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U. S. ARMY RESEARCH OFFICE (DURHAM)

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FINAL REPORT

May 15, 1966

Prepared by

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Scope of Project

To conduct a basic theoretical research program in stress waves and penetration mechanics, with particular emphasis on armor plate. To assist the experimental program in this field being conducted at the Arsenals by parallel theoretical investigations.

Project duration - from 1 January 1963 to 15 May 1966.

The technical work of this project has been reported by presentation at professional meetings, through publication in scientific journals, supported by reprints submitted to AROD, as far as is available, and by Interim Technical Reports. While recognizing the greater usefulness of journal articles, the purpose of the Interim Reports has been to make the results available much sooner than reprints of publications. At times the lag has been reduced by as much as a year thereby. A second purpose of the technical reports has been to provide a much fuller presentation than is possible in a paper and as a repository for data and details.

The contents of this final report include:

1) list of output documents,
2) general summary and review of project activities,
3) progress of last 6 month period, and
4) technical work of significance not previously reported.

List of Publications

1. "A Viscous Model for Plug Formation in Plates" by A. Pytel and N. Davids, J. Franklin Inst. 276, No. 5, Nov. 63, pp. 394 - 406

2. "Transient Analysis of Oblique Impact on Plates" by N. Davids and W. Lawhead, J. Mechanics and Physics of Solids, 1965, Vol. 13, pp. 199-212.

3. "A Penetration Method for Determining Impact Yield Strength" by N. Davids, R. Minnich and J. Sliney, Proc. VII Hypervelocity Impact Symposium, Tampa, pp. 261-297.

4. "Couple-Stress Effects on Stress Concentration Around a Cylindrical Inclusion" by Y. Weitsman, J. Appl. Mechanics, Vol. 32, Ser. E., No. 2, June 1965, pp. 424-428.

5. "Direct Numerical Analysis Method for Cylindrical and Spherical Elastic Waves" by P. Mehta and N. Davids, Jrl. Amer. Inst. Aerosp. Sc., Vol. 4, 1, Jan. 1966, pp. 112-117.

6. "Penetration Experiments with Fiberglass Reinforced Plastics" by B.P. Gupta and N. Davids, to appear shortly in the J. Exper. Mechs.

7. "Elastic Waves in Projectiles" by N. Davids, B.P. Gupta, H. R. Minnich, to appear in Marin Anniversary Volume, Toronto University Press.

8. "Elastic Stress Waves in Multilayered Cylindrical and Spherical Shells by a Direct Computational Analysis" by P.K.Mehta and N. Davids, The Indian Society of Theoretical and Applied Mechanics, 10th Annual Congress, Madras,India, Dec.20-24,1965.

[redacted]

List of Technical Reports

1. "Transient Analysis of Oblique Impact" by W. Lawhead and N. Davids, issued March 1, 1963.

2. "Experimental Investigation of Penetration in Fiberglass Reinforced Plastics" by B.P. Gupta and N. Davids, issued July 1, 1964.

3. "Plug Formation in Plates" by H.R. Minnich and N. Davids, issued September 1, 1964.

List of Theses

1. A. Pytel, Ph.D., Viscous Theory for Plug Formation.
2. W. Lawhead, M.S., Oblique Impact.
3. B. P. Gupta, M.S., Penetration of Fiberglass.
4. L. A. Grieb, M.S., Hertz Contact Dynamics.
5. M. L. Wenner, M.S., Viscoelastic Waves and Viscoplastic Impact, The Pennsylvania State University, March 1965.
6. H. Minnich, M.S., Plug Formation in Plates.
7. P. K. Mehta, Ph.D., Spherical Waves.

Other Investigators

Y. Weitsman - (Couple Stresses), paper no. 4.

Presentations at Scientific Meetings

1. Seventh Hypervelocity Impact Symposium, Tampa, Nov. 17-19, 1964, presentation of paper no. 3 by N. Davids.

2. S.E.S.A. Spring Meeting, Detroit, May 4,5, 1966, presentation of paper no. 6, by N. Davids.

3. Invited presentation of plug formation results at the Ballistic Research Labs, March 10, 1964.

4. Invited presentation of computer analysis methods, University of Texas Colloquium Series, March 2, 1966.

Other Visits

To AROD - November 16, 1964.

AMRA (Watertown) - April 2, 1963, October 22, 1963, April 9, 1964, September 11, 1964, May 21, 1965.

Frankford Arsenal- April [illegible], 1963.

Brief Review of Project

1. Oblique Impact

One of the phases of the research on this project has been the analytical study of transient stresses in a plate due to step impacts at a point on the surface. The point impact is a fair idealization representing a pointed projectile, and is applicable to other types of disturbances as well. Considerable work on this problem was carried out on an earlier AROD project for normal and shear impacts, using a mathematical procedure which has come to be known as "Caignard's Method", used extensively in geophysical analyses. The present effort continued the study and brought it to conclusion by working out the principal stresses induced for impacts at 30° and 60° angles of incidence. The results are reported in paper no. 1 and report no. 1, and a computer program is available for any additional numerical data required.

This method of analysis is valuable in that it follows both the dilatation and distortion wavefronts or pulses across the plates as well as their interaction due to reflections across the back face. This provides a direct understanding, unlike other methods based on "normal modes" or vibrational considerations. Further, it is able to assess realistically magnitudes of the induced tensile stresses, which are very sensitive to the reflected waves. For this reason, analyses based on semi-infinite media or on normal modes lead to considerable errors on this estimation. On the other hand, Caignard's approach is not realistic for studying actual penetration dynamics (for which it was not intended) and is limited to brittle-type materials which are elastic until failure.

Since penetration dynamics was considered the main theme of this project, the following further phases were studied.

2. Plug Formation

This phase was concerned with an area of penetration dynamics referred to as "plug formation". This is part of the more general area of plastic impact, where the impacting materials undergo a type of viscous flow. It may be considered as intermediate in impact velocity range between the elastic and hydrodynamic deformation theories, and is characterized by a paucity of results on account of the uncertainties in the constants of the material and complexity of the analyses heretofore presented in the literature. A reasonable assumption made by Pytel on a previously sponsored AROD project (also by some other literature) has been that of linear viscosity, so that the methods of the equation of diffusion or heat conduction were applicable. Although it becomes possible to solve this equation explicitly with the given boundary conditions, the old solutions failed to predict the time when motion is stopped or the final shape of the target.

The need for comparing results with these prime physical observables led to to undertake a new investigation of this problem. At this time specimens of deformed target plates of ½" rolled armor steel became available from Watertown Arsenal Laboratories, etched by Oberhoffer's reagent. These yielded values of deformation under the projected impact by direct measurement. Even though relatively few such specimens were available, they constituted an opportunity for an improved theoretical approach to the problem. Following suggestions in the literature which assumes the predominant forces in the deformation of a plate under impact to consist of frictional forces, the new feature is to suppose there is a threshold stress level or impact yield constant at which flow starts, and below which the material is either elastic or, more simply, rigid. This behavior prevents the material from flowing indefinitely.

The analysis of this non-linear problem is described in Technical Report no. 3, presentation no. 1 and publication no. 3. Values for impact yield constants of 210 x 10^3 psi were found for mild steel and deformations agreed very well with those shown on the etched specimens.

3. Penetration in Fiberglass Reinforced Plastics (FRP)

This investigation was undertaken to determine the basic knowledge of impact behavior of a class of materials (FRP) and was especially spurred on by the needs of the military for light personal armor. A series of 70 penetration tests were completed of small caliber projectiles through laminated fiberglass plates. The material, in proper combinations, was found to give a substantial saving of weight (14% to 30%) over steel for the same stopping power. A firing range was built and projectile velocities, both incident and residual, were measured at 6 stations by aluminum foils, in conjunction with a Polaroid camera and an externally-triggered oscilloscope. (Publication no. 6)

4. Couple-Stresses

The theory of couple-stresses, originally developed by Mindlin, was extended to a cylindrical inclusion of one material imbedded in another medium under uniaxial tension, and has calculated the stress-concentration factors at the interface. These come out to be about 15%-20% higher than that given by the classical theory of elasticity. (Publication no. 4).

5. Other Completed Work

Our work on viscoelastic waves and further studies in penetration dynamics, which have not previously been reported, are covered in detail further below in this report.

6. Recent Work (last 6 months) Not Completed

One of the really large problems in dynamics is the analysis of stress wave propagation in multi-dimensional bodies. The analytical solutions to date apply only to very simple geometries. Our success with the use of discrete approaches for the various plane and spherical geometries has led us to attempt to apply this method to the aforementioned problems. To date, some test cases have been solved.

The same general approach is also applicable to a class of statical plasticity problems. To date we have validated this method for some known problems in the literature.

The analysis of flexural travelling waves has produced solutions for step and ramp moment inputs, but step-shear input is still unsolved.

Viscoelastic Waves with Reflection

for Longitudinal Impact (Ch. I-IV)

by

M. L. Wenner

Armor Penetration Dynamics (CH. V-VIII)

by

R. Minnich

NOMENCLATURE

VISCOELASTIC WAVES

ρ	Mass density of bar material
x	Longitudinal coordinate
t	Time
$v(x,t)$	Velocity in x-direction
A	Cross-sectional area of bar
$\sigma(x,t)$	Tensile stress in x-direction
$\epsilon(x,t)$	Tensile strain in x-direction
p_o	Pressure at origin
c_g	Glassy (fastest) wave speed
t_p	Time duration of stress input
$E(t)$	Viscoelastic relaxation modulus
E	Spring constant-standard linear solid
E'	Spring constant-standard linear solid
$1/\mu$	Dashpot viscosity coefficient-standard linear solid
i_m	Number of bar elements
k_m	Number of time elements
dx	Change in x-coordinate
dt	Change in time
$E(t = 0)$	Glassy state relaxation modulus
$d\epsilon$	Change in strain
L	Length of bar
dv	Change in velocity

VISCOPLASTIC IMPACT

X	Longitudinal Coordinate
T	Time
$\sigma(X,T)$	Compressive stress in X-direction
$\epsilon(X,T)$	Compressive strain in X-direction
$V(X,T)$	Particle velocity
ρ	Mass density of bar material
σ_o	Static yield stress (compressive)
A	Cross-sectional area of bar
D	Material constant
p	Overstress exponent
G	Mass of striking body
sl_i	Slip factor
L	Length of bar
V_o	Initial velocity of striker
x	Dimensionless coordinate ($= X/L$)
t	Dimensionless time($= \sigma_o T/\rho D L^2$)
s	Dimensionless stress ($= \sigma/\sigma_o$)
v	Dimensionless velocity($= V/DL$)
v_o	Dimensionless impact velocity ($= V_o/DL$)
η	Dimensionless strain ($= \sigma_o \epsilon/\rho D^2 L^2$)
k	Dimensionless mass factor($= G/\rho AL$)
η_f	The quantity $\frac{1}{2} k v_o^2$
i_m	Number of bar elements
k_m	Number of time elements

t_o	Time at which unloading begins
t_1	Time at which all motion stops
dv	Change in dimensionless velocity
$d\eta$	Change in dimensionless strain
ds	Change in dimensionless stress
dt	Change in dimensionless time
$\leftarrow$	"is replaced by"

Chapter I

INTRODUCTION

1.1 One-Dimensional Impact

Since the problem of one-dimensional elastic stress waves in a slender bar was solved by Newton in 1685, it has been recognized that the one-dimensional problem is quite valuable for developing and checking further analyses on more complicated cases. As our store of knowledge increases, however, we find that the material representations which are most realistic are usually those which also present the most difficult mathematical obstacles. Thus, even the simplest cases of dynamic problems are difficult to treat analytically.

It is proposed to analyze in this thesis two problems of longitudinal impact of slender bars. The ultimate concern here is to obtain solutions for realistic materials, since only in this way may it be expected that a contribution of a quantitative or of an engineering nature may be made.

The problems to be analyzed are stress waves in a viscoelastic material and plastic impact of a viscoplastic material. The material representation for the viscoelastic material is one which can be directly obtained from experiments, namely, the relaxation modulus in tension. The conventional method of describing a material by a spring-dashpot model will be discarded. The law used herein to describe the viscoplastic material will be a nonlinear stress-strain rate law.

In the following, two problems will be explained concurrently since many of their aspects are quite similar. When there is a dissimilarity, separate treatments will be used.

1.2 Introduction--Viscoelastic Waves

With the increasing use of viscoelastic materials in such diverse applications as insulations, binders for solid fuel rocket propellants, and structures, there has occurred an increase in the number of theoretical investigations into the subject. These have been hampered, particularly in dynamic analyses, by formidable mathematical difficulties, which are normally overcome by representing the material by means of spring-dashpot models. These models simplify the analytical problems considerably, but at the expense of inadequately describing the materials. In fact, the commonly used models consisting of two or three elements exhibit nearly all of the change in a particular viscoelastic function in a single logarithmic decade of time, whereas experiments show that actual viscoelastic materials require at least seven to fourteen logarithmic decades of time to describe their full range.

The problem of one-dimensional wave propagation in viscoelastic bars is among the simplest of dynamic problems to state, but no analytic solution has yet been obtained which does not depend in one way or another upon a material representation of springs and dashpots. A general solution to this problem would determine stress (or strain) in a bar as a function of time and position, using only the experimentally obtained material data and the boundary conditions on each end of the bar. It is one of the purposes of this report to develop an analysis which yields such a solution.

In analyzing the problem of one-dimensional wave propagation in viscoelastic media the following assumptions are made:

a) The bar is assumed to be slender enough that we have plane waves. Hence, lateral effects are disregarded, and the stress is uniform on any cross section.

b) The displacements are infinitesimal.

c) The bar consists of a linear viscoelastic material and thus obeys Boltzmann's superposition principle. This principle, which forms the basis of all linear viscoelastic analysis, will be stated in Section 2.2 below.

d) The bar is assumed to have a uniform cross section.

A complete solution to this problem must show the response of a viscoelastic bar subjected to impulsive loading as a function of time and position. It will be readily adaptable to free or fixed ends, and will show the reflection of the waves from each end. It is hoped that this will provide a direct means by which to compare theoretical results with experimental results.

1.3 Introduction-Viscoplastic Impact

In order to adequately describe the pheomenon of deformation of materials in which plastic deformation occurs at a high rate, it has been found necessary to incorporate a constitutive law which takes into account the strain rate. A simple type of experiment is the axial impact of a prismatic bar by a finite mass.

One of the laws proposed for this problem is a power law relation between strain rate and dynamic overstress. The advantage of choosing this law is that some material data are available from test results and the law has shown to be effective in predicting deformations of cantilever beams even when the physical constants had to be crudely estimated. This past success should be a stimulus to further investigation in order to determine if the law in question adequately describes different problems. If this is the case, the theory could be used together with experiments, in order to determine the physical constants more accurately, thereby producing solutions for other problems.

The following assumptions and conditions are imposed on the analysis:

a) Uniaxial stress is assumed, and no lateral effects are admitted.

b) The material is rigid viscoplastic; that is, no strain increment occurs at a point unless the static yield stress there is exceeded. This is a realistic assumption if the plastic deformation at a point is much larger than the elastic deformation.

c) The striking mass moves parallel to the axis of the bar and after impact, it sticks to the end of the bar.

d) The constitutive law is a power relation between the strain rate and the dynamic overstress. This law will be stated explicitly in section 2.3 below.

The law to be used in this analysis is a more general one than has been used previously in the problem of longitudinal impact in viscoplastic rods. These results should enable researchers to more accurately evaluate the worth of this particular constitutive law.

1.4 Past Work--Viscoelastic Waves

Most of the analytic investigations accomplished to date have used Maxwell solids, Voigt solids, or a combination of the two. The Maxwell model, proposed in 1890, is a spring in series with a dashpot, while the Voigt model (1892) is a spring in parallel with a dashpot. These models have been used not only for stress wave problems but also for vibrations and quasi-static problems.

Hillier [1]* has used the Maxwell model, Voigt model, and two models using three elements each as material representations for the problem of longitudinal sinusoidal waves in a bar. This is of course a vibrations solution and no transient effects are considered.

The transient problem was treated by Lee and Morrison [2], also using simple model representations. Laplace transform

*Numbers in brackets refer to references listed in Bibliography.

techniques were used in order to solve the boundary value problems.

Kolsky [3] performed experiments on a viscoelastic material by subjecting a rod to explosives at one end and recording the response of the other end. An analytic solution was obtained for the same problem by using Fourier analysis. The theoretical results compared favorably with the experimental results.

A partial solution to the problem is given by Bland [4]. In this analysis a wave front expansion is used which yields a long time solution. We desire, however, to find a complete solution whenever possible.

Morrison [5] has given integral solutions to several model representations, including the standard linear solid which will be described below. Laplace transform methods were used, with a numerical inversion. Our computer solution will be applied to this model and the results compared to Morrison's.

The most comprehensive solution to this problem to date has been given by Arenz [6]. In this study the material is represented by a Kelvin model consisting of a glassy spring plus n Voigt elements in series. Arenz uses this model with nine Voigt elements to represent the real part of the complex shear compliance of a polyurethane material. Transform techriques are used with two different methods of inversion. These methods were formulated for other applications by Schapery [7,8].

Several review articles have recently been published concerning dynamic phenomena in viscoelastic materials. More references to these problems may be found in the papers by Zverev [9], Lee [10], and Hopkins [11].

1.5 Past Work--Viscoplastic Impact

The constitutive law, a form of which we shall use in this analysis, was first proposed by Hohenemser and Prager [12]. It is for a solid of the Bingham type in which elastic deformations are neglected and in which the strain rate is related to the amount by which the stress at a point exceeds the static yield stress. Several experimenters [13, 14, 15] have shown that this type of law satisfactorily describes the plastic impact of beams and other structures.

Lee and Wolf [16] made an investigation of longitudinal impact on a rigid-plastic bar in which the material was considered to be linearly strain-hardening but rate independent. In using their solution to analyze tests made by Habib [17], Lee and Wolf showed that rate dependence effects may become of importance if nonuniform strain distribution resulting from plastic wave propagation is ignored.

The linear form of the rigid-viscoplastic law has formed the basis of several studies. Sokolovskii [18] used it to solve several problems of plane shear waves in semi-infinite media, and Ting and Symonds [19] used it to analyze the problem of longitudinal impact of viscoplastic bars. The same two authors also give [20] some approximate methods for the nonlinear case and compare these results with both the linear case and with the simplest rigid-plastic, rate independent problem. They conclude that for the case of high impact velocity of a large impact mass the assumption of uniform final strain is reasonable. We shall have some comments on this assumption and shall show its limitations.

Some review articles which will provide more references are Lee's review [10] which treats elastic-plastic problems, and an article by Cristescu [21] which describes European contributions.

Chapter II

MATHFMATICAL FORMULATION

2.1 Pertinent Mathematical Relations

The standard method by which physical problems of this sort are solved is to write down the appropriate mathematical relations, combine them, and then solve the resulting boundary value problem. The approach used here also depends, of course, on a certain combination of physical laws, geometrical relations and definitions. In particular, what we need are: definition of strain, a geometrical relation between strain and velocity, the impulse-momentum law, and a law relating stress to some other parameters. These form the basis for each of the two analyses given.

2.2 Mathematical Formulation--Viscoelastic Waves

The physical conditions governing the one-dimensional viscoelastic wave problem is shown in Figure 1. The origin is located at the left end of a bar of length L. The right end may either be fixed or free. The left end is subjected to an impulsive stress loading at time zero; this stress may remain acting for any desired amount of time. Thus, the boundary condition at the origin is

$$\sigma(0,t) = -p_o[H(t) - H(t_p)] \tag{2.1}$$

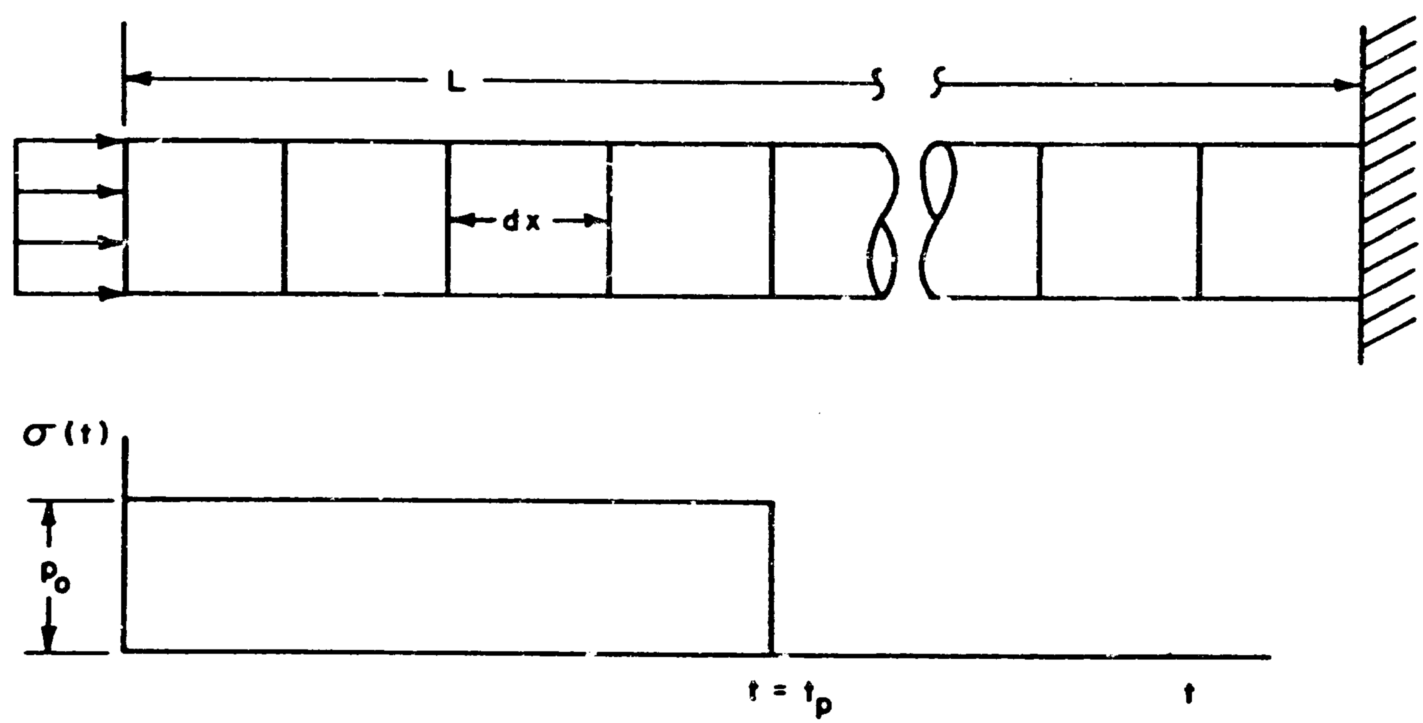


FIG. I BOUNDARY CONDITIONS FOR STRESS IMPACT ON VISCOELASTIC BAR WITH FIXED END where $H(t)$ is the Heaviside step function and t_p is the time at which the stress is removed. The boundary condition at the right end is

$$v(L,t) = 0 \tag{2.2}$$

if the right end is fixed, or

$$\sigma(L,t) = 0 \tag{2.3}$$

if the end is free.

The equation of motion governing the one-dimensional case is

$$\partial\sigma/\partial x = \rho\, \partial^2 u/\partial t^2 \; . \tag{2.4}$$

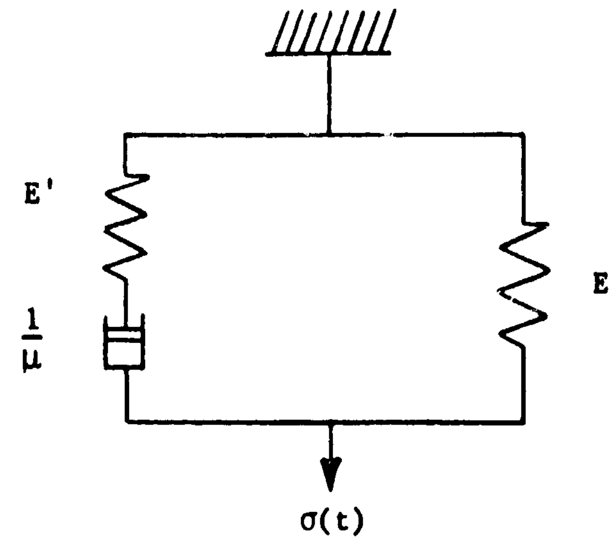
and the definition of strain is

$$\epsilon = \partial u/\partial x \tag{2.5}$$

The above is standard analysis, and may be found, for instance, in Timoshenko [27].

