

AD632718

TECHNICAL REPORT 231-10

A COMPARISON BETWEEN
THEORETICAL AND MEASURED WAVES
ABOVE A SUBMERGED RANKINE BODY

By

C. C. Hsu

And

B. Yim

February 1966

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

CLEARINGHOUSE FOR FEDERAL SCIENTIFIC AND TECHNICAL INFORMATION			
Hardcopy	Microfilm		
\$1.00	\$1.50	26 pp	a
ARCHIVE COPY			

Circle 1

HYDRONAUTICS, incorporated
research in hydrodynamics

Research, consulting, and advanced engineering in the fields of NAVAL
and INDUSTRIAL HYDRODYNAMICS. Offices and Laboratory in the
Washington, D. C., area: Pindell School Road, Howard County, Laurel, Md.

TABLE OF CONTENTS

	Page
SUMMARY.....	1
INTRODUCTION.....	1
THEORETICAL CONSIDERATIONS.....	3
COMPARISONS AND DISCUSSIONS.....	7
REFERENCES.....	11

LIST OF FIGURES

- Figure 1 - Surface Wave Pattern of a Submerged Body Started Impulsively from Rest
- Figure 2 - Definition Sketch
- Figure 3 - Comparison of Theoretical and Experimental Centerline Wave Profiles Due to a 4.5 Foot, 7 to 1, Rankine Ovoid Submerged 1.5 Feet below the Free Water Surface .
- Figure 4 - Comparison of Theoretical and Experimental Off Centerline Wave Profile Due to a 4.5 Foot, 7 to 1, Rankine Ovoid Submerged 1.5 Feet Below the Free Water Surface
- Figure 5 - Comparison of Non-dimensional Theoretical Calculations and Experimental Centerline Wave Profiles Due to 4.5 Feet and 9.0 Feet Long Models; $f/L = 1/3$

NOTATION

f	Depth of submergence measured from the centerline of the body
F	Froude number based on depth
g	Gravitational acceleration
M	Strength of point source
R	Radial distance from source or sink
t	Time
U	Uniform free stream speed
x,y,z	Rectangular cartesian coordinates
ζ	Surface elevation
ζ_r	Regular wave height
ζ_l	Local disturbance
μ	Fictitious frictional constant

SUMMARY

A comparison is made of theoretical and measured wave patterns produced by a submerged Rankine body (7:1) moving at constant speed. The measurements have been made by the staff of the David Taylor Model Basin, for a range of Froude numbers, but for a single depth/length ratio (1:3). Calculations were made for both the far and near fields. The former utilize the stationary phase approximation and the latter involve numerical integrations and include the local disturbance. In general, the agreement between theory and experiment is as good as the agreement between measurements taken at the same Froude number but for two different body lengths (4.5 and 9.0 feet). It is concluded from these tests that first-order wave theory predicts the shape and amplitude of the waves produced by a fully submerged body with sufficient accuracy for almost all practical purposes.

INTRODUCTION

In the past few years there has been a renewed interest in the problem of determining the surface wave pattern due to submerged bodies. Considerable progress in both the theoretical and experimental area has since been made. For a simple submerged body travelling at constant speed in deep water, an extensive analysis has been given by Yim (1). More recently, surface wave height measurements due to a Rankine ovoid in a towing tank have been undertaken at the David Taylor Model Basin (2), (3).

The object of this report is to compare the results between measured values and theoretical calculations.

The surface disturbance produced by a submerged body is usually analyzed into two parts: One is called the local disturbance which is, more or less, symmetrical and moves with the body; the other forms a group of regular waves which travels with group velocity within a vee-shaped region behind the body. In the case of towing tank experiments, there is an additional time-dependent disturbance which travels both upstream and downstream of the body due to its starting conditions. The fluid motion due to submerged bodies started from rest has been discussed in some detail in (4); the resulting surface disturbance is shown in Figure 1. The steady regular waves, in this case, travel downstream from the origin to $x = (1/2)Ut$ with group velocity $U/2$. It has been found that the part due to the initial acceleration diminishes rather rapidly with time. The effect of the unsteady disturbance, for sufficiently long time t , becomes appreciable, in general, only far downstream, i.e., $x \rightarrow (1/2)Ut$. Since the steady wave pattern due to a submerged Rankine ovoid is of primary concern here, the theoretical and experimental comparisons are limited to those tests which have unimportant unsteady effects.

The wave pattern of a submerged Rankine ovoid in a uniform stream, to a first approximation, may be calculated as due to distribution of a point source and point sink. The surface disturbances in the near field, are generally evaluated by numerical

integrations. The regular waves, in the far field, say 3 or 4 body-lengths away from the body, can easily be estimated by applying the method of stationary phase. In Figures 3 and 4, the test data of a 4.5 feet long, 7 to 1, Rankine ovoid towed at 10, 9, 7.3 and 6 feet per second and a given submergence depth of 1.5 feet (Reference 2) together with the corresponding calculations are shown. The calculated curves have the same general shape as the experimental measurements; the agreement between theory and experiment is discussed later. The centerline wave profile measurements of a 9 feet long, 7 to 1, Rankine ovoid towed at 10 feet per second and a submergence depth of 3 feet (Reference 3) is also examined. Except that the physical scale of the body length and submergence depth were doubled, this test was otherwise conducted under very similar conditions as that given in (2) for the case $U = 7.3$ feet per second. Comparison of non-dimensional theoretical calculations and experimental centerline wave profiles due to the two different length-scale models are shown in Figure 5. It is found that the theoretical calculations are in much better agreement with the larger model test data.

