the fourth power of the frequency. Hence for sufficiently high values of frequency the terms in q^3 and q may be neglected and the equation reduces to a simple quadratic in q.

$$(1.2) \quad a_4\, q^4 + a_2\, q^2 + a_o = 0$$

From this equation it is then relatively easy to obtain values of q. This introduces one obvious physical error in that it assumes that the effect of the earth's magnetic field on the upward and downward waves is symetric. However, for sufficiently high frequencies this asymetrical effect of the earth's field can for all practical purposes be neglected. The solution of the equation gives rise to two pairs of equal but opposite signed real values. We associate the positive values with the up going wave and the negative with the down coming. The lesser in numerical values is assigned to the ordinary wave since it is closer to the value obtained when the magnetic field is neglected. One notes that one of the effects of the magnetic field is to make the refractive indices closer to unity.

Perhaps at this point some remarks should be made as to what is meant by the rather arbitrary statement that one may use (Eq. 1.2) in place of (Eq. 1.1) for sufficiently high frequencies. Certainly the obtaining of exact solutions for q from Eq. 1.1 is of such difficulty as to warrant any not too unreasonable simplification. However, it is clear that if one is chiefly interested in an investigation of polarization and absorption effects resulting from a complex index of refraction, that even a very high frequency does not justify the use of Eq. 1.2, since Eq. 1.2 in the most part neglects these effects. On the other hand if the primary concern is with possible weakening of the signal due to the interaction of two magneto-ionic components of different phase, one needs a high enough frequency to make $\left|\frac{q_r}{q_i}\right| \gg 1$ where q_r is real part of q

q_i is imaginary part of q

to justify the use of Eq. 1.2. The complete mathematical investigation of what frequency, for a specific value of equivalent electron density and angle of incidence, is necessary to make $\left|\frac{q_r}{q_i}\right|$ equal any desired ratio is beyond the scope of this paper. However, Table I gives some indication of the value of frequency necessary. To set up the criteria to be used here one should concentrate on the ratio $\frac{a_0}{a_3}$. If this ratio is equal to or greater than 10^5 then one can feel reasonably sure that neglecting the terms in q^3 and q will have little effect on the numerical value of q. Hence, for frequencies that give this value for a_0/a_3 one can use Eq. 1.2. Under these conditions the q's obtained from Eq. 1.2 are essentially the vertical components of the Q's calculated from the expression given by Appleton

$$(1.3)\quad Q = \left\{1 - \frac{2}{2\alpha - \frac{\gamma_T^2}{\alpha - 1} \pm \left[\frac{\gamma_T^4}{(\alpha - 1)^2} + 4\gamma_L^2\right]^{1/2}}\right\}^{1/2}$$

H_T is the value of the magnetic field in direction of propagation

H_L is value of magnetic field perpendicular to direction of propagation,

other symbols as previously used,

and;

$$\alpha = \frac{\pi m f^2}{N e^2}$$

$$\gamma_T = \frac{f H_T}{2v_o N e}$$

$$\gamma_L = \frac{f H_L}{2v_o N e}$$

There are then two components of the form $A\, e^{i\omega(t - \frac{x_2}{v_o} \sin\theta - \frac{x_3}{v_o} q)}$ present in the ionized layer, where, for up going waves, the positive values of q as calculated from Eq. 1.2 are used to give the ordinary and extraordinary components. One wishes to consider what effect these components have upon each other. These two waves, considered as rays, will take different paths and for the most part have different spatial location at any particular time. There will exist cross modulation between the waves and further interference due to difference in polarization. Study of these effects has attracted considerable attention in the recent literature[1] and will not be considered here. Rather it is proposed to make the simplifying assumption, however naive, that the net resultant field at any point in space near the mean path of the waves at any particular time, is merely the sum of two waves of the same frequency, but of changing phase with respect to each other. Further, it will be assumed that this difference in phase is dependent only on the vertical distance of the chosen point from some initial point and the difference in vertical refractive index of the two waves. This leads to the consideration that a minimum field strength due to interaction of the two components will occur when the waves are 180° out of phase, that is when

$$\omega \left(t - \frac{x_2}{v_o} \sin\theta - \frac{x_3}{v_o} q_o\right) = (2n + 1)\pi + \omega \left(t - \frac{x_2}{v_o} \sin\theta - \frac{x_3}{v_o} q_x\right)$$

where n is an integer.

