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THE DESIGN OF VESSELS FOR UNDERWATER ORDNANCE

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WHITE OAK, MARYLAND

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ABSTRACT: The more important theories and concepts involved in the analysis and design of vessels subjected to external pressure are reviewed and discussed. Using the analysis of one investigator as a foundation, an equation is formulated which is subsequently employed in the construction of charts for practical design. The charts will yield an optimum design with a safety factor of unity since they incorporate a parameter for out-of-roundness, a fabrication anomaly heretofore accounted for by the application of factors to classical perfect-shell theory. Ranges of design parameters and operating depths that will find primary usage in the field of underwater ordnance are covered. A typical design example of a pressure hull for an undersea weapon is presented in conclusion.

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This report reviews classical pressure vessel theory and employs it as a foundation for the development of charts for the practical design of underwater ordnance pressure hulls. The work was performed under Task Number NOL-NM-41-55. Mr. Marvin Burns, formerly with the Naval Ordnance Laboratory, aided the author in conducting literature surveys and in reviewing prior investigations. The opinions and data presented herein represent the conclusions of the Underwater Ordnance Department of the U. S. Naval Ordnance Laboratory

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I. INTRODUCTION

1. The analysis and design of vessels subjected to external pressure has received extensive consideration in classical literature since the latter part of the 19th century. The path pursued by the majority of investigators has been one based on the exact mathematical methods of the theory of elasticity. As a logical corollary to this, mathematical perfection of form of a vessel is always assumed. Factors which may considerably influence the collapse pressure of a vessel, in particular out-of-roundness, are disregarded. As a consequence, the practical designer, aware that perfection of form is impossible to achieve even with the most precise manufacturing process, will invariably abandon the rigorous path in favor of a short cut. The latter takes the form of rule-of-thumb formulas, empiricism, and application of a "factor of safety". The result is a design which, though always adequate, is virtually never optimum.

2. This approach is entirely satisfactory where the nature of the vessel application permits or warrants the inevitable overdesign resulting from use of large safety factors. The A.S.M.E. Boiler Code, for example, recommends a factor of four.

3. Operational requirements of modern deep-running underwater ordnance (as well as their targets) are such, however, that overdesign is undesirable. Particularly in instances where an item's tactical use requires that it be airborne, a minimum weight investment is imperative. It follows, then, that an optimum vessel design for ordnance should be based on a factor of safety of unity with no sacrifice of reliability. To achieve this, expressions or "factors" heretofore used to wed classical perfect-shell theory to practical fabrication must be discarded and a more exacting design procedure pursued.

4. It is the purpose of this report to outline and present the results of such a procedure, with particular application to a shell design problem encountered at the U. S. Naval Ordnance Laboratory during the development of an underwater weapon requiring great depth of operation. The results are summarized in the form of charts which cover ranges of geometrical parameters and operating depths most commonly applicable to similar weapons in order that future designs may be readily carried out. In addition, classical theory will be reviewed and a bibliography appended to facilitate further pursuit of the subject.

II. ANALYSIS

A. Stresses in Thin Shells

5. The discussion is limited to the case of thin-walled vessels wherein the outer diameter of the shell is large in comparison to the wall thickness. The usual requirement is that the thickness be less than one tenth the radius. For such shells under pressure, whether internal or external, the state of stress in the wall is triaxial with a practically uniform distribution throughout the thickness provided that no abrupt changes in thickness, slope, or curvature exist. At any point there is a hoop stress acting along the circumference, a meridional stress in the longitudinal direction, and a radial stress. The latter is small and usually neglected.

6. For any figure of revolution the hoop stress is given by

$$S_H = \frac{PR}{2t}\left(2-\frac{R}{r}\right) \qquad (1)$$

and the meridional stress by

$$S_M = \frac{PR}{2t} \qquad (2)$$

where R is the hoop radius of curvature and r the meridional. The wall thickness is t and the pressure P. Internal and external pressure are distinguished by assigning the proper sign to P so as to yield tensile stresses for the case of internal pressure and compressive for external.

7. For a cylinder, where the meridional radius of curvature is infinite, equation (1) reduces to

$$S_H = \frac{PR}{t} \qquad (1a)$$

indicating that for this figure hoop stress is twice as large as meridional stress.

8. For a sphere, where hoop and meridional radii are equivalent, equation (1) reduces to

$$S_H = \frac{PR}{2t} \qquad (1b)$$

indicating that for this figure hoop and meridional stresses are equal in magnitude.

9. Equations (1) and (2), then, define one method of failure of pressure vessels, namely yielding. The destructive pressure is seen to be a function only of the mechanical strength of the material and the thickness to diameter ratio of the shell. The equations apply equally to cases concerning internal or external pressure, in the former failure being a result of bursting and in the latter of crushing.

10. It is here, however, that any similarity between the two types of loading conditions terminates. Whereas for internal pressure a single stress equation may completely define a vessel design, for external pressure failure is generally not predictable by a single formula.

B. Elastic Instability of Thin Cylindrical Shells

11. A majority of ordnance applications involve shells of cylindrical geometry and it will be seen that this is the most critical case. For a given major diameter and wall thickness a vessel of double curvature is sturdier. One evidence of this is apparent from the previous section where, for the limiting case of a sphere, hoop stress was found to be half that for a cylinder under equivalent loading.

12. Vessels under external pressure are analogous in behavior to columns.

13. One example of this analogy can be found in the vessel of proper geometry, i.e., short and relatively thick-walled, which fails by crushing when the stress as defined by equation (1a) exceeds the compressive yield strength of the material. This is akin to the short stocky column under end compressive forces. It undergoes a reduction in length parallel to the line of force application and stresses, uniform across any section, are found simply by dividing load by cross-sectional area.

14. It will be noted that in the above paragraph an adjective pertaining to vessel geometry has been introduced which was not previously mentioned, the term "short". For external pressure applications, the length to diameter ratio of a vessel is equally as important as its thickness to diameter ratio.

15. For vessels having closely spaced bulkheads or stiffening rings collapse is governed by the stress equations of the previous section which are independent of L/D. As the distance between end closures is increased, the analogy approaches that of the long, slender column and length becomes an important parameter. As supports are further and further removed a point is reached where they exert no appreciable influence on the central portion of the shell and, once again, collapse is

independent of L/D. Vessels of this type are more truly termed pipes or tubes and the minimum length of vessel for consideration as such is designated as the "critical length". This quantity has been variously defined by almost every investigator and may be of the order of 6 to 8 diameters in length. Since underwater ordnance applications with such high L/D ratios are uncommon, pipes will not be considered herein.

16. Attention, now, will be confined to the vessel category where length is a determinative quantity, as it is in the slender column.

17. The latter, when subjected to end compressive loads, suffers a reduction in length through lateral bending and/or twisting rather than axial compression. Continued application of load results in progressively increasing deflections which in turn increase the bending moment arm and thus further increase deflection, etc., until the entire train of events is climaxed by sudden and dramatic collapse of the structure. Failure is a result of buckling or elastic instability and occurs at stress levels lower than those corresponding to the yield strength of the material. The minimum value of load that makes the structure unstable is called the critical load and is predictable from column formulas such as the well known Euler equation. It is a function only of the column's geometry and the elastic modulus of the material.

18. The behavior of vessels under uniform external pressure is entirely similar. Failure may occur due to instability of the shell at stress values, as computed from the hoop stress equation, which are low. The unsupported length of shell is significant not only in establishing the nature of failure (yielding or buckling), and the magnitude of the critical pressure, but also in determining the pattern of the characteristic buckles or lobes which form around the circumference at collapse. The number of these bulges is a function of the rigidity of the shell and the energy available for deformation, the minimum number consistent with the existing energy level always being formed. The least rigid shell, i.e., the long pipe or tube, for instance, is merely squeezed flat at mid-span. This constitutes a two-lobe collapse. As the unsupported length is decreased, shell rigidity is increasingly obtained, and, obviously, the energy requirements for deformation increase. The consequence of this is that buckle frequency increases but amplitude decreases, for it takes less energy to form many small bulges than to squeeze a shell flat.

19. The analogy between columns and shells is apparently two-fold, for not only may similes be drawn as to their behavior, but apparently there exist just as many equations defining shell instability as there are column formulas. The present text shall be limited to those of chief interest in the design of

underwater ordnance, i.e., those which include the effect of both radially and axially applied uniform pressure.

20. One of the earliest and possibly best analyses of shell instability which considers all the significant parameters is that of von Mises (1)*:

$$P_{cr} = \left[\frac{1}{3} \left\{ \left[n^2 + \left(\frac{\pi D}{2L} \right)^2 \right]^2 - 2\mu_1 n^2 + \mu_2 \right\} \frac{2E}{1-\mu^2} \left(\frac{t}{D} \right)^3 + \frac{2E\left(\frac{t}{D}\right)}{\left[n^2 \left(\frac{2L}{\pi D} \right)^2 + 1 \right]^2} \right] \frac{1}{n^2 - 1 + \frac{1}{2}\left(\frac{\pi D}{2L} \right)^2} \tag{3}$$

where $2\mu_1 = \left[1 + (1+\mu)\rho\right]\left[2 + (1-\mu)\rho\right]$

$$\mu_2 = (1 - \rho\mu)\left[1 + (1+2\mu)\rho - (1-\mu^2)\left(1 + \frac{1+\mu}{1-\mu}\rho\right)\rho^2\right].$$

$$\rho = \frac{1}{n^2\left(\frac{2L}{\pi D}\right)^2 + 1}$$

and

P_{cr} = critical (collapse) pressure

μ = Poisson's ratio

E = modulus of elasticity

n = number of lobes in buckled pattern

L = unsupported length of shell

D = mean diameter of shell (taken as outer diameter for thin shells with negligible error)

t = thickness of shell material

21. The von Mises formula has been adapted and simplified in one form or another by many subsequent analysts. The most noteworthy of these is the contribution of Windenburg and Trilling (2) at the U. S. Experimental Model Basin. Their approximation is independent of the number of lobes and it is claimed by the authors that it checks the von Mises equation very closely, the average deviation being about one percent.

