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ANALYSIS OF EDGE NOTCHES IN A SEMI-INFINITE REGION

TECHNICAL REPORT

by

OSCAR L. BOWIE

JUNE 1966

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ANALYSIS OF EDGE NOTCHES IN A SEMI-INFINITE REGION

ABSTRACT

A procedure for solving the plane elastic problem for edge notches in a semi-infinite region is presented. A modification of the conventional Muskhelishvili technique is made by utilizing a mapping function which describes both the physical region and its reflection with respect to the straight edges. An effective power series development in the parameter plane is then described and illustrated by the numerical solution for semi-elliptical notches in a semi-infinite sheet in tension at infinity parallel to the edge.

CONTENTS

INTRODUCTION

Edge notches in a semi-infinite region identify a familiar problem class in plane elasticity. The difficulty of finding a representation of the solution which allows a systematic consideration of boundary conditions on both the straight line and notch sections of the geometry is a challenging matter. Several particular geometries have been successfully analyzed by a variety of rather ingenious methods. Perhaps the most familiar is the original Maunsell[1] solution for the problem of a semicircular notch in a semi-infinite plate in tension parallel to the edge. Later, Ling[2] discovered an integral representation for this problem as well as a class of problems corresponding to circular notches or mounds and straight boundaries. Edge cracks have been successfully studied by several investigators, e.g., References 3 and 4. Estimates of the maximum stress for edge notches have been made by approximating arguments by Neuber.[5]

For general notch shapes a more uniform approach would be desirable. One could, for example, consider the mapping technique of Muskhelishvili[6] as such an approach. However, several practical difficulties arise in the application of this technique to this problem class. Frequently the exact mapping function contains branch point singularities necessary for the description of the junctions of the notch and the straight line boundaries. Effective use of the exact mapping function in such cases is limited, as the problem ceases to be one of *linear relationship*. The alternative of polynomial approximation of the exact mapping function can be used, but frequently a suitably accurate polynomial representation is awkward to find. A good example of this difficulty is shown in the recent work of Mitchell[7] in his solution for a semicircular notch.

In this paper a power series technique is presented which appears to provide an effective and fairly uniform approach to single edge notches in a semi-infinite region. Initially, Muskhelishvili's reflection argument is used to translate the problem into one of finding a suitable analytic stress function defined exterior to the closed region bounded by the notch and its reflection with respect to the real axis. A departure from the conventional mapping approach is then made by introducing a mapping function defined on the unit circle and its exterior to describe *both* the given physical region and its reflection with respect to the real axis. A power series representation in the parameter plane is a natural consequence of this formulation. Furthermore, in many practical problems, the explicit occurrence of the branch point singularities in the mapping function referred to above is eliminated. Finally, the structure of the solution can be predicted in a fairly systematic manner by utilizing most of the consequences of the unmodified Muskhelishvili theory.

INITIAL FORMULATION OF THE PROBLEM

It will be assumed that the material occupies the notched lower half-plane S^- defined in the z-plane, Figure 1. The boundary of the notch C^- will be assumed to be an arbitrary continuous curve, and the straight edges are assumed to lie on the real axis, $y=0$.

