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APPLICATION OF LINEARIZED CHARACTERISTICS TO A THREE-DIMENSIONAL NOZZLE FLOW

ARTHUR J. SCHLESINGER

POLYTECHNIC INSTITUTE of BROOKLYN
FREEPORT, NEW YORK

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AEROSPACE RESEARCH LABORATORIES
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FOREWORD

This technical report was prepared by Polytechnic Institute of Brooklyn, Freeport, New York on Contract AF 33(657)-8286 for the Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force. The research reported herein was accomplished on Task 7065-01, "Fluid Dynamics Facilities Research" of Project 7065, "Aerospace Simulation Techniques Research" under the technical Cognizance of Lt. Arthur J. Wennerstrom of the Fluid Dynamics Facilities Laboratory of ARL. Mr. A. Schlesinger is a research assistant at the Polytechnic Institute of Brooklyn.

The author is pleased to acknowledge Professor R. J. Cresci for initiating this project and for his helpful discussions concerning the research presented herein.

ABSTRACT

The method of three-dimensional linearized characteristics has been applied to the design of annular supersonic nozzles. The equations and coordinate system used are presented along with a step-by-step calculation procedure of the method. A typical nozzle has been designed which produces a radial variation in the exit Mach number and flow deflection. The results are presented in terms of the two-dimensional and final t' ree-dimensional nozzle contours in three radial planes. The method and system of equations presented are applicable to guide vanes or stators with large turning angles and high tangential components of velocity.

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LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

a	speed of sound
B_1, B_2, C_1, C_2, H_1, H_2,	defined by equations (10)
u, v, w,	velocity components along the x- and r- axes and perpendicular to the meridian plane in cylindrical coordinates.
V	intensity of the velocity vector
x, r, φ,	cylindrical coordinates (Figure 3b)
γ	ratio of specific heats
θ	inclination of the velocity vector with respect to the x-axis
$\lambda_{1,2}$	inclination of the characteristic lines
μ	Mach angle
σ	angle defined by equation (13a)
o	zero subscripts refers to basic flow field quantities
$'$	primed quantities refer to perturbations

SECTION I

INTRODUCTION

Three-dimensional flow fields have given rise to many problems in the field of high speed fluid mechanics. These flow fields exist for three-dimensional bodies, axisymmetric bodies at angle of attack, in turbomachinery, and for flows with large boundary curvatures. In the latter case, the large curvature produces centrifugal forces which in turn create pressure gradients in the direction of the centrifugal forces. One example of this is an annular supersonic nozzle in which the flow in the tangent plane is inclined at an appreciable angle with respect to the axis of symmetry. The pressure gradients required to balance the centrifugal forces produce radial cross flows in the nozzle, thus giving rise to the three-dimensional effects. Therefore, in order to design such a nozzle for given exit conditions, these effects must be considered. It is the purpose of this paper to present an analysis for the design of this type of annular supersonic nozzle.

This investigation was initiated in connection with the design of a supersonic compressor for a high enthalpy and high pressure test facility as first proposed by Ferri.[1] For efficient operation of the compressor an annular nozzle is required to produce an entrance flow with a large tangential component of velocity entering the rotor. A more detailed description of the overall mechanical accelerator is presented in reference (2), wherein typical flow Mach numbers and nozzle angles are shown. A schematic of the nozzle under consideration for the present investigation is shown in Figure (1), along with the coordinate system used.

The centrifugal forces produced in the nozzle will create radial pressure gradients in the flow which are not necessarily uniform with x and $rd\varphi$. These in turn will generate cross flows in the field. If the pressure gradients are small then the cross flows resulting from them will also be small compared to the free stream velocity. If this is the case, the cross flow velocities can be considered as perturbations of the total velocity in the flow field. Ferri[3] refers to this type of flow field as a quasi-two-dimensional flow field and presents an analysis using the characteristics equations for flows of this type. The method of three-dimensional linearized characteristics is thereby obtained.

Vaglio-Laurin[4] derives the governing equations for linearized characteristics in cylindrical coordinates. In this treatment, the radial component of the total velocity is considered as one perturbation quantity. This velocity component will produce perturbations of the velocity in the original flow field. If these perturbations are added to the initial velocity components, then from the governing equation of motion a set of linearized equations are obtained. The total velocity thus becomes the velocity in a surface of constant radius plus a small radial velocity component.

LaRocca[5] has applied the method of linearized characteristics in a cartesian coordinate system to the design of supersonic stator passages. In this analysis, the entering flow into the passage is non-uniform and inclined with respect to the axial direction. The flow must be turned back to the axial direction with a uniform Mach number at the exit of the stator. The flow is essentially two-dimensional with a perturbation normal to the plane. The cartesian, rather than the cylindrical, coordinate system is used since the stator passages are located far from the axis and subtend a small angle of arc. The curvature effects are therefore negligible. In the present analysis, a cylindrical coordinate system must be used; this is a result of the large angle that the flow must be turned through when leaving the nozzle. Since the nozzle is annular and the effective turning angle of the flow is large, a relatively large peripheral angle of the annulus must be considered.

Using the equations of reference (4), a Fortran program was developed and a typical annular entrance nozzle was designed. This nozzle is typical of the type that will be used in the mechanical accelerator in that the radial momentum equation is satisfied at the nozzle exit. This produces a flow with a uniform peripheral Mach number distribution but a varying distribution in the radial direction. The hub radius was held constant and the shroud contour allowed to vary to account for the radial effects, upstream of the nozzle exit.

Although the particular problem of interest in the present report is related to the mechanical accelerator, this analysis can be readily applied to a stationary cylindrical system with relatively large curvature, such as may be present in inlet guide vanes, stators, etc. It can also be applied to rotating components if the governing equations are modified to include the additional centrifugal and Coriolus terms.

SECTION II

DESIGN ANALYSIS

The method of linearized three-dimensional characteristics is one in which a two-dimensional flow field is perturbed to allow for variations in a direction normal to the two dimensional field. Thus the first step in the design of the nozzle is to establish the two-dimensional basic flow fields. This was done for the present case in three radial planes. The basic flow fields are Busemann nozzles, designed by using a sharp corner and then choosing suitable streamlines for the given exit conditions. The basic nozzles including flow angles and design Mach numbers are shown in Figure (2). With the basic flow fields thus established, the velocity and flow deflection at each point in the field is known in all three planes. A condition is imposed at the exit station which requires the velocity and flow deflection to be that of the basic flow, i.e., the perturbation quantities at this station must be zero. The method assumes that the three-dimensional effects are small, and therefore that they may be determined by linearizing the difference between the two-dimensional and the actual three-dimensional flow. The basic simplification for this method is that the linearized three-dimensional flow field and the basic two-dimensional flow field have the same characteristic surfaces. Hence, the linearized flow may be analyzed along the characteristic net of the basic flow field.

