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## THE RELATIVE MOTION BETWEEN A RADAR AND A SATELLITE OBJECT

MAY 1966

S. H. Bickel

Prepared for

SPACE DEFENSE SYSTEM PROGRAM OFFICE (496L/474L)  
ELECTRONIC SYSTEMS DIVISION  
AIR FORCE SYSTEMS COMMAND  
UNITED STATES AIR FORCE  
L. G. Hanscom Field, Bedford, Massachusetts



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Project 4966

Prepared by

THE MITRE CORPORATION  
Bedford, Massachusetts  
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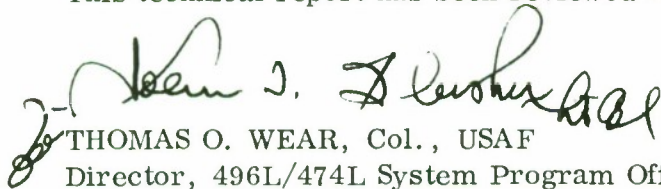
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## ABSTRACT

The relative motion between an earth-based radar and a satellite object is of interest for radar-tracking studies, signature simulation studies, and for developing inverse scattering techniques. Here, the motion analysis is accomplished with elementary vector algebra. This approach results in considerable simplification of the final equations and relationships.

## REVIEW AND APPROVAL

This technical report has been reviewed and is approved.



THOMAS O. WEAR, Col., USAF  
Director, 496L/474L System Program Office  
Deputy for Surveillance and Control Systems

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## SECTION I

### INTRODUCTION

The relative motion between an earth-based radar and a satellite object is of interest for radar-tracking studies, signature simulation studies, and for developing inverse scattering techniques. This relative motion can be decomposed into two component motions: the radar-tracking motion; and the rotation of the satellite about its center of mass. The radar-tracking motion is, in turn, caused by the relative motion between the satellite in its orbit and the radar site fixed on the surface of a rotating earth.

The equations and relations required in consideration of overall motion can be considered in terms of products of rotation matrices which transform from one coordinate system to another.\*† The approach here is to examine each of the component motions separately and to form the final combination with elementary vector manipulations.

The purpose of this report is to study the gross effects of the relative motion on radar observations, and consequently only first-order orbit perturbations are considered. The advantages of considering higher order perturbations are not great enough to justify the complications which they introduce.

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\* J. F. A. Ormsby, Motion Simulation of Ground Observed Earth Satellites, The MITRE Corporation, W-7289, Bedford, Mass., 4 September 1964.

† J. F. A. Ormsby and S. H. Bickel, A Generalized Radar Output Simulation, The MITRE Corporation, W-7346, Bedford, Mass., 19 October 1964.

Both stabilized and torque-free motion of the satellite about its center of mass are considered. In all cases, expressions for the relative orientation angles between the body and an earth-based radar are developed.

SECTION II  
ORBITAL MOTION

There are four different motions of a satellite in orbit:

- (1) the motion of the satellite which is restricted to move in an elliptical path due to the inverse square law for radial force;
- (2) the slipping motion of the perigee point within the plane of this ellipse;
- (3) the twisting motion of the plane regressing westward; and
- (4) the change in the size and shape of the ellipse due to atmospheric drag.

Effects (2) and (3) above are caused by the bulge of the earth at the equator. Five orbital elements (see Figure 1) are necessary to define these motions:

- (1) the perigee height  $h$ , which is the distance between the center of the earth and the closest point on the ellipse.
- (2) the inclination angle  $\delta$ , which is the angle that the orbital plane makes with the equator;
- (3) the right ascension of the orbit, which takes the form

$$\alpha = \alpha_0 + \omega_n t, \quad (1)$$

where  $\alpha_0$  is the right ascension at time,  $t = 0$ , and  $\omega_n$  is the rate of change in the westward direction of the right ascension due to the earth bulge;

- (4) the time of the ascending node, which takes the form

$$t = t_0 + T_n + D_1 n^2 + D_2 n^3, \quad (2)$$



where  $n$  is the revolution number,  $T$  is the satellite period, and  $D_1$  and  $D_2$  are drag coefficients; and

(5) the argument of perigee, which takes the form

$$\varphi = \varphi_0 + k_2 \left( \frac{4}{5} - \sin^2 \delta \right) t, \quad (3)$$

where  $k_2$  is a constant which specifies the rate of change in the argument of perigee due to the earth's bulge. At  $\sin^2 \delta = 4/5$ , or at an inclination of approximately 63.4 degrees, this rate of change is zero. At this inclination, a crossover exists where the rate of change shifts from positive to negative.