THEORETICAL CONSIDERATIONS

When a Rankine ovoid is submerged in a uniform stream, the fluid motion, to a first approximation, may be taken to be that due to a point source and point sink of constant strength $|M|$ distributed at points $(0, 0, -f)$ and $(l, 0, -f)$ respectively as shown in Figure 2. If the fluid is assumed to be inviscid,

incompressible and infinite in extent, the surface elevation in this case may be shown approximately to be

$$\zeta = - \sum_{n=1}^2 \frac{M_n}{2\pi U} \operatorname{Re} \int_{-\pi}^{\pi} \int_0^{\infty} \frac{x (i\omega_n - f) e^{x(1\omega_n - f)}}{x - x_0 \sec^2 \theta - i\mu \sec \theta} dx d\theta \quad [1]$$

in which

$$x_0 = \frac{g}{U^2}$$

$$M_1 = M, \quad M_2 = -M_1 = -M$$

$$\omega_1 = x \cos \theta + y \sin \theta = (R_1) \cos(\theta - \delta_1)$$

$$\omega_2 = (x - l) \cos \theta + y \sin \theta = (R_2) \cos(\theta - \delta_2)$$

$$R_1 = \sqrt{x^2 + y^2}, \quad R_2 = \sqrt{(x-l)^2 + y^2}$$

$$\delta_1 = \tan^{-1} \left(\frac{y}{x} \right), \quad \delta_2 = \tan^{-1} \left(\frac{y}{x-l} \right)$$

and μ is a fictitious frictional constant. The integral in Equation [1] is transformed by contour integration; when μ is made zero ultimately, the complete expressions are

$$\zeta = -\frac{2}{\pi} \sum_{n=1}^2 \frac{M_n}{Uf} \int_{-\pi/2+\delta_n}^{\pi/2+\delta_n} \int_0^{\infty} \frac{\exp(-m\omega_n) m \sec \theta}{\kappa_o^2 \sec^4 \theta + m^2} \left[\kappa_o f \sec^2 \theta \sin(mf) - \right. \\ \left. mf \cos mf \right] dm d\theta \\ + 4 \sum_{n=1}^2 \frac{M_n}{U} \kappa_o \int_{-\pi/2+\delta_n}^{\pi/2} \exp(-\kappa_o f \sec^2 \theta) \sec^3 \theta \cos(\kappa_o \omega_n \sec^2 \theta) d\theta \quad [2]$$

The first sum in Equation [2] is the local disturbance ζ_l , which diminishes with large $|x|$. The second sum represents the regular wave which travels only downstream and is contained within two straight lines radiating from the origin (0, 0, 0), each making with the line of motion an angle, δ_c , approximately $19^\circ 28'$.

Near the body the evaluation of ζ is not simple and can only be done numerically; the details of the calculations have been discussed in (1). Far behind the body Equation [2] may be expressed as

$$\zeta \approx \zeta_r = 4 \sum_{n=1}^2 \frac{M_n}{U} \kappa_o \int_{-\pi/2}^{\pi/2} \exp(-\kappa_o f \sec^2 \theta) \sec^3 \theta \cos(\kappa_o R F_n(\theta)) d\theta, R \gg l \quad [3]$$

with $F_n(\theta) = 3 \sec^2 \theta \cos(\theta - \delta_n)$. By applying the method of stationary phase, Equation [3] becomes

$$\zeta = 4\kappa_o \sum_{n=1}^2 \frac{M_n}{U} \sec^3 \theta_{n1} \exp(-\kappa_o r \sec^3 \theta_{n1}) \sqrt{\frac{2\pi}{\kappa_o R_n |F_n''(\theta_{n1})|}} \cos\left(\kappa_o R_n F_n(\theta_{n1}) + \frac{\pi}{4}\right) \\ + 4\kappa_o \sum_{n=1}^2 \frac{M_n}{U} \sec^3 \theta_{n2} \exp(-\kappa_o r \sec^3 \theta_{n2}) \sqrt{\frac{2\pi}{\kappa_o R_n |F_n''(\theta_{n2})|}} \cos\left(\kappa_o R_n F_n(\theta_{n2}) - \frac{\pi}{4}\right)$$

[4]

where

$$\theta_{n1} = -\frac{1}{4} \left[\cot \delta_n - \sqrt{\cot^2 \delta_n - 8} \right]$$

$$\theta_{n2} = -\frac{1}{4} \left[\cot \delta_n + \sqrt{\cot^2 \delta_n - 8} \right]$$

The first and second sum in [4] represent systems of transverse and diverging waves respectively travelling with the body; the amplitude for a given azimuth δ diminishes as $(\kappa_o R)^{-\frac{1}{2}}$. On the centerline, where $\delta_n = 0$, Equation [4] reduces to

$$\zeta \doteq \zeta_r = \zeta_r \text{ transverse}$$

$$= 4\kappa_o \sum_{n=1}^2 \frac{M_n}{U} \exp(-\kappa_o r) \sqrt{\frac{2\pi}{\kappa_o R_n}} \cos(\kappa_o R_n + \frac{\pi}{4}) \quad [5]$$

In the next section comparisons between the calculated values and experimental data are made.

COMPARISONS AND DISCUSSIONS

The main sets of experimental results are taken from a DTMB test report by Shaffer (2). In these tests a 4.5 feet long, 7 to 1, Rankine ovoid was towed at constant speed U and a given submergence depth, $f = 1.5$ feet, in the 360 feet long, 240 feet wide and 20 feet deep basin. The length of the test run was limited to 225 feet. The wave profiles were measured at five stations; approximately 48, 63, 95, 133 and 187 feet from the starting position of the model. The influence of the initial acceleration, as discussed in (4), diminishes rather rapidly as the running time t increases and becomes appreciable, for sufficiently long time, only far downstream. The starting conditions, except far downstream, i.e., $x \rightarrow (1/2)Ut$, would, in general, have little effect on the measurements made at the last two stations. The last (187 feet) measuring station, however, was near the end of the run, so that the major part of the measurements for this station may be affected by the deceleration of the model. Therefore, only the data measured at the 133 feet station are being used in the comparison between the experimental and the theoretical steady wave profile calculations.

The measured centerline wave profiles in test runs at $U = 10.0, 9.0, 7.3$ and 6 feet per second together with the corresponding calculated values are shown in Figure 3(a), (b), (c) and (d) respectively. It can be seen that the calculated curves

express the general variations in the same manner as the experimental measurements. The agreements are generally better at higher speeds or higher Froude numbers $F(= U/\sqrt{gf})$. It is to be noted that in these comparisons, the experimental reference point P near the body, at which $\zeta = 0$, is taken to be coincident with that of the calculated zero wave height point between $x = 0$ and l . The exact location of point P depends very much on the running speed U . It is found from the calculations that P is close to $(1/2 l, 0, 0)$, as postulated in (2) for all test speeds, only at F near 1. The exact behavior of this phase difference as a function of Froude number may be worth further experimental investigation.

