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THE THEORY AND PRACTICE OF TWO-DIMENSIONAL SUPERSONIC PRESSURE CALCULATIONS

BY

N. EDMONSON, F. D. MURNAGHAN, R. M. SNOW

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INTRODUCTION

The experience of the Analysis Group engaged in theoretical supersonic aerodynamic analysis in the Applied Physics Laboratory of The Johns Hopkins University has shown that the computations of pressure distributions on airfoils of infinite span in a two-dimensional flow and of the effects of shock fronts can be greatly speeded up by the use of appropriate tables. Brief tables for the calculation of pressure distributions have been given by Lighthill[1] but so far as was known to the Analysis Group, when its work was started, tables to facilitate the computation of oblique shock front effects did not exist. The Analysis Group has calculated extensive tables for facilitating oblique shock front computations.

At the same time the need for a concise and accurate discussion of the principles of supersonic aerodynamics became apparent. With the aid of its consultant, Dr. F. D. Murnaghan, Professor of Mathematics in The Johns Hopkins University, the Analysis Group prepared such a discussion for two-dimensional supersonic flows, giving careful attention to the precise definition of terms. An important feature of this study is the demonstration that an arbitrary curve intersecting once and only once each of the straight-line patching or characteristic curves in an expansive supersonic flow can be treated as if it were a characteristic curve. This fact leads at once to the formulas for calculating pressure distributions over airfoils immersed in two-dimensional supersonic flows, and hence to lift, drag, and moment computations. This explicit demonstration is not given in any of the literature that so far has come to the attention of the Analysis Group. Also it is explicitly demonstrated that the Ackeret linear theory and the Busemann second order theory are approximations to the more general theory.

The present Bumblebee report includes both of the studies just described—the computation tables and the discussion of the principles of two-dimensional supersonic flow. Thus it is essentially a manual, for the two-dimensional case, on the theory and practice of computation in supersonic, aerodynamic flow, as far as the present state of the subject is concerned.

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[1] F. M. Panel No. 654, Ac. 2420; A. R. C. No. 7384.

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PART I. GENERAL THEORY

1. *The theory of patching curves (characteristics) in two-dimensional supersonic flow.*

We shall confine our discussion to the case of steady two-dimensional flow of a perfect fluid. The basic equations of motion are, in the absence of mass forces,

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{du}{dt}, \qquad -\frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{dv}{dt}.$$

We postulate a relation

$$p = p(\rho, \eta)$$

where η is the entropy and assume that η does not vary with x and y; then

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial \rho}\frac{\partial \rho}{\partial x} = c^2\frac{\partial \rho}{\partial x}, \qquad \frac{\partial p}{\partial y} = \frac{\partial p}{\partial \rho}\frac{\partial \rho}{\partial y} = c^2\frac{\partial \rho}{\partial y}$$

where

$$c^2 = \frac{\partial p}{\partial \rho}$$

so that c is the "local sound-velocity". Since the flow is steady

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}, \qquad \frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

and so the equations of motion may be written in the form

$$-\frac{c^2}{\rho}\frac{\partial \rho}{\partial x} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}, \qquad -\frac{c^2}{\rho}\frac{\partial \rho}{\partial y} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}.$$

The equation of continuity (for steady flow) is

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0.$$

Upon multiplying this by $-\frac{c^2}{\rho}$ and substituting the values just given for $-\frac{c^2}{\rho}\frac{\partial \rho}{\partial x}$, $-\frac{c^2}{\rho}\frac{\partial \rho}{\partial y}$ we obtain

$$u\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + v\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) - c^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

i.e.

$$(c^2 - u^2)\frac{\partial u}{\partial x} - uv\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + (c^2 - v^2)\frac{\partial v}{\partial y} = 0. \qquad (1)$$

We now consider an arbitrarily given curve C:

$$x = x(\alpha); \qquad y = y(\alpha)$$

(α being any convenient parameter or independent variable) and suppose that the values of u, v and of the vorticity $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ are prescribed along this curve. In other words, u, v, and $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ are given functions of α; does it, then, follow that the values of the four space derivatives $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ of the two velocity components (u, v) are unambiguously determined along C? If not we can regard the curve C as a *patching curve* along which two different flows are patched together, there being *no* discontinuity in the velocity components (u, v) nor in the vorticity $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ across the patching curve. For this reason any such patching curve C must be clearly distinguished from a *shock-curve* across which the normal component of velocity is discontinuous.

Since u and v are given functions of α along C we have

$$\frac{\partial u}{\partial x}\frac{dx}{d\alpha} + \frac{\partial u}{\partial y}\frac{dy}{d\alpha} = \frac{du}{d\alpha}$$

$$\frac{\partial v}{\partial x}\frac{dx}{d\alpha} + \frac{\partial v}{\partial y}\frac{dy}{d\alpha} = \frac{dv}{d\alpha}.$$

These two equations together with the equation

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \omega(\alpha),$$

where $\omega(\alpha)$ is a given function of α, and the equation (1) constitute a system of four linear equations for the four space derivatives of the velocity components (u, v). These equations determine, without ambiguity, the four space derivatives of u and v unless the determinant of the coefficients happens to be zero. Thus in order that C may be a curve which patches together two flows for which, at the various points of C, at least one of the four derivatives $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ has different values for the two flows (the values of u, v and $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ being, however, the same for both flows along C, it is necessary that

$$(c^2 - v^2)\left(\frac{dx}{d\alpha}\right)^2 + 2uv\frac{dx}{d\alpha}\frac{dy}{d\alpha} + (c^2 - u^2)\left(\frac{dy}{d\alpha}\right)^2 = 0. \quad (2)$$

This quadratic equation for the ratio $\frac{dx}{d\alpha} : \frac{dy}{d\alpha}$ has real roots if, and only if,

$$u^2 + v^2 \geq c^2$$

(the roots being equal when the equality holds). Thus a point of the flow cannot be a point of a patching curve unless the flow is supersonic (or, at least, sonic) at that point; in other words a *necessary* condition for a point of the flow to be a point of a patching curve is

$$q \geq c$$

(where $q = (u^2 + v^2)^{\frac{1}{2}}$ is the speed of the flow). The essential difference between supersonic and subsonic flow is:

There are no patching curves in subsonic flow.[2]

It remains now to see what further condition, if any, must be imposed on u, v and $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ (which are given functions of α along C) in order that the four linear equations connecting $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ may have a solution when the determinant of their coefficients is zero. This condition is obtained by equating to zero the determinant obtained by replacing any one of the four columns of the determinant of the coefficients of the four equations by the column of numbers on the right-hand side of the four equations. Replacing the second column, for example, we obtain

$$\begin{vmatrix} \frac{dx}{d\alpha} & \frac{du}{d\alpha} & 0 & 0 \\ 0 & \frac{dv}{d\alpha} & \frac{dx}{d\alpha} & \frac{dy}{d\alpha} \\ 0 & \omega(\alpha) & 1 & 0 \\ c^2 - u^2 & 0 & -uv & c^2 - v^2 \end{vmatrix} = 0$$

or, equivalently,

$$(c^2 - v^2)\frac{dx}{d\alpha}\frac{dv}{d\alpha} + (c^2 - u^2)\frac{dy}{d\alpha}\frac{du}{d\alpha} = \left\{(c^2 - v^2)\frac{dx}{d\alpha} + uv\frac{dy}{d\alpha}\right\}\frac{dx}{d\alpha}\,\omega(\alpha).$$

We shall confine ourselves to the case where $\omega(\alpha) = 0$ so that the vorticity is taken to be zero along the patching curve; in particular, this will certainly be the case when the flow is *irrotational*, i.e. when the vorticity is zero throughout the entire flow. Then the sufficient condition for the existence of solutions of the four linear equations for the four space derivatives of u and v along C (when the determinant of these four equations is zero) is

$$(c^2 - v^2)\frac{dx}{d\alpha}\frac{dv}{d\alpha} + (c^2 - u^2)\frac{dy}{d\alpha}\frac{du}{d\alpha} = 0. \quad (3)$$

From this point on we shall suppose that the flow is *irrotational*, i.e., that $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$ and that the relation which expresses p as a function of the density ρ and the entropy η is of the form

$$p = k\rho^\gamma$$

where k is a function of the entropy η alone (so that k does not vary with x and y). Since, when the flow is irrotational,

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(\tfrac{1}{2}q^2)$$

$$\frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(\tfrac{1}{2}q^2)$$

the *two* equations of motion are equivalent to (i.e. imply and are implied by) the *single* relation

$$\tfrac{1}{2}q^2 + \int\frac{c^2}{\rho}d\rho = \text{const.} \quad (4)$$

Using the relation $p = k\rho^\gamma$ we have $c^2 = \frac{\partial p}{\partial \rho} = \gamma k\rho^{\gamma-1} = \frac{\gamma p}{\rho}$ so that $\int\frac{c^2}{\rho}d\rho = \frac{\gamma k}{\gamma - 1}\rho^{\gamma-1} = \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = \frac{c^2}{\gamma - 1}$. We may, then, write the single relation which is equivalent to the two equations of motion in the form

$$c^2 = \frac{\gamma - 1}{2}(q_M^2 - q^2) \quad (5)$$

where q_M is a constant $\geqslant q$ (it is the value of q at those points, if any, of the flow where $c = 0$).

The differential equation which determines the patching curves may be written in the form

$$(c^2 - v^2)(dx)^2 + 2\,uv\,dx dy + (c^2 - u^2)(dy)^2 = 0$$

---

[2] What are here termed patching curves are usually called *characteristics*; we prefer the name patching curve since there is danger of confusing characteristics with shock curves.

and, on dividing through by $ds^2 = (dx)^2 + (dy)^2$, we obtain

$$c^2 = \left(v\frac{dx}{ds} - u\frac{dy}{ds}\right)^2.$$

On denoting by θ the inclination of the flow we have

$$u = q\cos\theta,\ v = q\sin\theta$$

so that

$$\frac{c^2}{q^2} = \left(\sin\theta\frac{dx}{ds} - \cos\theta\frac{dy}{ds}\right)^2 = \sin^2\mu$$

where μ is the angle between the direction of flow and the direction of the patching curve; μ is the Mach angle and

$$\sin\mu = c/q.$$

The number $M = q/c = \operatorname{cosec}\mu$ is known as the *Mach number* of the flow at the point in question.

Through any point, then, where the flow is supersonic there pass two possible patching curves and the direction of flow bisects the angle between these two curves (the angle between the direction of flow and the patching curves being $\pm\mu$; it is convenient to direct the patching curves so that the direction of flow bisects *internally* the angle between the two curves). At a point where the flow is sonic, $\mu = \pi/2$ and the two possible patching curves through this point touch at the point (the direction of flow being along their common normal at the point).

The two slopes of the possible patching curves through a point where the flow is supersonic are

$$\frac{-uv \pm c\sqrt{q^2 - c^2}}{c^2 - u^2}.$$

Let us consider the curve for which

$$\frac{dy}{dx} = \frac{-(uv + c\sqrt{q^2 - c^2})}{c^2 - u^2}$$

and calculate the rate, $\frac{dq}{d\alpha}$, at which the speed q varies along this patching curve. Since $q^2 = u^2 + v^2$ and since $(c^2 - v^2)\,dx\,dv + (c^2 - u^2)dy\,du = 0$ we have

$$\begin{aligned} q\,dq = u\,du + v\,dv &= \left\{u - v\frac{c^2 - u^2}{c^2 - v^2}\frac{dy}{dx}\right\}du \\ &= \left\{u + v\frac{uv + c\sqrt{q^2 - c^2}}{c^2 - v^2}\right\}du \\ &= \frac{c}{c^2 - v^2}(uc + v\sqrt{q^2 - c^2})\,du \end{aligned}$$

so that

$$q\frac{dq}{d\alpha} = \frac{c}{c^2 - v^2}(uc + v\sqrt{q^2 - c^2})\frac{du}{d\alpha}.$$

Since $u = q\cos\theta$, $v = q\sin\theta$, we have

$$\begin{aligned} u\,dv - v\,du = u(\sin\theta\,dq + q\cos\theta\,d\theta) \\ - v(\cos\theta\,dq - q\sin\theta\,d\theta) = q^2 d\theta \end{aligned}$$

and since

$$\begin{aligned} u dv - v du &= -\left\{u\frac{c^2 - u^2}{c^2 - v^2}\frac{dy}{dx} + v\right\}du \\ &= \left\{\frac{u}{c^2 - v^2}(uv + c\sqrt{q^2 - c^2}) - v\right\}du \\ &= \frac{v(q^2 - c^2) + uc\sqrt{q^2 - c^2}}{c^2 - v^2}du \end{aligned}$$

we obtain

$$q^2\frac{d\theta}{d\alpha} = \frac{\sqrt{q^2 - c^2}}{c^2 - v^2}\{uc + v\sqrt{q^2 - c^2}\}\frac{du}{d\alpha}.$$

On combining this with the formula furnishing $q\frac{dq}{d\alpha}$ we find

$$\frac{1}{q}\frac{dq}{d\theta} = \frac{c}{\sqrt{q^2 - c^2}} = \tan\mu. \tag{6}$$

This is the fundamental relation as far as the application of the theory of patching curves (characteristics) to supersonic calculations is concerned. It is easy to show that if we had selected the other characteristic direction through the given point we should have obtained

$$\frac{1}{q}\frac{dq}{d\theta} = -\tan\mu.$$

Summarizing these results, we may say:

Along any patching curve (or characteristic) in supersonic flow

$$\frac{1}{q}\frac{dq}{d\theta} = \pm\tan\mu$$

where q is the speed, θ is the inclination of flow and μ is the Mach angle.

If we denote by t the component of velocity tangent to the patching curve, so that $t = q\cos\mu$, we have, since $c = q\sin\mu$,

$$q^2 = c^2 + t^2.$$

When this is substituted in the relation

$c^2 = \frac{\gamma - 1}{2}(q_M^2 - q^2)$ we get

$$\frac{\gamma + 1}{2} c^2 = \frac{\gamma - 1}{2}(q_M^2 - t^2)$$

so that

$$c^2 = \frac{\gamma - 1}{\gamma + 1}(q_M^2 - t^2).$$

Now the relation $t = q \cos \mu$ yields

$$dt = \cos \mu \, dq - q \sin \mu \, d\mu$$

and on substituting for dq its value $\pm q \tan \mu \, d\theta$, we obtain

$$dt = q \sin \mu \, (\pm d\theta - d\mu) = c(\pm d\theta - d\mu).$$

Hence

$$\pm\theta = \mu + \int \frac{dt}{c} = \mu + \sqrt{\frac{\gamma + 1}{\gamma - 1}} \int \frac{dt}{\sqrt{q_M^2 - t^2}}$$

$$= \mu + \sqrt{\frac{\gamma + 1}{\gamma - 1}} \sin^{-1} \frac{t}{q_M} + \text{const.}$$

Since

$$\sqrt{q_M^2 - t^2} = \sqrt{\frac{\gamma + 1}{\gamma - 1}}\, c,$$

$$\sin^{-1} \frac{t}{q_M} = \cot^{-1}\left(\sqrt{\frac{\gamma + 1}{\gamma - 1}} \frac{c}{t}\right) = \cot^{-1}\left(\sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan \mu\right)$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan \mu\right).$$

Thus the inclination of the flow at any point of a patching curve is connected with the Mach angle by the formula

$$\pm\theta = \mu - \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1}\left(\sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan \mu\right) + \text{constant}$$

$$= \text{constant} - f(\mu)$$

where

$$f(\mu) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1}\left(\sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan \mu\right) - \mu. \qquad (7)$$

It follows from this relation that if θ is known at every point of a patching curve and if μ is known at *one* point of this curve then μ may be determined at every point of the curve. All that is needed is a table of values of the function $f(\mu)$. For

$$\pm(\theta - \theta_0) = f(\mu_0) - f(\mu)$$

so that

$$f(\mu) = \pm(\theta_0 - \theta) + f(\mu_0). \qquad (8)$$

This relation determines $f(\mu)$ and, hence (by means of the table), μ, for any given value of θ.

Table 5 furnishes the values of $f(\mu)$, calculated for $\gamma = 1.4$.

The relation $c^2 = \frac{\gamma - 1}{2}(q_M^2 - q^2)$ implies the relation $q^2 - c^2 = \frac{\gamma + 1}{2}(q^2 - q_m^2)$ where $q_m = \sqrt{\frac{\gamma - 1}{\gamma + 1}}\, q_M$. Hence, at any point where the flow is supersonic,

$$q_M \geq q \geq q_m = \sqrt{\frac{\gamma - 1}{\gamma + 1}}\, q_M.$$

In a *velocity diagram* (where the coordinates of any point are the velocity components (u, v)) a point where the flow is supersonic lies in the circular ring

$$q_M \geq q \geq q_m$$

(the polar coordinates of any point in the velocity diagram being $q \angle \theta$). The relation

$$\frac{1}{q}\frac{dq}{d\theta} = \pm \tan \mu$$

shows that the graph, in the velocity diagram, of the points on a patching curve lie on an epicycloid traced by a point on the circumference of a circle of diameter $q_M - q_m$ which rolls externally on the circle of radius q_m whose center is at the origin. In fact if a circle of radius b rolls externally on a circle of radius a the coordinates of a point on its circumference are given by

$$x = (a + b) \cos \frac{b}{a} \tau - b \cos\left(1 + \frac{b}{a}\right)\tau,$$

$$y = (a + b) \sin \frac{b}{a} \tau - b \sin\left(1 + \frac{b}{a}\right)\tau$$

where τ is the angle through which the circle of radius b has rolled. The polar coordinates $r \angle \theta$ of this point are given by

$$r^2 = x^2 + y^2 = (a^2 + 2ab + 2b^2) - 2b(a + b) \cos \tau, \quad \tan \theta = y/x.$$

From the latter relation we deduce $\sec^2\theta \, d\theta = \frac{x\,dy - y\,dx}{x^2}$ so that $d\theta = \frac{x\,dy - y\,dx}{x^2 + y^2}$. Hence

$$
\begin{aligned}
r^2\,d\theta &= x\,dy - y\,dx \\
&= \frac{(a+b)b}{a} \times \Bigg\{\left[(a+b)\cos\frac{b}{a}\tau - b\cos\left(1+\frac{b}{a}\right)\tau\right] \\
&\qquad \times\left[\cos\frac{b}{a}\tau - \cos\left(1+\frac{b}{a}\right)\tau\right] \\
&\qquad +\left[(a+b)\sin\frac{b}{a}\tau - b\sin\left(1+\frac{b}{a}\right)\tau\right] \\
&\qquad \times\left[\sin\frac{b}{a}\tau - \sin\left(1+\frac{b}{a}\right)\tau\right]\Bigg\}\,d\tau \\
&= \frac{(a+b)b(a+2b)}{a}(1-\cos\tau)\,d\tau.
\end{aligned}
$$

In the case of the velocity diagram, q takes the place of r and for points on any member of one of the two families of patching curves we have

$$\pm\frac{1}{q}\frac{dq}{d\theta} = \tan\mu = \frac{c}{\sqrt{q^2-c^2}} = \sqrt{\frac{\gamma-1}{\gamma+1}}\sqrt{\frac{q_M^2-q^2}{q^2-q_m^2}}.$$

Then q_M, the radius of the outer circle, takes the place of $a + 2b$; and q_m, the radius of the inner circle, takes the place of a. We calculate, accordingly, $(a + 2b)^2 - r^2$ and $r^2 - a^2$:

$$(a+2b)^2 - r^2 = 2b(a+b)(1+\cos\tau)$$

$$r^2 - a^2 = 2b(a+b)(1-\cos\tau)$$

so that

$$\sqrt{\frac{(a+2b)^2-r^2}{r^2-a^2}} = \cot\frac{\tau}{2}.$$

Since

$$r\,dr = b(a+b)\sin\tau\,d\tau$$

we find, on using the value of $r^2 d\theta$ calculated above, that

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{a}{a+2b}\cot\frac{\tau}{2} = \frac{a}{a+2b}\sqrt{\frac{(a+2b)^2-r^2}{r^2-a^2}}.$$

Since $\frac{a}{a+2b} = \frac{q_m}{q_M} = \sqrt{\frac{\gamma-1}{\gamma+1}}$ it follows that the plot, in the velocity diagram, of any member of one of the two families of patching curves (namely, the family for which $\frac{1}{q}\frac{dq}{d\theta} = +\tan\mu$) is an epicycloid traced by a point on the circumference of a circle which rolls between two circles, of radii q_m and q_M, both circles having a common center (the origin).

