TIME-COST TRADEOFFS IN UNCERTAIN EMPIRICAL

RESEARCH PROJECTS*

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ABSTRACT

This paper explores the relationship between research project cost and expected time to completion under various scheduling strategies; it assumes that many potential technical approaches to the research problem can be identified; and that each approach has a low but finite subjective probability of success. It is shown that under a variety of assumptions, expected time to project completion can be reduced, but that as a result expected project cost rises at an increasing rate. Some cases in which this convex time-cost tradeoff relationship might not hold generally are identified. When the time-cost tradeoff function is convex, the desirability of concurrent as opposed to series scheduling of approaches depends crucially upon the depth of the stream of benefits expected to be realized upon successful project completion. The deeper the benefit stream is, the more desirable concurrent scheduling is.

INTRODUCTION

Recently the relationship between time and cost in research and development projects has attracted considerable interest. Empirical estimation of the time-cost tradeoff function is exceptionally difficult, since R&D projects can seldom be replicated under precisely controlled conditions. PERT-type analytic studies have suggested a convex relationship, with total cost decreasing at a decreasing rate as time is increased.[†] Most of the results presented thus far, however, depend upon an initial basic assumption that diminishing returns set in as more manpower is applied to individual project tasks. While plausible, this is surely not the only and perhaps not the most important mechanism causing cost to vary with time. Notably, the role of technological uncertainty cannot be overlooked.

In this paper, the relationship between time and cost in what may be called uncertain empirical research and development projects is explored. Specifically, we consider situations where a large number of potential technical approaches can be identified and where several of these approaches may lead to successful solutions, even though ex ante no single approach promises anything close to certainty of success. Many illustrations come to mind. Edison, for example, is said to have tested 1600 different filament materials before finding one which satisfied his incandescent lamp requirements, although his carbon filament solution was not

[†]For a survey of the PERT literature, see [6]. Other approaches to the problem are found in [5, chapter 9] and [7].

the only one which ultimately proved workable. The discovery of aureomycin followed testing of several thousand microorganisms, but later work disclosed a whole class of organic molecules with roughly similar therapeutic properties. Numerous different materials were tried in the search for suitable Jupiter IRBM reentry body ablative coatings and for a high-strength, low-weight Polaris rocket motor casing. Similarly, in the Polaris program a variety of approaches to the problem of thrust vectoring were pursued, and several acceptable solutions emerged.

Given the difficulty of ascertaining actual time-cost tradeoff relationships empirically, an effort is made here to proceed as far as possible on the basis of elemental a priori assumptions. Qualifications associated with real-world complications will be added later. The following greatly simplifying assumptions will be made initially: First, we assume that each approach to a technical problem has the same prior subjective probability of success p, where $0 < p < 1$. From this, we define $q = 1 - p$ to be the probability of failure for a given approach. Second, we assume that the success probability of any specific approach is independent of the number and sequence of other approaches pursued. Third, it is assumed that any successful approach will solve the required problem, and that additional successes are redundant. Fourth, we assume that each approach has the same dollar cost M of execution, which is independent of the number and sequence of other approaches pursued. Finally, it is assumed that each fully-staffed approach normally takes exactly one time period to execute.*

SETTING THE NUMBER OF APPROACHES

Under these assumptions, the probability of overall project success is defined by:

$$P(p, N) = 1 - q^N , \tag{1}$$

where N is the number of approaches executed. The more approaches pursued, the higher the probability of overall project success will be. To decide how large N will be, one must weigh the expected cost M of each additional approach against the expected gain from pursuing that approach. If B is the discounted present value of the benefits realizable upon successful solution of the technical problem, the expected value E(B) of a project with N approaches is $P(p, N) B = (1 - q^N) B$. As a first approximation (to be modified later), expected profits will be maximized by authorizing additional approaches as long as the cost M of the N-th approach is less than the incremental gain from conducting that approach, which is defined by:†

$$\Delta E(B) = \left[(1 - q^N) - (1 - q^{N-1}) \right] B = q^{N-1} (1 - q) B . \tag{2}$$