The inverse square law for the radial force field from the center of the earth results in an elliptical orbit for the satellite object with the earth's center at one focal point. Let  $\hat{e}_1, \hat{e}_2, \hat{e}_3$  be an orthonormal set of vectors in

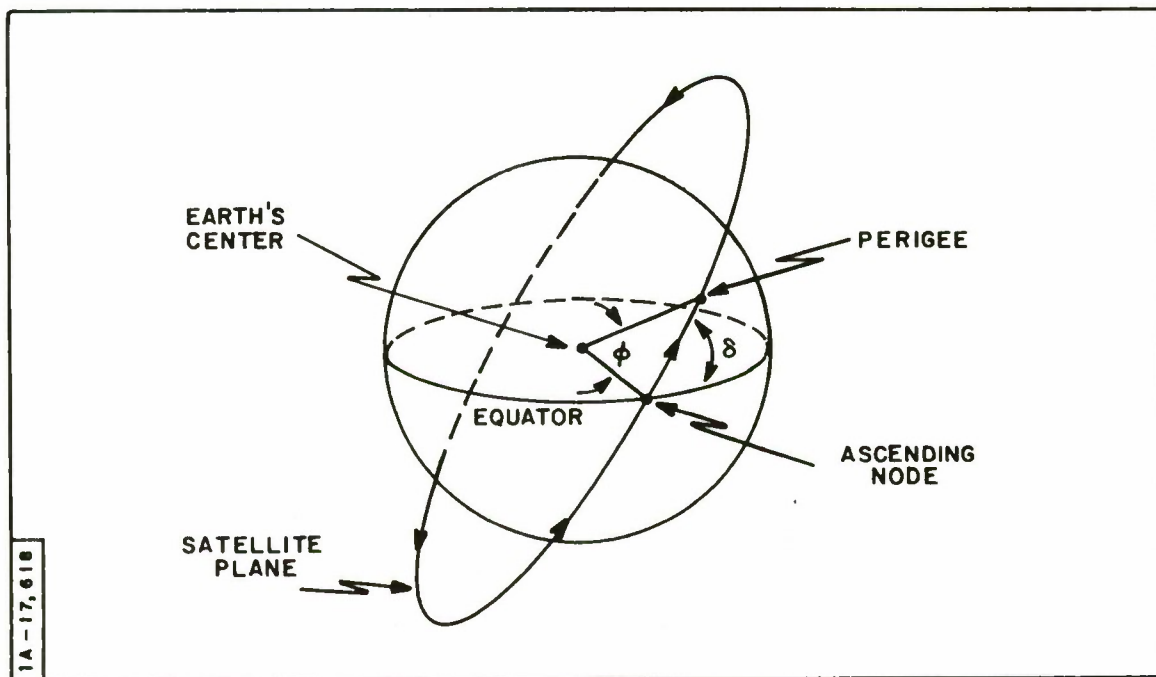


Figure 1. Orbital Element Geometry

an inertial coordinate system with the origin at the center of the earth. Let  $\hat{e}_1$  point in the direction of vernal equinox,  $\hat{e}_3$  in the direction of the north pole, and  $\hat{e}_2$  in the direction of  $\hat{e}_3 \times \hat{e}_1$ . In this coordinate system, the vector equation for an ellipse becomes

$$0 = \sum_{i=1}^3 \hat{e}_i \left[ (\bar{a} \cdot \hat{a}_i) (\cos E - e) + (\bar{b} \cdot \hat{e}_i) \sin E \right], \quad (4)$$

where  $e$  is the eccentricity of the orbit, and the vectors  $\bar{a}$  and  $\bar{b}$  have the length of point along the semi-major and semi-minor axes of the ellipse, respectively. (Note:  $\bar{a}$  points in the direction of perigee.) The eccentric anomaly, \* which is related to time in Kepler's equation, is  $E$ .

$$t - t_0 = \left( \frac{a^3}{d} \right)^{1/2} (E - e \sin E) = \frac{T}{2\pi} (E - e \sin E), \quad (5)$$

where  $t_0$  is the time of perigee crossing,  $d$  is the gravitational constant, and  $a$  is the length of  $\bar{a}$ . Substituting for  $d$  in Equation (5), the major semi-axis of any satellite is given in terms of its period by

$$a = 205.82 T^{2/3}, \quad (6)$$

where  $T$  is the period in minutes. The ellipticity of the orbit is given in terms of the semi-major axis, and the perigee height by

$$e = 1 - h/a. \quad (7)$$

The semi-minor axis is given by

$$b = a \left( 1 - e^2 \right)^{1/2}. \quad (8)$$

---

\* See page 1, first reference.