The members of the second family of characteristics also plot as epicycloids traced by a point on the circumference of the same rolling circle; all we have to do is to replace τ by $-\tau$ (or, equivalently, by $2\pi - \tau$). This merely amounts to the geometric fact that through any point in a circular ring there can be drawn two circles each of which touches the circles which determine the ring (the common radius of the constructed circles being the difference of the radii of these two circles).

When a patching curve serves to patch a *uniform* flow to a second flow (uniform or non-uniform) the plot, in the velocity diagram, of the patching curve is a point and the patching curve itself is a straight line. In fact u and v are constant along the patching curve so that the plot, in the velocity diagram, of the patching curve is a point. Since q is constant along the patching curve so also is c, in view of the relation $c^2 = \frac{\gamma-1}{2}(q_M^2 - q^2)$, and it follows from (2) that $\frac{dy}{dx}$ is constant along the patching curve; in other words the patching curve is a straight line. This result is of fundamental importance:

A curve which patches, in supersonic flow, a uniform flow onto a second flow (uniform or non-uniform) is a straight line whose plot, in the velocity diagram, is a point.

What can we say about the various members of the *other* family of patching curves which intersect such a straight line patching curve (whose plot in the velocity diagram is the point $P_0 = (u_0, v_0)$)? The plot of any such patching curve (of the second family) in the velocity diagram must pass through P_0 and hence must be the epicycloid (of the second type) which passes through P_0. In other words:

All patching curves of the second family which intersect a curve which patches a uniform flow to a second flow (uniform or non-uniform) have the same plot in the velocity diagram.

Let us now examine the various members of the first family of patching curves which intersect these members of the second family. The various points where a fixed member of the first family intersects different members of the second family all have the same plot in the velocity diagram (for the *various* members of the second family plot into a *single* curve in the velocity diagram). Hence the velocity vector is constant along the fixed member of the first family of patching curves and this implies that this member of the first family of patching curves is a straight line. In other words:

The patching curve which patches a uniform flow to a second flow (uniform or non-uniform) is not only a straight line but it is one of a family of straight lines each of which is a patching curve; furthermore the velocity vector is constant along each member of this family of straight lines.

Along each member of the family of straight line patching curves, both q and θ are constant so that the relation $\frac{1}{q}\frac{dq}{d\theta} = \pm \tan \mu$ is meaningless (the left-hand side of this relation being indeterminate). If the relation $\frac{1}{q}\frac{dq}{d\theta} = + \tan \mu$ is valid along the members of the second family, q decreases with θ and hence c, ρ and p *increase* as θ decreases. If $\frac{1}{q}\frac{dq}{d\theta} = - \tan \mu$, q increases as θ decreases, c, ρ and p decrease as θ decreases, and the flow is defined as an *expansive* flow. It will be assumed from now on in this paper that the flow is expansive.

Since the velocity vector is constant along each member of the family of straight-line patching curves so also is c, by virtue of the relation

$$c^2 = \frac{\gamma - 1}{2}(q_M^2 - q^2).$$

What is the practical significance of these facts? If we draw an *arbitrary* curve (not merely a member of the second family of patching curves) which intersects once and only once each member of the family of straight-line patching curves, the values of q, θ and μ at any point of this curve are the same as those at the corresponding point of any member of the second family of patching curves (two points being said to correspond when they lie on the same straight-line patching curve). In other words the plot in the velocity diagram of the *arbitrary* curve is the same as the plot of any one of the members of the second family of patching curves. This implies that the relation

$$\frac{1}{q}\frac{dq}{d\theta} = -\tan \mu,$$

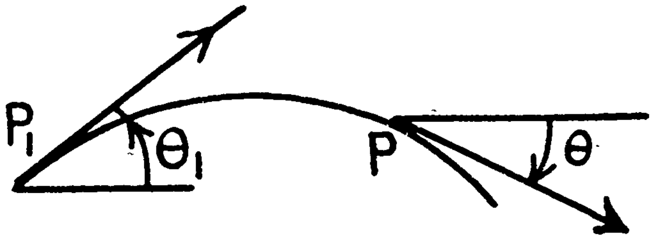
which is valid for any member of the second family of patching curves, is also valid for any curve which intersects once and only once each member of the family of straight-line patching curves. In particular, it is valid, in two dimensional flow, for the contour of an obstacle immersed in the flow if each component curve of the contour intersects once and only once each member of the family of straight-line patching curves. It is understood that θ does not increase as we move from the nose towards the trailing edge of the obstacle.

To avoid two sets of formulas, one set for the upper part of the contour and another for the lower part of the contour, we must keep in mind that, as usual, θ is measured positively in the counterclockwise direction. We then treat the lower part of the contour as if it were the upper part of a contour obtained by reflecting the given contour in a line through the nose and parallel to the direction of flow. This amounts to taking θ positive in the clockwise direction when we are working on the *lower* contour. We now have a basis for the following theorem on which rest the lift, drag and moment calculations in supersonic flow:

If P_1 *and* P *are two points which are both on the upper part, or both on the lower part, of the contour of the obstacle then*

$$\theta_1 - \theta = f(\mu_1) - f(\mu)$$

where θ, μ *are the inclination and the Mach angle,*



respectively, at P, and θ_1 , μ_1 *are the inclination and Mach angle, respectively, at* P_1. Since θ and θ_1 are given by the geometry of the obstacle (they are the inclinations of the tangents at P and P_1, respectively) this relation furnishes μ when μ_1 is given.

Once μ has been determined it is easy to calculate the ratio p/p_1 of the pressures at the points P and P_1. In fact the relation $p = k\rho^\gamma$ yields $c^2 = \frac{\partial p}{\partial \rho} = \gamma k \rho^{\gamma-1}$ so that $\frac{c^2}{c_1^2} = \left(\frac{\rho}{\rho_1}\right)^{\gamma-1}$ and $\frac{p}{p_1} = \left(\frac{\rho}{\rho_1}\right)^{\gamma} = \left(\frac{c^2}{c_1^2}\right)^{\frac{\gamma}{\gamma-1}}$. The relations

$$c = q \sin \mu; \qquad c^2 = \frac{\gamma - 1}{2}(q_M^2 - q^2)$$

yield

$$q^2(\gamma - \cos 2\mu) = (\gamma - 1)q_M^2$$

so that

$$c^2 = \frac{\gamma - 1}{2}\left\{1 - \frac{\gamma - 1}{\gamma - \cos 2\mu}\right\} q_M^2$$
$$= (\gamma - 1)q_M^2\left(\frac{\sin^2\mu}{\gamma - \cos 2\mu}\right).$$

On denoting by $g(\mu)$ the function $\left(\frac{\sin^2\mu}{\gamma - \cos 2\mu}\right)^{\frac{\gamma}{\gamma-1}}$ it follows that

$$\frac{p}{p_1} = \frac{g(\mu)}{g(\mu_1)} \qquad (9)$$

Table 5 furnishes values of $g(\mu)$ (calculated for $\gamma = 1.4$).

The determination of the pressure at any point of an obstacle in supersonic flow is, then, merely a matter of reading tables once μ_1 is known. It is convenient to take P_1 as the nose of the obstacle; once μ_1 is known at the nose $f(\mu)$ is calculated from

$$f(\mu) = f(\mu_1) - (\theta_1 - \theta)$$

and then μ is determined. $\frac{p}{p_1}$ is then read from a table. Since the uniform flow usually suffers a shock at the nose of the obstacle, μ_1 is not, in general, the Mach angle μ_0 of the undisturbed flow. Rather μ_1 must be determined by a separate *shock calculation*, the theory of which is explained in the following section.

2. *The theory of shock surfaces in supersonic flow.*

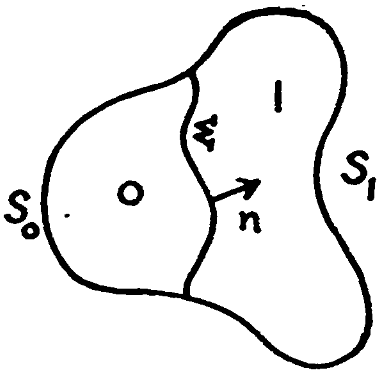
The conditions which hold at a shock surface may readily be found by considering the time derivative of a volume integral

$$I = \int_V f(x, y, z, t)dV$$

The surface S bounding the volume V of integration may vary with t and

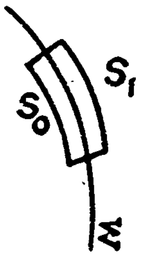
$$\frac{dI}{dt} = \int_V \frac{df}{dt}dV + \int_S fw_n\,dS$$

where w_n is the normal component of velocity of the surface (the normal being drawn away from the volume of integration). This result is valid if f possesses continuous derivatives with respect to x, y, z and t throughout V. If f does not satisfy these conditions the theorem is not necessarily valid. However if V is divided by a surface Σ into two regions V_0 and V_1 in each of which the conditions hold we may apply the theorem to V_0 and V_1 separately and combine the results; the only penalty is that we have to consider the surface integrals of fw_n over the two sides of Σ. We obtain, then,



$$\frac{dI}{dt} = \int_V \frac{\partial f}{\partial t}dV + \int_S fw_n\,dS + \int_\Sigma (f_0 - f_1)w_n\,d\Sigma$$

(the normal on Σ being drawn from V_0 to V_1). Applying this theorem to a volume where S_0 and S_1 are composed of particles of the fluid and are such that V_0 and V_1 are infinitesimal we obtain, up to infinitesimals,



$$\frac{dI}{dt} = \int_{S_0} fv_n\,dS + \int_S fv_n\,dS + \int_\Sigma (f_0 - f_1)w_n\,d\Sigma$$

where v denotes fluid velocity (as opposed to w which denotes velocity of the surface Σ); as S_0 and $S_1 \to \Sigma$ we obtain

$$\frac{dI}{dt} \to \int_\Sigma f_0(w - v_0)_n\,d\Sigma - \int_\Sigma f_1(w - v_1)_n\,d\Sigma.$$

If we know that $\frac{dI}{dt} = 0$ (for every V whose boundary consists of given fluid particles), it follows (in view of the arbitrary choice of the part of Σ over which the integration is carried out) that

$$f_0(w - v_0)_n = f_1(w - v_1)_n.$$

If, in particular, Σ is stationary we find

$$(f_0 v_0)_n = (f_1 v_1)_n.$$

In other words this means that the product of f by the normal component v_n of the fluid velocity is continuous across Σ. This is certainly the case (in view of the principle of conservation of mass) when f is the density ρ. This furnishes the *first shock condition* (for stationary shock fronts), namely

$$(\rho_0 v_0)_n = (\rho_1 v_1)_n, \qquad (10)$$

and it merely expresses the principle of *conservation of mass.*

We obtain the second and third shock conditions from the mechanical law that force = time rate of change of momentum. If ℓ is the x-direction cosine of the unit outward-drawn normal we have

$$\int_S p\ell\, dS = \frac{d}{dt}\int_V \rho u\, dV.$$

Setting $f = \rho u$ and proceeding as before we obtain

$$\int_\Sigma (p_0 - p_1)\ell\, d\Sigma = \int_\Sigma \{\rho_0 u_0(w_n - v_n)_0 - \rho_1 u_1(w_n - v_n)_1\}\, d\Sigma$$

(the outward drawn normal to S_0, for example, approaches, as $S \rightarrow \Sigma$, the normal to Σ drawn from V_2 towards V_0). Hence at any point of Σ

$$(p_0 - p_1)\ell = \rho_0 u_0(w_n - v_n)_0 - \rho_1 u_1(w_n - v_n)_1.$$

Taking the x-axis tangential to Σ we have $\ell = 0$ so that

$$\rho_0 u_0(w_n - v_n)_0 = \rho_1 u_1(w_n - v_n)_1.$$

On combining this with the first shock condition we obtain

$$(v_t)_0 = (v_t)_1. \qquad (11)$$

This is the *second shock condition;* the tangential component of fluid velocity is continuous across the shock (whether or not the shock is stationary). Note that this is a result of the combination of two physical laws: a) conservation of mass, and b) conservation of momentum under zero force.

The *third shock condition* follows on taking the x-axis normal to Σ so that $\ell = 1$. For a stationary shock we obtain

$$p_0 + \rho_0(v_n)_0^2 = p_1 + \rho_1(v_n)_0^2. \qquad (12)$$

The *fourth and final shock-condition* follows from the energy principle. The rate at which work is being done on the fluid is $-\int_s p v_n dS$ and we equate this to

$$\frac{d}{dt}\int_V \rho(\tfrac{1}{2}q^2 + U)dV$$

where U is the internal energy.

This gives

$$p_0(v_n)_0 + \rho_0(\tfrac{1}{2}q_0^2 + U_0)(v_n - w_n)_0$$
$$= p_1(v_n)_1 + \rho_1(\tfrac{1}{2}q_1^2 + U_1)(v_n - w_n)_1.$$

For stationary shocks this reduces, in view of the first shock condition, to

$$i_0 + \tfrac{1}{2}q_0^2 = i_1 + \tfrac{1}{2}q_1^2$$

where $i = U + \frac{p}{\rho}$ is the *heat content,* or enthalpy.

If the flow is isentropic in each region $di = dU + pd\left(\frac{1}{\rho}\right) + \frac{dp}{\rho} = \frac{dp}{\rho}$ and writing $p = k\rho^\gamma$ we have $di = \gamma k\rho^{\gamma-2}d\rho$ so that $i = \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = \frac{c^2}{\gamma - 1}$ (the constant of integration being determined by the convention that i vanishes with ρ). Now the argument which showed that in irrotational *two-dimensional* flow the two equations of motion are equivalent to the single relation

$$c^2 = \frac{\gamma - 1}{2}(q_M^2 - q^2)$$

may be repeated without change for three-dimensional flow; we find that, when the flow is irrotational, the three equations of motion are equivalent to the single relation

$$c^2 = \frac{\gamma - 1}{2}(q_M^2 - q^2).$$

Hence the shock condition,

$$\frac{c_0^2}{\gamma - 1} + \tfrac{1}{2}q_0^2 = \frac{c_1^2}{\gamma - 1} + \tfrac{1}{2}q_1^2 \qquad (13)$$

may be formulated as follows:

In irrotational flow the value of the constant q_M *is the same on both sides of a shock surface. In two-dimensional irrotational flow, then, the circular ring in the velocity diagram whose bounding circles have* *radii q_M and q_m respectively, is unaffected by the presence of shock lines (straight or curved).*

In view of the relations $c^2 = \frac{\gamma p}{\rho}$, $(v_t)_0 = (v_t)_1$ we may write the fourth shock condition in the form

$$\frac{\gamma}{\gamma - 1}\left(\frac{p_1}{\rho_1} - \frac{p_0}{\rho_0}\right) = \frac{1}{2}\left\{(v_n)_0^2 - (v_n)_1^2\right\}.$$

But the first and third shock conditions yield

$$p_1 - p_0 = \rho_0\left(1 - \frac{\rho_0}{\rho_1}\right)(v_n)_0^2 = \rho_1\left(\frac{\rho_1}{\rho_0} - 1\right)(v_n)_1^2$$

so that

$$(v_n)_0^2 - (v_n)_1^2 = \frac{p_1 - p_0}{\rho_1 - \rho_0}\left(\frac{\rho_1}{\rho_0} - \frac{\rho_0}{\rho_1}\right) = \frac{(p_1 - p_0)(\rho_1 + \rho_0)}{\rho_0\rho_1}$$

$$= \frac{p_1}{\rho_1}\left(1 + \frac{\rho_1}{\rho_0}\right) - \frac{p_0}{\rho_0}\left(1 + \frac{\rho_0}{\rho_1}\right).$$

Hence

$$\frac{p_1}{\rho_1}\left(\frac{\gamma + 1}{\gamma - 1} - \frac{\rho_1}{\rho_0}\right) = \frac{p_0}{\rho_0}\left(\frac{\gamma + 1}{\gamma - 1} - \frac{\rho_0}{\rho_1}\right)$$

or, equivalently,

$$\frac{p_1}{p_0} = \frac{(\gamma + 1)\rho_1 - (\gamma - 1)\rho_0}{(\gamma + 1)\rho_0 - (\gamma - 1)\rho_1} = \frac{(\gamma + 1)x - (\gamma - 1)}{(\gamma + 1) - (\gamma - 1)x} \qquad (14)$$

where $x = \frac{\rho_1}{\rho_0}$ is the density ratio across the shock surface. This is the *Rankine-Hugoniot relation.* It may be written in the equivalent form

$$\frac{\rho_1}{\rho_0} = \frac{(\gamma + 1)p_1 + (\gamma - 1)p_0}{(\gamma - 1)p_1 + (\gamma + 1)p_0} = \frac{(\gamma + 1)y + (\gamma - 1)}{(\gamma - 1)y + (\gamma + 1)} \qquad (15)$$

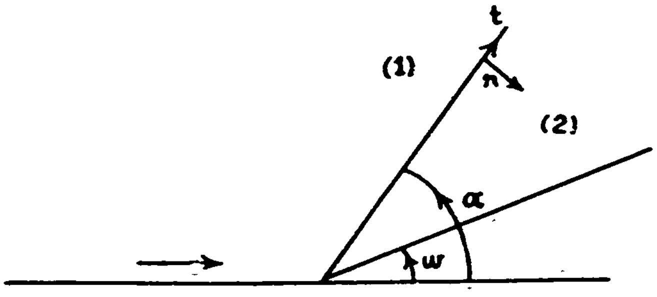
where $y = \frac{p_1}{p_0}$ is the pressure ratio across the shock. Taking p_0 as given ($\neq 0$), $x = \frac{\rho_1}{\rho_0}$ cannot surpass the ratio $\frac{\gamma + 1}{\gamma - 1} = 6$ (with $\gamma = 1.4$) nor be less than $\frac{\gamma - 1}{\gamma + 1} = \frac{1}{6}$.

3. Flow against the nose of a wedge.

We denote by $w < \frac{\pi}{2}$ the angle from the direction of the incident flow to the upper surface of the wedge and assume a plane shock inclined at angle α as shown. Then $(v_t)_0 = q_0 \cos \alpha$; $(v_n)_0 = q_0 \sin \alpha$; $(v_t)_1 = q_1 \cos(\alpha - w)$; $(v_n)_1 = q_1 \sin(\alpha - w)$. The first and second shock conditions yield

$$\rho_0 q_0 \sin \alpha = \rho_1 q_1 \sin(\alpha - w) \qquad (16)$$

$$q_0 \cos \alpha = q_1 \cos(\alpha - w) \qquad (17)$$



Eliminating q_1 and q_0 by division we obtain

$$x \tan(\alpha - w) = \tan \alpha \qquad (18)$$

where $x = \frac{\rho_1}{\rho_0}$ is the condensation (so that x lies in the range $\frac{1}{6} \leq x \leq 6$).

The relation (18) expresses either of the two variables x and α as a function of the other (the angle w being supposed given). In order to determine either of the variables x and α (and, hence, the other) we need a second relation between them. This is forthcoming when the Mach number M_0 of the incident flow is given. In fact since $(v_n)_0 = q_0 \sin \alpha$, $(v_n)_1 = q_1 \sin(\alpha - w)$ we obtain from (12) and (16)

$$p_1 - p_0 = \rho_0 q_0^2 \sin^2\alpha\left(1 - \frac{1}{x}\right) = \rho_1 q_1^2 \sin^2(\alpha - w)(x - 1).$$

Hence

$$\frac{p_1}{p_0} = 1 + \frac{\rho_0 q_0^2}{p_0}\sin^2\alpha\left(1 - \frac{1}{x}\right) = 1 + \gamma M_0^2 \sin^2\alpha\left(1 - \frac{1}{x}\right)$$

since $c^2 = \frac{\partial p}{\partial \rho} = \gamma\frac{p}{\rho}$ (on the assumption that $p = k\rho^\gamma$). From the Rankine-Hugoniot relation (14) we derive

$$\frac{p_1}{p_0} - 1 = \frac{2\gamma(x - 1)}{(\gamma + 1) - (\gamma - 1)x}$$

and so

$$M_0^2 \sin^2\alpha = \frac{2x}{(\gamma + 1) - (\gamma - 1)x}. \qquad (19)$$

Similarly

$$M_1^2 \sin^2(\alpha - w) = \frac{2}{(\gamma + 1)x - (\gamma - 1)}. \tag{20}$$

In order to facilitate the determination of the ratio p_1/p_0 when M_0 and w are given, Tables 1–4 have been prepared. Their use is discussed in detail in Sections 8 and 10.