For any given q and B, $\Delta E(B)$ must decline as N increases. For any given q and N, $\Delta E(B)$ increases with B. Therefore, the greater are the benefits B realizaole upon successful project completion, the more approaches it will pay to authorize.*

PROPERTIES OF THE TIME-COST TRADEOFF FUNCTION

Let us now assume provisionally that the number of approaches authorized N and hence the probability of overall project success $P(p, N)$ is decided upon. It is still necessary to determine how the execution of those N approaches will be phased over time. One possibility is to run the approaches serially, that is, one after the other. Another is to run all the approaches concurrently in the first period, guaranteeing project completion (with probability $P(p, N)$ of success) by the end of that period. Many other distributions are possible when N and the number of periods are fairly large.

Three quantities vary with the choice of a time sequence of approaches: (1) the number of periods T required to execute all N approaches, and hence the planned time before the probability of project success $P(p, N)$ is expected to attain its full anticipated value; (2) the expected value $E(T)$ of the time required to achieve a successful solution or terminate the project (the latter after N unsuccessful approaches); and (3) the expected cost $E(C)$ of the project. These three are functionally related, defining two different time-cost tradeoff functions: the relationship between T and $E(C)$, and the relationship between $E(T)$ and $E(C)$. Let us consider first the relationship between T and $E(C)$.

Plainly, if the N approaches are distributed over T periods, T periods are required before the anticipated probability of project success $P(p, N)$ is assured in an ex ante sense. For example, if 10 approaches, each with p of 0.20, are to be pursued, one can expect the overall success probability of 0.89 to be attained one period hence if all 10 approaches are run concurrently in the first period, but only 10 periods hence if the approaches are run serially. The expected cost of project execution must be 10 M in the fully concurrent strategy, since all 10 approaches will have been run before any approach is completed under the present assumptions. But expected cost must be lower if the series approach is pursued. This is so because success may come in an early period, making it unnecessary to continue with the remaining, originally planned approaches.

To study the properties of the function linking T and $E(C)$ further, let us begin with a special assumption: that the N planned approaches are to be distributed evenly over the planned project duration T, so that in any single period N/T approaches will be executed.[†] If the cost per approach is M, expected cost as a function of T is given by:

$$(3) \qquad E(C) = NM/T + q^{N/T} (NM/T) + \ldots + \left(q^{N/T}\right)^{N-1} (NM/T) .$$

Letting t be a running integral time variable, this simplifies to:

$$(4) \qquad E(C) = \sum_{:0}^{T-1} \left(q^{N/T}\right)^t (NM/T) .$$

Now the larger T is, the smaller the cost per period NM/T will be, but the more periods there will be in the summation. If the $(q^{N/T})$ term were equal to 1.0, which is possible only when $N/T = 0$, $E(C)$ would be constant with respect to T, since increases in T would simply mean dividing NM into T equal parts and summing T of those parts. $E(C)$ can increase with T only when $(q^{N/T}) > 1$, which is impossible, since $0 < q < 1$ in any uncertain approach worth trying. It must decrease with T when $(q^{N/T}) < 1$. And this must normally be true, since N must be positive and T is not likely to be infinite in any research project worth conducting. Therefore, expected project cost $E(C)$ must decline as T is increased. Or conversely, the more rapidly the project is to be conducted, the higher its expected cost will be.

If the probability of overall project success $P(p, N)$ is to be held constant, q and N must be held constant. Given q and N constant, the greater T is, the smaller N/T is and so the closer we approximate the constant cost case of $q^{N/T} = 0$. Thus as T increases, $E(C)$ decreases, but at a diminishing rate. This proves that for the special case of approaches distributed evenly over time, the T, $E(C)$ tradeoff function is strictly convex to its origin. Computed tradeoff functions for $P(p, N)$ of 0.99, 0.95, 0.90, and 0.80, each assuming an individual approach success probability p of 0.05 with a cost per approach of \$1,000, are illustrated in Figure 1.*