*Numbers in parentheses correspond to references listed in bibliography.

$$P_{cr} = \frac{2.42\, E \left(\frac{t}{D}\right)^{5/2}}{(1-\mu^2)^{3/4}\left[\frac{L}{D} - .45\left(\frac{t}{D}\right)^{1/2}\right]} \qquad (4)$$

22. Sturm (3) (4) in an extensive work based on shell theory proposed by Donnell (5) developed a relationship of very simple form for collapse pressure:

$$P_{cr} = K\, E \left(t/D\right)^3 \qquad (5)$$

where K is a coefficient dependent on L/D; t/D; n; and the end conditions of the vessel.

23. As previously noted, the number of lobes appearing at collapse is explicitly related to vessel geometry and must be the minimum integer for each case. This plus the fact that n is therefore an implicit function of K, has made it possible for Sturm to prepare charts unique in that they present the variation of four quantities on a single coordinate system. For any vessel L/D and t/D the values of K and n may be readily determined. It is then a simple matter to determine the collapse pressure from equation (5). The charts cover all possible loading conditions. These include vessels with fixed or simply supported ends for pressure applied either axially, radially, or in both directions.

24. The A.S.M.E. Boiler Code (6) recommends the following conservative formulas:

$$P_{cr} = \frac{2E}{1.27\,(1-\mu^2)}\left(\frac{t}{D}\right)^3 \quad \text{for } \frac{t}{D} < 0.023 \qquad (6)$$

$$P_{cr} = 2.34\, S_y\left(\frac{t}{D}\right) - 1.06\, S_y\left(\frac{S_y}{E}\right)^{1/2} \quad \text{for } \frac{t}{D} > 0.023 \qquad (7)$$

where S_y is material yield strength and other terms as previously defined.

25. It will be noted that as thickness is increased beyond a certain value, and consequently there is a gain in shell

rigidity, there is greater tendency for failure to occur by yielding. This is evidenced by the introduction of the yield strength.

26. The preceding outlines some of the major contributions in the field of analysis and design of thin cylindrical shells under external pressure. Many more original works and extensions of previous investigations may be found in the literature. The appended bibliography is offered as a guide.

C. Elastic Instability of Thin Shells of Double Curvature

27. Spherical shells may also suffer an instability collapse and a number of formulas for the prediction of their critical pressure exist. The two most commonly employed are those of Timoshenko (7):

$$P_{cr} = \left[2.2\left(\frac{D}{t}\right)\right] \frac{2E}{1-\mu^2} \left(\frac{t}{D}\right)^3 \tag{8}$$

And Tsien (8):

$$P_{cr} = \frac{8E}{3\sqrt{5(1-\mu^2)}} \left(\frac{t}{D}\right)^2 \tag{9}$$

These may be used to check the stability of hemispherical bulkheads frequently used as end closures on pressure vessels.

28. Shells other than those of cylindrical or spherical geometry have escaped the wealth of attention directed toward the latter two. No proven formula for the critical pressure of odd-shaped vessels of double-curvature is known to exist.

29. It is usually the case in practical applications that a double-curvature section is joined to a cylindrical one and that manufacturing expediency dictates that the two shall be of equal wall thickness. For this condition, it has already been pointed out that stress wise the vessel having double-curvature is superior in strength to the cylindrical one. The hoop stress equations previously cited show that a factor indicative of increased strength will vary as does the meridional radius of curvature, reaching a maximum value of two for a sphere.

30. Similarly, it can be shown that stability-wise the shell of double-curvature is more rigid than a cylinder. Comparison may be made with the least rigid cylindrical vessel (the infinitely long pipe) whose equation for critical pressure is (9) and (10):

$$P_{cr} = \frac{2E}{1-\mu^2}\left(\frac{t}{D}\right)^3 \qquad (10)$$

31. It may be seen from examination of equations (8) and (10) that the spherical vessel has a critical pressure greater than that of a pipe by a factor of 2.2 $\left(\frac{D}{t}\right)$.

32. It is thus evident that application of double-curvature to a design based on a simple cylinder will provide an increase in collapse pressure from both the standpoints of yielding and instability.

D. Limitations in the Application of Perfect Shell Theory

33. Thus far it has been presumed that the perfect shell has been complemented by the perfect material and that vessel collapse occurs by either of two distinct modes of failure; yielding or elastic instability. However, failure may also occur as a combination of both plastic flow and instability.

34. When a material is stressed beyond its elastic limit its usefulness as a load-carrier is by no means at an end, yet, at this point, it no longer follows Hooke's law. The proportionality between stress and strain is no longer the constant Young's modulus which has appeared in all the stability equations. The reduction in value of the elastic modulus progresses as the plastic region is increasingly penetrated.

35. The existing expressions for collapsing pressure may, however, be applied to shells stressed beyond the elastic limit if a suitable "effective modulus" is substituted for Young's modulus.

36. Sturm (3) has proposed three values for an effective modulus, each applicable to a particular stress level. His three cases are: (1) the average stress is between the elastic limit and the yield strength, (2) the average stress is between the yield strength and the ultimate strength, and (3) the average stress is below the elastic limit but because of eccentricities, the maximum stress in the shell exceeds it.

37. Sturm's third case is considered most applicable to designs where the maximum stress developed in the shell will be limited to the yield strength of the material. The equation for reduced modulus, E', under these conditions is:

$$E' = E\left[1 - \frac{1}{4}\left(\frac{S_T - S_e}{S_u - S_H}\right)^2\right] \tag{11}$$

where S_T is the maximum stress in the shell, S_e and S_u the elastic limit and ultimate strength of the material respectively, and the remaining terms as previously defined.

38. The major limitation of perfect shell theory stems from the fact that the perfect shell is virtually nonexistent. It has been observed repeatedly that test results, even on ostensibly identical vessels under equivalent conditions, are rarely duplicated and show considerable departure from perfect shell theory.

39. Lack of perfection in a shell may be due to any of numerous fabrication irregularities such as surface discontinuities, nonhomogeneity and anisotropism of material, and out-of-roundness.

40. In a stress analysis it is possible to accurately account for many factors which cause departure from the perfect structure. A stability calculation, on the other hand, since it does not represent the physical process actually taking place, makes inclusion of all irregular influences an impossibility. Thus, perfect shell theory substitutes for the actual conditions a greatly idealized limiting case and the uncertainty of the process is reckoned with through the application of a "safety factor".

41. The term "safety factor" when applied to a stability calculation is in reality a misnomer, for, to the machine designer, it is truly defined as the ratio of the strength of a material to the maximum calculated stress. It is obvious that when any of the perfect shell theories are applied no such concept enters the mind of the vessel designer. The factor he employs to transform theory into hardware is more aptly termed an "ignorance factor".

42. In machine design the use of a safety factor is standard practice and is based not only on manufacturing anomalies but on cost of replacement and possible damage to lives and property in the event of failure.

43. In underwater ordnance design, where operation is usually a one-time proposition, cost of replacement is no factor.

Damage is the ultimate goal, and the safety of personnel, at least from the standpoint of merely the pressure vessel, does not enter the picture. Personnel, that is, personnel whose "safety" it is desirable to perpetuate, are not supposed to be present when the structure is in its intended environment of criticality — great depths of sea water. Minimum weight investment is invariably of cardinal importance and this decrees that use of factors, whatever they be called, be kept to a minimum.

44. The most commonly encountered imperfection in pressure vessels, one which has marked effect on lowering their strength, is eccentricity, or out-of-roundness. It is the initial deviation of a circular cross-section of the structure from the ideal geometrical shape.

45. Out-of-roundness will occur even with the most precise manufacturing techniques. It must be "lived with" after fruition of a design, hence it is proper that it be accounted for during its conception. If this is done, and a rigid inspection for material flaws and fabrication irregularities is carried out, the ultimate product will be the optimum.

E. The Out-of-Round Shell

46. Two of the few contributions on the subject of eccentric vessels are those of Sturm (3) and Holt (11).

47. The latter is actually based on the work of the former and is primarily intended for modification of the A.S.M.E. Boiler Code. This has allowed Holt to incorporate in his analysis a factor proposed by the Boiler Code which limits vessel out-of-roundness to that value which reduces critical pressure to 80 percent of that of the corresponding perfect shell. Ensuing manipulations result in development of a parameter, Q, which is ostensibly only a function of vessel geometry and universally applicable to all materials. This is finally used in an expression relating out-of-roundness to vessel geometry, operating pressure, and material strength:

$$\Delta R = tQ\left(\frac{3S}{P}\,\frac{t}{D} - 1\right) \qquad (12)$$

where ΔR is the maximum deviation of the radius from that of a true circle, P is the design pressure, S the design stress, Q a parameter which a function only of L/D, and the other terms as previously defined.