4. *Approximate evaluation of the pressure ratio* p/p_1.

As we have seen in the previous sections, the determination of the pressure ratio p/p_0 involves two separate calculations. In the first place the shock tables furnish the pressure ratio p_1/p_0 and the Mach angle μ_1 after the shock when w and M_0 are given. Then Table 5 furnishes p/p_1 at any point of the contour. On multiplying p/p_1 by p_1/p_0 we obtain the desired pressure ratio p/p_0. The shock calculation is the more troublesome of the two and it may be completely avoided (if w is not too large) by means of the following observation. It may be verified[3] by Tables 2 and 3 that, when w is not too large, the relation

$$f(\mu_1) = f(\mu_0) + w$$

holds to a high degree of approximation. Since

$$w - \theta = f(\mu_1) - f(\mu)$$

it follows that, to a high degree of approximation,

$$-\theta = f(\mu_0) - f(\mu).$$

This means that we can ignore completely the shock calculation and evaluate the pressure by means of Table 5 starting from any point P_0 in the uniform flow (the value of θ at P_0 being zero). The following samples show how closely this "short-cut" calculation approximates the calculation made by using the shock tables. It was convenient to base this comparison on M_1, the Mach number behind the shock-wave. The result of the "short-cut" method is marked by an asterisk.

$M_0 = 2.13$

w	= 1°	2°	5°	10°	15°	20°	22°	24°	25.07°
M_1	= 2.091	2.055	1.946	1.759	1.563	1.340	1.233	1.096	.9262
M_1^*	= 2.091	2.055	1.946	1.771	1.601	1.432	1.361	1.289	1.250
deviation	= 0.0%	0.0%	0.0%	0.7%	2.4%	6.9%	10.4%	17.6%	35.0%

$M_0 = 1.85$

w	= 1°	2°	5°	10°	15°	20°	20.20°
M_1	= 1.815	1.780	1.676	1.498	1.298	.9802	.9178
M_1^*	= 1.815	1.780	1.678	1.509	1.336	1.140	1.131
deviation	= 0.0%	0.0%	0.1%	0.7%	2.9%	16.3%	23.2%

$M_0 = 1.40$

w	= 1°	2°	5°	8°
M_1	= 1.365	1.330	1.215	1.075
M_1^*	= 1.365	1.330	1.217	1.081
deviation	= 0.0%	0.0%	0.1%	0.6%

The behavior of the ratio M_1^*/M_1 shows that there is a high degree of continuity in the physical variables as w changes from positive through zero to negative, since the simplified method then becomes exact. (When w is negative there is no shock and the flow is expansive from the very beginning.) It should also be noticed that M_1^*/M_1 seems to be almost independent of M_0 for wedge angles not too large.

Two further properties of this solution are worthy of mention:

(1) The simplified method gives zero entropy increase on passing through the shock wave.

(2) The calculations using the shock wave tables cannot in general give a description of conditions in the wake, since there are two independent physical quantities to be matched on the two sides of the wake, and only one adjustable degree of freedom, namely the direction of the final flow. Using the simplified method, the difficulty is of a different sort since now an infinite number of solutions exist. Since p, ρ, M depend only on M_0 and on the angle between the incident and final flow directions, the two flows derived from the upper and lower surfaces always match, no matter what direction is assumed for the final uniform flow.

5. *The linear theory.*

It is easy to derive, from the rather complicated formulas of the preceding sections, simpler formulas which serve as approximations which are valid for

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[3] For example, it is found from Table 1 that $M_0 = 1.777$ for $\alpha = 39°$ and $w = 5°$, so that $\mu_0 = 34.25$ and $f(\mu_0) = 110.40$. The relation in question yields $f(\mu_1) = 115.40°$, whence $\mu_1 = 38.49°$. But from Table 2 it is found that when $\alpha = 39°$ and $w = 5°$, $M_1 = 1.605$, whence $\mu_1 = 38.54°$.

sufficiently small values of w and of θ. If w is so small that its square is negligible, we have

$$\tan(\alpha - w) = \tan\alpha - w \sec^2\alpha$$

so that, from formula (18)

$$x = 1 + w\frac{\sec^2\alpha}{\tan\alpha} = 1 + 2w \operatorname{cosec} 2\alpha.$$

Hence as $w \rightarrow 0$, $x \rightarrow 1$, and formula (19) then shows that $\sin^2\alpha \rightarrow \frac{1}{M_0^2} = \sin^2\mu_0$. Thus a *first* approximation to the shock angle α is the Mach angle μ_0 of the undisturbed uniform flow. Hence a *second* approximation to x is

$$x = 1 + 2w \operatorname{cosec} 2\mu_1\,. \tag{21}$$

It follows from formula (14) that a second approximation to the pressure ratio p_1/p_0 across the shock line is

$$\frac{p_1}{p_0} = \frac{1 + (\gamma + 1)w \operatorname{cosec} 2\mu_0}{1 - (\gamma - 1)w \operatorname{cosec} 2\mu_0}.$$

Since we are neglecting squares of w this may be written in the equivalent form

$$\frac{p_1}{p_0} = 1 + 2\,\gamma\, w \operatorname{cosec} 2\mu_0\,. \tag{22}$$

It follows from formula (20) that as $w \rightarrow 0$, $M_1 \rightarrow M_0$ so that a first approximation to μ_1 is μ_0. In order to obtain a second approximation to μ_1 we must first obtain from formula (19) a second approximation to the shock angle α. To do this we set $\alpha = \mu_0 + kw$ (where k is a multiplier which we wish to determine) and $x = 1 + 2w \operatorname{cosec} 2\mu_0$ in (19). Since $M_0 = \operatorname{cosec}\mu_0$ we obtain

$$1 + 2kw \cot\mu_0 = \frac{1 + 2w \operatorname{cosec} 2\mu_0}{1 - (\gamma - 1)w \operatorname{cosec} 2\mu_0}$$

$$= 1 + (\gamma + 1)w \operatorname{cosec} 2\mu_0$$

so that $k = \frac{(\gamma + 1)\operatorname{cosec} 2\mu_0}{2\cot\mu_0} = \frac{\gamma + 1}{4\cos^2\mu_0}$. Hence a second approximation to the shock angle α is

$$\alpha = \mu_0 + \frac{\gamma + 1}{4\cos^2\mu_0}w. \tag{23}$$

On setting $\mu_1 = \mu_0 + k'w$ in formula (20), after replacing M_1 by $\frac{1}{\sin\mu_1}$ and making use of (23) and (21) we find that $k' = \frac{\gamma + 1}{2\cos^2\mu_0} - 1 = \frac{\gamma - \cos 2\mu_0}{2\cos^2\mu_0}$. Thus a second approximation to μ_1 is

$$\mu_1 = \mu_0 + \frac{\gamma - \cos 2\mu_0}{2\cos^2\mu_0}w. \tag{24}$$

Now it follows from formula (7) that

$$f'(\mu) = \frac{2\cos^2\mu}{\gamma - \cos 2\mu}. \tag{25}$$

so that to a second approximation

$$f(\mu_1) = f(\mu_0) + w. \tag{26}$$

This is the basis for the short-cut calculation of Section 4.

It follows from formula (25) and the relation $\theta - w = f(\mu) - f(\mu_1)$ that

$$\theta - w = (\mu - \mu_1)\cdot\frac{2\cos^2\mu_1}{\gamma - \cos 2\mu_1}.$$

Hence

$$\mu - \mu_1 = \frac{\gamma - \cos 2\mu_1}{2\cos^2\mu_1}(\theta - w) = \frac{\gamma - \cos 2\mu_0}{2\cos^2\mu_0}(\theta - w) \tag{27}$$

since μ_0 is a first approximation to μ_1. On combining (24) and (27) we obtain

$$\mu = \mu_0 + \frac{\gamma - \cos 2\mu_0}{2\cos^2\mu_0}\theta. \tag{28}$$

On taking the logarithmic derivative of the function $g(\mu) = \left(\frac{\sin^2\mu}{\gamma - \cos 2\mu}\right)^{\frac{\gamma}{\gamma-1}}$ we obtain

$$\frac{g'(\mu)}{g(\mu)} = \frac{2\gamma}{\gamma - 1}\left\{\cot\mu - \frac{\sin 2\mu}{\gamma - \cos 2\mu}\right\} = 2\gamma\frac{\cot\mu}{\gamma - \cos 2\mu}. \tag{29}$$

It follows from formula (9) that

$$\frac{p}{p_1} = 1 + 2\gamma\frac{\cot\mu_1}{\gamma - \cos 2\mu_1}(\mu - \mu_1)$$

and this reduces, by means of formula (27), to

$$\begin{aligned}\frac{p}{p_1} &= 1 + 2\gamma \operatorname{cosec} 2\mu_1\cdot(\theta - w)\\ &= 1 + 2\gamma \operatorname{cosec} 2\mu_0\cdot(\theta - w)\end{aligned} \tag{30}$$

since we are neglecting $w(\theta - w)$. On combining formulas (22) and (30) we obtain

$$\frac{p}{p_0} = 1 + 2\gamma\theta \operatorname{cosec} 2\mu_0\,. \tag{31}$$

This simple formula for the pressure ratio $\frac{p}{p_0}$ is known as the *linear theory*. Since $c^2 = \frac{\gamma p}{\rho}$ formula (30) may be rewritten in the following equivalent form:

$$\Delta p = p - p_0 = \frac{p_0 \gamma}{\sin \mu_0 \cos \mu_0}\,\theta = \frac{c_0^2 \rho_0}{\sin \mu_0 \cos \mu_0}\,\theta$$

$$= \rho_0 q_0^2 \frac{\theta}{M_0^2 \sin \mu_0 \cos \mu_0} = \rho_0 q_0^2 \frac{\theta}{\cot \mu_0} \qquad (32)$$

$$= \rho_0 q_0^2 \frac{\theta}{\sqrt{M_0^2 - 1}}.$$

6. *The quadratic or Busemann theory.*

This theory is obtained from the method of this report by carrying out the various approximations as far as the w^2 terms. Since

$$\tan(\alpha - w) = \tan\alpha - \sec^2\alpha \cdot w + \sec^2\alpha \tan\alpha \cdot w^2$$

we have

$$\frac{\tan(\alpha - w)}{\tan\alpha} = 1 - \frac{\sec^2\alpha}{\tan\alpha} w + \sec^2\alpha \cdot w^2$$

so that

$$x = \frac{\tan\alpha}{\tan(\alpha - w)} = 1 + \frac{\sec^2\alpha}{\tan\alpha} w + \left(\frac{\sec^4\alpha}{\tan^2\alpha} - \sec^2\alpha\right) w^2$$

$$= 1 + \frac{\sec^2\alpha}{\tan\alpha} w + \operatorname{cosec}^2\alpha \cdot w^2.$$

On substituting for α, in the coefficient of w, the second approximation $\alpha = \mu_0 + \frac{\gamma + 1}{4\cos^2\mu_0} w$ (see (23)) we obtain, since

$$\tan\alpha = \tan\mu_0 + \frac{\gamma + 1}{4\cos^4\mu_0} w$$

$$\frac{\sec^2\alpha}{\tan\alpha} = \tan\alpha + \frac{1}{\tan\alpha} = \frac{\sec^2\mu_0}{\tan\mu_0} + \frac{\gamma + 1}{4\cos^4\mu_0}(1 - \cot^2\mu_0) w.$$

Hence the third approximation to x is

$$x = 1 + \frac{\sec^2\mu_0}{\tan\mu_0} w + \left\{\operatorname{cosec}^2\mu_0 + \frac{\gamma + 1}{4\cos^4\mu_0}(2 - \operatorname{cosec}^2\mu_0)\right\} w^2 \qquad (33)$$

On substituting this value of x in (14) we obtain

$$\frac{p_1}{p_0} = 1 + 2\gamma \operatorname{cosec} 2\mu_0 \cdot w + \gamma\left\{\operatorname{cosec}^2\mu_0 + \frac{4 + (\gamma - 3)\operatorname{cosec}^2\mu_0}{4\cos^4\mu_0}\right\} w^2. \qquad (34)$$

On differentiating (29) with respect to μ we obtain

$$\frac{g''(\mu)}{g(\mu)} - \left\{\frac{g'(\mu)}{g(\mu)}\right\}^2 = \frac{-2\gamma}{(\gamma - \cos 2\mu)^2}\{(\gamma - \cos 2\mu)\operatorname{cosec}^2\mu + 2\cot\mu \sin 2\mu\}$$

so that

$$\frac{g''(\mu)}{g(\mu)} = \frac{2\gamma}{(\gamma - \cos 2\mu)^2}\{(\gamma + 1)\cos 2\mu \operatorname{cosec}^2\mu - 4\cos^2\mu\}.$$

Hence

$$\frac{p}{p_1} = 1 + \frac{2\gamma \cot\mu_1}{\gamma - \cos 2\mu_1}(\mu - \mu_1) + \frac{\gamma}{(\gamma - \cos 2\mu_1)^2} \times \{(\gamma + 1)\cos 2\mu_1 \operatorname{cosec}^2\mu_1 - 4\cos^2\mu_1\} \times (\mu - \mu_1)^2 \cdots \qquad (35)$$

On differentiating (25) we obtain

$$f''(\mu) = \frac{-2(\gamma + 1)\sin 2\mu}{(\gamma - \cos 2\mu)^2}$$

and so the relation $\theta - w = f(\mu) - f(\mu_1)$ yields

$$\theta - w = \frac{2\cos^2\mu_1}{\gamma - \cos 2\mu_1}(\mu - \mu_1) + \frac{(\gamma + 1)\sin 2\mu_1}{(\gamma - \cos 2\mu_1)^2}(\mu - \mu_1)^2.$$

Inverting this relation we obtain

$$\mu - \mu_1 = \frac{\gamma \cos 2\mu_1}{2\cos^2\mu_1}(\theta - w) + \frac{(\gamma + 1)\sin 2\mu_1(\gamma - \cos 2\mu_1)}{8\cos^6\mu_1} \times (\theta - w)_2$$

and on substituting this in (35) we find

$$\frac{p}{p_1} = 1 + 2\gamma \operatorname{cosec} 2\mu_1(\theta - w) + \frac{\gamma \sec^4\mu_1}{4} \times [(\gamma + 1)\operatorname{cosec}^2\mu_1 - 4\cos^2\mu_1](\theta - w)^2. \qquad (36)$$

We replace in this formula μ_1 by μ_0 in the coefficient of $(\theta - w)^2$ and by $\mu_0 + \frac{\gamma - \cos 2\mu_0}{2\cos^2\mu_0} w$ (see formula (24)) in the coefficient of $\theta - w$. In this way we obtain

$$\frac{p}{p_1} = 1 + 2\gamma \operatorname{cosec} 2\mu_0 \cdot (\theta - w) - 2\gamma \sec^2\mu_0\{\operatorname{cosec} 2\mu_0 \cot 2\mu_0 \times (\gamma - \cos 2\mu_0)\} w(\theta - w) + \frac{\gamma}{4}\sec^4\mu_0\{(\gamma + 1)\operatorname{cosec}^2\mu_0 - 4\cos^2\mu_0\}(\theta - w)^2$$

$$= 1 + 2\gamma \operatorname{cosec} 2\mu_0 \cdot (\theta - w) + \gamma \sec^2\mu_0(\gamma \operatorname{cosec}^2 2\mu_0 + 2\gamma \operatorname{cosec} 2\mu_0 \cot 2\mu_0 - \cot^2 2\mu_0) w^2$$
$$- 2\gamma \sec^2\mu_0(\gamma \operatorname{cosec}^2 2\mu_0 + \gamma \operatorname{cosec} 2\mu_0 \cot 2\mu_0) w\theta$$
$$+ \gamma \sec^2\mu_0(\gamma \operatorname{cosec}^2 2\mu_0 + \cot^2 2\mu_0)\theta^2.$$

Upon multiplying this by (34) we obtain

$$\frac{p}{p_0} = 1 + 2\gamma\,\theta \operatorname{cosec} 2\mu_0 + \gamma \sec^2\mu_0 \operatorname{cosec}^2 2\mu_0 \times (\gamma + \cos^2 2\mu_0)\theta^2. \qquad (37)$$

This approximation to the pressure ratio is known as the quadratic or Busemann Theory.

PART II. APPLICATION OF THE GENERAL THEORY

7. *Limitations on method.*

This part of the report describes in detail the calculation of the pressure at any point of an airfoil in a supersonic stream. The lift, drag and moment may then be obtained by numerical integration, when the pressure is known at enough points on the surface of the airfoil. In making the application it should be remembered that the method is subject to the following limitations:

(1) The effect of viscosity in forming a boundary layer is neglected. It is assumed in effect that the flow past the surface is not retarded by frictional forces.

(2) The calculations are strictly two-dimensional. No attempt is made here to correct for the effect of finite span.

(3) The procedure breaks down for wedge angles which are too large. For a given incident Mach number, there is a maximum wedge angle through which the flow may be turned by a plane shock wave issuing from the corner. For greater wedge angles, the shock is presumably curved and detached from the corner. As applied to airfoil calculations this limitation means not only that the leading edge of the airfoil must be fairly acute (which is likely to be true of any airfoil worthy of consideration), but also that the angle of attack must not be too great. No matter how thin the airfoil, this limits the permissible range in angle of attack. This limitation is more restrictive for lower Mach numbers; in fact, it is always possible for the Mach number to be so low (yet greater than unity) that the calculations cannot be carried out for *any* angle of attack. As an example, for the symmetrical bi-convex GU2 profile with a semi-angle of 11.42° at the leading edge, the computations can only be carried out for angles of attack up to 13.43° for $M_0 = 2.13°$, and up to 8.45° for $M_0 = 1.85°$; for M_0 less than 1.489, the calculations cannot be carried out for *any* angle of attack including zero.

(4) This method is not exact even for the ideal two-dimensional non-viscous case. Unless the obstacle is a wedge formed by two half planes, the shock wave is necessarily curved and the motion has vorticity. However, the initial slope of the shock wave is given by the present method, and the curvature of the shock wave is small if the curvature of the airfoil is small; hence, the effect of vorticity will operate chiefly near the trailing edge and will probably be small. It seems probable that this effect is small in comparison with the effect of the boundary layer near the trailing edge, which is likewise ignored.

In all of these tables the ratio of the specific heats (γ) is taken as 1.4. The formulas on which the tables are based are restated below for this value of γ.

8. *Plane shock wave solution for flow in a corner.*

The following formulas describe two-dimensional, non-viscous supersonic flow in a corner, throughout the range of Mach numbers and corner angles for which the shock wave is plane. By juxtaposition of two such corner flows with, in general, different wedge angles we have the case of a uniform stream impinging on an infinite wedge, not necessarily placed symmetrically with respect to the stream. On either side of the shock wave the flow is uniform. The angle w through which the flow must turn in order to stay in the corner is called the wedge angle. The Mach number, pressure density and velocity are denoted by M, p, ρ, q, with subscript 0 or 1 used to refer respectively to the state before or after the shock wave. The angle which the shock wave makes with the incident flow is denoted by α. The relations between these quantities are as follows:

$$\left.\begin{aligned}
\frac{\rho_1}{\rho_0} &= \frac{\tan\alpha}{\tan(\alpha - \mathrm{w})} \equiv \mathrm{x} \\
\mathrm{M}_0 &= \sqrt{\frac{5\mathrm{x}}{6 - \mathrm{x}}} \Big/ \sin\alpha \\
\mathrm{M}_1 &= \sqrt{\frac{5}{6\mathrm{x} - 1}} \Big/ \sin(\alpha - \mathrm{w}) \\
\frac{\mathrm{p}_1}{\mathrm{p}_0} &= \frac{6\mathrm{x} - 1}{6 - \mathrm{x}} \\
\frac{\mathrm{q}_1}{\mathrm{q}_0} &= \frac{\cos\alpha}{\cos(\alpha - \mathrm{w})}
\end{aligned}\right\} \quad (38)$$

Tables 1–4 give x, M_0, M_1, and $\frac{p_1}{p_0}$ respectively as functions of α and w. The usual problem is to find M_1 and $\frac{p_1}{p_0}$, given w and M_0. Since w and M_0 are known, α may be found from the table of M_0 as a function of α and w (Table 2). Then M_1 and $\frac{p_1}{p_0}$ may be read from Tables 3 and 4. In general a two way interpolation is required in each table. A specific numerical illustration will be given later.

