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THE INTEGRAL EQUATION FOR THE CURRENT IN A THICK TUBULAR ANTENNA

By

Tai Tsun Wu and Ronold W. P. King

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ABSTRACT

The conventional integral equation for the current in a cylindrical antenna is not correct for a tubular antenna when driven by an emf maintained in a narrow circumferential region along its outside surface. The basic electromagnetic theory of the doubly-connected region represented by such an antenna is reviewed. It is shown that the scalar potential and the normal derivative of the vector potential must be discontinuous across the air cylinder within the tube and that this discontinuity contributes an additional term to the integral equation. This term is negligible for electrically thin antennas but of major importance in determining the current and admittance for antennas that are electrically thick.

INTRODUCTION

Cylindrical antenna with cross-sectional dimensions that are comparable with the wavelength are of practical importance in applications that include structurally rigid microwave elements, broad-band antennas, and certain problems of weapon vulnerability in which bombs and missiles are treated as radiating elements. Significantly, the electromagnetic boundary-value problem associated with the electrically thick tubular transmitting antenna involves fundamental considerations that bring into focus implicit limitations of the well-known integral equation for center-driven cylinders.

When a tubular antenna of circular cross section with outer radius a is not electrically thin and, hence, does not satisfy the inequality $ka = 2\pi a/\lambda << 1$ (as assumed in all of the many available studies of the cylindrical antenna [1,2]) significant transverse currents may be generated by any asymmetry in the excitation. The complications introduced by such currents are not considered in this investigation and complete rotational symmetry is assumed. This means that practical methods of driving involve, for example, a radial transmission line as shown in Fig. 1a or a coaxial line operated in the conventional TEM mode in arrangements like those shown in Figs. 2a,b,c. For simplicity, it is assumed that the antenna is perfectly conducting and either base-driven over an infinite perfectly conducting ground plane as in Figs. 1a and 2a,b or center driven as in Fig. 2c. Fig. 1b is the physically unavailable image equivalent of Fig. 1a. The axial length of the center-driven antenna is 2h, of the base-driven element h; the outer radius of the tube is a, the inner radius is a_i.

When antennas are electrically thin ($a << h$, $ka << 1$), the geometrical structure of the ends has little effect on the current distribution, the admittance and the electromagnetic field. These are virtually the same [3] for all elements

