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IMPEDANCE OF AN INSULATED LINEAR ANTENNA FOR LAYERS OF COMPRESSIBLE PLASMA

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AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
Office of Aerospace Research
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ABSTRACT

A flat strip antenna is embedded in a planar dielectric slab, which is surrounded on both sides by layers of compressible isotropic electron plasma. Several closed form expressions are obtained for the impedance with a sinusoidal current distribution along the antenna. The antenna impedance is computed numerically when considering the perturbations of the antenna current by surface waves. Except for thin plasma layers, the antenna impedance can be computed using the same current distribution as for a cold plasma. This supports the validity of earlier work which neglects the perturbation of the antenna current by plasma waves. However, it is essential to consider the finite transverse dimension of the antenna and the presence of an insulating layer or of an ion sheath.

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1.0 INTRODUCTION

The variational calculation of the admittance of a slot antenna immersed in a two-layer compressible plasma was recently reported by Galejs [1966a] and this paper contains also a detailed discussion of past work in this problem area. The surface waves supported by the plasma layers were shown to modify the field distribution along the antenna and to affect the antenna admittance. A similar effect can be anticipated also for a finite linear antenna, which is treated in the present paper.

A flat strip antenna, shown in Fig. 1, is embedded in a planar dielectric slab of thickness 2Δ, which is surrounded on both sides by a compressible isotropic plasma of thickness h. The dielectric layer is intended to approximate the effects of the ion sheath which is formed around the antenna. The expressions for the fields are derived starting out with linearized field equations [Oster, 1960] as indicated in Section 2. The antenna impedance is formulated in Section 3 in the same way as for a linear antenna in an incompressible plasma or dielectric layers [Galejs, 1965b]. The antenna current distribution is assumed to be the sum of two terms. The first term is a sine wave with the same wave number k_A as in incompressible plasma. The second term is representative of surface waves supported by this plasma geometry. The wave number k_B of the surface wave is determined from the solution of the transcendental equation, which describes the poles of the integrands of the field expressions. The resulting impedance expressions are evaluated numerically, as is done for an incompressible plasma, except that the integrals consider in detail the contribution of the surface wave poles. In the special case of an antenna immersed in a homogeneous compressible plasma with an assumed sinusoidal current distribution, closed form approximations of the antenna impedance are derived in Section 5, which compare closely with the results of other investigations [Seshadri, 1965; Balmain, 1965]. Numerical results are discussed in Section 6.

2. FIELD EXPRESSIONS

The field equations for a suppressed $\exp(-i\omega t)$ time variation can be written following Oster [1960] as

$$\nabla \times \underline{E} = i\omega\mu_o \underline{H} \tag{1}$$

$$\nabla \times \underline{H} = - i\omega\epsilon_o \underline{E} + N e \underline{V} \tag{2}$$

$$(- i\omega + \nu) m N\underline{V} = + Ne\underline{E} - \nabla p \tag{3}$$

$$u^2 m N \nabla.\underline{V} = i\omega p \tag{4}$$

where $\underline{E}$ and $\underline{H}$ are the electric and magnetic field vectors, N, $\underline{V}$, ν are the average values of particle density, velocity and collision frequency respectively; p, u, e, and m are the time varying component of the scalar pressure, the acoustic velocity and particle charge and mass respectively.

Eliminating $\underline{V}$ from (2) and (3) gives

$$\underline{E} = - \frac{1}{i\omega\epsilon_o \epsilon} \nabla \times \underline{H} - \frac{1-\epsilon}{\epsilon Ne} \nabla p \tag{5}$$

with

$$\epsilon = 1 - \frac{\omega_p^2}{\omega(\omega+i\nu)} \tag{6}$$

$$\omega_p^2 = n e^2/(\epsilon_o m) \tag{7}$$

Substituting (5) in (1) results in

$$\nabla^2 \underline{H} + k_e^2 \underline{H} = 0 \tag{8}$$

with $k_e^2 = \omega^2\mu_o\epsilon_o\epsilon$. For the later work it is convenient to separate the vector components of $\underline{H}$ into the TE and TM parts that can be derived from two scalar functions Ψ and Φ each of which satisfies a wave equation similar to (8). H_x, H_y and H_z are related to Ψ and Φ by

$$H_x = \frac{1}{i\omega\mu_o} \left(\frac{\partial^2}{\partial z \partial x} \Psi + k_e^2 \frac{\partial}{\partial y} \Phi \right) \tag{9}$$

$$H_y = \frac{1}{i\omega\mu_o} \left(\frac{\partial^2}{\partial z \partial y} \Psi - k_e^2 \frac{\partial}{\partial x} \Phi \right) \tag{10}$$

$$H_z = - \frac{1}{i\omega\mu_o} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Psi \tag{11}$$

Eliminating $\underline{E}$ from (2) and (3) gives

$$\underline{V} = - \frac{1-\epsilon}{Ne\epsilon} \left[\nabla \times \underline{H} + \frac{i\omega\epsilon_o}{Ne} \nabla p \right] \tag{12}$$

and substituting (12) in (4) it follows that

$$\nabla^2 p + k_p^2 p = 0 \tag{13}$$

with $k_p^2 = [\omega(\omega+i\nu) - \omega_p^2]/u^2$.

The solutions Ψ, Φ and p for the three regions of Fig. 1 can be expressed in terms of the integrals

$$\Psi_n = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_n(u,v)e^{-iux} e^{-ivy} (e^{\gamma_{en}z} + R_{an}e^{-\gamma_{en}z}) \, du \, dv \tag{14}$$

$$\Phi_n = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B_n(u,v)e^{-iux} e^{-ivy} (e^{\gamma_{en}z} + R_{bn}e^{-\gamma_{en}z}) \, du \, dv \tag{15}$$

$$p_n = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_n(u,v)e^{-iux} e^{-ivy} (e^{\gamma_{pn}z} + R_{cn}\, e^{-\gamma_{pn}z}) \, du \, dv \tag{16}$$

where the subscript n = 1 or 2 designates the region, the amplitude $C_1 = C_3 = 0$, the reflection coefficients $R_{a3} = R_{b3} = 0$ and where $\gamma_{en} = i\sqrt{k_{en}^2 - u^2 - v^2}$ and $\gamma_{pn} = i\sqrt{k_{pn}^2 - u^2 - v^2}$ are defined in the second quadrant of the complex plane including the positive imaginary and the negative real axes. Also, $k_{e1} = k_{e3} = k_o = \omega\sqrt{\mu_o\epsilon_o}$. After computing the components of $\underline{H}$ from (14) and (15) with the aid of (9) to (11), the components of $\underline{E}$ and $\underline{V}$ are obtained from (5) and (12) by making use of (16).