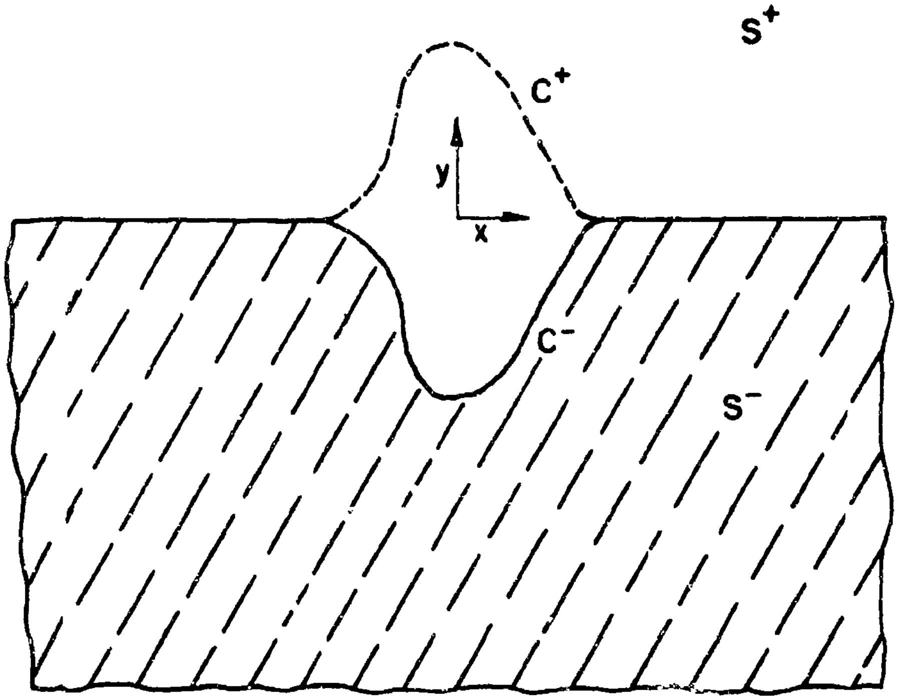

Figure I. SINGLE EDGE NOTCH IN A SEMI-INFINITE REGION

In the initial stage of the formulation, the reflection argument of Muskhelishvili[6] will be utilized. The essential details of this argument will be briefly summarized in this paragraph. The stress components in cartesian coordinates can be expressed in terms of two analytic functions $\Phi(z)$ and $\Psi(z)$ in the following manner:

$$\sigma_y + \sigma_x = 2\,[\Phi(z) + \Phi(z)] \tag{1}$$

$$\sigma_y - \sigma_x + 2i\tau_{xy} = 2[\bar{z}\Phi'(z) + \Psi(z)], \tag{2}$$

where primes denote differentiation and bars denote complex conjugates.

In the present problem, the region of definition of $\Phi(z)$ and $\Psi(z)$ is S^-. On the other hand, using the extension principle of Muskhelishvili, the definition of $\Phi(z)$ can be extended into S^+ by defining

$$\Phi(z) = -\bar{\Phi}(z) - z\bar{\Phi}'(z) - \bar{\Psi}(z) \text{ for } z \epsilon S^+, \tag{3}$$

where

$$\bar{f}(z) \equiv \overline{f(\bar{z})}. \tag{4}$$

The stress function $\Psi(z)$ can now be expressed in terms of the extended definition of $\Phi(z)$ as

$$\Psi(z) = -\Phi(z) - \bar{\Phi}(z) - z\Phi'(z) \text{ for } z \epsilon S^-. \tag{5}$$

For simplicity of presentation, self-equilibrating load systems acting on the boundary of the notch will be considered at this time. Actually, this is a minor restriction since loading at infinity or along the straight edges can be handled directly for the unnotched half-plane in terms of elementary functions and Cauchy integrals, e.g., Reference 6. By considering the superposition of solutions, it is clear that the problem difficulty rests in solving the cases of self-equilibrating load systems acting on the notch boundary. For self-equilibrating load systems,

$$\Phi(z) \text{ and } \Psi(z) = O(1/z^2) \text{ for large } |z|. \tag{6}$$

Thus, from (3) it is evident that $\overline{\Phi}(z)$ is analytic in both S^- and S^+.

The character of $\Phi(z)$ on the straight edges of the boundary is clear from

$$\sigma_y - i\tau_{xy} = \Phi(z) - \Phi(\overline{z}) + (z-\overline{z})\overline{\Phi'(z)}. \tag{7}$$

If the straight sections along the real axis are considered as load-free, then, since $z-\overline{z} = 0$ on the real axis,

$$\Phi^-(x) - \Phi^+(x) = 0, \text{ (on the straight boundary)}. \tag{8}$$

The notation $\Phi^-(x)$ and $\Phi^+(x)$ denotes the value of $\Phi(z)$ as $z \to x$ through S^- and S^+, respectively. From (8) and the previous discussion, $\Phi(z)$ can now be considered as an analytic function in the region of the plane exterior to the contour $C = C^- + C^+$ where C^+ is the reflection of C^- with respect to the real axis, Figure 1.