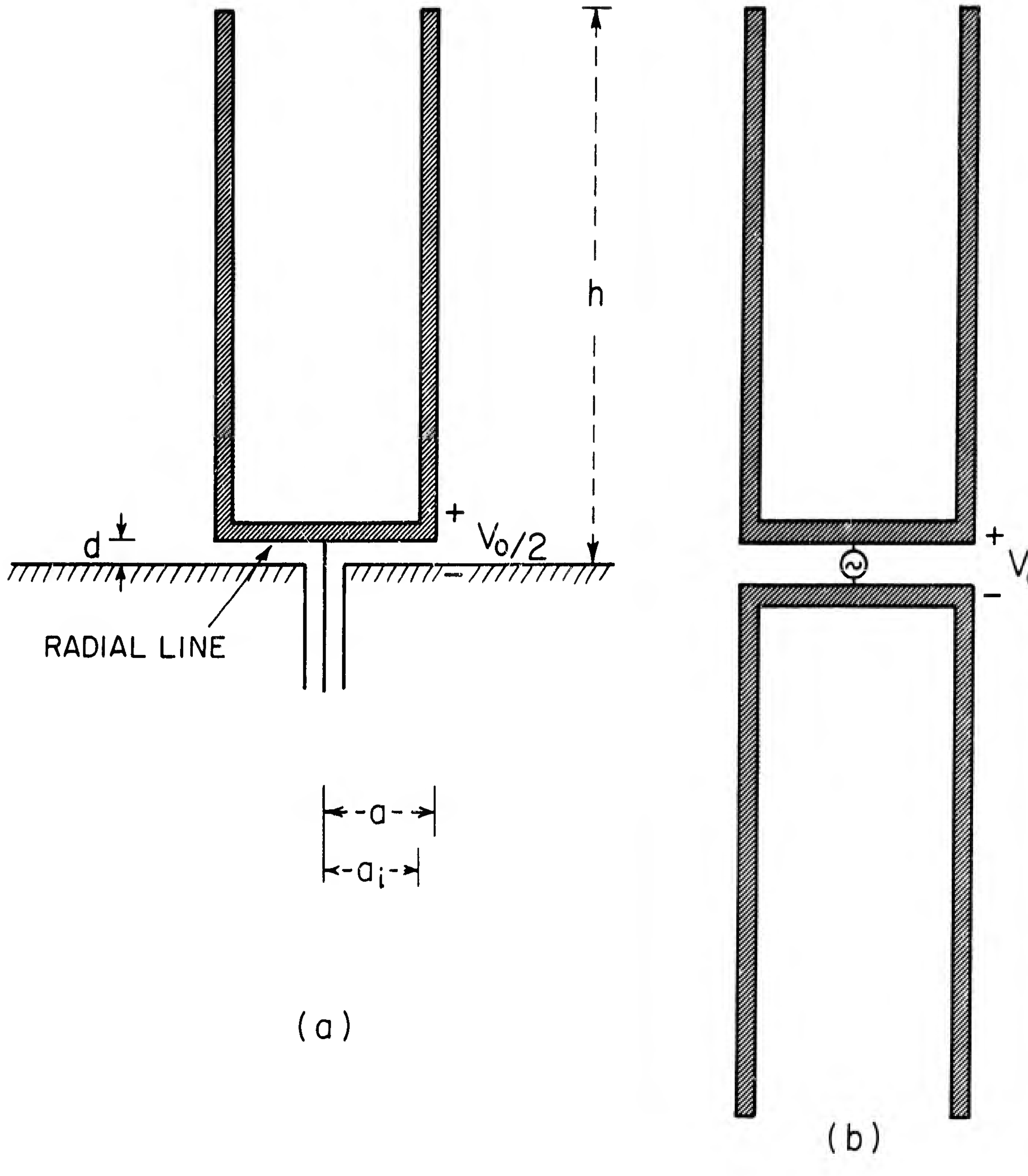

FIG. 1 ELECTRICALLY THICK ANTENNAS DRIVEN BY RADIAL TRANSMISSION LINES.