From (4) it follows that the length of the vector  $\vec{0}$  is given by

$$|\vec{0}| = a|1 - e \cos E|. \quad (9)$$

Figure 2 illustrates the geometrical relationships between these orbital elements.

In order to complete the description of the orbit, the vector dot products indicated by (4) must be represented in terms of the inclination angle and the argument of perigee. Let  $\alpha$  be the angle between ascending node  $\hat{A}$  and the unit vector  $\hat{e}_1$  (see Figures 1 and 3).

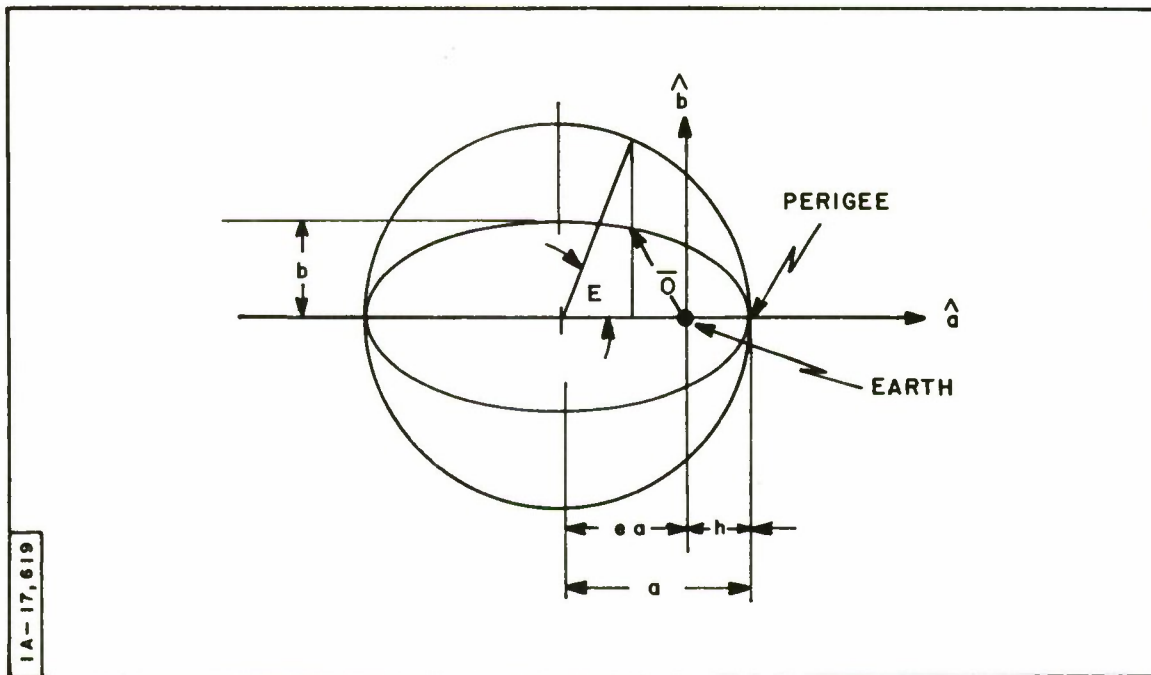


Figure 2. Basic Ellipse Orbit

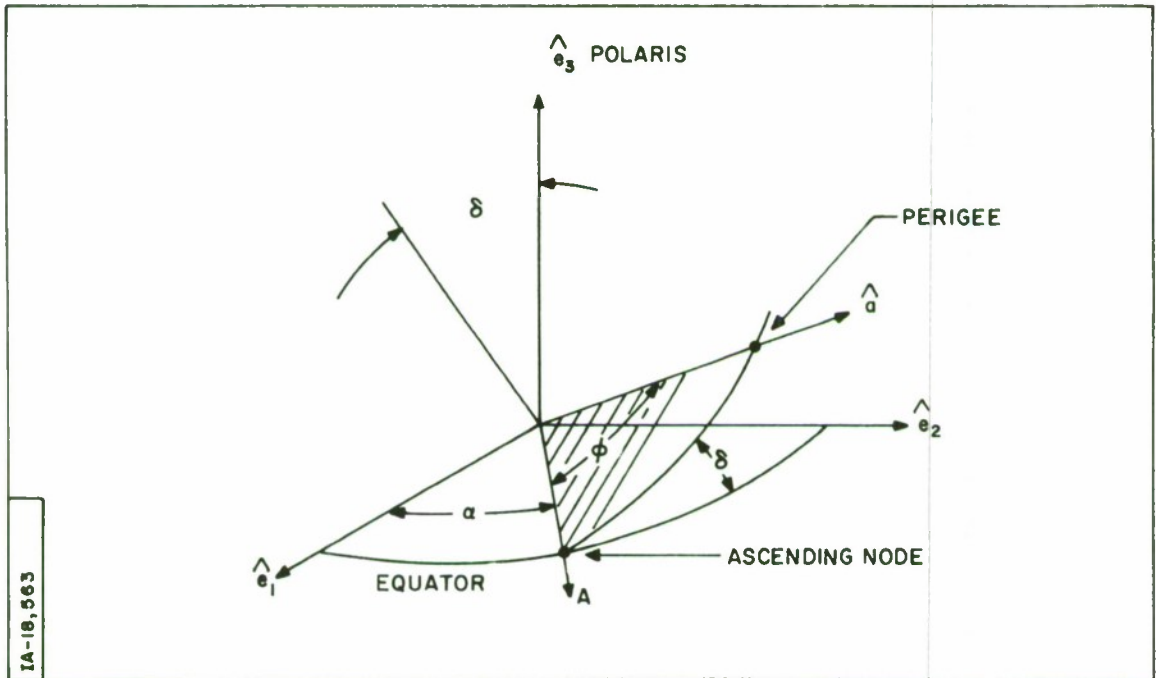


Figure 3. Earth-Orbital Orientation

From the geometry of Figure 3, it follows that:

$$(\hat{a} \cdot \hat{e}_1) = \cos \alpha \cos \varphi - \sin \varphi \sin \alpha \cos \delta; \quad (10)$$

$$(\hat{a} \cdot \hat{e}_2) = \sin \alpha \cos \varphi + \sin \varphi \cos \alpha \cos \delta; \quad (11)$$

$$(\hat{a} \cdot \hat{e}_3) = \sin \delta \sin \varphi. \quad (12)$$

Since  $\hat{b}$  is perpendicular to  $\hat{a}$ , it is only necessary to replace  $\varphi$  by  $\varphi + \pi/2$  in order to develop the corresponding equations for  $\hat{b}$ .

### SECTION III

#### SITE ORIENTATION AND RADAR-TRACKING MOTION

Let  $\lambda_s$  be the latitude of the site and  $\alpha_s$  the right ascension of the site (i. e. , the angle between the site longitude and the vernal equinox. This angle is time varying due to the earth's rotation). The unit vectors in the direction of the zenith, east, and north as measured at the site are given by:

$$\hat{Z} = \cos \lambda_s (e_1 \cos \alpha_s + \hat{e}_2 \sin \alpha_s) + \sin \lambda_s \hat{e}_3; \quad (13)$$

$$\hat{E} = -\hat{e}_1 \sin \alpha_s + \hat{e}_2 \cos \alpha_s; \quad (14)$$

$$\hat{N} = -\sin \lambda_s (\hat{e}_1 \sin \alpha_s + \hat{e}_2 \cos \alpha_s) + \cos \lambda_s \hat{e}_3 \quad (15)$$

(see Figure 4 for a description of the geometry involved).

