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**THE ROLE OF THE ELECTROSTATIC FIELD  
IN THE COAGULATION OF FOG AND CLOUD DROPLETS**

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ABSTRACT

The electrostatic field of the atmosphere is normally enhanced in fog and cloud. This field, even when quite small, can effect the combination of colliding but otherwise noncoalescing droplets. Stronger fields produce forces of mutual attraction among the droplets. Preliminary results on the degree to which electrostatic forces act to modify the motion of cloud droplets are presented. Methods for the possible modification of the field are discussed.

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## I. INTRODUCTION

The effect of the average charge in modifying the motion of neighboring droplets in clouds or fog has been estimated many times. Generally this effect and, consequently, that of the field also is concluded to be of negligible significance, except possibly in thunderstorms. The equations used in these deductions have restricted validity for small droplet separation and velocity. Within the past several months, however, rigorous solutions to both the electrostatic and hydrodynamic problems have become available. A RAND colleague, M. H. Davis (1960) has obtained a complete solution for the force between two cloud droplets in a uniform field. The other half of the problem, that of the hydrodynamic forces, is now possible also. The viscous forces between two cloud droplets can be derived from a solution to the two-body hydrodynamic problem given by L. M. Hocking (1959). Together, these results provide the necessary components for an accurate evaluation of the role of electrostatic fields and charges in the collision and coalescence of cloud drops.

## II. ELECTROSTATIC FORCES

First let us consider only electrostatic forces. For most purposes, water droplets with dielectric constant of 81 can be considered as conducting spheres. Coulomb's law is not valid for this problem, since the distance separating two droplets is comparable with their diameter. As an illustration, Figure 1 shows the forces on two 10-micron drops computed according to Coulomb's law and computed from the complete solution for charged drops of equal size. The abscissa is used to designate the separation of the near surfaces of the droplets in units of droplets radius. Along the

ordinate is plotted the force in dynes. The letter  $S$  is used to denote the separation of droplet surfaces while  $R$  refers to droplet radius.

Curve  $F_1$  represents Coulomb's law,  $F_1 = \frac{q_1 q_2}{(S+2R)^2}$ ; and  $F_2$  represents a fictitious solution in the form of Coulomb's law,  $F_2 = \frac{q_1 q_2}{S^2}$ .

$q_1 = -q_2 = 10$  elementary units. Curve  $F$  is computed from a complete solution for the force between two droplets with equal and opposite charges of the same magnitude. This charge (10e) is of the order of magnitude of charges observed on cloud droplets.

All the forces due to charges on the droplets coincide at large separations.

$F_2$  is fictitiously large and represents some sort of a maximum possible theoretical force for charged droplets. Coulomb's law,  $F_1 = \frac{q_1 q_2}{(S+2R)^2}$ , becomes insensitive to separation and incorrect at small separations. The true force,  $F$ , falls in between and continues to increase exponentially as the separation decreases. The force of attraction due to a weak field is shown by the lower heavy line. This force arising from the polarization of the droplets in the field increases more rapidly with decreasing separation than does that due to weak charges on the drops. An order-of-magnitude change in the ambient field strength produces a change of two orders of magnitude in the force. The same is true of the charge on oppositely charged drops. However, the chance that two drops have high charges of opposite sign is small while all drops will be affected similarly by the electrostatic field. The direct effect of charges on the droplets is of secondary importance in the following work so we will concentrate our attention on the effect of the electrostatic field.

### III. HYDRODYNAMIC FORCES

Until Hocking's work was published last year, no suitable solution was available for the motion of two neighboring cloud droplets or for the relative forces between them. In the past, estimates of the combined effect of electrostatic charges and viscous motion had to be made using the relative terminal velocities of the cloud droplets. The relative terminal velocity was used to compute the time required for freely falling droplets to pass one another. Dividing the time of passage into the radial distance two drops must move in order to effect collision gave the radial velocity. Employing this velocity in Stokes' law for the viscous drag allowed one to estimate the electrostatic force of attraction required to offset the viscous force resisting the motion of the droplets toward each other.

Figure 2 shows the relative trajectory of a smaller droplet of 6 microns radius about a larger droplet of 20 microns radius, computed from the rigorous solution to the two-body hydrodynamic problem. The collision efficiency is defined as the square of the ratio of the critical horizontal-droplet separation at a large vertical distance to the large-droplet radius. The critical droplet separation is the maximum horizontal separation at large vertical distances from which the two droplets will eventually collide. The hydrodynamic collision efficiency for these droplets is zero. One can see from the diagram that only a slight motion towards one another as they pass the point of minimum separation is required to effect

collision. At great distances considerably greater motion does not result in collision. If sufficient electrostatic force is added throughout the entire trajectory for collision to occur, the new collision efficiency may be called the electrostatic collision efficiency.

#### IV. SOME ELECTROSTATIC COLLISION EFFICIENCIES

At this point in the work of computing hydrodynamic and electrostatic collision efficiencies, it is possible to give the maximum field required to produce collision in a few cases of normally noncolliding droplet pairs.