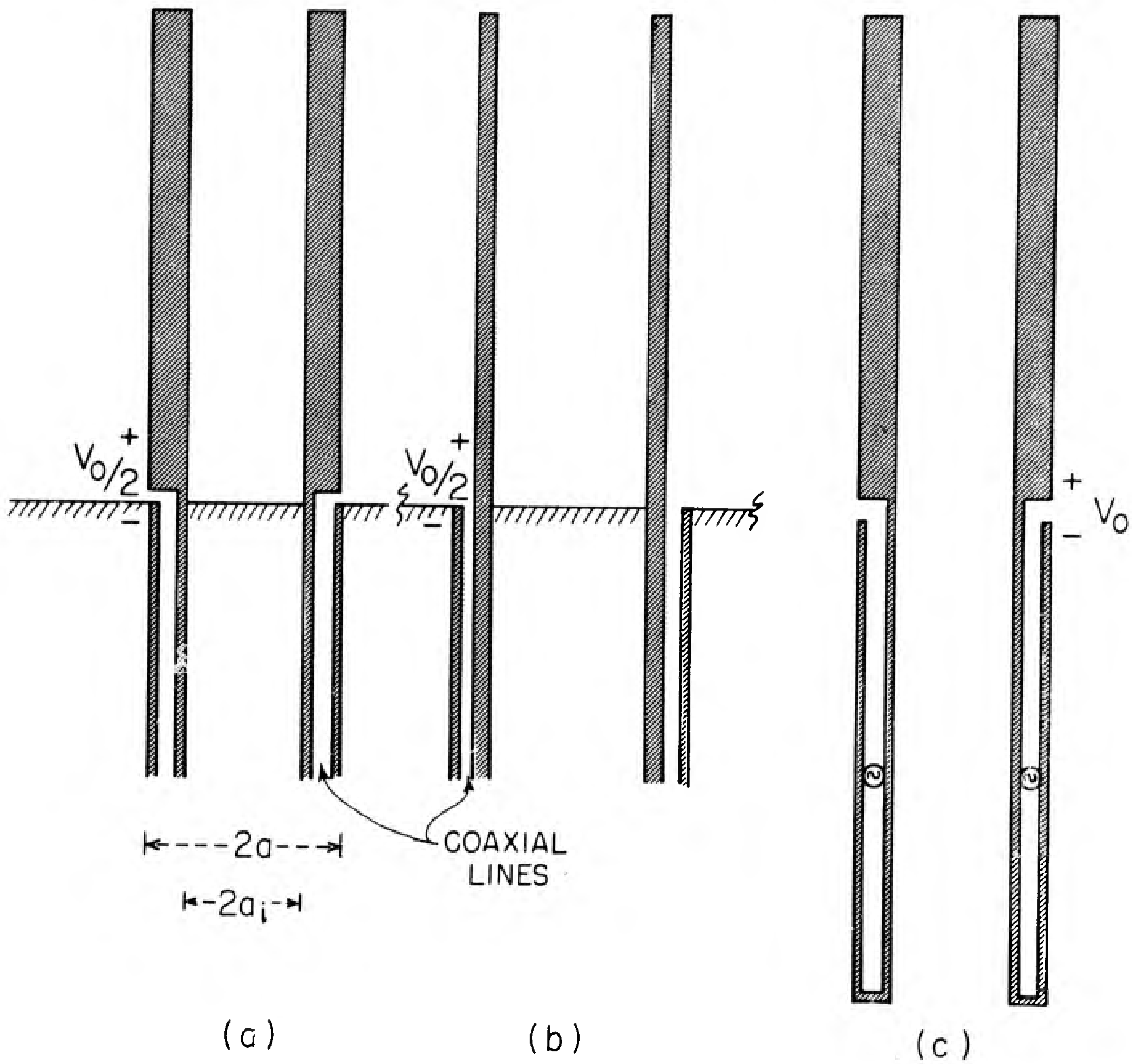

FIG. 2 ELECTRICALLY THICK ANTENNAS DRIVEN BY COAXIAL LINES

with equal values of h and a, regardless of whether the tubular ends are open, or closed with flat or hemispherical caps. When the conditions $a << h$, $ka << 1$ are not satisfied, this is no longer true. The properties of an open tubular antenna are changed significantly when the ends are capped and the analysis involves different complications. The present study is concerned with the thick tubular antenna with open ends as shown in Figs. 1a,b and 2a,b,c.

If the thicknesses of the conducting walls of the radial lines in Figs. 1a,b, and of the coaxial lines in Figs. 2a,b,c are assumed to be very small compared to the wavelength, that is, $kd << 1$, $k(a-a_i) << 1$, the approximate theoretical model shown in Fig. 3 is useful. It consists of a perfectly conducting tube (length 2h, inner radius a_i, outer radius a, and negligible wall thickness $a-a_i$) that is center-driven by a discontinuity in the rotationally symmetrical electric field,

$$E_z(a,\theta,z) = E_z(a,z) = -V_0\delta(z); \quad -h \leq z \leq h \tag{1}$$

on the <u>outside</u> surface at $\rho = a$ only. In (1), $\delta(z)$ is the Dirac delta function and the cylindrical coordinates ρ,θ,z are used. The inside surface at $\rho = a_i$ is an uninterrupted perfect conductor so that

$$E_z(a_i,\theta,z) = E_z(a_i,z) = 0; \qquad -h \leq z \leq h. \tag{2}$$

Evidently (1) and (2) are idealized representations of the actual boundary conditions for Figs. 1a,b and Figs. 2a,c, where the driving voltage is maintained across a narrow circumferential gap along the outside surface of the antenna while the inside surface is a continuous conductor. (A radial gap as in Fig. 2b is not fundamentally different.)

When the driving voltage is maintained across a circumferential gap in both the outside and inside surfaces, currents are excited on both of these surfaces directly by the generator. On the other hand, in the actual driving conditions shown in Figs. 1a,b and 2a,b,c, the generator (transmission line) excites currents