48. It will be noted that the material property of determinative influence in stability calculations, the elastic modulus, is absent. In reality, its influence is concealed in the "universal" parameter Q.

49. Consider the hypothetical example wherein two vessels, identical in geometry (including out-of-roundness) and made of materials identical in mechanical strength but having different elastic moduli, are investigated for collapsing pressure. It will be found that results from the original Sturm equation vastly differ from those obtained from equation (12). Obviously, for a fixed geometry and material strength the latter will yield equivalent collapse pressures whether the vessel has the high elastic modulus of steel or the low one of aluminum.

50. It may be concluded, then, that Holt's work is limited in scope to finding the singular value of out-of-roundness for a vessel which will reduce its critical pressure to 80 percent of that for a perfect shell. Any attempt to apply it to calculate collapse pressures for arbitrary values of out-of-roundness will prove meaningless.

51. The original equation for out-of-round vessels from Sturm (3) is:

$$P'_{cr} = \frac{2S\left(\frac{t}{D}\right)}{1 + 4\frac{\Delta R}{t}\left[\frac{E\left(n^2 - 1 + \mu\frac{\pi R^2}{L^2}\right)\left(\frac{t}{D}\right)^3}{(1-\mu^2)(P_{cr} - P'_{cr})}\right]} \tag{13}$$

where P'_{cr} is the collapse pressure for the out-of-round shell, P_{cr} the collapse pressure for the corresponding perfect shell, S the stress level existing at collapse, and the other terms as previously defined.

52. It will be modified slightly to improve its manipulatability and applicability to underwater ordnance design, and then used as a foundation for the construction of vessel design charts. It is apparent from examination of the equation that to arrive at solutions for P'_{cr} or t/D directly would involve tedious trial and error.

53. Before proceeding, however, it would be well to return to column analogy for the equation of Sturm bears what is perhaps the most vivid of all similarities to a column formula.

. It may be compared to the secant formula for eccentrically loaded columns:

$$F_{cr} = \frac{aS}{1 + \frac{ec}{\rho^2} \sec \frac{L}{2\rho} \sqrt{F_{cr}/aE}} \qquad (14)$$

here F_{cr} is the critical value of axially applied force, a the cross-sectional area, c the distance to the ventral axis, the length of column, ρ the radius of gyration of the cross-section about the ventral axis, e the eccentricity, and the remaining terms as previously defined.

5. The similarity between equations (13) and (14) is readily apparent. The critical load appears in both the left and right hand sides making trial and error or graphical means the most feasible approach for solution. For the perfect column, $e = 0$, equation (14) has two solutions corresponding to the two modes of failure possible: $F_{cr} = aS$ (direct compression), and secant $\frac{L}{2\rho}\sqrt{F_{cr}/aE} = \infty$ which reduces to Euler's equation for a perfect column: $F_{cr} = \pi^2 aE/(L/\rho)^2$ (instability). For the perfect shell, $\Delta R = 0$, equation (13) also has two solutions: $P'_{cr} = 2S(t/D)$ (hoop compression), and $P'_{cr} = P_{cr}$ (critical pressure for instability of perfect shell).

56. The effect of eccentricity in a shell is, as in a column, to introduce bending in addition to direct stress. The maximum stress is therefore the sum of the two. Sturm gives as the maximum direct stress the hoop stress S_H, as defined by equation (1a), and as the maximum bending stress resulting from an initial out-of-roundness ΔR:

$$S_B = \frac{Et}{2(1-\mu^2)} \; \frac{P'_{cr}\,\Delta R}{P_{cr} - P'_{cr}} \left(\frac{n^2-1}{R^2} + \mu \frac{\pi^2}{L^2} \right) \qquad (15)$$

57. Equation (15) is based on the flexure formula which assumes that stress varies directly with distance from the neutral axis and is not valid past the elastic limit. For ductile materials failure occurs in the plastic range and the actual stress in a beam is not that defined by $S = Mc/I$ but some lower value. It follows that greater ultimate and yield strengths may be developed in flexure than in simple stress. Beyond the elastic limit the flexure formula may be considered an empirical relationship used to define the maximum bending strength, or so-called modulus of rupture, of a beam. The modulus of rupture has been suggested to be 1.5 times greater than the yield strength for ductile materials (12).

58. From the preceding it may be concluded that the "maximum bending stress as defined by Sturm using the flexure formula is too high by a factor of 1.5 or that

$$S'_B = \frac{2}{3} S_B \qquad (15a)$$

where S'_B is the true bending stress due to application of maximum moment, and S_B is defined by equation (15).

59. It may be pointed out that Holt (11) applies a 20 percent apparent strength increase factor to total stress (sum of S_H and S_B). This is approximately equivalent to the application of the 1.5 factor to only the bending component for the condition of equal stress levels in both bending and direct stress. It is felt, however, that the latter procedure is more correct since apparent increases in strength are limited to flexural loading.

60. If the sum of the hoop stress, equation (1a), and the bending stress, equation (15a), is assigned as a maximum value the yield strength of the material (presuming this to be the limit of its usefulness), then an expression defining the critical pressure for an out-of-round vessel is obtained that is similar to that of Sturm:

$$P'_{cr} = \frac{2 S_y \left(\frac{t}{D}\right)}{1 + \frac{8}{3}\frac{\Delta R}{D} \frac{E\left(n^2 - 1 + \mu \frac{\pi^2 R^2}{L^2}\right)\left(\frac{t}{D}\right)^2}{(1-\mu^2)(P_{cr} - P'_{cr})}} \qquad (16)$$

61. The difference between equations (13) and (16) lies in a factor of 2/3 appearing in the denominator of the right-hand side as a result of the increased flexural strength, and in use of the dimensionless ratio $\Delta R/D$ rather than $\Delta R/t$. This has been adopted since in most design problems wall thickness is the unknown quantity to be solved for, and a nominal outer diameter is usually specified.

62. Equation (16) may be modified to facilitate computation as follows:

let

$$\phi = \frac{8\left[n^2 - 1 + \frac{\mu \pi^2}{4\left(\frac{L}{D}\right)^2}\right]\left(\frac{t}{D}\right)^2}{3(1-\mu^2)} \qquad (17)$$

and $$P_y = 2\ S_y\ t/D \qquad (18)$$

Py is the pressure at which hoop stress equals the yield strength of the material. Substituting equations (17) and (18) in equation (16) and solving for $\Delta R/D$:

$$\frac{\Delta R}{D} = \frac{(P_y - P'_{cr})(P_{cr} - P'_{cr})}{\phi\ E\ P'_{cr}} \qquad (19)$$

63. Presentation of equation (16) in the form of equation (19) makes the result of reducing eccentricity to zero immediately apparent. The two solutions corresponding to the two modes of failure of perfect shells are: $P'_{cr} = P_y$ (hoop compression), and $P'_{cr} = P_{cr}$ (instability collapse).

64. For the materials commonly used in pressure vessel fabrication Poisson's ratio may be assigned the constant value 0.3. The quantity ϕ, therefore, is a function only of t/D and L/D since n, it will be recalled, is a function of the same parameters. P_{cr} is defined by Sturm's equation for the perfect shell, equation (5).

65. Vessels used as pressure hulls for underwater ordnance are subjected to axial and radial loading and the worst case for end conditions is that of simple support. The appropriate chart from Sturm for these conditions, reproduced herein as Figure 1, may be used to define K and n for specified values of t/D and L/D. These in turn, with equations (5) and (17), are employed to prepare charts showing the variation of P_{cr}/E and ϕ with t/D and L/D, as are shown in Figures 2 and 3 respectively.

66. For a vessel of known proportions and of known material (E and S_y defined), Figures 1, 2, and 3 and equations (18) and (19) may be utilized to arrive at many quantities of interest. The critical pressure for instability failure of the perfect shell, the pressure required for failure in direct stress, the maximum allowable out-of-roundness for any operating pressure, and the number of lobes to be expected at collapse are all readily deduced. In addition, equation (19), when used in conjunction with Figures 2 and 3, lends itself to more practicable computation by trial and error of the critical pressure for a vessel of specified out-of-roundness.

67. Out-of-roundness of a pressure vessel is significant not only in magnitude but in orientation as well. It will be recalled that the buckled pattern of a shell at collapse is predetermined by its proportions. Any initial embryonic lobular formations corresponding to the inherent collapse pattern will have maximum effect in precipitating early failure. Observed initial deviations

from the round to be applied in equation (19) should therefore be limited to the worst reading over a specific arc or chord length that corresponds to the natural collapse pattern. Holt (11) presents a chart for determination of this arc or chord length as a function of t/D and L/D. It is reproduced in the present report as Figure 4.

F. Reinforcing Rings

68. For vessels shorter than the critical length, the parameter L/D enters into stability equations. The term L is defined as the length of shell between transverse or circumferential supports. These supports may take the form of either stiffening rings or the actual end closure or bulkhead. Their function is to preserve the circular form of the hull so that shell failure will not occur prematurely, and they must therefore possess adequate rigidity. When a bulkhead is used, reinforcement is automatic and the section will invariably be more rigid than required. When a ring or flange is used as a support, its required moment of inertia must be determined.