Finally, the boundary conditions on C^- must be considered. For the present purpose, we assume that the normal and tangential stress components, N and T, are prescribed on C^-. Then, if α denotes the angle between the x-axis and the outward normal,

$$N - iT = \Phi(z) + \overline{\Phi(z)} - e^{2i\alpha}[(\overline{z}-z)\Phi'(z) - \Phi(z) - \Phi(z)] \text{ for } z \epsilon C^-. \tag{9}$$

A POWER SERIES FORMULATION OF THE PROBLEM

For a power series representation, it is obviously desirable to deal with a parametric region consisting of a circle and its exterior (or interior). Therefore, consider an auxiliary complex plane, the ζ-plane, such that the unit circle $\zeta = \sigma = e^{i\theta}$ and its exterior are mapped into C and its exterior, respectively, by the analytic function

$$z = \omega(\zeta). \tag{10}$$

A departure from the conventional mapping approach is made here since *both* the physical region and its reflected image are described by the auxiliary plane. The stress functions $\Phi(z)$ and $\Psi(z)$ can be considered as analytic functions of the parameter ζ since z and ζ are related by the analytic function (10). The necessity of introducing considerable new notation can be avoided by designating $\Phi(z)=\Phi[\omega(\zeta)]$ as $\Phi(\zeta)$, etc., which leads to such relationships as $\Phi'(z)=\Phi'(\zeta)/\omega'(\zeta)$, etc. Due to the symmetry of C with respect to the x-axis, $\overline{\Phi}(z)=\overline{\Phi}(\zeta)$ in the notation described.

The boundary condition (9) can now be expressed as a function of σ. Due to the symmetry of C, the range $0\leq\theta\leq\pi$ will be considered as corresponding to C^+ and the range $\pi\leq\theta\leq 2\pi$, corresponding to C^-. The applied load can be considered as a function of σ, i.e.,

$$N - iT = g(\sigma), \quad \pi\leq\theta\leq 2\pi. \tag{11}$$

Thus,

$$\Phi(\sigma) + \overline{\Phi(\sigma)} - e^{2i\alpha}\{[\overline{\omega(\sigma)} - \omega(\sigma)]\Phi'(\sigma)/\omega'(\sigma) - \Phi(\sigma) - \overline{\Phi}(\sigma)\} = g(\sigma),$$
$$\pi\leq\theta\leq 2\pi \tag{12}$$

where

$$e^{2i\alpha} = \sigma^2\omega'(\sigma)/\overline{\omega'(\sigma)}. \tag{13}$$

On the unit circle itself, it is possible to encounter singularities in $\Phi(\zeta)$ either by load distribution or as a result of geometric irregularities in the notch shape. For severe types of singularities, e.g., poles occurring at crack roots, logarithmic singularities corresponding to point loads, etc., it is generally possible to anticipate and introduce the proper structure as a separate part of the representation. The remaining arguments will, therefore, be confined to the cases of nonsevere boundary singularities, i.e.,

$$\Phi(\zeta) = \sum_{n=2}^{\infty} A_n\zeta^{-n}, \quad |\zeta|>1 \tag{14}$$

with at least conditional convergence assumed on the unit circle τ.

DETERMINATION OF THE SERIES COEFFICIENTS

The analyticity of $\Phi(\zeta)$ permits the expression of the coefficients in terms of the well-known integrals

$$2\pi i A_n = \int_\tau \Phi(1/\zeta)d\zeta/\zeta^{n+1}, \quad n = 2, 3, \ldots \tag{15}$$

or

$$2\pi A_n = \int_0^{2\pi} \sigma^{-n}\Phi(\overline{\sigma})d\theta, \quad n = 2, 3, \ldots \tag{16}$$

In addition,

$$0 = \int_0^{2\pi} \sigma^{-n}\Phi(\overline{\sigma})d\theta, \quad n = 1, 0, -1, -2, \ldots, \tag{17}$$

but if the solution exists in the hypothesized form, the conditions (17) can be considered as redundant.

If the function $F(\sigma)$ is defined as

$$F(\sigma) = \Phi(\bar{\sigma}) - \Phi(\bar{\sigma}) = 0, \quad 0 \leq \theta \leq \pi$$

$$= \Phi(\bar{\sigma}) - e^{2i\alpha}[\overline{g(\sigma)} - \overline{\Phi(\sigma)} - \Phi(\sigma)] - \overline{\Phi(\sigma)}$$

$$+[\omega(\sigma) - \overline{\omega(\sigma)}]\ \overline{\Phi'(\sigma)/\omega'(\sigma)}, \quad \pi \leq \theta \leq 2\pi, \tag{18}$$

then the boundary conditions of the present problem are equivalent to

$$F(\sigma) = 0, \quad 0 \leq \theta \leq 2\pi. \tag{19}$$

The Fourier criteria for the vanishing of $F(\sigma)$ is thus

$$\int_0^{2\pi} \sigma^{-n} F(\sigma)\, d\theta = \int_\pi^{2\pi} \sigma^{-n} F(\sigma)\, d\theta = 0, \quad n = 0, \pm 1, \pm 2, \ldots \tag{20}$$