Unfortunately for the cause of simplicity, the equal-number-of-approaches-per-time-period scheduling strategy is generally not the most efficient strategy if one is interested primarily in achieving a certain probability of project success $P(p, N)$ by time T. Where N_i is the number of approaches scheduled for the i-th time period, the problem of efficient scheduling in this case is to minimize the more general expected cost function:

$$(5) \qquad E(C) = MN_1 + q^{N_1} MN_2 + \ldots + \left(q^{N_1 + \ldots + N_{T-1}}\right) MN_T$$

subject to the constraint

$$\sum_{i=1}^{T} N_i = N ,$$

with N set at a level which yields the desired probability of overall project success. Under these assumptions, the most efficient strategy is to schedule a greater number of approaches in each successive time period. To see this, the scheduling problem can be viewed as a problem of allocating resources (N approaches) to T activities (periods). Cost is generally minimized in such a case with an allocation which lets each activity come as close as possible (within limits imposed by the integral number of projects) to having the same incremental cost.†

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*The tendency for $E(C)$ to be less variable for lower $P(p, N)$ is quite general. For p ranging from 0.01 to 0.50, the ratio of $E(C)$ for $T = 1$ to $E(C)$ for $T = 25$ was found to be approximately 4.3 for $P(p, N) = 0.99$, 3.0 for $P(p, N) = 0.95$, 2.5 for $P(p, N) = 0.90$, 2.0 for $P(p, N) = 0.80$, and 1.4 for $P(p, N) = 0.50$. Thus, the more confident of ultimate project success one seeks to be, the more sensitive one's costs are to scheduling decisions.
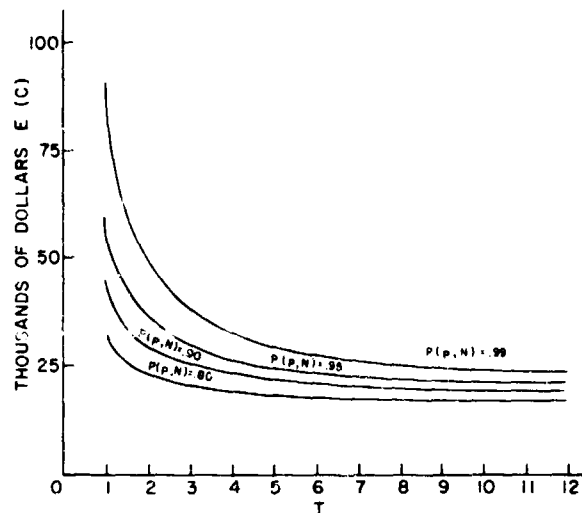†See for example [1], pp. 21-22.

Figure 1

In the present instance, where ΔN_i is the marginal allocation of resources to the i-th period, expected marginal costs are equalized when:

$$(6) \qquad M \Delta N_1 q^0 = M \Delta N_2 q^{N_1} = \ldots = M \Delta N_T \left(q^{N_1 + \ldots + N_{T-1}} \right).$$

Since the M's cancel out, and since each successive q term is smaller than its predecessor, each successive ΔN_i term must be larger to preserve the equality. And since this must be true for any N (that is, at any overall level of resource allocation), each successive period i must have a higher total allocation N_i (the sum of its marginal allocations ΔN_i) than the preceding period i - 1. In common sense terms, this means that when interested in achieving success by the end of two or more periods, one saves the bulk of his trials for later periods, hoping that the few early trials will yield a success making the expense of later approaches unnecessary.

A numerical example of Eq. (5) was computed by dynamic programming methods [2, pp. 14-18]. A 0.05 individual approach success probability was assumed, N was set at 60 to yield an overall project success probability of 0.95, and optimal schedules were computed for T from two through six periods. The most efficient six-period schedule was found to require a 6, 7, 8, 10, 12, 17 time pattern of approaches — a substantial departure from the equal-approaches-per-period strategy. The computation, however, also revealed that normally this additional scheduling sophistication does not buy a particularly large reduction in costs. For p = 0.05 and N = 60, the optimal two-period schedule has expected costs E(C) only 5.9 percent lower than the equal-approaches-per-period schedule. The percentage saving from optimal scheduling declines as T increases; thus, with T = 6, the optimal strategy yields an E(C) only 3.3 percent less than the equal-approaches strategy.*