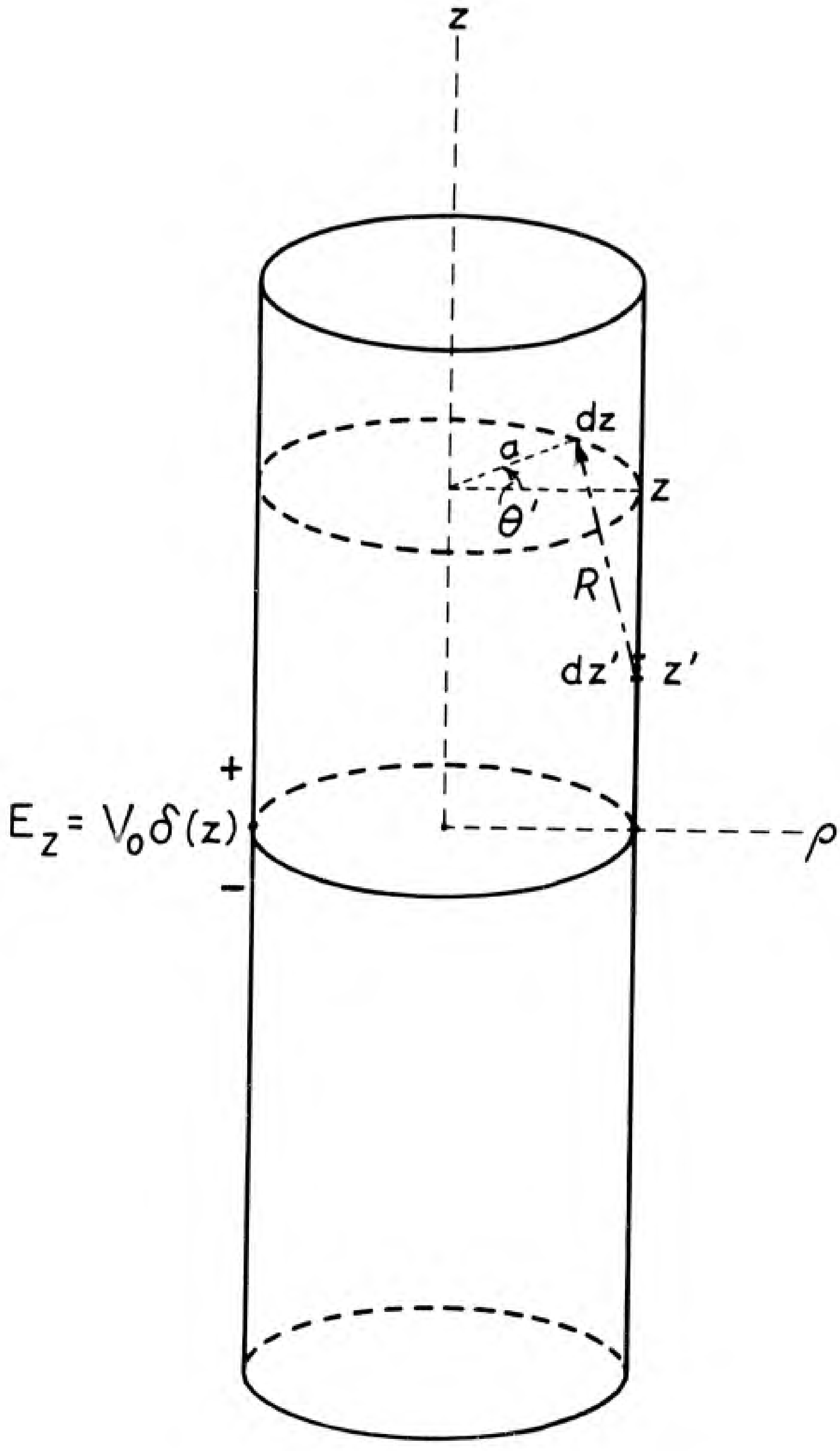

FIG. 3 THEORETICAL MODEL OF ELECTRICALLY THICK ANTENNA DRIVEN ON OUTSIDE SURFACE ONLY BY DELTA-FUNCTION GENERATOR.

directly only on the outside surface. However, currents can exist on the inside if there is a significant flow of charges around the tubular ends at $z = \pm h$ from the outside to the inside.