69. Christensen (13) presents the following formula defining the required moment of inertia of a stiffening ring in terms of shell geometry and operating pressure:

$$I_r = LD^3\left[\frac{(1-\mu^2)}{24}\left(\frac{P}{E}\right) - \frac{1}{12}\left(\frac{t}{D}\right)^3\right] \qquad (20)$$

III. CHARTS FOR DESIGN

70. In section (e) of the Analysis the various quantities that could be readily deduced for a known shell using Figures 1, 2, and 3 and equations (18) and (19) were enumerated. When the situation is reversed, i.e., some function of the geometry of a shell of specified out-of-roundness is desired, it is apparent that a calculation involving successive trials must be resorted to. For example, if t/D were the unknown for all other quantities known, successive estimates of t/D would define successive values of P_{cr}, ϕ, and P_y to be tested in equation (19) until ultimately the one value of t/D that satisfies the equation is found.

71. To circumvent this, charts have been prepared that are primarily intended for the designer of hulls for underwater ordnance.

72. Since required depth of operation is usually specified for an undersea weapon, the charts show pressure in terms of feet of sea water. A depth range of 500 to 3000 feet is covered in intervals of 500 feet. At each depth the variation of t/D with ΔR/D is indicated for the specific values of L/D that have found most common usage in this type of application.

73. Two materials finding most common acceptance in the fabrication of vessels for underwater operation are the SAE 4 --- series of steels and 61S aluminum, usually in the T-4 condition. The design charts are based on the use of these materials.

74. The design charts for steel vessels are presented as Figure 5, those for aluminum vessels as Figure 6.

75. In construction of the design charts, yield strengths were taken as 60,000 psi and 21,000 psi as typical values for the steel (14) and aluminum (15) respectively.

76. Structures under combined stress, such as the vessels under consideration, display an increased yield strength over that in simple stress, as predicted by the distortion-energy theory. This increase is of the same order of magnitude as the difference between the yield point and the elastic limit for the materials considered. Since the yield strengths used in construction of the charts are those in simple compression, it was felt that, so long as the maximum stress under combined loading was limited to these values, linearity of the stress-strain curve would be maintained. Hence, the values for elastic modulus used in chart construction were Young's modulus, 30×10^6 psi and 10×10^6 psi for the steel and aluminum respectively.

77. Since the design charts take into account out-of-roundness, the primary motivation for application of a design factor, they are, of course, based on a "safety" factor of unity. The operating depths indicated are the critical depths for a vessel of geometry defined by a point on a curve. Conversely, for any specified operating depth, a point on a curve defines the maximum allowable L/D and ΔR/D and the minimum allowable t/D.

IV. A DESIGN PROBLEM

78. The following is a review of an actual design problem adjunctive to the general development program of an undersea weapon at the U. S. Naval Ordnance Laboratory.

A. Statement of Problem

79. An underwater weapon case, whose tactical use requires that it be airborne and therefore of minimum weight and limited outer diameter and length, is to be designed. The following is specified:

a. Geometry. The case is to consist of a cylindrical mid-section whose outer diameter must not exceed 31.5 inches and whose length, free of any protrusions on the inner surface due to the nature of internal components, shall be equal to one diameter. Attached to the mid-section on one end will be an ogival forward section of approximately the same length and whose wall thickness is permitted to be equal to that of the mid-section. At the other end of the mid-section, closure shall be effected by means of a hemispherical bulkhead.

b. Operating Depth. It is desired to investigate designs for operation at both 2500 and 3000 feet of sea water.

c. Materials. In the interest of possible weight saving, aluminum, in addition to steel, is to be investigated as a material for construction.

d. Fabrication. Since the weapon case shall be the carrying vehicle for costly internal components, as near perfect a shell as possible will be the goal, with 100 percent inspection to follow.

B. Execution of Design

80. a. Mid-section. The restriction that the inner surface of the shell be "clean" means that no reinforcing rings may be used within the midsection and therefore the unsupported length equals the outer diameter:

$$L = D = 31.5 \text{ inches}; \quad L/D = 1$$

Consideration of manufacturing techniques and discussions with fabricators of precision vessels established the fact that, for diameters of this order of magnitude, concentricity can be maintained to ± 0.015 inch on the diameter. The worst possible case of out-of-roundness will be that within the appropriate arc length specified by Figure 4, both the high and low sides of the tolerance will be encountered and therefore:

$$\Delta R = 0.015 \text{ inch}; \quad \Delta R/D = 0.000476$$

Using the appropriate design charts, the following may now be obtained for the defined values of L/D and $\Delta R/D$:

steel at 2500': $t/D = 0.01233$; $t = 0.388$ in.
steel at 3000': $t/D = 0.01380$; $t = 0.435$ in.
aluminum at 2500': $t/D = 0.02811$; $t = 0.886$ in.
aluminum at 3000': $t/D = 0.03327$; $t = 1.048$ in.

The ratio of weight of the aluminum shell to weight of the steel shell will be in direct proportion to the product of wall thickness and material density. Using 0.1 lb./cu.in. as the density of aluminum and 0.284 lb./cu.in. as that of steel it is found:

$$\frac{\text{wt. of aluminum mid-section}}{\text{wt. of steel midsection}} \text{ at } 2500' = 0.80$$

$$\frac{\text{wt. of aluminum mid-section}}{\text{wt. of steel mid-section}} \text{ at } 3000' = 0.85$$

It may be noted that for the same increase in operating depth a greater percentage increase in wall thickness (and therefore weight) is required for the aluminum than for the steel vessel.

b. Forward Section. Since the ogival forward section will be of the same wall thickness as the mid-section and of the same unsupported length, it will be superior in strength to the mid-section because of its double-curvature. No calculation is necessary.

c. Bulkhead. The bulkhead is to be made of the same material as the remainder of the vessel. It is probable that holes will be cut in it at sometime for insertion of components. It is desirable, therefore, that the blank be considered in the light of this eventuality. The effect of holes will be to act as stress-raisers. Using equation (1b) and applying a stress-concentration factor of 1.5 as reported in reference (16) it may be found that:

steel bulkhead at 2500': $t = 0.220$ in.
steel bulkhead at 3000': $t = 0.263$ in.
aluminum bulkhead at 2500': $t = 0.627$ in.
aluminum bulkhead at 3000': $t = 0.750$ in.

A check for stability of the bulkhead may be made using equation (9). For the steel bulkhead of 0.220 inch thickness it is found that:

$$P_{cr} = 1829 \text{ psi.}$$

This is considerably greater than the corresponding pressure at either the 2500' depth or at 3000'. The latter case, therefore, need not be checked. Similarly, for the aluminum bulkhead it is found that the minimum critical pressure is:

$$P_{cr} = 4953 \text{ psi.}$$

It may be concluded that here, too, both bulkhead designs are quite safe in stability.

d. Reinforcing Ring. The flanging used to connect the forward section to the midsection will have to act as a reinforcing ring for the latter, since the mid-section was designed on the basis that one end of its unsupported length coincides with the juncture of the two pieces. Equation (20) may be used to specify the minimum moment of inertia required for this flanging:

$$\begin{aligned} \text{Steel at } 2500'&: \quad I_r = 1.231 \text{ in}^4. \\ \text{Steel at } 3000'&: \quad I_r = 1.447 \text{ in}^4. \\ \text{Aluminum at } 2500'&: \quad I_r = 2.343 \text{ in}^4. \\ \text{Aluminum at } 3000'&: \quad I_r = 1.969 \text{ in}^4. \end{aligned}$$

Unlike the steel hull, the aluminum hull requires flanging of lesser moment of inertia at 3000 feet than at 2500 feet. This is due to the increased shell rigidity provided by the greater percentage increase in wall thickness required at the lower depth in aluminum over steel (as previously noted). The thickness to diameter ratio appears to the cube power as a subtractive term in equation (20).

e. Number of Lobes. The number of lobes to be expected at collapse is determined from Figure 1:

for steel mid-section: $n = 5$
for aluminum mid-section: $n = 4$

C. Recapitulation

81. For the following fixed conditions:

$L = D = 31.5$ inches for the mid-section
Minimum S_y for steel = 60,000 psi
Minimum S_y for aluminum = 21,000 psi
Maximum out-of-roundness of mid-section = ± 0.015 inch on diameter.

the following quantities have been defined:

	2500'		3000'	
operating (crit.) depth	2500'		3000'	
Material	Steel	Alum.	Steel	Alum.
midsection: L/D	1		1	
midsection: max. $\Delta R/D$	0.000476		0.000476	
midsection: min. t/D	0.01233	0.02811	0.01380	0.03327
midsection: min. t (in.)	0.388	0.886	0.435	1.048
midsection: no. of lobes	5	4	5	4
midsection: wt. ratio: al./stl.	0.80		0.85	
bulkhead: min. t (in.)	0.220	0.627	0.263	0.750
ring: min. I_r (in.^{4})	1.251	2.343	1.447	1.969

D. Inspection

82. No matter how tight the tolerances, how thorough the control of material, and how precise the manufacturing process, a design in terms of hardware is very often not a reflection of what appeared on the drafting board. However, inspection and evaluation may prove that many of the anomalies acquired during fabrication are tolerable. Some may even cancel each other in effect.

83. The latter can well be the case for pressure vessels for it has been seen how the many parameters entering into their behavior are interrelated. An inspection program should take this into account, and, to better do so, a guide establishing limits on quantities that may vary from the specified would be helpful.

84. The above can best be illustrated by using the design example just presented. For the weapon case designed the following quantities may very well turn out to be other than as specified initially: t/D, $\Delta R/D$, and S_y. (Any variation in L/D due to the usual manufacturing tolerances will prove insignificant.)