We now require a stress strain law for the viscoelastic material. Normally, this is obtained from a model representation. For instance, the stress strain law for the standard linear solid shown below is

$$(1/E')d\sigma/dt + \mu\sigma = (1 + E/E')d\epsilon/dt + E\mu\epsilon . \tag{2.6}$$



The standard procedure is to insert equations (2.6) and (2.7) into (2.4) and solve the resulting partial differential equation subject to the boundary conditions, usually by transform techniques.

However, the general stress strain law for a linear viscoelastic material is given by the Boltzmann superposition principle, which is, in fact, the definition of a linear viscoelastic material. This is an integral equation and as given by several authors [4, 22, 23] may take either of two equivalent forms. One representation is

$$\sigma(t) = \epsilon(t)E(t = 0) - \int_0^t \epsilon(\tau) \frac{dE(t-\tau)}{d\tau} d\tau. \qquad (2.7)$$

This is a Riemann-Stieltjes integral, and when we apply integration by parts,

$$\sigma(t) = \int_0^t E(t-\tau) \frac{d\epsilon(\tau)}{d\tau} d\tau. \qquad (2.8)$$

There are corresponding integral relations giving the strain when the stress history is known. In (2.7) and (2.8) the function E(t) is the relaxation modulus in tension and it is the stress/strain ratio as determined from a relaxation test. In this test the material is suddenly subjected to a strain which is subsequently held constant, and the stress required to maintain this deformation is measured as a function of time.

Inserting one of equations (2.7) or (2.8) with (2.5) into (2.4) gives an integro-differential equation which has not yet been solved mathematically. Instead of attacking this equation, we propose to use the Boltzmann superposition principle as the basis of a so-called "computer analysis" to solve the one-dimensional wave problem for any linear viscoelastic material.

2.3 Mathematical Formulation-Viscoplastic Impact

Figure 2 shows a rod of length L, fixed to a rigid wall at the right end and subjected to the uniform impact of a body of mass G on the left end. The origin is located at the left end and the impact occurs at time zero. The boundary conditi at the left end is obtained by applying the impulse-momentum law at X = 0. That is,

$$G\,\partial V(0,T)/\partial T + A\sigma(0,T) = 0. \qquad (2.9)$$

The boundary condition at X = L is

$$V(L,T) = 0. \qquad (2.10)$$

The impulse-momentum equation is

$$\partial\sigma/\partial X = -\rho\,\partial V/\partial T \qquad (2.11)$$

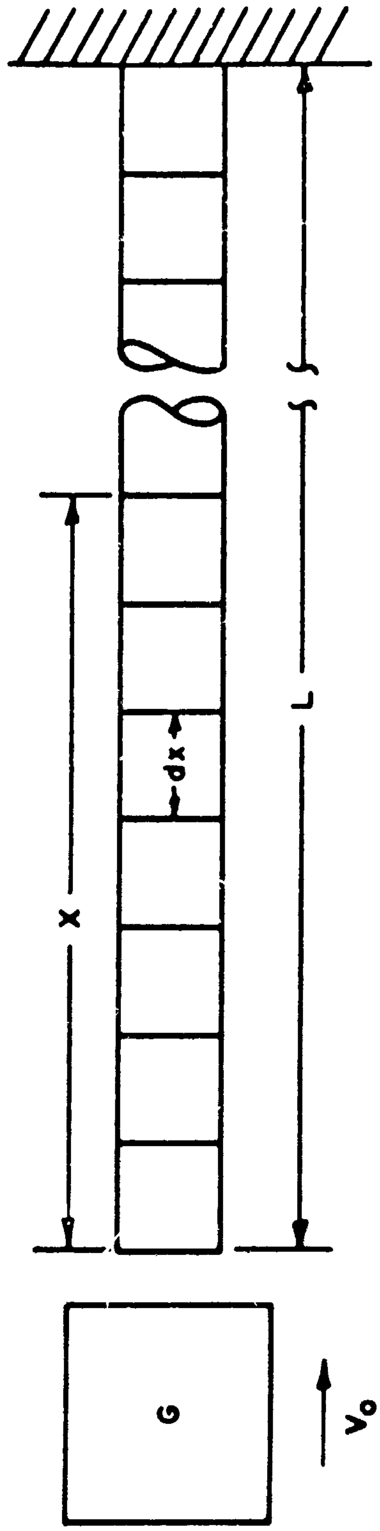


FIG. 2 LONGITUDINAL IMPACT OF MASS ON VISCOPLASTIC BAR

and the equation relating the velocity to the strain (where they are continuous) is

$$\partial \epsilon / \partial T = -\partial V / \partial X. \tag{2.12}$$

The equation relating the strain rate to the dynamic over-stress is

$$\frac{\partial \epsilon}{\partial T} = D\left(\frac{\sigma}{\sigma_o} - 1\right)^p \quad \text{if } |\sigma| > \sigma_o \tag{2.13a}$$

$$\frac{\partial \epsilon}{\partial T} = 0 \quad \text{if } |\sigma| \leq \sigma_o \tag{2.13b}$$

where D and p are material constants. Several consequences of this law should be noticed. First, upon being loaded, a material which obeys equations (2.13) exhibits no deformation until the stress exceeds the static yield stress. Second, when a particular region is unloaded from a stress which is higher than the static yield stress to a stress which is less than the yield stress, that region will move as a rigid body.

Following Ting and Symonds [19] we introduce nondimensional variables in order to simplify the analysis. These nondimensional quantities are recorded in the list of symbols, and using these terms, the equations (2.11), (2.12) and (2.13) become

$$\partial v / \partial t = -\partial s / \partial x \tag{2.14}$$

$$\partial v / \partial x = -\partial \eta / \partial t \tag{2.15}$$

$$\partial\eta/\partial t = (s-1)^p \quad \text{if } |s| > 1 \qquad (2.16a)$$

$$\partial\eta/\partial t = 0 \quad \text{if } |s| \leq 1 \qquad (2.16b)$$

respectively. In the case of $p = 1$ the combination of equations (2.14), (2.15) and (2.16a) yields

$$\partial^2 v/\partial x^2 - \partial v/\partial t = 0 \quad \text{if } |s| > 1 \qquad (2.17)$$

$$\partial^2 s/\partial x^2 - \partial s/\partial t = 0 \quad \text{if } |s| > 1 . \qquad (2.18)$$

This is the one-dimensional heat conduction equation and analytical solutions may be obtained for a number of problems. In a region where $|s| \leq 1$,

$$\partial\eta/\partial t = 0 \qquad (2.19a)$$

$$\partial v/\partial x = 0 \qquad (2.19b)$$

$$\partial^2 s/\partial x^2 = 0. \qquad (2.19c)$$

In nondimensional notation the boundary conditions at the right end becomes

$$v(1,t) = 0. \qquad (2.20)$$

The boundary condition at $x = 0$ which was given by equation (2.9) is

$$k\,\frac{\partial v(0,t)}{\partial t} - \frac{\partial v(0,t)}{\partial x} + 1 = 0 \qquad (2.21)$$

when combined with (2.16a) using $p = 1.0$. In nondimensional notation the initial conditions are

$$v(0,0) = v_o \tag{2.22a}$$

$$v(x,0) = \eta(x,0) = 0 \tag{2.22b}$$

$$s(x,0) = 1 \qquad \text{for } x > 0. \tag{2.22c}$$

The solution of this problem is given by Ting and Symonds [19] with solutions to three similar problems.

It is proposed in this report to give a means by which to solve the viscoplastic impact problem using the nonlinear case of equation (2.13). This will be accomplished by means of a computer analysis using the above relations as a basis. The means by which these relations are written in finite form and the method by which we incorporate them into a computer approach is given in the following chapter.

Chapter III

COMPUTER ANALYSIS

3.1 General Computer Approach

The purpose of this section is to give a brief account of the method by which both the viscoelastic wave problem and the viscoplastic impact problem will be solved. Instead of deriving complicated equations from basic principles and then using the methods of numerical analysis in order to obtain results, we shall write down the physical laws, definitions, and assumptions for a particular problem and use them in finite form. This is called "computer analysis". Some of these relations are given in this section, and the method by which we combine them for computational purposes is illustrated in the following two sections.

This method has several advantages over standard numerical procedures. First, the program is physically meaningful because the physical laws appear explicitly and the program actually follows the physical processes as they occur. This fact makes this method more desirable than the standard procedure in which equations are derived from simple laws and then subjected to finite-difference analysis which usually renders them unrecognizable. Secondly, the prgram is easily adaptable to changes because often only one statement need be modified with no change to the rest of the program. In the use of a computer to solve finite-difference equations which have been derived from the basic principles, the entire logic must usually be changed when one of these basic laws is changed. Thirdly, the resulting program is more efficient because finite methods are used immediately rather than as a means to solve complicated integral or differential equations.

Since we are using a finite process to simulate a continuous process, some further features of the approach must be explained. Unless stated otherwise the following comments apply to both the viscoelastic and the viscoplastic programs.

1. The bar is divided into a finite number of cells and the basic equations are applied to each of these cells; the time variable is periodically changed and the variables such as stress, strain, displacement, and velocity are recorded for each cell and for each time. We consider here only bars of uniform cross sectional area A. The length of a cell is dx, which is related to the element of time dt for the viscoelastic program.

2. It has been found that the order in which the basic equations are stated is of great importance and the order given in our programs is the only one which has yielded a solution. The reason for this fact is that our finite analysis must necessarily do one thing at a time whereas in the actual physical process several changes may occur at one time. The stability of the procedure is directly dependent on the order of the steps.

3. Another factor which affects stability is the cell size and the size of the time increment. Detailed comments on this problem will be found below in the discussion of each problem

4. The program should be constructed in such a manner that minor changes may be incorporated in both the data and the procedure. If possible, a single program should be able to solve several problems. For instance, a program might have a feature by which it may solve a bar with a fixed or free end.

5. The program mechanism should be constructed so that the physical process is as closely simulated as possible. This gives an insight into the problem and may lead to further understanding and study of related problems.

Figure 3 shows the notation by which the cells and stresses are labelled. It should be noted that the stresses are shown in the positive direction for the viscoelastic program, but in the negative direction for the viscoplastic program in which compressive stresses are regarded as positive. In each of the programs the index i runs from 1 to i_m, and the time index k runs from 1 to k_m.

The cores of the two problems are similar and may be outlined as follows.

1. The law relating stress to strain is stated in finite form and the stress in cell i + 1 is calculated and recorded. For the viscoelastic problem this step is a numerical integration while a finite form of equation (2.16a) serves the purpose for the viscoplastic program.

2. The strain increment acting on cell i + 1 may be written in terms of displacement as

$$d\epsilon_{i+1} = (u_{i+1} - u_i)/dx_{i+1} \tag{3.1}$$

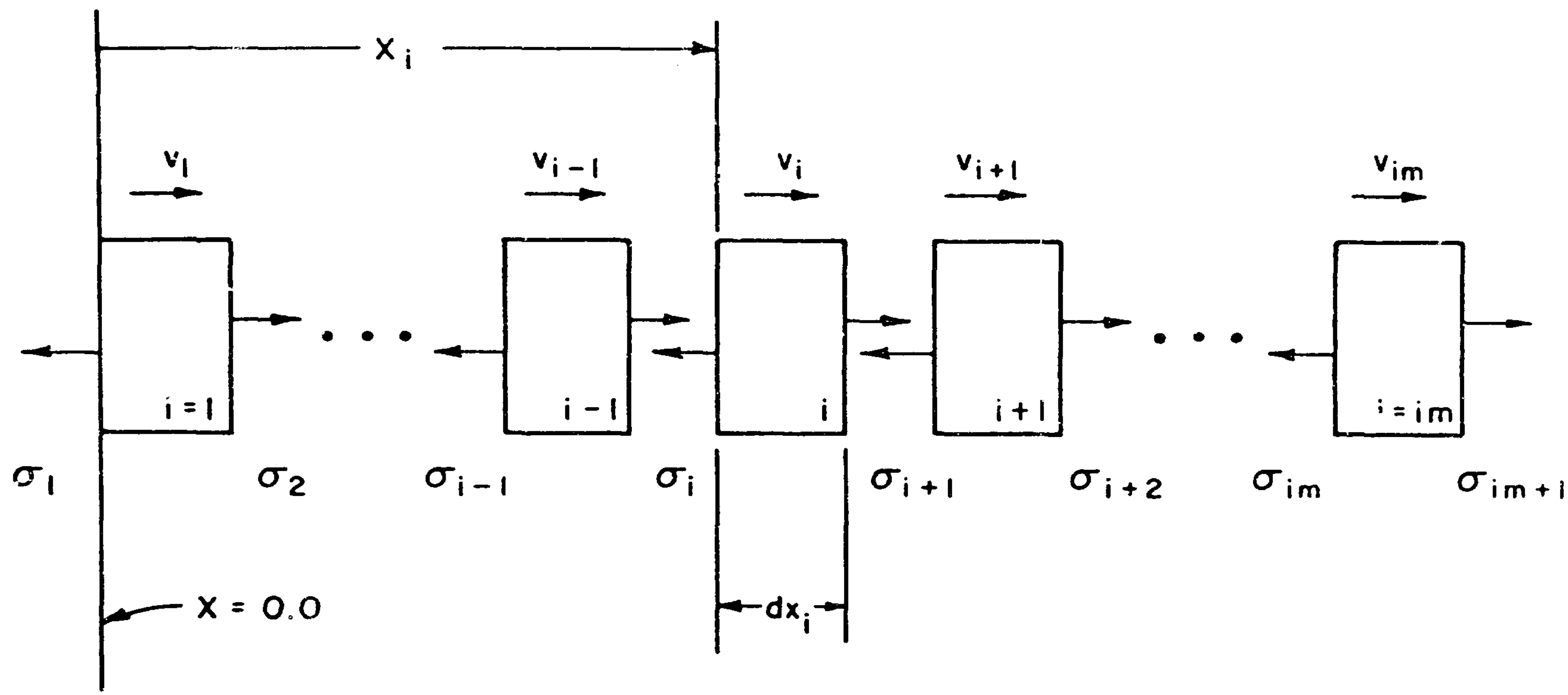


FIG. 3 BAR CELLS AND NOTATION AS USED IN COMPUTER ANALYSIS

where the displacements are given by

$$u_{i+1} = v_{i+1}\, dt \qquad (3.2)$$

and

$$u_i = v_i\, dt. \qquad (3.3)$$

Combining (3.1), (3.2) and (3.3) yields

$$d\epsilon_{i+1} = (v_{i+1} - v_i)dt/dx_{i+1} \qquad (3.4)$$

which is the form we shall use.

3. A cumulation process follows in which the strain in cell i + 1 is replaced by the addition of $d\epsilon_{i+1}$ to the original strain ϵ_{i+1}. This may be written symbolically as

$$\epsilon_{i+1} \leftarrow \epsilon_{i+1} + d\epsilon_{i+1}. \qquad (3.5)$$

The symbol ← is understood to mean "is replaced by".

4. The impulse-momentum law may be written as

$$\sum F\, dt = \text{change in momentum} \qquad (3.6)$$

where the left hand side of (3.6) is the sum of the impulses. We apply this law to cell i by noticing that the impulse on the right end of the cell is σ_{i+1} A dt and the impulse on the left end is σ_i A dt, where A is the cross sectional area of the bar. The change in momentum of the cell is given by the mass times the increment of velocity,

$$M_i \, dv = (\rho \, A \, dx_i) \, dv. \tag{3.7}$$

Using these quantities in (3.6) gives

$$(\rho \, A \, dx_i) \, dv = \sigma_{i+1} \, A \, dt - \sigma_i \, A \, dt$$

or, in the form which we shall use,

$$dv = (\sigma_{i+1} - \sigma_i) \, dt/\rho \, dx_i. \tag{3.8}$$

Note that this gives an increment of velocity in cell i as a result of a net impulsive force acting on that element.

5. In order to calculate the actual velocity of cell i we use the relation

$$v_i \leftarrow v_i + dv. \tag{3.9}$$

This is actually an integration process which recalculates the velocity in cell i.

6. The position along the bar is calculated by another integration process which we write as

$$x_{i+1} = x_i + dx_i. \tag{3.10}$$

These six relations, taken in the given order, form the basis of each of the two solutions. There are only minor revisions in the problems which will be explained further below. Note that two of the relations are mechanical or physical laws (1 and 4), while the others are geometrical definitions or integrations.

In each of the programs, these six relations, (1) to (6) are repeatedly calculated for values of the index ranging from 1 to i_m. Physically, i_m is the number of elements making up the entire length of the bar. This repetition for the index i is then nested in another repetition procedure which increments the time variable by means of the index k. This index runs from 1 to k_m, where k_m is the total number of time elements. It should be noted that all boundary conditions such as stress at x = 0, momentum interchange at x = 0 or a velocity condition at x = L are included in one or both of these steps. Any initial conditions (as on the stress field for the viscoplastic problem) are included before-hand.

The above is a general sketch of the method to be employed. The details of the solution for each of the programs are given in the following sections.

3.2 Computer Theory-Viscoelastic Waves

As previously stated, a spring-dashpot model of two to four elements is not sufficient to describe a realistic viscoelastic material. Although the number of elements in the model could theoretically be increased so that the model would represent any given material very closely, this procedure is not recommended because it leads to a numerical analysis scheme of probhibitive length and complexity. Therefore, we shall ignore models of this sort entirely and shall use the Boltzmann superposition principle.

Instead of writing the integrals cited previously [equations

(2.7) and (2.8)] we shall deduce the proper form of the law directly from the definition of the relaxation modulus and the superposition principle. Recall that the stress at any time in a viscoelastic material due to a constant strain suddenly applied at zero time is given by

$$\sigma(t) = E(t)\,\epsilon_o \tag{3.11}$$

where ϵ_o is the constant strain. If the strain had been applied not at time zero but at time $t = \tau$, the relation would be

$$\sigma(t) = E(t - \tau)\epsilon_o \tag{3.12}$$

since we must use the value of the relaxation modulus whose argument is the elapsed time since the strain was applied. Now let us apply (3.12) to a particular cell, say cell $i + 1$. Then the increment of stress on cell $i + 1$ at time t_k due to an applied strain $d\epsilon_{i+1}$ at time t_{m+1} will be given by

$$d\sigma_{i+1} = d\epsilon_{i+1}(t_{m+1}) \times E(t_k - t_{m+1}). \tag{3.13}$$

In order to calculate the stress on the cell due to a series of successively applied strains the Boltzmann superposition principle is used. This states that we may use (3.13) to compute the quantity $d\sigma_{i+1}$ for each strain applied and add these quantities in order to obtain the value of the total stress. Thus, the total stress is given by

$$\sigma_{i+1} = d\epsilon_{i+1}(t_1) \times E(t_k - t_1) + d\epsilon_{i+1}(t_2) \times E(t_k - t_2) + \dots + d\epsilon_{i+1}(t_k) \times E(t = 0) \tag{3.14}$$

If we take the limit as k tends to infinity and $d\epsilon_{i+1}$ tends to zero we obtain the familiar representation of equation (2.8):

$$\sigma(t) = \int_0^t E(t - \tau) \frac{d\epsilon(\tau)}{d\tau} d\tau. \tag{2.8}$$

The equation (3.14) is not quite the form which is used in the analysis. This is because the propagation of stress waves is a continuous process and (3.14) was written for a finite number of constant valued strain increments. Therefore, in order to obtain the correct value of wave-front stress and to more accurately describe the continuous process, the terms $E(t_k - t_{m+1})$ are replaced by $E[t_k - \frac{1}{2}(t_m + t_{m+1})]$. That is, the strain is regarded as being applied at a time halfway between the times t_m and t_{m+1}. It is imperative to do this in order that a correct value of the wave front stress is obtained. If $E(t_k - t_m)$ were used, the wave front stress would not be relaxed at all and if $E(t_k - t_{m+1})$ were used, the stress would be relaxed too greatly.

A different form of the integral equation can be obtained by applying an integration by parts to equation (2.8). The result is

$$\sigma(t) = \epsilon(t) E(t = 0) - \int_0^t \epsilon(t) \frac{dE(t - \tau)}{d\tau} d\tau. \tag{2.7}$$

This representation is attractive because it quite strikingly shows two physical phenomena. The first term on the right represents the stress acting on the element as a result of the strain which has deformed it at the instant considered. This part is completely analogous to the case of elastic waves in which the function E(t) would be a constant. The integral, however, represents in our problem the process of relaxation in which the stress which has been acting on the element for some finite time is reduced in value because of the nature of the relaxation modulus. In order to apply the analysis of this report to the integral, we again write increments of stress and add these increments. In this case the quantity $d\sigma_{i+1}$ is calculated by

$$d\sigma_{i+1} = \epsilon_{i+1}(t_{m+1})[E(t_{k+1} - t_{m+1}) - E(t_{k+1} - t_{m+2})]. \tag{3.15}$$

Inherent in this equation is the assumption that the strain on element i + 1 is constant from time t_m to time t_{m+1}. This equation is calculated for the entire strain history of the element and the stress is thus integrated. The complete the calculation of equation (2.7), the "elastic" part of the stress is calculated by means of

$$d\sigma_{i+1} = \epsilon_{i+1}(t_{k+1}) E(t = 0). \tag{3.16}$$

It was found that an effective way to increase stability in this program was to compute the integral in two places in the procedure of the problem. Therefore, the order of the laws given in section 3.1 was modified as follows. (1) The first calculation is the evaluation of the integral as described above. (2) The increment of strain $d\epsilon_{i+1}(t_k)$ is calculated and added to the strain on cell i + 1 at the last time index, k - 1. Thus we calculate the strain $\epsilon_{i+1}(t_k)$, which had not as yet been available. (3) This newly calculated strain is then used to obtain the elastic part of the stress by means of equation (3.16). (4) The integral is again evaluated, but now the strain $\epsilon_{i+1}(t_{m+1})$ has been calculated for $m + 1 = k$, whereas it had not been available for the first integral calculation. (5) The stress σ_{i+1} is now calculated by adding the elastic part of the stress to the average of the two integral calculations.

The reason that it was found necessary to incorporate this somewhat artificial device is that if it were not used, the integral would be evaluated either by using all of the strain $\epsilon_{i+1}(t_k)$, or by using none of it. Either of these two alternatives may lead to serious errors. For instance, a stress may be calculated which is higher in magnitude than the input stress, or a change of signs may occur. Either of these errors tend to be magnified as the calculation proceeds, rendering the results meaningless. In addition, our method is probably closer than either alternative to the actual physical process in which several changes occur simultaneously.

The only other exceptional feature of this program is that the elements of time need not be equal. Since elements of time are related to elements of distance (cell size) by the relation

$$dx_i = c_g \times dt_i \tag{3.17}$$

it follows that the cell sizes may likewise be unequal. If these sizes are unequal, an interpolation procedure is required in order that the proper values of the relaxation modulus are available. A linear interpolation is used which introduces some errors into the glassy and transition regions of the modulus; these are not serious if the time elements are small enough.

Two schemes have thus been devised for the calculation of viscoelastic stress waves. The remainder of what follows in this section applies to both representations.

In order to allow these schemes to be used for bars with fixed or free ends, an "end factor" is employed. This is simply a number, either one or zero, which is multiplied with the stress σ_{im+1}. If the bar is free at the right end, EF = 0.0, and if the right end is fixed, EF = 1.0. Thus, the stress at the end of the bar is made to be zero for a free end, and is undisturbed for a fixed end. These conditions lead, by means of the impulse-momentum law, to the proper type of reflection at the right end.

The left end condition is decided by means of a test in the time repetition loop, but outside the position loop. This test applies a given constant stress, either tensile or compressive, to the first cell if the time is less than t_p and applies no stress if the time is greater than t_p. It would be a simple matter to apply any given stress at any time to the bar, but since this would add essentially no new information or new understanding to the problem, it was not done in our program.

The basic laws as they appear in the double integral program are shown in Figure 4. Note that this diagram is merely the logical "skeleton" of the program.