In previous work [Galejs, 1965b] a stationary expression for the antenna impedance was derived in terms of the electric fields and currents of the antenna which were related to the coefficients A_1 and B_1 of the functions Ψ_1 and Φ_1 for the region just outside the aperture. In the present problem of a compressible plasma the fields can be specified with the same coefficients A_1 and B_1 after relating them to the remaining constants R_{a1}, R_{b1}, A_2, B_2, C_2, R_{a2}, R_{b2}, R_{c2}, A_3

and B_3 of the field expressions (14) to (16) from a total of 10 boundary conditions (E_x, E_y, H_x, H_y, continuous and $V_z = 0$ at $z = z_1$ and z_2). These equations can be solved for the reflection coefficients R_{aj} and R_{bj}. A lengthy algebraic manipulation results in

$$R_{a1} = e^{2\gamma_{e1}z_1} \frac{\gamma_{e1}\left(e^{2\gamma_{e2}z_1} + R_{a2}\right) - \gamma_{e2}\left(e^{2\gamma_{e2}z_1} - R_{a2}\right)}{\gamma_{e1}\left(e^{2\gamma_{e2}z_1} + R_{a2}\right) + \gamma_{e2}\left(e^{2\gamma_{e2}z_1} - R_{a2}\right)} \tag{17}$$

$$R_{a2} = e^{2\gamma_{e2}z_2} \frac{\gamma_{e2} - \gamma_{e3}}{\gamma_{e2} + \gamma_{e3}} \tag{18}$$

$$R_{b1} = e^{2\gamma_{e1}z_1} \frac{P - Q}{P + Q} \tag{19}$$

with

$$P = \left(\frac{k_{e2}}{k_{e1}}\right)^2\left(e^{2\gamma_{e2}z_1} + R_{b2}\right) \tag{20}$$

$$Q = \frac{\gamma_{e2}}{\gamma_{e1}} \left(e^{2\gamma_{e2}z_1} - R_{b2}\right) - d_1 Ct \left(e^{2\gamma_{e2}z_1} + R_{b2}\right) + d_1 Cs\, e^{-\gamma_{e2}h} \left(e^{2\gamma_{e2}z_2} + R_{b2}\right) \tag{21}$$

$$R_{b2} = e^{2\gamma_{e2}z_2} \frac{-\left(\frac{k_{e2}}{k_{e3}}\right)^2 + d_3 Ct - d_3 Cs\, e^{-\gamma_{e2}h} + \frac{\gamma_{e2}}{\gamma_{e3}}}{\left(\frac{k_{e2}}{k_{e3}}\right)^2 - d_3 Ct + d_3\, Cs\, e^{\gamma_{e2}h} + \frac{\gamma_{e2}}{\gamma_{e3}}} \tag{22}$$

where

$$d_j = - \frac{(1 - \epsilon_2)(u^2 + v^2)}{\gamma_{ej}\, \gamma_{p2}} \tag{23}$$

$$Ct = \coth\,(\gamma_{p2}h) \tag{24}$$

$$Cs = 1/\sinh(\gamma_{p2}h) \tag{25}$$

The reflection coefficient of the TE modes R_{aj} of (17) and (18) are the same as for an incompressible plasma. [Galejs, 1965b]. The compressibility of the plasma introduces the terms proportional to d_j in the expressions (19) and (22) for the reflection coefficients R_{bj} of the TM modes. R_{b1} of (19) is the reflection coefficient for radiation in a compressible plasma layer of thickness h. In the limit of $h \to \infty$, $R_{b1} = 0$ and R_{b2} of (22) is the reflection coefficient for radiation into a plasma halfspace which is separated from the antenna by an ion sheath of thickness $z_1 = \Delta$. These two limiting forms of reflection coefficients of the TM modes have been obtained previously [Galejs, 1966a]. It can be also shown that R_{a1} and $R_{b1} \to 0$ as $h \to 0$, if $k_{e1} = k_{e3}$.

The assumption of a rigid boundary of the plasma layers with $V_z = 0$ can be replaced by more elaborate boundary conditions of continuous scalar pressure p and continuous normal particle velocity V_z. This leads to increased algebraic complexity of the derivations, but give essentially the same results as the simpler and less accurate boundary condition of $V_z = 0$ [Galejs, 1966a].

3.0 ANTENNA IMPEDANCE AND RADIATION RESISTANCE

The ion sheath surrounding the antenna is approximated by free space and the antenna impedance can be formulated in terms of the TE and TM modes of this region as for a linear antenna in a stratified dielectric [Galejs, 1965b]. The surface current of the antenna is assumed to have only an x-component J_x, which in addition, is an even function about x=0, y=0 and the driving point impedance of a flat strip antenna may be computed from the expression

$$Z = \frac{i\omega\mu_o}{[2\pi\ I(x=0)]^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) \left[\iint J_x(x,y) \cos ux \cos vy \, dx \, dy \right] \tag{26}$$

where

$$F(u,v) = \frac{(1/2)}{u^2 + v^2} \left[\frac{v^2}{\gamma_{e1}} \frac{1+R_{a1}}{1-R_{a1}} - \frac{u^2 \gamma_{e1}}{k_{e1}^2} \frac{1-R_{b1}}{1+R_{b1}} \right] \tag{27}$$

The antenna impedance Z depends on the current distribution $J_x(x,y)$ along the antenna, which is assumed to be in the form

$$J_x(x,y) = \left\{ A \sin[k_A(\ell-|x|)] + B \sin[k_B(\ell-|x|)] \right\} f(y) \tag{28}$$

with $f(y) = 1/(2\epsilon)$ = const. The phase constant k_A is selected in such a way that the impedance Z computed with B=0 approximates the impedance Z of the corresponding cold plasma geometry, which was derived in an earlier formulation using a more accurate two term trial function [Galejs, 1965b]. The second term of the current distribution (28) should approximate the effects of the finite plasma temperature. The phase constant k_B is representative of surface waves that are supported by the antenna in the presence of the compressible plasma and it will be discussed in Section 4.

Substituting (28) in (26) and utilizing the stationary character of the impedance expression for determining the complex amplitudes A and B, it follows that [Galejs, 1965b]

$$\frac{A}{B} = \frac{\gamma_{BB} F_A - \gamma_{AB} F_B}{\gamma_{AA} F_B - \gamma_{AB} F_A} \tag{29}$$

$$Z = \frac{\gamma_{AA}\gamma_{BB} - \gamma_{AB}^2}{\Delta} \tag{30}$$

with

$$\Delta = F_B^2 \gamma_{AA} - 2F_A F_B \gamma_{AB} + F_A^2 \gamma_{BB} \tag{31}$$

$$F_N = \sin k_N \ell \tag{32}$$

$$\gamma_{NM} = \frac{4i\omega\mu_o}{\pi^2} \int_0^\infty dv \left(\frac{\sin \epsilon v}{\epsilon v} \right)^2 \int_0^\infty du\, F(u,v) g_N(u) g_M(u) \tag{33}$$

$$g_N(u) = \frac{k_N}{k_N^2 - u^2} (\cos u\ell - \cos k_N \ell) \tag{34}$$

The impedance can be computed also for the sinusoidal field distribution term by setting B = 0 in (28). This gives

$$Z_s = \frac{\gamma_{AA}}{F_A^2} \tag{35}$$

4.0 SURFACE WAVES

The phase constant k_B is assumed to be the same as for surface waves which are supported by the plasma layers, and $k_B = \sqrt{u^2 + v^2}$ is determined from the poles of the integrand (26), where

$$1 + R_{b1} = 0 \tag{36}$$

Substituting (19) to (22) in (36) results in

$$\frac{\gamma_{e2}}{\gamma_{e1}}\left(\frac{k_{e2}}{k_{e3}}\right)^2 - \frac{\gamma_{e2}}{\gamma_{e3}}\left(\frac{k_{e2}}{k_{e1}}\right)^2 \coth \gamma_{e1}z_1 + \left(d_1 \frac{\gamma_{e2}}{\gamma_{e3}} + d_3 \frac{\gamma_{e2}}{\gamma_{e1}}\right)\left(\frac{Cs}{\cosh\gamma_{e2}h} - Ct\right)$$

$$+ \tanh \gamma_{e2}h \left\{\left[\left(\frac{k_{e2}}{k_{e3}}\right)^2 - d_3Ct\right]\left[\left(\frac{k_{e2}}{k_{e1}}\right)^2 \coth \gamma_{e1}z_1 + d_1Ct\right]\right.$$

$$\left. - \frac{\gamma_2}{\gamma_3}\frac{\gamma_2}{\gamma_1} + d_1d_3(Cs)^2\right\} = 0 \tag{37}$$

where the symbols Cs and Ct are defined in (24) and (25). In the limiting case of $h \to 0$ and $k_B \ll k_{p2}$, (38) is reduced to

$$\coth \gamma_{e1}z_1 = \left(\frac{k_{e1}}{k_{e3}}\right)^2 \frac{\gamma_{e3}}{\gamma_{e1}} \tag{38}$$

Equation (38) can be seen to be identical with (26) of Galejs [1965b], which denotes the characteristic values of surface waves for TM modes in a dielectric layer of relative dielectric constant $\epsilon_r = (k_{e1}/k_{e3})^2$ and of total thickness $2z_1$.