Recognition that E(C) is minimized for any T by scheduling more approaches in later than in earlier periods does not alter the previous conclusion about the time-cost tradeoff

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*The percentage saving from optimal scheduling increases rapidly as higher success probabilities P(p, N) are sought. For instance, the saving jumps to about 16 percent in the two-period case if N is increased to 100, letting P(p, N) = 0.994.

function's convexity. For the $T = 1$ case, there can be no difference in scheduling strategies, and so E(C) is unaltered. But as T increases, strategies superior to the equal-approaches method can be found. It follows that expected cost must decrease as T increases even more under the efficient strategy than under the equal-approaches strategy of Eq. (4). And since the percentage gain from efficient as opposed to equal-approaches scheduling declines as t increases, the convex curvature must be more pronounced with the efficient strategy.

Convexity of the tradeoff function is also preserved when all costs are discounted to present value by a multiplier such as $1/(1+r)^t$, where r is the appropriate interest rate. The greater T is, the more the cost of the last period's approaches will be discounted, and so discounted expected cost must decrease as T increases. But because the second derivative of the discount multiplier with respect to $t = T$ is positive, the rate of decrease in cost must decline as T increases.

In sum, under the assumptions set forth thus far, we find that expected project cost E(C) decreases at a declining rate with increases in the number of periods T over which the project's technical approaches are distributed.

The task remains of deducing the properties of the function relating expected time and expected cost. This is perhaps a more empirically meaningful relationship, decision-makers are probably concerned more about the expected or average length of time to project completion than they are with the maximum time T required to reach a predetermined confidence level $P(p, N)$, assuming the worst possible luck in early approaches. "Project completion" in this context means either the achievement of a successful approach or abandonment of the project after N unsuccessful trials. Expected time to project completion E(T) is defined as:

$$(7) \qquad E(T) = 1 + q^{N_1} + \ldots + q^{N_1 + \ldots + N_{T-1}}.$$

Intuitively, E(T) is the sum of the units of time for each of the T periods, each unit deflated by the probability that a prior success will have made that period's effort unnecessary.

Since E(T) increases monotonically with T, given N and q, the function relating E(C) and E(T) must have a shape similar to the function relating E(C) and T. The values of the E(C), E(T) relationship are simply shifted nearer the E(C) axis for all values of $E(T) \geq 1$. Therefore E(C) must decrease as E(T) increases, although the fall in E(C) per unit of E(T) will be more rapid than the fall in E(C) per unit of T. And as in previous cases, the decline in E(C) as E(T) increases must occur at a diminishing rate.*

MODIFICATIONS OF THE BASIC MODEL

The time-cost tradeoff functions studied thus far have been based upon assumptions which obviously violate reality through oversimplification. Most fundamental have been the

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*It is appropriate once again to ask, is there a strategy better than the equal-approaches-per-time-period strategy? That is, is there a strategy which maintains the same value of E(T) with a lower E(C)? The problem is one of minimizing E(C), subject to N and also to E(T) (as defined in Eq. (7)) being held constant. The two-period case of this problem turns out to be overdetermined. Cases for T ≥ 2 are determinate, but too complicated to yield much in the way of analytic insights. Numerical trials involving many different scheduling strategies for the N = 60 case suggested that the equal-approaches strategy is inferior to a strategy which has the number of approaches scheduled in successive periods decline slightly. The difference in costs is quite small, however.

assumptions of equal success probabilities for each approach, equal costs per approach, inde-
pendence of approach success probabilities and costs, and redundancy of second and subsequent
successes.* Let us consider now whether the time-cost tradeoff function's convexity is pre-
served when these assumptions are relaxed.