This reduces to

$$(q_x - q_o)\, X_3 = (2n + 1)\frac{\lambda}{2}$$

with q_o, q_x = the vertical indices of refraction of the ordinary and extraordinary waves respectively, x_3 = vertical distance of point considered above initial plane, λ = wave length.

1. See for example - Scott, Proc. I.R.E., Vol. (28), No. 9, (1950), P-1057.

Let reference now be made to Table II which gives the q's calculated from Eq. 1.2 using the coefficients of Table I. Consider only those q's that are obtained when the criteria for the use of Eq. 1.2 is satisfied; that is $\frac{a_o}{a_3} \geqq 10^5$. These values of q are so nearly equal that minimums resulting from phase interference must be a relatively large vertical distance apart. Hence, it seems reasonable, considering all the assumptions made and realizing that the values of the q's are probably accurate only to three figures and certainly not more than four that one can, for frequencies satisfying the criteria on the use of Eq. 1.2, ignore these minimums and considered the two components as one wave of the same frequency and phase. .

REFLECTED AND REFRACTED COMPONENTS OF THE WAVE

It is now wished to study the effect of reflection on the wave. Treating a plane polarized wave incident on a stratified ionized medium an attempt will be made to determine the reflection and refraction ratios in terms of the index of refraction and the initial angle of incidence. Consider for the moment, only a single layer of the ionosphere with total refractive index Q and assume a plane polarized wave incident on the layer at an angle Θ. Part of the wave will be transmitted thru the medium and the remainder will be reflected from the layer. Assume that the reflection takes place at the boundary of the layer.

One may take the angle of incidence to be in the 2-3 plane as before without loss of generality and one may further assume that the magnetic vector H of the wave is in the plane of incidence and the electric vector E normal to it. Let I_H, I_E be the incident magnetic and electric vectors respectively; R_H, R_E the reflected vectors and D_H, D_E the refracted vectors. The usual boundary conditions that at the reflecting surface the tangential components of E and H must be continuous hold, that is,

$$(2.1) \quad I_{E_t} + R_{E_t} = D_{E_t}$$

$$(2.2) \quad I_{H_t} + R_{H_t} = D_{H_t}$$

where t refers to tangential component.

Since E is normal to the plane of incidence, it will be tangent to the boundary so one can write 2.1 as

$$(2.3) \quad I_E + R_E = D_E$$

If one takes ϕ as the angle of refraction and takes the angle of reflection as equal to the angle of incidence one can write

$$(2.4) \quad I_{H_t} = I_H \cos \theta, \; R_{H_t} = R_H \cos \theta, \; D_{H_t} = D_H \cos \phi$$

Furthermore, for a plane wave $\eta H = E$ where η is the intrinsic impedance of the medium. Eq. (2.2) can then be written as

$$(2.5) \quad \frac{I_E \cos \theta}{\eta_o} - \frac{R_E \cos \theta}{\eta_o} = \frac{D_E \cos \theta}{\eta_1}$$

The minus sign arising from the fact that E and H are of opposite sign after reflection. Combining 2.3 and 2.4 one obtains

$$(2.6) \quad \frac{R_E}{I_E} = \frac{\eta_1 \cos \theta - \eta_o \cos \phi}{\eta_1 \cos \theta + \eta_o \cos \phi}$$

$$(2.7) \quad \frac{D_E}{I_E} = \frac{2 \eta_1 \cos \theta}{\eta_1 \cos \theta + \eta_o \cos \phi}$$

and in a similar manner

$$(2.8) \quad \frac{R_H}{I_H} = - \frac{\eta_1 \cos \theta - \eta_o \cos \phi}{\eta_1 \cos \theta + \eta_o \cos \phi}$$

$$(2.9) \quad \frac{D_H}{I_H} = \frac{2 \eta_o \cos \theta}{\eta_1 \cos \theta + \eta_o \cos \phi}$$

