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RM-4673-ARPA
APRIL 1966

INTERPRETATION OF RECOVERY-TEMPERATURE MEASUREMENTS AT LOW REYNOLDS NUMBERS

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PREFACE

An area of great interest to the ballistic-missile-defense community is the study of wake flows. Recent experimental work has been directed toward the measurement of basic parameters which could be used to characterize these flows. The experimental data thus obtained must then be compared with the theoretical data that are available. The purpose of this Memorandum is to clarify the interpretation of the measurements so that comparison with theoretical results will be more meaningful.

This Memorandum is a part of a study of reentry aerodynamics for the Advanced Research Projects Agency.

SUMMARY

This Memorandum demonstrates that the wire recovery temperature in a steady-state flow may be accurately determined without a detailed knowledge of the convective heat-transfer rate to the wire. The analysis is valid even for large conduction losses at the wire supports, and a similar analysis could be made for transient conditions.

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I. INTRODUCTION

In interpreting recovery-temperature measurements in conventional steady-state wind tunnels, it is common practice[1,2] to find the average temperature of a fine wire with no electrical heating and to infer the wire recovery temperature by a detailed accounting of the heat transferred to the wire by convection and of the heat lost from the wire by conduction to the supports and by radiation. Todisco and Pallone[3] have recently proposed a variation of this method, applicable to shock tunnels, in which the recovery temperature is determined in the presence of simultaneous electrical heating.

The purpose of this analysis is to demonstrate that the wire recovery temperature (and hence the local stagnation temperature of the stream) may be accurately determined without a detailed knowledge of the convective heat-transfer rate to the wire. The analysis remains valid even in the presence of large conduction losses to the supports. This technique may be considered to be the steady-state analogue of the method proposed in Ref. 3.

II. STEADY-STATE HEAT BALANCE

The technique to be analyzed consists of two parts. Before flow initiation, the wire is placed in a vacuum with a fixed current, and the mean wire temperature is measured. Flow is then initiated, and the change in mean wire temperature (mean wire resistance) with the same current is noted. The experiment is repeated with different currents until no change in wire resistance before and after establishment of flow is observed.† The ideal recovery temperature of the wire is then inferred from a detailed heat balance.

For simplicity, assume that radiation is negligible and that the wire resistivity varies linearly with temperature according to the relation

$$\frac{R(T)}{R^*} = \frac{1 + \alpha(T - T_r)}{1 + \alpha(T^* - T_r)} \tag{1}$$

where T^* is a scaling temperature to be specified and T_r is the temperature at which the resistivity coefficient α is evaluated. Assume also that the wire thermal conductivity k_w is independent of temperature, that each cross section of the wire is at a uniform temperature, and that the convective heat-transfer coefficient h is constant across the wire, i.e.,

$$Nu_o = \frac{hd}{k_o} = \text{constant}$$

where k_o is the thermal conductivity of the stream at the local stagnation temperature and d is the wire diameter.

The heat balance for the wire, using the assumptions listed above, is

†This tedious repetition may be circumvented in practice by recording the wire resistance as a function of current in both a vacuum and the flow and by noting the intersection of the two curves.

$$i^2R(x) + \left(\frac{\pi d^2 \ell}{4}\right)\frac{d}{dx}\left(k_w \frac{dT}{dx}\right) = \pi d \ell h(T - T_{aw}) \qquad (2)$$

where R(x) is the resistance along the wire and T_{aw} is the recovery temperature of the flow. This equation is to be solved with the boundary conditions $T = T_s$ (the temperature of the supports) at $x = \pm (\ell/2)$.

Define the new variables

$$z = (x/\ell)$$

$$s = \frac{\alpha\widetilde{T}}{1 + \alpha(T^* - T_r)} \qquad (3)$$

$$t = (T - T^*)/\widetilde{T}$$

where T^* and $\widetilde{T}$ are scaling temperatures. Then

$$\frac{R}{R^*} = 1 + st \qquad (4)$$

and Eq. (2) may be written

$$\frac{d^2t}{dz^2} + at(1 - c) + a(1 + bc)/s = 0 \qquad (5)$$

where

$$a = \frac{4i^2R^*s\ell}{\pi d^2 k_w \widetilde{T}} = \frac{4}{\pi}\left(\frac{\ell}{d}\right)^2\left(\frac{i^2R_r\alpha}{k_w\ell}\right) \qquad (6)$$

$$b = s(T_{aw} - T^*)/\widetilde{T} = \alpha(T_{aw} - T^*)/[1 + \alpha(T^* - T_r)] \qquad (7)$$

$$c = \frac{1}{a}\left[4\ Nu_o\left(\frac{\ell}{d}\right)^2\left(\frac{k_o}{k}\right)\right] = \frac{\pi Nu_o k_o \ell}{i^2 R_r \alpha} \tag{8}$$

and R_r is the wire resistance for a uniform temperature T_r.

In many wind-tunnel investigations, the convective heat-transfer rates are quite low, so that the nondimensional parameter c is less than unity. The solution of Eq. (5) is then a cosine distribution of temperature between the supports, with the wire temperature equal to the support temperature at $z = \pm 1/2$.