85. A chart intended for the inspection of the mid-section of the vessel just designed has been prepared and is presented as Figure 7. It shows the allowable variation of $\Delta R/D$ with t/D for various values of steel yield strength at each of the operating depths considered. Any number of fortuitous combinations of the parameters involved will yield a vessel that meets operational requirements. The inspector may measure wall thickness, check out-of-roundness, convert a hardness reading to yield

strength, and he is ready to consult the chart after forming the proper dimensionless ratios. It will be recalled that when he is measuring out-of-roundness his readings should be confined to the proper arc lengths. He should be provided with a copy of Figure 4.

86. Figure 7 is for the design in steel. Heat treatment and cold-working that occur in steel fabrication make a wide range of yield strengths possible for a given alloy. Aluminum differs in that once an alloy and temper are specified the yield strength is specified. The design in aluminum was based on use of 61S-T4 and the yield strength is therefore fixed at 21,000 psi. The remaining variables are t/D and ΔR/D. The design charts for aluminum vessels are therefore the inspection charts as well.

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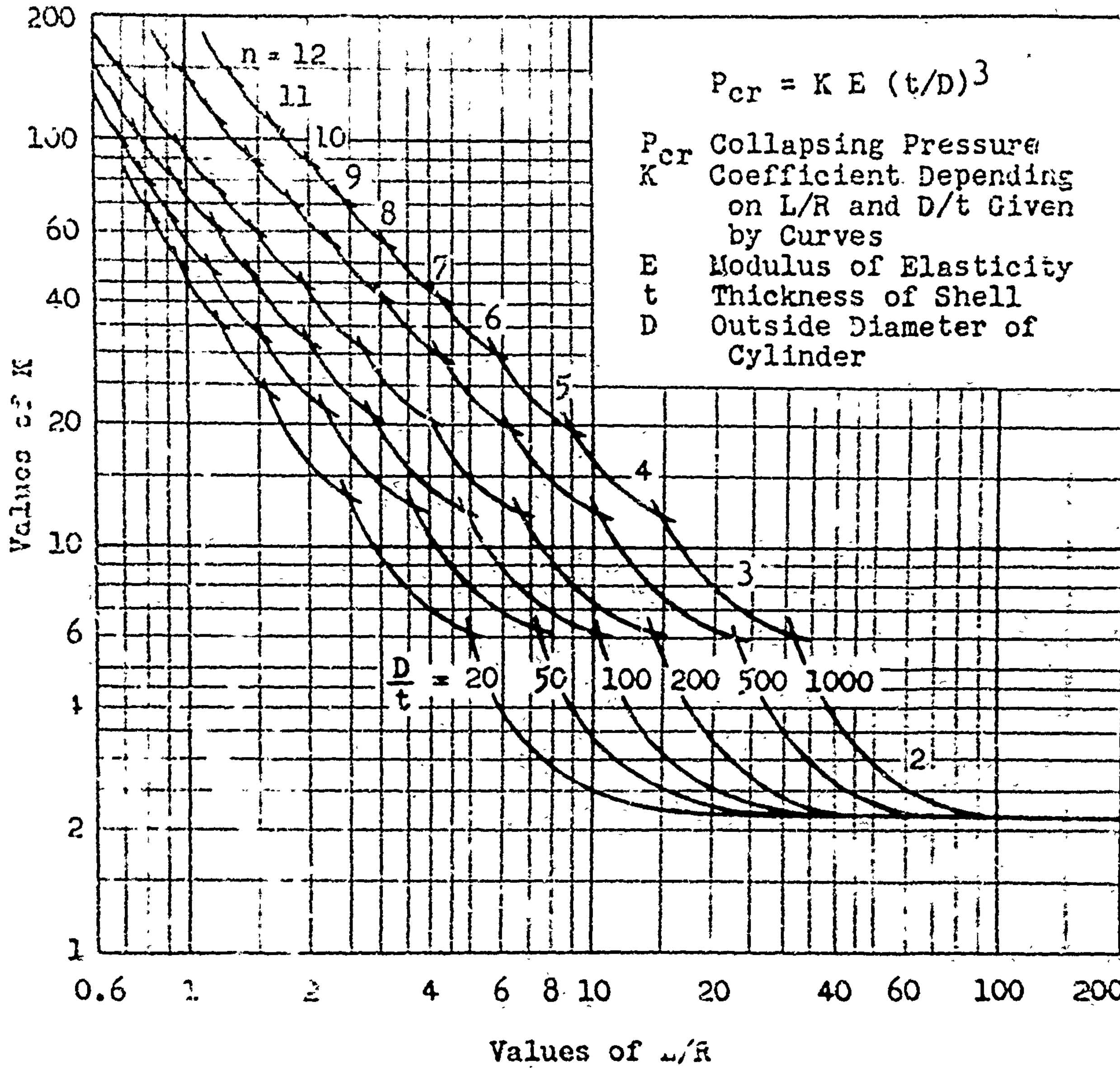

FIG. 1 COLLAPSE-COEFFICIENTS. ROUND CYLINDER WITH PRESSURE ON SIDES AND ENDS, EDGES SIMPLY SUPPORTED; μ = 0.30.

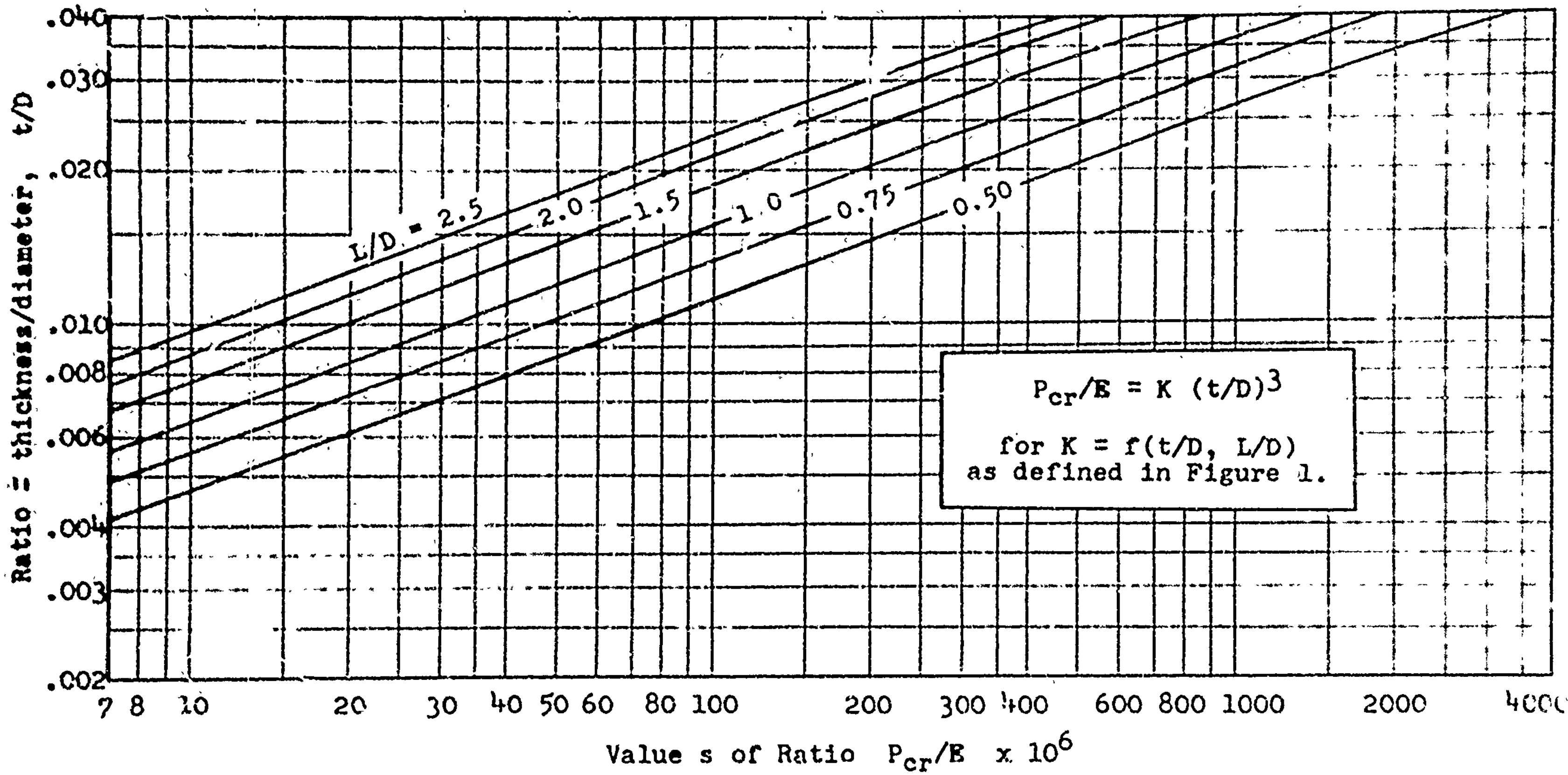

FIG. 2 VARIATION OF DIMENSIONLESS COLLAPSE PRESSURE RATIO P_{cr}/E WITH t/D FOR FIXED VALUES OF L/D; ROUND CYLINDER WITH PRESSURE ON SIDES AND ENDS, EDGES SIMPLY SUPPORTED.