If we denote by F_j,

$$F_j = \int_\pi^{2\pi} \sigma^{-j} F(\sigma)\, d\theta, \qquad j = 0, \pm 1, \pm 2, \ldots \tag{21}$$

it is evident that computationally (16) and (17) are equivalent to

$$F_j = 0, \qquad j = 2, 3, 4, \ldots \tag{22}$$

$$F_j = 0, \qquad j = 1, 0, -1, -2, \ldots \tag{23}$$

respectively. For an infinite system, the redundancy of (17) implies the redundancy of (23).

A reasonable plan of solution conceptually is to introduce the series (14) into the set of conditions (22). This leads to an infinite set of linear equations in the unknowns A_n, which can be solved by successive truncations of the system. Early numerical solutions indicated, however, that the rate of convergence of this truncation procedure was too slow to be practical. The difficulty shows up numerically in a lack of dominance of the principal diagonal of the coefficient matrix.

Conceptually, the difficulty can be traced to the series truncation. Let $\Phi_A(\zeta)$ be defined as a truncation of (14),

$$\Phi_A(\zeta) = \sum_{n=2}^{M+1} A_n \zeta^{-n}. \tag{24}$$

In general, the corresponding function $F_A(\sigma)$ is a polynomial (or a rational function) of σ involving both positive and negative powers of σ and (19) cannot be identically satisfied by a choice of the M available coefficients A_n. Furthermore, the Fourier conditions (23) which cannot be considered as redundant for a truncated stress function are ignored in the plan described above.

A combination of two procedures was found to be the most effective computational plan. For the leading coefficients, the system

$$F_j = 0, \quad j = 0, \pm 1, \pm 2, \ldots, \pm P \tag{25}$$

was solved for several values of P with M = 2P + 1. A rather dramatic improvement in convergence of the leading coefficients was found by this plan. In the illustrative problems, the first twenty coefficients were found with reliable accuracy with values of P<20. On the other hand, a critical system size is reached where the higher ordered coefficients begin a rapid growth in inaccuracy and the determinant of the coefficient matrix becomes very small. This can be anticipated due to the redundancy in the limit of P + 2 of the equations in (25).

The procedure used to calculate the higher order coefficients consisted of solving the set of conditions

$$\frac{\partial}{\partial A_i} \int_0^{2\pi} \{F(\sigma)\}^2 \, d\theta = 0, \; i = 2, 3, \ldots, M + 1. \tag{26}$$

This set of conditions corresponds to the requirement that (19) be satisfied for a finite set of coefficients in the least square sense. The system (26) eliminates the previous stability difficulties for larger systems and can be compactly expressed as a suitable linear combination of the elements F_j. Although (26) could be used to determine the entire set of coefficients, the leading coefficients converge somewhat more slowly than in (25), and the maximum system size is reduced by using a combination of the two procedures.

SEMICIRCULAR EDGE NOTCH

The simplest illustration of the preceding analysis is provided by Maunsell's problem, i.e., a semicircular notch in a semi-infinite plate in tension, T, parallel to the edge, Figure 2. The radius of the semicircle will be chosen as the unit length, and polar coordinates (r,θ) are introduced in the manner shown. For illustrative purposes, the mapping notation will be retained, thus,

$$z = \omega(\zeta) = \zeta. \tag{27}$$

The stress functions will be denoted by $\Phi_1(\zeta)$ and $\Psi_1(\zeta)$ where

$$\begin{aligned} \Phi(\zeta) &= T/4 + \Phi(\zeta) = T/4 + T \sum_{n=2}^{\infty} A_n \zeta^{-n} \\ \Psi_1(\zeta) &= -T/2 + \Psi(\zeta). \end{aligned} \tag{28}$$