The off centerline wave profile measurements in these tests were made at $y = 11.375$, 22.750 , and 34.125 feet. The regular wave pattern begins to develop only after $x \approx y/\tan(19^\circ 28')$ which is quite far downstream in the latter cases; the measurements at $y = 22.750$ and 34.125 feet are largely distorted by the transient effects before the steady portion of the wave train can be developed. The only meaningful experimental data which can be compared with the theoretical calculation is that measured at $y = 11.375$ feet. In Figure 4 comparison of theoretical and experimental off centerline wave profiles at $U = 7.3$ feet per second, $y = 11.375$ feet is given; the results are in fairly good agreement.

Before discussing these data further, a different set of experimental results, taken from a previous DIMB test report (3), should be examined. In this test, a 9 feet long, 7 to 1, Rankine

ovoid was towed at $U = 10.0$ feet per second and $f = 3$ feet. Except that the physical scale of the body length and submergence depth were doubled, the experiment was otherwise conducted under very similar test conditions as that given in (2) for the case $U = 7.3$ feet per second and $f = 1.5$ feet. The non-dimensional centerline wave height, $\zeta/(M/Uf)$, versus x/l for both the experiments and their corresponding calculations are shown in Figure 5. The theoretical calculations, as seen from Figure 5, are in much better agreement with the data given in (3) than that in (2). The measured nondimensional wave amplitude for the 9 foot body is generally 15 percent higher than that for the 4.5 foot body. This discrepancy may not be due entirely to instrumentation difficulties, since the two tests were conducted at similar Froude but quite different Reynolds scales.

In general the theoretical predictions are in fairly good agreement with the test results. At the higher Froude numbers and for the larger test body, the agreement between the predicted and measured wave lengths and amplitudes behind the body are excellent. In all cases the shape of the wave over the body is well predicted by the theory, but the amplitude of both the first crest and the first trough are overestimated, sometimes by a factor of 2. The reason for this is not clear, but it is to be noted that in the case of the large body (9 feet length) at 10 feet per second the agreement between the theory and experiment for the first trough amplitude is really excellent, while the first peak is overestimated by only about 25 percent.

We should imagine that as far as the wave theory itself is concerned, that the agreement between theory and measurement might become worse with increasing Froude number, as the effect of the surface waves on the submerged body become more pronounced; this is, however, not observed to be the case. In fact, the test results may be interpreted to suggest that for too small scales and speeds, viscous effects in the flow about the body begin to influence the wave pattern.

In any case, it seems safe to conclude that existing wave theory predicts the shape and magnitude of the waves above a fully submerged body with sufficient accuracy for almost all practical purposes.

REFERENCES

1. Yim, B., "Waves Due to a Submerged Body," HYDRONAUTICS, Incorporated Technical Report 231-3, May 1963.
2. Shaffer, D. A., "Surface waves Generated by a Submerged Rankine Ovoid," DTMB Test Report 105-H-01, October 1965.
3. "Preliminary Kelvin Wake Study," DTMB Test Report 039-H-01, December 1964.
4. Hsu, C. C., "On the Surface Wave Pattern of Submerged Bodies Started from Rest," HYDRONAUTICS, Incorporated Technical Report 231-7, April 1965.