If the probability of success p varies from approach to approach, the conclusion that
E(C) decreases at a declining rate as E(T) increases remains unaltered. The principal change
is that the time savings attainable by increasing E(C) will be less than in a comparable equal-
success-probabilities case. To illustrate, we may compare two projects, one with N ap-
proaches whose failure probabilities q are all equal, and the other with N approaches whose
failure probabilities $q_1, \ldots, q_N$ are unequal. Let the overall probability of project success
be held constant; that is, let

$$1 - q^N = 1 - \prod_{i=1}^{N} q_i.$$

Assume also that the cost per approach is constant in both cases. If in each situation all N
approaches are scheduled concurrently for the first period, E(T) = 1 in both cases. If on the
other hand one wishes to minimize expected cost, the approaches must be scheduled serially.
In the unequal probabilities case it will furthermore be advantageous to schedule the approach
with the highest success probability first, and so on. Since the probability that approaches in
any subsequent period will have to be executed must be lower in the unequal probabilities case
than in the equal probabilities case, E(T) for the minimum-cost serial strategy with unequal
probabilities must also be lower.† Thus, the E(C), E(T) tradeoff function will initially have a
steeper slope in the unequal probabilities case, but it still must be of the general convex form
illustrated in Figure 1.

The same conclusion applies when the costs of different approaches, or both costs and
success probabilities, vary from approach to approach. Here the analysis becomes much
more complicated, since an unusual nonlinear integer programming problem must be solved.
But as long as at least some approaches may be deferred to later periods, E(C) must decline
as E(T) increases. This is so quite generally because whenever the probability of incurring a
cost can be reduced by deferring the planned incurrence of that cost, expected cost will be re-
duced, even though expected time to incurrence of all planned costs must be increased.

A third modification in the direction of greater realism is to assume that the probabil-
ity of success or the cost of any given approach is related in some systematic way to the exe-
cution of previously executed approaches — i.e., that learning takes place. Each approach car-
ried out may, for instance, raise the probability of success for subsequent approaches or
permit the identification of approaches with higher success probabilities. It is not necessary
to explore specific Markov process models of this phenomenon to see that expected cost must
continue to be inversely related to expected time. If benefits are to be realized through learn-
ing, two things are necessary: at least one approach must be scheduled for the first period,
and there must be approaches scheduled for later periods whose subjective success probabilities

*The assumption that approaches must be conducted in exactly one time period has alread,
been relaxed in the footnote on page 73. It does, however, raise an economics of scale ques-
tion similar to the one to be explored shortly in connection with the independence assumptions.
†A proof of this assertion is developed easily from Eq. (7) and from the fact that the sum of
variables (the failure probabilities) whose product is constant is minimized when the varia-
bles are equal.

will be enhanced or whose costs will be reduced by the prior experience. No learning is assumed to take place with the minimum-time strategy — with all approaches scheduled concurrently during the first period.* Only as trials are deferred into later periods, increasing E(T), can the cost-reducing (and to some extent also time-reducing) benefits of learning be reaped. Therefore, E(C) must decline as E(T) increases. What learning does is shift the curves such as those in Figure 1 to the left for values of E(T) (on the horizontal axis) greater than 1.

A fourth possibility is that the success probabilities of individual approaches, or their costs, may be interdependent within a single time period. Beneficial cross-fertilization of ideas may occur between concurrently scheduled approaches; specialized equipment or talent may be available on favorable terms only if some critical scale of concurrently executed approaches is exceeded; or incentives for vigorous creative effort may be enhanced as the number of competing approaches is increased. Any of these influences might cause approach success probabilities to rise or approach costs to fall as one moves from serial towards some concurrent scheduling. As a result there may be a tendency for E(C) actually to fall as E(T) is decreased within certain limits — notably, within the range of relatively high E(T) values. The final result depends upon the strength of these tendencies compared to the strength of the opposing probabilistic tendencies explored in the section entitled "Properties of the Time-Cost Tradeoff Function." This is essentially an empirical question on which relatively little evidence is available. However, there are a priori grounds for believing that the forces causing E(C) to decline with E(T) are not overpowering. Cross-fertilization over time (e.g., interactions following from learning) is probably more effective than cross-fertilization within a single time period. Specialized talent can often be hired advantageously on a consulting basis, and independent research organizations offer access to special experimental equipment. I have argued elsewhere [8, pp. 44-49] that the incentive benefits of competition reach a peak with relatively few competing R&D projects and probably decline when more projects vie for a single prize. And when approaches to a technical problem are pursued concurrently within the confines of a single organization, there is a propensity for all to hew to a fairly narrow conceptual line favored by the organization's professional leaders.† If promising but unconventional approaches are thereby excluded from the menu, the overall probability of success will fall and expected cost to successful completion will rise as a result of concurrent scheduling. In short, while institutional and organizational factors may well cause the time-cost function to bend back upward beyond some value of E(T), in contrast to the continuously declining configuration shown in Figure 1, it is not likely that cost will become a continuously increasing function of time.