Usually the x-table (Table 1) is not used. It was a necessary step in the preparation of Tables 2–4, and is included here because it gives a quantity of physical interest, namely the ratio of densities behind and in front of the shock wave.

Several properties of this solution may be pointed out.

The density is greater, the Mach number smaller, the pressure greater, the velocity smaller, and the velocity of sound greater behind the shock wave (subscript 1) than in front of the shock wave (subscript 0).

The shock angle α is approximately equal to the Mach angle μ for small wedge angles, as would be expected on general grounds. (In general small wedge angles give weak shocks.)

For a given Mach number, there is a maximum wedge angle w_m. For wedge angles greater than w_m the problem has no solution in terms of a plane shock wave; presumably the shock moves forward from the edge and is curved. There is another, slightly smaller, critical wedge angle w_s for which the Mach number M_2 is just equal to unity; for wedge angles between w_s and w_m the flow behind the shock wave is subsonic. In this case the procedure of the next two sections is not applicable since the Mach angle is not real; furthermore, it seems likely that the solution is unstable even for the case of an infinite wedge, if the flow behind the shock wave is subsonic. It therefore seems likely that the critical angle[4] for which the shock breaks away from the nose is nearer to w_s than to w_m. Practically, the distinction between w_m and w_s is quite unimportant. These angles are tabulated in Table 6. They approach equality for both small and large Mach numbers; the maximum difference $w_m - w_s$ is 0.420° (occurring for $M_0 = 1.52$). As M_0 increases, w_m and w_s both increase; but it should be noted that for any Mach number, no matter how large, w_m and w_s are each less than 45.584°.

Table 6 was constructed from the following formulas given by Crocco[5].

$$\sin^2\alpha_m = \frac{1}{\gamma M^2}\left[\frac{\gamma+1}{4}M_0^2 - 1 + \sqrt{(\gamma+1)\left(1 + \frac{\gamma-1}{2}M_0^2 + \frac{\gamma+1}{16}M_0^4)\right)}\right]$$

$$\sin^2\alpha_s = \frac{1}{\gamma M_0^2}\left[\frac{\gamma+1}{4}M_0^2 - \frac{3-\gamma}{4} + \sqrt{(\gamma+1)\left(\frac{9+\gamma}{16} - \frac{3-\gamma}{8}M_0^2 + \frac{\gamma+1}{16}M_0^4\right)}\right]$$

With either subscript,

$$w = \alpha - \beta,$$

$$\frac{\tan\beta}{\tan\alpha} = \frac{2}{\gamma+1}\left(\frac{\sin^2\mu_0}{\sin^2\alpha} + \frac{\gamma-1}{2}\right)$$

An alternative formula may be derived for the determination of w_m, by eliminating α between the first two of equations (38). Writing $z \equiv \left(\frac{\tan w_m}{5}\right)^2$, $m \equiv M_0^2$,

$$az^2 + bz + c = 0$$

$$\begin{aligned}\text{where } a &= 4(6m+5)^3(m+5)\\ b &= 27(m+5)^2 - (m-1)^2(6m+5)^2 + 18(m+5)(m-1)(6m+5)\\ c &= -4(m-1)^3\end{aligned}$$

A similar formula may be obtained for w_s, also of the fourth degree in tan w_s; but it cannot be solved explicitly.

There is a wedge angle w*, analogous to w_s or w_m, which limits the existence of the "short cut method" described. Since $f(\mu) = f(\mu_0) + w$, and since

M_0	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.5	3	4	5	6	10	∞
w*	0°	1.3°	3.6°	6.2°	9.0°	11.9°	14.9°	17.8°	20.7°	23.6°	26.4°	39.1°	49.8°	65.8°	76.9°	85.0°	102.3°	130.5°
w_s	0°	1.4°	3.7°	6.3°	9.0°	11.7°	14.2°	16.6°	18.8°	20.9°	22.7°	29.7°	34.0°	38.7°	41.1°	42.4°	44.4°	45.6°
w_m	0°	1.5°	3.9°	6.7°	9.4°	12.1°	14.6°	17.0°	19.2°	21.2°	23.0°	29.8°	34.1°	38.8°	41.1°	42.4°	44.4°	45.6°

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[4] It is not certain that such a critical angle exists. Since the plane shock wave solution is not really "exact", the shock may be detached slightly and have some curvature even for small wedge angles.

[5] L. Crocco, Singolarita della corrente gassosa iperacustica nell'intorno di una prora a diedro. L'aerotecnica. Vol. XVII, fasc. 6, 1937.

$f(\mu) \leqq 90° (\sqrt{6} - 1) = 130.45°$, w must be less than $130.45° - f(\mu_0)$ if μ is to be real. Therefore $w^* = 130.45° - f(\mu_0)$. The preceding table shows that w^* is of the same order of magnitude as w_s and w_m for small Mach numbers, but diverges radically at very high Mach numbers.

Inspection of Table 2 shows that, for fixed w and varying α, M_0 decreases as α increases up to a point in the neighborhood of 65°, after which it increases. Thus for each wedge angle, there is a critical value of M_0, below which the plane shock wave solution does not exist.[6] In Table 2, the smallest value of M_0 for each wedge angle is enclosed in a box. Similarly the value of M_0 for which the flow behind the shock becomes sonic is underlined and in italics. For any wedge angle and for any M_0 above the minimum, *two* values of α may be read from Table 2, or at least could be if the table were extended. The smaller of these two values of α should always be chosen. The solution obtained on using the other value involves subsonic flow behind the shock wave and is probably unstable. In any case, the method of this report is applicable only to supersonic flow.

9. *Flow around a corner or convex curve ("expansive flow").*

The Mach angle of the initial uniform flow is $\mu_1 = \sin^{-1} \dfrac{1}{M_1}$. The Mach angle $\mu = \sin^{-1} \dfrac{1}{M}$ at any point where the flow has turned through an angle η is determined by

$$f(\mu_1) - f(\mu) = \eta$$

where $f(\mu) \equiv \sqrt{6} \tan^{-1} (\sqrt{6} \tan \mu) - \mu$.

Table 5 gives the numerical value of $f(\mu)$ in degrees. Since $f(\mu)$ is a monotonically increasing function of μ, it is seen that μ steadily decreases, and hence M steadily increases as we move with the flow. Since μ is an essentially positive quantity, the maximum angle through which the flow may turn is $\eta_{\max} = f(\mu_1)$. Practically, however, this is no restriction since $f(\mu_1)$ is greater than a right angle for Mach numbers less than 2.5, and greater than 60° for Mach numbers less than 4.3.

After μ has been determined at any point, the pressure (p), velocity (q) and density (ρ) are determined by the following formulae:

$$\frac{p}{p_1} = \frac{g(\mu)}{g(\mu_1)} \qquad g(\mu) = \left(\frac{\sin^2 \mu}{0.2 + \sin^2 \mu}\right)^{7/2}$$

$$\frac{q}{q_1} = \sqrt{\frac{0.2 + \sin^2 \mu_1}{0.2 + \sin^2 \mu}}$$

$$\frac{\rho}{\rho_1} = \left(\frac{p}{p_1}\right)^{5/7} \equiv \frac{h(\mu)}{h(\mu)} \quad \text{where} \quad h(\mu) = [g(\mu)]^{5/7}$$

$$= \left(\frac{\sin^2 \mu}{0.2 + \sin^2 \mu}\right)^{5/2}$$

It was convenient to use the above definition of $g(\mu)$, which differs by a (non-essential) constant from the function $g(\mu)$ defined in Section 1. It is to be noted that p, ρ, and $\sqrt{\dfrac{dp}{d\rho}}$ (the local velocity of sound) decrease, and q increases, in flow around a corner or convex surface. Of the functions $f(\mu)$, $g(\mu)$, $h(\mu)$ tabulated in Table 5, only f and g are used in obtaining the pressure. The function $h(\mu)$ was a necessary step in calculating $g(\mu)$, and is given in the table since it may occasionally be of interest to know the density changes in expansive flow.

10. *Specific numerical illustration.*

The calculations for two-dimensional airfoils are based on the two problems just discussed. The first problem, with the aid of Tables 1–4, gives the pressure and Mach number on either surface of the airfoil at the leading edge, just behind the shock wave (the pressure and Mach number in the undisturbed stream being given). The Mach number thus obtained is converted into a Mach angle $\left(\mu = \sin^{-1} \dfrac{1}{M}\right)$ and the flow around the curved profile is followed by means of the solution to the second problem. In this way it is possible to calculate a value for the pressure and the Mach angle at any point of the profile. The shock waves proceeding from the trailing edge, and the conditions in the wake, are ignored, on the principle that they do not affect the pressure on the airfoil.

As a numerical example, consider a GU2 profile (formed by two 22.84° circular arcs), and specifically the 30% chord station on the upper surface, with +5° angle of attack and Mach number 1.77. The semi-angle at the leading edge is 11.42°, and this

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[6] The relation between this critical M_0 and w is precisely the same as the relation between M_0 and w_m given by Table 6. In constructing Table 6, a very slightly different approach was made, in that the maximum w was found for fixed M_0 (instead of the minimum M_0 for fixed w).

would be the wedge angle to use in connection with a zero angle of attack; but as a result of the 5° angle of attack, the wedge angle to be used for calculations on the upper surface is $w = 11.42° - 5° = 6.42°$. The shock angle α is then to be found from Table 1 by double interpolation. Inspection of Table 2 shows that $M_0 = 1.77$ and $w = 6.42°$ determines α to be between 40° and 41°. For $w = 6.42°$, interpolation gives $M_0 = .58 \times 1.779 + .42 \times 1.822 = 1.797$ for $\alpha = 40°$ and $M_0 = .58 \times 1.740 + .42 \times 1.783 = 1.758$ for $\alpha = 41°$. Another linear interpolation shows that for $M_0 = 1.77$, $\alpha = 40 + \frac{27}{39} = 40.69°$. Now that α is known, M_1 and p_1/p_0 are obtained by interpolating in Tables 3 and 4.

$$\begin{aligned} M_1 = {} & .31 \times (.58 \times 1.573 + .42 \times 1.580) \\ & + .69 \times (.58 \times 1.536 + .42 \times 1.541) = 1.550 \end{aligned}$$

$$\begin{aligned} p_1/p_0 = {} & .31 \times (.58 \times 1.359 + .42 \times 1.434) \\ & + .69 \times (.58 \times 1.354 + .42 \times 1.429) = 1.387 \end{aligned}$$

This describes the state just behind the shock wave. To follow the subsequent changes, find the Mach angle μ_1 corresponding to the Mach number M_1:

$$\sin \mu_1 = M_1 = .6452$$

$$\mu_1 = 40.18°,$$

and determine μ at any point by

$$f(\mu_1) - f(\mu) = \eta$$

where η is the angle between the tangent at the point in question and the tangent at the leading edge. For example, the geometry of the wing shows that at the 30% chord station $\eta = 6.88°$, so that at this point (using Table 5)

$$f(\mu) = f(40.18°) - 6.88° = 117.08° - 6.88° = 110.20°$$

$$\mu = 34 + \tfrac{13}{136} = 34.10°$$

Then $\frac{p^a}{p_1} = \frac{g(\mu)}{g(\mu_1)} = \frac{.17843}{.25328} = .7045$. This is the ratio of the pressure at any point to the pressure just behind the shock wave. To find the ratio of the pressure to the pressure in the undisturbed stream, this must be multiplied by p_1/p_0. In the above example, the ratio of the pressure at the 30% chord station to the pressure in the undisturbed stream is

$$p/p_0 = 1.387 \times .7045 = .9771$$

The calculations for the lower surface proceed similarly, but with a wedge angle $w = 11.42° + 5° = 16.42°$.

After the pressure has been obtained at a sufficient number of points on both upper and lower surface, the lift, drag, and moment forces are obtained by numerical integration. For the GU2 profile it is sufficient to calculate the pressure at 10% chord intervals, including both leading and trailing edges, and integrate by Simpson's rule. For profiles in the shape of a polygon, the integrations may be effected without approximation. In any case, the integrals to be evaluated are

$$\int p\,dx$$

$$\int p \tan \beta\,dx$$

$$\int (x + y \tan \beta)\,p\,dx.$$

on both surfaces. Here x is the distance back along the chord (measured from any fixed point on the chord line; the moment derived is then understood to be the moment about this fixed origin), y the distance of the point on the surface above the chord line, β the angle which the tangent to the surface makes with the chord line. All of these quantities must be taken with correct sign: y is positive on the upper surface and negative on the lower surface; β is positive on the front of the upper surface and the rear of the lower surface, but negative on the rear of the upper surface and the front of the lower surface; x takes on negative values unless the origin is taken at the leading edge. The moment per unit span is

$$\int_{\text{upper}} (x + y \tan \beta) p\,dx - \int_{\text{lower}} (x + y \tan \beta) p\,dx$$

which is considered as positive if it tends to rotate the wing about the origin in the sense of increasing angle of attack. The lift and drag, measured perpendicular and parallel to the chord line, are

$$\lambda = \int_{\text{lower}} p\,dx - \int_{\text{upper}} p\,dx$$

$$\delta = \int_{\text{lower}} p \tan \beta\,dx - \int_{\text{upper}} p \tan \beta\,dx$$

per unit span. The lift and drag, measured perpendicular and parallel to the undisturbed stream, are per unit span

$$L = \lambda \cos \phi - \delta \sin \phi$$

$$D = \lambda \sin \phi - \delta \cos \phi$$

where ϕ is the angle of attack.

Although this procedure is occasionally referred to as an "exact" method, it is actually an approximation, although presumably a very good one. Two facts must be neglected in the development of this method:

(1) The mathematical solution for the flow around a corner, or around a cylindrical surface which is never concave outward, is based on the assumption that the characteristic lines proceed from the body to infinity without meeting any barrier or locus of discontinuity. In the wedge problem, all of the characteristics on both sides of the shock wave intersect the shock wave. Hence the flow around a corner (or convex surface) cannot be treated independently of a preceding flow *in* a corner involving shock. Likewise the shape of the shock wave is not independent of the flow following the shock wave. The theory of the plane shock wave is developed on the assumption that the magnitude and direction of flow just behind the shock wave is everywhere the same. This is impossible if expansive flow follows. It appears certain that for an airfoil such as the GU2, a general theory taking account of this interaction would lead to a curved[7] shock wave, perhaps slightly detached from the leading edge, even for small wedge angles; in this case the critical wedge angle discussed earlier would have no exact meaning.

(2) The conditions in the wake have been ignored, presumably on the theory that the wake is bounded by shock waves proceeding from the trailing edge, and hence can have no effect on the pressure at any point on the surface. In general, however, this method does not yield a solution for the flow in the wake, since it is necessary to match three physical quantities such as p, ρ, M, of which two are independent (because of the relation $\frac{q^2}{2} + \frac{c^2}{\gamma - 1} =$ constant) and there is only one degree of freedom in applying the method of this report, namely the direction of the final flow. Therefore the assumption, that conditions in the wake do not affect the flow over the airfoil, is without any satisfactory foundation.

Neglecting these two effects might be justified, partly by photographic evidence and partly by the consideration that the pressure distribution is affected mainly near the trailing edge, where the pressure is small, and therefore the total lift, drag and moment are affected only slightly. However, both of these faults in this method should be recognized as mathematical inconsistencies, quite distinct from objections that may be raised on physical grounds (e.g., the neglect of viscosity).

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[7] There are physical reasons for believing that, at distances which are large compared with the maximum diameter of a body disturbing a uniform flow, the shock angle approaches the Mach angle of the undisturbed stream. This leads to a shock which continually decreases in intensity with distance from the disturbance, as compared with the present solution which gives a shock wave which has the same "strength" at every point.

TABLE I

$$\frac{\rho_1}{\rho_0} \equiv X = \frac{\tan \alpha}{\tan(\alpha - w)} \; ; X =$$

α→ w↓	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°		18°
1°		2.001	1.501	1.334	1.251	1.201	1.168	1.145	1.127	1.113	1.102	1.094	1.086	1.080	1.075	1.070	1.066	1.063	1.060	1.057	1	1.063
2°			3.002	2.002	1.669	1.503	1.403	1.337	1.290	1.255	1.227	1.205	1.188	1.173	1.161	1.150	1.141	1.133	1.126	1.120	1	1.13
3°				4.006	2.506	2.006	1.753	1.606	1.507	1.436	1.383	1.342	1.309	1.283	1.261	1.242	1.226	1.213	1.201	1.190	1	1.213
4°					5.012	3.010	2.343	2.010	1.810	1.678	1.583	1.512	1.458	1.414	1.378	1.349	1.324	1.303	1.285	1.269	1	1.30
5°						6.021	3.516	2.682	2.265	2.015	1.849	1.731	1.643	1.574	1.520	1.475	1.438	1.407	1.381	1.358	1	1.407
6°							7.034	4.025	3.022	2.522	2.222	2.022	1.860	1.774	1.692	1.626	1.573	1.529	1.491	1.460	1	1.529
7°								8.052	4.536	3.365	2.780	2.430	2.197	2.031	1.907	1.810	1.734	1.672	1.620	1.577	1	1.672
8°									9.074	5.049	3.709	3.040	2.659	2.372	2.182	2.040	1.930	1.843	1.771	1.712	1	1.843
9°										10.102	5.566	4.056	3.302	2.850	2.549	2.335	2.175	2.051	1.953	1.872	1	2.051
10°											11.136	6.087	4.405	3.566	3.063	2.728	2.490	2.312	2.174	2.064		2.312
11°												12.177	6.611	4.757	3.832	3.278	2.909	2.646	2.450	2.298		2.64
12°													13.226	7.140	5.113	4.101	3.495	3.091	2.804	2.590		3.091
13°															7.673	5.471	4.372	3.714	3.276	2.964	2	3.714
14°																8.211	5.834	4.647	3.936	3.463	3	4.64
15°																	8.755	6.200	4.924	4.160	3	6.20
16°																		9.304	6.570	5.205	4	9.304
17°																			9.660	6.945	5	
18°																				10.423	7	
19°																						
20°																						
21°																						
22°																						
23°																						
24°																						
25°																						
26°																						
27°																						
28°																						
29°																						
30°																						