For convenience, choose the two scaling temperatures T^* and $\widetilde{T}$ and the reference temperature T_r to be equal to the support temperature T_s (assumed known in a given experiment). Then $s = \alpha T_s$, $t_s = 0$, $b = \alpha(T_{aw} - T_s)$, and the temperature distribution t(z) across the wire is

$$1 + st(1 - c)/(1 + bc) = \cos\{[a(1 - c)]^{1/2}z\}/\cos\{[a(1 - c)]^{1/2}/2\} \tag{9}$$

The temperature distribution without convection (c = 0) before flow initiation is $t_o(z)$:

$$1 + st_o = \cos(a^{1/2}z)/\cos(a^{1/2}/2) \tag{10}$$

Integrating these results across the wire to find average wire temperatures, $\bar{t}$ and $\bar{t}_o$, yields

$$1 + s\bar{t}(1 - c)/(1 + bc) = 2\tan\{[a(1 - c)]^{1/2}/2\}/[a(1 - c)]^{1/2} \tag{11}$$

$$1 + s\bar{t}_o = 2\tan(a^{1/2}/2)/a^{1/2} \tag{12}$$

III. INTERPRETATION OF THE RECOVERY TEMPERATURE

The proposed experiment is conducted in such a way as to equate the average wire resistances before and after initiation of flow, i.e., $\bar{R} = \bar{R}_o$. Since the resistance varies linearly with temperature, this requires $\bar{t} = \bar{t}_o$. Assuming that the wire properties and the Nusselt number Nu_o are known, Eqs. (11) and (12) may be combined to solve for the recovery temperature T_{aw}. After some manipulation, the recovery temperature is found to be given by the solution of the following expression:

$$g/(1 + bc) = (\tan \beta g - \beta g)/(\tan \beta - \beta) \tag{13}$$

$$b = \alpha(T_{aw} - T_s) \tag{14}$$

$$\beta = (a/2)^{1/2} = (\ell/d)(i^2 R_r \alpha/\pi k_w \ell)^{1/2} \tag{15}$$

$$g = (1 - c)^{1/2} \tag{16}$$

and β and c are defined, with R_r and α being determined from Eq. (1) and with $T_r = T_s$.

The solution of Eq. (13) may be put into dimensionless form by using the measured mean wire temperature $\bar{T}_o$ without flow:

$$\mathcal{R} \equiv \frac{T_{aw} - T_s}{\bar{T}_o - T_s} \tag{17}$$

Then $\mathcal{R} = \mathcal{R}(\beta, c)$ and in general will depend on the local Nusselt number Nu_o through c. The parameter β is restricted to values less than $\pi/2$, since $\bar{t}_o$ is finite (see Eq. (12)). The cosine temperature distribution has been taken as $0 < c < 1$.

The ratio $\mathcal{R}(\beta, c)$ has been computed for pairs of values $\beta = 0.1, 0.2, \ldots 1.5$, and $c = 0.1, 0.2, \ldots 0.9$. Somewhat surprisingly, the value of $\mathcal{R}$ is very insensitive to the values of β and c chosen, with

$\mathcal{R}_{min} = 1.1995$ ($\beta = 0.1$, $c = 0.1$) and $\mathcal{R}_{max} = 1.228$ ($\beta = 1.5$, $c = 0.1$). An asymptotic expansion for $\beta^2 << 1$ yields $\mathcal{R} = 1.2$ exactly, independent of the value of c. The interpretation of this result is very important: The recovery temperature T_{aw} may be accurately determined without a precise knowledge of the local Nusselt number Nu_o.

An <u>a posteriori</u> physical argument for the insensitivity of $\mathcal{R}$ to the local heat-transfer rate may be constructed. The heat lost to the supports with and without flow has been denoted by Q and Q_o, respectively. Without flow and neglecting radiation, the Joulian heating $i^2\bar{R}_o$ is equal to Q_o. After flow initiation, the temperature distribution will become more uniform across the center of the wire, but the temperature gradient at the tips will increase, i.e., $Q > Q_o$. The difference $Q - Q_c$ must be equal to the net convective heat input in the wire or

$$Q - Q_o \sim Nu_o(T_{aw} - \bar{T})$$

As the Nusselt number Nu_o becomes small, the difference $Q - Q_o$ also decreases so that the temperature difference $(T_{aw} - \bar{T}) \equiv (T_{aw} - \bar{T}_o)$ remains essentially constant. Therefore, the ratio $\mathcal{R}$ is nearly independent of Nu_o, and to the accuracy of most experiments, $\mathcal{R}$ may be taken to be 1.2.

For this analysis to be valid, c must be less than unity, and a steady-state condition must have been established. A similar analysis may be constructed for $c > 1$ (the temperature distribution follows a cos h βz law) and for transient conditions (the problem is linear, and variables are separable).

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10. ABSTRACT

A demonstration that the wire recovery temperature in a steady-state flow may be accurately determined without a detailed knowledge of the convective heat-transfer rate to the wire. The analysis is valid even for large conduction losses at the wire supports, and a similar analysis could be made for transient conditions.

11 KEY WORDS

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