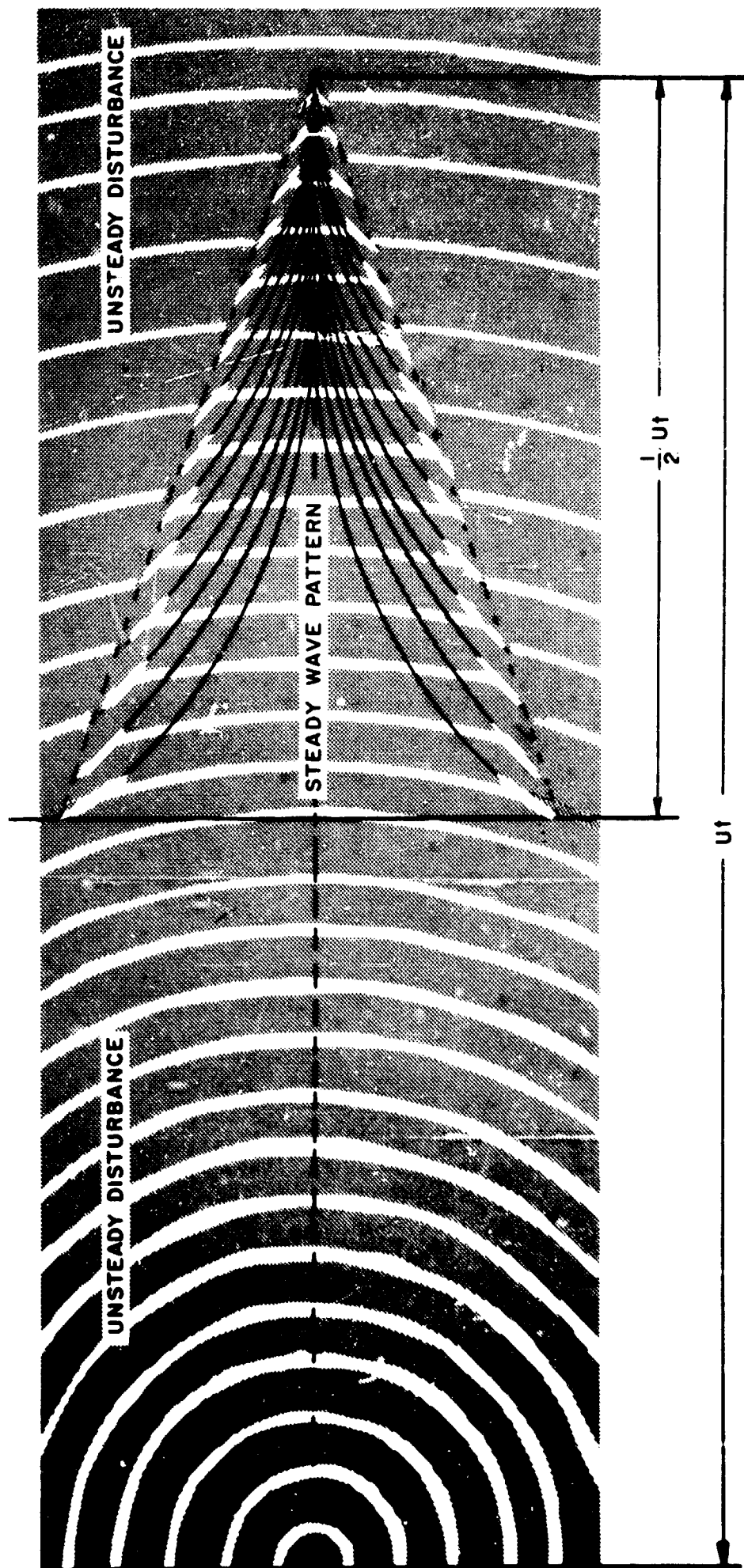


FIGURE 1 - SURFACE WAVE PATTERN OF A SUBMERGED BODY STARTED IMPULSIVELY FROM REST

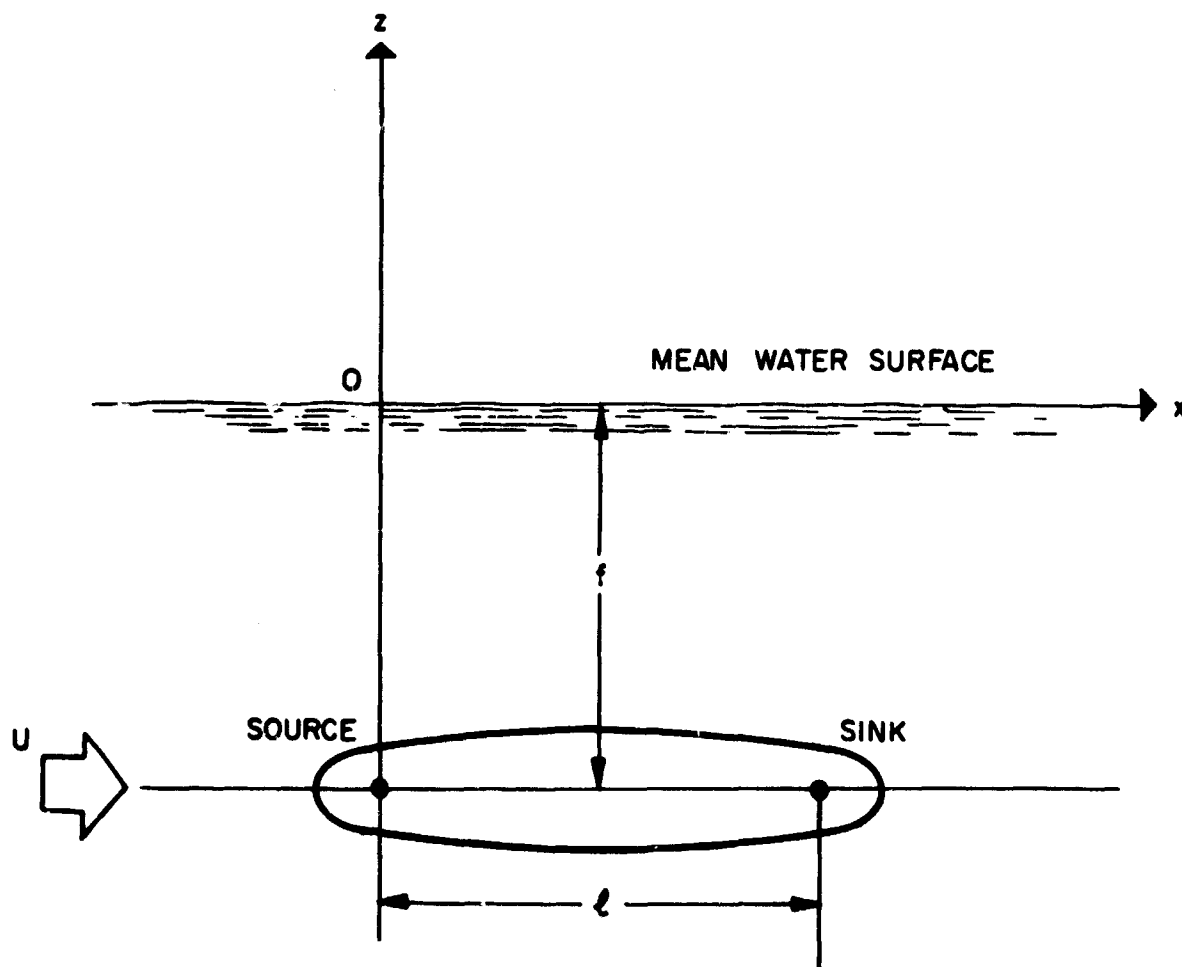


FIGURE 2- DEFINITION SKETCH