	39°	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	58°		56
1°	1.036	1.036	1.036	1.036	1.036	1.036	1.036	1.036	1.036	1.036	1.036	1.036	1.036	1.036	1.037	1.037	1.038	1.038	1.039	1.039		1.0
2°	1.075	1.074	1.073	1.073	1.073	1.073	1.072	1.072	1.072	1.073	1.073	1.073	1.073	1.074	1.075	1.075	1.076	1.077	1.078	1.079		1.0
3°	1.115	1.114	1.113	1.112	1.111	1.111	1.111	1.110	1.110	1.111	1.111	1.111	1.112	1.113	1.114	1.115	1.116	1.117	1.119	1.121		1.1
4°	1.156	1.155	1.154	1.152	1.152	1.151	1.150	1.150	1.150	1.150	1.150	1.151	1.152	1.152	1.154	1.155	1.156	1.158	1.160	1.163		1.1
5°	1.201	1.198	1.196	1.195	1.194	1.193	1.192	1.191	1.191	1.191	1.191	1.192	1.193	1.194	1.195	1.196	1.198	1.201	1.203	1.206		1.2
6°	1.247	1.244	1.241	1.239	1.237	1.236	1.235	1.234	1.234	1.233	1.234	1.234	1.235	1.236	1.237	1.239	1.241	1.244	1.247	1.250		1.2
7°	1.298	1.292	1.289	1.286	1.283	1.282	1.280	1.279	1.278	1.278	1.278	1.278	1.279	1.280	1.282	1.283	1.286	1.289	1.292	1.296		1.2
8°	1.348	1.343	1.339	1.335	1.332	1.329	1.327	1.325	1.324	1.324	1.323	1.324	1.324	1.325	1.327	1.329	1.332	1.335	1.339	1.343		1.3
9°	1.403	1.396	1.391	1.387	1.383	1.379	1.376	1.374	1.373	1.371	1.371	1.371	1.371	1.373	1.374	1.376	1.379	1.383	1.387	1.391		1.3
10°	1.461	1.453	1.447	1.441	1.436	1.432	1.428	1.425	1.423	1.422	1.421	1.420	1.421	1.422	1.423	1.425	1.428	1.432	1.436	1.441		1.4
11°	1.523	1.514	1.506	1.499	1.492	1.487	1.483	1.479	1.476	1.474	1.472	1.472	1.472	1.472	1.474	1.476	1.479	1.483	1.487	1.492		1.4
12°	1.589	1.578	1.568	1.560	1.552	1.545	1.540	1.535	1.532	1.529	1.527	1.525	1.525	1.525	1.527	1.529	1.532	1.535	1.540	1.545		1.5
13°	1.660	1.647	1.635	1.624	1.615	1.607	1.600	1.595	1.590	1.586	1.583	1.582	1.581	1.581	1.582	1.583	1.586	1.590	1.595	1.600		1.5
14°	1.737	1.720	1.706	1.693	1.682	1.673	1.664	1.657	1.651	1.647	1.643	1.640	1.639	1.638	1.639	1.640	1.643	1.647	1.651	1.657		1.6
15°	1.819	1.799	1.782	1.767	1.754	1.742	1.732	1.723	1.716	1.710	1.705	1.702	1.700	1.699	1.699	1.700	1.702	1.705	1.710	1.716		1.7
16°	1.908	1.885	1.864	1.846	1.830	1.816	1.804	1.794	1.785	1.777	1.771	1.767	1.764	1.762	1.761	1.762	1.764	1.767	1.771	1.777		1.7
17°	2.004	1.977	1.952	1.931	1.912	1.896	1.881	1.868	1.857	1.848	1.841	1.835	1.831	1.828	1.827	1.827	1.828	1.831	1.835	1.841		1.8
18°	2.110	2.077	2.048	2.022	2.000	1.980	1.963	1.948	1.935	1.924	1.915	1.907	1.902	1.898	1.895	1.894	1.895	1.898	1.902	1.907		1.8
19°	2.225	2.186	2.152	2.121	2.094	2.071	2.050	2.032	2.017	2.004	1.992	1.983	1.976	1.971	1.967	1.966	1.966	1.967	1.971	1.976		1.9
20°	2.352	2.305	2.265	2.229	2.197	2.169	2.145	2.123	2.105	2.089	2.075	2.064	2.055	2.048	2.043	2.041	2.040	2.041	2.043	2.048		2.0
21°	2.492	2.437	2.388	2.346	2.308	2.275	2.246	2.221	2.199	2.180	2.164	2.150	2.139	2.130	2.124	2.119	2.117	2.117	2.119	2.124		2.1
22°	2.649	2.582	2.525	2.474	2.429	2.390	2.356	2.326	2.300	2.277	2.258	2.241	2.228	2.217	2.209	2.203	2.199	2.198	2.199	2.203		2.1
23°	2.824	2.745	2.675	2.615	2.562	2.516	2.475	2.440	2.409	2.382	2.359	2.339	2.323	2.309	2.299	2.291	2.286	2.283	2.283	2.286		2.2
24°	3.022	2.926	2.843	2.771	2.708	2.653	2.605	2.563	2.526	2.494	2.467	2.443	2.424	2.407	2.394	2.384	2.377	2.373	2.371	2.373		2.3
25°	3.248	3.132	3.032	2.945	2.870	2.805	2.747	2.698	2.654	2.616	2.584	2.556	2.532	2.512	2.496	2.483	2.474	2.467	2.464	2.464		2.4
26°	3.508	3.365	3.244	3.140	3.050	2.972	2.904	2.845	2.794	2.749	2.710	2.677	2.648	2.624	2.604	2.589	2.576	2.568	2.563	2.561		2.5
27°	3.810	3.636	3.487	3.360	3.252	3.159	3.078	3.007	2.946	2.893	2.847	2.808	2.774	2.745	2.721	2.701	2.686	2.675	2.667	2.663		2.6
28°	4.166	3.948	3.765	3.611	3.480	3.368	3.271	3.187	3.114	3.061	2.997	2.950	2.909	2.875	2.846	2.822	2.803	2.788	2.778	2.772		2.7
29°	4.593	4.317	4.090	3.900	3.740	3.604	3.487	3.387	3.300	3.225	3.161	3.105	3.056	3.015	2.981	2.952	2.928	2.910	2.896	2.887		2.9
30°	5.113	4.759	4.472	4.236	4.039	3.873	3.732	3.611	3.508	3.418	3.341	3.274	3.217	3.168	3.126	3.091	3.063	3.040	3.022	3.010		3.0

A

TABLE I

$$X = \frac{\tan \alpha}{\tan(\alpha - w)}\ ;\ \alpha > w$$

[redacted]

18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	31°	32°	33°	34°	35°	36°	37°	38°
1.063	1.060	1.057	1.056	1.053	1.051	1.049	1.047	1.046	1.045	1.044	1.043	1.042	1.041	1.040	1.039	1.039	1.038	1.038	1.037	1.037
1.133	1.126	1.120	1.115	1.110	1.106	1.102	1.099	1.096	1.093	1.090	1.088	1.086	1.084	1.082	1.081	1.079	1.078	1.077	1.076	1.075
1.213	1.201	1.190	1.181	1.173	1.166	1.160	1.154	1.149	1.144	1.140	1.137	1.133	1.130	1.127	1.125	1.123	1.121	1.119	1.117	1.116
1.303	1.265	1.269	1.256	1.243	1.233	1.223	1.215	1.207	1.200	1.194	1.189	1.184	1.179	1.175	1.172	1.168	1.166	1.163	1.160	1.158
1.407	1.381	1.358	1.339	1.322	1.308	1.293	1.281	1.271	1.261	1.253	1.245	1.238	1.232	1.226	1.221	1.217	1.213	1.209	1.206	1.203
1.529	1.491	1.450	1.433	1.409	1.388	1.370	1.354	1.340	1.327	1.316	1.306	1.297	1.289	1.281	1.275	1.269	1.263	1.258	1.254	1.250
1.672	1.620	1.577	1.540	1.508	1.480	1.456	1.435	1.416	1.400	1.386	1.372	1.360	1.350	1.340	1.331	1.324	1.317	1.311	1.305	1.300
1.843	1.771	1.712	1.663	1.620	1.584	1.553	1.525	1.501	1.480	1.461	1.444	1.429	1.416	1.403	1.393	1.383	1.374	1.366	1.359	1.353
2.051	1.953	1.872	1.806	1.750	1.702	1.662	1.628	1.595	1.568	1.544	1.523	1.504	1.487	1.472	1.459	1.448	1.436	1.426	1.417	1.409
2.312	2.174	2.084	1.975	1.901	1.839	1.786	1.740	1.701	1.667	1.636	1.610	1.586	1.565	1.547	1.530	1.515	1.502	1.490	1.479	1.469
2.646	2.450	2.298	2.177	2.079	1.997	1.928	1.870	1.820	1.777	1.739	1.706	1.677	1.651	1.628	1.607	1.589	1.573	1.568	1.545	1.533
3.091	2.804	2.590	2.424	2.291	2.184	2.096	2.020	1.956	1.902	1.854	1.813	1.777	1.745	1.717	1.692	1.669	1.650	1.632	1.616	1.602
3.714	3.276	2.964	2.731	2.551	2.407	2.291	2.194	2.113	2.044	1.954	1.933	1.888	1.849	1.815	1.784	1.757	1.733	1.712	1.693	1.675
4.647	3.936	3.463	3.126	2.875	2.680	2.525	2.399	2.295	2.207	2.133	2.069	2.013	1.965	1.923	1.886	1.853	1.824	1.798	1.775	1.755
6.200	4.924	4.160	3.652	3.291	3.020	2.811	2.645	2.509	2.397	2.303	2.223	2.155	2.095	2.044	1.999	1.959	1.924	1.893	1.865	1.841
9.304	6.570	5.205	4.368	3.844	3.457	3.168	2.944	2.768	2.621	2.501	2.401	2.316	2.242	2.179	2.124	2.076	2.034	1.996	1.963	1.934
	9.860	6.945	5.490	4.618	4.039	3.626	3.318	3.079	2.890	2.735	2.608	2.501	2.410	2.332	2.265	2.206	2.155	2.110	2.070	2.035
		10.423	7.325	5.778	4.852	4.236	3.798	3.470	3.217	3.015	2.852	2.716	2.603	2.506	2.424	2.352	2.290	2.236	2.188	2.147
				7.709	6.070	5.089	4.437	3.972	3.625	3.357	3.144	2.970	2.827	2.707	2.605	2.517	2.442	2.378	2.319	2.287
					8.099	6.367	5.330	4.640	4.150	3.783	3.500	3.274	3.091	2.940	2.813	2.705	2.613	2.534	2.465	2.405
						8.496	6.669	5.575	4.848	4.330	3.944	3.645	3.408	3.215	3.055	2.922	2.808	2.711	2.628	2.555
							8.698	6.975	5.824	5.059	4.514	4.108	3.794	3.544	3.341	3.173	3.033	2.914	2.812	2.725
								9.306	7.287	6.077	5.274	4.702	4.275	3.945	3.683	3.470	3.294	3.147	3.022	2.916
										7.604	6.336	5.493	4.894	4.446	4.100	3.825	3.602	3.418	3.264	3.134
											7.927	6.599	5.717	5.089	4.621	4.259	3.971	3.738	3.545	3.394
												8.256	6.568	5.945	5.289	4.799	4.421	4.120	3.877	3.676
													8.593	7.142	6.179	5.493	4.992	4.587	4.274	4.019
															7.423	6.418	5.703	5.170	4.758	4.431
																	6.662	5.917	5.362	4.933
																		6.913	6.137	5.559

56°	57°	58°	59°	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°
1.038	1.039	1.039	1.040	1.041	1.042	1.043	1.044	1.045	1.046	1.047	1.049	1.051	1.053	1.055	1.057	1.060	1.063	1.066	1.070	1.075
1.077	1.078	1.079	1.081	1.082	1.084	1.086	1.088	1.090	1.093	1.095	1.099	1.102	1.106	1.110	1.115	1.120	1.126	1.133	1.141	1.150
1.117	1.119	1.121	1.123	1.125	1.127	1.130	1.133	1.137	1.140	1.144	1.149	1.154	1.160	1.166	1.173	1.181	1.190	1.201	1.213	1.226
1.158	1.160	1.163	1.165	1.168	1.172	1.175	1.179	1.184	1.189	1.194	1.200	1.207	1.215	1.223	1.233	1.243	1.256	1.269	1.285	1.303
1.201	1.203	1.206	1.209	1.213	1.217	1.221	1.226	1.232	1.238	1.245	1.253	1.261	1.271	1.281	1.293	1.306	1.322	1.339	1.358	1.381
1.244	1.247	1.250	1.254	1.258	1.263	1.269	1.275	1.281	1.289	1.297	1.308	1.316	1.327	1.340	1.354	1.370	1.388	1.409	1.433	1.460
1.289	1.292	1.296	1.300	1.305	1.311	1.317	1.324	1.331	1.340	1.350	1.360	1.372	1.386	1.400	1.416	1.435	1.456	1.480	1.508	1.540
1.335	1.339	1.343	1.348	1.353	1.359	1.366	1.374	1.383	1.393	1.403	1.416	1.429	1.444	1.461	1.480	1.501	1.525	1.553	1.584	1.620
1.383	1.387	1.391	1.396	1.403	1.409	1.417	1.426	1.436	1.446	1.459	1.472	1.487	1.504	1.523	1.544	1.568	1.595	1.626	1.662	1.702
1.432	1.436	1.441	1.447	1.453	1.461	1.469	1.479	1.490	1.502	1.515	1.530	1.547	1.565	1.586	1.610	1.636	1.667	1.701	1.740	1.786
1.483	1.487	1.492	1.499	1.506	1.514	1.523	1.533	1.545	1.558	1.573	1.589	1.607	1.628	1.651	1.677	1.706	1.739	1.777	1.820	1.870
1.535	1.540	1.545	1.552	1.560	1.568	1.578	1.589	1.602	1.616	1.632	1.650	1.669	1.692	1.717	1.746	1.777	1.813	1.864	1.902	1.956
1.590	1.595	1.600	1.607	1.615	1.624	1.635	1.647	1.660	1.675	1.693	1.712	1.733	1.757	1.784	1.815	1.849	1.888	1.933	1.984	2.044
1.647	1.651	1.657	1.664	1.673	1.682	1.693	1.706	1.720	1.737	1.755	1.775	1.798	1.824	1.853	1.886	1.923	1.965	2.013	2.069	2.133
1.705	1.710	1.716	1.723	1.732	1.742	1.754	1.767	1.782	1.799	1.819	1.841	1.865	1.893	1.924	1.959	1.999	2.044	2.095	2.155	2.223
1.767	1.771	1.777	1.785	1.794	1.804	1.816	1.830	1.846	1.864	1.885	1.908	1.934	1.963	1.996	2.034	2.076	2.124	2.179	2.242	2.316
1.831	1.835	1.841	1.848	1.857	1.868	1.881	1.895	1.912	1.931	1.952	1.977	2.004	2.035	2.070	2.110	2.155	2.206	2.265	2.332	2.410
1.898	1.902	1.907	1.915	1.924	1.935	1.948	1.963	1.980	2.000	2.022	2.048	2.077	2.110	2.147	2.188	2.238	2.290	2.352	2.424	2.506
1.967	1.971	1.976	1.983	1.992	2.004	2.017	2.032	2.050	2.071	2.094	2.121	2.152	2.186	2.225	2.269	2.319	2.376	2.442	2.517	2.605
2.041	2.043	2.048	2.055	2.064	2.075	2.089	2.105	2.123	2.145	2.169	2.197	2.229	2.265	2.305	2.352	2.405	2.465	2.534	2.613	2.706
2.117	2.119	2.124	2.130	2.139	2.150	2.164	2.180	2.199	2.221	2.246	2.275	2.308	2.346	2.388	2.437	2.492	2.556	2.626	2.711	2.808
2.198	2.199	2.203	2.209	2.217	2.228	2.241	2.258	2.277	2.300	2.326	2.356	2.390	2.429	2.474	2.525	2.582	2.649	2.725	2.812	2.914
2.283	2.283	2.286	2.291	2.299	2.309	2.323	2.339	2.359	2.382	2.409	2.440	2.475	2.516	2.562	2.615	2.675	2.746	2.824	2.916	3.022
2.373	2.371	2.373	2.377	2.384	2.394	2.407	2.424	2.443	2.467	2.494	2.526	2.563	2.605	2.653	2.708	2.771	2.843	2.926	3.022	3.134
2.467	2.464	2.464	2.467	2.474	2.483	2.496	2.512	2.532	2.556	2.584	2.616	2.654	2.698	2.747	2.805	2.870	2.945	3.032	3.132	[illegible]
2.568	2.563	2.561	2.563	2.568	2.576	2.589	2.604	2.624	2.648	2.677	2.710	2.749	2.794	2.845	2.904	2.972	3.050	3.140	3.244	3.365
2.675	2.667	2.663	2.663	2.667	2.675	2.686	2.701	2.721	2.745	2.774	2.808	2.847	2.893	2.946	3.007	3.078	3.159	3.252	3.360	3.487
2.788	2.778	2.772	2.770	2.772	2.778	2.788	2.803	2.822	2.846	2.875	2.909	2.950	2.997	3.061	3.114	3.187	3.271	3.368	3.480	3.611
2.910	2.896	2.887	2.883	2.883	2.887	2.896	2.910	2.928	2.952	2.981	3.015	3.056	3.105	3.161	3.225	3.300	3.387	3.487	3.604	3.740
3.040	3.022	3.010	3.002	3.000	3.002	3.010	3.022	3.040	3.063	3.091	3.126	3.168	3.217	3.274	3.341	3.418	3.508	3.611	3.732	3.873

TABLE 2

$$M_0 = \left(\frac{5X}{6-X}\right)^{1/2} \operatorname{cosec} \alpha$$

α →

w ↓

	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	31°	32°	33°
1°	4.086	3.779	3.565	3.358	3.182	3.023	2.892	2.764	2.637	2.531	2.435	2.344	2.262	2.186	2.116	2.050	1.989	1.932	1.879
2°	4.232	3.950	3.706	3.491	3.301	3.132	2.981	2.844	2.721	2.608	2.505	2.410	2.324	2.244	2.171	2.102	2.039	1.979	1.926
3°	4.467	4.145	3.876	3.643	3.436	3.252	3.089	2.945	2.811	2.691	2.582	2.482	2.391	2.307	2.230	2.158	2.091	2.029	1.972
4°	4.717	4.369	4.070	3.811	3.586	3.386	3.211	3.051	2.910	2.782	2.666	2.560	2.465	2.374	2.295	2.217	2.147	2.082	2.025
5°	5.032	4.632	4.294	4.006	3.755	3.536	3.344	3.173	3.019	2.881	2.757	2.644	2.541	2.447	2.360	2.280	2.207	2.138	2.075
6°	5.414	4.946	4.559	4.232	3.949	3.708	3.496	3.307	3.139	2.990	2.856	2.735	2.625	2.526	2.433	2.349	2.271	2.198	2.135
7°	5.887	5.332	4.876	4.428	4.177	3.904	3.666	3.459	3.276	3.112	2.966	2.835	2.717	2.609	2.511	2.421	2.339	2.263	2.192
8°	6.531	5.823	5.267	4.818	4.445	4.131	3.864	3.630	3.427	3.249	3.069	2.946	2.818	2.702	2.597	2.500	2.413	2.331	2.256
9°	7.425	6.475	5.767	5.215	4.771	4.403	4.094	3.830	3.601	3.403	3.226	3.069	2.930	2.804	2.690	2.587	2.492	2.406	2.327
10°	8.823	7.407	6.442	5.729	5.177	4.734	4.371	4.065	3.806	3.579	[illegible]	3.209	3.055	2.916	2.793	2.681	2.579	2.487	2.402
11°	11.486	8.902	7.419	6.427	5.706	5.151	4.709	4.346	4.042	3.786	3.560	3.366	3.196	3.043	2.907	2.785	2.675	2.575	2.485
12°	20.743	11.921	9.054	7.459	6.435	5.696	5.137	4.691	4.329	4.027	3.769	3.547	3.356	3.185	3.035	2.901	2.780	2.672	2.576
13°		28.089	12.533	9.223	7.532	6.460	5.703	5.134	4.684	4.321	4.017	3.761	3.540	3.348	3.180	3.030	2.898	2.779	2.671
14°			45.340	13.410	9.485	7.638	6.507	5.725	5.142	4.686	4.319	4.015	3.757	3.537	3.346	3.178	3.030	2.898	2.780
15°					14.698	9.830	7.782	6.579	5.761	5.161	4.698	4.324	4.017	3.759	3.538	3.348	3.180	3.033	2.902
16°						16.729	10.295	7.970	6.672	5.815	5.193	4.717	4.338	4.027	3.767	3.546	3.353	3.187	3.059
17°							20.472	10.911	8.213	6.794	5.885	5.237	4.748	4.369	4.044	3.781	3.567	3.365	3.197
18°								30.482	11.755	8.519	6.949	5.974	5.295	4.787	4.390	4.067	3.800	3.574	3.380
19°										12.994	8.915	7.132	6.065	5.368	4.859	4.428	4.098	3.826	3.598
20°											14.923	9.422	7.377	6.222	5.487	4.901	4.475	4.136	3.857
21°												18.474	10.104	7.669	6.388	5.564	4.978	4.534	4.182
22°													[illegible]	11.044	8.089	6.590	5.694	5.069	4.602
23°															12.431	8.512	6.835	5.846	5.175
24°																14.720	9.133	7.137	6.031
25°																	19.614	9.973	7.516
26°																		43.870	11.196
27°																			
28°																			
29°																			
30°																			