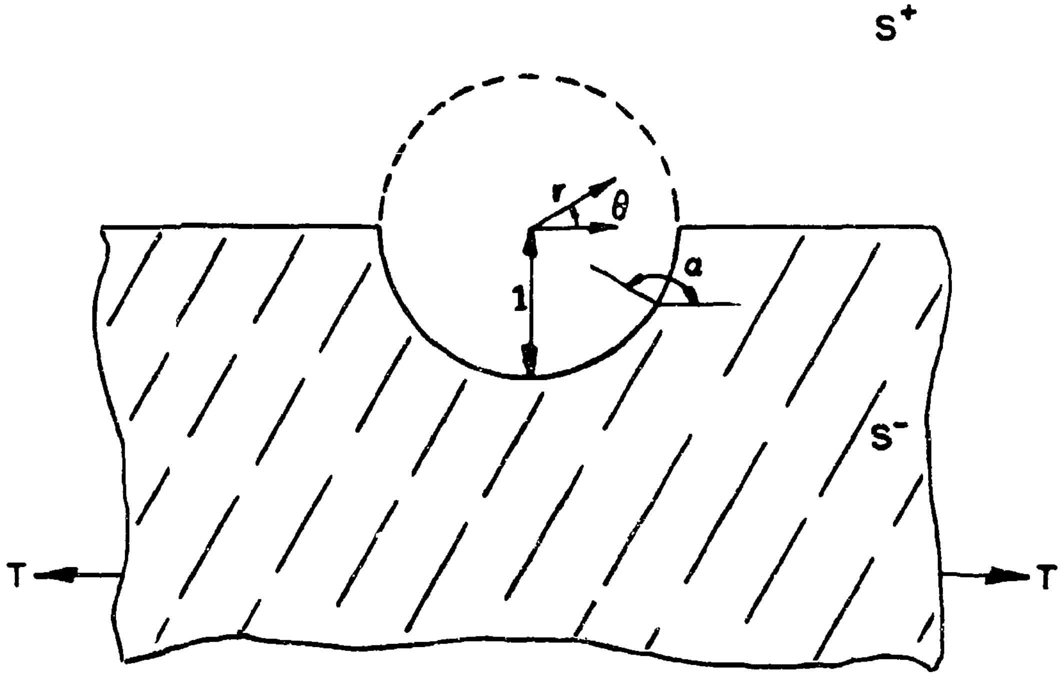

Figure 2. SEMICIRCULAR EDGE NOTCH

Since the particular solution in (28) corresponds to $\sigma_x = T$ everywhere, it is clear that the determination of $\Phi(\zeta)$ and $\Psi(\zeta)$ is a problem type with the properties assumed in the previous analysis. Furthermore, since $\alpha = \pi + \theta$,

$$g(\sigma) = -T/2 \ (1+\sigma^2). \tag{29}$$

From symmetry considerations, the series coefficients in (28) can be written in the form

$$\begin{aligned} A_{2k} &= B_{2k}, \qquad && k = 1, 2, 3, \ldots \\ A_{2k-1} &= iC_{2k-1}, \qquad && k = 2, 3, 4, \ldots \end{aligned} \tag{30}$$

where B_{2k} and C_{2k-1} are real.

Corresponding to (18),

$$\begin{aligned} F(\sigma) &= 0, \qquad 0 \le \theta \le \pi \\ &= \Phi(\overline{\sigma}) + \overline{\Phi(\sigma)} - (\sigma - \sigma^{-1})\overline{\Phi'(\sigma)} + \sigma^2[\Phi(\sigma) + \overline{\Phi(\sigma)}] \\ &\quad + (T/2)\ (1+\sigma^2), \qquad \pi \le \theta \le 2\pi. \end{aligned} \tag{31}$$