The equation of motion for an isentropic flow referred to a cylindrical coordinate system (x, r, φ) is:

$$\left[1 - \frac{u^2}{a^2}\right]\frac{\partial u}{\partial x} + \left[1 - \frac{w^2}{a^2}\right]\frac{\partial w}{r\partial\varphi} + \left[1 - \frac{v^2}{a^2}\right]\frac{\partial v}{\partial r} - \frac{uv}{a^2}\left[\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right]$$

$$- \frac{wu}{a^2}\left[\frac{\partial u}{r\partial\varphi} + \frac{\partial w}{\partial x}\right] - \frac{wv}{a^2}\left[\frac{\partial v}{r\partial\varphi} + \frac{\partial w}{\partial r}\right] + \frac{v}{r} = 0 \qquad (1)$$

where u is the velocity component along the x axis, v is the velocity component along r axis, and w is the velocity component normal to the meridian plane. Now, the flow field is represented by a velocity $V_o(u_o, w_o)$ and superposed three-dimensional flow velocity $V'(u', v', w')$ such that

$$u = u_o + u'$$

$$v = v' \qquad (2)$$

$$w = w_o + w'$$

where the primed quantities are small with respect to the basic two-dimensional flow corresponding to the quantities having zero subscript.

In the basic two-dimensional flow field the velocity components u_o

and w_o can be expressed as:

$$u_o = V_o \cos \theta_o$$

$$w_o = V_o \sin \theta_o \qquad (3)$$

where θ_o is the flow inclination with respect to the x-axis at the point in question. Now, since

$$V = V_o + V' \qquad (4)$$

the perturbation velocity components can be expressed by

$$u' = V' \cos \theta_o - \theta' V_o \sin \theta_o$$

$$w' = V' \sin \theta_o + \theta' V_o \cos \theta_o \qquad (5)$$

neglecting higher order terms and where θ' is the additional flow inclination due to the three-dimensional effects.

Substituting equations (2), (3) and (5) into equation (1) it can be shown that the characteristic lines of the three-dimensional flow field will be coincident with those of the basic two-dimensional flow field. Expansion of the resulting partial differential equations into total differential equations will yield, on surfaces of r = constant, the following equations. Along the characteristic line defined by;

$$r \frac{\partial \varphi}{\partial x} = \lambda_1 = \tan (\theta_o + \mu_o) \qquad (6)$$

the following holds,

$$- \frac{dv'}{dr} \frac{1}{V_o} \frac{\sin \mu_o \tan \mu_o}{\cos (\theta_o + \mu_o)} + \frac{1}{V_o} \frac{dV'}{dx} - \tan \mu_o \frac{d\theta'}{dx} + B_1 \theta' + \frac{V'}{V_o} C_1 + \frac{v'}{V_o} H_1 = 0 \qquad (7)$$

Similarly, along the characteristic line defined by,

$$r \frac{\partial \varphi}{\partial x} = \lambda_2 = \tan(\theta_o - \mu_o) \tag{8}$$

the following holds,

$$- \frac{dv'}{dr} \frac{1}{V_o} \left(\frac{\sin\mu_o \tan\mu_o}{\cos(\theta_o - \mu_o)}\right) + \frac{1}{V_o} \frac{dV'}{dx} + \tan\mu_o \frac{d\theta'}{dx} + B_2 \theta' + \frac{V'}{V_o} C_2 + \frac{v'}{V_o} H_2 = 0 \tag{9}$$

where the coefficients B_1, B_2, C_1, C_2, H_1, and H_2 are functions only of the basic flow field and are defined as:

$$B_1 = - \frac{1}{V_o} \left(\frac{dV_o}{dx}\right)_{\lambda_2} \left[\frac{\cos(\theta_o - \mu_o)}{\sin\mu_o \cos\mu_o \cos(\theta_o + \mu_o)}\right]$$

$$B_2 = \frac{1}{V_o} \left(\frac{dV_o}{dx}\right)_{\lambda_1} \left[\frac{\cos(\theta_o + \mu_o)}{\sin\mu_o \cos\mu_o \cos(\theta_o - \mu_o)}\right]$$

$$C_1 = \frac{1}{V_o} \left(\frac{dV_o}{dx}\right)_{\lambda_1} \left[\tan^2\mu_o + \frac{\gamma - 1}{2 \sin^2\mu_o \cos^2\mu_o}\right] - B_1 \left[1 + \frac{\gamma - 1}{2 \sin^2\mu_o}\right] \tag{10}$$

$$C_2 = \frac{1}{V_o} \left(\frac{dV_o}{dx}\right)_{\lambda_2} \left[\tan^2\mu_o + \frac{\gamma - 1}{2 \sin^2\mu_o \cos^2\mu_o}\right] + B_2 \left[1 + \frac{\gamma - 1}{2 \sin^2\mu_o}\right]$$

$$H_1 = \frac{\sin\mu_o}{\cos(\theta_o + \mu_o)} \left[\frac{1}{V_o \sin\mu_o \cos\mu_o} \frac{\partial V_o}{\partial r} - \frac{1}{r} (\tan\mu_o + \sin\theta_o \cos\theta_o) - \frac{\partial \theta_o}{\partial r}\right]$$

$$H_2 = \frac{\sin \mu_o}{\cos(\theta_o - \mu_o)} \left[\frac{1}{V_o \sin\mu_o \cos\mu_o} \frac{\partial V_o}{\partial r} + \frac{\partial \theta_o}{\partial r} - \frac{1}{r} (\tan \mu_o - \sin\theta_o \cos\theta_o) \right]$$