If one now sets $\eta = (\frac{\mu}{\epsilon'})^{1/2}$ where $\epsilon' = \epsilon \, (1 - \frac{N e^2}{\omega^2 \epsilon \, m})$

ϵ = dielectric constant μ = permeability, the other symbols as before, one may write

$$(2.10) \quad \eta_o = (\frac{\mu_o}{\epsilon_o})^{1/2} \text{ for free space}$$

$$\eta_1 = (\frac{\mu_1}{\epsilon'_1})^{1/2} \text{ for ionized medium}$$

The permeability of the ionosphere is very nearly equal to that of free space, hence

$$(2.11) \quad \frac{\eta_o}{\eta_1} = (\frac{\epsilon_1'}{\epsilon_o})^{1/2}$$

And for a low conducting medium one has

$$(2.12)\quad Q \approx \left(\frac{\epsilon_1}{\epsilon_o}\right)^{1/2}$$

So that

$$(2.13)\quad Q \approx \frac{\eta_o}{\eta_1}$$ where Q is the total refractive index of the medium.

By Snell's law one can write $\frac{\sin\theta}{\sin\phi} = \frac{Q}{Q_o}$ where $Q_o = 1$, hence

$$(2.14)\quad \cos\phi = \frac{1}{Q}\left(Q^2 - \sin^2\theta\right)^{1/2}$$

Substituting these relations in 2.6 - 2.9 it follows that

$$(2.15)\quad \frac{R_E}{I_E} = \frac{Q_o\cos\theta_o - Q_1\cos\theta_1}{Q_o\cos\theta_o + Q_1\cos\theta_1} = \frac{\cos\theta_o - q_1}{\cos\theta_o + q_1}$$

$$(2.16)\quad \frac{L_E}{I_E} = \frac{2\cos\theta_o}{Q_o\cos\theta_o + Q_1\cos\theta_1} = \frac{2\cos\theta_o}{\cos\theta_o + q_1}$$

$$(2.17)\quad \frac{R_H}{I_H} = -\left(\frac{Q_o\cos\theta_o - Q_1\cos\theta_1}{Q_o\cos\theta_o + Q_1\cos\theta_1}\right) = -\left(\frac{\cos\theta_o - q_1}{\cos\theta_o + q_1}\right)$$

$$(2.18)\quad \frac{D_H}{I_H} = \frac{2Q_1\cos\theta_o}{Q_o\cos\theta + Q_1\cos\theta_1} = \frac{2Q_1\cos\theta_o}{\cos\theta_o + q_1}$$

One may note that these ratios are functions of the angle of incidence and the index of refraction which itself is a function of N, f and the magnetic field.

It is wished now to extend this development to a stratified ionosphere. Let there be (n) layers numbered consecutively (1, 2 - - n); to each layer assign a Q_i (i = 1 - - - n). For free space assign $Q_o = 1$ and let I_{E_o} be the wave incident on the boundary between free space and the first layer at angle θ_o. Then R_{E_o} and D_{E_o} will be the reflected and refracted waves respectively associated with the incident wave I_{E_o}, R_{E_j}, and D_{E_j} will be reflected and refracted waves associated with I_{E_j}, the incident wave on the boundary between the j and j + 1 layer. Neglecting absorption through the layer $D_{E_j} = I_{E_{j+1}}$. The equations can then be written in the following form

$$D_{E_o} = M_o I_{E_o}$$

$$D_{E_1} = M_1 I_{E_1} = M_1 D_{E_o} = M_o M_1 I_{E_o}$$

$$D_{E_n} = M_n I_{E_n} = M_o M_1 \text{ - - - - } M_n I_{E_o} = \prod_{i=o}^{i=n} M_i I_{E_o} \tag{2.19}$$

where

$$M_o = \frac{2 \cos \theta_o}{Q_o \cos \theta_o + Q_1 \cos \theta_1}$$

$$M_1 = \frac{2 Q_1 \cos \theta_1}{Q_1 \cos \theta_1 + Q_2 \cos \theta_2}$$

$$M_n = \frac{2 Q_n \cos \theta_n}{Q_n \cos \theta_n + Q_{n+1} \cos \theta_{n+1}} \tag{2.20}$$