Thus,

$$\left.\begin{aligned}
F_o &= \pi B_2 - 8 \sum_{k=2}^{\infty} C_{2k-1}/(2k-3)(2k+1) + \pi/2 \\
F_1 &= 0 \\
F_2 &= 8 \sum_{k=2}^{\infty} C_{2k-1}/(2k-3)(2k-1) + \pi/2 \\
F_{2j} &= 2\pi(1-j)B_{2j} + (2j-1)\pi\, B_{2j-2} \\
&\quad + 8 \sum_{k=2}^{\infty} C_{2k-1}(2k-1)(2j-1)/[(2k-2j)^2-1][2k+2j-3], \quad j = 2, 3, \ldots \\
-iF_{2j-1} &= \pi(2j-1)C_{2j-1} - 2\pi(j-1)C_{2j-3} - 16 \sum_{k=1}^{\infty} B_{2k}(2j-3)(j-1)/[(2j-3)^2-4k^2][2k-2j+1] \\
&\quad + 4(j-1)/(2j-1)(2j-3), \quad j = 2, 3, \ldots \\
F_{-2j} &= \pi B_{2j+2} - 8 \sum_{k=2}^{\infty} C_{2k-1}(2j+1)(2k-1)/[(2k+2j)^2-1][2k-2j-3], \quad j = 1, 2, \ldots \\
-iF_{-2j+1} &= \pi C_{2j+1} + 16 \sum_{k=1}^{\infty} B_{2k} j(2j+1)/[(2k+2j)^2-1][2k-2j-1] - 4j/(4j^2-1), \\
&\quad j = 1, 2, \ldots
\end{aligned}\right\} \quad (32)$$

The conditions corresponding to (26) can be conveniently expressed in terms of F_n. Since,

$$\frac{\partial}{\partial A_i} \int_{\pi}^{2\pi} \{F(\sigma)\}^2 d\theta = 2 \int_{\pi}^{2\pi} F(\sigma) \frac{\partial F(\sigma)}{\partial A_i} d\theta = 0, \qquad (33)$$

we find from the even and odd coefficients, respectively,

$$(2-2j)F_{-2j} + (2j+1)\, F_{-2j-2} + F_{2j-2} = 0, \qquad j = 1, 2, \ldots, M$$

$$(2j-1)F_{1-2j} - 2j\, F_{-1-2j} + F_{2j-3} = 0, \qquad j = 2, 3, \ldots, M+1. \qquad (34)$$

The first twenty coefficients were found to five decimal place accuracy by using (25) for several values of $P \leq 20$. The higher order coefficients were then calculated by solving the system (34) for several values of M. Forty of the higher order coefficients were obtained to five decimal place accuracy with the use of systems in which $M<50$. The comparatively rapid decay of the coefficients is shown in Table I.

Due to some recent controversy as to the accuracy of Ling's[2] numerical results, e.g., Reference 7, the stress at the notch root, $\theta = -\pi/2$, was calculated. In terms of the present formulation,

Table I. STRESS FUNCTION COEFFICIENTS FOR SEMICIRCULAR EDGE NOTCH

k	B_{2k}	C_{2k+1}	k	B_{2k}	C_{2k+1}
1	-0.79248	-0.66283	11	-0.00147	-0.00074
2	+0.65206	+0.33908	12	-0.00110	-0.00070
3	-0.04406	+0.05755	13	-0.00084	-0.00065
4	-0.02325	+0.01765	14	-0.00065	-0.00059
5	-0.01358	+0.00616	15	-0.00051	-0.00053
6	-0.00850	+0.00199	16	-0.00040	-0.00047
7	-0.00561	+0.00032	17	-0.00032	-0.00042
8	-0.00385	-0.00038	18	-0.00025	-0.00038
9	-0.00272	-0.00066	19	-0.00020	-0.00034
10	-0.00198	-0.00074	20	-0.00016	-0.00030

$$\sigma_x(\theta = -\pi/2) = T\,\{1-4\,[\sum_{k=1}^{\infty} (C_{2k+1}-B_{2k})(-1)^k]\}. \tag{35}$$

Using the coefficients in Table I, $\sigma_x/T = 3.0656$ at the notch root. Allowing for round-off in the present data, it would be reasonable to estimate that $\sigma_x/T = 3.065 \pm 0.001$ which can be compared with Ling's result, $\sigma_x/T = 3.065$.

Estimates of the accuracy of the solution at the free corner, $\theta = 0$, were made from the data of Table I and additional higher order B_{2k} coefficients not recorded here. Excellent agreement was found which further indicates that reliable peak stresses at notch roots can be obtained by this procedure, particularly since series convergence would be expected to be more rapid at such points.

SEMI-ELLIPTIC NOTCH IN A SEMI-INFINITE SHEET

A final illustration of the analysis is provided by a semi-elliptic notch in a semi-infinite sheet in tension at infinity parallel to the edge, Figure 3. Only an approximation of this problem has been previously available. Again, the straight edges of the sheet are assumed to lie on the real axis, $y = 0$.