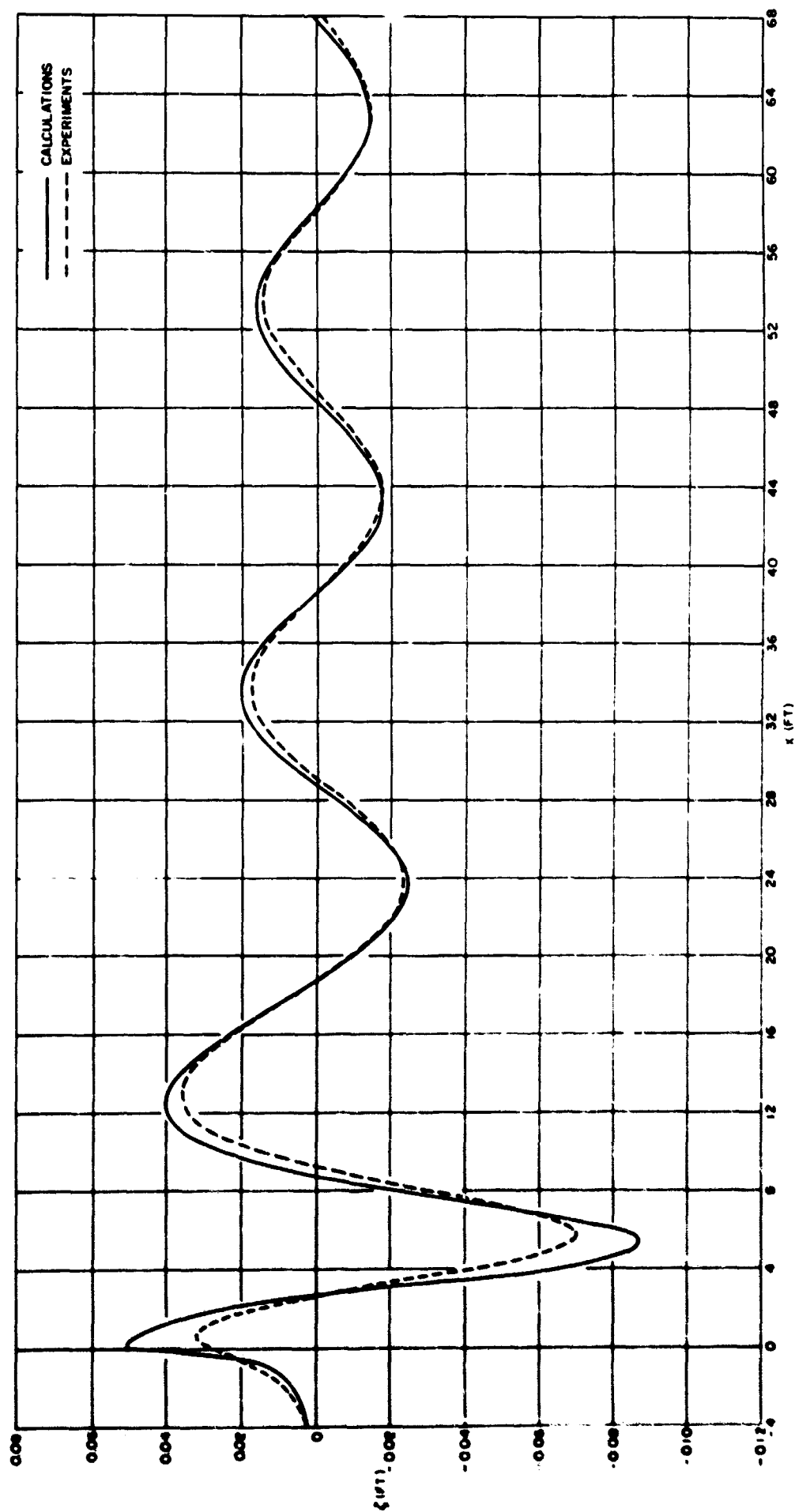


FIG. 3.10.1 - COMPARISON OF THEORETICAL AND EXPERIMENTAL CENTERLINE WAVE PROFILES DUE TO A 4.5 FT. 7 TO 1, RANKINE OVOID SUBMERGED 1.5 FT. BELOW THE FREE WATER SURFACE $U=10$ FT/SEC