	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	61°	62°	63°	64°	65°	66°
1°	1.375	1.354	1.334	1.314	1.296	1.280	1.263	1.249	1.234	1.220	1.207	1.195	1.183	1.172	1.162	1.152	1.142	1.134	1.126
2°	1.404	1.383	1.362	1.343	1.325	1.308	1.291	1.276	1.262	1.248	1.235	1.223	1.211	1.201	1.191	1.181	1.172	1.164	1.156
3°	1.434	1.412	1.391	1.372	1.354	1.337	1.320	1.305	1.290	1.277	1.264	1.252	1.240	1.230	1.220	1.211	1.203	1.196	1.188
4°	1.465	1.443	1.422	1.403	1.383	1.366	1.349	1.334	1.319	1.305	1.293	1.281	1.269	1.260	1.250	1.241	1.234	1.227	1.220
5°	1.497	1.474	1.453	1.433	1.414	1.396	1.379	1.363	1.349	1.335	1.322	1.310	1.300	1.290	1.280	1.272	1.265	1.258	*1.252*
6°	1.530	1.508	1.485	1.466	1.445	1.427	1.410	1.394	1.379	1.366	1.353	1.341	1.330	1.320	1.312	1.304	1.296	*1.291*	1.288
7°	1.565	1.541	1.519	1.498	1.478	1.459	1.441	1.426	1.411	1.397	1.384	1.372	1.361	1.352	1.343	1.335	*1.328*	1.323	1.319
8°	1.601	1.576	1.553	1.531	1.511	1.492	1.474	1.458	1.443	1.429	1.416	1.404	1.393	1.383	1.375	*1.368*	1.362	1.357	1.352
9°	1.638	1.612	1.589	1.566	1.546	1.526	1.508	1.491	1.476	1.462	1.449	1.436	1.426	1.416	1.408	*1.401*	1.395	1.390	1.387
10°	1.677	1.651	1.625	1.603	1.581	1.561	1.543	1.526	1.510	1.496	1.482	1.471	1.460	1.450	*1.442*	1.435	1.430	1.426	1.423
11°	1.717	1.689	1.664	1.641	1.618	1.598	1.579	1.561	1.545	1.530	1.517	1.505	1.495	1.485	*1.477*	1.470	1.465	1.461	1.459
12°	1.760	1.731	1.704	1.680	1.656	1.636	1.616	1.598	1.581	1.567	1.553	1.541	1.530	1.521	*1.513*	1.506	1.502	1.498	1.496
13°	1.804	1.774	1.747	1.721	1.697	1.675	1.655	1.636	1.620	1.604	1.590	1.578	1.567	*1.557*	1.550	1.544	1.539	1.535	1.535
14°	1.851	1.819	1.790	1.764	1.739	1.716	1.696	1.676	1.659	1.643	1.629	1.616	1.605	*1.596*	1.588	1.582	1.577	1.575	1.574
15°	1.900	1.867	1.837	1.809	1.783	1.760	1.738	1.718	1.699	1.683	1.669	1.656	1.645	*1.635*	1.628	1.621	1.617	1.615	1.614
16°	1.952	1.917	1.886	1.857	1.830	1.805	1.782	1.762	1.743	1.725	1.710	1.698	1.686	*1.676*	1.668	1.662	1.658	1.656	1.657
17°	2.007	1.971	1.937	1.907	1.878	1.853	1.829	1.807	1.787	1.770	1.754	1.740	1.729	*1.719*	1.711	1.705	1.701	1.700	1.700
18°	2.067	2.029	1.992	1.960	1.930	1.902	1.877	1.855	1.835	1.816	1.800	1.786	1.774	*1.764*	1.756	1.750	1.746	1.745	1.745
19°	2.131	2.089	2.051	2.013	1.985	1.955	1.930	1.906	1.884	1.865	1.848	1.833	1.820	*1.811*	1.802	1.796	1.792	1.791	1.792
20°	2.199	2.154	2.114	2.077	2.043	2.012	1.985	1.959	1.937	1.916	1.898	1.883	1.870	*1.859*	1.851	1.845	1.841	1.840	1.842
21°	2.273	2.225	2.181	2.142	2.105	2.073	2.042	2.016	1.992	1.970	1.952	1.935	1.922	*1.911*	1.902	1.896	1.892	1.891	1.893
22°	2.353	2.302	2.254	2.211	2.172	2.137	2.105	2.076	2.051	2.028	2.008	1.991	1.977	*1.965*	1.955	1.949	1.946	1.945	1.946
23°	2.441	2.385	2.333	2.287	2.244	2.207	2.172	2.142	2.114	2.090	2.069	2.050	2.035	*2.022*	2.013	2.006	2.003	2.002	2.005
24°	2.538	2.476	2.419	2.369	2.323	2.281	2.244	2.211	2.182	2.155	2.133	2.113	2.096	*2.083*	2.073	2.066	2.062	2.062	2.064
25°	2.646	2.577	2.515	2.459	2.408	2.363	2.322	2.287	2.254	2.226	2.201	2.180	2.163	*2.148*	2.137	2.130	2.126	2.125	2.129
26°	2.767	2.689	2.620	2.557	2.502	2.452	2.408	2.368	2.333	2.302	2.275	2.253	2.233	*2.218*	2.206	2.198	2.193	2.193	2.197
27°	2.903	2.815	2.738	2.668	2.608	2.551	2.501	2.458	2.419	2.386	2.355	2.330	2.310	2.293	*2.280*	2.271	2.266	2.266	2.270
28°	3.061	2.960	2.871	2.791	2.722	2.660	2.605	2.556	2.513	2.476	2.443	2.416	2.393	2.374	*2.359*	2.350	2.344	2.344	2.348
29°	3.244	3.126	3.023	2.932	2.852	2.782	2.720	2.665	2.617	2.575	2.539	2.509	2.483	2.462	*2.446*	2.435	2.429	2.428	2.432
30°	3.462	3.321	3.199	3.094	3.001	2.920	2.849	2.788	2.738	2.686	2.646	2.610	2.582	2.558	*2.541*	2.528	2.521	2.520	2.523

A

TABLE 2

$$M_0 = \left(\frac{5X}{6-X}\right)^{1/2} \operatorname{cosec} \alpha$$

[redacted]

28°	29°	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	43°	44°	45°	46°	47°
2.186	2.116	2.050	1.989	1.932	1.879	1.830	1.783	1.740	1.698	1.660	1.623	1.589	1.557	1.527	1.498	1.471	1.445	1.420	1.397
2.244	2.171	2.102	2.039	1.979	1.925	1.872	1.824	1.779	1.737	1.697	1.660	1.624	1.591	1.559	1.530	1.502	1.475	1.450	1.426
2.307	2.230	2.158	2.091	2.029	1.972	1.919	1.869	1.821	1.777	1.736	1.698	1.661	1.627	1.594	1.563	1.534	1.507	1.481	1.457
2.374	2.293	2.217	2.147	2.082	2.023	1.966	1.914	1.865	1.819	1.776	1.736	1.698	1.663	1.629	1.598	1.566	1.540	1.514	1.489
2.447	2.360	2.280	2.207	2.138	2.078	2.017	1.962	1.911	1.864	1.819	1.777	1.738	1.701	1.667	1.634	1.604	1.575	1.547	1.522
2.525	2.433	2.349	2.271	2.198	2.133	2.071	2.013	1.959	1.910	1.865	1.820	1.779	1.740	1.705	1.672	1.640	1.610	1.580	1.556
2.609	2.511	2.421	2.339	2.263	2.192	2.128	2.067	2.012	1.959	1.910	1.865	1.822	1.783	1.745	1.710	1.678	1.647	1.618	1.591
2.702	2.597	2.500	2.413	2.331	2.258	2.189	2.125	2.065	2.011	1.960	1.913	1.868	1.827	1.788	1.751	1.717	1.686	1.655	1.627
2.804	2.690	2.587	2.492	2.406	2.327	2.253	2.187	2.124	2.066	2.012	1.963	1.918	1.872	1.832	1.794	1.758	1.725	1.694	1.665
2.916	2.793	2.681	2.579	2.487	2.402	2.324	2.253	2.187	2.125	2.068	2.016	1.966	1.921	1.879	1.839	1.802	1.767	1.735	1.705
3.043	2.907	2.785	2.675	2.575	2.483	2.400	2.324	2.253	2.188	2.128	2.072	2.021	1.973	1.929	1.886	1.848	1.812	1.778	1.746
3.185	3.035	2.901	2.780	2.672	2.573	2.482	2.401	2.325	2.256	2.192	2.133	2.078	2.027	1.981	1.937	1.896	1.858	1.823	1.790
3.348	3.180	3.030	2.898	2.779	2.671	2.573	2.484	2.404	2.330	2.260	2.197	2.140	2.086	2.036	1.990	1.947	1.907	1.871	1.836
3.537	3.346	3.178	3.030	2.898	2.780	2.673	2.576	2.488	2.408	2.335	2.266	2.205	2.148	2.095	2.046	2.002	1.959	1.920	1.884
3.759	3.538	3.348	3.180	3.033	2.902	2.784	2.678	2.583	2.495	2.416	2.344	2.276	2.215	2.159	2.107	2.059	2.014	1.973	1.935
4.027	3.767	3.546	3.353	3.187	3.039	2.909	2.792	2.686	2.591	2.506	2.426	2.354	2.288	2.228	2.172	2.121	2.073	2.030	1.990
4.359	4.044	3.781	3.557	3.365	3.197	3.049	2.919	2.802	2.697	2.602	2.516	2.439	2.367	2.302	2.242	2.187	2.137	2.090	2.047
4.787	4.390	4.057	3.800	3.574	3.380	3.211	3.063	2.932	2.815	2.711	2.617	2.531	2.454	2.382	2.318	[illegible]	2.205	2.155	2.109
5.368	4.839	4.428	4.098	3.826	3.596	3.399	3.230	3.080	2.949	2.832	2.728	2.634	2.549	2.471	2.401	2.337	2.278	2.224	2.176
6.222	5.457	4.901	4.475	4.136	3.857	3.623	3.424	3.253	3.108	2.971	2.853	2.748	2.654	2.569	2.492	2.422	2.359	2.300	2.248
7.669	6.368	5.564	4.978	4.534	4.182	3.896	3.656	3.454	3.280	3.128	2.995	2.877	2.771	2.678	2.592	2.516	2.446	2.383	2.326
1.044	8.089	6.590	5.694	5.069	4.602	4.236	3.942	3.697	3.490	3.313	3.159	3.023	2.905	2.799	2.704	2.619	2.543	2.473	2.411
	12.431	8.512	6.835	5.846	5.173	4.683	4.301	3.996	3.748	3.532	3.350	3.195	3.067	2.937	2.830	2.735	2.650	2.573	2.504
		14.720	9.133	7.137	6.031	5.303	4.778	4.377	4.068	3.798	3.579	3.394	3.234	3.096	2.974	2.866	2.770	2.684	2.607
			19.514	9.973	7.516	6.254	5.454	4.890	4.465	4.131	3.860	3.635	3.445	3.281	3.140	3.016	2.908	2.810	2.723
				43.870	11.198	7.993	6.523	5.632	5.021	4.568	4.216	3.931	3.698	3.502	3.334	3.189	3.063	2.962	2.854
						13.162	8.524	6.654	5.847	5.173	4.687	4.313	4.015	3.770	3.567	3.394	3.246	3.116	3.003
							17.063	9.495	7.272	6.104	5.366	4.825	4.424	4.108	3.853	3.641	3.462	3.309	3.176
								22.120	10.771	7.809	6.420	5.571	4.986	4.554	4.218	3.948	3.725	3.559	3.380
										12.896	8.531	6.812	5.831	5.178	4.705	4.344	4.056	3.822	3.628

61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°
1.172	1.162	1.152	1.142	1.134	1.125	1.118	1.111	1.105	1.099	1.094	1.089	1.086	1.081	1.078	1.077	1.075	1.076	1.075	1.077
1.201	1.191	1.181	1.172	1.164	1.156	1.150	1.144	1.139	1.134	*1.130*	1.126	1.124	1.122	1.122	1.122	1.124			
1.230	1.220	1.211	1.203	1.195	1.188	1.182	1.177	*1.173*	1.169	1.166	1.164	1.163	1.164	1.165	1.168				
1.260	1.250	1.241	1.234	1.227	1.220	*1.215*	1.210	1.207	1.204	1.203	1.202	1.203	1.206	1.208	1.214				
1.290	1.280	1.272	1.265	1.258	*1.252*	1.248	1.244	1.242	1.240	1.239	1.240	1.243	1.247	1.252	1.260				
1.320	1.312	1.304	1.296	*1.291*	1.285	1.281	1.278	1.276	1.276	1.277	1.279	1.283	1.289	1.297	1.307				
1.352	1.343	1.335	*1.328*	1.323	1.319	1.315	1.313	1.312	1.313	1.314	1.318	1.324	1.331	1.341	1.354				
1.383	1.375	*1.368*	1.362	1.357	1.352	1.350	1.348	1.348	1.350	1.353	1.358	1.365	1.375	1.386	1.402				
1.416	1.408	*1.401*	1.395	1.390	1.387	1.385	1.384	1.385	1.388	1.392	1.398	1.407	1.418	1.433	1.450				
1.450	*1.442*	1.436	1.430	1.426	1.423	1.421	1.421	1.423	1.428	1.432	1.440	1.450	1.463	1.479	1.500				
1.485	*1.477*	1.470	1.465	1.461	1.459	1.458	1.459	1.462	1.466	1.473	1.482	1.494	1.509	1.525	1.551				
1.521	*1.513*	1.506	1.502	1.498	1.496	1.496	1.497	1.501	1.507	1.514	1.525	1.539	1.556	1.577	1.603				
1.557	1.550	1.544	1.539	1.535	1.535	1.535	1.537	1.541	1.548	1.557	1.569	1.584	1.604	1.627	1.657				
1.596	1.588	1.582	1.577	1.575	1.574	1.575	1.578	1.583	1.591	1.601	1.615	1.632	1.653	1.679	1.712				
1.635	1.628	1.621	1.617	1.615	1.614	1.616	1.620	1.626	1.635	1.647	1.662	1.681	1.704	1.733	1.768				
1.676	1.668	1.662	1.658	1.656	1.657	1.659	1.663	1.670	1.680	1.694	1.710	1.731	1.757	1.788	1.827				
1.719	1.711	1.705	1.701	1.700	1.700	1.703	1.708	1.716	1.727	1.742	1.760	1.783	1.811	1.846	1.888				
1.764	1.756	1.750	1.746	1.745	1.745	1.749	1.755	1.764	1.776	1.792	1.812	1.837	1.868	1.906	1.952				
1.811	1.802	1.796	1.792	1.791	1.792	1.796	1.804	1.813	1.827	1.844	1.866	1.893	1.927	1.968	2.019				
1.859	1.851	1.845	1.841	1.840	1.842	1.846	1.854	1.865	1.879	1.899	1.923	1.953	1.989	2.033	2.086				
1.911	1.902	1.896	1.892	1.891	1.893	1.898	1.907	1.919	1.935	1.956	1.982	2.014	2.054	2.102	2.161				
1.965	1.955	1.949	1.946	1.945	1.948	1.953	1.962	1.975	1.993	2.016	2.043	2.079	2.122	2.174	2.239				
2.022	2.013	2.006	2.003	2.002	2.005	2.011	2.021	2.035	2.054	2.079	2.109	2.147	2.195	2.251	2.321				
2.083	2.073	2.066	2.062	2.062	2.064	2.071	2.083	2.098	2.119	2.146	2.178	2.219	2.269	2.332	2.410				
2.148	2.137	2.130	2.126	2.125	2.129	2.136	2.148	2.165	1.187	2.216	2.251	2.296	2.351	2.419	2.504				
2.218	2.206	2.198	2.193	2.193	2.197	2.205	2.218	2.236	2.260	2.290	2.329	2.378	2.437	2.512	2.604				
2.293	*2.280*	2.271	2.266	2.266	2.270	2.278	2.292	2.311	2.337	2.370	2.413	2.466	2.531	2.612	2.715				
2.374	*2.359*	2.350	2.344	2.344	2.348	2.357	2.372	2.395	2.420	2.457	2.503	2.560	2.631	2.720	2.833				
2.462	*2.446*	2.435	2.429	2.428	2.432	2.441	2.457	2.481	2.511	2.549	2.599	2.652	2.740	2.839	2.965				
2.558	*2.541*	2.528	2.521	2.520	2.523	2.533	2.561	2.575	2.608	2.651	2.705	2.774	2.860	2.970	3.110				

B

TABLE 3

$$M_1 = \left(\frac{5}{6X-1}\right)^{\frac{1}{2}} \operatorname{cosec}(\alpha - w)$$

α→ w↓	24°	25°	26°	27°	28°	29°	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	43°	44°	45°	[illegible]
1°	2.487	2.392	2.303	2.222	2.147	2.077	2.014	1.953	1.897	1.844	1.795	1.749	1.705	1.665	1.626	1.590	1.556	1.523	1.492	1.463	1.438	1.409	.863
2°		2.420	2.329	2.244	2.167	2.095	2.028	1.966	1.908	1.864	1.804	1.752	1.711	1.669	1.630	1.592	1.557	1.524	1.492	1.461	1.433	1.407	.866
3°			2.357	2.270	2.189	2.114	2.045	1.961	1.921	1.866	1.812	1.763	1.718	1.675	1.633	1.596	1.558	1.524	1.492	1.461	1.432	1.404	.881
4°				2.298	2.214	2.136	2.065	1.998	1.936	1.878	1.826	1.774	1.726	1.682	1.640	1.600	1.562	1.527	1.494	1.461	1.431	1.403	.898
5°					2.242	2.161	2.057	2.018	1.954	1.894	1.837	1.786	1.736	1.690	1.646	1.605	1.567	1.531	1.496	1.463	1.432	1.403	.816
6°						2.189	2.111	2.039	1.973	1.910	1.852	1.798	1.748	1.700	1.656	1.613	1.573	1.535	1.500	1.466	1.434	1.403	.889
7°							2.139	2.063	1.994	1.930	1.869	1.813	1.760	1.711	1.666	1.621	1.580	1.541	1.504	1.470	1.436	1.406	.063
8°								2.090	2.018	1.961	1.888	1.830	1.776	1.724	1.676	1.631	1.588	1.548	1.510	1.474	1.441	1.408	.080
9°									2.045	1.974	1.910	1.848	1.792	1.739	1.689	1.642	1.599	1.557	1.517	1.480	1.445	1.412	
10°										2.001	1.933	1.869	1.810	1.755	1.704	1.655	1.610	1.566	1.526	1.488	1.451	1.417	
11°											1.959	1.893	1.831	1.774	1.720	1.670	1.622	1.578	1.536	1.496	1.459	1.42	
12°												1.918	1.854	1.794	1.738	1.586	1.637	1.591	1.547	1.506	1.467	1.	
13°													1.879	1.817	1.759	1.704	1.653	1.606	1.560	1.517	1.477	1.4	
14°														1.842	1.781	1.724	1.671	1.621	1.574	1.530	1.488	1.448	
15°															1.806	1.746	1.691	1.638	1.589	1.543	1.500	1.459	
16°																1.770	1.712	1.358	1.607	1.559	1.514	1.472	
17°																	1.736	1.680	1.626	1.576	1.529	1.486	
18°																		1.708	1.646	1.595	1.546	1.500	
19°																		1.729	1.671	1.617	1.565	1.517	
20°																		1.759	1.697	1.640	1.586	1.536	
21°																		1.791	1.726	1.665	1.609	1.556	
22°																		1.826	1.757	1.694	1.634	1.579	
23°																		1.865	1.792	1.726	1.662	1.604	
24°																		1.909	1.831	1.759	1.693	1.631	
25°																		1.957	1.873	1.797	1.726	1.662	
26°																		2.011	1.921	1.839	1.764	1.695	
27°																		2.071	1.974	1.885	1.805	1.731	
28°																		2.139	2.033	1.938	1.851	1.772	
29°																		2.217	2.100	1.996	1.902	1.818	
30°																		2.306	2.177	2.082	1.950	1.868	