Introducing infinitesimal plasma losses ($\nu \neq 0$) in the limit of $h \to \infty$, $\tanh \gamma_{e2}h$ and $Ct \to (-1)$ and $Cs \to 0$. Equation (37) simplifies now to

$$\left(\frac{k_{e2}}{k_{e1}}\right)^2 \coth \gamma_{e1}z_1 = d_1 + \frac{\gamma_{e2}}{\gamma_{e1}} \tag{39}$$

which can be also obtained from (36) and (19) by setting $R_{b2} = 0$ in (20) and (21). Equation (39) can be expected to have solutions for $k_B \approx k_{p2} \gg k_{ej}$. Letting $\epsilon_2 = (k_{e2}/k_{e1})^2$ (39) can be expressed as

$$\epsilon_2 \coth k_B z_1 = \frac{1 - \epsilon_2}{\sqrt{1 - (k_{p2}/k_B)^2}} \quad 1 \tag{40}$$

For thick insulating layers, $k_B z_1 \gg 1$ (40) is changed to

$$k_B = k_{p2} \; \frac{1 + \epsilon_2}{2\sqrt{\epsilon_2}} \tag{41}$$

which represents surface waves guided along the boundary of a semi-infinite plasma (Eq. (54), Hessel et al. [1962]; Eq. (55), Galejs [1966a]). For thin insulating layers, $k_B \, z_1 \ll 1$ (40) becomes

$$k_B = k_{p2} \left[1 + \frac{1}{2} \left(\frac{1-\epsilon_2}{\epsilon_2} k_{p2} z_1 \right)^2 + \ldots \right] \tag{42}$$

and the wave number of the surface wave k_B approaches the wave number k_{p2} of plasma waves in a homogeneous plasma. In the limit of $\epsilon_2 \to 0$ and $\epsilon_2 \ll k_B z_1$ both (41) and (42) are replaced by

$$k_B = k_o \; \frac{c}{2u} \tag{43}$$

where $k_o = \omega \sqrt{\mu_o \epsilon_o}$. A more detailed examination of (39) shows that (40) represents its only solution and that k_B lies within the limits indicated by (41) to (43) if ϵ_2 is in the range $0 < \epsilon_2 < 1$.

For a plasma layer of finite thickness 'h' (37) will be simplified by assuming $k_B \approx k_{p2} \gg k_{ej}$, and h of the order of the free space wavelength $2\pi/k_o$. This gives

$$\left[1 + \epsilon_2 \coth k_b z_1 - \frac{(1-\epsilon_2) k_B}{\sqrt{k_B^2 - k_{p2}^2}} \coth \left(\sqrt{k_B^2 - k_{p2}^2} \, h \right) \right] \tag{44}$$

$$\cdot \left[1 + \epsilon_2 - \frac{(1-\epsilon) k_B}{\sqrt{k_B^2 - k_{p2}^2}} \coth \left(\sqrt{k_B^2 - k_{p2}^2} \, h \right) \right]$$

$$= \frac{(1-\epsilon_2)^2 \, k_B^2}{k_B^2 - k_{p2}^2} \; \frac{1}{\sinh^2 \left(\sqrt{k_B^2 - k_{p2}^2} \, h \right)} \; .$$

For h large and $|k_B| > |k_{p2}|$ the right-hand side of (44) approaches zero. The zero of the first square bracket represents a wave guided along the plasma boundary near the antenna (40), while the zero of the second square bracket represents k_B of a wave guided along the outer plasma surface which is the same as k_B of the wave guided along the inner boundary, if $k_B z_1 \gg 1$. These two waves are coupled, if the right-hand side of (44) is finite, but the resulting value of k_B will approximate k_B computed from (40). Equation (44) has further solutions with $|k_B| < |k_{p2}|$. However, it can be shown that these waves are excited with small amplitudes if the plasma layer has a thickness comparable to the free space wavelength [Galejs, 1966b]. These waves are not considered at present.

A real root k_B is determined first from the solution of (44) for a lossless plasma. For a lossy plasma the complex root is found by applying Newton's iteration method and by using the former real root for the initial estimate. For $h \gg z_1$ the roots of (44) differ negligibly from the roots of (40), which lie in the range indicated by (41) to (43).

5. IMPEDANCE OF A SHORT ANTENNA IN AN UNBOUNDED PLASMA

The impedance of a short antenna ($k_o \ell \ll 1$) will be computed from (35) for an unbounded plasma with an assumed sinusoidal current distribution of $k_A = k_{e2}$. For $z_1 = 0$ and $h \to \infty$ $R_{a2} = R_{b2} = 0$, and the substitution of (17) and (19) in (27) shows that

$$F(u,v) = \frac{1}{2(u^2+v^2)} \left(\frac{v^2}{\gamma_{e2}} - \frac{u^2 \gamma_{e2}}{k_{e2}^2} \right) + \frac{u^2}{2\,k_{e2}^2} \, \frac{1-\epsilon_2}{\gamma_{p2}} \qquad (45)$$

The first term of (45) does not depend on the finite temperature of the plasma, and substituting it in (33) the impedance due to the electromagnetic waves (subscript e) can be computed from

$$Z_e = R_e + i\,X_e = \frac{2\,i\omega\mu_o}{(\pi k_A \ell)^2} \int_0^\infty du \, \frac{(\cos u\ell - \cos k_A \ell)^2}{k_A^2 - u^2} \int_0^\infty dv \left(\frac{\sin \epsilon v}{\epsilon v} \right)^2 \frac{1}{\gamma_{e2}} \qquad (46)$$

The antenna resistance R_e is computed by confining the range of integration to values of u and v where $u^2 + v^2 < k_A^2$. Over this range of integration the sine and cosine functions can be replaced by their small angle approximations. The integrations are elementary and

$$R_e = 20 \sqrt{\epsilon_2} \, (k_o \ell)^2 \tag{47}$$

The reactance is computed as

$$X_e = - \frac{2\,\omega\mu_o}{(\pi k_A \ell^2)} \left[\int_0^{k_A} du \int_{\sqrt{k_A^2-u^2}}^{\infty} dv + \int_{k_A}^{\infty} du \int_0^{\infty} dv \right] \frac{(\cos u\ell - \cos k_A\ell)^2}{(k_A^2-u^2)\sqrt{u^2+v^2-k_A^2}} \left(\frac{\sin \epsilon v}{\epsilon v} \right)^2 \tag{48}$$

The v-integrations become elementary by noting that $\sin \epsilon v \approx \epsilon v$ over the range of v where $\sqrt{u^2 + v^2} \neq v$, which applies strictly if $\epsilon \to 0$ and $u \leq u_o$ is finite. The v integrals are evaluated to give

$$X_e = - \frac{2\,\omega\mu_o}{(\pi k_A \ell)^2} \int_0^{\infty} du \, \frac{(\cos u\ell - \cos k_A\ell)^2}{(k_A^2 - u^2)} \left[\frac{3}{2} - \bar{C} \log (\epsilon k_A) - \frac{1}{2} \log \left| 1 - \left(\frac{u}{k_A} \right)^2 \right| \right] \tag{49}$$

where C = 0.5772...is Euler's constant. The integral which is proportional to the constant term of the square brackets is evaluated directly. The remaining term of the integral which involves a logarithmic function is reduced to tabulated integrals after expanding the denominator in partial fractions and changing the variables of integration to $y = (u/k_A) \pm 1$. This results in

$$X_e = \frac{120}{\epsilon_2 k_o \ell} \left[\log \frac{2\ell}{\epsilon} + 0.5 - 2 \log 2 \right] \tag{50}$$

The contribution of the plasma waves (subscript p) to the antenna impedance is computed using the second term of (45). This results in

$$Z_p = R_p + iX_p = \frac{2\omega\mu_o}{\pi^2 \ell^2} \frac{1-\epsilon_2}{\epsilon_2 k_o^2} \int_0^{\infty} du \, \frac{(\cos u\ell - 1)^2}{u^2} \int_0^{\infty} dv \left(\frac{\sin \epsilon v}{\epsilon v} \right)^2 \frac{1}{\sqrt{k_{p2}^2 - u^2 - v^2}} \tag{51}$$

The computations will be made first for very thin antennas, $(k_{p2}\epsilon) \ll 1$.