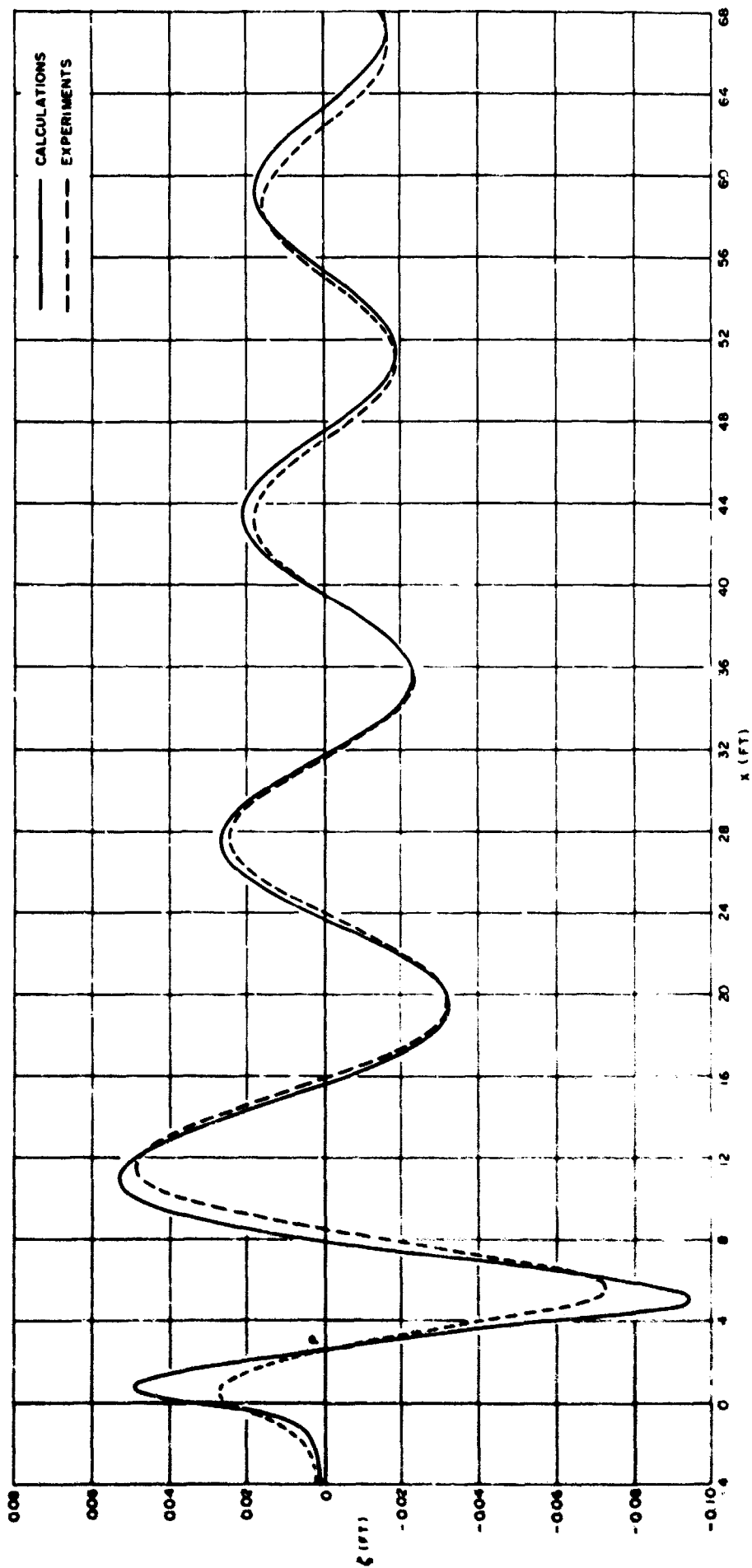


FIGURE 3(b) - CONTINUED U=9.0 FT/SEC

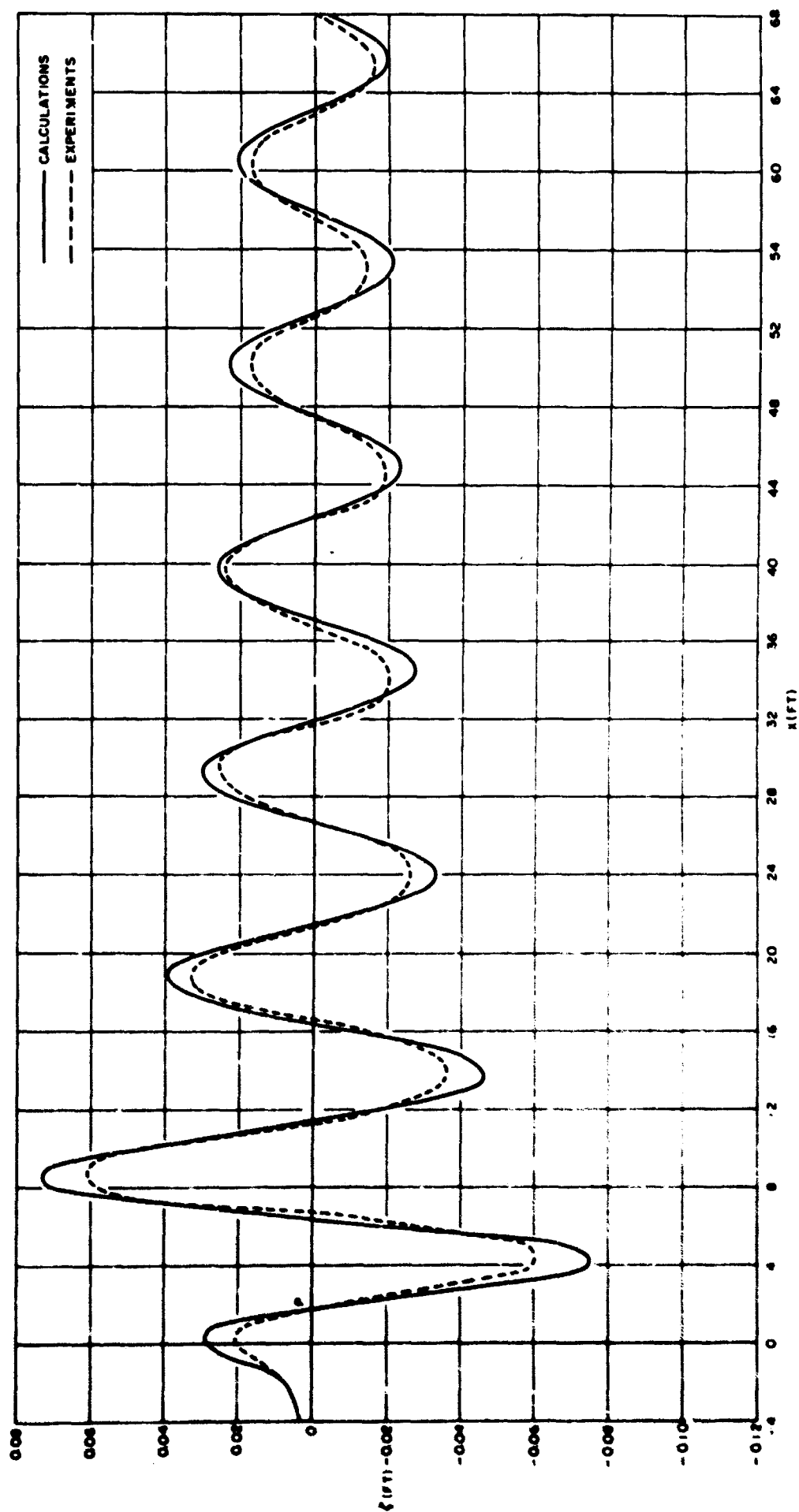


FIGURE 3(c) - CONTINUED $U = 7.3$ FT/SEC

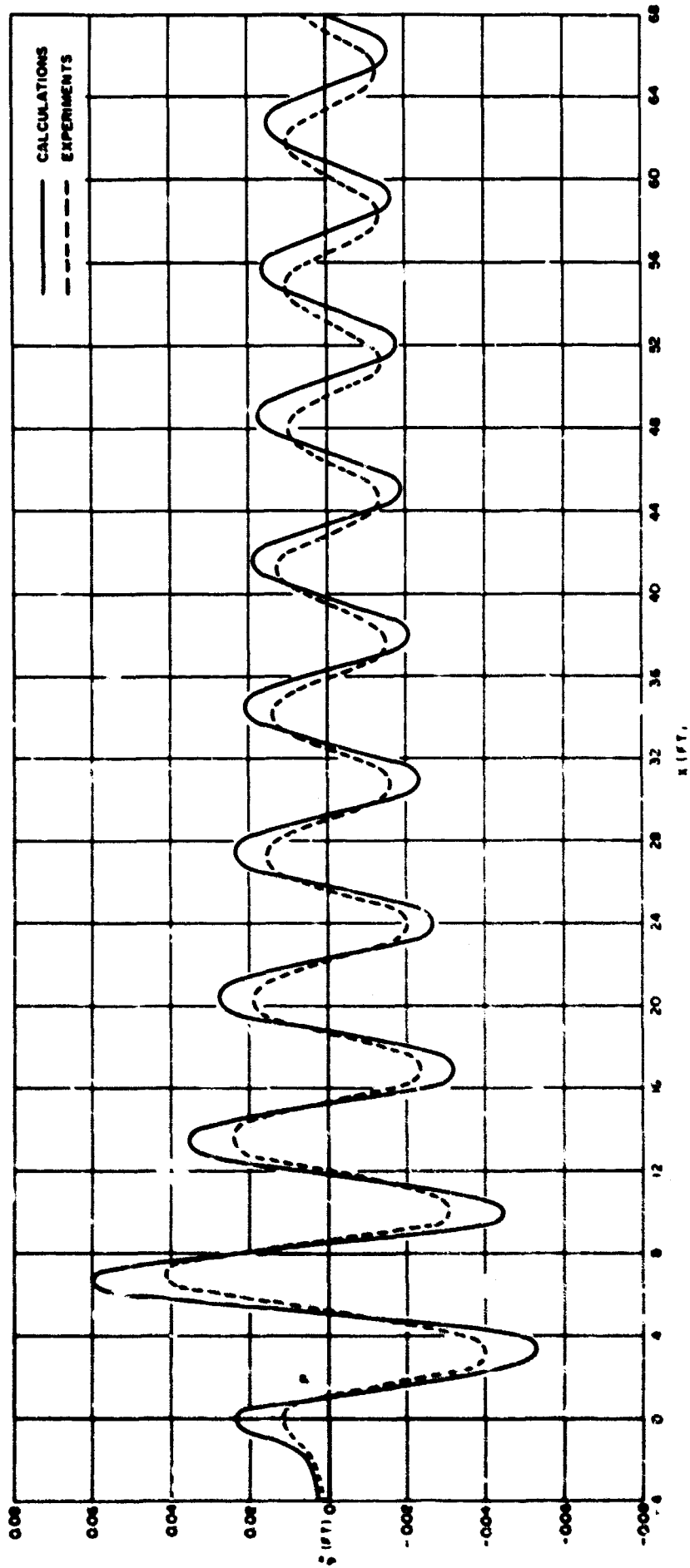


FIGURE 3(d) - CONCLUDED U = 6.0 FT/SEC