Now

$$\sin \theta_n = \frac{\sin \theta_o}{Q_n}, \quad \cos \theta_n = \frac{1}{Q_n} (Q_n^2 - \sin^2 \theta_o)^{1/2} \tag{2.21}$$

from which

$$M_n = \frac{2 (Q_n^2 - \sin^2 \theta_o)^{1/2}}{(Q_n^2 - \sin^2 \theta_o)^{1/2} + (Q_{n+1}^2 - \sin^2 \theta_o)^{1/2}} = \frac{2 q_n}{q_n + q_{n+1}} \tag{2.22}$$

In a similar manner

$$R_{E_o} = K_o I_{E_o}$$

$$R_{E_1} = K_1 I_{E_1} = K_1 D_{E_o} = M_o K_1 I_{E_o}$$

$$R_{E_n} = M_o M_1 \text{ - - - } M_{n-1} K_n I_{E_o} = \left(\prod_{i=1}^{n-1} M_i\right) K_n I_{E_o} \tag{2.23}$$

where $$K_o = \frac{\cos\theta_o - Q_1 \cos\theta_1}{\cos\theta_o + Q_1 \cos\theta_1}$$

$$K_1 = \frac{Q_1 \cos\theta_1 - Q_2 \cos\theta_2}{Q_1 \cos\theta_1 + Q_2 \cos\theta_2}$$

$$K_n = \frac{Q_n \cos\theta_n - Q_{n+1} \cos\theta_{n+1}}{Q_n \cos\theta_n + Q_{n+1} \cos\theta_{n+1}} \qquad (2.24)$$

or

$$K_n = \frac{(Q_n^{\ 2} - \sin^2\theta_o)^{1/2} - (Q^2_{\ n+1} - \sin^2\theta_o)^{1/2}}{(Q_n^{\ 2} - \sin^2 Q_o)^{1/2} + (Q^2_{\ n+1} - \sin^2\theta_o)^{1/2}} = \frac{q_n - q_{n+1}}{q_n + q_{n+1}} \qquad (2.25)$$

Finally one has for the ratio of the reflected and refracted wave at the n^{th} Layer boundary to the incident wave in free space.

$$\frac{R_{E_n}}{I_{E_o}} = 2^n \left(\frac{q_n - q_{n+1}}{q_n + q_{n+1}}\right)^{1/2} \prod_{j=o}^{n-1} \frac{q_j}{q_j + q_{j+1}} \qquad (2.26)$$

$$\frac{D_{E_n}}{I_{E_D}} = 2^{n+1} \prod_{j=o}^{n} \frac{q_j}{q_j + q_{j+1}} \qquad (2.27)$$

The energy in the transmitted wave is dependent on the index of refraction and the area of the ray. If one lets $I_{\mathcal{E}_o}$ be the energy in the incident wave and $D_{\mathcal{E}_o}$ be the energy in the refracted wave, one can write

$$\frac{D_{\mathcal{E}_o}}{I_{\mathcal{E}_o}} = \left(\frac{D_{E_o}}{I_{E_o}}\right)^2 \left(Q_1 \frac{\cos\theta_1}{\cos\theta_o}\right) = \left(\frac{D_{E_o}}{I_{E_o}}\right)^2 \left(\frac{q_1}{\cos\theta_o}\right)$$

$$\frac{D_{\mathcal{E}_n}}{I_{\mathcal{E}_o}} = \left(\frac{D_{E_n}}{I_{E_o}}\right)^2 \left(\frac{q_{n+1}}{\cos\theta_o}\right) \qquad (2.28)$$

In a similar manner if $R_{\mathcal{E}}$ is the energy in the reflected wave then

$$\frac{R_{\mathcal{E}_o}}{I_{\mathcal{E}_o}} = \left(\frac{R_{E_o}}{I_{E_o}}\right)^2$$

$$(2.29)\quad \frac{R_{\mathcal{E}_n}}{I_{\mathcal{E}_o}} = \left(\frac{R_{E_n}}{I_{E_o}}\right)^2 \frac{q_n}{\cos\theta_o}$$

NUMERICAL EXAMPLE

It is now possible to apply the equations derived to certain idealized, but representative conditions. Let there first be set up a stratified ionosphere of four layers and let the equivalent electron density in the layers one to four be $.1 \times 10^6$, $.2 \times 10^6$, $.5 \times 10^6$ and 1×10^6 electrons per cubic centimeters respectively. Then a wave of frequency f and incident at angle θ on layer one will be partially transmitted and partially reflected. The energy transmitted in the fourth layer for frequencies of 40 mc. and 100 mc. incident at 30° and 40° respectively will be calculated.