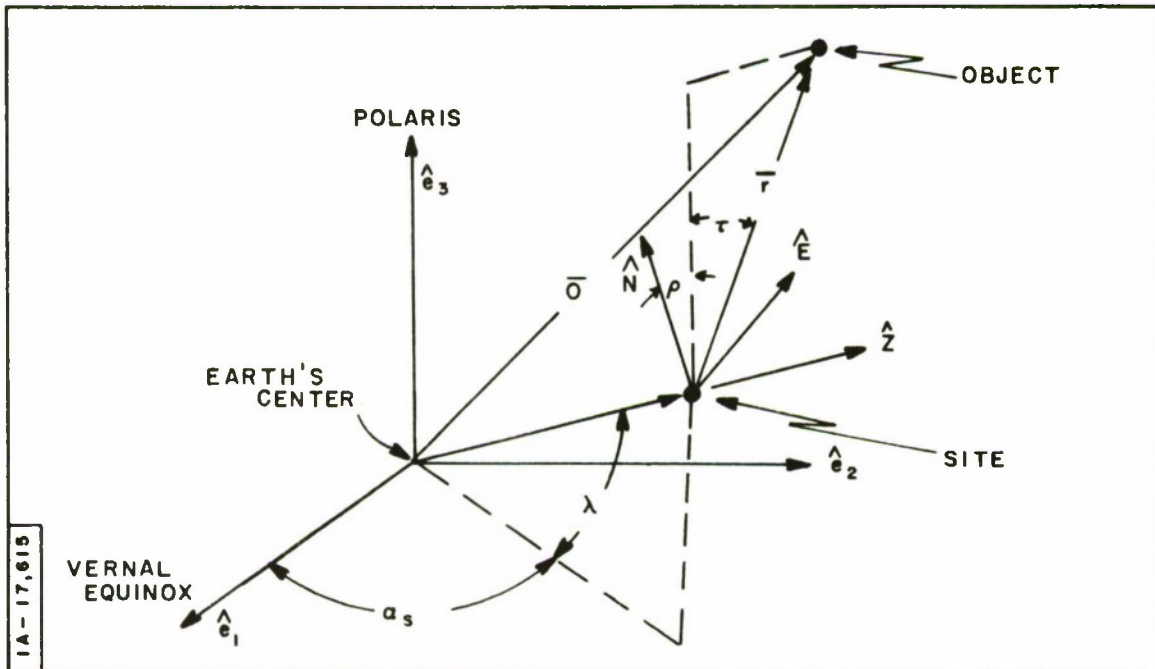


Figure 4. Site-Earth Geometry

The vector from the site to the object is given by

$$\bar{r} = \bar{0} - r_e \hat{Z}, \quad (16)$$

where  $r_e$  is the radius of the earth. From Equation (16), it follows that the distance or radar range from the site to the object is

$$r = \sqrt{|\bar{0}|^2 - 2r_e(\bar{0} \cdot \hat{Z}) + r_e^2}. \quad (17)$$

The elevation and azimuth angles of the radar (i. e., the angle which the target makes with the horizon measured at the site and the angle between due north and the target measured in a clockwise direction when looking down on the earth) are given by:

$$\text{Tan } \tau = \frac{(\bar{0} \cdot \hat{Z}) - r_e}{\left[|\bar{0}|^2 - [(\bar{0} \cdot \hat{Z})^2]\right]^{1/2}} - \frac{\pi}{2} \leq \tau \leq \frac{\pi}{2}; \quad (18)$$

$$\text{Tan } \rho = \frac{\bar{0} \cdot \hat{E}}{\bar{0} \cdot \hat{N}}, \quad 0 \leq \rho \leq 2\pi. \quad (19)$$

A simple and useful acquisition scheme would be to test for the zero elevation angle. Positive angles indicate that the satellite is above the horizon and visible, while the satellite is below the horizon for negative elevations. From Equation (4),  $\bar{0} \cdot \hat{Z}$  can be written as

$$\bar{0} \cdot \hat{Z} = OZ \cos (E - EZ) - e(\bar{a} \cdot \hat{Z}), \quad (20)$$

where

$$OZ = \sqrt{(\bar{a} \cdot \hat{Z})^2 + (\bar{b} \cdot \hat{Z})^2}, \quad \text{Tan } EZ = \frac{\bar{b} \cdot \hat{Z}}{\bar{a} \cdot \hat{Z}}. \quad (21)$$

Thus, it follows from setting the numerator of (18) equal to zero that the horizon plane (i. e. , the plane tangent to the earth at the site) intersects the orbit when E is given by either

$$E_1 = EZ + EC \quad (22)$$

or by

$$E_2 = EZ + 2\pi - EC, \quad (23)$$

where

$$EC = \cos^{-1} \left( \frac{r_e + e(\bar{a} \cdot \hat{Z})}{OZ} \right). \quad (24)$$

The first solution,  $E_1$ , indicates the last point where the satellite is visible before it sinks below the horizon, while  $E_2$  indicates the first point where the satellite becomes visible above the horizon (see Figure 5).