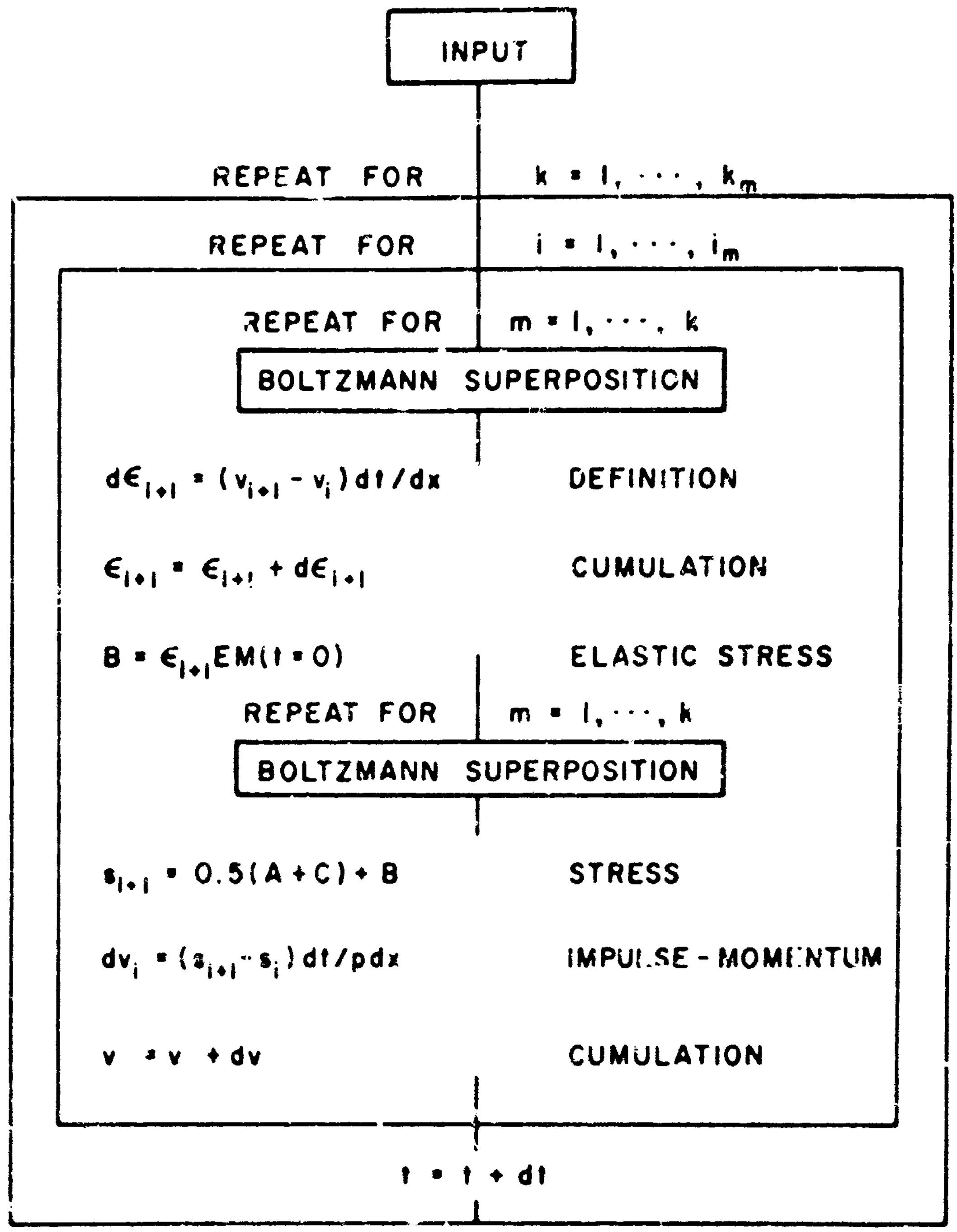


FIG. 4 VISCOELASTIC COMPUTER SCHEME USING TWO BOLTZMANN SUPERPOSITIONS

3.3 Computer Theory-Viscoplastic Impact

This program was constructed along the general lines given in section 3.1. A minor difference is that the strain increment is calculated and integrated before the stress-strain rate law is applied. This order was chosen because the stress-strain rate law is not used when there is no deformation occurring, whereas the strain is always calculated. Thus, the order is merely a matter of convenience, since this particular choice does not alter the logic. The basis of the scheme is outline in Figure 5.

In the computer procedure we write the stress-strain rate law, equation (2.13) as

$$s_{i+1} = 1.0 + [(v_i - v_{i+1})/dx]^{1/p} \quad \text{if } v_i \geq v_{i+1} \qquad (3.18a)$$

$$s_{i+1} = 1.0 \quad \text{if } v_i < v_{i+1} \qquad (3.18b)$$

This representation uses a somewhat different criterion than does equation (2.13). In (3.18) it is the velocity that determines whether or not deformation occurs whereas in (2.13) the stress is the determining factor. This difference arises because the initial input to the bar is a velocity on the left end. This, of course, gives rise to stresses, but we follow the diffusion of velocity through the bar and use it as a "deformation criterion".

The means by which this deformation decision is made is the use of "slip factors". This is a factor (called si_i in the program) which takes the value of 1.0 if cells i and i - 1 are

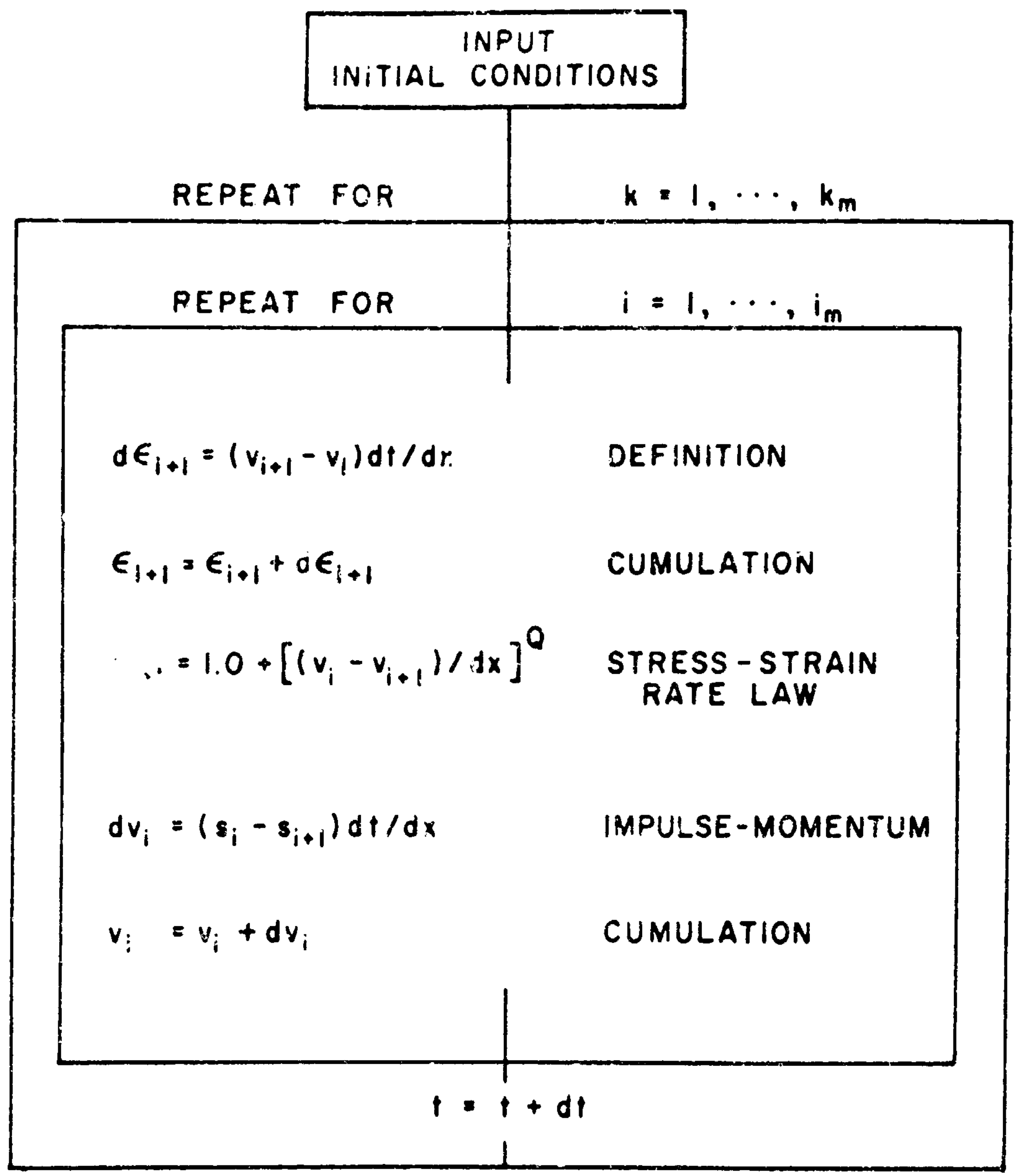


FIG. 5 VISCOPLASTIC COMPUTER SCHEME

deforming relative to each other, and the value 0.0 if the cells i and $i - 1$ are moving with the same velocity. We thus have

$$sl_i = 1.0 \quad \text{for} \quad v_{i-1} \neq v_i$$

$$sl_i = 0.0 \quad \text{for} \quad v_{i-1} = v_i .$$

The dynamics of cell number i is thus determined by the relative values of the slip factors sl_i and sl_{i+1}. We compare these two slip factors by computing their difference, $sl_{i+1} - sl_i$. The results of this subtraction may lead to any one of three possibilities; that is, the difference may be zero, positive, or negative. There are actually, then, four dynamic conditions:

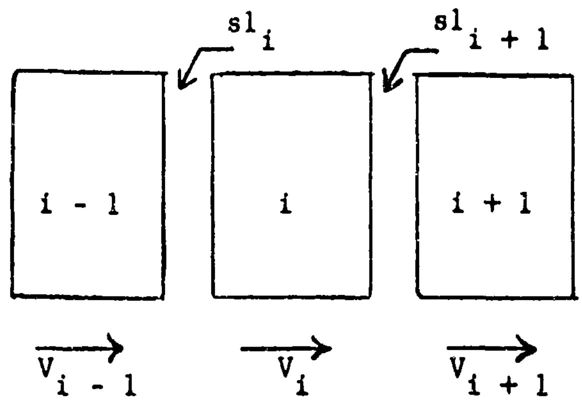
A. $sl_{i+1} - sl_i = 0$ (A1. $sl_i = sl_{i+1} = 0.0$)
(A2. $sl_i = sl_{i+1} = 1.0$)

B. $sl_{i+1} - sl_i > 0$ $(sl_{i+1} = 1.0,\ sl_i = 0)$

C. $sl_{i+1} - sl_i < 0$ $(sl_{i+1} = 0.0,\ sl_i = 1.0)$

This method was devised by Minnich and Davids [25] for another application, and the details of their method are similar in some respects to those used here. However, the method will be given explicitly here since the physical conditions of the two applications are different in a number of circumstances.

<u>Condition A1</u> $sl_i = sl_{i+1} = 0.0$

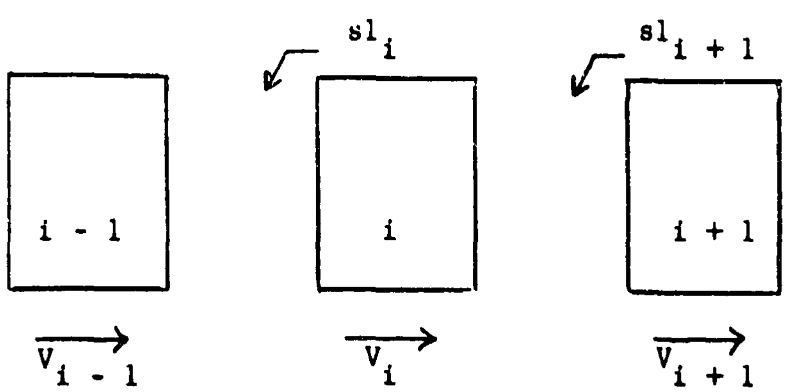


In this case, since each slip factor is zero, the three velocities shown above are equal and there is no net force on cell i. Therefore, its velocity increment is zero and, if we write the impulse-momentum law in the form

$$dv = (s_i \times sl_i - s_{i+1} \times sl_{i+1})dt/dx \qquad (3.19)$$

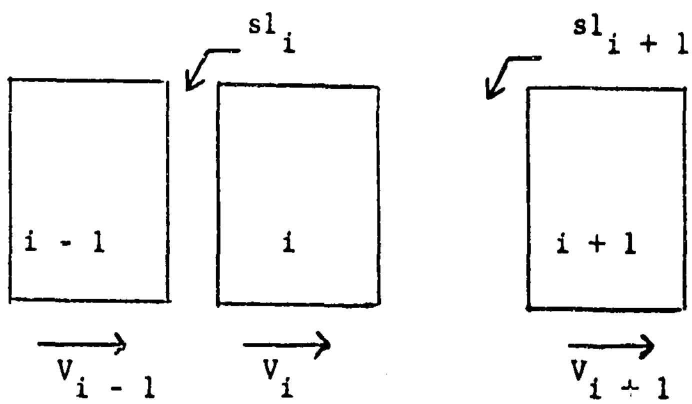
we may use (3.19) to calculate dv.

<u>Condition A2</u> $sl_i = sl_{i+1} = 1.0$



Now deformation occurs at both ends of the i - th cell and therefore there are stresses on each end of it. Thus, equation (3.19) may again be used.

<u>Condition B</u> $\quad sl_i = 0.0, \; sl_{i+1} = 1.0$

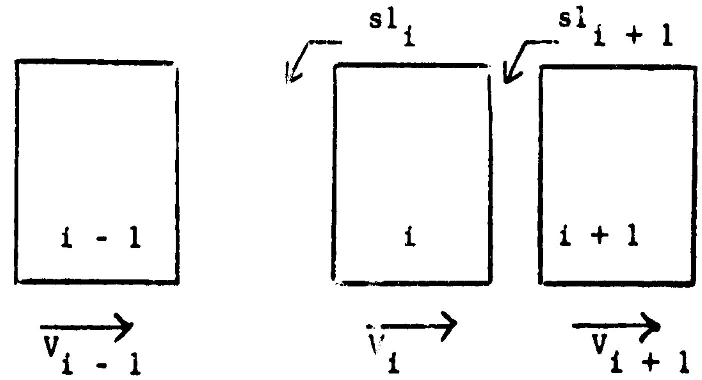


Physically, this means that the material immediately to the left of the i - th cell is not deforming, while the material to the right is deforming. Thus, the region in question is in the "unloading" process in which the boundary between rigid and deforming material is moving to the right. Ting and Symonds[19] prove that this unloading must start at the impact end of the bar, and since this is a diffusion phenomenon, once a region has unloaded, it will not deform again. We thus use a form of the impulse-momentum law in which the mass of the striker must be taken into account. This is

$$dv = -s_{i+1}\, dt/k. \tag{3.20}$$

This velocity increment is added to the first i cells.

Condition C $\qquad sl_i = 1.0, \; sl_{i+1} = 0.0$



In this instance the material to the left of the i - th cell is deforming while the material to the right is rigid. This corresponds to a loading process where the deformation field is moving to the right. Since s_{i+1} must be 1.0 and s_i must be greater than 1.0, deformation will be initiated to the right of cell i. Therefore, sl_{i+1} is set equal to one and equation (3.19) is used.

The structure of the logic for this program is indicated in the schematic diagram, Figure 6.

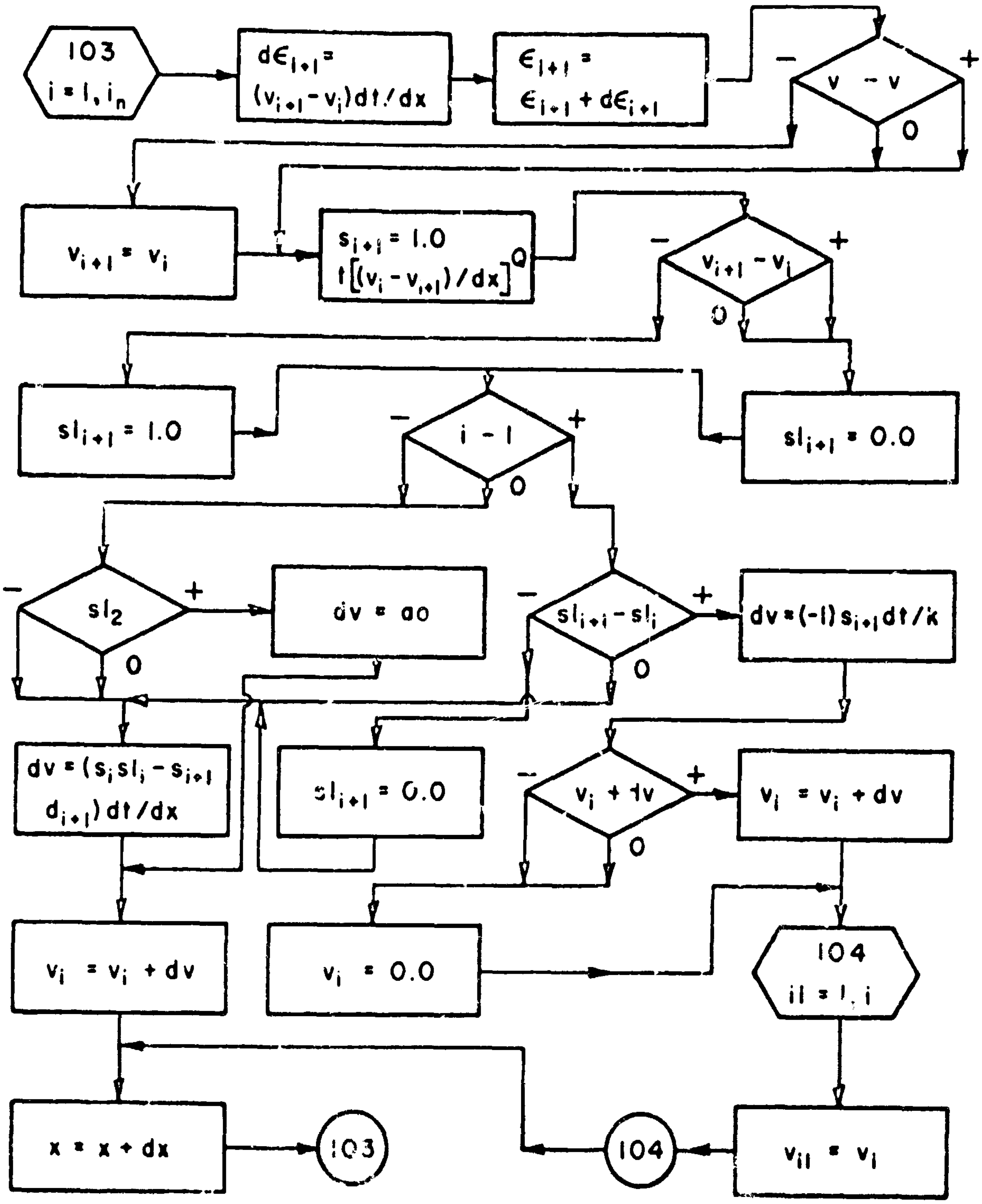


FIG. 6 LOGIC OF SLIP FACTORS

Chapter IV

RESULTS OF CALCULATION AND DISCUSSION

4.1 Standard Linear Solid Material

Two computer programs based on the theory given in sections 2.2, 3.1 and 3.2 were written in the Fortran language. These programs are given in their entirety in Appendices A and B.

In order to check the validity of the programs, runs were made using the standard linear solid to represent the material. The values of the system parameters were chosen as follows:

$$E = E' = 1.0$$

$$1/\mu = 1.0$$

$$\rho = 1.0$$

$$p_o = -1.0$$

The relaxation modulus in this case is

$$E(t) = 1 + e^{-t} \qquad (4.1)$$

Actually these values were chosen in order to facilitate a comparison with both Morrison [5] and Arenz [6]. As Figure 7 shows, the integral solution by Morrison, the Laplace transform method of Arenz, and the computer solutions agree very well.

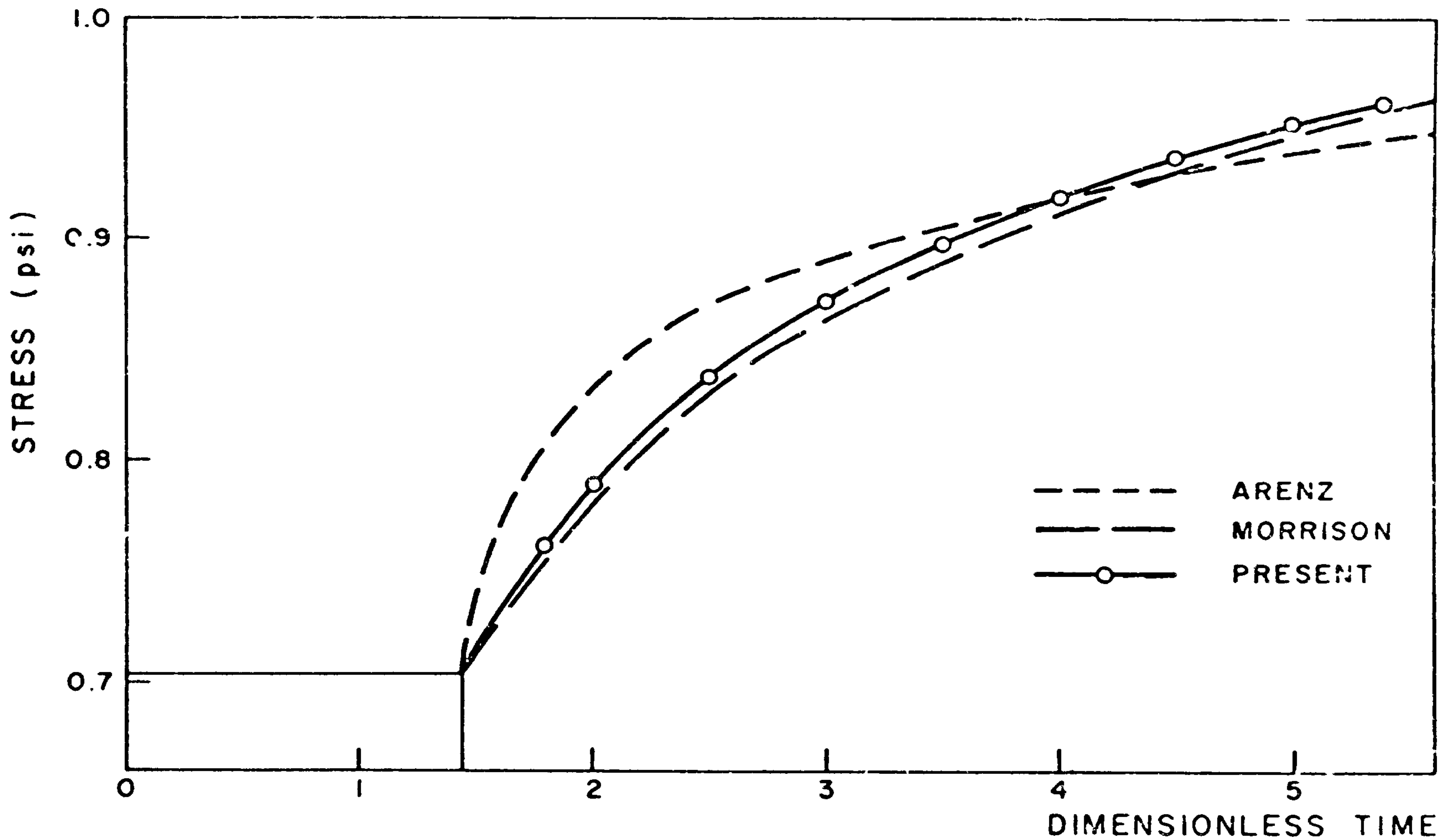


FIG. 7 COMPARISON OF SOLUTIONS FOR STANDARD LINEAR SOLID WITH UNIT STEP STRESS INPUT

As a point of interest, we show in Figure 8 the response of a bar of the same material and at the same location. The difference this time is that the bar has a finite length ($L = 2.828$), and the right end is fixed. A constant unit stress is applied at the origin. The step discontinuities in the response are due to the reflections from the ends of the bar.

Several facts should be noted from the diagrams. First, the response of the bar takes place in less than two decades of log time, thus bearing out what was stated previously: that the standard linear solid is a fictitious material. Also, if the response of two bar locations are plotted, the slope becomes less steep, as we should expect.