In the resistance calculations $u^2 + v^2 < k_{p2}^2$ and sine $\epsilon v \approx \epsilon v$ over this range of integration. It follows that

$$R_p = \frac{120(1-\epsilon_2)}{\epsilon_2 k_o \ell} \left\{ 2\,\mathrm{Si}(k_{p2}\ell) - \mathrm{Si}(2\,k_{p2}\ell) - \frac{4\sin^4(k_{p2}\ell/2)}{k_{p2}\ell} \right\} \tag{52}$$

For $k_{p2}\ell \gg 1$ (52) simplifies to

$$R_p = \frac{60\pi}{\epsilon_2 k_o \ell} (1 - \epsilon_2) \tag{53}$$

and for $k_{p2}\ell \ll 1$ (52) is approximated by

$$R_p = 10\,(1-\epsilon_2)\,\sqrt{\epsilon_2}\,\left(\frac{c}{u_2}\right)^3 (k_o\ell)^2 \tag{54}$$

Also, R_p of (54) can be obtained directly from (51) by using small argument approximations of the cosine function. In the reactance computations $u^2 + v^2 > k_{p2}^2$. The v-integrals are evaluated using the same approximations as in (48), which gives

$$X_p = -\frac{2\omega\mu_o(1-\epsilon_2)}{\epsilon_2(\pi k_o\ell)^2} \int_0^\infty du\, \frac{(1-\cos u\ell)^2}{u^2} \left[\frac{3}{2} - C - \log(\epsilon k_{p2}) - \frac{1}{2}\log\left|1 - \left(\frac{u}{k_{p2}}\right)^2\right|\right] \tag{55}$$

The integral which is proportional to the constant term of the square brackets is evaluated directly. The remaining term of the integral which involves a logarithmic function is evaluated using the relation

$$\int_0^\infty dx\, \frac{\log|1-a^2x^2|}{x^2} (\cos xy_2 - \cos xy_1) = \pi \int_{y_1}^{y_2} \mathrm{Ci}\left(\frac{y}{a}\right) dy \tag{56}$$

where Ci(x) is the cosine integral. (Equation (56) is obtained by integrating a tabulated pair of Fourier sine transforms [Oberhettinger, 1957].) For $k_{p2}\ell \gg 1$, (55) becomes

$$X_p = -\frac{120(1-\epsilon_2)}{\epsilon_2 k_o \ell} \left\{\frac{3}{2} - C - \log(\epsilon k_{p2}) - \frac{1}{(k_{p2}\ell)^2}\left[2\cos(k_{p2}\ell) - \frac{1}{2}\cos(2k_{p2}\ell)\right]\right\} \tag{57}$$

For antennas which are wide relative to the plasma wavelength, $k_{p2}\epsilon \gg 1$, only those values of u and v where $u^2 + v^2 \ll k_{p2}^2$ will contribute significantly to the integrals and u and v can be neglected relative to k_{p2}. The integrations become elementary in (51) and

$$Z_p \approx R_p = \frac{60\pi\,(1-\epsilon_2)}{\epsilon_2 k_o \ell\ \epsilon k_{p2}} \tag{58}$$

It may be noted that (54) has been obtained first by Hessel and Shmoys [1962], and (52) has also been derived by Seshadri [1965]. Balmain [1965] has considered a cylindrical dipole antenna of a finite radius r, and expresses the antenna impedance in terms of Bessel functions. For r small his resistance agrees with (53), and the reactance is the same as the leading term of (57) if the term (3/2) is replaced by 2 log 2. However, there are differences for r large. His resistance is proportional to $\cos^2(k_{p2}r - \pi/4)$ with a peak value 4 times larger than shown in (58). His reactance can have either sign and has a peak value two times larger than R_p of (58). However the reactance X_p of the present calculations is negligible relative to R_p for $k_{p2}\epsilon_1 \gg 1$.

6. DISCUSSION OF NUMERICAL RESULTS

The antenna impedance has been computed for very thick plasma layers ($h \to \infty$). The antenna has a finite width ($\Omega = 2 \log 4\ell/\epsilon = 10$) and the medium has slight losses ($\tan \delta_2 = 0.03$). The calculations are made for very thin ($\Delta = 10^{-5}\lambda$) and for ($\Delta = 10^{-2}\lambda$) layers of insulation and the results for a warm plasma with $c/u_2 = 100$ are compared with a cold plasma ($c/u_2 = \infty$). For small values of ϵ_2 the antenna is thin relative to the acoustic wavelength ($k_{p2}\epsilon \approx 0.43$ for $\epsilon_2 = 0.01$) and the appropriate closed form approximations are indicated by heavy lines in Fig. 2. The finite plasma temperature tends to increase the antenna resistance and to decrease the capacitive reactance. Also the net change due to the finite plasma temperature appears to be larger for small values of ϵ_2.

Landau damping of the longitudinal plasma oscillations may become significant if k_B becomes comparable to ω_{p2}/u_2. Noting that $k_{p2} \sim k_B$, this damping will be small if $k_{p2} < \omega_{p2}/u_2$. This condition can be rearranged into $\omega < \sqrt{2}\ \omega_{p2}$ or $\epsilon_2 < 0.5$. The damping can be neglected for the smaller values of ϵ_2 shown in Fig. 2.

The further numerical calculations are restricted to a constant value of $\epsilon_2 = 0.03$ and the variation of the antenna impedance with the velocity ratio c/u_2 is shown in Fig. 3. The closed form approximations of $k_{p2}\epsilon \ll 1$ and $\gg 1$ apply to c/u_2 small and large respectively and are indicated by heavy lines. The finite width of the antenna can be neglected only if $k_{p2}\epsilon \ll 1$. Approximation of a finite antenna by a current filament will introduce considerable errors if $k_{p2}\epsilon \gg 1$, which corresponds to $c/u_2 > 1000$ in the present numerical examples.

The data shown in Figs. 2 and 3 refer to an antenna of a constant length $\ell = 0.25\lambda$ and the impedance Z of (30) computed with a two term trial function differs by less than a few percent from Z_s of (35) calculated for the sinusoidal field distribution. Further investigations show that the current ripple with wave number k_B has practically no effect on the antenna impedance if the antenna length 2ℓ is an integer multiple of the wavelength of the surface waves or if $\mathrm{Re}\ k_B\ell = n\pi$, where n is an integer. The plasma wave components of the current distribution cause a maximum perturbation of the antenna impedance if $\mathrm{Re}\ k_B\ell = (n + 0.5)\pi$. For a lossless plasma the wavenumber k_B is real, and $k_B\ell = n\pi$ corresponds to a standing wave pattern with a null at the antenna center. Such a standing wave does not perturb the antenna current at the feed point and will not be excited. Similarly $k_B\ell = (n + 0.5)\pi$ causes a standing wave pattern with a maximum at the antenna center, it will perturb the antenna current at the feed point and will affect the antenna impedance.