The dot product between  $\bar{0}$  and any general vector  $\bar{V}$  is given by (20) and (21), where  $\hat{Z}$  is now replaced by  $\bar{V}$ . Hence, in order to complete the description of range, elevation, and azimuth, it is necessary to find the vector dot products between the set  $\hat{Z}$ ,  $\hat{N}$ ,  $\hat{E}$ , and the orbit axes  $\bar{a}$  and  $\bar{b}$ . After combining (10), (11), and (12) with (13), (14), and (15), the desired products for  $\bar{a}$  become

$$\bar{a} \cdot \hat{Z} = C_1 \cos \lambda_s \cos \alpha + a \sin \lambda_s \sin \delta \sin \varphi, \quad (25)$$

$$\bar{a} \cdot \hat{E} = -C_1 \sin \alpha, \quad (26)$$

$$\bar{a} \cdot \hat{N} = -C_1 \sin \lambda_s \cos \alpha + \cos \lambda_s \sin \delta \sin \varphi, \quad (27)$$



where

$$\alpha = (\omega_e + \omega_n) t + \alpha_1 - \alpha_n - \text{Tan}^{-1} (\cos \delta \tan \varphi);$$

$\omega_e$  = angular velocity of the earth;

$\omega_n$  = rate of change of the right ascension of the ascending node due to earth's bulge;

$\alpha_1$  = right ascension of the site at  $t = 0$ ;

$\alpha_n$  = right ascension of the ascending node at  $t = 0$ ;

$\delta$  = inclination of the orbital plane;

$\varphi$  = argument of perigee;

$\lambda_s$  = latitude of site; and

$$C_1 = a \sqrt{\cos^2 \varphi + \sin^2 \varphi \cos^2 \delta}.$$

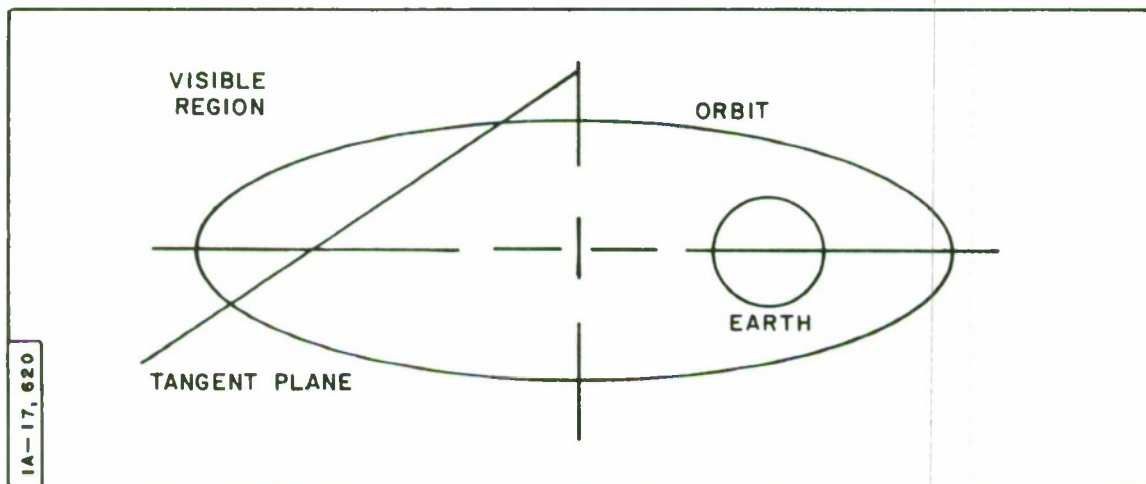


Figure 5. Region of Satellite Visibility



The corresponding equations for  $b$  can be developed by replacing  $\varphi$  by  $\varphi + \pi/2$  in the above equations.

In order to relate the radar orientation to the target orientation, it is convenient to relate the radar to the  $\hat{e}_1, \hat{e}_2, \hat{e}_3$  coordinate system or, more specifically, to specify the latitude and right ascension of unit vectors in the radar range direction  $\hat{r}$ , the radar horizontal  $\hat{H}$ , and the radar vertical  $\hat{V}$ . From inspection of Figure 4, the unit vector in the radar range direction is given by

$$\hat{r} = \sin \tau \hat{z} + \cos \tau (\cos \rho \hat{N} + \sin \rho \hat{E}). \quad (28)$$