4.2 Realistic Viscoelastic Material

We now apply the program to a realistic material. Viscoelastic data was taken from a thesis by Arenz [6] for a polyurethane synthetic rubber, a low modulus polymer. The relaxation modulus is shown in Figure 9. There are some graphs given in Arenz's work showing stress wave behavior as calculated by an approximate Laplace inversion technique. The single integral program was applied to this material and a comparison of results is shown in Figure 10. The high frequency response as calculated by Arenz is slower than our data. This seems to be true generally. Also, Arenz obtained some oscillation in the high frequency response, supposedly due to alternate reinforcement and interference of waves of differing frequency and therefore differing speeds of propagation. No such

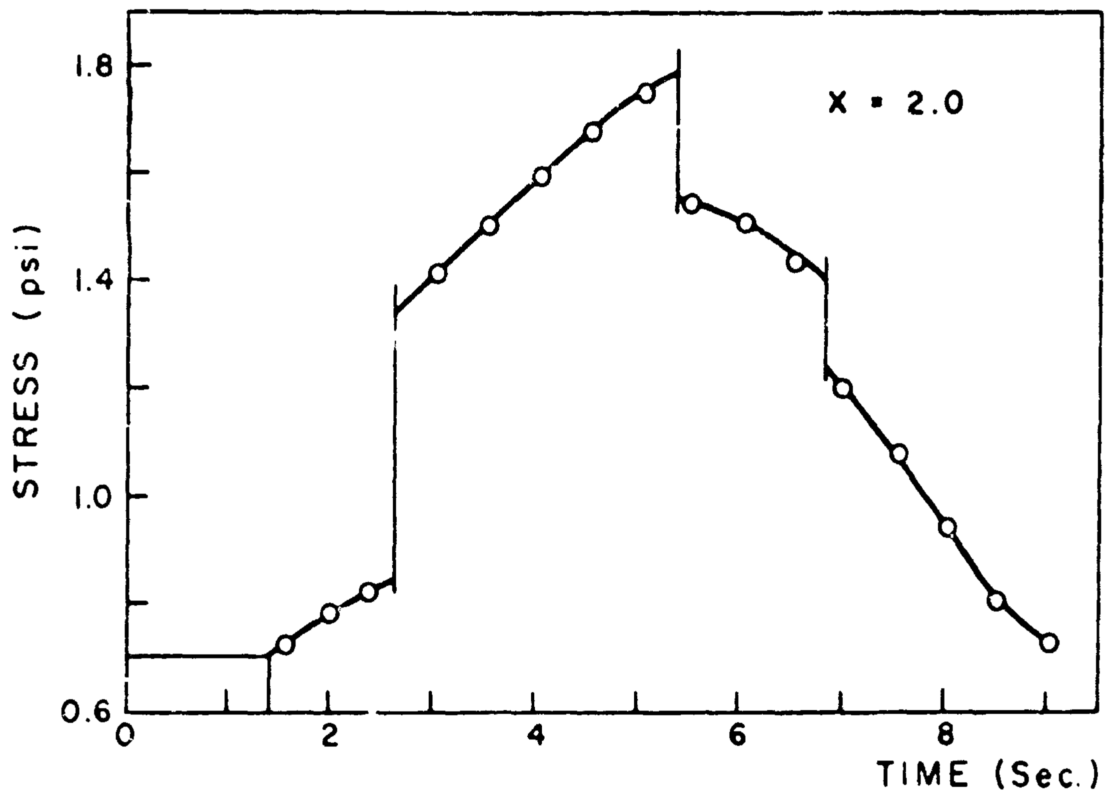


FIG. 8 REFLECTION OF STRESS WAVE IN BAR OF STANDARD LINEAR SOLID MATERIAL - CONSTANT UNIT STEP STRESS INPUT

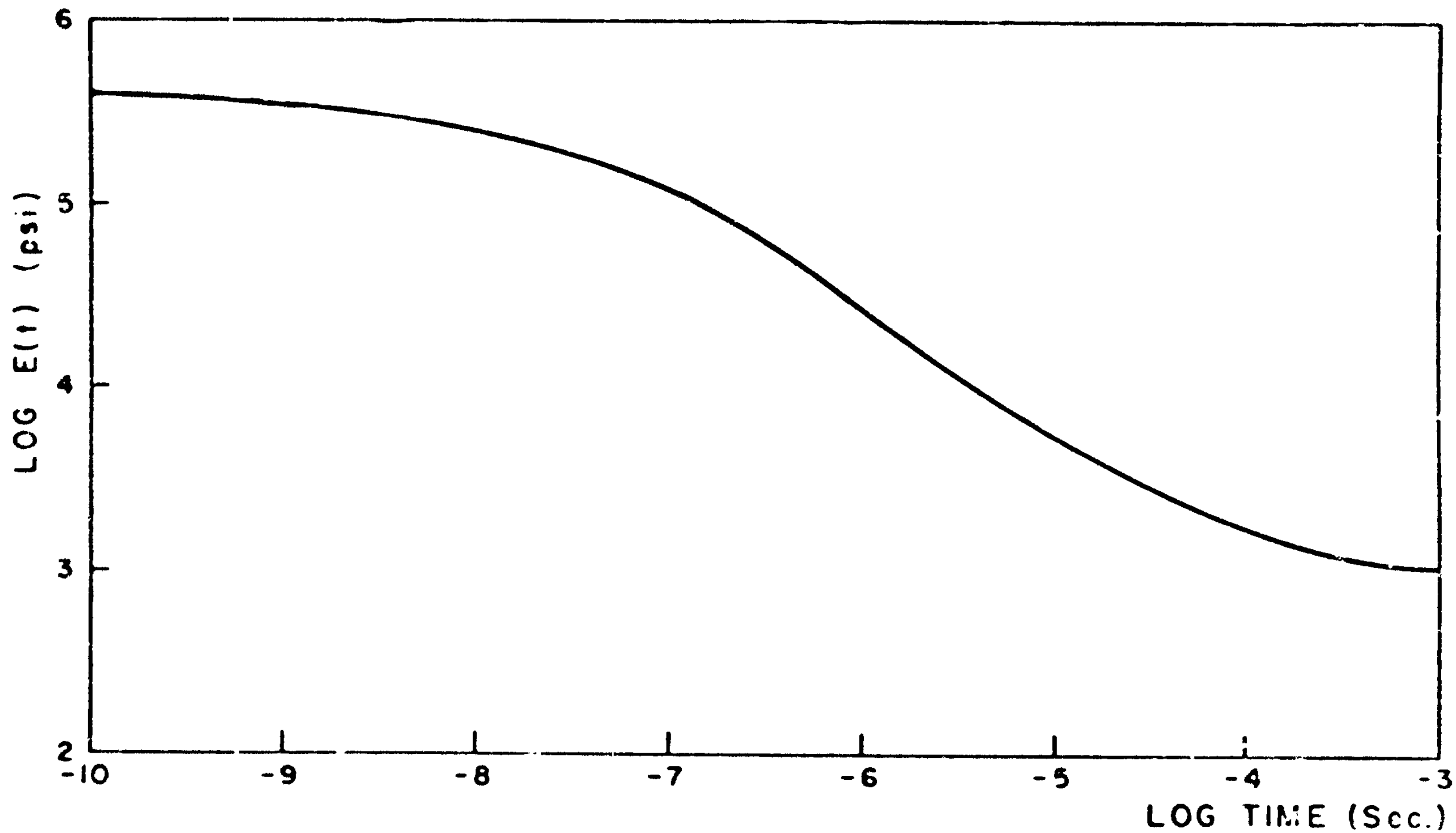
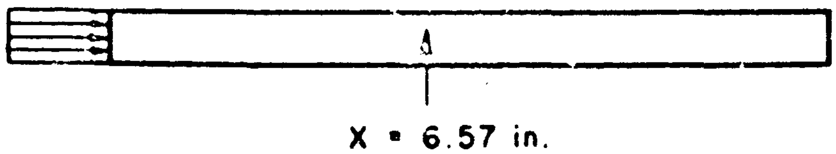


FIG. 9 TENSILE RELAXATION MODULUS FOR POLYURETHANE MATERIAL; TEMPERATURE = 14 °C



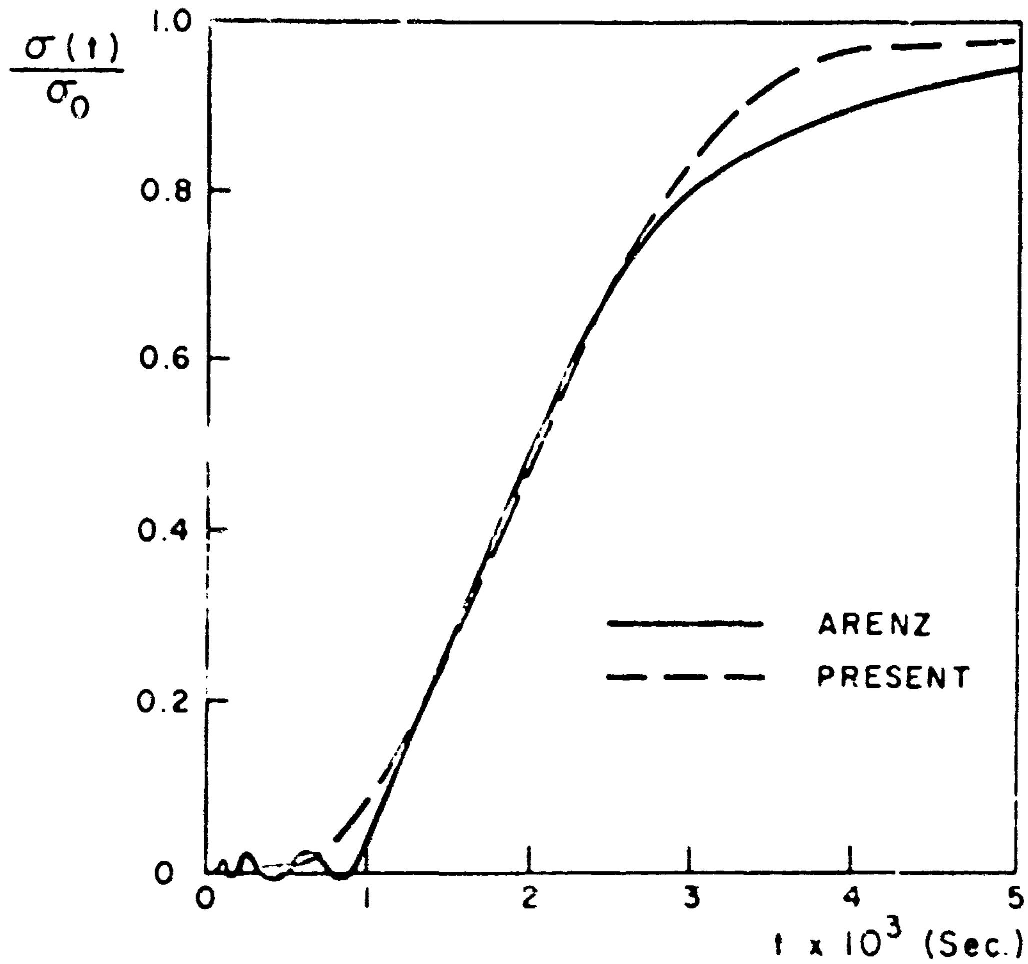


FIG. 10 COMPARISON OF SOLUTIONS OF STRESS WAVE RESPONSE IN POLYURETHANE VISCOELASTIC MATERIAL

occurrence was obtained in our analysis, and indeed it seems likely that the oscillation reported by Arenz is not a genuine physical fact, but a result of some mathematical approximation. The transition region of the response is quite well matched for the two solutions but some divergence is apparent at large time. It is believed that this is due to an inaccurate low frequency material representation in this analysis.

The double integral program was applied to another, similar material in reference [24]. This program does not seem to operate as effectively as the single integral program, and there was some scattering of results. Figure 11 shows the response of this material (Hysol 8705) at two positions along the bar x = 1.77 inches and x = 3.29 inches.

It was found that several factors could give rise to instability in either of the programs. First, the time element must be chosen small enough so that enormous changes in the relaxation modulus do not take place. This factor is far more critical for the double integral program than for the single integral solution. The time interval was taken as 3.0×10^{-3} sec. for the results in Figure 10 and as 1.0×10^{-7} sec. for those in Figure 11. Secondly, since each of the programs incorporates a linear interpolation, the material data must be given to the program at quite a few points. Normally, the data was given at log time increments of 0.1 and even this is not enough for response calculations at positions where the glassy wave speed arrival time is less than 10^{-6} sec.

The double integral program was so written that variable

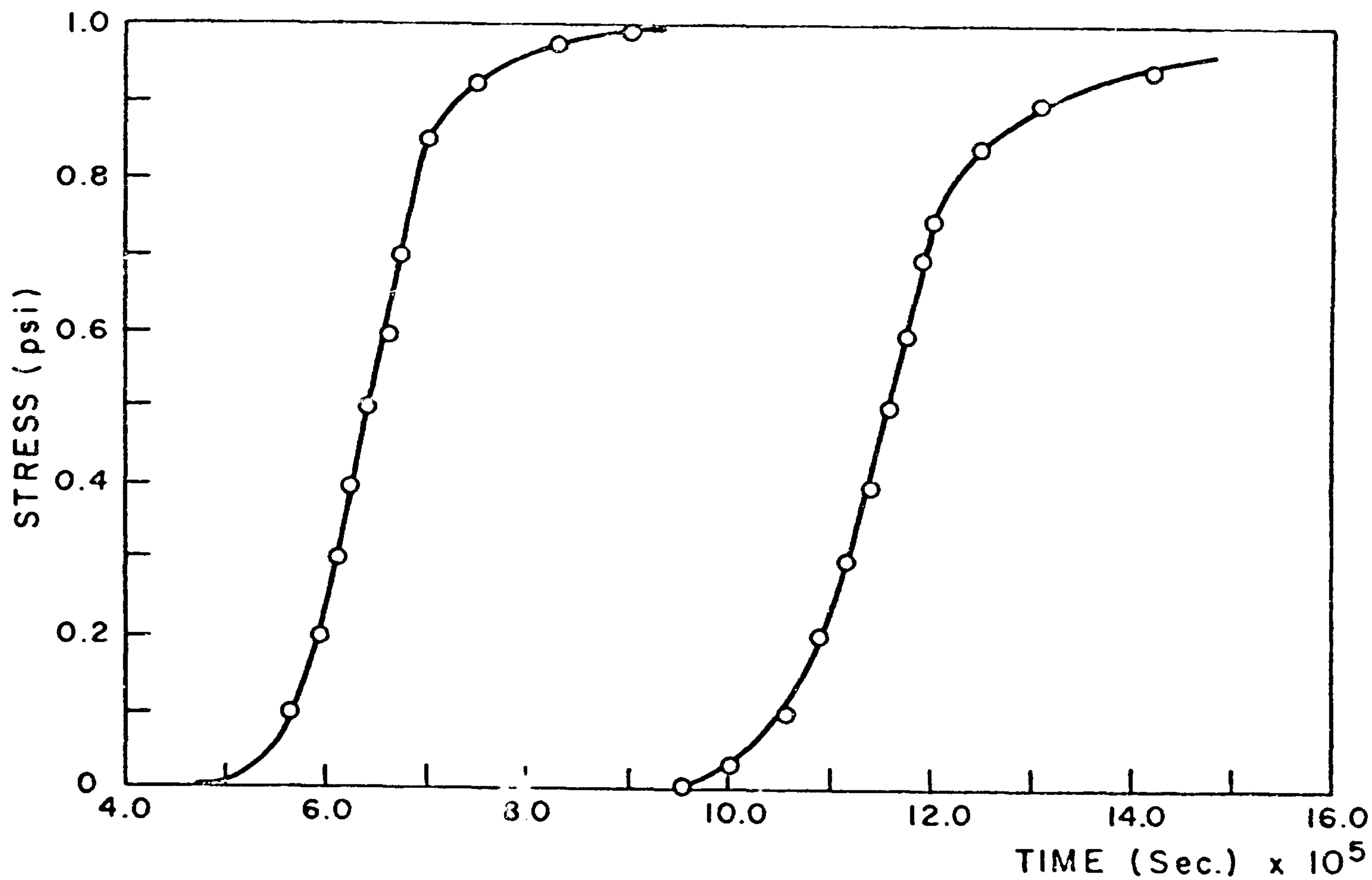


FIG. 11 RESPONSE OF POLYMER ROD TO UNIT STEP STRESS INPUT

elements of time could be chosen. It was found, however, that actually using variable time elements leads to serious errors in most cases. Generally, it may be said that the time increments should be decreased in size as the time increases. This, however, is not an advantage over taking equal time increments, insofar as required computer time or representation of material are concerned.

It may be said that for all cases tested, the single integral program outperformed the double integral program in every way. Furthermore, it is more efficient than the double integral program.

4.3 Viscoplastic Impact

A program incorporating the theory of sections 2.3, 3.1 and 3.3 was written in Fortran. This program is shown in Appendix C.

The case of the overstress exponent equal to unity was performed first, and the results were compared with those of Ting and Symonds [19]. Figure 12 shows a comparison of our stress calculations with those of Ting and Symonds. This plot is the quantity $(s - 1)/v_o$ versus the dimensionless distance x. The calculation is for values of $k = 1.0$ and $v_o = 1.0$. Calculations for other values of k and v_o showed similar agreement with Ting and Symonds. Figure 13 shows a plot of dimensionless strain, η, divided by η_f (which is defined as the uniform strain which could absorb the initial kinetic energy at stress $s = 1$) versus the dimensionless distance. The quantity η_f is equal to one-half the product $k\ v_o^2$. Again agreement was as close for other values of k and v_o. In Figures 12 and 13 the values of t_o and t_1 are the times at which the striker stops moving

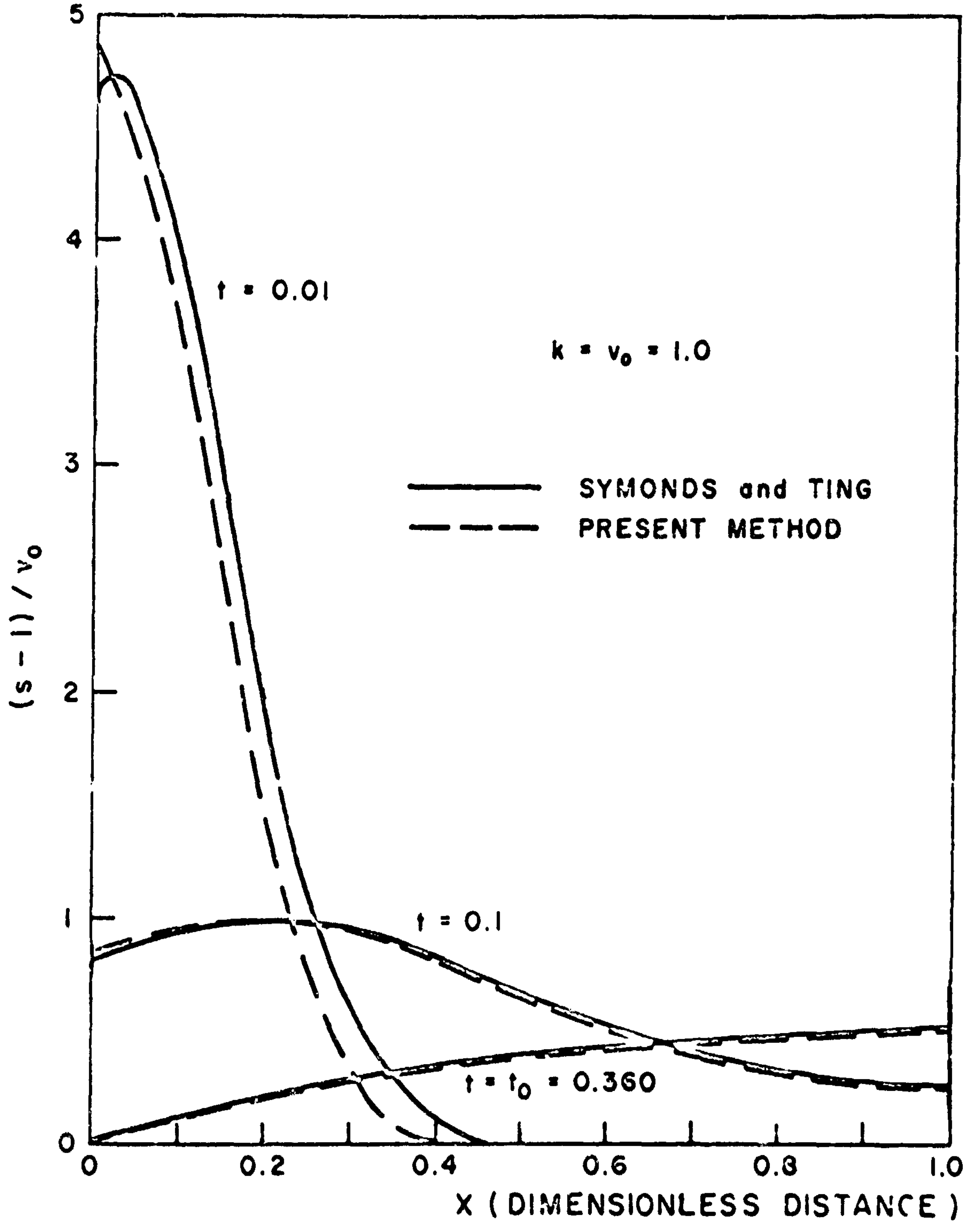


FIG. 12 COMPARISON OF STRESS RESPONSE OF VISCO-PLASTIC IMPACT - OVERSTRESS EXP. OF 2.0

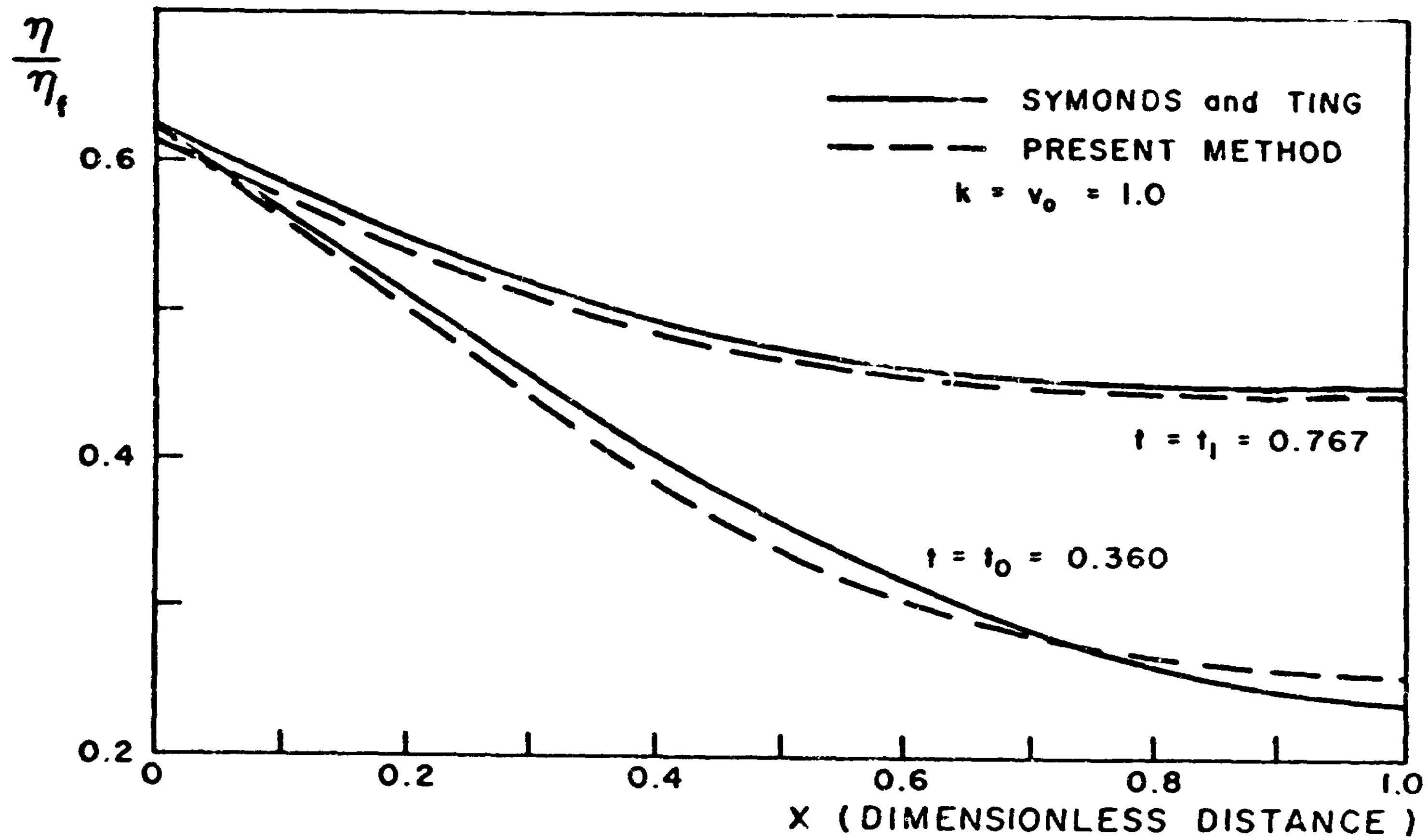


FIG. 13 COMPARISON OF STRAIN RESPONSE – VISCOPLASTIC IMPACT, $p = 1.0$

and the velocity in the entire bar vanishes, respectively.