The antenna impedance is shown in Fig. 4 for a two-term trial function (Z) and for an assumed sinusoidal current distribution (Z_s) for various amounts of plasma losses. The antenna length is made equal to $\ell = 8.5\pi/\mathrm{Re}\ k_B$. Both sets of computations give nearly the same resistance for $\tan\delta_2 < 0.1$. The amplitude ratio A/B of the two components of the antenna current distribution is computed from (29). It is nearly 500 in magnitude for small values of $\tan\delta_2$ and is approximately 160 for $\tan\delta_2 = 0.3$.

The effects of gradually increasing the thickness of the antenna insulation are shown in Fig. 5. The antenna resistance is decreased by increasing thickness of the insulation Δ. However the oscillatory peaks of the antenna resistance which are observed for larger values of Δ are observed also for a cold plasma. For small values of Δ the antenna resistance in the cold plasma case is due principally to plasma losses and $R \approx \tan\delta_2 X$. The impedance data shown in Fig. 5 has been computed for an assumed sinusoidal current distribution from (35) and the use of a two-term trial function gives a change of less than 5 percent in the impedance figures.

Antenna impedance for plasma layers of various thicknesses h is shown in Fig. 6. The impedances $Z=R + iX$ and $Z_s = R_s + iX_s$ are both indicated. For $h > 0.3\lambda$ the antenna impedance is nearly the same as for $h = \infty$. However for small values of h the antenna impedance exhibits several high resonance peaks. The largest peak occurs near $h = 0.028\lambda$ where $k_{p2}h \approx \pi$. These peaks may be attributed to the resonance of the plasma waves in the transvers dimension of the plasma slab. The surface waves which are guided along the plasma slab have a considerable effect on the antenna impedance near these resonances. The computed reactance X undergoes seven sign changes in the h/λ interval between 0.03 and 0.1, but this could not be shown in Fig. 6.

The antenna impedance near the resonance peak of $k_{p2}h = \pi$ is further illustrated in Fig. 7. The impedance is computed with a gradual change of the antenna length. The impedance components R_s and X_s which are computed based on an assumed sinusoidal current distribution remain nearly constant, but the impedance component R and X which are computed using a two-term trial function exhibit significant changes. The two sets of computations give nearly the same impedance figures for $\mathrm{Re}\, k_B \ell = n\pi$, but the largest differences in impedance occur near $\mathrm{Re}\, k_B \ell = 10.5\, n\pi$, where the antenna reactance is changed from capacative to inductive. However the impedance curves are not symmetrical with respect to Re $k_B \ell = 10.5\ n\pi$ and the deviations of the antenna impedance Z from Z_s values are decreased in magnitude with an increased antenna length. The amplitude A of the fundamental sine wave is less than the amplitude B of the surface waves in the vicinity of Re $k_B \ell =$ 10.5π. However the surface wave have considerably less effect for larger values of the layer thickness h and for longer antenna lengths ℓ, when Z_s of (35) can be expected to indicate the correct order of magnitude.

7. SUMMARY OF CONCLUSIONS

The impedance of an antenna has been computed in the presence of a compressible plasma by considering the perturbations of the antenna current distribution by plasma waves. With the exception of very thin plasma layers the antenna impedance can be determined by neglecting the presence of surface waves. The impedance of a finite antenna was shown to approach the impedance in the presence of cold plasma as the acoustic velocity is decreased. The presence of an insulating layer (or of an ion sheath) around the antenna also tends to decrease the compressibility effects.

The present impedance computations tend to justify past work which has been carried out by assuming that the antenna current distribution is the same as for cold plasma. On the other hand, calculations which neglect the presence of the electromagnetic waves and attribute the antenna current variations solely to plasma waves [Cook and Edgar, 1966] are obviously in error.

This behaviour of a linear antenna is quite different from a wide slot antenna, where the compressibility of the plasma affects the antenna impedance only when considering the surface waves in a direction transverse to the antenna aperture [Galejs, 1966a]. The waveguide aperture is wide in terms of the plasma wavelength and the presence of a compressible plasma would have negligible effects on the impedance of a linear antenna of comparable dimensions. Furthermore the surface waves in the linear antenna case would perturb only the longitudinal current distribution which has been shown to have small effects on the antenna impedance.

The antenna of the present investigations was idealized as a current sheath. For thin insulating layers the surface waves supported in this antenna geometry are different from the waves near a perfectly conducting antenna. However these differences become small for insulating layers of a thickness of the order of the plasma wavelengths [Galejs, 1965a], and the conclusions of the present analysis will apply also to metallic antennas if the ion sheath is of such a thickness.

8. ACKNOWLEDGMENTS

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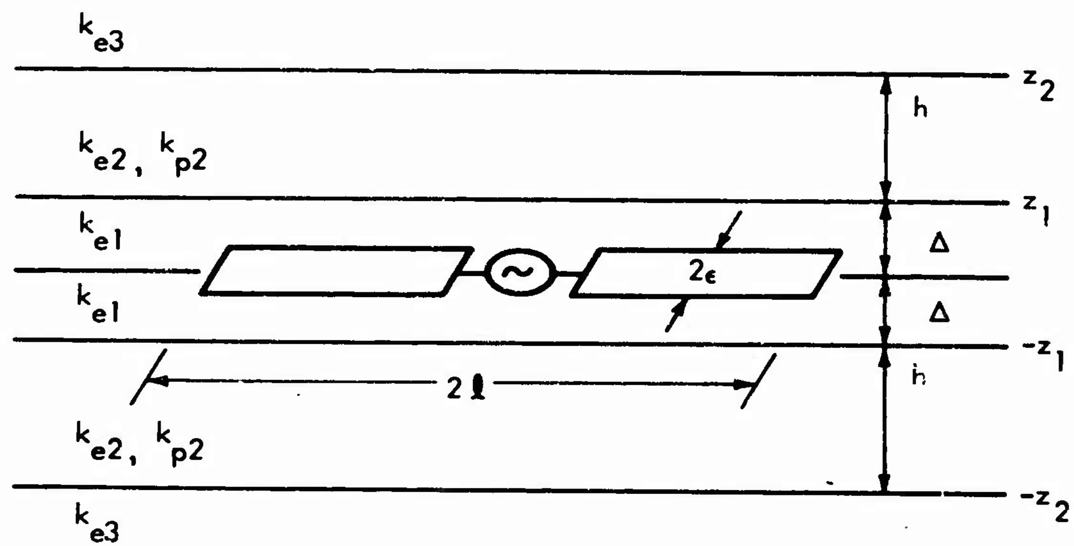

Figure 1. Antenna Geometry.