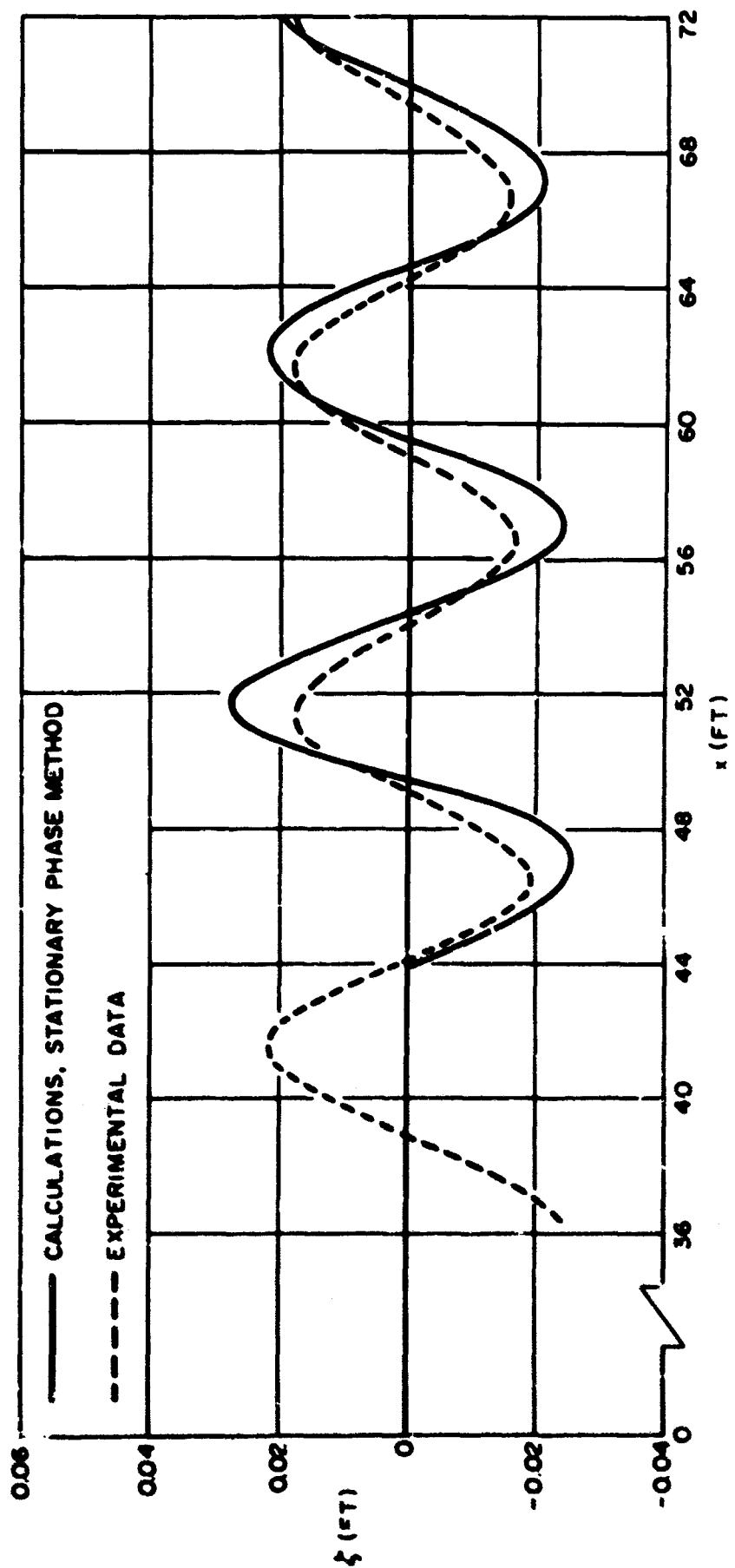


FIGURE 4-- COMPARISON OF THEORETICAL AND EXPERIMENTAL OFF CENTERLINE WAVE PROFILE DUE TO A 4.5 FT. 7 TO 1, RANKINE OVOID SUBMERGED 1.5 FT BELOW THE FREE WATER SURFACE
U = 7.3 FT./SEC. y = 11.375 FT.