From Equations (13), (14), and (15), it follows that the latitude of  $\hat{r}$  is given by

$$\sin \lambda_r = \hat{r} \cdot \hat{e}_3 = \sin \tau \sin \lambda_s + \cos \tau \cos \lambda_s \cos \rho, \quad (29)$$

and the right ascension by

$$\tan \alpha_r = \frac{\sin \tau \sin \alpha_s \cos \lambda_s + \cos \tau (\sin \rho \cos \alpha_s - \sin \lambda_s \cos \rho \sin \alpha_s)}{\sin \tau \cos \alpha_s \cos \lambda_s - \cos \tau (\sin \rho \sin \alpha_s + \sin \lambda_s \cos \rho \cos \alpha_s)}. \quad (30)$$

The latitude and right ascension of the radar vertical vector can be found by replacing  $\tau$  by  $\tau + \pi/2$  in (29) and (30), while the horizontal vector is defined by letting  $\tau = 0$  and  $\rho = \rho + \pi/2$ . In this case, the vectors  $\hat{r}$ ,  $\hat{V}$ , and  $\hat{H}$  form a right-handed system.

SECTION IV  
TARGET MOTION

In general, three Eulerian angles are necessary to specify the orientation of an earth-based radar with respect to the satellite. They are:

- (1) aspect angle, the angle between an axis fixed in the satellite (satellite axis) and the radar line-of-sight;
- (2) polarization angle, the angle which the projection of the satellite axis on the plane normal to the radar line-of-sight makes with the radar horizontal polarization axis; and
- (3) roll angle, the angle which denotes rotation about the satellite axis.

These angles are labeled  $\theta$ ,  $\beta$ , and  $\xi$ , respectively, in Figure 6.

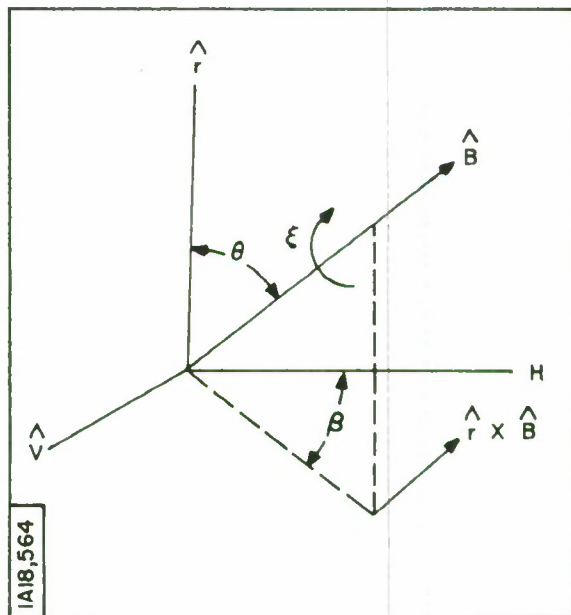


Figure 6. Euler Angle Geometry

$\hat{B}$  points in the direction of the satellite axis, and  $\xi$  is referenced from a unit vector in the  $\hat{r} \times \hat{B}$  direction to a fixed axis  $\hat{B}_x$  in the body perpendicular to  $\hat{B}$ . The horizontal axis  $\hat{H}$  is taken in the  $\hat{r} \times \hat{V}$  direction (i. e.,  $\hat{H}$  is pointing east when the radar is pointing north). It follows from the geometry of Figure 6 that these angles are given by:

$$\cos \theta = \hat{r} \cdot \hat{B}, \quad 0 \leq \theta \leq \pi; \quad (31)$$

$$\tan \beta = \frac{\hat{B} \cdot \hat{V}}{\hat{B} \cdot \hat{H}}, \quad 0 \leq \beta \leq 2\pi; \quad (32)$$

$$\tan \xi = \frac{\hat{B}_x \cdot \hat{r}}{\hat{B} \times \hat{B}_x \cdot \hat{r}}, \quad 0 \leq \xi \leq 2\pi. \quad (33)$$

In order to work out these relationships explicitly, the satellite motion about its center of gravity must be studied. The principal types of stabilized and unstabilized motion are summarized herein.

The six principal types of satellite stabilization are:

- (1) spin, the satellite is fixed in inertial space;
- (2) earth center, one axis of the satellite points toward the center of the earth;
- (3) earth horizon, the satellite is stabilized with respect to the local horizon;
- (4) magnetic, an axis of the satellite points along the lines of magnetic force (often the magnetic field is used for damping for earth center stabilization); and

- (5) inertial guidance, the satellite orientation is governed by some predetermined program or by signals from ground-based transmitters.

The four principal types of unstabilized motion are:

- (1) tumbling,
- (2) spin,
- (3) precession, and
- (4) precession with nutation.