The walls of the tube in Fig. 3 are so thin that strictly only the sum $I_z(z)$ of the outside and inside currents is meaningful. Nevertheless, since the magnetic field $B_\theta(\rho,z)$ is discontinuous across the tube, $B_\theta(a,z)$ may be associated with the outside currents $I_{zo}(z)$, $B_\theta(a_i,z)$ with the inside currents $I_{zi}(z)$ in the range $-h \leq z \leq h$. That is

$$I_z(z) = 2\pi a\mu_o^{-1}[B_\theta(a,z) - B_\theta(a_i,z)] = I_{zo}(z) + I_{zi}(z). \qquad (3)$$

Although even the thinnest tubular antennas are also often driven in the manner shown in Fig. 2b, which is equivalent to an outside generator only, the use of a theoretical model with a δ-function generator that extends through the walls does not involve a significant error when $ka \ll 1$, since currents on the inside surface are then negligible except very near $z = 0$. As the delta-function generator is approached along both inside and outside surfaces, the currents rise steeply to infinity at $z = 0$. Since this charging current for the knife-edge capacitance of the generator must be subtracted out in any case if a useful susceptance is to be obtained [4], it is immaterial whether currents on the inside are included or not. It follows that with sufficiently thin tubular antennas, the error introduced by the use of an incorrect model with a generator that directly excites currents on the inside as well as the outside surface is negligible. On the other hand, when ka is unrestricted, currents in the well-known wave-guide modes may be excited by a generator on the inside surface. The admittance $Y = V_o/I_z(0)$ is then the sum of Y_{inside} and $Y_{outside}$ and, when ka_i is sufficiently large, this differs greatly from the value of $Y_{outside}$ for an antenna driven as shown in Figs. 1a,b and 2a,b,c.

REVIEW OF ELECTROMAGNETIC THEORY

The usual derivation [5] of the integral equation for the current in a cylindrical antenna makes use of the vector potential $\vec{A}(\vec{r})$ which satisfies the relation

$$\nabla \times \vec{A}(\vec{r}) = \vec{B}(\vec{r}) \tag{4}$$

When this is substituted in Maxwell's equation, $\nabla \times \vec{E}(\vec{r}) = -j\omega\vec{B}(\vec{r})$, the result is

$$\nabla \times [\vec{E}(\vec{r}) + j\omega\vec{A}(\vec{r})] = 0 \tag{5a}$$

It is then possible to define the scalar potential $\varphi(\vec{r})$ as follows:

$$-\nabla \varphi(\vec{r}) = \vec{E}(\vec{r}) + j\omega\vec{A}(\vec{r}). \tag{5b}$$

The following conditions, which are consistent with the Maxwell eqautions and the associated boundary conditions for the electromagnetic vectors [6], are usually imposed on the potential functions and their normal derivatives at boundaries between air (region 1) and a perfect non-magnetic conductor (region 2):

a) $$\vec{A}_1(\vec{r}) - \vec{A}_2(\vec{r}) = 0 \tag{6a}$$

b) $$\left[\frac{\partial \varphi(\vec{r})}{\partial n}\right]_1 + \left[\frac{\partial \varphi(\vec{r})}{\partial n}\right]_2 = \varepsilon_o^{-1}\, \eta(\vec{r}) \tag{6b}$$

$$\left[\frac{\partial \vec{A}_t(\vec{r})}{\partial n}\right]_1 + \left[\frac{\partial \vec{A}_t(\vec{r})}{\partial n}\right]_2 = \mu_o\, \vec{K}(\vec{r}) \tag{6c}$$

$$\left[\frac{\partial \vec{A}_n(\vec{r})}{\partial n}\right]_1 + \left[\frac{\partial \vec{A}_n(\vec{r})}{\partial n}\right]_2 = 0 \tag{6d}$$

c) $$\lim_{r\to\infty} \varphi(\vec{r}) = 0, \qquad \lim_{r\to\infty} \vec{A}(\vec{r}) = 0 \tag{6e}$$

$\vec{K}(\vec{r})$ and $\eta(\vec{r})$ are, respectively, the surface densities of current and charge on a perfect conductor; $\vec{A}_t(\vec{r})$ and $\vec{A}_n(\vec{r})$ are the tangential and normal components of $\vec{A}(\vec{r})$ on the boundary specified by the external normal with respect to the region indicated by the subscript.