Since the flow is isentropic, two other equations may be obtained along the characteristic lines. They are:

along λ_1,

$$\frac{\cos(\theta_o + \mu_o)}{V_o \cos\mu_o} \left(\frac{dv'}{dx}\right)_{\lambda_1} = \frac{1}{V_o} \frac{\partial}{\partial r} (V_o + V') - \theta' \frac{\partial \theta_o}{\partial r} + \tan\mu_o \left[(1 + \frac{V'}{V_o}) \frac{\partial \theta_o}{\partial r} + \right.$$

$$\left. + \frac{\partial \theta'}{\partial r} + \frac{\theta'}{V_o} \frac{\partial V_o}{\partial r} \right] + \frac{\sin(\theta_o + \mu_o)}{r \cos\mu_o} \left[(1 + \frac{V'}{V_o}) \sin\theta_o + \theta' \cos\theta_o \right] \tag{11}$$

and along λ_2,

$$\frac{\cos(\theta_o - \mu_o)}{V_o \cos\mu_o} \left(\frac{dv'}{dx}\right)_{\lambda_2} = \frac{1}{V_o} \frac{\partial}{\partial r} (V_o + V') - \theta' \frac{\partial \theta_o}{\partial r} - \tan\mu_o \left[(1 + \frac{V'}{V_o}) \frac{\partial \theta_o}{\partial r} + \right.$$

$$\left. + \frac{\partial \theta'}{\partial r} + \frac{\theta'}{V_o} \frac{\partial V_o}{\partial r} \right] + \frac{\sin(\theta_o - \mu_o)}{r \cos\mu_o} \left[(1 + \frac{V}{V_o}) \sin\theta_o + \theta' \cos\theta_o \right] \tag{12}$$

Hence, by using equations (7), (9), (11) and (12) the three-dimensional flow field can be obtained.

The velocity vector at a point (x, r, φ,) is equal to $\overline{V}_o$ in a plane r = constant plus an additional velocity $\overline{V}'$ in the same plane r = constant due to the three-dimensional effects, plus a small velocity component v' perpendicular to the plane. The flow inclination is equal to θ_o in the plane r = constant plus θ', which is a correction factor for the three-dimensional flow field. A schematic representation of the flow at a point in cylindrical coordinates is found in Figure (3).

The boundary conditions of the present problem require the velocity component normal to the hub surface of the nozzle to vanish.

The streamlines must also be tangent to the side walls of the nozzle thus:

$$\frac{w - v \tan \sigma}{u} = \tan \theta_o \tag{13}$$

where

$$u = (V_o + V') \cos (\theta_o + \theta_o') \tag{14}$$

$$w = (V_o + V') \sin (\theta_o + \theta')$$

and σ denotes the angle between the r direction and that of the tangent to the nozzle cross section at the considered point. Substitution of equations (14) into equation (13) yields:

$$\theta' = \frac{v'}{V_o} \tan \sigma \cos \theta_o \tag{15}$$

The above equations will now allow the design of the nozzle, with three-dimensional effects included, to be accomplished.

A step-by-step procedure can be performed which considers two unit problems. The first is a point in the flow field noted by the intersection of two characteristic lines. The second is a point on the side wall of the nozzle determined by the intersection of one characteristic line and a streamline.

In Figure (4) the first unit problem is considered. It is assumed that at points 1 and 2 all parameters are known from a previous calculation.

The parameters are $V_o, \theta_o, V', \theta'$, and v'. The flow variables at point 3 must be determined. From the basic two-dimensional flow field the location of point 3 is known; hence V_o and θ_o are also known. The first step is to determine the value of v' at point 3. Equation (11) can be solved using a finite-difference scheme. For the first approximation equation (11) is evaluated at point 1. Thus,

$$\left[\frac{\cos(\theta_o + \mu_o)}{V_o \cos \mu_o}\right]_{P_1} \left(\frac{dv'}{dx}\right)_{\lambda_1} = \left[\frac{1}{V_o} \frac{\partial}{\partial r}(V_o + V') - \theta' \frac{\partial \theta_o}{\partial r}\right]_{P_1} +$$

$$+ \tan \mu_o \left[(1 + \frac{V'}{V_o}) \frac{\partial \theta_o}{\partial r} + \frac{\partial \theta'}{\partial r} + \frac{\theta'}{V_o} \frac{\partial V_o}{\partial r}\right]_{P_1} + \tag{11a}$$

$$+ \left[\frac{\sin(\theta_o + \mu_o)}{r \cos \mu_o}\right]_{P_1} \left[(1 + \frac{V'}{V_o}) \sin \theta_o + \theta' \cos \theta_o\right]_{P_1}$$

In order to calculate the derivatives along the r - direction, the values of the considered quantities at the points 1 and 2 must be determined in the adjacent (in the radial direction) planes by projection of the points onto these planes. The radial derivatives may then be found using a second-order polynomial equation as a curve fit to the points in the three tangent planes.

Equation (11) is solved for $\left(\frac{dv'}{dx}\right)_{\lambda_1}$ the

$$v_{3\lambda_1} = v_1' + \left(\frac{dv'}{dx}\right)_{\lambda_1} (x_3 - x_1) \tag{16}$$

Similarly, $\left(\frac{dv'}{dx}\right)_{\lambda_2}$ can be obtained from equation (12) and

$$v'_{3_{\lambda_2}} = v'_2 + \left(\frac{dv'}{dx}\right)_{\lambda_2} (x_3 - x_2) \tag{17}$$

Therefore, as a first approximation:

$$v'_3 = \frac{1}{2}\left[v'_{3_{\lambda_1}} + v'_{3_{\lambda_2}}\right] \tag{18}$$

Using the above value of v'_3 the values of V'_3 and θ'_3 can be calculated from equations (7) and (9). For brevity, equations (7) and (9) can be written as:

$$-a\,\frac{dv'}{dr} + \frac{dV'}{dx}\,b \pm \frac{d\theta'}{dx}\,c + \theta' d + V' e + v' f = 0 \tag{7a}$$

The boundary condition is:

$$v' = 0 \text{ at } r = r_1$$

Now along

$$\lambda_1 = \tan(\theta_o + \mu_o)$$

equation (7a) becomes

$$-\left[\left(\frac{dv'}{dr}\right)_1 + \left(\frac{dv'}{dr}\right)_3\right](a_1 + a_3) + 2\left[\frac{V'_3 - V'_1}{x_3 - x_1}\right](b_1 + b_3) - 2\left[\frac{\theta'_3 - \theta'_1}{x_3 - x_1}\right](c_1 + c_3) +$$

$$+ (\theta'_1 + \theta'_3)(d_1 + d_3) + (V'_1 + V'_3)(e_1 + e_3) + (v'_1 + v'_3)(f_1 + f_3) = 0 \tag{19}$$

and along

$$\lambda_2 = \tan(\theta_o - \mu_o)$$

equation (7a) becomes:

$$-\left[\left(\frac{dv'}{dr}\right)_2 + \left(\frac{dv'}{dr}\right)_3\right](a_2 + a_3) + 2\left[\frac{V_3' - V_2'}{x_3 - x_2}\right](b_2 + b_3) + 2\left[\frac{\theta_3' - \theta_2'}{x_3 - x_2}\right](c_2 + c_3) +$$

$$+ (\theta_2' + \theta_3')(d_2 + d_3) + (V_2' + V_3')(e_2 + e_3') + (v_2' + v_3')(f_2 + f_3) = 0 \qquad (20)$$

Solving equations (19) and (20) simultaneously will yield solutions for V_3' and θ_3'. With these values, $\left(\frac{dV'}{dr}\right)_3$ and $\left(\frac{d\theta'}{dr}\right)_3$ can be found using the second-order equation for the three planes. Now equations (11) and (12) can be re-evaluated at point 3 and a new value of v_3' obtained. The iteration procedure is continued until two successive values of v_3' are within a predetermined tolerance of each other. The values of V_3' and θ_3' obtained with the last value of v_3' are taken as the actual values.