An appropriate form of the mapping function is

$$z = \omega(\zeta) = \zeta - B\zeta^{-1}, \quad 0 \leq B < 1. \tag{36}$$

The ratio of the axes of the ellipse, $\lambda = a/b$, is clearly

$$\lambda = (1-B)/(1+B). \tag{37}$$

For illustrative purposes,[6] the analysis was carried out in terms of the stress function $\phi(\zeta)$ and $\psi(\zeta)$, where

$$\phi'(\zeta) = \Phi(\zeta)\omega'(\zeta), \quad \psi'(\zeta) = \Psi(\zeta)\omega'(\zeta). \tag{38}$$

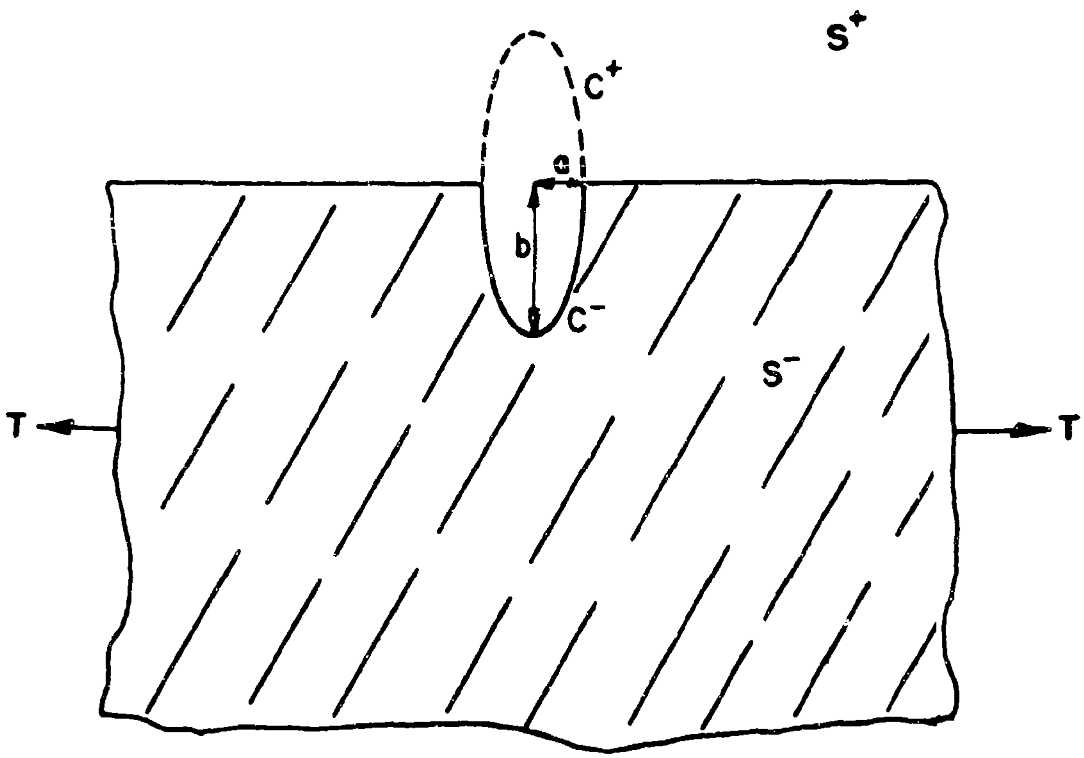

Figure 3. SEMI-ELLIPTIC EDGE NOTCH

The preceding arguments carry through in a straightforward manner and the boundary condition can be written as

$$\overline{\omega'(\sigma)}\;\phi(\sigma) = \overline{\omega'(\sigma)}\;\phi(\sigma) + \omega(\sigma)\overline{\phi'(\sigma)} - \overline{\omega(\sigma)}\;\overline{\phi'(\sigma)}$$

$$-\overline{\omega'(\sigma)}\int_{-1}^{\sigma}\omega'(\sigma)\;\overline{g(\sigma)}\,d\sigma,\quad \pi\leq\theta\leq 2\pi. \tag{39}$$