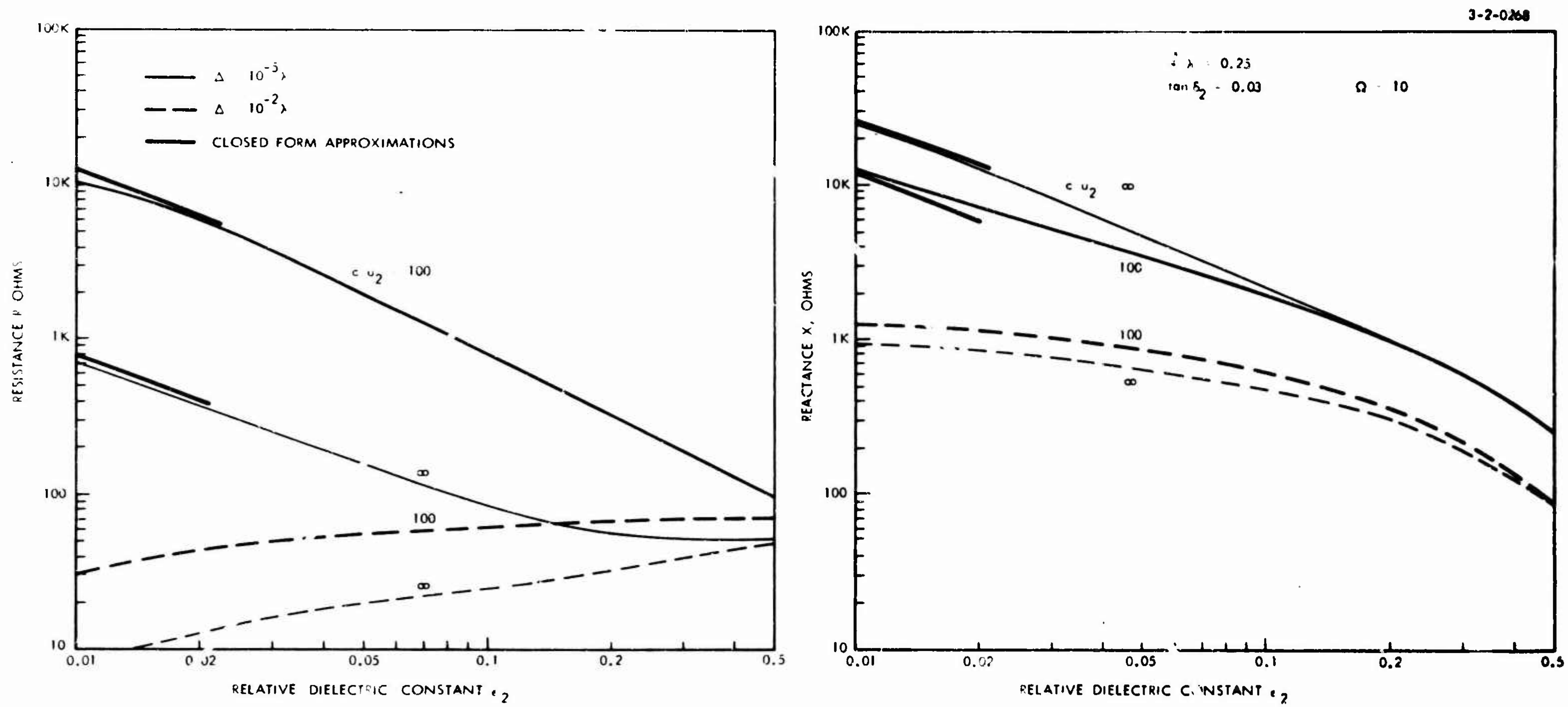

Figure 2. Antenna Impedance. Effects of Dielectric Constant.

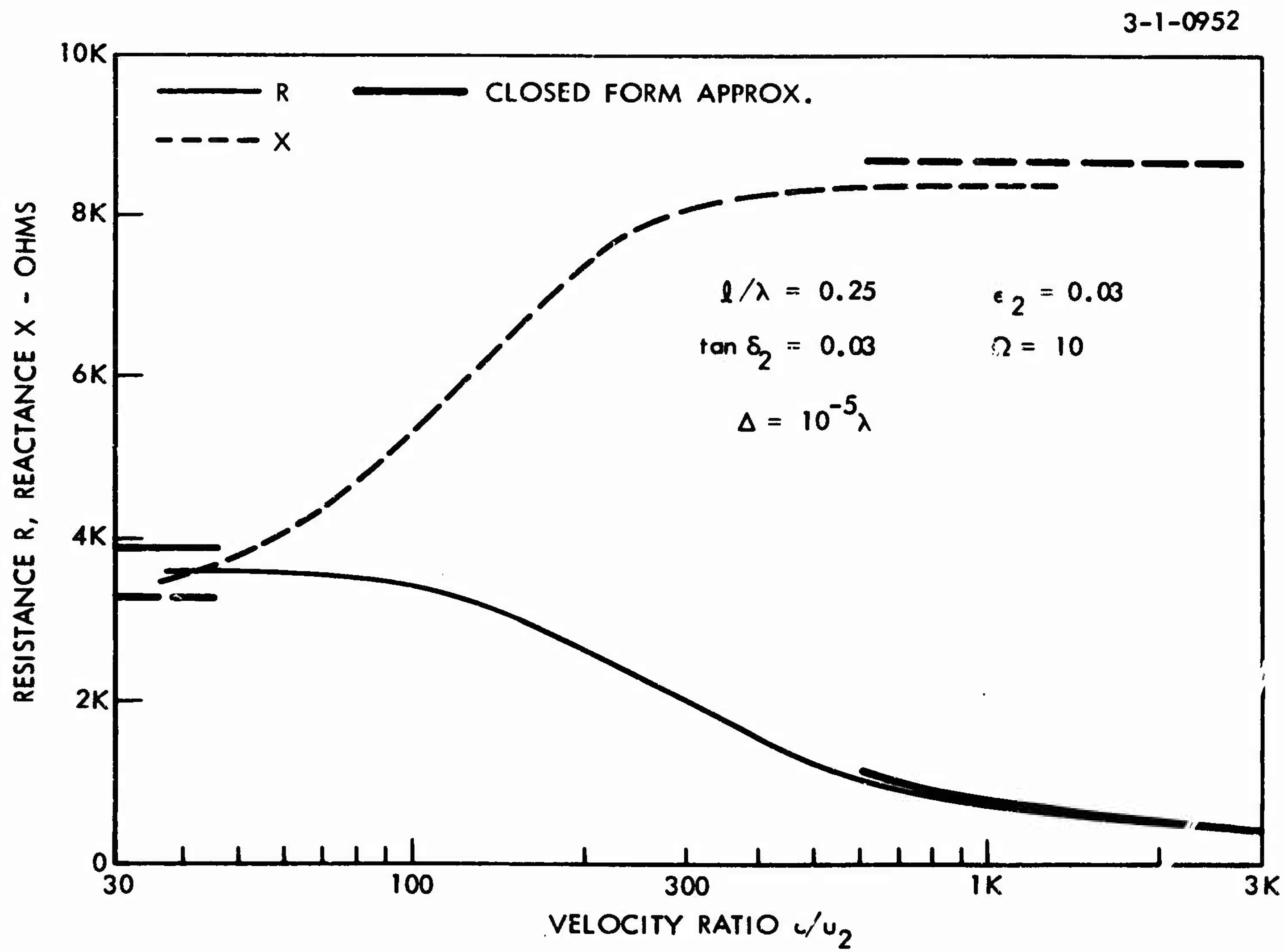

Figure 3. Antenna Impedance. Effects of Acoustic Velocity.