The case of overstress exponent equal to unity is quite unrealistic. In engineering situations, we would require an analysis using the correct value of p. For mild steels, p = 5, and for aluminum alloys, p = 4; therefore, the analysis for p = 1 does not give a quantitative answer to the impact problem.

Accordingly, we introduced the value of p = 4 into the program. Thus the stress-strain rate law becomes

$$s_{i+1} = 1 + (v_i - v_{i+1}/dx)^Q \tag{4.2}$$

where Q = 1/p. No serious difficulties were encountered as long as the value of the time increment was kept sufficiently small. For instance, for dx = 0.050, the program operated satisfactorily for dt = 0.001, but became unstable for dt = 0.01. This arises because the computer performs one operation at a time. If the time increment is too large relative to the distance increment, one parameter may accumulate errors. The velocity field, for instance, may reverse directions, or the strains become impossibly high.

The results of the calculations for p = 4.0 are shown in Figures 14 and 15, where they are compared to the solutions for p = 1.0. As could be seen from the general law, the strains are larger for this case. Also, the stresses are lower initially but increase to a higher value at times t = 0.1 and t = 0.36. The time for complete cessation of motion for the case p = 1.0 was t = 0.770. At this time there was still plastic deformation occurring in the bar when p = 4.0. It was also noticed that the velocity of the

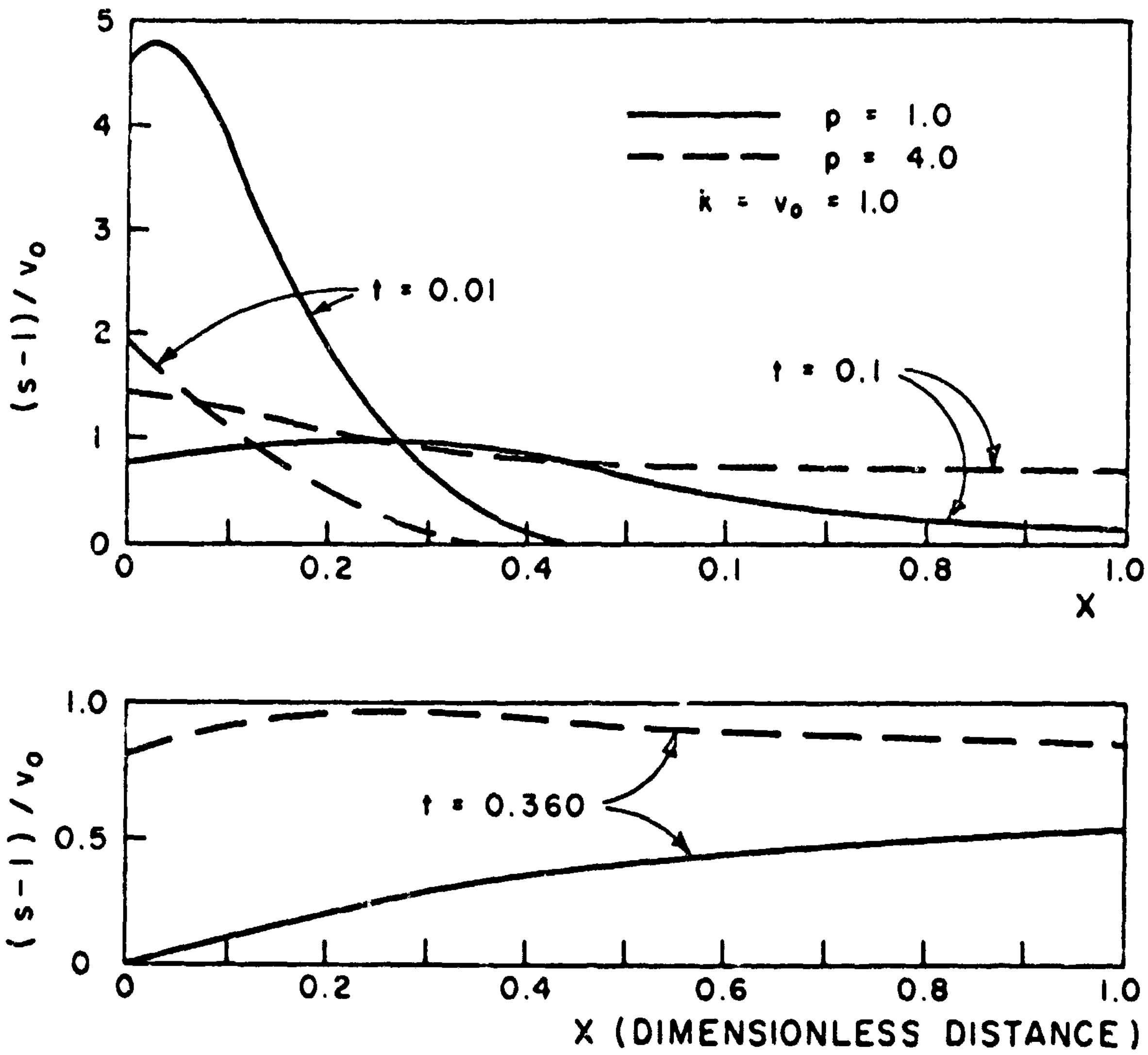


FIG. 14 COMPARISON OF VISCOPLASTIC STRESS SOLUTIONS FOR $p = 1.0$, $p = 4.0$

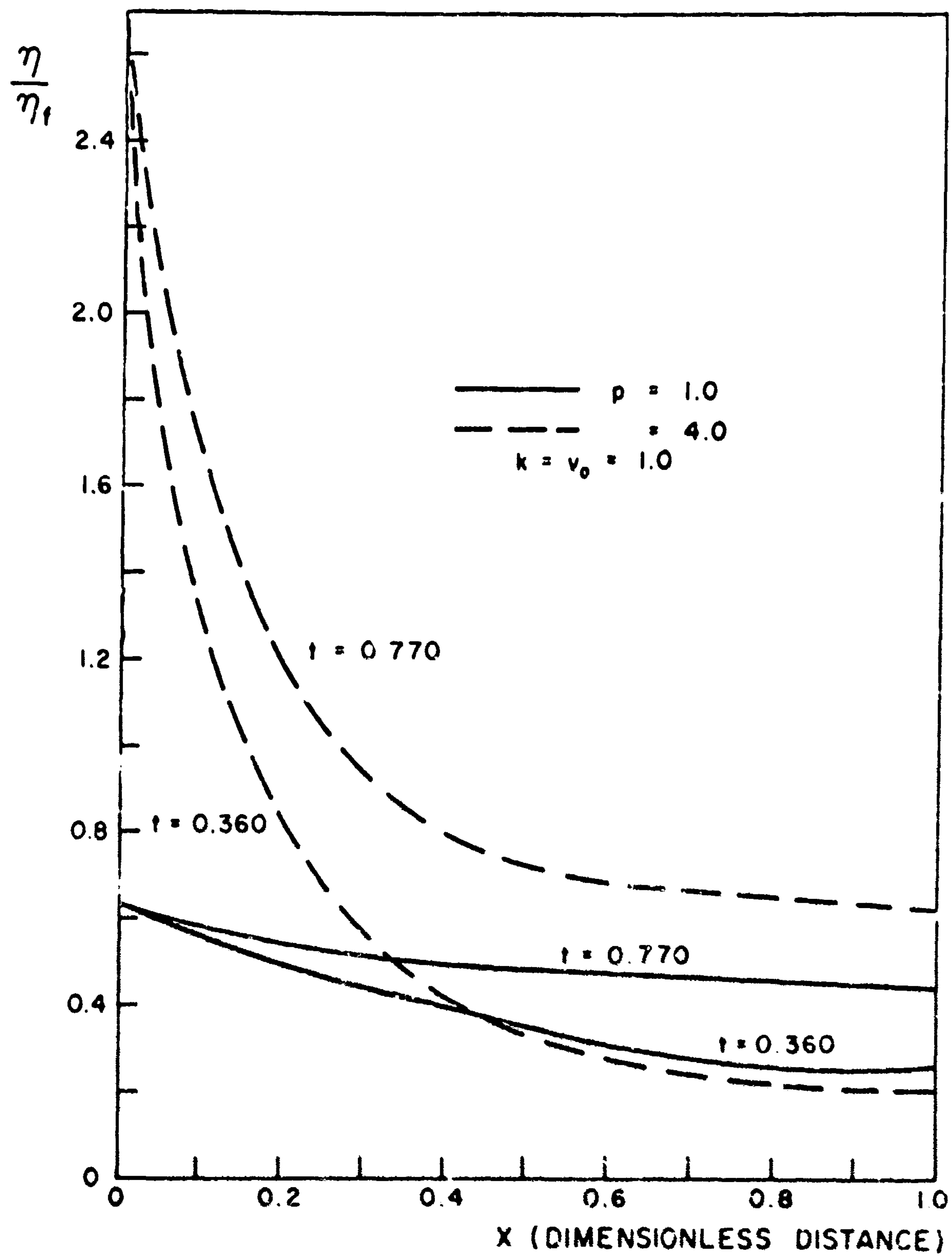


FIG. 15 COMPARISON OF VISCOPLASTIC STRAIN SOLUTIONS FOR p = 1.0, p = 4.0

striking mass slows down much faster for $p = 1$ than for $p = 4$. For instance, with $v_o = 1.0$, $k = 1.0$ the striker velocity at $t = 0.360$ was 0.31 for $p = 1.0$ and 0.50 for $p = 4$; at $t = 0.600$, the velocity was 0.07 for $p = 1.0$ and 0.25 for $p = 4$.

SUMMARY AND CONCLUSIONS

7.1 Summary

The problems of longitudinal impact are important because of their relative simplicity. Their study indicates the pertinent physical laws of a problem and often indicates the direction in which further research should be directed. Even more important, they provide a simple and direct method by which a particular law may be experimentally verified. The two problems presented here belong to that class of problems which have received considerable attention in recent years. They are: viscoelastic waves in a longitudinal bar and viscoplastic impact of a longitudinal bar.

Viscoelastic investigations typically begin with a model representation of the material. Some of these studies are described in the first Chapter. When simple models are used, however, the material is highly fictitious. If a model is used which does represent a viscoelastic material, the solution usually involves an extremely difficult numerical program. In this work a finite numerical scheme has been devised, using the Boltzmann superposition principle as the stress strain law. Spring-dashpot models have been eliminated altogether, and the actual material data is used in graphical form. This method has been shown to solve problems represented by models as well as problems represented by more realistic materials.

The current situation for plastic impact of bars may not be so easily summarized. There exist many mathematical models for these phenomena, with varying degrees of complexity. We have chosen to analyze the case of plastic impact of a finite mass on a bar of length L, where the material is rigid-viscoplastic. That is, the bar does not deform until the stress at a point exceeds the static yield strength. When it does deform, it does so according to a power law relation between the rate of strain and the amount by which the stress exceeds the static yield stress. The solution has been shown to agree very well with existing analytical results using the linear law. The nonlinear case has also been solved, and a great difference is shown between the linear and nonlinear cases. In addition, the final strain for the nonlinear case has been shown to differ greatly from that obtained by assuming uniform strain, when the impact mass and velocity are small. Our solution also yields the value of stress, strain and velocity at any point of the bar at any time, instead of just the final strain.

A.2 Conclusions

The model representation of viscoelastic materials is inadequate to describe the phenomenon of stress waves. The definition of a linear viscoelastic material is the Boltzmann superposition principle and this should be used to calculate any short time effects. The response of a realistic viscoelastic material takes place over a large number of decades of log time. This indicates that phenomena occurring in material which is more rigid than that used here will require a great deal of time to reach equilibrium. We conclude that the study of viscoelastic problems other than stress waves should also use the Boltzmann principle.

The linear law of viscoplastic impact does not give quantitatively correct results for common materials such as steels and aluminum alloys. The nonlinear law also differs greatly from the simplifying assumption of uniform strain cases of low impact mass and velocity. This is an important example for experimenters, since it would be easier to conduct a test with small parameters than with very large ones. Also, the nonlinear law extends the time of the problem; since this analysis gives the complete stress, strain and velocity distributions in the bar at any time, tests could be made to check all these quantities for any period of time.

A.3 Suggestions for Further Study

The method of viscoelastic wave analysis presented here should be used in an attempt to solve other problems of more direct engineering and research value. The problems of two dimensional waves and of long time duration, complicated geometry and with accompanying creep are examples of other engineering applications. In the area of research, a program of this nature might be used in reverse to calculate material data with given stress wave response. The interesting Fourier analysis of Kolsky [3] in which he calculated the stress wave response to an explosive discharge at one end of a bar is a potential check on our method. This study is particularly valuable because experimental data is also given. Finally, the problem of impact by a finite mass would be a valuable extension of this problem.

The most immediate and important use of the viscoplastic program would be to compare it with extensive experimental data. If justified, it could be immediately applied to other geometries and structures. If the law is found lacking in any way, it might be combined with strain hardening effects in order to decide what type of constitutive equations are most applicable for certain problems. This could then be used as an aid in designing and interpreting experiments. Eventually, criterions for failure by various means could be added.

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APPENDIX A

```
      COMPILE   RUN  FORTRAN
C       VISCOELASTIC WAVES IN BAR
C       BOLTZMANN SUPERPOSITION PRINCIPLE
C       DOUBLE INTEGRAL PROGRAM
C           LIST  OF  SYMBOLS
C       X         = LONGITUDINAL COORDINATE
C       T         = TIME
C       S(I)      = TENSILE STRESS IN X-DIRECTION
C       E(I,K)    = TENSILE STRAIN IN X-DIRECTION
C       V(X,T)    = VELOCITY IN X-DIRECTION
C       AREA      = CROSS-SECTIONAL AREA OF BAR
C       RHO       = MASS DENSITY OF BAR MATERIAL
C       CG        = GLASSY (FASTEST) WAVE SPEED
C       TP        = TIME DURATION OF STRESS INPUT
C       PO        = PRESSURE AT ORIGIN
C       EM(K)     = VISCOELASTIC RELAXATION MODULUS
C       T1(M)     = INTERMEDIATE TIME (=T(K+1)-T(M+1))
C       EMO       = GLASSY STATE RELAXATION MODULUS
C       EM1(M)    = EM EVALUATED AT TIME=T1(M)
C       TL        = LOG (TIME)
C       IM        = NUMBER OF BAR ELEMENTS
C       KM        = NUMBER OF TIME ELEMENTS
C       DX        = CHANGE IN X-COORDINATE
C       DT        = CHANGE IN TIME
C       DE(I,K)   = CHANGE IN STRAIN
C       DV        = CHANGE IN VELOCITY
      DIMENSION E(22,120),DE(22,120),T(200),DT(200),DX(200),X(200)
      DIMENSION S(200),V(200),IDENT(16),EM(200),TL(200)
      DIMENSION T1(200),EM1(200)
    1 READ 801, IDENT
    2 READ 802,RHO,TP,PO
    3 READ 802,AREA,EF
    4 READ 804,I[illegible] KM,IB,IE
    5 READ 803, EM(1)
  801 FORMAT(16A5)
  802 FORMAT (5F10.0)
  803 FORMAT (E10.4)
  804 FORMAT (8I10)
      EMO=EM(1)
      CG = SQRTF (EMO/RHO)
      T(1)=0.0
      DO 101 N=1,KM
      READ 805,EM(N+1),TL(N+1),A
  805 FORMAT (E10.4,F10.4,I10)
      T(N+1)=10.0**TL(N+1)
      DT(N)=T(N+1)-T(N)
      DX(N)=CG*DT(N)
      IF (A) 20,101,20
  101 CONTINUE
      GO TO 17
   20 N1=N+1
      DO 104 I=N1,KM
      EM(I+1)=EM(N1)
      DT(I)=DT(N)
      DX(I)=CG*DT(I)
      T(I+1)=T(I)+DT(N)
  104 CONTINUE
   17 EMO = EM(1)
      PRINT 901, IDENT
      PRINT 902,RHO,TP,PO
```

APPENDIX A (continued)

```
      PRINT 903,AREA
      PRINT 904, IM,KM
      PRINT 908
  908 FORMAT (1H1,20HTENSILE MODULUS DATA/)
      DO 105 K=1,IM
      PRINT 909,TL(K),EM(K)
  909 FORMAT (1H ,F10.2,5X,F8.0)
  105 CONTINUE
      DO 102 K=1,KM
      WRITE TAPE 2,T(K),(S(I),I=1,K)
      X(1)=0.0
      IF (K-1) 41,41,40
C     INTERPOLATION PROCEDURE
   40 DO 110 N=1,K
      T1(N)=T(K+1)-T(N+1)
      DO 111 J=1,KM
      IF(T1(N)-T(J+1)) 30,30,111
  111 CONTINUE
   30 EM1(N)=(EM(J+1)-EM(J))*(T1(N)-T(J))/(T(J+1)-T(J))+EM(J)
  110 CONTINUE
C     STEP PRESSURE INPUT, LEFT END
   41 IF (T(K)-TP) 11,12,12
   11 P=P0
      GO TO 13
   12 P=0.0
C     PROPAGATION PROCEDURE
   13 S(1)=-P/AREA
      DO 103 I=1,IM
      K1=K-1
      IF (K-1) 27,27,25
   25 A=0.0
C     ANELASTIC PART, BOLTZMANN SUPERPOSITION
   32 DO 107 M=1,K1
      A=A+E(I+1,M+1)*(EM1(M)-EM1(M+1))
  107 CONTINUE
C     DEFINITION OF STRAIN
      DE(I+1,K)=(V(I+1)-V(I))*DT(K)/DX(I+1)
      E(I+1,K)=E(I+1,K1)+DE(I+1,K)
C     STRESS-STRAIN LAW, ELASTIC PART
      B=E(I+1,K)*EM0
      C=0.0
      DO 109 M=1,K1
      C=C+E(I+1,M+1)*(EM1(M)-EM1(M+1))
  109 CONTINUE
      S(I+1)=0.5*(A+C)+B
      IF (I-IM) 27,14,14
   14 S(I+1)=S(I+1)*EF
C     IMPULSE-MOMENTUM LAW
   27 DV=(S(I+1)*AREA-S(I)*AREA)*DT(K)/(RHO*AREA*DX(I))
      V(I)=V(I)+DV
      X(I+1)=X(I)+DX(I)
  103 CONTINUE
  102 CONTINUE
      REWIND 2
      PRINT 905, (X(I),I=IB,IE)
  905 FORMAT (1H5,11X,E10.4)
      DO 106 K=1,KM
      READ TAPE 2,T(K),(S(I),I=1,K)
      PRINT 906,T(K),(S(I),I=IB,IE)
  906 FORMAT (1H5,E10.4,1X,19F6.3)
```

APPENDIX A (continued)

```
106  CONTINUE
 500 STOP
 901 FORMAT(1H1,40X,25HVISCOELASTIC WAVES IN BAR/1H0,16
 902 FORMAT(
    132H0MASS DENSITY                      =,  F10.1,10H           //
    232H PULSE DURATION                    =,  F10.1,10H           //
    332H PULSE INTENSITY                   =,  F10.1,10H           /)
 903 FORMAT(
    132H0AREA OF BAR                       =,  F10.1,10H           /)
 904 FORMAT(
    132H0NUMBER OF CELLS                   =,  I3,    //
    232H NUMBER OF TIME ELEMENTS           =,  I3/1H1)
     END
```

APPENDIX B

```
      [illegible] FORTRAN
C     VI[illegible] IN BAR
C     BOLTZMAN[illegible]ITION PRINCIPLE
C     SINGLE INTEGRAL PROGRAM
C         LIST  OF  SYMBOLS
C     X        = LONGITUDINAL COORDINATE
C     T        = TIME
C     S(I)     = TENSILE STRESS IN X-DIRECTION
C     E(I,K)   = TENSILE STRAIN IN X-DIRECTION
C     V(X,T)   = VELOCITY IN X-DIRECTION
C     AREA     = CROSS-SECTIONAL AREA OF BAR
C     RHO      = MASS DENSITY OF BAR MATERIAL
C     CG       = GLASSY (FASTEST) WAVE SPEED
C     TP       = TIME DURATION OF STRESS INPUT
C     PO       = PRESSURE AT ORIGIN
C     EM(K)    = VISCOELASTIC RELAXATION MODULUS
C     EMO      = GLASSY STATE RELAXATION MODULUS
C     T1(N)    = INTERMEDIATE TIME
C     EM1(M)   = EM EVALUATED AT TIME=T1(M)
C     TL       = LOG (TIME)
C     IM       = NUMBER OF BAR ELEMENTS
C     KM       = NUMBER OF TIME ELEMENTS
C     DX       = CHANGE IN X-COORDINATE
C     DT       = CHANGE IN TIME
C     DE(I,K)  = CHANGE IN STRAIN
C     DV       = CHANGE IN VELOCITY
      DIMENSION E(22,120),DE(22,120),T(200),TT(200)
      DIMENSION S(200),V(200),IDENT(16),EM(200),TL(200)
      DIMENSION T1(200),EM1(200)
    1 READ 801, IDENT
    2 READ 802,RHO,TP,PO
    3 READ 802,AREA,EF
    4 READ 804,IM,KM,IB,IE
    5 READ 803,EM(1),DT
  801 FORMAT(16A5)
  802 FORMAT (5F10.0)
  803 FORMAT (2E10.4)
  804 FORMAT (8I10)
      EMO=EM(1)
      CG = SQRTF (EMO/RHO)
      DX=CG*DT
      T(1)=0.0
   17 EMO = EM(1)
      TT(1)=0.0
      DO 101 J=1,200
      READ 809,TL(J+1),EM(J+1),A
  809 FORMAT (F10.4,E10.4,I10)
      IF (A) 90,90,91
   90 TT(J+1)=10.0**TL(J+1)
      T(J+1)=T(J)+DT
  101 CONTINUE
   91 KMM=J-1
      PRINT 901,IDENT
      PRINT 902,RHO,TP,PO
      PRINT 903,AREA,DT
      PRINT 904, IM,KM
      PRINT 908
  908 FORMAT (1H1,20HTENSILE MODULUS DATA/)
      DO 105 K=1,IM
      PRINT 909,TT(K),EM(K)
```

APPENDIX E (continued)

```
  909 FORMAT (1H ,E10.4,5X,F10.2)
  105 CONTINUE
      DO 102 K=1,KM
      WRITE TAPE 2,T(K),(S(I),I=1,K)
      X=0.0
C     INTERPOLATION PROCEDURE
      K1=K-1
      IF (K-1) 41,41,40
   40 T1(1)=T(K)-0.5*DT
      IF (K-2) 42,42,70
   70 DO 110 N=2,K1
      T1(N)=T1(N-1)-DT
  110 CONTINUE
   42 DO 111 N=1,K
      DO 112 J=1,KMM
      IF (T1(N)-TT(J+1)) 30,30,112
  112 CONTINUE
   30 EM1(N)=(EM(J+1)-EM(J))*(T1(N)-TT(J))/(TT(J+1)-TT(J))+EM(J)
  111 CONTINUE
C     STEP PRESSURE INPUT, LEFT END
   41 IF (T(K)-TP) 11,12,12
   11 P=P0
      GO TO 13
   12 P=0.0
C     PROPAGATION PROCEDURE
   13 S(1)=-P/AREA
      DO 103 I=1,IM
      K1=K-1
      IF (K-1) 27,27,50
C     DEFINITION OF STRAIN
   50 DE(I+1,K)=(V(I+1)-V(I))*DT/DX
      E(I+1,K)=E(I+1,K1)+DE(I+1,K)
C     ANELASTIC PART, BOLTZMANN SUPERPOSITION
      CN=0.0
      DO 109 M=1,K1
      CN=CN+DE(I+1,M+1)*(EM1(M))
  109 CONTINUE
      S(I+1)=CN
      IF (I-IM) 27,14,14
   14 S(I+1)=S(I+1)*EF
C     IMPULSE-MOMENTUM LAW
   27 DV=(S(I+1)*AREA-S(I)*AREA)*DT/(RHO*AREA*DX)
      V(I)=V(I)+DV
      X=X+DX
  103 CONTINUE
  102 CONTINUE
      REWIND 2
      DO 106 K=1,KM
      READ TAPE 2,T(K),(S(I),I=1,K)
      PRINT 906,T(K),(S(I),I=IB,IE)
  906 FORMAT (1H5,E10.4,1X,19F6.3)
 106  CONTINUE
 550  STOP
  901 FORMAT(1H1,40X,25HVISCOELASTIC WAVES IN BAR/1H0,16A5)
  902 FORMAT(
     132H0MASS DENSITY                    = , F10.1,10H          //
     232H PULSE DURATION                  = , F10.1,10H          //
     332H PULSE INTENSITY                 = , F10.1,10H          /)
  903 FORMAT(
     132H0AREA OF BAR                     = , F10.1,10H          //
```