When (4) and (5) are substituted in the Maxwell equations,

$$\nabla \times \vec{B}(\vec{r}) = \mu_o[\vec{J}(\vec{r}) + j\omega\epsilon_o\vec{E}(\vec{r})] \tag{7a}$$

$$\nabla \cdot \vec{E}(\vec{r}) = \rho(\vec{r})/\epsilon_o \tag{7b}$$

these become

$$(\nabla^2 + k^2)\vec{A}(\vec{r}) = -\mu_o\vec{J}(\vec{r}) + \nabla\chi(\vec{r}) \tag{8}$$

$$(\nabla^2 + k^2)\vec{\varphi}(\vec{r}) = -\epsilon_o^{-1}\rho(\vec{r}) - j\omega\chi(\vec{r}) \tag{9}$$

where

$$\chi(\vec{r}) = \nabla \cdot \vec{A}(\vec{r}) + j\frac{k^2}{\omega}\varphi(\vec{r}). \tag{10}$$

Since only the curl of $\vec{A}(\vec{r})$ has been specified in (4), $\nabla \cdot \vec{A}(\vec{r})$ may be so defined that $\chi(\vec{r}) = 0$ in (10) and hence the variables $\vec{A}(\vec{r})$ and $\varphi(\vec{r})$ separated in (8) and (9). The gauge equation obtained from (10) with $\chi(\vec{r}) = 0$ is known as the Lorentz condition. The equations (8) and (9) may be solved with (6a-d) to obtain the general Helmholtz integrals:

$$\vec{A}(\vec{r}) = \mu_o\int_V \vec{J}(\vec{r})G(|\vec{r}-\vec{r}'|)dV' + \mu_o\int_S \vec{K}(\vec{r})G(|\vec{r}-\vec{r}'|)dS' \tag{11}$$

$$\varphi(\vec{r}) = \epsilon_o^{-1}\int_V \rho(\vec{r})G(|\vec{r}-\vec{r}'|)dV' + \epsilon_o^{-1}\int_S \eta(\vec{r})G(|\vec{r}-\vec{r}'|)dS' \tag{12}$$

where

$$G(|\vec{r}-\vec{r}'|) = \frac{e^{-jk|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|} \tag{13}$$

is the Green's function. The vector $\vec{r}$ is drawn from an arbitrary origin, e.g. the center of the tube, to the point where $\vec{A}(\vec{r})$ and $\varphi(\vec{r})$ are calculated; the vector $\vec{r}'$ extends from the same origin to the location of the volume element dV' or the surface element dS'. In the case at hand, $\vec{J}(\vec{r})$ and $\rho(\vec{r})$ are zero since currents and charges are confined to the surfaces of the thin perfectly conducting tube. Moreover, with rotational symmetry, $\vec{K}(\vec{r}) = \hat{z}K_z(z) = \hat{z}I_z(z)/2\pi a$, $\eta(z) = q(z)/2\pi a$ where $I_z(z)$ and $q(z)$ are the total axial current and the charge per unit length.

It follows that $dS' = a\, d\theta'\, dz'$ and,

$$\vec{A}(\vec{r}) = \hat{z}A_z(\rho,z) = \frac{\mu_o}{2\pi}\int_{-h}^{h} I_z(z')dz' \int_{-\pi}^{\pi} G(|\vec{r}-\vec{r}'|)d\theta' \tag{14}$$

$$\varphi(\vec{r}) = \varphi(\rho,z) = \frac{1}{2\pi\epsilon_o}\int_{-h}^{h} q(z')dz' \int_{-\pi}^{\pi} G(|\vec{r}-\vec{r}'|)d\theta' \tag{15}$$

The corresponding specialized form of the Lorentz condition is

$$\frac{\partial A_z(\rho,z)}{\partial z} + j\,\frac{k^2}{\omega}\,\varphi(\rho,z) = 0. \tag{16}$$

MODIFICATION FOR DOUBLY-CONNECTED REGION

Owing to the discontinuity in $E_z(\vec{r})$ at $\rho = a$, $z = 0$, (5a) is certainly meaningless in the tubular antenna - at least at its center. This is illustrated by the application of Stokes' theorem to (5a) with (5b) to give:

$$N = \int_{S_{cap}} \hat{n} \cdot \nabla \times [\vec{E}(\vec{r}) + j\omega\vec{A}(\vec{r})]dS = \oint_{s} [\vec{E}(\vec{r}) + j\omega\vec{A}(\vec{r})]\cdot d\vec{s} = \oint d\,\varphi(\vec{r}) \tag{17}$$