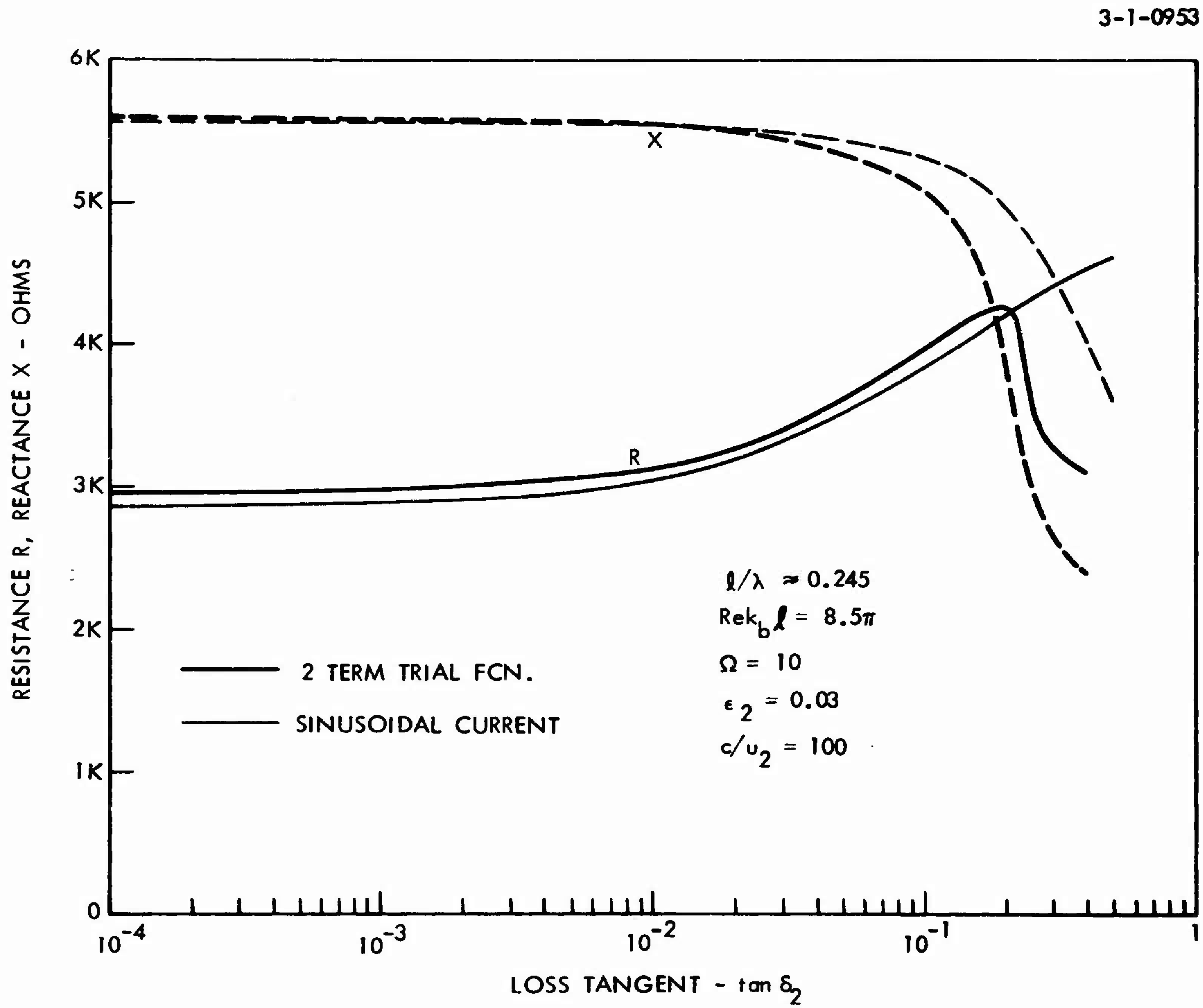

Figure 4. Antenna Impedance. Effects of Plasma Losses.

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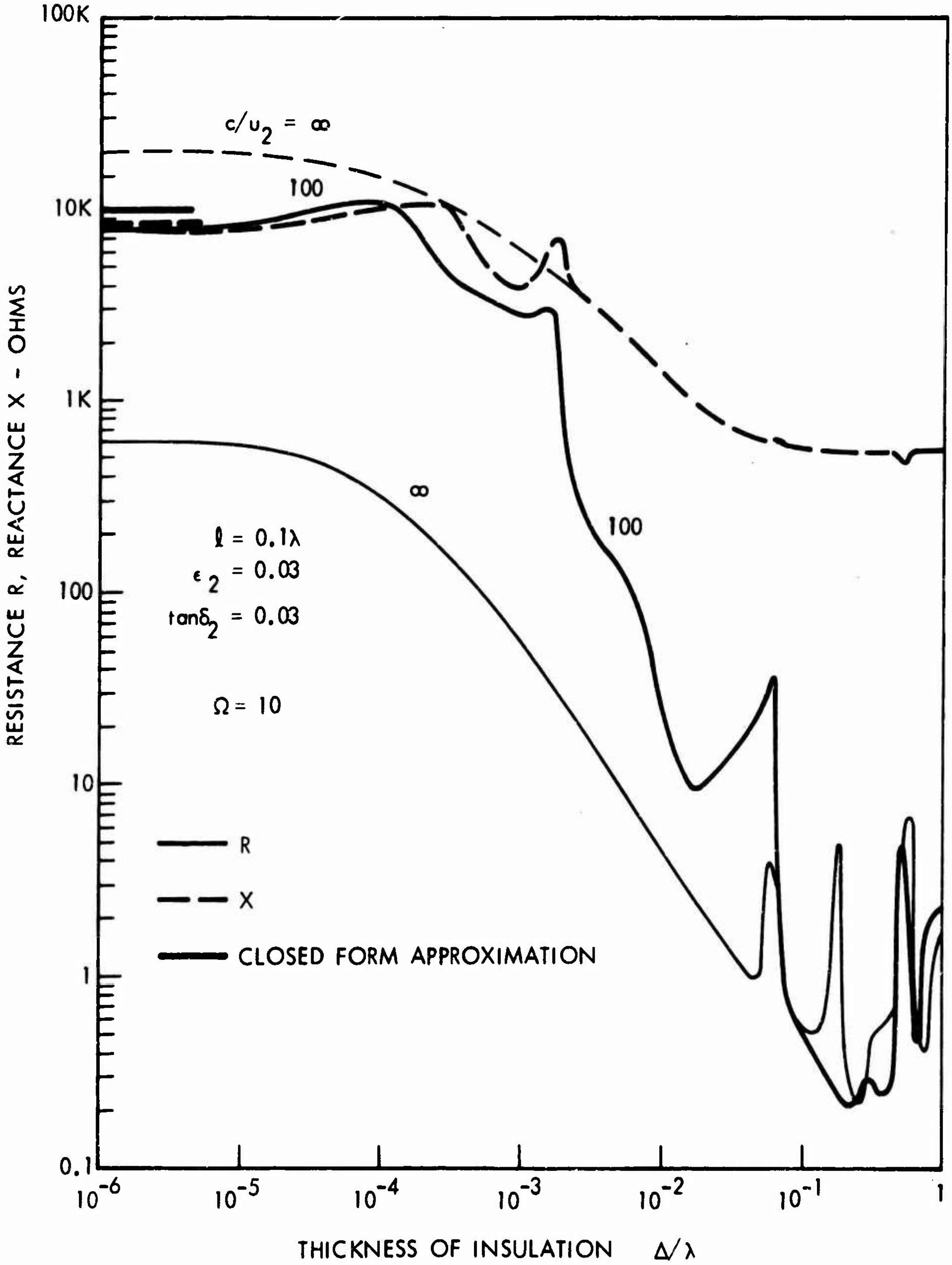

Figure 5. Impedance of an Insulated Antenna.