From Table II the q's corresponding to an ordinary wave of 40 mc and 30° angle of incidence are

$$q_1 = .8630 \qquad q_3 = .8509$$

$$q_2 = .8600 \qquad q_4 = .8356$$

then from Eq. (2.25)

$$K_o = \frac{.8660 - .8630}{.8660 + .8630} = .0017 \qquad M_o = \frac{2\,(.8660)}{.8660 + .8630} = 1.0017$$

$$K_1 = \frac{.8630 - .8600}{.8630 + .8600} = .0017 \qquad M_1 = \frac{2\,(.8630)}{.8630 + .8600} = 1.0017$$

$$K_2 = \frac{.8600 - .8509}{.8600 + .8509} = .0053 \qquad M_2 = \frac{2\,(.8600)}{.8600 + .8509} = 1.0053$$

$$K_3 = \frac{.8509 - .8356}{.8509 + .8356} = .0091 \qquad M_3 = \frac{2\,(.8509)}{.8509 + .8356} = 1.0091$$

The total reflected energy will be the sum of the energy reflected from each boundary of the medium. That is

$$R_{\mathcal{E}_T} = R_{\mathcal{E}_o} + R_{\mathcal{E}_1} + R_{\mathcal{E}_2} + R_{\mathcal{E}_3} = \sum_{i=o}^{3} R_{\mathcal{E}_i}$$

From Eq. (2.29) this may be written as

$$R_{\mathcal{E}_T} = \left\{\left(\frac{R_{E_o}}{I_{E_o}}\right)^2 + \frac{R_{E_1}}{I_{E_o}}^2 \frac{q_1}{\cos\theta_o} + \frac{R_{E_2}}{I_{E_o}}^2 \frac{q_2}{\cos\theta_o} + \frac{R_{E_3}}{I_{E_o}}^2 \frac{q_3}{\cos\theta_o}\right\} I_{\mathcal{E}_o}$$

$$= \left\{ K_o^{\,2} + (K_1 M_o)^2 \frac{q_1}{\cos \theta_o} + (K_2 M_o M_1)^2 \frac{q_2}{\cos \theta_o} + (K_3 M_o M_1 M_2)^2 \frac{q_3}{\cos \theta_o} \right\} I_{\mathcal{E}_o}$$

$$= \left\{ (.0017)^2 + (.0017 \times 1.0017)^2 \frac{.8630}{.8660} + (.0053 \times 1.0017 \times 1.0017)^2 \frac{.8600}{.8660} + (.0091 \times 1.0017 \times 1.0017 \times 1.0053)^2 \frac{.8509}{.8660} \right\} I_{\mathcal{E}_o}$$

$$R_{\mathcal{E}_T} = .00012\ I_{\mathcal{E}_o}$$

Then the energy transmitted to the fourth layer (since absorption is being neglected) is merely

$$D_{\mathcal{E}_3} = I_{\mathcal{E}_o} - .00012\ I_{\mathcal{E}_o} = .99988\ I_{\mathcal{E}_o}$$

Or the energy transmitted can be found directly from Eq. (2.28)

$$D_{\mathcal{E}_3} = \frac{D_{E_3}^{\ 2}}{I_{E_o}} \frac{q_4}{\cos \theta_o} I_{\mathcal{E}_o} = (M_o M_1 M_2 M_3)^2 \frac{q_4}{\cos \theta_o} I_{\mathcal{E}_o}$$

$$= (1.0017 \times 1.0017 \times 1.0053 \times 1.0091)^2 \frac{8356}{866} I_{\mathcal{E}_o}$$

$$D_{\mathcal{E}_3} = .99984\ I_{\mathcal{E}_o}$$

This agreement between the transmitted energy is well within the accuracy of the figures used. Furthermore it appears that one can for practical purposes assume that all the energy is transmitted.