By combining Equations (31) through (33) with the body motion equations for a particular satellite, the three orientation angles (aspect, polarization, roll) can be found explicitly. For example, consider an earth-centered stabilized body. In this case, the vector  $\hat{B}$  points along  $\bar{0}$  and consequently

$$\cos \theta = \frac{1}{r} \left( |\bar{0}| - r_e \sin \tau \right) \quad (34)$$

$$\beta = \frac{\pi}{2}. \quad (35)$$

The roll angle will depend upon the type of stabilization assumed for  $\hat{B}_x$ . One possibility for earth center stabilization would be to take  $\hat{B}_x$  in the direction to the unit vector normal to the orbital plane. In this case, from Equation (33)

$$\text{Tan } \xi = \frac{\hat{n} \cdot \hat{Z}}{(\bar{0} \cdot \hat{a}) (\hat{b} \cdot \hat{Z}) - (\bar{0} \cdot \hat{b}) (\hat{a} \cdot \hat{Z})}, \quad (36)$$

where  $\hat{n} = \hat{a} \times \hat{b}$ . Explicit formulas for the dot products indicated in (36) were presented earlier.

Another common method of stabilization is spin-stabilization. Here, the  $\hat{B}$  axis is considered as fixed in space. Taking  $\lambda_p$  and  $\alpha_p$  as the

latitude and the right ascension of the body axis, respectively (which corresponds to the angular momentum vector for spin-stabilized bodies), the aspect angle is given by

$$\cos \theta_{pr} = \sin \lambda_p \sin \lambda_r + \cos \lambda_p \cos (\alpha_r - \alpha_p). \quad (37)$$

Hence,  $\lambda_r$  and  $\alpha_r$  correspond to the latitude and right ascension of the radar in inertial coordinates. \* Similarly, the dot products in (32) for the angle  $\beta$  may be determined from (37) by replacing  $\lambda_r$  and  $\alpha_r$  by the corresponding latitudes and right ascension for the radar horizontal and vertical vectors.

Since the angular momentum vector is fixed in space for torque-free motion, the  $\theta_{pr}$  given by (37) is the angle between the radar and the angular momentum vector. Torque-free motion of an axially symmetric body results in precession about this vector. If we take  $\theta_p$  and  $\dot{\phi}$  as the precession cone angle and precession rate, respectively, then the radar aspect angle becomes

$$\cos \theta = \hat{B} \cdot \hat{r} = \cos \theta_{pr} \cos \theta_p + \sin \theta_{pr} \sin \theta_p \cos (\dot{\phi}t + \phi_0 - \phi_1), \quad (38)$$

where

$$\tan \phi_1 = \frac{\sin \lambda_r \cos \lambda_p - \cos \lambda_r \sin \lambda_p \cos (\alpha_r - \alpha_p)}{\cos \lambda_y \sin (\alpha_r - \alpha_p)} \quad (39)$$

and  $\phi_0$  is the initial location at time  $t = 0$  of the body axis on the precession cone. The particular cases of spin-stabilized or tumbling motion may be obtained by setting the precession angle equal to 0 or  $\pi/2$ , respectively.

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\* See Equations (29) and (30).

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The roll angle will depend upon the type of stabilization assumed for  $\hat{B}_x$ . One possibility for earth center stabilization would be to take  $\hat{B}_x$  in the direction to the unit vector normal to the orbital plane. In this case, from Equation (33)

$$\tan \xi = \frac{\hat{n} \cdot \hat{Z}}{(\bar{0} \cdot \hat{a}) (\hat{b} \cdot \hat{Z}) - (\bar{0} \cdot \hat{b}) (\hat{a} \cdot \hat{Z})}, \quad (36)$$

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\* See Equations (29) and (30).

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11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY Space Defense System Program Office (496L/474L), Electronic Systems Division, L. G. Hanscom Field, Bedford, Massachusetts	
13. ABSTRACT The relative motion between an earth-based radar and a satellite object is of interest for radar-tracking studies, signature simulation studies, and for developing inverse scattering techniques. Here, the motion analysis is accomplished with elementary vector algebra. This approach results in considerable simplification of the final equations and relationships.			

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
RADAR TECHNIQUES Tracking, Satellites Earth-Based Radar, Relative Motion vs Satellite MATHEMATICS Algebra, Vector Motion Analyses, Earth-Based Radar vs Satellite						

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