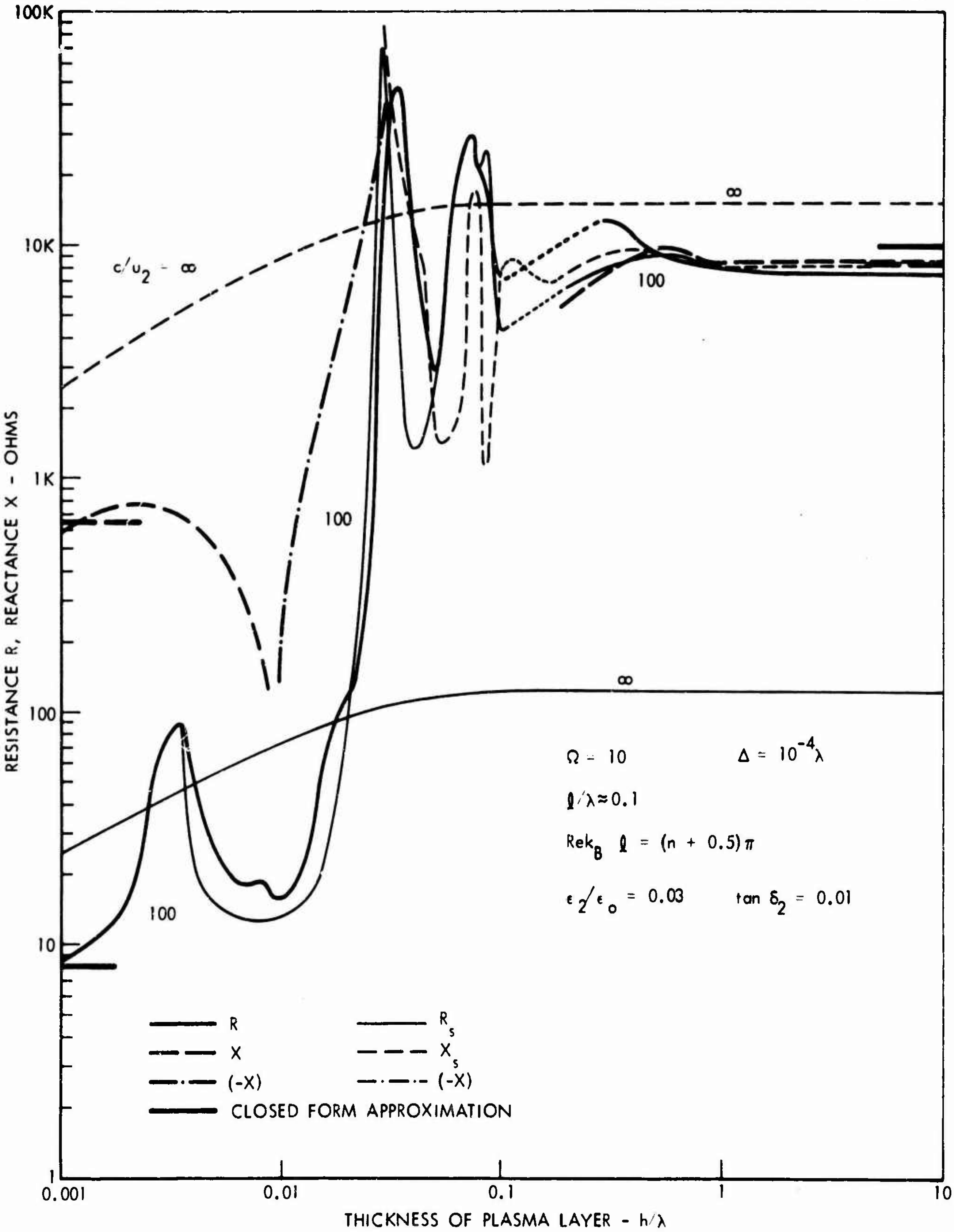

Figure 6. Antenna Impedance for Plasma Layers.

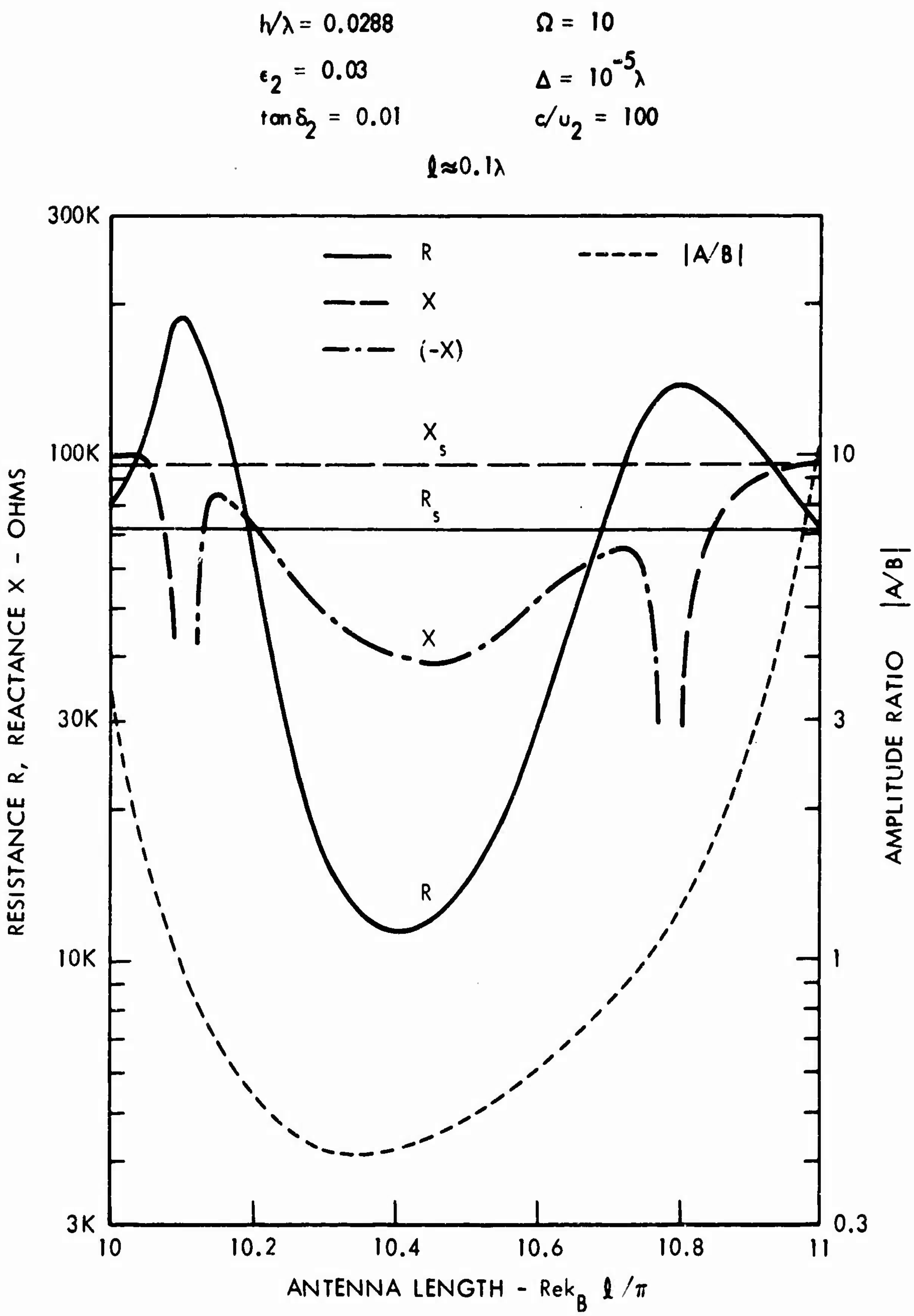

Figure 7. Antenna Impedance and Current Amplitudes for Radiation in Plasma Layer.

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13. ABSTRACT

A flat strip antenna is embedded in a planar dielectric slab, which is surrounded on both sides by layers of compressible isotropic electron plasma. Several closed form expressions are obtained for the impedance with a sinusoidal current distribution along the antenna. The antenna impedance is computed numerically when considering the perturbations of the antenna current by surface waves. Except for thin plasma layers, the antenna impedance can be computed using the same current distribution as for a cold plasma. This supports the validity of earlier work which neglects the perturbation of the antenna current by plasma waves. However, it is essential to consider the finite transverse dimension of the antenna and the presence of an insulating layer or of an ion sheath.

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