The second unit problem is now considered in Figure (5). All quantities are known at points 2 and 3. Since point 4 is reached by only one characteristic line another equation must be utilized. First the change in the velocity along the characteristic line from point 3 to point 4 is calculated from equation (11). Then, the velocity v_4' is calculated using an equation similar to equation (16), i.e.,

$$v_4' = v_3' + \left(\frac{dv'}{dx}\right)_{\lambda_1} (x_4 - x_3) \qquad (16a)$$

The additional equation may now be used to determine the additional flow inclination at point 4. From equation (15)

$$\theta_4' = \left(\frac{v'}{V_o}\right)_4 \tan \sigma_4 \cos \theta_{o4} \qquad (15a)$$

where

$$\sigma = r \frac{\partial \varphi}{\partial r} \tag{13a}$$

and φ is the arc angle shown in Figure (3b). The derivative is evaluated with a second - order equation.

With v_4' and θ_4' known, V_4' can be calculated from equation (7a) in finite difference form as follows:

$$- \left[\left(\frac{dv'}{dr}\right)_3 + \left(\frac{dv'}{dr}\right)_4 \right] (a_3 + a_4) + 2 \left[\frac{V_4' - V_3'}{x_4 - x_3} \right] (b_3 + b_4) - 2 \left[\frac{\theta_4' - \theta_3'}{x_4 - x_3} \right] (c_3 + c_4)$$

$$+ (\theta_3' + \theta_4')(d_3 + d_4) + (V_3' + V_4')(e_3 + e_4) + (v_3' + v_4')(f_3' + f_4) = 0 \tag{21}$$

Upon calculation of $(\frac{dV'}{dr})$ and $(\frac{d\theta'}{dr})$ the rate of change $(\frac{dv'}{dx})_{\lambda_1}$ at point 4 may be obtained from equation (11) and v_4' can again be calculated from equation (16a). The iteration procedure is continued until two successive values of v' are coincident. At that time the values V_4' and θ_4' are taken as exact and $(\frac{dV'}{dr})_4$ and $(\frac{d\theta'}{dr})_4$ are calculated.

The method of calculation for the design of the nozzle is then completed, since any point can be referred to the scheme of either point 3 or point 4.

SECTION III

DISCUSSION OF RESULTS

The inlet nozzle designed by the method outlined in the previous section corresponds to a radial variation in exit Mach number between 3.4 at the hub and 3.01 at the tip, with exit flow angles of 39.7° and 41.7° respectively. The hub radius was assumed to be 9.0 inches and the tip 10.0 inches; an intermediate radius (9.5 inches) was also included in the analysis. For this radial variation of flow conditions, the three radial planes were considered sufficient although more may be required for a design with larger radial gradients in the flow parameters. The basic flow field, two dimensional, nozzles were designed such that at the exit their axes lie along a radial line. The tangential component of the exit flow is designed to correspond to a compressible vortex so that in the exit plane, there is no radial component of velocity. The analysis proceeds in an upstream direction (in the actual computation procedure the x axis was considered positive upstream) and thereby determines the corrected wall contours required to produce the specified exit flow conditions.

Figure (6) presents the final contours obtained in the three radial planes along with the characteristic nets and the original two dimensional contours superimposed for comparison. It is evident that over a large portion of the nozzle, the original contours are reasonably close to the final contours obtained. As the throat region is approached, however, the nozzle twists as the radius increases so that the effective tangential component of velocity decreases in the radial direction. The flow is thereby approaching a vortex distribution in the radial direction even though it was not initially designed (in the two-dimensional contours) to produce such a variation in the throat region. Figure (7) presents the streamlines along the nozzle walls and along the centerline, which correspond to constant radius surfaces in the basic flow field. The hub ($R = 9.0''$) is still a cylindrical surface since this was a specified boundary condition in the analysis. The surfaces initially at the $9.5''$ and $10.0''$ radius, however, are slightly warped due to the three dimensional effects. It is evident again that at the nozzle exit the flow is essentially that of the basic two dimensional flow field while in the throat region significant radial velocity components are required to balance the centrifugal force field.

From these results, it is evident that a simplifed design technique can be utilized in such nozzles by using an initial radial distribution of tangential velocity which corresponds to a compressible vortex, both at the throat and at the exit plane of the nozzle. In this manner, the flow in between should remain in the cylindrical surface with a relatively low radial flow component. This is the design utilized in reference (6) wherein the basic two dimensional nozzle designs matched the pure vortex flow distribution at both the throat and the nozzle exit.