	54°	55°	56°	57°	58°	59°	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	[illegible]
1°	1.225	1.209	1.194	1.179	1.165	1.152	1.139	1.127	1.115	1.104	1.093	1.083	1.074	1.064	1.055	1.046	1.037	1.030	1.021	1.014	1.007	.9992	.127
2°	1.215	1.199	1.183	1.167	1.153	1.138	1.125	1.112	1.099	1.087	1.076	1.064	1.054	1.043	1.033	1.023	1.014	1.004	.9949	.9857	.9764	.9671	.112
3°	1.206	1.189	1.173	1.156	1.141	1.126	1.112	1.098	1.085	1.072	1.060	1.048	1.036	1.025	1.014	1.003	.9920	.9815	.9709	.9603	.9493	.9384	.098
4°	1.199	1.181	1.164	1.147	1.130	1.115	1.100	1.086	1.072	1.058	1.045	1.032	1.020	1.008	.9958	.9837	.9722	.9604	.9492	.9369	.9253	.9130	.086
5°	1.192	1.173	1.155	1.138	1.121	1.105	1.089	1.074	1.060	1.046	1.032	1.018	1.005	.9920	.9794	.9665	.9542	.9416	.9291	.9160	.9031	.8900	.074
6°	1.186	1.167	1.148	1.130	1.113	1.096	1.080	1.064	1.049	1.034	1.020	1.006	.9915	.9778	.9644	.9511	.9376	.9244	.9109	.8974	.8833	.8689	.064
7°	1.181	1.161	1.142	1.123	1.105	1.088	1.071	1.055	1.039	1.024	1.009	.9938	.9790	.9649	.9507	.9367	.9225	.9037	.8944	.8800	.8654	.8501	.065
8°	1.177	1.156	1.136	1.117	1.099	1.081	1.064	1.047	1.030	1.014	.9984	.9829	.9681	.9528	.9382	.9235	.9088	.8910	.8793	.8642	.8487	.8330	.047
9°	1.174	1.153	1.132	1.112	1.093	1.075	1.056	1.039	1.022	1.005	.9891	.9735	.9575	.9422	.9268	.9115	.8962	.8810	.8655	.8498	.8338	.8172	.059
10°	1.171	1.150	1.128	1.108	1.088	1.069	1.051	1.032	1.015	.9978	.9809	.9644	.9483	.9322	.9162	.9006	.8848	.8688	.8529	.8364	.8200	.8030	.062
11°	1.170	1.147	1.125	1.104	1.084	1.064	1.045	1.027	1.009	.9911	.9736	.9566	.9397	.9233	.9070	.8905	.8741	.8577	.8412	.8245	.8074	.7899	.027
12°	1.169	1.146	1.123	1.102	1.081	1.060	1.041	1.022	1.003	.9849	.9669	.9495	.9321	.9150	.8984	.8813	.8645	.8477	.8307	.8135	.7959	.7777	.022
13°	1.169	1.145	1.122	1.100	1.078	1.057	1.037	1.018	.9982	.9794	.9612	.9433	.9252	.9077	.8904	.8732	.8559	.8384	.8211	.8034	.7853	.7669	.018
14°	1.170	1.145	1.121	1.099	1.076	1.055	1.034	1.014	.9943	.9749	.9561	.9374	.9192	.9013	.8834	.8656	.8479	.8301	.8122	.7942	.7758	.7567	.014
15°	1.171	1.146	1.122	1.098	1.075	1.053	1.032	1.011	.9907	.9710	.9517	.9327	.9138	.8953	.8771	.8588	.8407	.8225	.8041	.7856	.7669	.7475	.011
16°	1.174	1.148	1.123	1.099	1.075	1.052	1.030	1.009	.9881	.9678	.9479	.9284	.9091	.8902	.8714	.8528	.8343	.8155	.7969	.7780	.7588	.7393	.009
17°	1.177	1.150	1.124	1.100	1.075	1.052	1.030	1.007	.9860	.9653	.9448	.9248	.9053	.8857	.8666	.8475	.8285	.8094	.7903	.7711	.7514	.7315	.007
18°	1.182	1.154	1.127	1.101	1.077	1.052	1.029	1.007	.9846	.9632	.9424	.9219	.9018	.8819	.8622	.8426	.8232	.8040	.7844	.7648	.7449	.7245	.007
19°	1.186	1.158	1.130	1.104	1.078	1.054	1.030	1.006	.9840	.9622	.9407	.9196	.8991	.8787	.8584	.8386	.8187	.7990	.7791	.7591	.7388	.7182	.006
20°	1.192	1.163	1.134	1.107	1.081	1.056	1.031	1.007	.9840	.9614	.9395	.9179	.8968	.8760	.8554	.8350	.8149	.7946	.7743	.7540	.7334	.7125	.007
21°	1.200	1.169	1.140	1.112	1.084	1.058	1.033	1.008	.9846	.9615	.9389	.9169	.8953	.8740	.8530	.8321	.8116	.7909	.7703	.7496	.7286	.7074	.006
22°	1.207	1.176	1.145	1.116	1.088	1.061	1.036	1.010	.9861	.9622	.9391	.9164	.8943	.8725	.8611	.8299	.8087	.7876	.7668	.7455	.7243	.7028	.010
23°	1.216	1.183	1.152	1.122	1.093	1.066	1.039	1.013	.9878	.9636	.9398	.9166	.8939	.8716	.8497	.8279	.8065	.7851	.7637	.7421	.7206	.6967	.013
24°	1.226	1.192	1.160	1.129	1.099	1.071	1.043	1.016	.9906	.9655	.9413	.9174	.8943	.8714	.8489	.8268	.8048	.7830	.7612	.7394	.7174	.6951	.016
25°	1.237	1.202	1.169	1.137	1.106	1.076	1.048	1.020	.9939	.9682	.9432	.9188	.8949	.8717	.8487	.8260	.8037	.7813	.7592	.7370	.7146	.6920	.020
26°	1.249	1.213	1.178	1.145	1.113	1.083	1.053	1.025	.9979	.9716	.9459	.9209	.8963	.8725	.8490	.8258	.8030	.7803	.7577	.7351	.7124	.6895	.025
27°	1.263	1.226	1.189	1.155	1.122	1.090	1.060	1.031	1.003	.9756	.9491	.9234	.8983	.8738	.8499	.8262	.8029	.7797	.7566	.7336	.7106	.6874	.031
28°	1.278	1.238	1.201	1.165	1.131	1.099	1.067	1.037	1.008	.9802	.9531	.9267	.9010	.8759	.8513	.8271	.8033	.7797	.7562	.7327	.7093	.6857	.037
29°	1.294	1.253	1.214	1.177	1.142	1.108	1.075	1.044	1.015	.9856	.9578	.9306	.9042	.8786	.8534	.8285	.8041	.7801	.7562	.7323	.7085	.6845	.044
30°	1.312	1.269	1.228	1.190	1.153	1.118	1.085	1.053	1.022	.9919	.9631	.9352	.9082	.8818	.8559	.8305	.8057	.7810	.7566	.7323	.7081	.6837	.062

TABLE 3

$$M_1 = \left(\frac{5}{6X-1}\right)^{1/2} \operatorname{cosec}(\alpha - w)$$

[redacted]

°	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	51°	52°	53°
9 .863	1.897	1.844	1.795	1.749	1.705	1.665	1.626	1.590	1.566	1.523	1.492	1.463	1.433	1.409	1.366	1.361	1.339	1.317	1.297	1.278	1.280	1.242
7 .868	1.908	1.854	1.804	1.756	1.711	1.669	1.630	1.592	1.557	1.524	1.492	1.461	1.433	1.407	1.381	1.357	1.333	1.311	1.290	1.271	1.251	1.232
4 .881	1.921	1.865	1.812	1.763	1.718	1.675	1.633	1.596	1.558	1.524	1.492	1.461	1.432	1.404	1.378	1.353	1.329	1.308	1.284	1.263	1.243	1.224
3 .[illegible]98	1.936	1.878	1.826	1.774	1.726	1.682	1.640	1.600	1.562	1.527	1.494	1.461	1.431	1.403	1.376	1.350	1.325	1.302	1.279	1.257	1.237	1.217
3 .[illegible]6	1.954	1.894	1.837	1.786	1.736	1.690	1.646	1.605	1.567	1.531	1.496	1.463	1.432	1.403	1.375	1.348	1.323	1.298	1.275	1.253	1.231	1.211
3 .[illegible]9	1.973	1.910	1.862	1.798	1.748	1.700	1.656	1.613	1.573	1.535	1.500	1.466	1.434	1.403	1.375	1.347	1.321	1.298	1.272	1.249	1.227	1.208
6 .[illegible]63	1.994	1.930	1.869	1.813	1.760	1.711	1.665	1.621	1.580	1.541	1.504	1.470	1.436	1.405	1.375	1.347	1.320	1.294	1.270	1.246	1.224	1.202
8 .[illegible]0	2.018	1.961	1.888	1.830	1.776	1.724	1.676	1.631	1.588	1.548	1.510	1.474	1.441	1.406	1.372	1.348	1.320	1.294	1.268	1.244	1.221	1.198
12	2.045	1.974	1.910	1.848	1.792	1.739	1.669	1.642	1.539	1.557	1.517	1.480	1.445	1.412	1.380	1.350	1.322	1.294	1.268	1.243	1.219	1.192
17		2.001	1.933	1.869	1.810	1.756	1.704	1.665	1.610	1.566	1.526	1.488	1.451	1.417	1.384	1.353	1.323	1.296	1.269	1.242	1.219	1.194
2[illegible]			1.959	1.893	1.831	1.774	1.720	1.670	1.622	1.578	1.536	1.496	1.459	1.423	1.389	1.357	1.327	1.298	1.270	1.243	1.216	1.193
[illegible]				1.918	1.654	1.794	1.738	1.686	1.637	1.591	1.547	1.506	1.467	1.430	1.398	1.362	1.331	1.301	1.272	1.245	1.219	1.193
[illegible]					1.879	1.817	1.759	1.704	1.663	1.606	1.560	1.517	1.477	1.439	1.402	1.368	1.336	1.305	1.275	1.247	1.220	1.194
18						1.542	1.781	1.724	1.671	1.621	1.574	1.530	1.488	1.448	1.411	1.376	1.342	1.310	1.279	1.250	1.222	1.195
59							1.806	1.746	1.691	1.638	1.589	1.543	1.500	1.459	1.421	1.384	1.349	1.316	1.284	1.254	1.225	1.198
72								1.770	1.712	1.358	1.607	1.559	1.514	1.472	1.431	1.393	1.358	1.323	1.290	1.259	1.230	1.201
36									1.736	1.680	1.626	1.576	1.529	1.486	1.444	1.404	1.367	1.331	1.298	1.266	1.235	1.205
00										1.705	1.648	1.596	1.546	1.500	1.457	1.416	1.377	1.340	1.306	1.272	1.241	1.211
17										1.729	1.671	1.617	1.565	1.517	1.472	1.429	1.389	1.351	1.315	1.281	1.248	1.217
36										1.759	1.697	1.640	1.586	1.536	1.489	1.444	1.402	1.363	1.325	1.290	1.256	1.224
53										1.791	1.726	1.665	1.609	1.556	1.507	1.461	1.417	1.376	1.337	1.300	1.265	1.231
79										1.826	1.757	1.694	1.634	1.579	1.527	1.479	1.433	1.390	1.350	1.311	1.275	1.240
04										1.865	1.792	1.725	1.662	1.604	1.550	1.499	1.451	1.406	1.364	1.324	1.286	1.250
31										1.909	1.831	1.759	1.693	1.631	1.574	1.521	1.471	1.424	1.380	1.338	1.299	1.262
32										1.957	1.873	1.797	1.726	1.662	1.601	1.545	1.493	1.444	1.397	1.354	1.313	1.274
95										2.011	1.921	1.839	1.764	1.695	1.631	1.572	1.516	1.465	1.417	1.371	1.328	1.288
31										2.071	1.974	1.885	1.805	1.731	1.664	1.601	1.543	1.488	1.438	1.390	1.345	1.303
72										2.139	2.033	1.938	1.851	1.772	1.700	1.633	1.572	1.514	1.461	1.411	1.364	1.320
18										2.217	2.100	1.996	1.902	1.818	1.740	1.669	1.603	1.542	1.466	1.434	1.384	1.338
68										2.306	2.177	2.062	1.960	1.866	1.785	1.708	1.638	1.574	1.514	1.459	1.407	1.356

° 61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°			
92 .127	1.115	1.104	1.093	1.083	1.074	1.064	1.055	1.046	1.037	1.030	1.021	1.014	1.007	.9932	.9916	.9844	.9771	.9697				
71 .112	1.099	1.087	1.076	1.064	1.054	1.043	1.033	1.023	1.014	1.004	.9949	.9857	.9764	.9671	.9577							
84 .098	1.085	1.072	1.060	1.048	1.036	1.025	1.014	1.003	.9920	.9815	.9709	.9603	.9493	.9384	.9274							
30 .086	1.072	1.058	1.045	1.032	1.020	1.008	.9958	.9837	.9722	.9604	.9490	.9369	.9253	.9130	.9004							
00 .074	1.060	1.046	1.032	1.018	1.005	.9920	.9794	.9665	.9542	.9416	.9291	.9160	.9031	.8900	.8761							
89 .064	1.049	1.034	1.020	1.006	.9916	.9778	.9644	.9511	.9376	.9244	.9109	.8974	.8833	.8689	.8542							
01 .055	1.039	1.024	1.009	.9938	.9790	.9649	.9507	.9367	.9225	.9037	.8944	.8800	.8654	.8501	.8344							
30 .047	1.030	1.014	.9984	.9829	.9681	.9528	.9382	.9235	.9088	.8910	.8793	.8642	.8487	.8330	.8167							
72 .059	1.022	1.005	.9891	.9735	.9575	.9422	.9268	.9115	.8962	.8810	.8655	.8498	.8338	.5172	.8004							
30 .052	1.016	.9978	.9809	.9644	.9483	.9322	.9162	.9006	.8848	.8688	.8529	.8364	.8200	.8030	.7853							
99 .027	1.009	.9911	.9736	.9566	.9397	.9233	.9070	.8906	.8741	.8677	.8412	.8245	.8074	.7899	.7718							
77 .022	1.003	.9849	.9669	.9496	.9321	.9150	.8984	.8813	.8645	.8477	.8307	.8135	.7959	.7777	.7593							
69 .018	.9982	.9794	.9612	.9433	.9252	.9077	.8904	.8732	.8559	.8384	.8211	.8034	.7853	.7689	.7478							
67 .014	.9943	.9749	.9561	.9374	.9192	.9013	.8834	.8656	.8479	.8301	.8122	.7942	.7758	.7567	.7373							
75 .011	.9907	.9710	.9517	.9327	.9138	.8953	.8771	.8588	.8407	.8225	.8041	.7856	.7669	.7475	.7279							
93 .009	.9881	.9678	.9479	.9284	.9091	.8902	.8714	.8528	.8343	.8155	.7969	.7780	.7588	.7393	.7190							
15 .007	.9860	.9653	.9448	.9248	.9053	.8857	.8666	.8475	.8265	.8094	.7903	.7711	.7514	.7315	.7110							
45 .007	.9846	.9632	.9424	.9219	.9018	.8819	.8622	.8426	.8232	.8040	.7844	.7648	.7449	.7245	.7038							
32 .006	.9840	.9622	.9407	.9196	.8991	.8787	.8584	.8386	.8187	.7990	.7791	.7591	.7388	.7182	.6971							
25 .007	.9840	.9614	.9395	.9179	.8968	.8760	.8554	.8350	.8149	.7946	.7743	.7540	.7334	.7125	.6911							
74 .006	.9846	.9615	.9389	.9169	.8963	.8740	.8530	.8321	.8116	.7909	.7703	.7496	.7286	.7074	.6857							
28 .010	.9861	.9622	.9391	.9164	.8943	.8725	.8511	.8299	.8087	.7876	.7668	.7455	.7243	.7028	.6808							
67 .013	.9878	.9636	.9398	.9166	.8939	.8716	.8497	.8279	.8065	.7851	.7637	.7421	.7206	.6987	.6764							
51 .016	.9906	.9655	.9413	.9174	.8943	.8714	.8489	.8268	.8048	.7830	.7612	.7394	.7174	.6951	.6725							
20 .020	.9939	.9682	.9432	.9188	.8949	.8717	.8487	.8260	.8037	.7813	.7592	.7370	.7146	.6920	.6692							
395 .025	.9979	.9716	.9459	.9209	.8963	.8725	.8490	.8258	.8030	.7803	.7577	.7361	.7124	.6895	.6663							
374 .031	1.003	.9756	.9491	.9234	.8983	.8738	.8499	.8262	.8029	.7797	.7566	.7336	.7106	.6874	.6638							
357 .037	1.008	.9802	.9531	.9267	.9010	.8759	.8513	.8271	.8033	.7797	.7562	.7327	.7093	.6857	.6619							
345 .044	1.015	.9856	.9578	.9306	.9012	.8786	.8534	.8285	.8041	.7801	.7562	.7323	.7085	.6845	.6603							
337 .063	1.022	.9919	.9631	.9352	.9032	.8818	.8559	.8305	.8057	.7810	.7566	.7323	.7081	.6837	.6592							

TABLE AE

$$\frac{p_1}{p_0} = \frac{6X-}{6-}$$

w ↓ α →	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°
1°		2.753	1.760	1.511	[illegible]	1.237	1.242	[illegible]	[illegible]	[illegible]	[illegible]	1.134	[illegible]	1.114	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2°			6.674	[illegible]	[illegible]	1.783	1.414	[illegible]	1.451	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3°				11.553	[illegible]	2.763	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4°					23.425	[illegible]	3.571	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5°							[illegible]	4.545	[illegible]	[illegible]	2.438	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6°								11.722	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.114	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7°									[illegible]	7.253	[illegible]	[illegible]	[illegible]	[illegible]	2.551	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8°										[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9°											[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10°													[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11°														[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12°															[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
13°																[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14°																	[illegible]	[illegible]	[illegible]	[illegible]
15°																			[illegible]	[illegible]
16°																				[illegible]
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30°																				

	39°	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	58°
1°	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	[illegible]	[illegible]	[illegible]	1.052	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2°	1.107	1.105	1.104	1.104	1.104	1.104	1.102	1.102	1.102	1.104	1.104	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3°	1.165	1.163	1.162	1.160	1.159	1.159	1.159	1.157	1.157	1.154	1.154	[illegible]	[illegible]	[illegible]	[illegible]	1.165	[illegible]	[illegible]	[illegible]	[illegible]
4°	1.225	1.224	1.222	1.219	1.219	[illegible]	[illegible]	1.216	1.216	1.216	1.216	1.216	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5°	1.293	1.289	1.286	1.284	[illegible]	[illegible]	1.280	1.278	1.278	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6°	1.364	1.359	1.354	1.351	1.348	1.347	1.345	1.344	1.344	1.342	1.344	1.344	[illegible]	[illegible]	1.349	1.351	[illegible]	[illegible]	[illegible]	[illegible]
7°	1.440	1.434	1.429	1.425	1.420	[illegible]	1.415	1.414	1.412	1.412	1.412	1.412	1.414	[illegible]	[illegible]	1.42	[illegible]	[illegible]	[illegible]	[illegible]
8°	1.524	1.516	1.509	1.503	1.498	1.493	1.490	1.487	1.485	1.495	1.483	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9°	1.614	1.602	1.594	1.587	1.581	1.574	[illegible]	[illegible]	1.584	1.581	1.581	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10°	1.711	1.697	1.687	1.677	1.669	1.662	1.655	[illegible]	1.647	1.648	1.644	[illegible]	1.644	[illegible]	1.647	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11°	1.818	1.801	1.788	1.776	1.764	1.755	1.749	1.742	[illegible]	1.743	[illegible]	1.750	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12°	1.935	1.915	1.897	1.893	1.889	1.856	1.848	1.839	1.933	[illegible]	1.825	1.821	1.821	[illegible]	[illegible]	[illegible]	1.836	[illegible]	[illegible]	[illegible]
13°	2.065	2.040	2.016	1.998	1.982	1.967	1.965	1.946	[illegible]	[illegible]	1.924	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14°	2.210	2.178	2.151	2.126	[illegible]	2.089	2.072	2.059	2.049	2.040	2.033	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
15°	2.371	2.331	2.298	2.269	2.243	2.220	2.201	[illegible]	2.170	2.159	2.149	2.142	[illegible]	[illegible]	[illegible]	2.14	[illegible]	[illegible]	[illegible]	[illegible]
16°	2.524	2.505	2.481	2.427	2.393	2.365	2.341	2.321	2.304	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
17°	2.739	2.700	2.646	2.602	2.562	[illegible]	[illegible]	[illegible]	2.448	[illegible]	2.415	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
18°	2.937	2.922	2.856	2.798	2.750	2.708	2.670	[illegible]	[illegible]	2.587	[illegible]	2.551	2.541	2.522	2.526	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
19°	3.272	3.177	3.096	[illegible]	[illegible]	2.908	2.861	2.921	2.787	[illegible]	[illegible]	2.714	[illegible]	2.697	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20°	3.594	3.472	3.371	3.281	3.203	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.417	[illegible]	[illegible]	[illegible]	[illegible]	2.841	[illegible]	[illegible]	[illegible]	[illegible]
21°	3.977	3.922	3.690	3.579	[illegible]	[illegible]	[illegible]	3.262	[illegible]	[illegible]	3.124	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
22°	4.445	4.240	[illegible]	[illegible]	3.901	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
23°	5.020	4.752	[illegible]	4.740	[illegible]	[illegible]	[illegible]	3.521	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.41	[illegible]
24°	5.753	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
25°	6.718	[illegible]	5.742	5.457	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
26°	8.045	7.283	[illegible]	6.278	5.864	5.569	[illegible]	[illegible]	4.917	[illegible]	4.838	4.633	4.442	4.387	[illegible]	4.261	4.223	4.198	[illegible]	[illegible]
27°	9.992	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
28°	13.084	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5.475	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
29°	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5.728	[illegible]	5.485	[illegible]	[illegible]	[illegible]	[illegible]
30°	33.459	[illegible]	[illegible]	14.841	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7.163	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