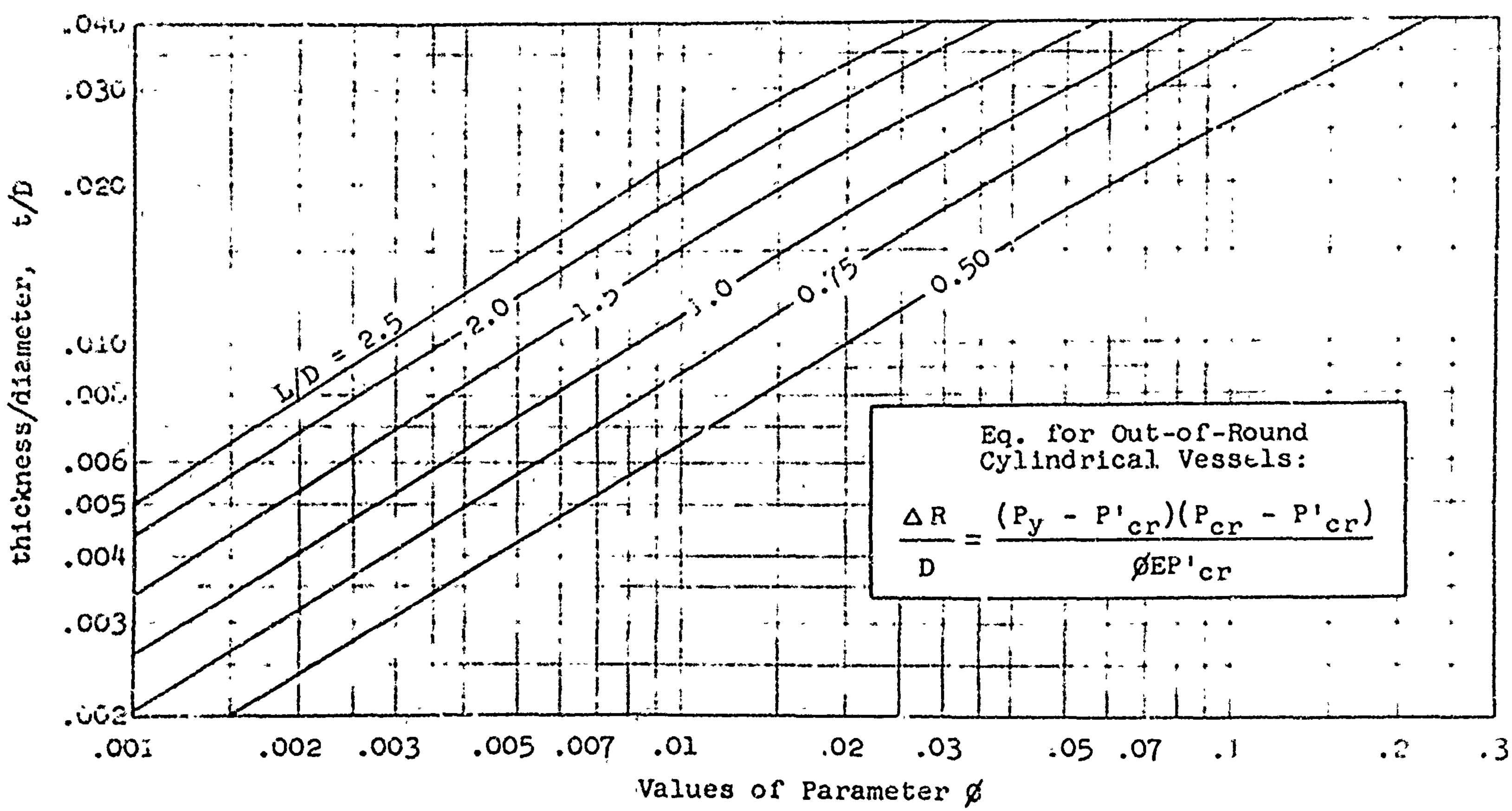

FIG. 3 VARIATION OF PARAMETER ø IN OUT-OF-ROUND VESSEL EQUATION WITH t/D FOR FIXED VALUES OF L/D.

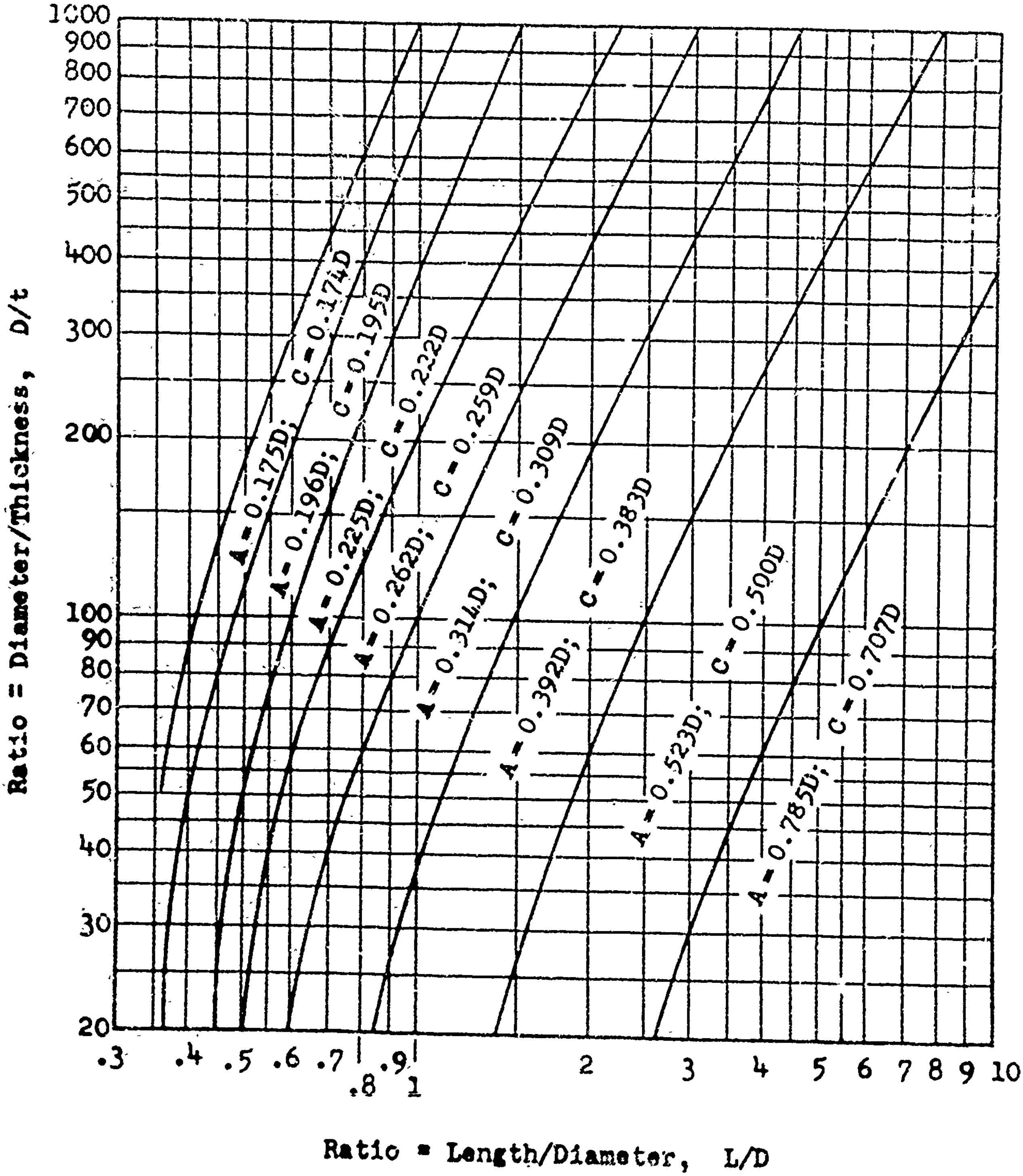

FIG. 4 CHART FOR DETERMINING ARC LENGTH OR CHORD LENGTH OVER WHICH OUT-OF-ROUNDNESS IS TO BE MEASURED.