APPENDIX B (continued)

```
   232H SIZE OF TIME ELEMENT             = , E10.4,10H           /)
204 FORMAT(
   132HONUMBER OF CELLS                  = , I3,    //
   232H NUMBER OF TIME ELEMENTS          = , I3/1H1)
    END
```

APPENDIX C

```
      COMPILE   RUN  FORTRAN
C      VISCOPLASTIC IMPACT OF BARS
C      FINITE MASS IMPACT ON LEFT END
C      POWER STRESS - STRAIN RATE LAW
C      DIMENSIONLESS FORM
C        LIST  OF  SYMBOLS
C      X    = DIMENSIONLESS COORDINATE
C      TT   = DIMENSIONLESS TIME
C      S    = DIMENSIONLESS STRESS
C      V    = DIMENSIONLESS VELOCITY
C      VO   = DIMENSIONLESS IMPACT VELOCITY
C      E    = DIMENSIONLESS STRAIN
C      P    = OVERSTRESS EXPONENT
C      D    = MATERIAL CONSTANT
C      XK   = DIMENSIONLESS MASS FACTOR
C      ETA  = THE QUANTITY 0.5*XK*(VO)**2.0
C      EE   = E/ETA
C      SS   = (S-1.0)VO
C      SL   = SLIP FACTOR
C      IM   = NUMBER OF BAR ELEMENTS
C      KM   = NUMBER OF TIME ELEMENTS
C      DV   = CHANGE IN DIMENSIONLESS VELOCITY
C      DE   = CHANGE IN DIMENSIONLESS STRAIN
C      DT   = CHANGE IN DIMENSIONLESS TIME
C      DS   = CHANGE IN DIMENSIONLESS STRESS
       DIMENSION  S(500),V(500),IDENT(16),SS(500),E(500),DE(500),EE(500)
       DIMENSION SL(500)
     1 READ 801, IDENT
     2 READ 802,VO,DT,DX,XK,P
     3 READ 803,IM,KM
   801 FORMAT(16A5)
   802 FORMAT (5F10.5)
   803 FORMAT (2I10)
       TT=0.0
       PRINT 901,IDENT
       PRINT 902,VO,DT,DX,XK,P
       PRINT 903,IM,KM
       DO 101 I=1,IM
       S(I)=1.0
       V(I)=0.0
   101 CONTINUE
       Q=1.0/P
       V(1)=VO
       DO 102 K=1,KM
       WRITE TAPE 2,TT,(S(I),I=1,IM),S(IM+1),(V(I),I=1,IM),(E(I),I=1,IM)
       X=0.0
C      IMPACT OF FINITE MASS ON LEFT END
       IF (I-1) 17,17,18
    18 DV1=(-DT/XK)*(1.0-(V(2)-V(1))/DX)**Q
       IF (V(1)+DV1) 19,17,17
    19 V(1)=0.0
       GO TO 20
    17 V(1)=V(1)+DV1
    20 SL(1)=0.0
       DO 103 I=1,IM
    14 DE(I+1)=(V(I+1)-V(I))*DT/DX
       E(I+1)=E(I+1)+DE(I+1)
       IF (V(I)-V(I+1)) 29,28,28
    29 V(I+1)=V(I)
C      VISCOPLASTIC STRESS STRAIN RATE LAW
```

APPENDIX C (continued)

```
   28 S(I+1)=1.0+((V(I)-V(I+1))/DX)**Q
C     TEST FOR SLIPPING
   27 IF (V(I+1)-V(I)) 31,30,30
   30 SL(I+1)=0.0
      GO TO 32
   31 SL(I+1)=1.0
   32 IF (I-1) 11,11,36
   11 IF (SL(2)) 33,33,12
   12 DV=0.0
      GO TO 35
   36 IF (SL(I+1)-SL(I)) 37,33,34
   37 SL(I+1)=1.0
      GO TO 33
   34 DV=(-1.0)*S(I+1)*DT/XK
      IF (V(I)+DV) 40,40,41
   41 V(I)=V(I)+DV
      GO TO 42
   40 V(I)=0.0
   42 DO 104 I1=1,I
      V(I1)=V(I)
  104 CONTINUE
      GO TO 15
C     IMPULSE MOMENTUM LAW
   33 DV=(S(I)*SL(I)-S(I+1)*SL(I+1))*DT/DX
   35 V(I)=V(I)+DV
   15 X=X+DX
  103 CONTINUE
      TT=TT+DT
  102 CONTINUE
      REWIND 2
      DO 105 K=1,KM
      READ TAPE 2,TT,  (S(I),I=1,IM),S(IM+1),(V(I),I=1,IM),(E(I),I=1,IM)
      IM1=IM+1
      DO 107 I=1,IM1
      SS(I)=(S(I)-1.0)/V0
  107 CONTINUE
      PRINT 906,TT,(SS(I),I=1,IM1)
  906 FORMAT(1HS,F5.3,1X,21F5.2)
  105 CONTINUE
      REWIND 2
      DO 106 K=1,KM
      READ TAPE 2,TT,  (S(I),I=1,IM),S(IM+1),(V(I),I=1,IM),(E(I),I=1,IM)
      PRINT 907,TT,  (V(I),I=1,IM)
  907 FORMAT (1HS,F5.3,3X,20F5.2)
  106 CONTINUE
      REWIND 2
      ETA=0.5*XK*(V0**2.0)
      DO 108 K=1,KM
      READ TAPE 2,TT,  (S(I),I=1,IM),S(IM+1),(V(I),I=1,IM),(E(I),I=1,IM)
      DO 109 I=1,IM
      EE(I)=E(I)/ETA
  109 CONTINUE
      PRINT 908,TT,(EE(I),I=1,IM)
  908 FORMAT (1HS,F5.3,3X,20F5.2)
  108 CONTINUE
  500 STOP
  901 FORMAT (1H1,40X,27HVISCOPLASTIC IMPACT OF BARS/1H0,16A5)
  902 FORMAT(
     132H0IMPACT VELOCITY                = ,  F10.3,10H          //
     232H ELEMENT OF TIME                = ,  F10.3,10H          //
```

APPENDIX C (continued)

```
   332H ELEMENT OF BAR LENGTH          = ,  F10.3,10H          //
   432H DIMENSIONLESS MASS FACTOR      = ,  F10.3,10H          //
   532H OVERSTRESS EXPONENT            = ,  F10.3,10H          /)
903 FORMAT(
   132HONUMBER OF BAR ELEMENTS         = ,  I3,    //
   232H NUMBER OF TIME ELEMENTS        = ,  I3/1H1)
    END
```

CHAPTER V

AN ANALYSIS OF ARMOR PENETRATION DYNAMICS

by R. Minnich

5.1 Introduction and Assumptions

A reasonable set of assumptions which may be made in analyzing the motion of a projectile as it penetrates armor material are:

i) The projectile is assumed to be a non-deforming body of arbitrary, but known, geometry and mass.

ii) The projectile's motion during penetration is resisted by a system of forces which depend upon geometry, initial velocity, and the material properties of the armor. These forces are assumed to be of two types: the resistance of the material to penetration due to its compressive resistance and the inertial resistance of the material as it is displaced by the projectile.

iii) Frictional effects are neglected at present but could be added.

5.2 Derivation of the Governing Equation

Because frictional effects are neglected, the resisting force components are assumed to be acting normal to the projectile surface. Figure 12 shows the force component acting on the elemental surface area, $d\hat{A}s$.

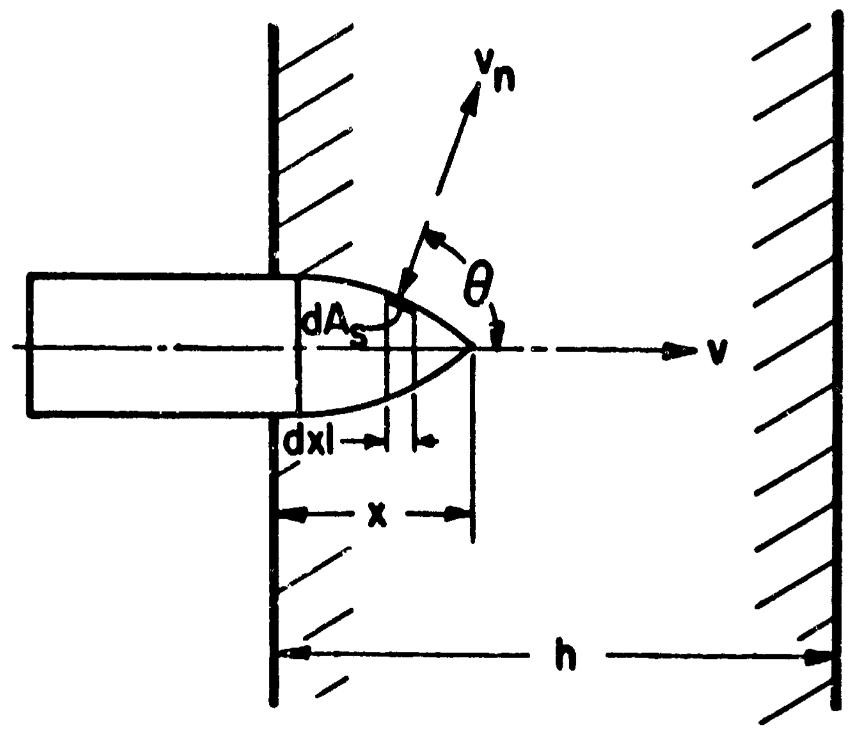

FIG. 12 PROJECTILE PENETRATING AN ARMOR MATERIAL

The compressive resistance force is considered to be uniformly distributed over the surface area of the projectile tip. This force per unit area, a property of the armor material, will be denoted as σ.

The inertial resistance is not uniformly distributed over the surface area of the projectile but is dependent upon the shape of the projectile. To find this force it is assumed that the change in kinetic energy of the armor material being displaced is equal to the work done by the inertial force on an element of armor material. This relation can be expressed as

$$df_n \ dx_n = 1/2 \ v_n^2 \ dm \tag{5.1}$$

where:

df_n = normal force acting on a differential area (dAs) of the projectile surface

dx_n = displacement of the element of mass of the armor material normal to the projectile surface.

v_n = velocity of the element of mass in a direction normal to the surface of the projectile (equal to the normal component of the projectile velocity).

dm = mass of the differential element of the armor material being displaced (equal to $\rho \ dx_n \ dAs$, where ρ is the mass density of the armor material).

The substitution of $\rho \ dx_n \ dAs$ for dm and $v \cos \theta$ for v_n yields

$$df_n = 1/2 \ \rho \ (\cos^2\theta) \ v^2 \ dAs \tag{5.2}$$

From these equations Adams and Tsai (11) derive an equation of motion given by

$$Mv \frac{dv}{dx} = - \int_{A_s} \sigma \cos \theta \, dA_s - \int_{A_s} \frac{1}{2} \rho \, (\cos^3\theta) \, v^2 \, dA_s \qquad (5.3)$$

where the component of each force per unit area in the direction of motion is integrated over the frontal surface area of the projectile. They then proceed to solve this problem in three parts for three conditions which arise, depending upon the location of the projectile nose in the armor material. The three positions are: the entrance phase when the surface area of the projectile increases with the depth of penetration, the phase where the nose is completely imbedded, and the exit phase where the area decreases to zero as the nose emerges.

CHAPTER VI

COMPUTER ANALYSIS OF PENETRATION

6.1 Introduction

The theory developed in Chapter V is not limited to the calculation of residual velocities for complete penetration. Much more complicated mathematical procedures would be required to solve the governing equation to predict depth of penetration if complete penetration does not occur. To avoid these limitations, a computer analysis was developed which combines the derived expressions for the forces with the physical laws governing the system and sidesteps the mathematics. The program has all the advantages mentioned in Section 3.1. The resulting program is general, and enables one to predict the depth of penetration or residual velocity depending upon the initial conditions.

6.2 Development of the Program

The projectile was divided into cells by cross-sections normal to its axis. A typical cell is shown in Fig. 12, its length being dxl and its surface area being dAs. The method employed was to sum the forces acting on each cell and use an impulse-momentum relationship to calculate the velocity change of the projectile for each time interval.

As was stated in Chapter V the forces are assumed to consist of two kinds, an inertial force and a compressive resistance force. Let f_1 and f_2 denote the sum of the components in the direction of motion, of the compressive force and the inertia force respectively. Then df_1 represents the component in the direction of motion acting on an individual cell of surface area dAs and is defined as

$$df_1 = \sigma \cos \theta_i \, dAs_i \tag{6.1}$$

where the subscript i denotes the i-th cell. Similarly, df_2 is given by

$$df_2 = \frac{1}{2} \rho \cos^3\theta_i v^2 dAs_i \tag{6.2}$$

The sum of f_1 and f_2 will then represent the total force acting on the projectile during a time dt. This sum is given as

$$f = f_1 + f_2 \tag{6.3}$$

The velocity change dv will then be calculated using the impulse-momentum law. This is expressed as

$$dv = - fdt/M \tag{6.4}$$

Because the forces act opposite the direction of motion, a minus sign is included in the above equation. This velocity increment is then added to the velocity of the projectile to obtain the velocity at a given time or

$$v \leftarrow v + dv \tag{6.5}$$

The distance the projectile travels during each time interval, dx, is obtained by integrating the velocity over the time interval dt. This is expressed as

$$dx = vdt \tag{6.6}$$

The total depth of penetration or the distance from the surface of the plate where the initial contact is made to the nose of the projectile is the sum of all these incremental dx's.

$$x \leftarrow x + dx \tag{6.7}$$

Equations 6.1 - 6.7 are combined in a program which calculates the force acting on the projectile during each time interval and finds the velocity change during this interval. This process begins when the projectile first penetrates the plate and stops when the projectile velocity becomes zero (for partial penetration) or when the projectile leaves the plate. We again add a DO loop which directs the repetitive operations from $i = 1$ to $i = i_m$ to obtain f and use another DO loop to repeat this process for successive time intervals. The program then appears as below.

Input

DO $k = 1, \cdots k_m$

DO $l = 1, \cdots l_m$

$df_1 = \sigma \cos \theta_l \, dAs_l$ compressive resistance force

$df_2 = \frac{1}{2} \rho \cos^3\theta_l v^2 dAs_l$ inertia force

$f_1 \leftarrow f_1 + df_1$

$f_2 \leftarrow f_2 + df_2$

$f = f_1 + f_2$

$dx = vdt$ definition

$dv = - fdt/M$ impulse-momentum law

$v \leftarrow v + dv$

$x \leftarrow x + dx$

$t \leftarrow t + dt$

output

The above is the basic program except for geometry calculations, which will be discussed below, input statements and output statements, and various tests to determine if the plate has been completely penetrated or if the projectile has stopped.

6.3 Geometry

The various projectile configurations dealt with and the dimensions which need to be specified to completely define them are shown in Fig. 13. Two quantities had to be determined for each cell in order for the force calculations to be carried out, namely the surface area, dAs_i, and the cosine of the angle between the normal of the cell and the direction of motion, $\cos \theta_i$. These quantities were determined at the beginning of the program and stored until they were needed. Their calculation for each projectile configuration follows.

6.3.1 <u>The Ogive</u>

Fig. 14 shows the ogive with its detailed dimensions. The quantities y_o, y_3 and y were required to be computed for the calculation of the cosine and the surface area for each cell. OB represents the entire surface of the nose while BN is the cylindrical surface. OP is the axis of the projectile. dAs_i is the surface area of the i-th cell and dxl is its length. The origin of the x and y axes was taken as shown. The needed quantities can now be expressed in terms of the known quantities for each cell. That is

$$y_o = (R^2 - xl_3^2)^{1/2} \tag{6.8}$$

$$y_3 = (R^2 - (xl_3 - xls_i)^2)^{1/2} \tag{6.9}$$

$$y = y_3 - y_o \tag{6.10}$$

dAs_i can therefore be expressed as

$$dAs_i = 2\pi R(1.0 - y_o/y_3)dxl \tag{6.11}$$

and the cosine as

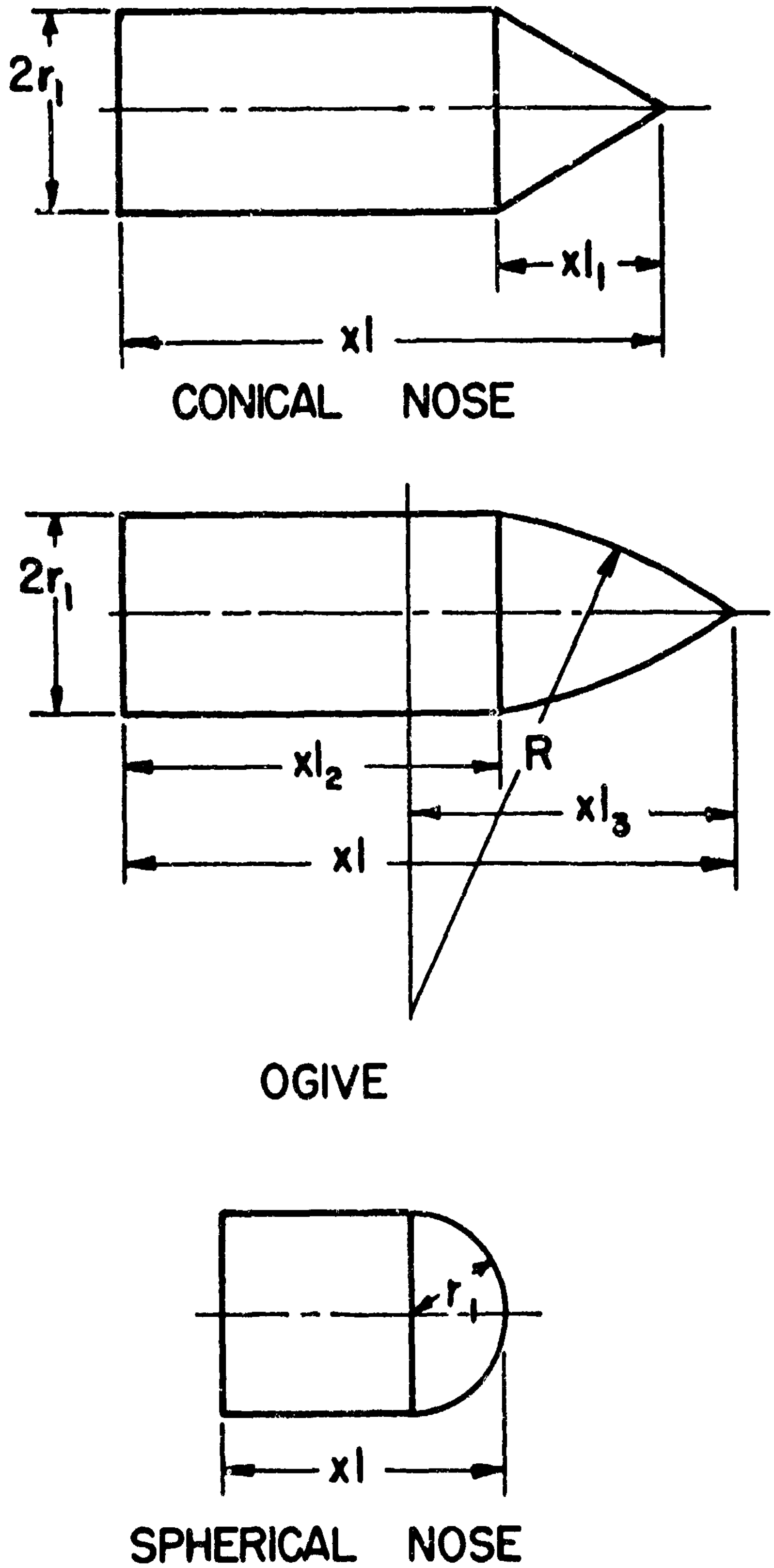

FIG. 13 VARIOUS PROJECTILE CONFIGURATIONS WITH DIMENSIONS

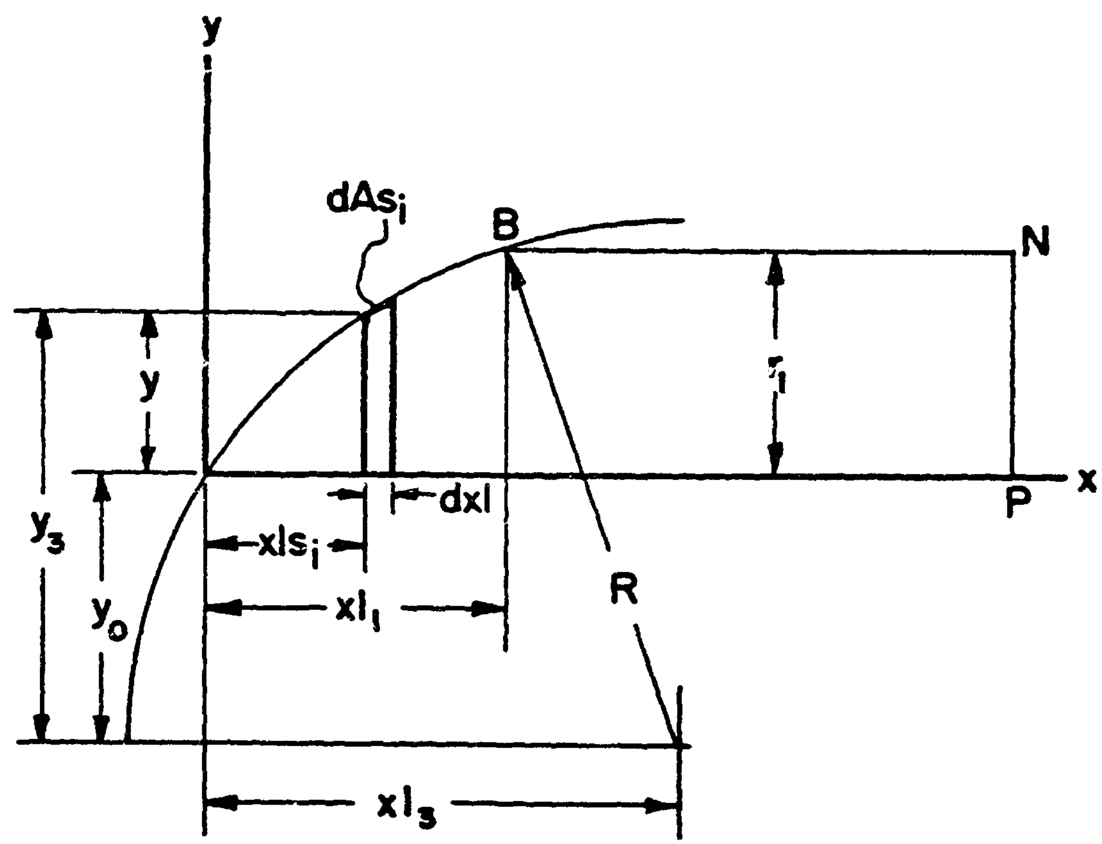

FIG. 14 GEOMETRY OF OGIVE IN DETAIL

$$\cos \theta_i = (xl_3 - xls_i)/R \tag{6.12}$$

For the cylindrical portion of the projectile the surface area of each cell is constant and is given by

$$dAs_i = 2\pi \; r_1 \; dxl \tag{6.13}$$

the $\cos \theta_i$ in this region is equal to zero.

6.3.2 Hemispherical End Cap

The hemispherical end cap projectile was useful to study because of the availability of experimental results.

The cylinder with hemispherical cap is a special case of the ogive with $R = r_1$ and $xl_3 = r_1$. Therefore the only dimensions required to define it are r_1 and xl. The surface area for elements of equal width is the same for a sphere and is given by

$$dAs_i = 2\pi \; r_1 \; dxl \tag{6.14}$$

This is also the expression for dAs_i in the cylindrical portion.

The cosine of the spherical portion is defined as

$$\cos \theta_i = 1.0 - (xls_i/r_1) \tag{6.15}$$

and again the cosine of the cylindrical portion is zero.