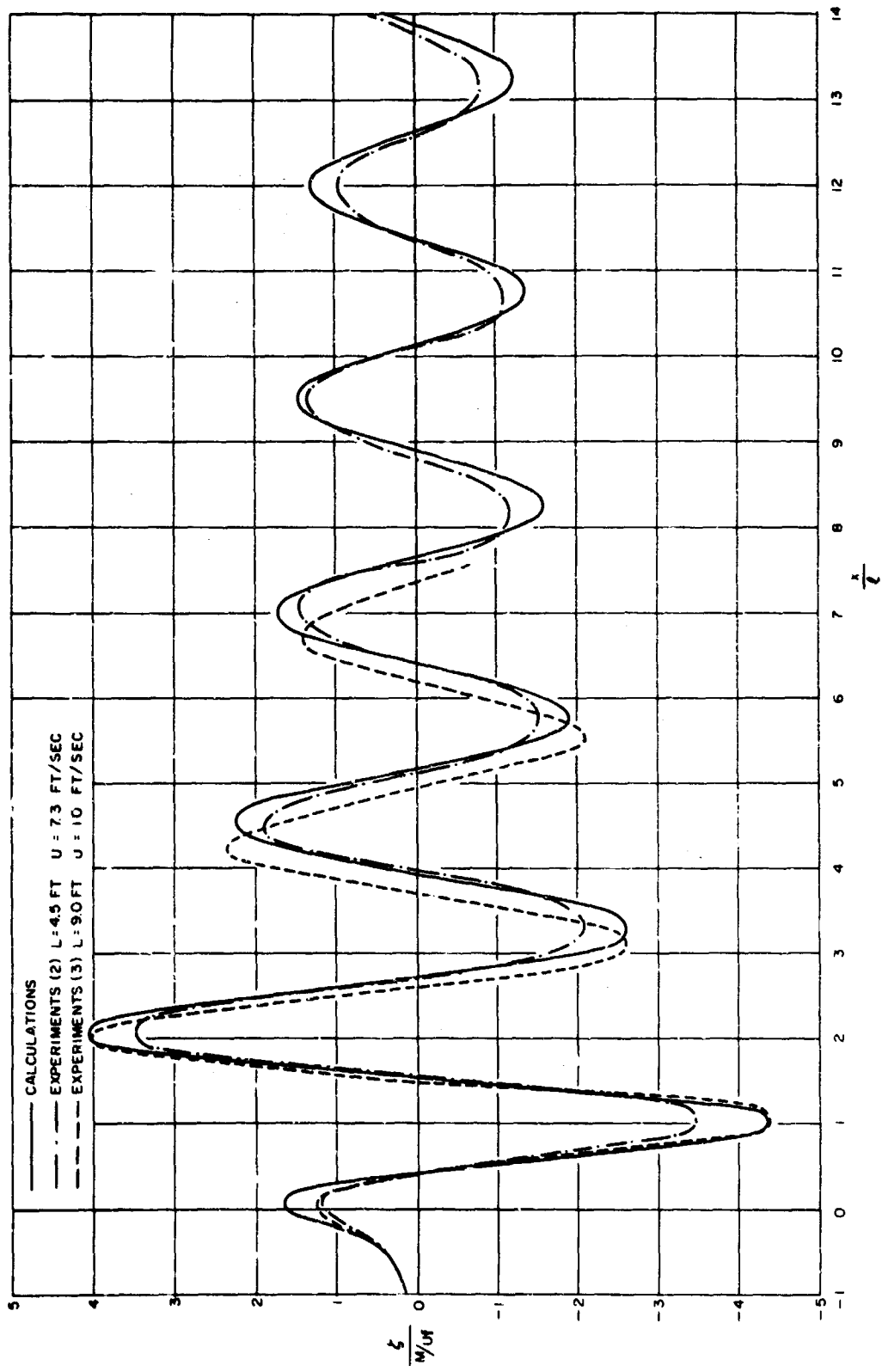


FIGURE 5— COMPARISON OF NON DIMENSIONAL THEORETICAL CALCULATIONS AND EXPERIMENTAL CENTERLINE WAVE PROFILE DUE TO 4.5 FT AND 9.0 FT LONG MODELS; $1/L = 1/3$

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) HYDRONAUTICS, Incorporated, Pindell School Road, Howard County, Laurel, Maryland		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	
		2b. GROUP	
3. REPORT TITLE A COMPARISON BETWEEN THEORETICAL AND MEASURED WAVES ABOVE A SUBMERGED RANKINE BODY			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Technical Report			
5. AUTHOR(S) (Last name, first name, initial) Hsu, C. C., and Yim, B.			
6. REPORT DATE February 1966	7a. TOTAL NO. OF PAGES 23	7b. NO. OF REFS 4	
8a. CONTRACT OR GRANT NO. Nonr 3688(00) NR 220-016	9a. ORIGINATOR'S REPORT NUMBER(S) Technical Report 231-10		
b. PROJECT NO. c. c	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)		
10. AVAILABILITY/LIMITATION NOTICES Distribution of this document is unlimited.			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY Office of Naval Research Department of the Navy	
13. ABSTRACT A comparison is made of theoretical and measured wave patterns produced by a submerged Rankine body (7:1) moving at constant speed. The measurements have been made by the staff of the David Taylor Model Basin, for a range of Froude numbers but a single depth/length ratio (1:3). Calculations were made for both the far and near fields. The former utilize the stationary phase approximation and the latter involve numerical integrations and include the local disturbance. In general, the agreement between theory and experiment is as good as the agreement between measurements taken at the same Froude number but for two different body lengths (4.5 and 9.0 ft.). It is concluded from these tests that first-order wave theory predicts the shape and amplitude of the waves produced by a fully submerged body with sufficient accuracy for almost all practical purposes.			

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
1. Submerged body						
2. Surface waves						
3. Theoretical considerations						
4. Experiments						
5. Comparisons and discussions						

INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (*corporate author*) issuing the report.

2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (*either by the originator or by the sponsor*), also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those

imposed by security classification, using standard statements such as:

- (1) "Qualified requesters may obtain copies of this report from DDC."
- (2) "Foreign announcement and dissemination of this report by DDC is not authorized."
- (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through _____."
- (4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through _____."
- (5) "All distribution of this report is controlled. Qualified DDC users shall request through _____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (*paying for*) the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.