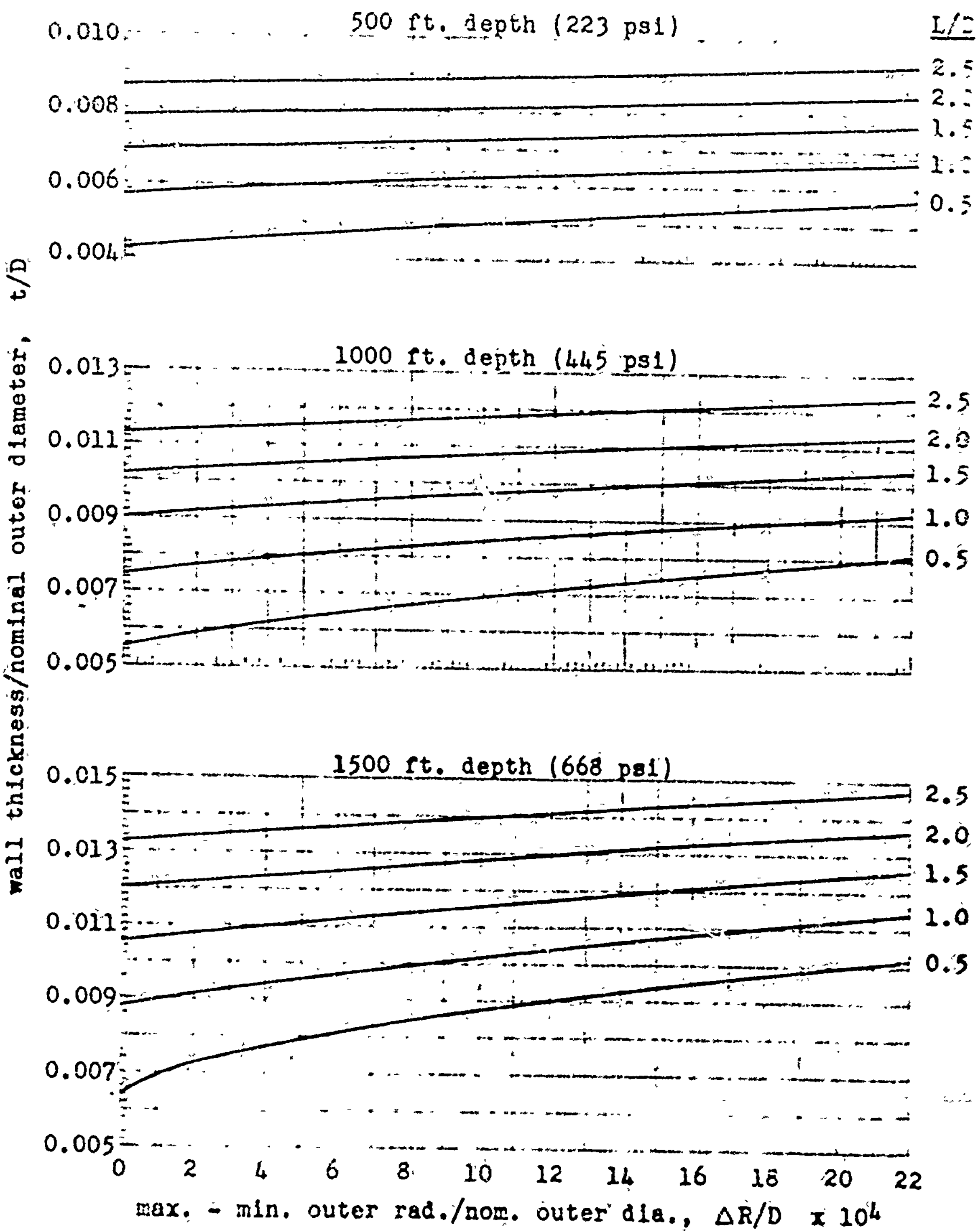

FIG. 5 DESIGN CHARTS. CYLINDRICAL VESSELS OF KNOWN OUT-OF-ROUNDNESS SUBJECTED TO SPECIFIED DEPTHS OF SEA-WATER ON SIDES AND ENDS WITH EDGES SIMPLY SUPPORTED. BASED ON USE OF STEEL: $E = 30 \times 10^6$ PSI, $S_y = 60,000$ PSI.

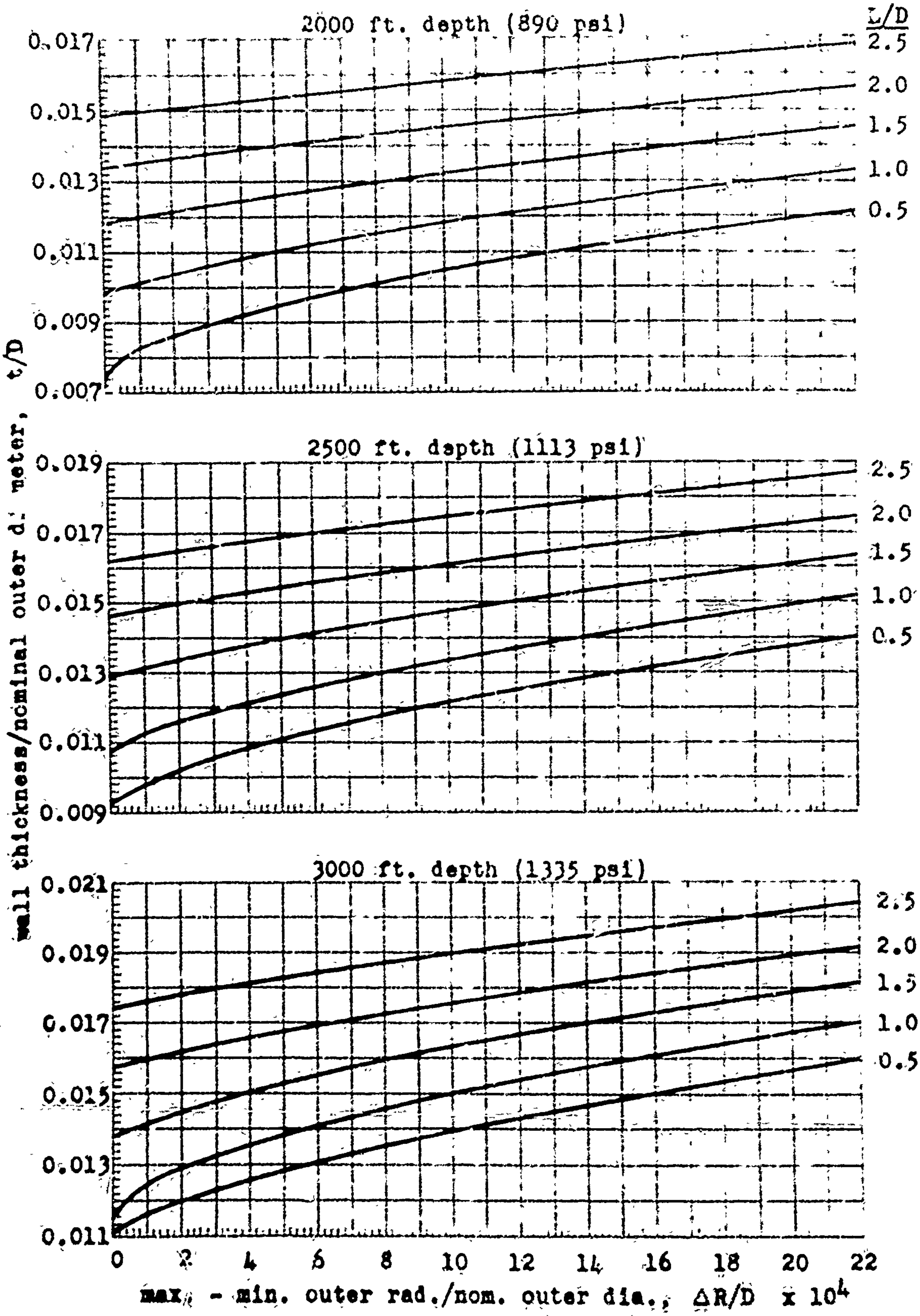

FIG. 5 CONCLUDED.

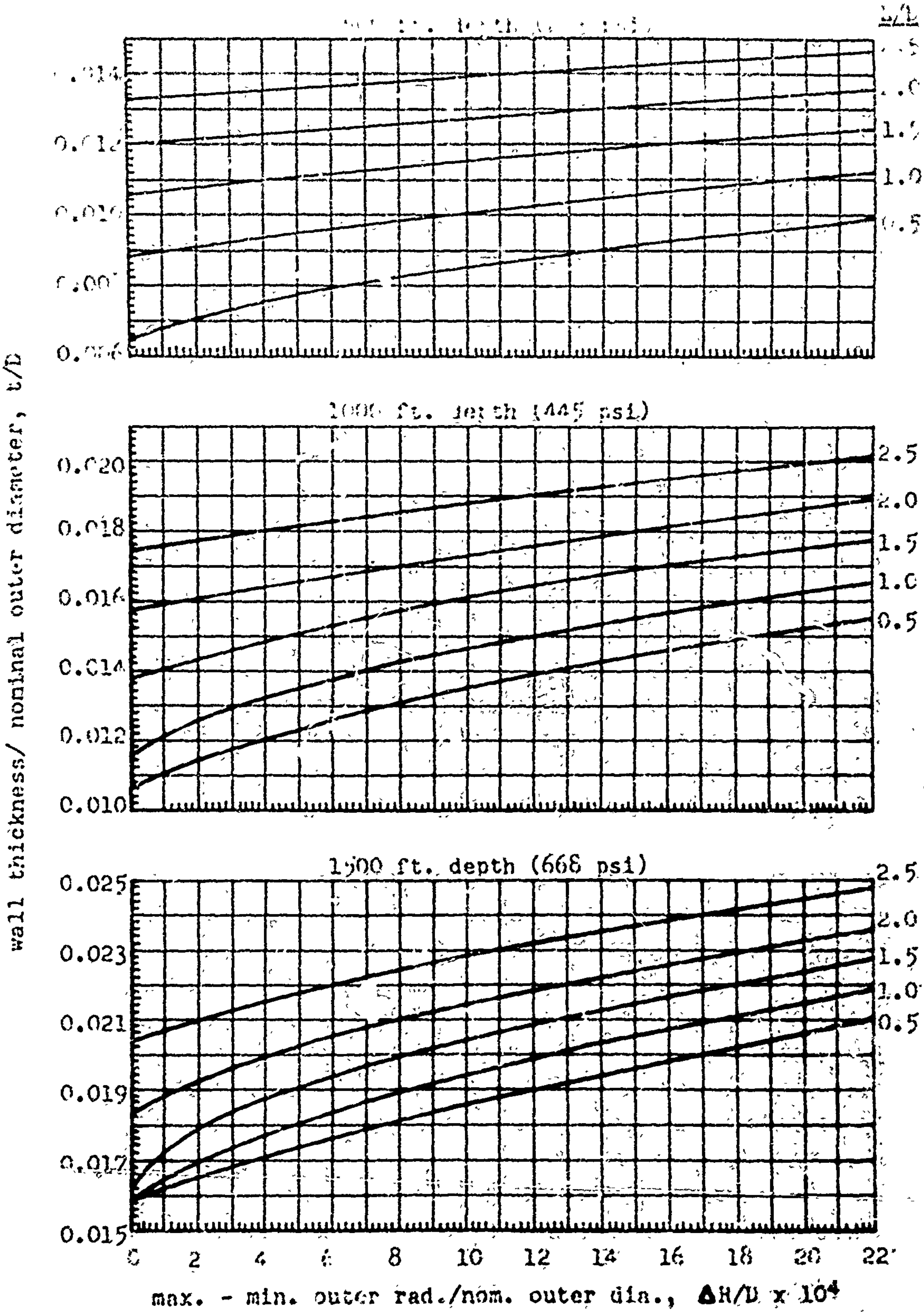

FIG. 6 DESIGN CHARTS. CYLINDRICAL VESSELS OF KNOWN OUT-OF-ROUNDNESS SUBJECTED TO SPECIFIED DEPTHS OF SEA-WATER ON SIDES AND ENDS WITH EDGES SIMPLY SUPPORTED. BASED ON USE OF ALUMINUM: $E = 10 \times 10^6$ PSI, $S_y = 21{,}000$ PSI.