SECTION IV

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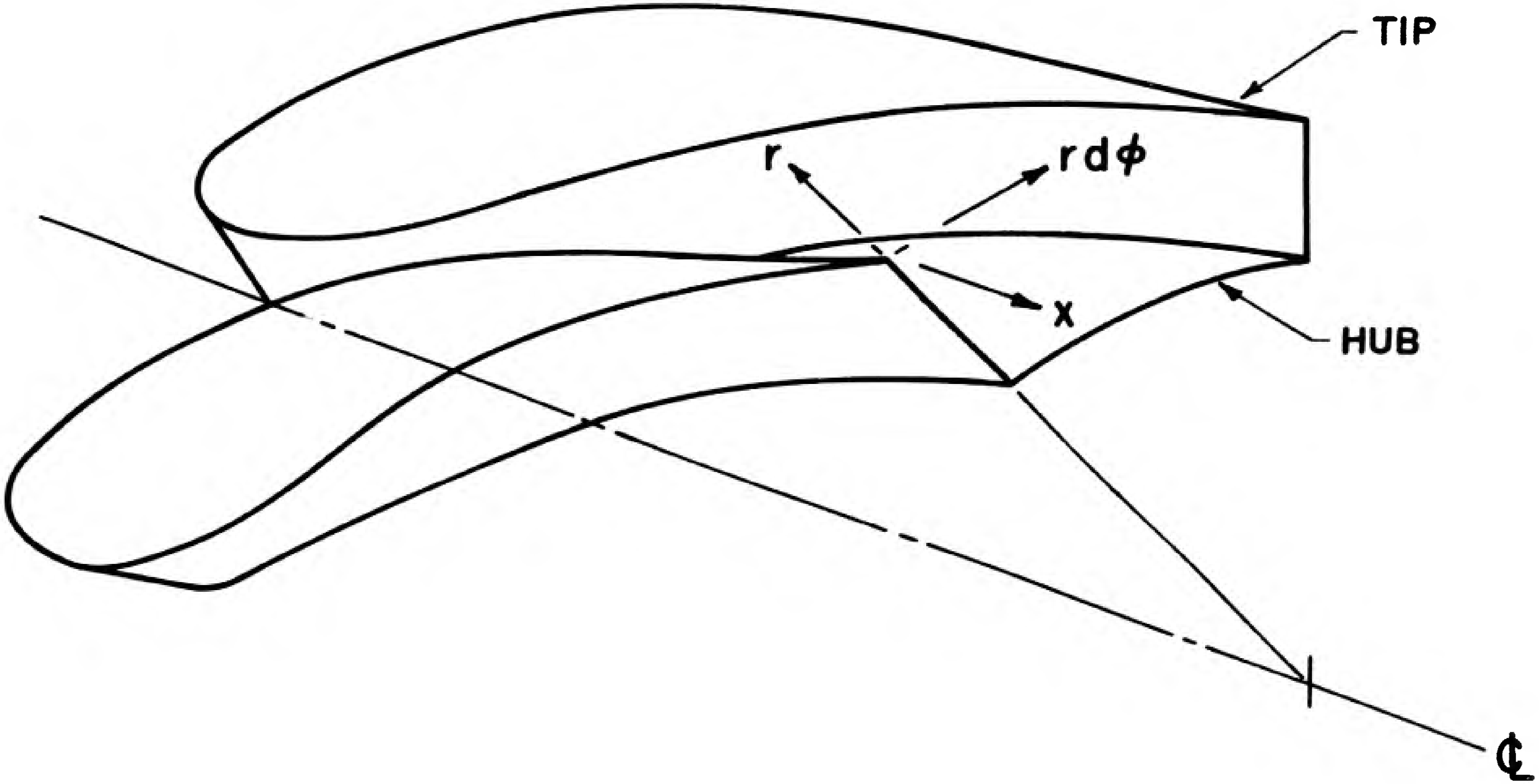
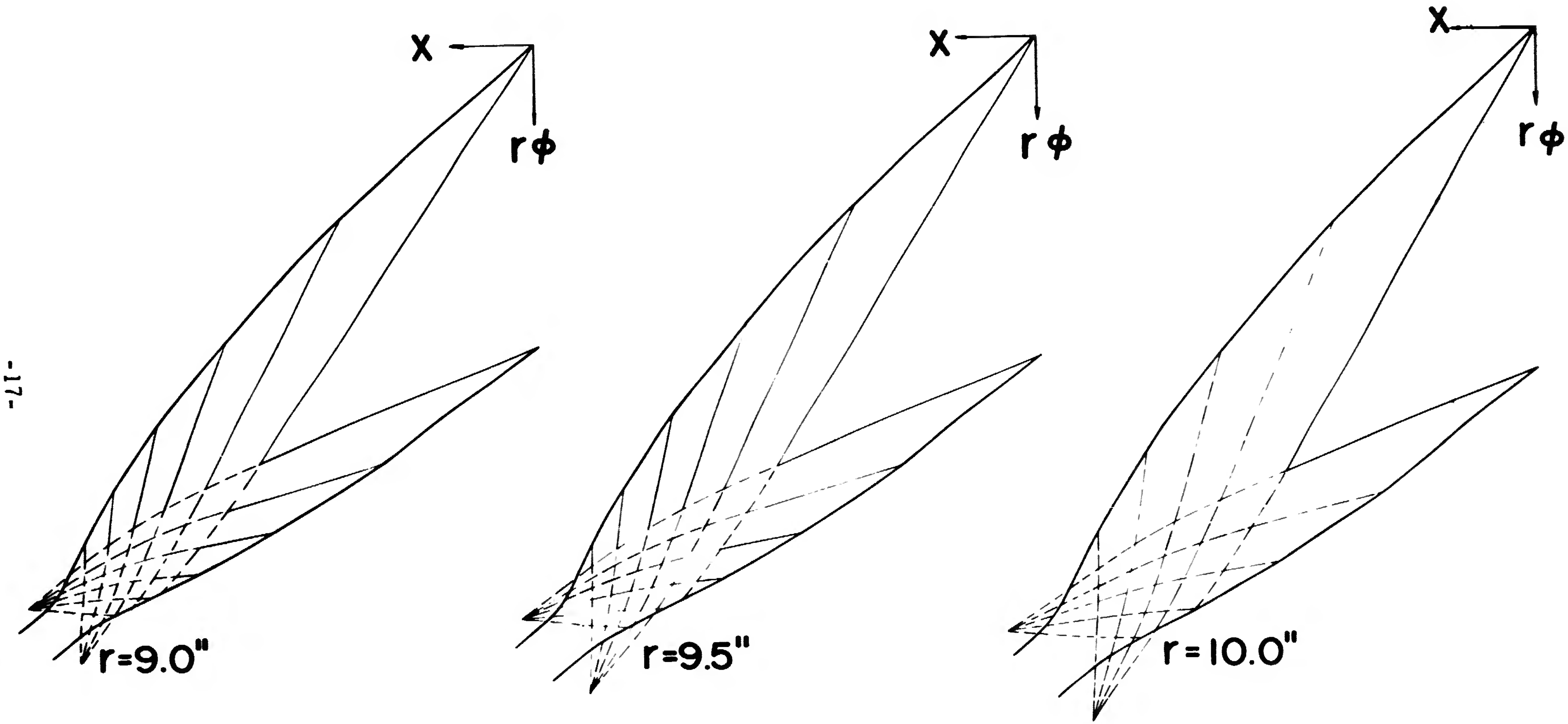


FIGURE (1) SCHEMATIC DIAGRAM OF THE ANNULAR SUPERSONIC NOZZLE.