In a similar manner for f = 100 mc, $\theta = 45°$, from Table II we determine the energy transmitted to the 4th layer.

$q_1 = .7065$

$q_2 = .7060$

$q_3 = .7042$

$q_4 = .7013$

$$K_o = \frac{.7071 - .7065}{.7071 + .7065} = \frac{.0006}{1.4136} \qquad M_o = \frac{1.4142}{1.4136} = 1.0004$$

$$K_1 = \frac{.7005 - .7060}{.7065 + .7060} = \frac{.0005}{1.4125} \qquad M_1 = \frac{1.4130}{1.4125} = 1.0004$$

$$K_2 = \frac{.7060 - .7042}{.7060 + .7042} = \frac{.0018}{1.4102} \qquad M_2 = \frac{1.4120}{1.4102} = 1.0013$$

$$K_3 = \frac{.7042 - .7013}{.7042 + .7013} = \frac{.0029}{1.4056} \qquad M_3 = \frac{1.4084}{1.4055} = 1.0020$$

The energy transmitted to the fourth layer will then be

$$D_{\mathcal{E}_3} = (M_o\, M_1\, M_2\, M_3)^2 \, \frac{q_4}{\cos\theta_o} \, I_{o} = .99993 \; I_{\mathcal{E}_o}$$

CONCLUSIONS

As was previously stated, absorption in the ionized medium has been neglected. However, the energy lost by absorption is approximately inversely proportional to the square of the frequency. Hence, for the frequencies under consideration, this loss will be small and can, for most practical purposes, be ignored. This has for the most part been born out by previous experience. The net result of the investigation then is that for sufficiently high frequencies, and for angles of incidence not exceeding 40° to 50°, the loss of signal strength due to magneto-ionic splitting and to reflection from the ionosphere is of a negligible nature.

John C Mester

JOHN C. MESTER

Coefficients of (q's) in $a'_4 q^4 + a_3 q^3 + a_2 q^2 + a_1 q + a_o = 0$

f = 10mc

	No. of Elec./cc		x 10^6	x 10^6	x 10^6	x 10^6
	a_1 x 10^{-24}		.1	.2	.5	10
θ=10°	a_4	=	8988.301	8195.740	5818.057	1855.251
	a_3		.5775757	1.1551514	2.887878	5.775757
	a_2		-15956.98	-13201.98	-6496.264	-517.676
	a_1		-.5601597	-1.120319	-2.800798	-5.601597
	a_o		7081.143	5312.953	1797.549	17.170
θ=20°	a_4	=	8988.301	8195.740	5818.057	1855.251
	a_3		1.1360584	2.2921168	5.680292	11.360584
	a_2		-14397.33	-11781.11	-5491.731	-207.040
	a_1		-1.0031648	-2.0063296	-5.015824	-10.031648
	a_o		5764.467	4230.520	1282.464	-3.614
θ=30°	a_4	=	8988.301	8195.740	5818.057	1855.251
	a_3		1.6608063	3.3216126	8.304031	16.608063
	a_2		-12007.81	-9604.213	-3952.698	+268.881
	a_1		-1.2060473	-2.4120946	-6.030236	-12.060473
	a_o		4009.674	2811.042	661.513	14.916
θ=45°	a_4		8988.301	8195.740	4818.057	1855.251
	a_3		2.3487348	4.6974696	11.743674	23.487348
	a_2		-7516.981	-5512.981	-1060.263	+1163.321
	a_1		-1.1743674	-2.3487348	-5.0871837	-11.743674
	a_o		1571.249	925.541	45.264	214.684