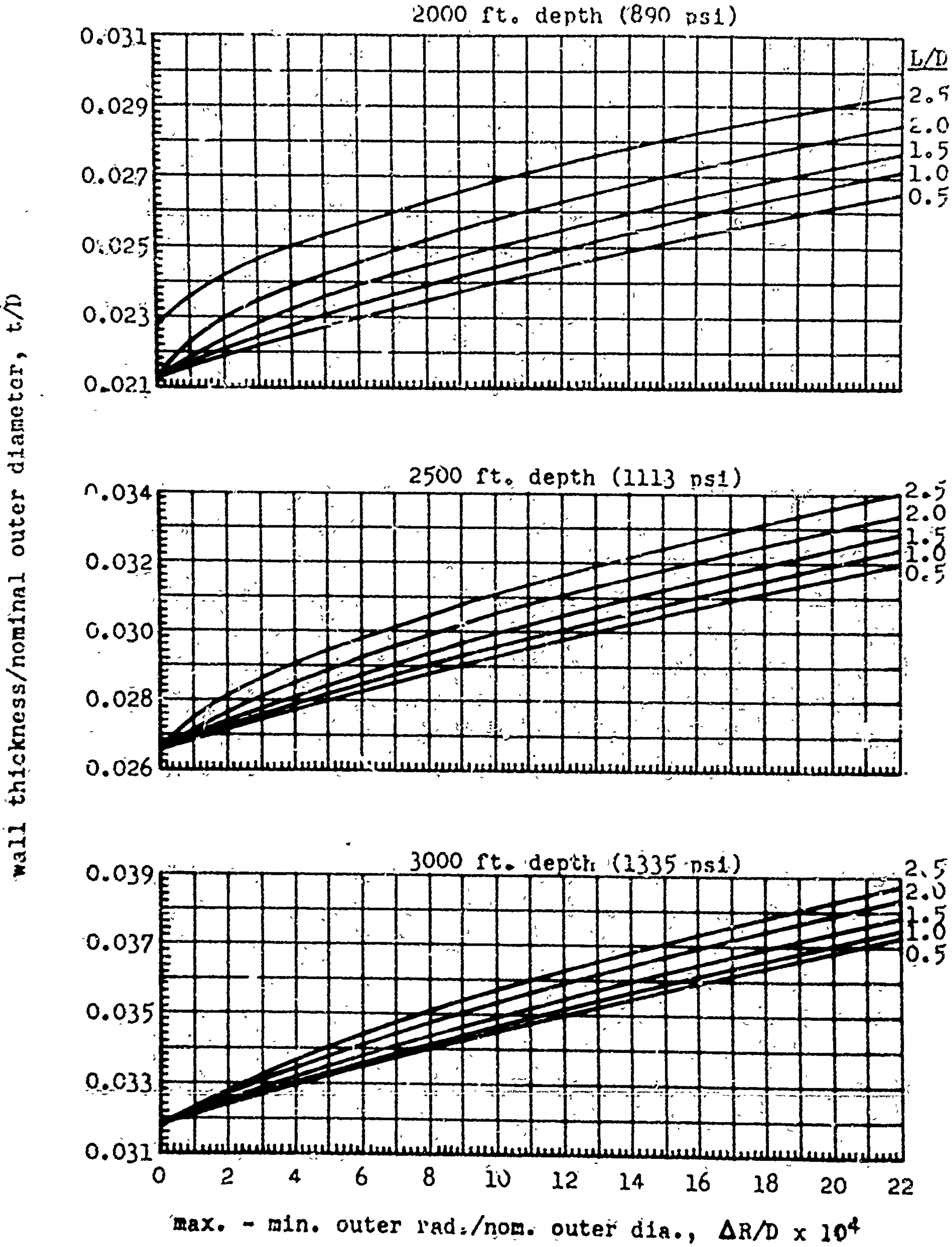

FIG. 6 CONCLUDED.

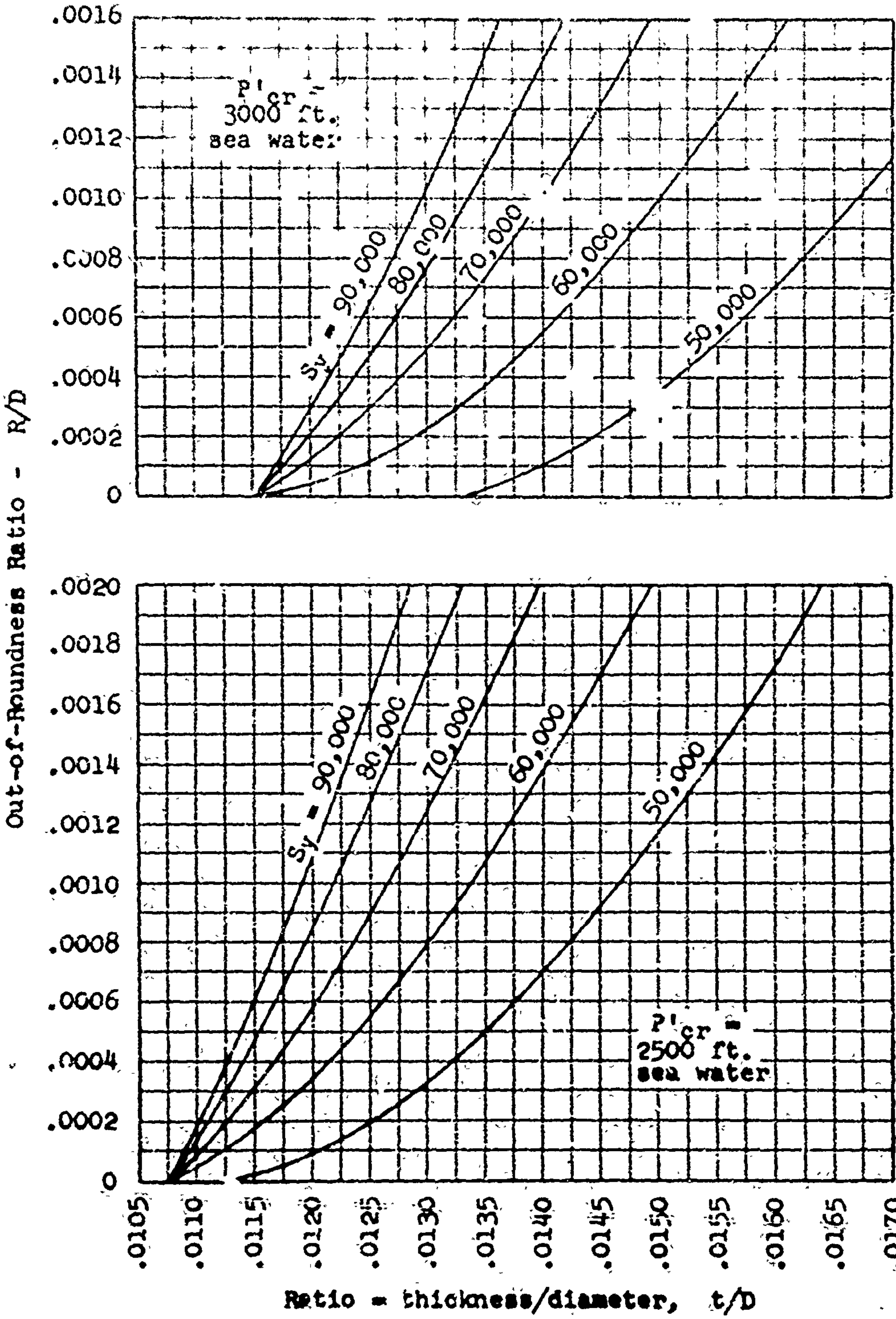

FIG. 7 MINIMUM REQUIRED WALL THICKNESS AND MAXIMUM ALLOWABLE OUT-OF-ROUNDNESS FOR CYLINDRICAL VESSEL MADE OF STEEL OF VARYING YIELD STRENGTH, L/D = 1, SUBJECTED TO 2500 AND 3000 FEET OF SEA-WATER ON SIDES AND ENDS, EDGES SIMPLY SUPPORTED.