It should be pointed out that a cylinder with a hemispherical nose was programmed rather than a sphere because it is more general. Since the forces only depend upon the surface area of the nose, a sphere can be made a special case of the hemispherical cylinder by setting the length of the cylindrical portion equal to 2/3 of the radius of the sphere. In this way the mass will be equal to that of a sphere.

6.3.3 Conical End Cap

The cone is defined if r_1, xl, and xl_1 are known. The distance from the projectile axis to the surface at xls_i is

$$y_i = r_1\, xls_i / xl_1 \tag{6.16}$$

The surface area of the i-th cell which is the surface area of the frustum of a cone is

$$dAs_i = \pi\,(y_i + y_{i+1})(dxl^2 + (y_{i+1} - y_i)^2)^{1/2} \tag{6.17}$$

The cosine of all the cells is a constant value equal to

$$\cos\theta_i = r_1/(r_1^{\,2} + xl_1^{\,2})^{1/2} \tag{6.18}$$

The surface area and the cosine for the cylindrical portion is the same as the other two cases.

These three configurations are related so that a separate program for each is not required. The given dimensions read into the program determine which configuration is being considered. The details of this are shown as comments in the program itself found in Appendix B.

CHAPTER VII

RESULTS AND CONCLUSIONS OF PENETRATION STUDY

7.1 Verification of the Program

The initial computations were concerned with checking the program's results with those of Adams and Tsai. The easiest method of doing this was to see if a computed curve of residual velocity vs initial velocity agreed with theirs using the same initial conditions. The initial conditions were

plate material	=	polyethylene
plate thickness	=	0.65 inches
plate mass density	=	0.89×10^{-4} lb-sec^2/in^4
compressive resistance	=	25,670 lb/in^2
projectile shape	=	sphere

The resulting curve from the program is shown in Fig. 15 as the curve for the spherical projectile. Because it coincided very closely with the Adams and Tsai's curve there was no need to include both.

All the work in (11) was shown to agree with experimental studies. However, because they conducted the experiments themselves it was decided to check the program with other experimental results. Gupta and Davids (17) performed studies on the penetration of fiberglass reinforced plastics.

Because the compressive resistance, σ, is a property of the armor material it must first be found. In order to find σ one must know all the conditions of a ballistics test. If the initial velocity

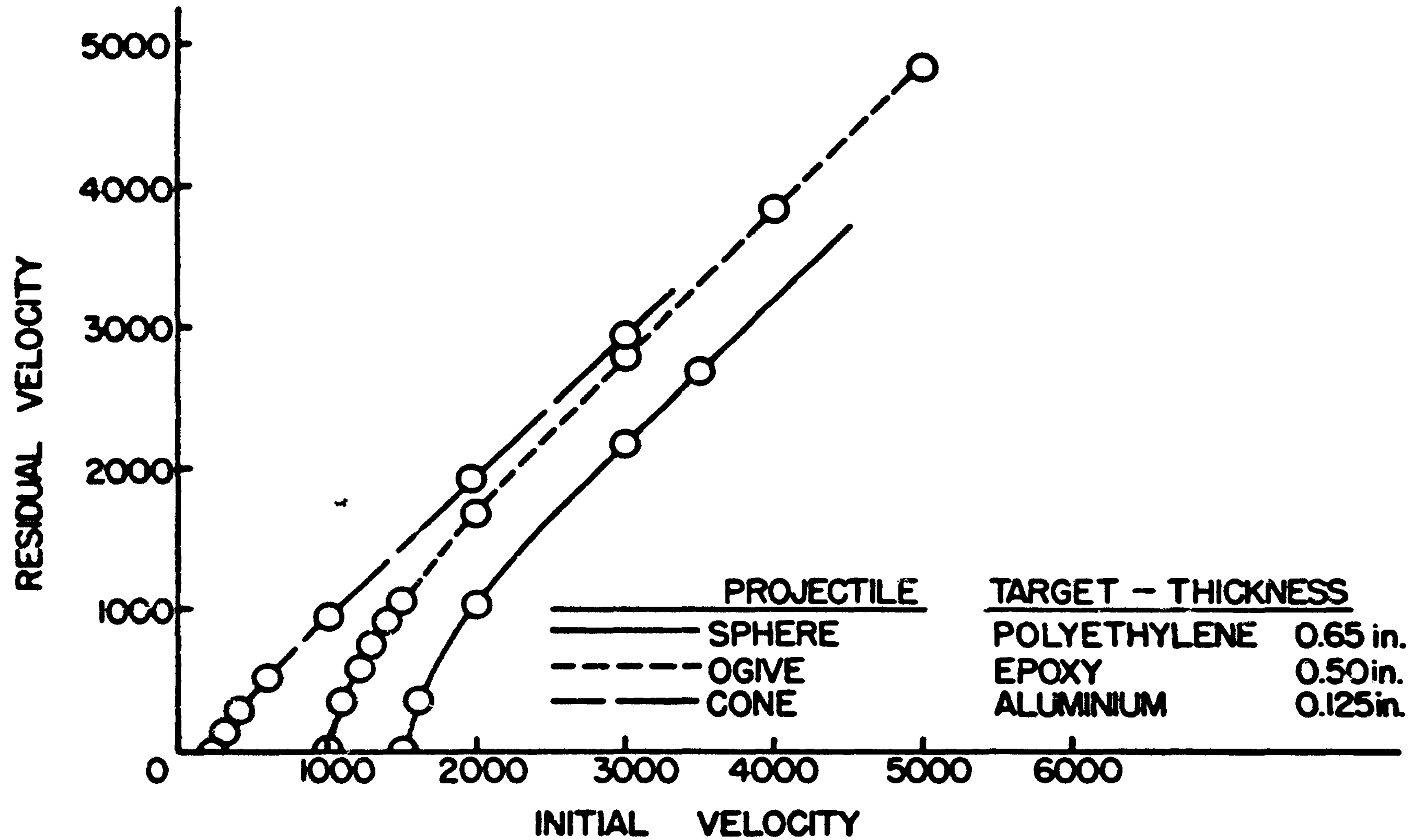

FIG. 15 RESIDUAL VELOCITY vs. INITIAL VELOCITY

and the residual velocity are known, different values of σ may be tried in the program with the intention of finding one which yields the known residual velocity.

This process is not so much of a trial and error effort as it might seem. A curve of residual velocity vs σ may be drawn which will enable one to find the correct value of σ from the known value of v_r.

The program was checked with the first three entries in Gupta and Davids' Table 2. The third entry which was for a plate 0.22 inches thick impacted by a 0.22 caliber projectile with an initial velocity of 1270 ft/sec was used in the calculation of σ, the intent being to determine a value which gave a residual velocity of 750 ft/sec. This value was found to be 127,500 lb/in^2. The program then correctly predicted the residual velocities for the different plate thicknesses found in (17). The results are summarized in Table 3.

Table 3

Comparison with Experimental Results of (17)

No	Thickness (inches)	Initial Velocity (ft/sec)	Residual Velocity ft/sec	
			Experimental	Program
1	0.22	1270	750	750
2	0.09	1270	1090	1085
3	0.13	1270	1000	995

When the value of σ is found the program becomes capable of predicting results of different initial conditions for any projectile configuration. It should be remembered that this value is only a property of that plate material for which it was found.

Because of the excellent agreement shown in Table 3 with experimental results it was concluded that the program is valid.

7.2 Review of Significant Runs

Curves showing residual velocity vs initial velocity were desired for the other two projectile configurations (the ogive and the cone). They are shown in Fig. 14 also. The compressive resistance constant was obtained from (11) for epoxy and from (12) for aluminum.

All of the curves in Fig. 15 approach a straight line. It can be seen that this straight line portion has the same slope regardless of plate material or projectile configuration. This straight line portion has a slope of unity which means that a change in the initial velocity will produce the same change in the residual velocity.

Figures 16, 17, and 18 show the force vs time plots for the three projectile configurations. It is seen that they are all of the same basic shape with the only differences being the entrance and exit regions. The rapid rise to the maximum force which the sphere exhibits indicates that this configuration is not a very good penetrator as compared to the ogive. The cone's ability to penetrate varies from being the best to the worst penetrator depending upon the angle the surface area of its nose makes with its axis.

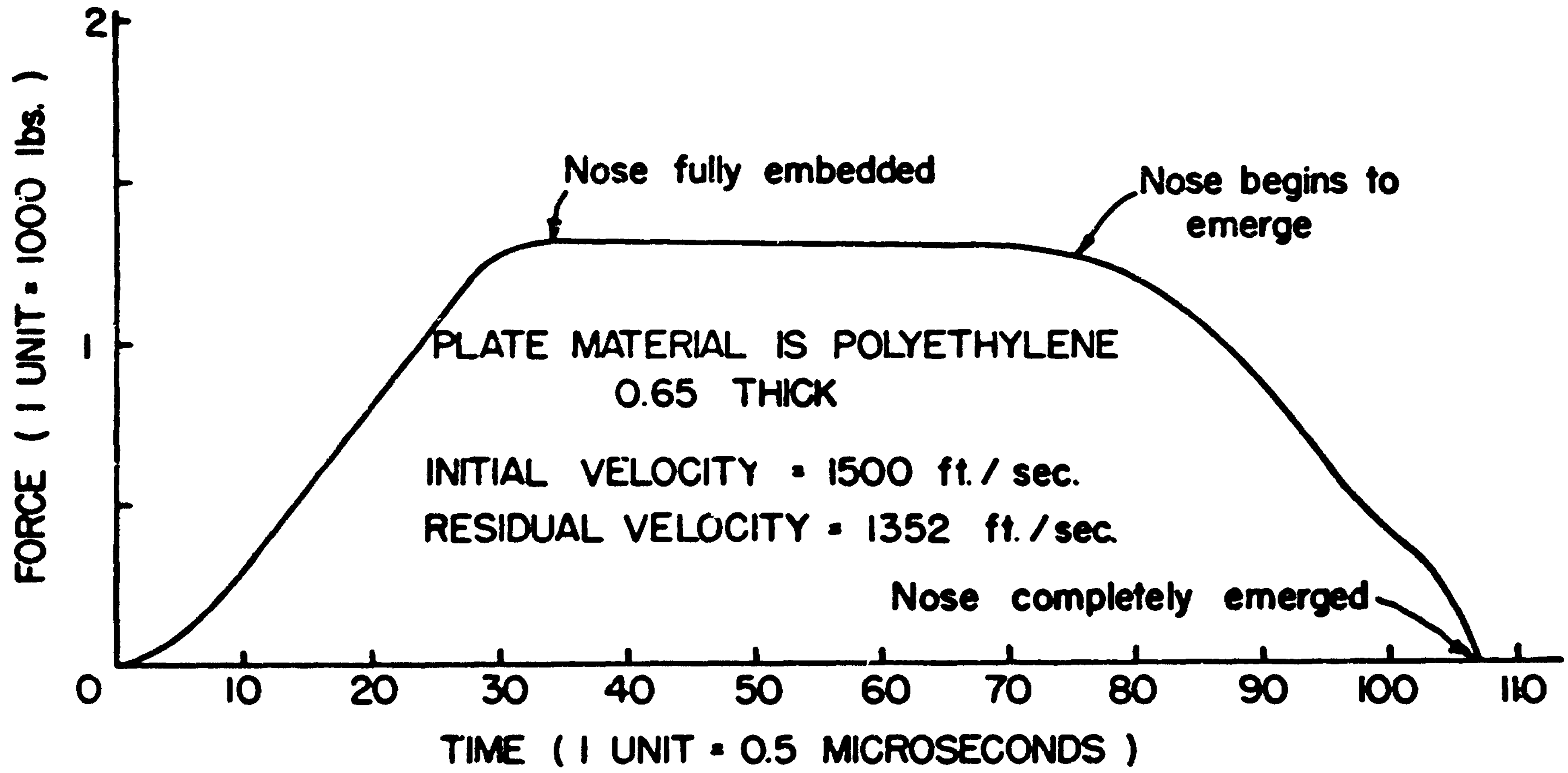

FIG. 16 FORCE vs.TIME FOR OGIVE PROJECTILES

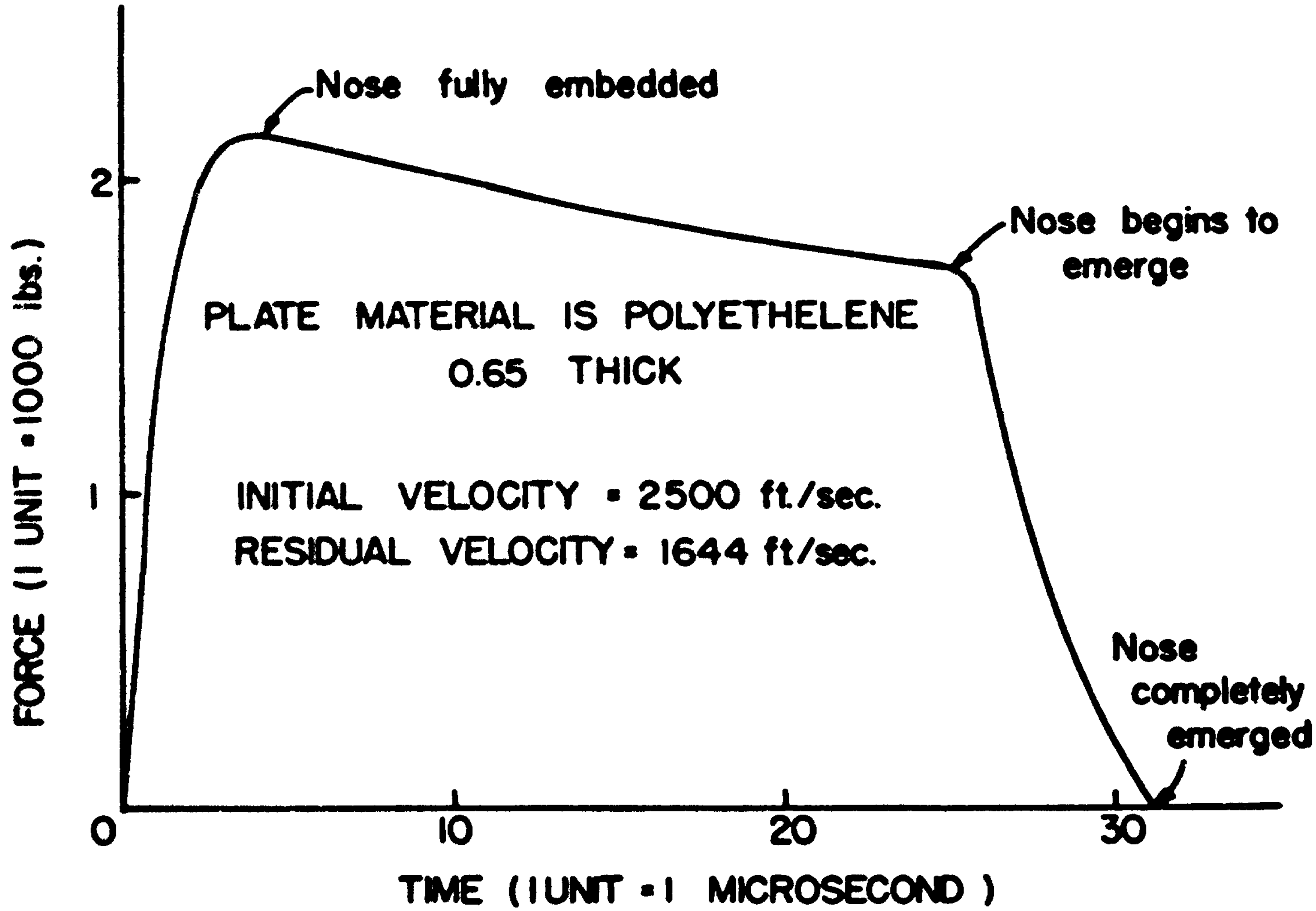

FIG. 17 FORCE vs. TIME FOR SPHERICAL PROJECTILES

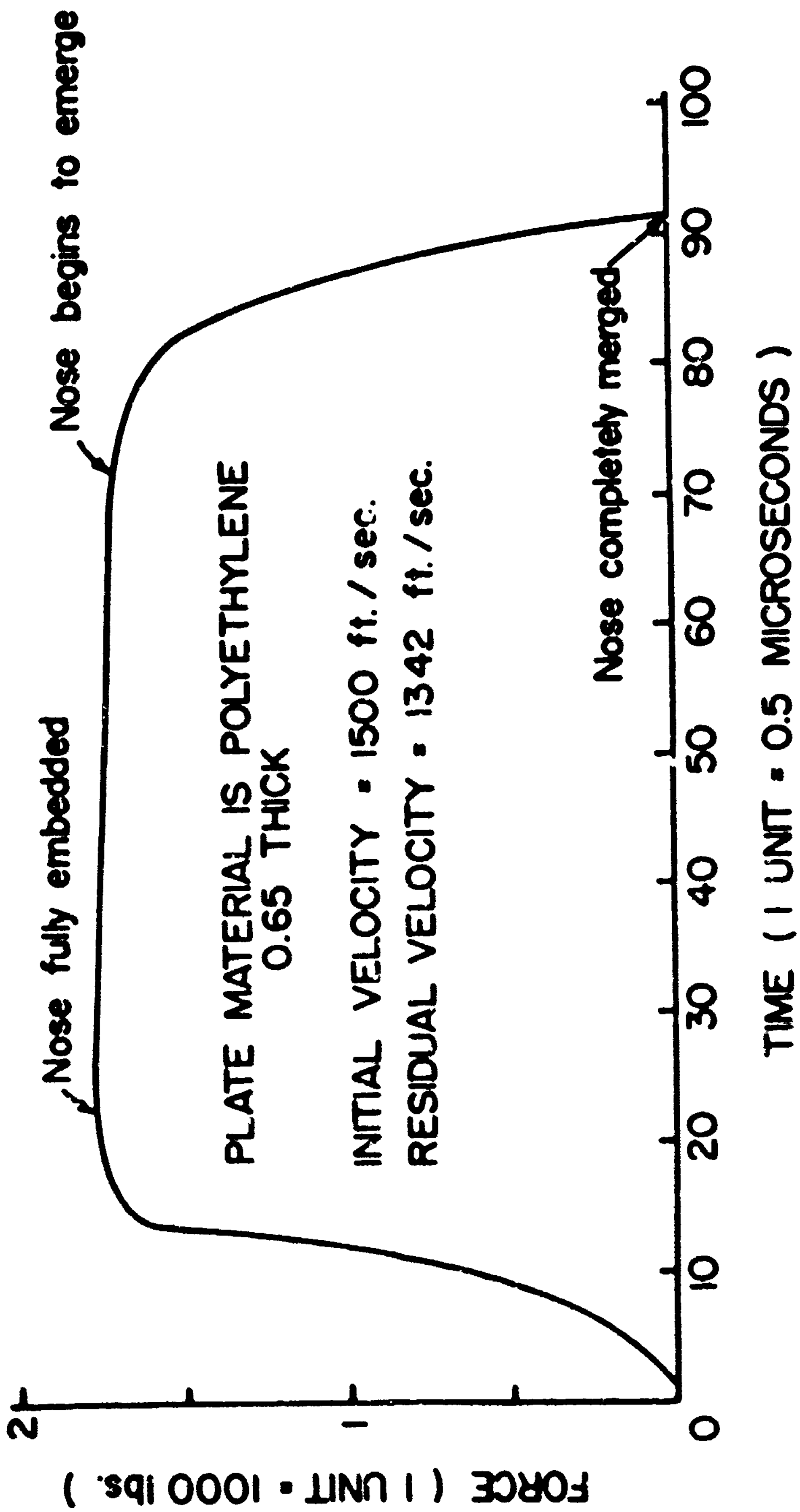

FIG. 18 FORCE vs. TIME FOR CONICAL PROJECTILE

Tables 4, 5, and 6 contain the results of all the runs made for ogives, spheres, and cones respectively. These tables contain the runs previously mentioned plus some other general runs for other plate materials and initial conditions. These were made merely to show the applicability of the program.

It can be concluded that for penetration in which the projectile is not deformed, this analysis is valid both in predicting depth of penetration and residual velocities, and is very efficient in computational accuracy.

Table 4

Summary of Computations for Ogive Projectiles

No.	Initial Velocity (ft/sec)	Residual Velocity (ft/sec)	Plate Material	Plate Thickness (inches)	Remarks
1	1000	0	Epoxy	0.5	Tip traveled 0.66 inches
2	1100	358	Epoxy	0.5	Complete Penetration
3	1200	596	Epoxy	0.5	Complete Penetration
4	1300	775	Epoxy	0.5	Complete Penetration
5	1400	928	Epoxy	0.5	Complete Penetration
6	1500	1068	Epoxy	0.5	Complete Penetration
7	2000	1688	Epoxy	0.5	Complete Penetration
8	3000	2781	Epoxy	0.5	Complete Penetration
9	4000	3819	Epoxy	0.5	Complete Penetration
10	5000	4835	Epoxy	0.5	Complete Penetration
11	1000	0	Polyethylene	0.65	Tip traveled 0.67 inches
12	1500	1351	Polyethylene	0.65	Complete Penetration
13	2000	1885	Polyethylene	0.65	Complete Penetration

Table 5

Summary of Computations for Hemispherical End Cap

No.	Initial Velocity (ft/sec)	Residual Velocity (ft/sec)	Plate Material	Plate Thickness (inches)	Remarks
1	1490	0	Polyethylene	0.65	Tip traveled 0.66 inches
2	1600	382	Polyethylene	0.65	Complete Penetration
3	2000	1020	Polyethylene	0.65	Complete Penetration
4	2500	1644	Polyethylene	0.65	Complete Penetration
5	3000	2178	Polyethylene	0.65	Complete Penetration
6	3500	2673	Polyethylene	0.65	Complete Penetration
7	1270	750	F.R.P.*	0.22	Complete Penetration
8	1270	1085	F.R.P.	0.09	Complete Penetration
9	1270	995	F.R.P.	0.13	Complete Penetration

*F.R.P. refers to fiberglass reinforced plastic

Table 6

Summary of Computations for Conical End Cap

No.	Initial Velocity (ft/sec)	Residual Velocity (ft/sec)	Plate Material	Plate Thickness (inches)	Remarks
1	100	0	Aluminum	0.126	Tip traveled 0.099 inches
2	200	0	Aluminum	0.126	Tip traveled 0.165 inches
3	300	150	Aluminum	0.126	Complete Penetration
4	400	301	Aluminum	0.126	Complete Penetration
5	600	534	Aluminum	0.126	Complete Penetration
6	652	566	Aluminum	0.126	Complete Penetration
7	1000	954	Aluminum	0.126	Complete Penetration
8	1968	1911	Aluminum	0.126	Complete Penetration
9	3000	2951	Aluminum	0.126	Complete Penetration
10	3281	3221	Aluminum	0.126	Complete Penetration
11	500	0	Polyethylene	0.65	Tip traveled 0.54 inches
12	1500	1342	Polyethylene	0.65	Complete Penetration
13	2500	2372	Polyethylene	0.65	Complete Penetration

Conclusions

The assumptions and the analytical solution of ductile hole enlargement are presented in Chapter V. The forces acting on the projectile are assumed to be of two kinds, the inertial resistance of the plate material and the compressive resistance. The expressions for these forces are given and are then used to derive the governing equation. The equation for predicting the residual velocity after penetration is also given.

The next chapter contains the development of the computer program used to solve the ductile hole enlargement problem. The first section describes the incorporation of the expressions for the forces into a program which will yield the velocity change during a set time interval. The next section discusses the geometry of the three projectiles considered and the method of dividing them into cells. The expressions for the surface area of each cell and the cosine of the angle between the normal to the surface of the cell and the horizontal axis are also developed.

The results of the investigation for ductile hole enlargement are contained in Chapter VII. The first results presented are those needed to verify the computer program. Both experimental and theoretical ballistic data are compared with the output of the program. The comparison in all cases is excellent. Curves showing force vs time and residual velocity vs initial velocity for all the projectile configurations are given. Also contained in this chapter are tables with the results of all the computer runs.

The ability of various projectile configurations to penetrate a plate is determined. It is concluded that a sphere is a much poorer penetrator than an ogive. The cone varies in penetration ability from the best penetrator to the worst depending upon the angle the surface of the nose makes with the axis.

8.2 Suggestions for Further Study

The most obvious suggestion for further work in the plug formation problem would be to extend this investigation to other materials. In order for this task to be undertaken, photographs must be obtained from which experimental deflection curves can be made. All the conditions of the impact must be known, i.e., initial velocity of the projectile, mass of the projectile, plate thickness and density of the plate.