If $\phi_1(\zeta)$ denotes the required stress function, then

$$\phi_1(\zeta) = (T/4)(\zeta - B\zeta^{-1}) + \phi(\zeta) \tag{40}$$

where

$$\phi(\zeta) = T\sum_{n=2}^{\infty} a_n\zeta^{-n+1}. \tag{41}$$

The previous argument was applied to the function $\omega'(\zeta)\phi(\zeta)$. Again from symmetry,

$$a_{2k} = b_k, \qquad k = 1,\ 2,\ \ldots.$$

$$a_{2k-1} = ic_{2k-1}, \quad k = 2,\ 3,\ \ldots. \tag{42}$$

Since

$$g(\sigma) = -(T/2)[1+\sigma^2\omega'(\sigma)/\overline{\omega'(\sigma)}], \tag{43}$$

it follows that in the boundary condition (39),

$$\int_{-1}^{\sigma}\omega'(\sigma)\overline{g(\sigma)}d\sigma = -(T/2)(1+B)(\sigma-\sigma^{-1}). \tag{44}$$

Thus,

$$f_j = \int_{\pi}^{2\pi} \sigma^{-j}\{\overline{\omega'(\sigma)}\phi(\sigma) - \overline{\omega'(\sigma)}\phi(\sigma) + \overline{\phi'(\sigma)}\,[\overline{\omega(\sigma)} - \omega(\sigma)]$$
$$-\overline{\omega'(\sigma)}\,(T/2)(1+B)(\sigma-\sigma^{-1})\}d\theta, \quad j = 0, \pm 1, \pm 2, \ldots \tag{45}$$

The least square conditions similar to (26) can then be written as

$$[2+B-2(1+B)j]\ f_{1-2j} + [2j(1+B) - 1]\ f_{-2j-1}$$
$$-\ f_{2j-1} - Bf_{2j-3} = 0, \qquad j = 1, 2, \ldots$$
$$[2(1+B)j - (1+2B)]\ f_{2-2j} - [2(1+B)j - (2+3B)]\ f_{-2j}$$
$$-\ f_{2j-2} - Bf_{2j-4} = 0, \quad j = 2, 3, \ldots \tag{46}$$

The numerical analysis was then carried out for several values of B in the manner previously outlined. For brevity, only the peak notch stresses for various values of $\lambda = a/b$ will be presented. In terms of the present formulation,

$$\sigma_{max} = \sigma_x(\zeta=-i) = T\{1 + 4\sum_{k=1}^{\infty}[(1-2k)b_{2k} + 2kc_{2k+1}](-1)^k/(1-B)\}. \tag{47}$$

Comparison with the conventional approximation to this problem was made by calculating

$$Q = \sigma_{max}/\sigma^*_{max}, \tag{48}$$

where σ^*_{max} is the corresponding peak stress for an elliptical hole in an infinite sheet with a corresponding uniaxial tension at infinity. The latter solution is well known, e.g., Reference 8, and

$$\sigma^*_{max} = T(1+2\lambda^{-1}). \tag{49}$$

The results are shown in Table II where the limiting value of Q for $\lambda = 0$ is based on Koiter's result.[4] The results listed can be considered accurate to within 0.1 of one percent. In the calculation of the coefficients on a digital computer, it was found advisable to use double precision.

Table II. MAXIMUM STRESSES FOR SEMI-ELLIPTIC NOTCHES

B	λ	σ_{max}/T	σ^*_{max}/T	Q
0.0	1.0000	3.065	3.000	1.022
0.1	0.8182	3.540	3.444	1.028
0.2	0.6667	4.136	4.000	1.034
0.3	0.5385	4.910	4.714	1.042
0.4	0.4286	5.948	5.667	1.050
0.5	0.3333	7.412	7.000	1.059
0.6	0.2500	9.625	9.000	1.069
0.7	0.1765	13.320	12.333	1.080
0.8	0.1111	By interpolation	→	1.092
0.9	0.0526	By interpolation	→	1.106
1.0	0.0000	Koiter's result	→	1.1215

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13. ABSTRACT

A procedure for solving the plane elastic problem for edge notches in a semi-infinite region is presented. A modification of the conventional Muskhelishvili technique is made by utilizing a mapping function which describes both the physical region and its reflection with respect to the straight edges. An effective power series development in the parameter plane is then described and illustrated by the numerical solution for semi-elliptical notches in a semi-infinite sheet in tension at infinity parallel to the edge. (Author)

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