A

ABLE 4

$$= \frac{6X-1}{6-X}$$

[redacted]

19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	31°	32°	33°	34°	35°	36°	37°	38°
1.085	1.081	1.078	1.075	1.072	1.069	1.066	1.065	1.064	1.062	1.061	1.059	1.058	1.056	1.055	1.055	1.054	1.054	1.052	[illegible]
1.181	1.172	1.165	1.157	1.152	1.146	1.141	1.136	1.133	1.128	1.125	1.122	1.120	1.117	1.115	1.112	1.111	1.109	[illegible]	1.107
1.293	1.277	1.263	1.249	1.240	1.231	1.222	1.215	1.208	1.202	1.197	1.192	1.187	1.182	1.180	1.177	1.174	1.171	[illegible]	[illegible]
1.423	1.398	1.378	1.358	1.342	1.327	1.315	1.302	1.292	1.283	1.275	1.267	1.260	1.254	1.249	1.243	1.239	1.236	1.231	[illegible]
1.577	1.540	1.509	1.482	1.456	1.436	1.417	1.401	1.386	1.373	1.361	1.350	1.341	1.331	1.324	1.318	1.312	1.306	[illegible]	[illegible]
1.762	1.709	1.652	1.624	1.589	1.559	1.533	1.511	1.490	1.472	1.456	1.442	1.429	1.417	1.407	1.398	1.389	1.381	[illegible]	1.369
1.991	1.913	1.842	1.792	1.743	1.702	1.667	1.635	1.609	1.584	1.565	1.543	1.527	1.511	1.496	1.485	1.474	1.464	1.455	1.447
2.276	2.162	2.070	1.991	1.926	1.870	1.821	1.780	1.743	1.711	1.682	1.658	1.635	1.614	1.597	1.581	1.568	1.553	1.541	[illegible]
2.649	2.478	2.345	2.225	2.143	2.088	2.002	1.948	1.897	1.855	1.818	1.785	1.755	1.730	1.708	1.686	1.669	1.662	[illegible]	1.624
3.147	2.891	2.698	2.540	2.410	2.306	2.216	2.141	2.078	2.020	1.973	1.929	1.892	1.860	1.830	1.804	1.781	1.761	1.744	1.725
3.859	3.455	3.155	2.915	2.743	2.596	2.475	2.373	2.288	2.214	2.151	2.098	2.048	2.005	1.957	1.935	1.906	1.879	[illegible]	1.835
4.950	4.264	3.786	3.438	3.171	2.963	2.794	2.655	2.541	2.442	2.359	2.288	2.226	2.172	2.124	2.081	2.046	2.013	1.984	1.958
6.650	5.527	4.708	4.149	3.741	3.437	3.196	3.004	2.847	2.715	2.608	2.512	2.432	2.363	2.302	2.249	2.202	2.162	[illegible]	2.092
10.957	7.797	6.179	5.200	4.542	4.072	3.720	3.447	3.228	3.051	2.904	2.779	2.674	2.585	2.508	2.440	2.381	2.329	2.284	2.246
26.528	13.022	8.305	6.921	5.745	4.977	4.432	4.028	3.714	3.467	3.257	3.103	2.963	2.847	2.748	2.661	2.587	2.522	2.464	2.415
	38.025	15.712	10.234	7.763	6.359	5.453	4.823	4.258	4.005	3.725	3.501	3.313	3.160	3.030	2.919	2.825	2.741	2.670	2.625
		62.627	19.326	11.848	9.743	7.050	5.982	5.254	4.720	4.318	4.003	3.749	3.542	3.371	3.225	3.103	2.997	2.908	2.827
			151.658	24.488	13.841	9.896	7.634	6.576	5.725	5.118	4.658	4.303	4.017	3.787	3.594	3.434	3.299	3.182	3.064
					32.419	16.393	11.258	8.737	7.243	6.255	5.551	5.031	4.629	4.309	4.049	3.837	3.658	3.508	3.391
						46.239	19.735	12.919	9.787	8.000	6.839	6.032	5.438	4.982	4.622	4.334	4.098	3.901	3.738
							76.353	24.322	14.958	11.023	8.862	7.503	6.567	5.885	5.371	4.965	4.642	4.350	4.160
								192.864	31.194	17.553	12.499	9.658	8.251	7.163	6.381	5.798	5.342	4.979	4.627
										42.209	20.965	14.220	11.032	9.106	7.834	6.934	6.268	5.753	5.349
											63.034	25.646	16.525	12.421	10.092	8.595	7.555	6.792	6.212
												117.675	32.419	19.381	14.103	11.250	9.473	8.257	7.379
													530.364	43.226	23.142	16.166	12.617	10.485	9.060
															63.034	28.381	18.770	14.278	11.868
																111.845	26.169	22.160	16.307
																	115.587	48.659	26.602
																			73.305

57°	58°	59°	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°
1.055	1.055	1.056	1.058	1.059	1.061	1.062	1.064	1.065	1.066	1.069	1.072	1.075	1.078	1.081	1.085	1.089	1.093	1.099	1.107
1.111	1.112	1.115	1.117	1.120	1.122	1.125	1.128	1.133	1.136	1.141	1.146	1.152	1.157	1.165	1.172	1.181	1.191	1.203	1.216
1.171	1.174	1.177	1.180	1.182	1.187	1.192	1.197	1.202	1.208	1.215	1.222	1.231	1.240	1.249	1.263	1.277	1.293	1.311	1.331
1.231	1.236	1.239	1.243	1.249	1.254	1.260	1.267	1.275	1.283	1.292	1.302	1.315	1.327	1.342	1.358	1.378	1.398	1.423	1.452
1.296	1.301	1.305	1.312	1.318	1.324	1.331	1.341	1.350	1.361	1.373	1.386	1.401	1.417	1.436	1.456	1.482	1.509	1.540	1.577
1.364	1.368	1.375	1.381	1.389	1.398	1.407	1.417	1.429	1.442	1.456	1.472	1.490	1.511	1.533	1.559	1.589	1.624	1.662	1.709
1.434	1.440	1.447	1.455	1.464	1.474	1.485	1.496	1.511	1.527	1.543	1.563	1.584	1.609	1.635	1.667	1.702	1.743	1.792	1.848
1.509	1.516	1.524	1.532	1.541	1.553	1.566	1.581	1.597	1.614	1.635	1.658	1.682	1.711	1.743	1.780	1.821	1.870	1.926	1.991
1.587	1.594	1.602	1.614	1.624	1.637	1.652	1.669	1.686	1.708	1.730	1.755	1.785	1.818	1.855	1.897	1.946	2.002	2.068	2.143
1.669	1.677	1.687	1.697	1.711	1.725	1.742	1.761	1.781	1.804	1.830	1.860	1.892	1.929	1.973	2.020	2.078	2.141	2.216	2.306
1.755	1.764	1.776	1.788	1.801	1.818	1.835	1.856	1.879	1.906	1.935	1.967	2.006	2.048	2.096	2.151	2.214	2.288	2.373	2.475
1.848	1.856	1.869	1.883	1.897	1.915	1.935	1.958	1.984	2.013	2.046	2.081	2.124	2.172	2.228	2.288	2.359	2.442	2.541	2.655
1.945	1.955	1.967	1.982	1.998	2.016	2.040	2.065	2.092	2.126	2.162	2.202	2.249	2.302	2.363	2.432	2.512	2.606	2.715	2.947
2.048	2.059	2.072	2.089	2.106	2.126	2.151	2.178	2.210	2.245	2.284	2.329	2.381	2.440	2.508	2.585	2.674	2.779	2.904	3.051
2.158	2.170	2.183	2.201	2.220	2.243	2.269	2.298	2.331	2.371	2.415	2.464	2.522	2.587	2.661	2.748	2.847	2.963	3.103	3.287
2.276	2.288	2.304	2.321	2.341	2.365	2.393	2.427	2.461	2.505	2.524	2.608	2.670	2.741	2.825	2.919	3.030	3.160	3.313	3.501
2.403	2.415	2.430	2.448	2.470	2.497	2.528	2.562	2.602	2.646	2.700	2.759	2.827	2.906	2.997	3.103	3.225	3.371	3.542	3.749
2.541	2.551	2.568	2.587	2.610	2.638	2.670	2.706	2.750	2.798	2.856	2.922	2.997	3.064	3.182	3.299	3.434	3.594	3.787	4.017
2.687	2.698	2.713	2.737	2.759	2.787	2.821	2.861	2.908	2.963	3.023	3.096	3.177	3.272	3.381	3.508	3.658	3.837	4.049	4.309
2.845	2.856	2.872	2.892	2.917	2.949	2.986	3.028	3.079	2.961	3.203	3.281	3.371	3.472	3.594	3.738	3.901	4.098	4.334	4.622
3.018	3.030	3.044	3.065	3.091	3.124	3.162	3.208	3.262	3.323	3.396	3.480	3.579	3.690	3.824	3.977	4.160	4.380	4.642	4.965
3.208	3.218	3.232	3.252	3.252	3.279	3.311	3.353	3.401	3.459	3.526	3.605	3.695	3.801	3.926	4.240	4.445	4.687	4.979	5.342
3.416	3.424	3.437	3.457	3.483	3.519	3.560	3.613	3.674	3.747	3.831	3.929	4.046	4.180	4.340	4.526	4.753	5.020	5.349	5.753
3.645	3.650	3.661	3.679	3.706	3.741	3.787	3.840	3.907	3.983	4.075	4.163	4.309	4.457	4.632	4.839	5.086	5.386	5.753	6.212
3.898	3.898	3.907	3.926	3.952	3.989	4.034	4.092	4.163	4.246	4.343	4.460	4.600	4.759	4.955	5.182	5.457	5.792	6.204	6.718
4.163	4.177	4.183	4.198	4.222	4.261	4.306	4.367	4.442	4.533	4.638	4.766	4.917	5.094	5.305	5.559	5.864	6.238	6.700	7.283
4.501	4.488	4.488	4.501	4.526	4.561	4.609	4.674	4.753	4.849	4.965	5.101	5.265	5.460	5.694	5.978	6.320	6.737	7.258	7.928
4.863	4.843	4.838	4.843	4.863	4.897	4.948	5.013	5.097	5.200	5.323	5.475	5.655	5.868	6.128	6.442	6.825	7.298	7.889	8.650
5.276	5.243	5.229	5.229	5.243	5.276	5.327	5.393	5.483	5.593	5.725	5.889	6.090	6.328	6.613	6.963	7.395	7.928	8.608	9.487
5.753	5.708	5.674	5.667	5.674	5.706	5.753	5.824	5.917	6.032	6.178	6.359	6.576	6.839	7.163	7.565	8.045	8.650	9.432	10.455

TABLE 5

Functions for Calculations of Expansive Flow

μ	f(μ)	g(μ)	h(μ)	μ	f(μ)	g(μ)	h(μ)
90°	130.45°	.5283	.6339	44°	120.32°	.2971	.4203
89°	130.45	.5282	.6339	43°	119.54	.2860	.4089
88°	130.45	.5279	.6336	42°	118.71	.2746	.3972
87°	130.45	.5274	.6332	41°	117.83	.2630	.3852
86°	130.45	.5268	.6326	40°	116.90	.2512	.3727
85°	130.44	.5259	.6319	39°	115.92	.2391	.3599
84°	130.44	.5249	.6310	38°	114.88	.2269	.3467
83°	130.42	.5236	.6300	37°	113.77	.2146	.3331
82°	130.41	.5222	.6287	36°	112.61	.2022	.3192
81°	130.39	.5206	.6274	35°	111.37	.1897	.3050
80°	130.37	.5188	.6258	34°	110.07	.1772	.2905
79°	130.34	.5168	.6241	33°	108.70	.1647	.2757
78°	130.31	.5146	.6222	32°	107.23	.1522	.2607
77°	130.27	.5122	.6201	31°	105.69	.1399	.2454
76°	130.22	.5096	.6178	30°	104.08	.1278	.2301
75°	130.16	.5067	.6154	29°	102.36	.1159	.2145
74°	130.10	.5037	.6127	28°	100.56	.1043	.1990
73°	130.03	.5005	.6099	27°	98.65	.09313	.1835
72°	129.95	.4970	.6069	26°	96.64	.08237	.1681
71°	129.85	.4933	.6037	25°	94.53	.07211	.1529
70°	129.75	.4894	.6003	24°	92.30	.06243	.1379
69°	129.64	.4853	.5966	23°	89.96	.05338	.1233
68°	129.51	.4809	.5928	22°	87.50	.04502	.1092
67°	129.37	.4763	.5887	21°	84.91	.03739	.09562
66°	129.22	.4715	.5845	20°	82.19	.03054	.08274
65°	129.05	.4664	.5800	19°	79.34	.02447	.07063
64°	128.87	.4611	.5752	18°	76.31	.01919	.05937
63°	128.67	.4555	.5702	17°	73.21	.01469	.04906
62°	128.45	.4497	.5650	16°	69.94	.01095	.03977
61°	128.22	.4436	.5595	15°	66.51	.00791	.03153
60°	127.97	.4372	.5538	14°	62.95	.00552	.02438
59°	127.69	.4306	.5478	13°	59.23	.00370	.01832
58°	127.40	.4237	.5415	12°	55.37	.00237	.01331
57°	127.08	.4165	.5349	11°	51.37	.00143	.00931
56°	126.74	.4090	.5281	10°	47.22	.00081	.00621
55°	126.38	.4013	.5209	9°	42.94	.00043	.00392
54°	125.99	.3933	.5135	8°	38.53	.00020	.00232
53°	125.57	.3850	.5057	7°	34.00	.00009	.00126
52°	125.12	.3763	.4976	6°	29.36	.00003	.00061
51°	124.64	.3675	.4891	5°	24.63	.00001	.00026
50°	124.14	.3583	.4804	4°	19.81	.00000	.00009
49°	123.59	.3488	.4713	3°	14.92	.00000	.00002
48°	123.02	.3390	.4618	2°	9.98	.00000	.00000
47°	122.40	.3289	.4520	1°	5.00	.00000	.00000
46°	121.75	.3186	.4418	0°	0.00	.00000	.00000
45°	121.06	.3080	.4312				

TABLE 6

Wedge Angle for Sonic Flow Behind Shock Wave (w_s). Maximum Wedge Angle (w_m)

M_0	w_s	w_m
1.01	0.05°	0.05°
1.02	0.13	0.15
1.03	0.24	0.26
1.04	0.37	0.40
1.05	0.52	0.56
1.06	0.67	0.73
1.07	0.84	0.91
1.08	1.02	1.10
1.09	1.21	1.30
1.10	1.41	1.51
1.11	1.61	1.73
1.12	1.82	1.96
1.13	2.04	2.19
1.14	2.26	2.43
1.15	2.49	2.67
1.16	2.73	2.92
1.17	2.97	3.17
1.18	3.21	3.42
1.19	3.45	3.68
1.20	3.70	3.94
1.21	3.95	4.21
1.22	4.21	4.48
1.23	4.46	4.74
1.24	4.72	5.01
1.25	4.99	5.29
1.26	5.25	5.56
1.27	5.52	5.83
1.28	5.78	6.11
1.29	6.05	6.39
1.30	6.32	6.66
1.31	6.59	6.94
1.32	6.86	7.22
1.33	7.13	7.49
1.34	7.40	7.77
1.35	7.67	8.05
1.36	7.94	8.32
1.37	8.21	8.60
1.38	8.48	8.88
1.39	8.76	9.15
1.40	9.03	9.43
1.41	9.30	9.70
1.42	9.57	9.97
1.43	9.84	10.25
1.44	10.10	10.52
1.45	10.37	10.79
1.46	10.64	11.05
1.47	10.90	11.32
1.48	11.17	11.59
1.49	11.43	11.85
1.50	11.69	12.11
1.51	11.96	12.37
1.52	12.21	12.63
1.53	12.47	12.89
1.54	12.73	13.15
1.55	12.99	13.40
1.56	13.24	13.66
1.57	13.49	13.91
1.58	13.74	14.16
1.59	13.99	14.41
1.60	14.24	14.65
1.61	14.49	14.90
1.62	14.73	15.14
1.63	14.98	15.38
1.64	15.22	15.62
1.65	15.46	15.86
1.66	15.70	16.09
1.67	15.93	16.32
1.68	16.17	16.55
1.69	16.40	16.78
1.70	16.63	17.01
1.71	16.86	17.24
1.72	17.09	17.46
1.73	17.31	17.68
1.74	17.54	17.90
1.75	17.76°	18.12°
1.76	17.98	18.34
1.77	18.20	18.55
1.78	18.41	18.76
1.79	18.63	18.97
1.80	18.84	19.18
1.81	19.05	19.39
1.82	19.26	19.59
1.83	19.46	19.80
1.84	19.67	20.00
1.85	19.87	20.20
1.86	20.07	20.40
1.87	20.27	20.59
1.88	20.47	20.78
1.89	20.67	20.98
1.90	20.86	21.17
1.91	21.05	21.36
1.92	21.24	21.54
1.93	21.43	21.73
1.94	21.62	21.91
1.95	21.81	22.09
1.96	21.99	22.27
1.97	22.17	22.45
1.98	22.35	22.63
1.99	22.53	22.80
2.00	22.71	22.97
2.01	22.88	23.14
2.02	23.05	23.31
2.03	23.23	23.48
2.04	23.40	23.65
2.05	23.56	23.81
2.06	23.73	23.98
2.07	23.90	24.14
2.08	24.06	24.30
2.09	24.22	24.46
2.10	24.38	24.61
2.11	24.54	24.77
2.12	24.79	24.92
2.13	24.85	25.08
2.14	25.01	25.23
2.15	25.16	25.38
2.16	25.31	25.52
2.17	25.46	25.67
2.18	25.61	25.82
2.19	25.76	25.96
2.20	25.90	26.10
2.21	26.05	26.24
2.22	26.19	26.38
2.23	26.33	26.52
2.24	26.47	26.66
2.25	26.61	26.80
2.3	27.28	27.45
2.4	28.53	28.68
2.5	29.67	29.80
2.6	30.70	30.81
2.7	31.64	31.74
2.8	32.50	32.59
2.9	33.29	33.36
3.0	34.01	34.07
4	38.75	38.77
5	41.11	41.12
6	42.44	42.44
8	43.79	43.79
10	44.43	44.43
15	45.07	45.07
20	45.29	45.29
∞	45.58	45.58

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It has been demonstrated that an arbitrary curve intersecting once and only once each of the straight-line patching or characteristic curves in an expansive supersonic flow can be treated as if it were a characteristic curve. This fact leads to the formules for calculating pressure distributions over airfoils immersed in two-dimensional supersonic flows, and hence to lift, drag, and moment computations. The Analysis Group has calculated also extensive tables for facilitating oblique shock front computations. The Bumblebee report includes these computation tables and the discussion of the principles of two-dimensional supersonic flow.

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Extensive tables are calculated for facilitating oblique shock front calculation on airfoils of infinite span at supersonic speeds. Explicit demonstration of mathematical approach to formulas for calculating pressure distributions and hence to lift, drag, and moment computations is presented for the first time in a concise and accurate discussion of principles of supersonic aerodynamics. Report is essentially a manual for the two-dimensional case on theory and practice of computation in supersonic, aerodynamic flow as far as present state of subject has advanced.

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