If the cap surface S_{cap} which is bounded by the closed contour s crosses the tubular antenna, N in (17) does not vanish as it does when the antenna is not crossed. Specifically, if the contour s is chosen up along the outer surface $\rho = a$ of the tube and down along the inner surface at $\rho = a_i$ so that S_{cap} is cut by the antenna, the conditions (1), (2), and (6a) applied to the middle integral in (17) yield $N = V_o^e$ and not $N = 0$ as required by (5a). This difficulty arises from the fact that the infinite region bounded by the tubular antenna is doubly connected. It can be avoided if the region is made singly connected by means of an actual or ficticious surface across the cylindrical region within the tube across which $\varphi(\vec{r})$ is required to be discontinuous by V_o^e. This surface is conveniently chosen at $z = 0$ so that

$$\lim_{z\to 0} [\varphi(\rho,z) - \varphi(\rho,-z)] = V_o^e \quad \text{with } \rho \le a_i. \tag{18}$$

In the arrangements in Figs. 1a and 2a,b there is a perfectly conducting wall across the tube near $z = 0$. This wall also exists in the image model in Fig. 1b, where the theorem of images requires that if $\varphi = \varphi_0$ on the bottom of the upper half of the tube, it must be $\varphi = -\varphi_0$ on the top of the lower half of the tube. There is no actual wall across the tube in Fig. 2c but, from symmetry, a perfectly conducting disk could be placed across the inside of the tube at $z = 0$ without affecting anything. With or without this disk, the scalar potential must be defined to have the form given in (18).

In order to preserve (16) everywhere except inside the tubular conductor (18) requires that (6d) be replaced by

$$\lim_{z\to 0}\left[\frac{\partial A_z(\rho,z)}{\partial z} - \frac{\partial A_z(\rho,-z)}{\partial z}\right] = -j\,\frac{k^2}{\omega}\,V_0^e; \qquad \rho \le a_1. \tag{19}$$

It then follows, for later use, that

$$\lim_{z\to 0}\left[\frac{\partial^2 A_z(\rho,z)}{\partial z^2} - \frac{\partial^2 A_z(\rho,-z)}{\partial z^2}\right] = -j\,\frac{k^2}{\omega}\,V_0^e\delta(z)H(a_1-\rho) \tag{20}$$

where $H(a_1-\rho)$ is the heaviside function defined by

$$H(x) = \begin{cases} 1, & x \ge 0 \\ 0, & x < 0 \end{cases}. \tag{21}$$

The discontinuity (18) in the scalar potential combined with the discontinuity (19) in the normal derivative of the normal component of the vector potential, maintains the validity of the Lorentz condition everywhere except in the walls of the tube itself and on the surface $z = 0$, $\rho \le a_1$.

THE MODIFIED DIFFERENTIAL EQUATION FOR THE VECTOR POTENTIAL

With $\chi(\vec{r}) = 0$, the differential equation (8) for $\vec{A}(\vec{r}) = \hat{z}A_z(\vec{r})$ reduces to

$$\nabla^2 A_z(\vec{r}) + k^2 A_z(\vec{r}) = -\mu_o J_z(\vec{r}). \tag{22}$$

But this equation is not correct for the modified vector potential that satisfies (19) instead of (6d). Owing to the discontinuity (20) in $\partial^2 A_z(\vec{r})/\partial z^2$ on the disk $z = 0$, $\rho \leq a_1$ which is included in $\nabla^2 A_z(\vec{r})$, the right side of (22) must be augmented by this same discontinuity. Thus,

$$\nabla^2 A_z(\vec{r}) + k^2 A_z(\vec{r}) = -\mu_o J_z(\vec{r}) - j\frac{k^2}{\omega} V_o^e \delta(z) H(a_1-\rho). \tag{23}$$

The Helmholtz integral for this equation due to the currents $I_z(z) = 2\pi a K_z(z)$ in the tube is

$$A_z(\vec{r}) = \frac{\mu_o}{2\pi} \int_{-h}^{h} I(z')dz' \int G(|\vec{r}-\vec{r}\,'|)d\theta' + j\frac{k^2}{\omega} V_o^e \int_0^{a_1} \rho' d\rho' \int_{-\pi}^{\pi} G(|\vec{r}-\vec{r}\,'|)d\theta'. \tag{24}$$

This differs from the usual form (11) by the addition of the second integral in (24) which is evaluated over the surface of the disk $z' = 0$, $\rho' \leq a_1$.