Finally, we must relax the assumption that only the first successful solution obtained in a multiple-approach project is not redundant. If many approaches are pursued concurrently,

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*In [4], Richard Nelson has formulated a somewhat different model of uncertain hardware development in which learning occurs within each approach. He considers only the cost-reducing and time-reducing benefits of running approaches concurrently when internal learning takes place, and not the benefits of series scheduling. But in fact, when the latter possibility is introduced into his model, one finds a relationship between expected cost and expected time similar to the one obtained here. Using his numerical example, and assuming that all but terminal projects are abandoned unless a first flight is made within 20 months, E(C) for the two-approach case is $84 million with series scheduling compared to $88 million with concurrent scheduling. For N = 3, E(C) is $80.4 million with series as opposed to $91 million with concurrent scheduling. For N = 4, the comparable cost figures are $78.2 million and $96.5 million. Naturally, E(T) increases as one moves from Nelson's concurrent strategy to series scheduling. But that is precisely the point: that tradeoffs between time and cost exist and must be made.

†The Air Force's early emphasis on heat sink reentry techniques is a glaring example.

more than one success may emerge, and the quality of the solutions meeting minimum standards of success will undoubtedly vary.* Presumably, the more solutions meeting minimum standards one achieves, the higher will be the quality of the best solution. In this respect minimum-time strategies and series strategies cannot be completely comparable, since the concurrent execution of many or all approaches will yield more successes than will series execution of the same approaches, with project termination after the first success. Indeed, the fundamental cost-saving property of the series strategy is the opportunity it provides for avoiding additional trials after one success emerges. Potential quality variation therefore raises definite complications for the analysis given here. Nevertheless, the series approach does allow greater flexibility to forego planned future approaches once a high-quality solution is in hand, and so expected cost must continue to decrease as expected time is increased, quality being held constant. The most that can presently be said in addition is that the more important better-than-minimum quality is to the decision-maker, the more advantageous concurrent scheduling is likely to be, other things being equal.

In sum, the time-cost tradeoff function appears to be convex for a much broader set of assumptions than those upon which the analysis in the third section was based, although in certain cases an increasing cost – increasing time relationship may set in for higher time values.

CHOOSING AN OPTIMAL SCHEDULE

If the relationship between time and cost is in fact generally convex – that is, if within some range decreases in expected time to project completion can be secured only by accepting accelerating increases in expected project cost – then tradeoffs between conflicting cost minimization and time minimization desires must be made. For a preliminary insight into these tradeoff decisions, let us assume (risking minor errors) that expected cost $E(C_d)$, discounted to present value at an appropriate interest rate, is a smooth continuous function of expected time $E(T)$, holding the probability of overall project success $P(p, N)$ constant:

$$(8) \qquad E(C_d) = f\,[E(T)\,|\,P(p, N)].$$

For the range within which expected cost decreases as expected time increases, the first derivative of (8) with respect to $E(T)$ is negative. When, assuming strict convexity, the decrease in $E(C)$ occurs at a declining rate, the second derivative of (8) with respect to $E(T)$ is positive.

To determine how far expected time should be compressed by increasing expected cost, we must know how much will be gained by saving time. Typically, the benefits realizable from a successful research project will flow in for many periods after the project's completion. We may therefore approximate the benefit stream by the expression:

$$(9) \qquad B\,[E(T), t] = P(p, N) \int_{E(T)}^{h} b(t)\, e^{-rt}\, dt;$$

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*For example, if $p = 0.05$ and $N = 50$, the probability that at least one success will be had is 0.92, assuming that all 50 approaches are executed. But there is also a 0.10 probability of five or more successes.