A criterion for complete penetration would be a worthwhile extension. As was mentioned before the program does not contain this important aspect. By inspection of the final deflection curves, one can obtain a fairly accurate guess as to what initial conditions will cause complete penetration; but specific results are needed.

Worthwhile studies could be begun on other types of failures caused by impact. A computer program incorporating the material laws associated with scabbing or dishing would be of interest. Some computer work (18) has been done for cratering.

Also minor revisions can be made in the present program to give radial strains and strain rates. A condition which prescribes the initial velocity around a hole in the plate might be of some value.

The program for the penetration of nonmetallic materials and ductile hole enlargement in metals is fairly complete. As is shown the correlation with experiment is excellent. Some approaches in this area take into account a friction force also. The program could very easily be extended to include this if it were deemed necessary.

It is believed that an extension of this program can be used to solve problems of water entry. For this problem water is considered incompressible so that the compressive force would be zero.

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APPENDIX A

COMPUTER PROGRAM FOR PLUG FORMATION

```
      COMPILE   RUN  FORTRAN
C      RIGID-VISCOUS MODEL FOR PLUG FORMATION IN PLATES
C      UNITS IN IN-LB-SEC
C      V         = VELOCITY IN Z-DIRECTION
C      F(I)      = VISCOUS FORCE ON LATERAL AREA OF I-TH CELL
C      XMR       = MASS OF EACH CELL
C      R         = RADIAL COORDINATE
C      AREA      = INSIDE LATERAL AREA OF CELL
C      W         = DISPLACEMENT IN Z-DIRECTION
C      S         = SHEAR STRESS ON R-PLANE IN Z-DIRECTION
C      XMOD      = STABILITY MODULUS FOR EACH CELL
C      XM        = CUMULATIVE MASS OF FIRST I CELLS
C      S1        = NONDIMENSIONAL SHEARING STRESS
C      VV        = NONDIMENSIONAL VELOCITY
C      WW        = NONDIMENSIONAL DISPLACEMENT
C      TT        = NONDIMENSIONAL TIME
C      DEN       = WEIGHT DENSITY OF PLATE MATERIAL
C      GNU       = COEFFICIENT OF VISCOSITY OF PLATE MATERIAL
C      SY        = IMPACT YIELD CONSTANT
C      H         = PLATE THICKNESS
C      VB        = INITIAL VELOCITY OF PROJECTILE
C      RO        = RADIUS OF PROJECTILE
C      XMB       = MASS OF PROJECTILE
C      IM        = NUMBER OF CELLS
C      KM        = NUMBER OF TIME INTERVALS
C      DT        = CHANGE IN TIME
C      VS        = VELOCITY SCALE FACTOR
C      WS        = DISPLACEMENT SCALE FACTOR
C      SS        = STRESS SCALE FACTOR
C      DR        = CHANGE IN RADIUS
C      FMT1      = VARIABLE FORMAT FOR PRINT STATEMENTS
C      RHO       = MASS DENSITY OF PLATE MATERIAL
C      XMRO      = MASS OF PLATE MATERIAL UNDER IMPACT
C      VO        = INITIAL VELOCITY OF PLATE MATERIAL UNDER IMPACT
       DIMENSION V(100),F(100),XMR(100),R(100),AREA(100),VV(100),
      1WW(100),W(100),S(100),S1(100),IDENT(16),FMT1(14),XMOD(100),
      2 SL(100),XM(100)
    1  READ 801,(IDENT(I),I=1,14)
  801  FORMAT(14A5)
    2  READ 802, NAME1,NAME2,DEN,GNU,SY,H
  802  FORMAT(2A5,3F10.0,F10.4)
    3  READ 803,VB,RO,XMB
  803  FORMAT(2F10.3,E10.4)
    4  READ     804,IM,KM,DT,VS,WS,DR,SS,K1
  804  FORMAT(2I5,2E10.4,2F10.4,1E10.4,I5)
       IF (IM)99,99,5
    5  READ 801,FMT1
       PI=3.1415927
       PHO=DEN/384.
         XMRO=PI*RO**2*H*RHO
       VO=(VB*XMB)/(XMB+XMRO)
       REWIND 2
       PRINT 900,(IDENT(I),I=1,14)
       PRINT 901,NAME1,NAME2,DEN,RHO,GNU,SY
       PRINT 902,VO,RO,H,VB,XMB
       PRINT 903,IM,KM,DT,VS,DR,WS,SS
       PRINT 905
       R(1)=0.00
       DO 101 I=1,IM
       AREA(I)=0.0
```

APPENDIX A. Cortinued

```
      R(I+1)=R(I)+DR
      AREA(I+1)=PI*R(I+1)*2.0*H
      W(I)=0.0
      WW(I)=W(I)/WS
      S(I)=0.0
      S1(I)=S(I)/SS
      F(I)=0.0
      IF(R(I+1)-RO)21,21,22
   21 V(I)=VO
      IF(I-1)15,15,16
   15 XM(1)=(PI*R(2)**2*RHO*H)*(XMB/XMRO+1.0)
      XMR(1)=XM(1)
      GO TO 23
   16 XM(I)=(PI*R(I+1)**2*RHO*H)*(XMB/XMRO+1.0)
      XMR(I)=XM(I)-XM(I-1)
      GO TO 23
   22 V(I)=0.0
          XMR(I)=PI *(R(I)+R(I+1))*(R(I+1)-R(I))*RHO*H
      XM(I)=XM(I-1)+XMR(I)
   23 VV(I)=V(I)/VS
C     MODULUS TEST FOR STABILITY
      XMOD(I)=GNU*AREA(I+1)*DT/(XMR(I)*DR)
      PRINT 906,I,XMR(I),XM(I),AREA(I),XMOD(I)
  101 CONTINUE
      T=0.0
      PRINT 904
      DO 103  K=1,KM
      TT=T/DT
      PRINT FMT1,TT,(VV(I),I=1,IM)
      WRITE TAPE 2,TT,(WW(I),I=1,IM),(S1(I),I=1,IM)
      DO 106 I=1,IM
      W(I)=W(I)+V(I)*DT
      WW(I)=W(I)/WS
  106 CONTINUE
      SL(1)=0.0
      DO 102  I=1,IM
      IF(K -K1)40,40,41
   40 IF(I-1)42,42,43
   42 PRINT 909
   43 PRINT 910,I,V(I),S(I),F(I),SL(I),DV
C   RIGID-PLASTIC-VISCOUS STRESS-STRAIN LAW
   41 S(I+1)=-SY+GNU*(V(I+1)-V(I))/DR
      S1(I+1)=S(I+1)/SS
      F(I+1)=S(I+1)*AREA(I+1)
C     TEST FOR SLIPPING
      IF(V(I)-V(I+1))24,24,25
   24 SL(I+1)=0.0
      GO TO 26
   25 SL(I+1)=1.0
   26 IF(SL(I)-SL(I+1))27,28,29
   27 IF(F(I+1)-F(I)*XM(I)/XM(I-1))30,31,31
   30 SL(I)=1.0
      GO TO 28
   31 DV= F(I+1)/XM(I)*DT
      IF(V(I)+DV)60,60,61
   61 V(I)=V(I)+DV
      GO TO 62
   60 V(I)=0.0
   62 DO 108 I1=1,I
      V(I1)=V(I)
```

APPENDIX A. Continued

```
      VV(I1)=V(I1)/VS
  108 CONTINUE
      GO TO 35
   29 IF(F(I+1)-F(I))32,32,33
   32 V(I)=0.0
      DV=0.0
      GO TO 35
   33 SL(I+1)=1.0
C     IMPULSE MOMENTUM LAW
      DV=(F(I+1)*SL(I+1)-F(I)*SL(I))/XMR(I)*DT
      SL(I+1)=0.0
      GO TO 34
C     IMPULSE MOMENTUM LAW
   28 DV=(F(I+1)*SL(I+1)-F(I)*SL(I))/XMR(I)*DT
   34 IF(V(I)+DV)50,50,51
   51 V(I)=V(I)+DV
      GO TO 35
   50 V(I)=0.0
   35 VV(I)=V(I)/VS
  102 CONTINUE
      T=T+DT
  103 CONTINUE
      PRINT 907
      REWIND 2
      DO 104 K=1,KM
      READ TAPE 2,TT,(WW(I),I=1,IM),(S1(I),I=1,IM)
      PRINT FMT1,TT,(WW(I),I=1,IM)
  104 CONTINUE
      REWIND 2
      PRINT 908
      DO 105 K=1,KM
      READ TAPE 2,TT,(WW(I),I=1,IM),(S1(I),I=1,IM)
      PRINT FMT1,TT,(S1(I),I=2,IM)
  105 CONTINUE
      GO TO 1
   99 STOP
  900 FORMAT(1H1,14A5///////)
  901 FORMAT(1H ,35X,31HVISCO-PLASTIC ANALYSIS OF PLATE   //
     132H MATERIAL                             = ,2A5//
     232H WEIGHT DENSITY                   DEN = ,F10.5,9H LB/IN**3    //
     332H RHO=DEN/G                        RHO = ,E10.4,16H LB-SEC**2/IN**4    //
     432H COEFFICIENT OF VISCOSITY GNU = ,F10.3,13H LB-SEC/IN**2 //
     532H IMPACT YIELD CONSTANT         SY = ,E10.4,9H LB/IN**2 )
  902 FORMAT(1H0,31HINITIAL VELOCITY               VO = ,E10.3,7H IN/SEC  //
     132H RADIUS OVER WHICH VO ACTS RO = ,F10.3,3H IN //
     232H THICKNESS OF PLATE             H = ,F10.3,3H IN //
     332H VELOCITY OF BULLET            VB = ,E10.3,7H IN/SEC //
     432H MASS OF BULLET               XMB = ,E10.3,13H LB-SEC**2/IN )
  903 FORMAT(1H0,31HTHE NUMBER OF CELLS            IM = ,I10  //
     132H NUMBER OF TIME INTERVALS  KM = ,I10     //
     232H CHANGE IN TIME                DT = ,E10.4,4H SEC  //
     332H VELOCITY SCALE FACTOR         VS = ,E10.4,7H IN/SEC  //
     432H CHANGE IN RADIUS              DR = ,F10.3,3H IN  //
     532H DISPLACEMENT SCALE FACTOR WS = ,F10.4,3H IN  //
     632H STRESS SCALE FACTOR           SS = ,E10.4,9H LB/IN**2 )
  904 FORMAT(1H1,4HTIME,50X,20HVELOCITY PROPAGATION  //)
  905 FORMAT(1H1,4HCELL,8X,9HCELL MASS, 8X,10HTOTAL MASS,9X,4HAREA,12X,
     1 7HMODULUS)
  906 FORMAT(1H ,I3,8X,E10.4,8X,E10.4,4X,F10.4,8X,F10.4)
  907 FORMAT(1H1,4HTIME,50X,12HDISPLACEMENT  //)
```

APPENDIX A. Continued

```
908 FORMAT(1H1,4HTIME,55X,6HSTRESS  //)
909 FORMAT(1H ,1X,1HI,12X,4HV(I),16X,4HS(I),16X,4HF(I),9X,
   1 2HSL,15X,2HDV)
910 FORMAT(1H ,I2,5X,F15.4,5X,F15.4,5X,F15.4,5X,F3.1,5X,F15.4)
    END
```

APPENDIX B

COMPUTER PROGRAM FOR ARMOR PENETRATION

```
      MULTIFILE RUN
      COMPILE   RUN  FORTRAN
C      MAINPROGRAM
C      PROJECTILE ARBITRARY SHAPE, NON-DEFORMING
C      COMPRESSIVE AND INERTIAL RESISTANCE BY TARGET
C      SUBROUTINES BULLET AND VELO REQUIRED
C      IF SPHERE SET R=R1 AND XL3=R1
C      IF CONE SET R=1000.0 AND XL3=XL1
C      SYMBOL TABLE   (UNITS IN IN-LB-SEC SYSTEM)
C      BRHO      = MASS DENSITY OF PROJECTILE
C      R         = RADIUS OF CIRCULAR ARC
C      R1        = RADIUS OF CYLINDRICAL PORTION OF PROJECTILE
C      XL        = TOTAL LENGTH OF PROJECTILE
C      XLO       = HOR. DISTANCE BET. R AND TIP OF PROJECTILE
C      XL1       = LENGTH OF PROJECTILE NOSE
C      XL2       = LENGTH OF CYLINDRICAL PART OF PROJECTILE
C      XL3       = LENGTH OF THE SEMICORD
C      DXL       = WIDTH OF A CELL
C      H         = PLATE THICKNESS
C      TRHO      = MASS DENSITY OF TARGET MATERIAL
C      SIG       = COMPRESSIVE STRENGTH OF TARGET MATERIAL
C      DT        = TIME INTERVAL
C      FS        = FORCE SCALE FACTOR
C      VS        = VELOCITY SCALE FACTOR
C      TS        = TIME SCALE FACTOR
C      VI        = INITIAL VELOCITY OF PROJECTILE
C      XLS       = DISTANCE OF A CELL FROM TIP OF PROJECTILE
C      DAS       = SURFACE AREA OF EACH CELL
C      ASD       = SURFACE AREA OF CELLS IN CYLINDRICAL PART OF PROJ.
C      COS       = COS. OF ANG. BET. HOR. DIA. AND NORMAL ON SURFACE
C      SIN       = SQRTF (1.0-COS**2)
C      COT       = COS/SIN
C      BM        = PROJECTILE MASS
C      IM        = NUMBER OF CELLS
C      T         = TIME
C      V         = VELOCITY OF PROJECTILE
C      X         = DISTANCE BET. TIP OF PROJ. AND FRONT EDGE OF PLATE
C      DX        = DISTANCE PROJECTILE TRAVELS IN TIME DT
C      XP        = DEPTH OF PENETRATION
C      VR        = RESIDUAL VELOCITY OF PROJECTILE
C      F1        = TOTAL FORCE DUE TO COMPRESSIVE RESISTANCE OF TARGET
C      F2        = TOTAL FORCE DUE TO INERTIAL RESISTANCE OF TARGET
C      F         = SUM OF F1 AND F2
C      YO        = DISTANCE BET. PROJ. AXIS AND HOR. DIAMETER
C      Y         = RADIUS OF ANY SECTION OF PROJECTILE
C      Y3        = VERTICAL DISTANCE OF SURFACE FROM HOR. DIAMETER
C      VOL1      = VOLUME OF NOSE OF PROJECTILE
C      VOL2      = VOLUME OF CYLINDRICAL PORTION OF PROJECTILE
C      VOL       = VOLUME OF PROJECTILE EQUAL TO VOL1+VOL2
C      TT        = NONDIMENSIONAL TIME
C      VV        = NONDIMENSIONAL VELOCITY
C      FF        = NONDIMENSIONAL FORCE
    1 DIMENSION XLS(200), DAS(200), COS(200), SIN(200), COT(200),
     1IDENT(16), FMT1(16)
    2 READ 800,(IDENT(I), I = 1,16 )
  800 FORMAT (16 A5)
    3 READ 805, BNAME1,BNAME2,BRHO
  805 FORMAT ( 2A5,E10.2 )
    4 READ 810, R,R1,XL3,XL,DXL
  8.0 FORMAT(5F10.0)
```

APPENDIX B. Continued

```
  5 READ 815, TNAME1,TNAME2,H,TRHO,SIG
815 FORMAT(2A5,3E10.2)
  6 READ 820, DT,FS,VS,IN,NRUN
820 FORMAT(3E10.2,2I10)
  7 READ 830,FMT1
830 FORMAT(16A5)
    DO 400 N=1,NRUN
  8 READ 825,VI
825 FORMAT(E10.2)
    IF(N-1)30,30,40
 30 PRINT 900, (IDENT (I),I = 1,16 )
900 FORMAT(1H1,48X,33HRESIDUAL VELOCITY IN A PROJECTILE /
   11H0,25X,16A5////)
    CALL       BULLET(R,R1,XL,XL1,XL2,XL3,DXL,IN,IM,DAS,COS,BM,BRHO)
C     SCALE FACTORS
    TS = DT
    PRINT 905,BNAME1,BNAME2,TNAME1,TNAME2,BRHO,BM,TRHO,SIG,H
905 FORMAT(26H0MATERIAL OF PROJECTILE = ,1X,2A5       //
   1          26H MATERIAL OF TARGET      = ,1X,2A5       //
   2          26H MASS DEN.OF B-MAT.BRHO = ,E13.4,16H LB-SEC**2/IN**4//
   3          26H MASS OF PROJECTILE  BM = ,E13.4,13H LB-SEC**2/IN//
   4          26H MASS DEN.OF T-MAT.TRHO = ,E13.4,16H LB-SEC**2/IN**4//
   5          26H COMP.RESIS.OF T-MAT.SIG= ,E13.4,9H LB/IN**2 //
   6          26H THICKNESS OF TARGET  H = ,F10.5,7H INCHES /)
    PRINT 910,XL,XL1,XL2,XL3,R,R1,DXL,IM
910 FORMAT (26H0TOTAL LENGTH OF CELL XL = ,F10.5,7H INCHES //
   1          26H LENGTH OF VAR.SECT. XL1 = ,F10.5,7H INCHES //
   2          26H LENGTH OF CON.SECT. XL2 = ,F10.5,7H INCHES //
   3          26H LENGTH OF SEMICHORD XL3 = ,F10.5,7H INCHES //
   4          26H RADIUS OF CIRC. ARC   R = ,F10.5,7H INCHES //
   5          26H RADIUS OF CON.SECT.  R1 = ,F10.5,7H INCHES //
   6          26H LENGTH OF THE CELL   DXL= ,E13.4,7H INCHES //
   7          26H NUMBER OF CELLS       IM = ,  I10    /)
    PRINT 950, VS,FS,DT
950 FORMAT(26H0VEL. SCALE FACTOR   VS = ,E13.4,7H IN/SEC //
   1          26H FORCE SCALE FACTOR  FS = ,E13.4,4H LB  //
   2          26H TIME INCREMENT TS = DT = ,E13.4,4H SEC  /)
 40 PRINT 915,VI
915 FORMAT(26H1INITIAL VELOCITY    VI = ,E13.4,7H FT/SEC /)
    CALL       VFLO (XL,DXL,COS,DAS,BM,IM,VI,DT,H,SIG,TRHO,FS,VS,
   1TS,TT,X,XP,VV,FF,XL1,R1,KM)
    REWIND 2
    PRINT 920
920 FORMAT(1H1,4HTIME,9X,11HPENETRATION,9X,8HTRAVEL-X,9X,
   18HVELOCITY,9X,5HFORCE ////)
    DO 400 K = 1,KM
    READ TAPE 2,TT,XP,X,VV,FF
    PRINT FMT1,TT,XP,X,VV,FF
400 CONTINUE
1000 STOP
    END

   COMPILE   RUN  FORTRAN
    SUBROUTINE BULLET(R,R1,XL,XL1,XL2,XL3,DXL,IN,IM,DAS,COS,BM,BRHO)
    DIMENSION XLS(200),DAS(200),COS(200), SIN(200),COT(200),Y(200)
    PI = 3.14159
    XLS(1) = 0.0
    ASC = 2.0*PI*R1*DXL
C     TEST TO DETERMINE IF PROJECTILE IS A CONE
    IF(R-1000.0)20,30,30
```

```
C     LOGIC FOR OGIVE AND HEMISPHERICAL NOSE
   20 YO  = SQRTF (R**2-XL3**2)
      XLO = SQRTF(R**2-(R1+YO)**2)
      XL1 = XL3-XLO
      XL2 = XL-XL1
      VOL1 = 0.0
      VOL2 = PI*R1**2*XL2
      DO 101 I = 1,IN
      IF (XLS(I)-XL1)  10,10,11
C     NON UNIFORM PORTION OF PROJECTILE
   10 Y3 = SQRTF(R**2-(XL3-XLS(I))**2)
      DAS(I) = 2.0*PI*R*(1.0-YO/Y3)*DXL
      COS(I) = (XL3-XLS(I))/R
      SIN(I) = Y3/R
      COT(I) = COS(I)/SIN(I)
      Y(I) = Y3-YO
      DVOL1= PI*DXL*(Y(I)**2+Y(I)*COT(I)*DXL+(COT(I))**2*DXL**2/3.0)
      VOL1 = VOL1+DVOL1
      GO TO 14
   11 IF (XLS(I)-XL) 13,13,12
   12 IM = I-1
      GO TO 501
C     UNIFORM PORTION
   13 DAS(I) = ASD
      COS(I) = 0.0
   14 XLS(I+1) = XLS(I)+DXL
  101 CONTINUE
  501 VOL = VOL1+VOL2
      BM = BRHO*VOL
      GO TO 400
C     LOGIC FOR CONICAL NOSE
   30 Y(1) = 0.0
      XL1 = XL3
      XL2 = XL - XL1
      DO 102 I = 1,IN
      XLS(I+1) = XLS(I)+DXL
      IF (XLS(I) - XL1 ) 40,40,41
   40 Y(I+1) =R1*XLS(I+1)/XL1
      DAS(I) =   PI   *(Y(I) + Y(I+1))*SQRTF(DXL**2+(Y(I)-Y(I+1))**2)
      COS(I) = R1/SQRTF(R1**2+XL1**2)
      GO TO 102
   41 IF (XLS(I)-XL) 43,43,42
   42 IM = I-1
      GO TO 50
   43 DAS(I) = ASD
      COS(I) = 0.0
  102 CONTINUE
   50 VOL = PI*R1**2*(XL1/3.0+XL2)
      BM = BRHO*VOL
  400 RETURN
      STOP
      END

    COMPILE   RUN  FORTRAN
      SUBROUTINE VELO (XL,DXL,COS,DAS,BM,IM,VI,DT,H,SIG,TRHO,FS,VS,
     1TS,TT,X,XP,VV,FF,XL1,R1,KM)
      DIMENSION XLS(200),COS(200),DAS(200)
      T = 0.0
      TT=T/TS
      V = VI*12.0
```

APPENDIX B. Continued

```
      VV = V/(VS*12.0)
      FF=0.0
      X = 0.0
      XLS(1) = 0.0
      REWIND 2
      DO 410 K = 1,1000
      IF(X-H-XL1)17,25,25
   17 F1 = 0.0
      F2 = 0.0
      DO 415 J = 1,IM
      IF (V) 24,24,18
   18 IF(X-H)19,19,20
   19 I = J
      XP = X
      GO TO 21
   20 IL = (X-H)/DXL
      I = J+IL
      XP = H
      IF(I-IM)21,21,23
   21 IF (XLS(I)-X) 22,22,23
C     COMPRESSIVE RESISTANCE OF PLATE MATERIAL
   22 DF1 = SIG*COS(I)*DAS(I)
C     INERTIAL RESISTANCE OF PLATE MATERIAL
      DF2 = 0.5*TRHO*(COS(I))**3*(V)**2*DAS(I)
      F1 = F1+DF1
      F2 = F2+DF2
      F = F1+F2
      XLS(I+1) = XLS(I)+DXL
  415 CONTINUE
   23 WRITE TAPE 2,TT,XP,X,VV,FF
      DX=V*DT
C     IMPULSE MOMENTUM LAW
      DV =-F *DX/(BM*V)
      V = V+DV
      X = X+DX
      T = T+DT
      FF = F/FS
      TT = T/TS
      VV=V/(VS*12.0)
  410 CONTINUE
   24 V=0.0
      VV = V/(VS*12.0)
   25 VR = V/12.0
      WRITE TAPE 2,TT,XP,X,VV,FF
      XPM = XP
      XM = X
      TM = T
      TTM = TT
      KM=K
      PRINT 950, VR,XPM,XM,TM,TTM
  950 FORMAT(26H0R.VELOCITY OF BULLET VR= ,E13.4,7H FT/SEC  //
     1       26H MAX.PENETRATION    XPM = ,E13.4,7H INCHES  //
     2       26H TOTAL TRAVEL        XM = ,E13.4,7H INCHES //
     3       26H TOTAL TIME FOR XM   TM = ,E13.4,4H SEC  //
     4       26H SCALED TIME FOR TM TTM = ,F10.3    /)
      RETURN
      STOP
      END

    MULTIFILE END
```

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13. ABSTRACT

A basic theoretical research program in stress waves and penetration mechanics, with particular emphasis on armor plate, is described. The contents of this report include (1) list of output documents, (2) general summary and review of project activities, (3) Progress of last 6 months period, and (4) technical work of significance not previously reported.

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Penetration Mechanics						
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