FIGURE(2) BASIC FLOW FIELD AND NOZZLE CONTOUR

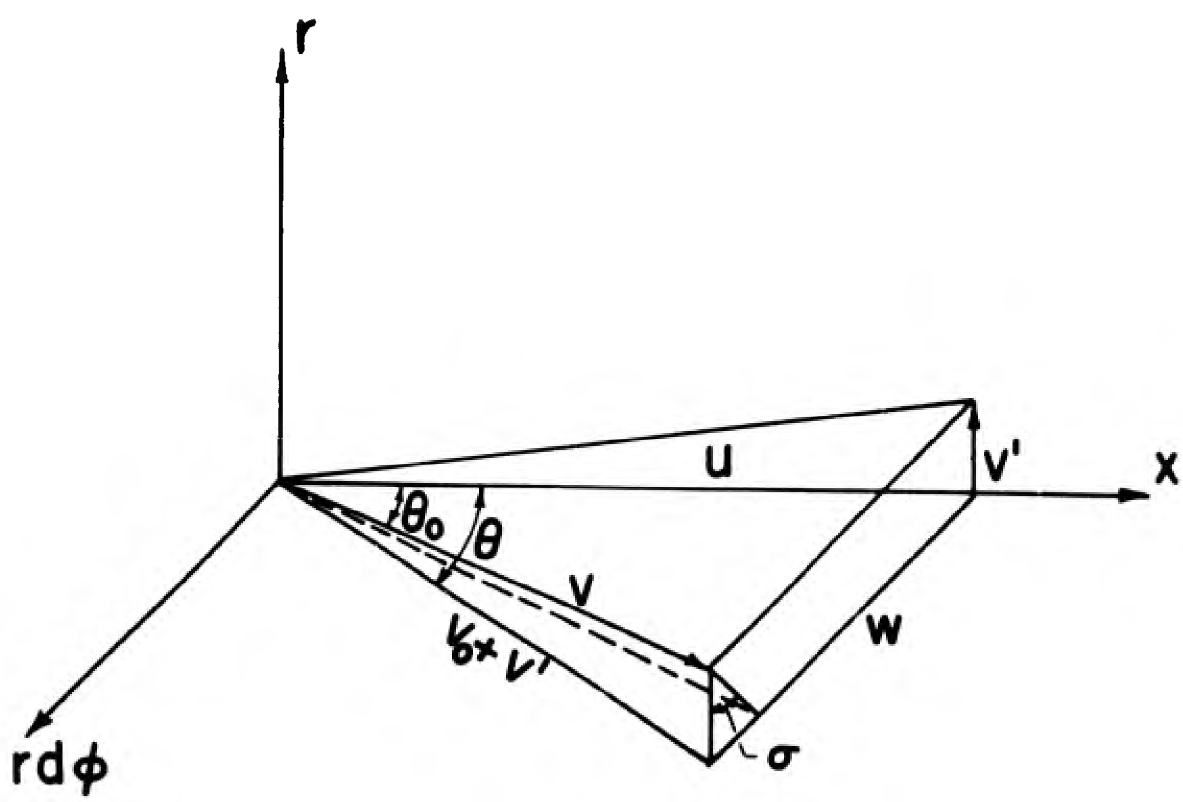


FIGURE (3a) VELOCITY DIAGRAM OF A POINT ON THE BOUNDARY.

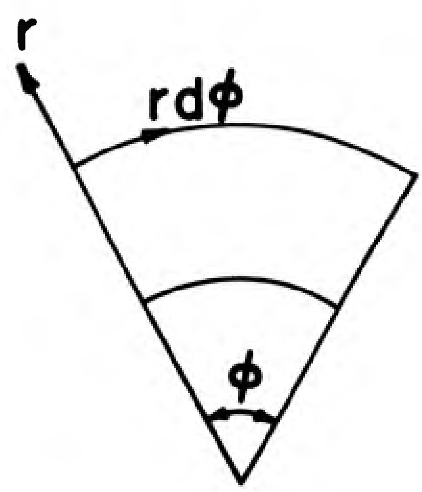


FIGURE (3b) CYLINDRICAL COORDINATE SYSTEM.