where $b(t)$ is the instantaneous rate at which benefits are received per unit of time, h is the decision-maker's horizon or the last year during which positive benefits can be reaped, and r is the interest or discount rate. [*] The conventional criterion of economic rationality calls for maximization of discounted benefit minus discounted costs: $B[E(T), t] - E(C_d)$. The first order condition for a local maximum in this case is:

$$(10) \qquad f'[E(T) \mid P(p, N)] = P(p, N) \, b[E(T)] \, e^{-rT}.$$

Intuitively, the expected increase in benefits due to anticipating research project completion one period earlier must, at the optimum, equal the expected increase in project cost due to compressing the schedule by one period. As long as the second derivative of (8) is positive, the second order conditions for a local maximum will normally be satisfied. If so, then the greater $b(t)$ is in the neighborhood of the optimal $E(T)$ — i.e., the deeper the stream of potential benefits is — the lower the optimal $E(T)$ will be. In other words, the greater the payoff in future time periods contingent upon successful research project completion, the more concurrent rather than series scheduling of research approaches should be emphasized.

One final complication must now be introduced. The preceding case assumed the probability of overall project success to be given. In the second section it was tentatively suggested that $P(p, N)$ be determined by equating the expected cost of the last approach with the expected gain from conducting that approach. But this optimizing method breaks down when $E(T)$ is variable, since the total amount of benefits to be realized [Eq. (9)] varies inversely with $E(T)$. The deeper the stream of benefits is, the shorter the optimal $E(T)$ will be, ceteris paribus. But the shorter $E(T)$ is, the longer benefits will be reaped, and so the more approaches it will pay to authorize. And the greater N is, the higher $P(p, N)$ will be, and so the deeper the expected (probability-weighted) stream of benefits will be. A simultaneous determination of N and T is required, unless one is willing to set N (and hence the level of confidence in eventual success) on the basis of purely subjective attitudes towards risk.

This simultaneous decision-making problem can be formulated as follows: Where t is a running integral time variable, B_t is the dollar value of the benefits realizable in the t-th period contingent upon success, M is the cost per approach, q is the probability of failure of each approach, N is the number of approaches, and r is the interest or discount rate, we wish to maximize net present value, defined as:

$$(11) \qquad V = \sum_{t=2}^{T} \left[1 - q^{N(t-1)/T} \right] \left(\frac{1}{1+r} \right)^t B_t + \sum_{t=T+1}^{h} (1 - q^N) \left(\frac{1}{1+r} \right)^t B_t$$

$$- \sum_{t=1}^{T} \left[q^{N(t-1)/T} \right] \left(\frac{1}{1+r} \right)^{t-1} \left(\frac{NM}{T} \right),$$

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[*] This formulation assumes that the project is to be initiated at $t = 0$. A different assumption would require only a slight change in notation.

with respect to N and T. Since this expression is impregnable to assault by analytic methods, a numerical simulation analysis to determine the properties of optimal solutions was executed. The value of (11) was computed for a wide range of possible combinations of N and T for each of eight benefit stream depths (assumed constant per period through 25 periods) for success probabilities of 0.01, 0.05, 0.10, and 0.20. M was assumed to equal $1,000 per approach and the discount rate r was 0.06. The results can be summarized as follows: First, for any given success probability, the optimal N increased monotonically with the value of the benefits B_t realizable per time period. No such consistent pattern was evident with respect to the optimal T's. Second, for any given q and B_t, many different combinations of N and T tended to give net present values very close to the maximum observed value. In other words, project profitability was insensitive to certain changes in the schedule variables. But third, profitability was very sensitive to changes in N/T — that is, to the number of approaches scheduled per time period. As long as the optimal N/T ratio was maintained, net present value V did not depart much from its maximum as N and T were varied together by substantial proportions away from their optimal values. Movement away from the optimal N/T ratio on the other hand caused a rapid decline in V. * Finally, the deeper the stream of benefits B_t was, given any q, the more approaches per period it was profitable to schedule. This last finding is illustrated in Figure 2, which plots the optimal N/T ratios as a function of dollar benefits per period B_t for four different approach success probabilities.
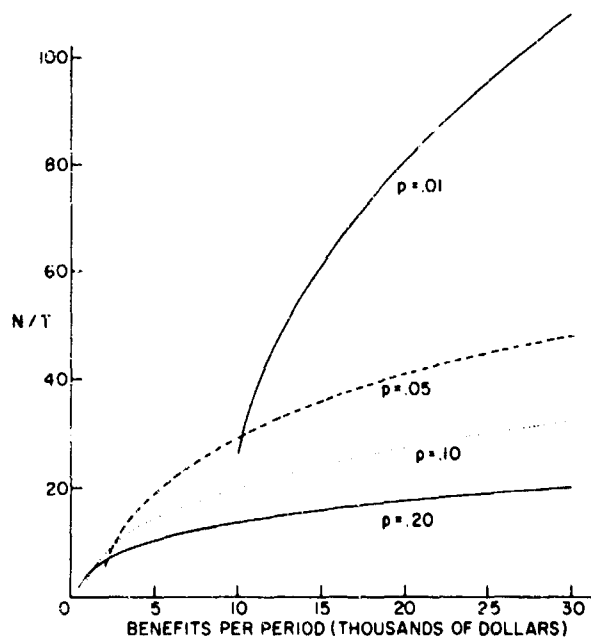