The vector potential at $\rho = a$ and z on the surface of the antenna is obtained from (24) in the explicit form

$$A_z(a,z) = \frac{\mu_o}{4\pi} \int_{-h}^{h} I(z')\mathcal{K}(z,z')dz' + j\frac{k}{\omega}\frac{2V_o^e}{4\pi} \int_0^{a_1} \int_{-\pi}^{\pi} M(a,z;\rho',\theta')\rho' d\rho' d\theta' \tag{25}$$

where

$$\mathcal{K}(z,z') = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{-jkR(\theta)}}{R(\theta)} d\theta; \qquad R(\theta) = \sqrt{(z-z')^2 + (2a \sin\tfrac{\theta}{2})^2} \tag{26}$$

and

$$M(a,z;\rho',\theta') = \frac{e^{-jkR(\rho',\theta')}}{R(\rho',\theta')}; \qquad R(\rho',\theta') = \sqrt{z^2+a^2+\rho'^2-2a\rho'\cos\theta'} \tag{27}$$

THE INTEGRAL EQUATION FOR THE CURRENT

The integral equation for the current in the tubular antenna is obtained from (5b) in the coordinate form,

$$E_z(a,z) = -\frac{\partial}{\partial z}\varphi(a,z) - j\omega A_z(a,z) \tag{28}$$

together with (16) and (1). Thus, at $\rho = a$, $-h \leq z \leq h$,

$$\left(\frac{\partial^2}{\partial z^2} + k^2\right)A_z(a,z) = -j\,\frac{k^2}{\omega}\,V_o^e\delta(z). \tag{29}$$

The well-known solution [1,2] is simply

$$A_z(a,z) = \frac{-j}{c}\left[C_1\cos kz + \frac{1}{2}V_o\sin k|z|\right] \tag{30}$$

where C_1 is to be determined from the condition $I_z(h) = 0$. If this solution is combined with (25), the resulting integral equation for the total current in the antenna is

$$\int_{-h}^{h} I_z(z')\mathcal{K}(z,z')dz' = \frac{-j4\pi}{\zeta_o}\left\{C_1\cos kz + \frac{1}{2}V_o[\sin k|z| + D_o(a,z)]\right\} \tag{31}$$

where

$$D_o(a,z) = \frac{k}{2\pi}\int_0^a\int_{-\pi}^{\pi} M(a,z;\rho',\theta')\,\rho'd\rho'd\theta'. \tag{32}$$

This is the integral equation for the current in a tubular antenna when center driven by a delta-function generator at $z = 0$ on the outside surface.

If the diameter of the antenna is small so that $ka << 1$, $D_o(a,z)$ is negligible. On the other hand, when $ka \sim 1$, $D_o(a,z)$ contributes very significantly to the right side of (25), and this contribution decreases outward from $z = 0$ in a manner that approaches $e^{-jk|z|}/|z|$ when $z^2 >> a^2$. It follows that the current near the driving point and, hence, the admittance of an electrically thick tubular antenna that is driven by an emf maintained in a narrow circumferential zone on the <u>outside</u> surface only, depends greatly on the presence of the term $D_o(a,z)$ in the integral equation (31). If this term is omitted, the equation implies that the driving voltage is maintained across a circumferential gap on the inside and outside surfaces of the tube. In this case the total current and the associated admittance include contributions from waveguide modes excited directly by the generator. These are absent in the structures illustrated in Figs. 1a,b and 2a,b,c.

Since it has been shown [7] that iterative procedures for solving the integral equation (31) even without the term $D_0(a,z)$ are not satisfactory when ka is not quite small, a numerical method of solution is indicated. The results of such an evaluation and corresponding measurements on electrically thick antennas will be presented in subsequent papers.

CONCLUSION

The integral equation for a tubular antenna center driven by a localized circumferential source on the outside surface only has been derived. It differs from the conventional equation (which implies a generator that excites both inside and outside surfaces) by an additional term that is negligible when $ka \ll 1$ but is of major importance in determining the current and the admittance of an electrically thick tubular antenna.

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