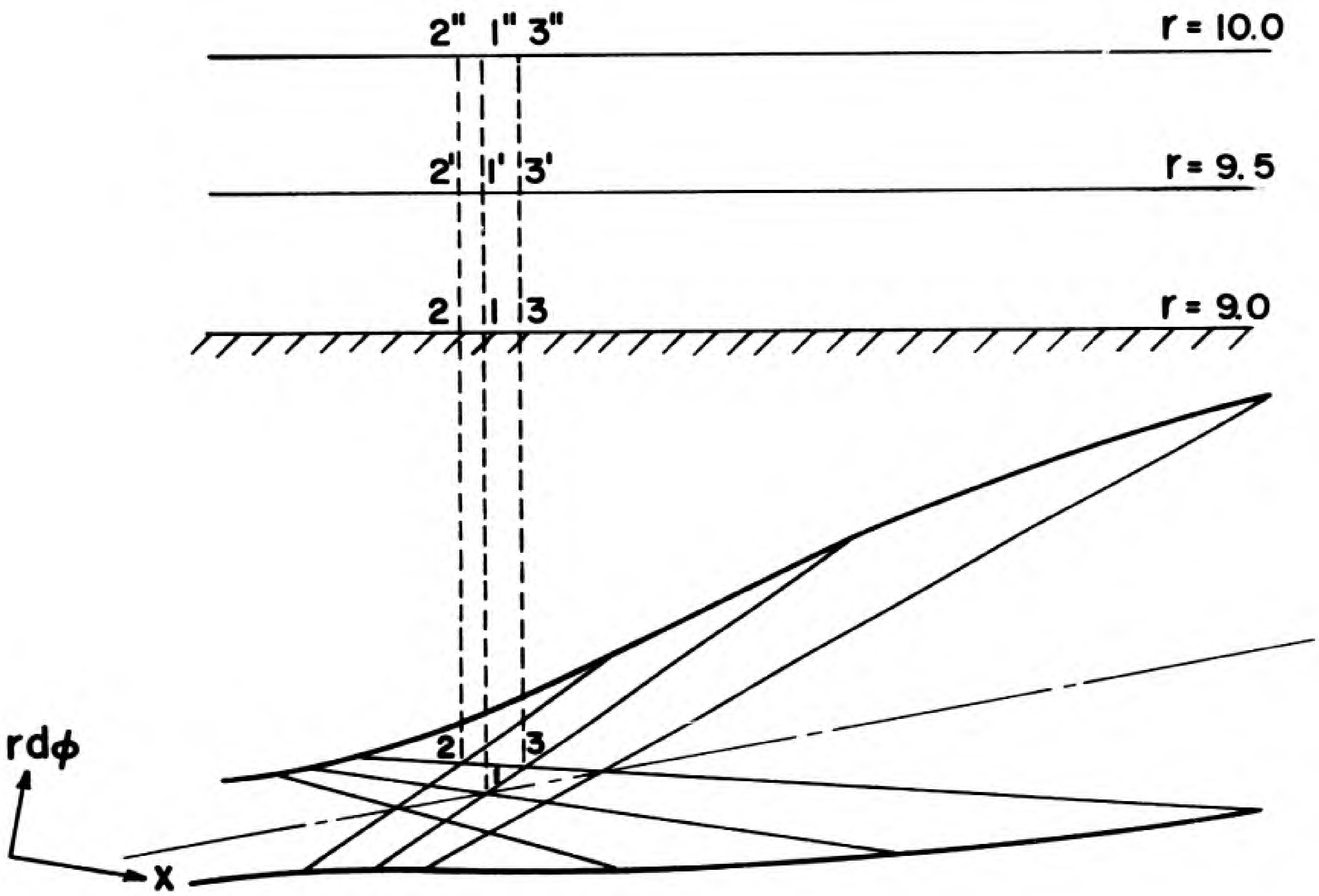


FIGURE (4) SCHEME OF FIRST UNIT PROBLEM (INTERSECTION OF TWO CHARACTERISTIC LINES)

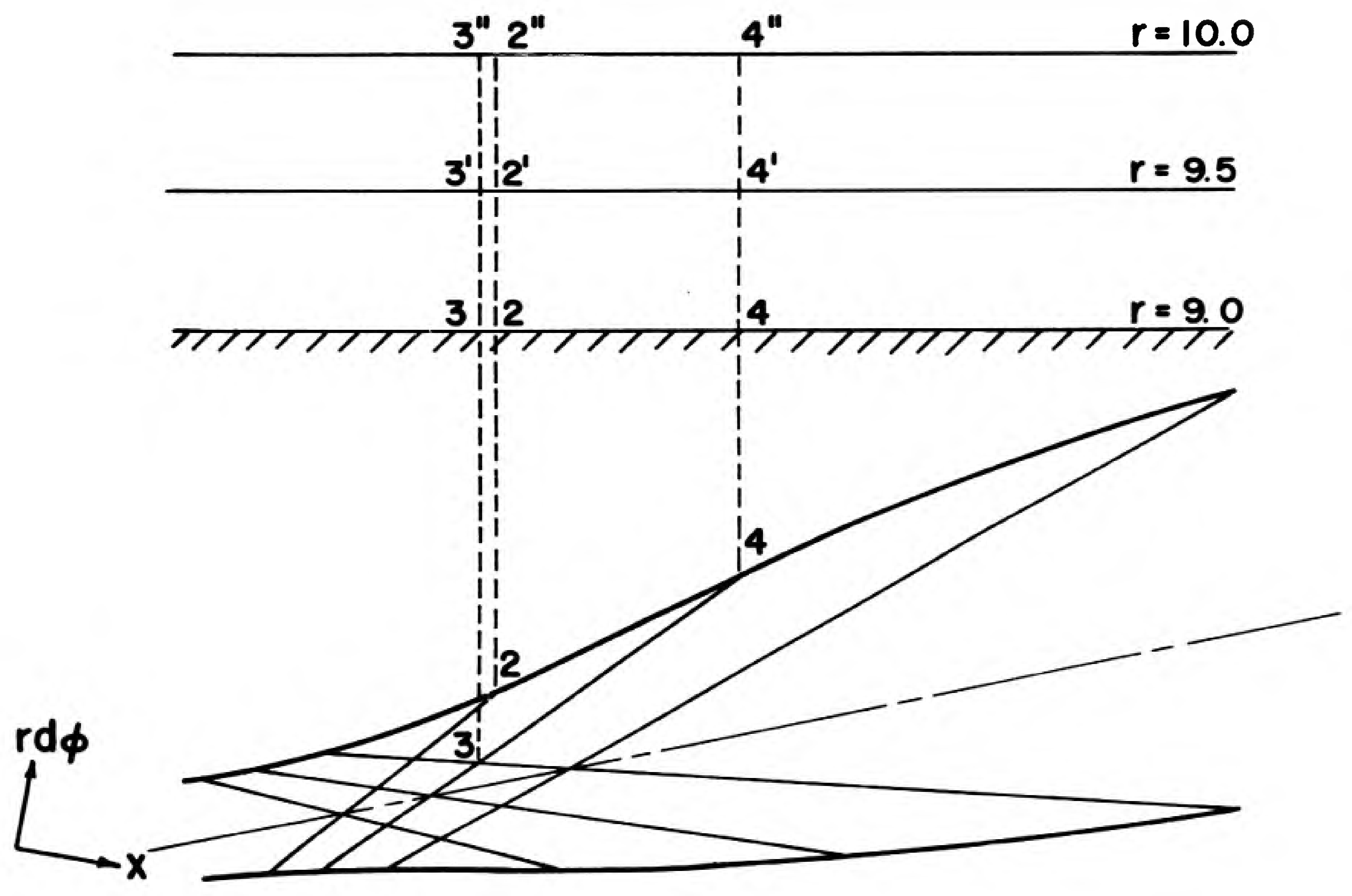
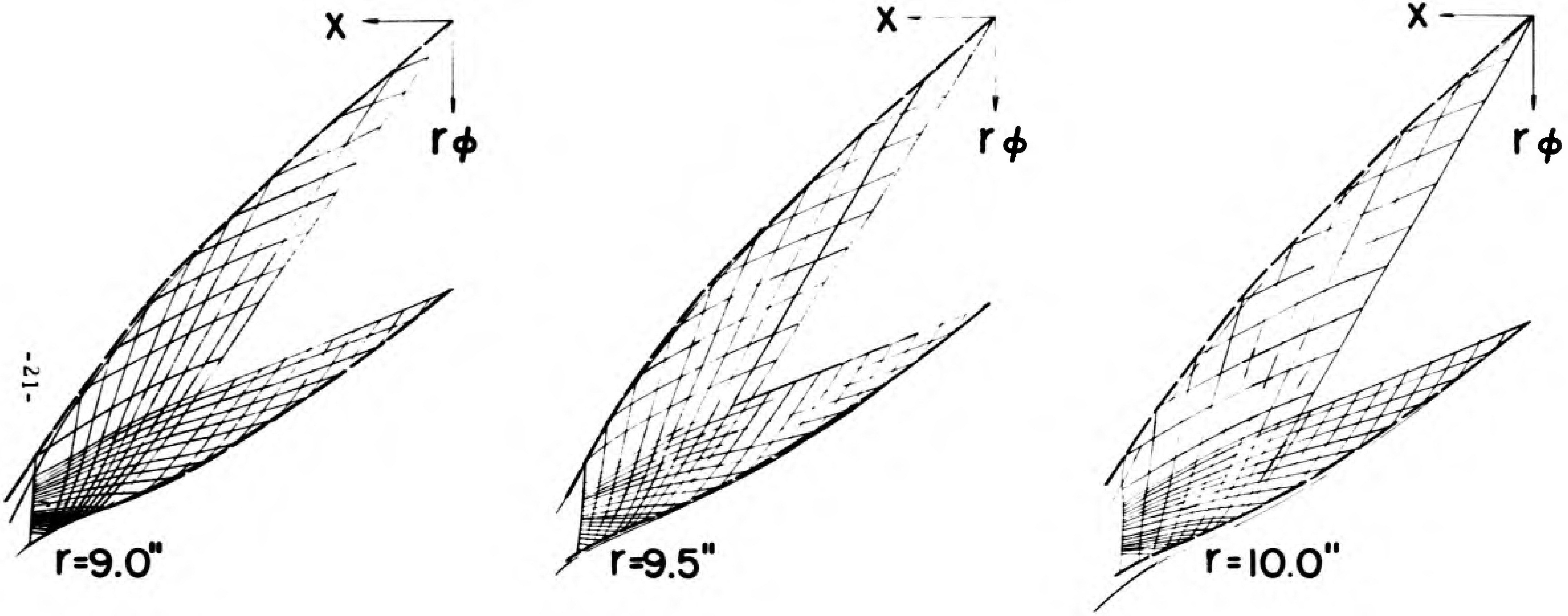