f = 40mc

	a_i x 10^{-24}	.1 x 10^6	.2 x 10^6	.5 x 10^6	1.0 x 10^6
10°	a_4	2543611.	2530728.	2492079.	2427663.
	a_3	.5775757	1.1551514	2.887878	5.775757
	a_2	-4908167.	-4857773.	-4708170.	-4464025.
	a_1	-.5601597	-1.102319	-2.800798	-5.601597
	a_o	2367706.	2331145.	2223731.	2052125
20°	a_4	2543611.	2530728.	2497079.	2427663
	a_3	1.1360584	2.2721168	5.680292	11.360584
	a_2	-4466472	-4418322	-4275431	-4042478
	a_1	-1.0031648	-2.0063296	-5.015824	-10.031648
	a_o	1960734	1928454	1833740	1682851
30°	a_4	2543611.	2530728.	2492079	2427663
	a_3	1.6608063	3.3216126	8.304031	16.608063
	a_2	-3789760	-2745039	-3612436	-3396629
	a_1	-1.2060473	-2.4120946	-6.030236	-12.060473
	a_o	1411604	1385503	1309117	1188082.
45°	a_4	2543611.	2530728.	2492079	2427663
	a_3	2.3487348	4.6974696	11.743674	23.487348
	a_2	-2517958	-2479682	2366414	2182831
	a_1	-1.1743674	-2.3487348	-5.0871837	-11.743674
	a_o	623141.0	607416.5	561770.9	490669.9

TABLE I (Cont'd)

f = 100mc

	$\times 10^{-24}$	$.1 \times 10^6$	$.2 \times 10^6$	$.5 \times 10^6$	1.0×10^6
10°	a_4	99897496.	99816905.	99575133	99172180
	a_3	.5775757	1.1551514	2.887878	5.775757
	a_2	-193609360	-193292226	-192342381	-190764504
	a_1	-.5601597	-1.120319	-2.800798	-5.601597
	a_0	93807618	93576044	92883610	91737157
20°	a_4	99897496	99816905	99575133	99172180
	a_3	1.1360584	2.2721168	5.680292	11.360584
	a_2	-176262344	-175959205	-175051347	-173543448
	a_1	-1.0031648	-2.0063296	-5.015824	-10.031648
	a_0	77750733	77546087	76934303	75921816

	10^{-24}	$.1 \times 10^6$	$.2 \times 10^6$	$.5 \times 10^6$	1.0×10^6
30°	a_4	99897496	99816905	99575133	99172180
	a_3	1.6608063	3.3216126	8.304031	16.608063
	a_2	-149685173	-149403476	-148559945	-147159258
	a_1	-1.2060473	-2.4120946	-6.030236	-12.060473
	a_0	56071603	55905857	55410564	54591537
45°	a_4	99897496	99816905	99575133	99172180
	a_3	2.3487348	4.6974696	11.743674	23.487348
	a_2	-99736428	-99495030	-98772395	-97573201
	a_1	-1.1743674	-2.3487348	-5.0871837	-11.743674
	a_0	24893905	24793548	24494032	24000000

TABLE II

Determined Values of q

θ	F \ N	.1 x 10^6	.2 x 10^6	.5 x 10^6	1 x 10^6
10°	10 x 10^6	.93657 .94771	.88569 .90905	.71142 .78131	.19614 .49047
	40 x 10^6	.98216 .98233	.97951 .97984	.97151 .97233	.95802 .95969
	100 x 10^6	.98439 .98441	.98398 .98400	.98273 .98279	.98065 .98076
20°	10 x 10^6	.88933 .90048	.83601 .85940	.65104 .72115	.35629 only one real root
	40 x 10^6	.93692 .93709	.93415 .93447	.92576 .92654	.91162 .91330
	100 x 10^6	.93926 .93926	.93883 .93884	.93752 .93757	.93534 .93545
30°	10 x 10^6	.81169 .82286	.75364 .77710	.54642 .61710	Imaginary
	40 x 10^6	.86303 .86319	.86002 .86035	.85093 .85176	.83565 .83724
	100 x 10^6	.86556 .86556	.86509 .86509	.86367 .86367	.86131 .86131
45°	10 x 10^6	.64099 .65226	.56794 .59170	.26126 .33761	Imaginary
	40 x 10^6	.70345 .70361	.69977 .70010	.68863 .68946	.66966 .67135
	100 x 10^6	.70653 .70654	.70596 .70598	.70422 .70428	.70137 .70149

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