Figure 2

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*The optimum was sensitive to changes in N/T, but not to proportional changes in N and T, because what happens in the later years of an uncertain planned project has little bearing on originally expected costs and returns. This is so because the costs and contingent returns associated with later years are discounted heavily by both the probability of success in prior periods and by the conventional interest discount term. The most important feature of optimal project scheduling appears to be that of having the correct number of approaches per time period in early periods, whose costs and contingent benefits are only weakly discounted. This clearly facilitates a dynamic, heuristic approach to R&D scheduling. One does the best he can this year, and then reoptimizes next year on the basis of new data and expectations.

Thus, we find our previous conclusion essentially unaltered in the more complex case of permitting N and T to vary simultaneously. The deeper the stream of benefits is, the more approaches one should schedule per time period, and therefore the sooner successful project completion can be expected.

IMPLICATIONS

Even though the models discussed here are gross simplifications of the real world, they seem to suggest def'nite implications for the budgeting of actual research projects. Convexity of the relationship between time and cost tends to emerge from the existence of substantial empirical uncertainties. And when the time-cost tradeoff function is convex, concurrent scheduling of all approaches to a technologically difficult problem is not necessarily optimal, despite frequent assertions to this effect, especially in military circles. Nor is serial scheduling necessarily a good thing. It all depends upon the stream of benefits realizable through a successful solution: the deeper the stream, the more desirable concurrent scheduling is, and the shallower the stream, the more desirable the series strategy is. Even when success probabilities and payoffs are difficult to estimate (as they always are), rough and ready recognition of this simple principle will undoubtedly lead to improved allocation of scarce research resources.

REFERENCES

[1] Baumol, W. J., Economic Theory and Operations Analysis (Prentice-Hall, Englewood Cliffs, N. J., 1961).

[2] Bellman, R. E. and Dreyfus, S. E., Applied Dynamic Programming (Princeton University Press, Princeton, N. J., 1962).

[3] Josephson, Matthew, "The Invention of the Electric Light," Scientific American (Nov. 1959), pp. 98-118.

[4] Nelson, R. R., "Uncertainty, Learning, and the Economics of Parallel Research and Development Projects," Review of Economics and Statistics (Nov. 1961), pp. 351-368.

[5] Peck, M. J. and Scherer, F. M., The Weapons Acquisition Process: An Economic Analysis (Harvard Business School Division of Research, Boston, 1962).

[6] Rosenbloom, R. S., "Notes on the Development of Network Models for Resource Allocation in R&D Projects," IEEE Transactions on Engineering Management (June 1964), pp. 58-63.

[7] Scherer, F. M., "Government Research and Development Programs," in Measuring Benefits of Government Investments, Robert Dorfman, ed. (Brookings Institution, Washington, D. C., 1965), pp. 34-56.

[8] Scherer, F. M., The Weapons Acquisition Process: Economic Incentives (Harvard Business School Division of Research, Boston, 1964).

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