FIGURE (5) SCHEME OF SECOND UNIT PROBLEM (INTERSECTION OF A CHARACTERISTIC LINE WITH A STREAMLINE)



FIGURE(6) TWO DIMENSIONAL AND THREE DIMENSIONAL CONTOURS

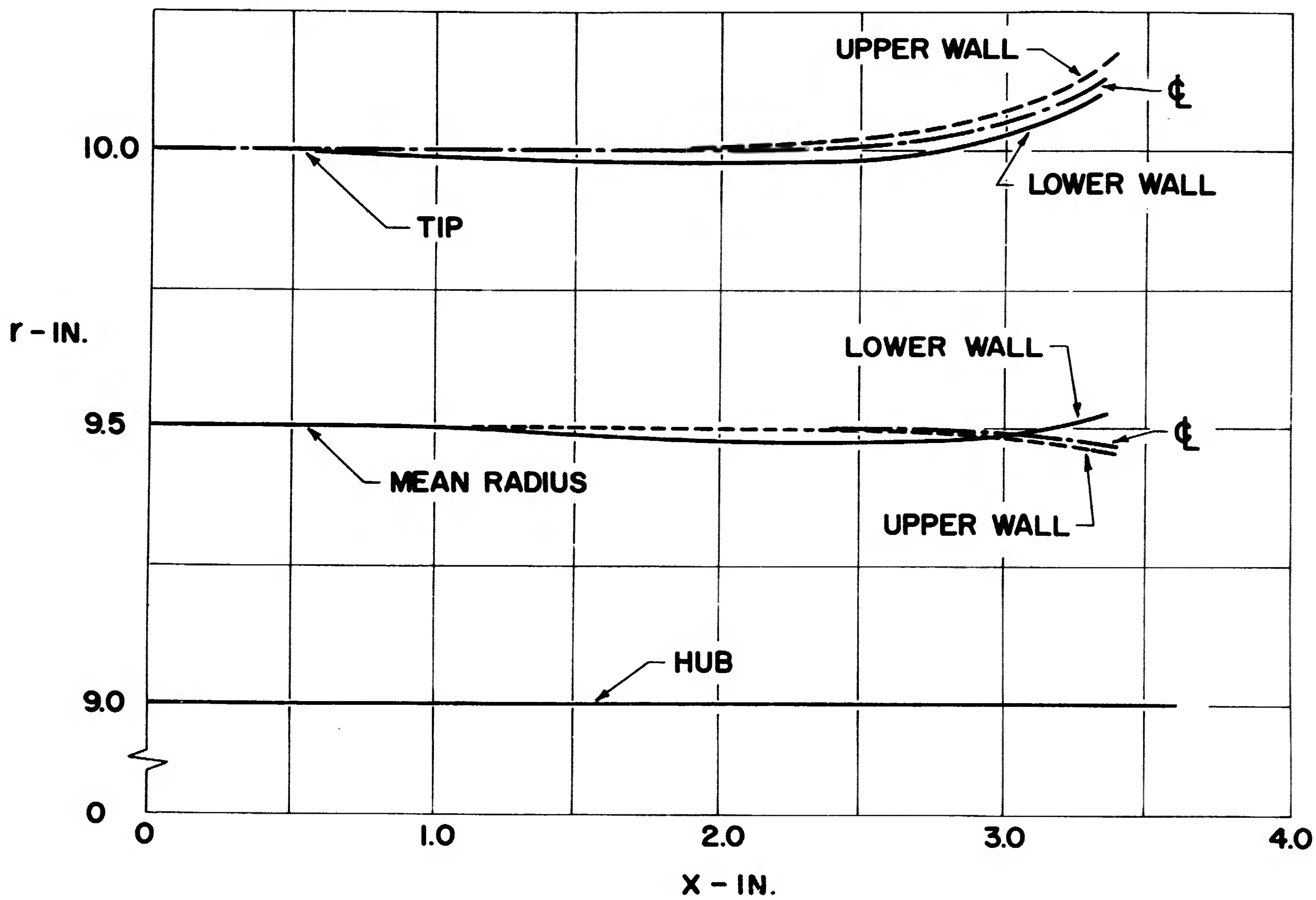


FIGURE (7) SURFACE CONTOURS IN RADIAL PLANE

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13. ABSTRACT

The method of three-dimensional linearized characteristics has been applied to the design of annular supersonic nozzles. The equations and coordinate system used are presented along with a step-by-step calculation procedure of the method. A typical nozzle has been designed which produces a radial variation in the exit Mach number and flow deflection. The results are presented in terms of the two-dimensional and final three-dimensional nozzle contours in three radial planes. The method and system of equations presented are applicable to guide vanes or stators with large turning angles and high tangential components of velocity.

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Nozzle Contours Supersonic Compressors Linearized Characteristics						

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