The integration of the hydrodynamic equation describing the motion of droplet pairs that incorporates the electrostatic forces step-by-step is not quite ready. In lieu of this, the maximum electrostatic fields required have been obtained in a manner similar to that previously described except that actual velocities and distances are used. The time required for the center of the small drop to move from a position opposite the bottom of the large drop to a position horizontally opposite the center of the large drop is divided into the minimum droplet separation giving the radial velocity with which the small droplet must move in order to effect a collision. The force required to produce this velocity is obtained by using this velocity "V" in the expression for the viscous drag  $6\pi\eta aV$ , where  $\eta$  is the viscosity and "a" the droplet radius. Table 1 lists some of the resulting electrostatic collision efficiencies and the maximum field required in each situation.

Note that fields of the order of the atmospheric fair weather field are not sufficient to increase significantly the hydrodynamic collision efficiencies, although they may strongly affect coalescence. Larger fields do produce increased collision efficiencies. The electrostatic force acting on the droplets is a function of the square of the product of the field and the radius of the droplets. From this relationship one estimates that collision efficiencies can be enhanced significantly for fields of the order of several tens of volts per centimeter. Fields of the order of hundreds of volts per centimeter (such as exist in actively precipitating clouds) can be expected to provide collision efficiencies as much as an order of magnitude greater than the hydrodynamic efficiencies. It is difficult to estimate the effect of fields of the order of 1000 volts per centimeter, since electrostatic discharge will occur between droplets before collision is effected.

These results demonstrate that the electrostatic field in clouds is of fundamental importance in the colloidal stability of clouds and in the quantitative evaluation of the precipitation mechanism.

#### V. ELECTROSTATIC FIELDS IN FOG AND CLOUDS

The fair-weather field over land is of the order of one volt per centimeter decreasing logarithmically upward. In fog or in precipitation-free cloud, the normal field is usually larger, sometimes by an order of magnitude. Precipitating clouds contain very strong fields, of course.

The field in nonprecipitating fog or clouds is enhanced because atmospheric conductivity is less within a cloud than in cloud-free air. Applying an equation due to Gish (1939) for sudden changes in conductivity of an



atmospheric layer, one can compute the new steady-state field. It can be shown that the field within the cloud should eventually increase by the same factor by which the cloud causes the ambient conductivity to decrease. Fog has been observed by Gunn (1954) and others to decrease the conductivity by 5 to 20 times.

#### VI. MODIFICATION OF THE ELECTROSTATIC FIELD

It is natural from the above results to think of the possibility of fog or cloud modification by increasing the electrostatic field. As suggested above, initially the field is enhanced through the difference in conductivity of the air in the cloud and outside. Because the conductivity is decreased within the cloud, the cloud boundary is a region of charge accumulation due to the sudden decreased ionic mobility as ions try to proceed through the cloud. The increased charge at the boundary builds the field within the cloud. Modification would appear possible through increasing the conductivity in the air above the fog by the addition of ions in some manner.

Something else may be possible also. Several years ago I proposed a cloud electrification hypothesis involving cloud droplets which almost collide in an electrostatic field. The recent work substantiates this hypothesis. If one refers to Figure 2, it will be easier to visualize the situation involved in the following reasoning. In a vertical uniform field the drops will be polarized so that opposite charges appear on the near surface of the droplets as the small drop approaches the underneath side of the large drop. For drops which just miss colliding, the new theoretical results now show that a breakdown potential or a force sufficiently great

to shear off a small portion of one drop appears between the closest points on the surface. Either way, the droplets exchange charge when the small drop is underneath the large drop. Due to the vertical asymmetry of the relative trajectory shown by Figure 2, charge is not transferred back when the small drop moves around the top side of the larger drop. When the droplets separate, they are charged in such a way that the existing field is enhanced as their separation increases.

Applying this theory to the modification of the field within the cloud, it appears that one should spray the top of a cloud layer or fog with droplets slightly larger than those of the cloud to increase the field within the cloud and incidentally provide charged droplets to further enhance the possibility of collision. The fields necessary and the most suitable drop size undoubtedly depend on the fog itself. Further research is required to determine these. Without practical evaluations, the probability of positive results in fog modification is extremely small.

COMPARISON OF HYDRODYNAMIC AND ELECTROSTATIC COLLISION EFFICIENCIES AND MAXIMUM FIELD REQUIRED

	1	2	3	4	5	6
Case Number	Large Drop	Small Drop	Initial Separation	Hydrodynamic Collision Efficiency	Electrostatic Collision Efficiency	Approximate Field Required
34	19 $\mu$	9.5 $\mu$	12.8 $\mu$	0.03	0.45	24
35	19	11.3	15.8 $\mu$	0.12	0.69	101
36	19	13.3	15.5 $\mu$	0.09	0.66	34
33	19	13.3	7.9 $\mu$	0.09	0.17	02
1	20	6	9.4 $\mu$	0	0.22	88
2	20	6	3.0 $\mu$	0	0.02	24
24	30	27	43.5 $\mu$	0.87	2.04	110
16	30	27	30.8	0.87	1.1	11
23	30	24	51.8 $\mu$	1.4	2.9	204
15	30	24	38.8	1.4	1.7	17
22	30	21	46.3	1.5	2.9	200
14	30	21	39.3	1.5	1.7	28

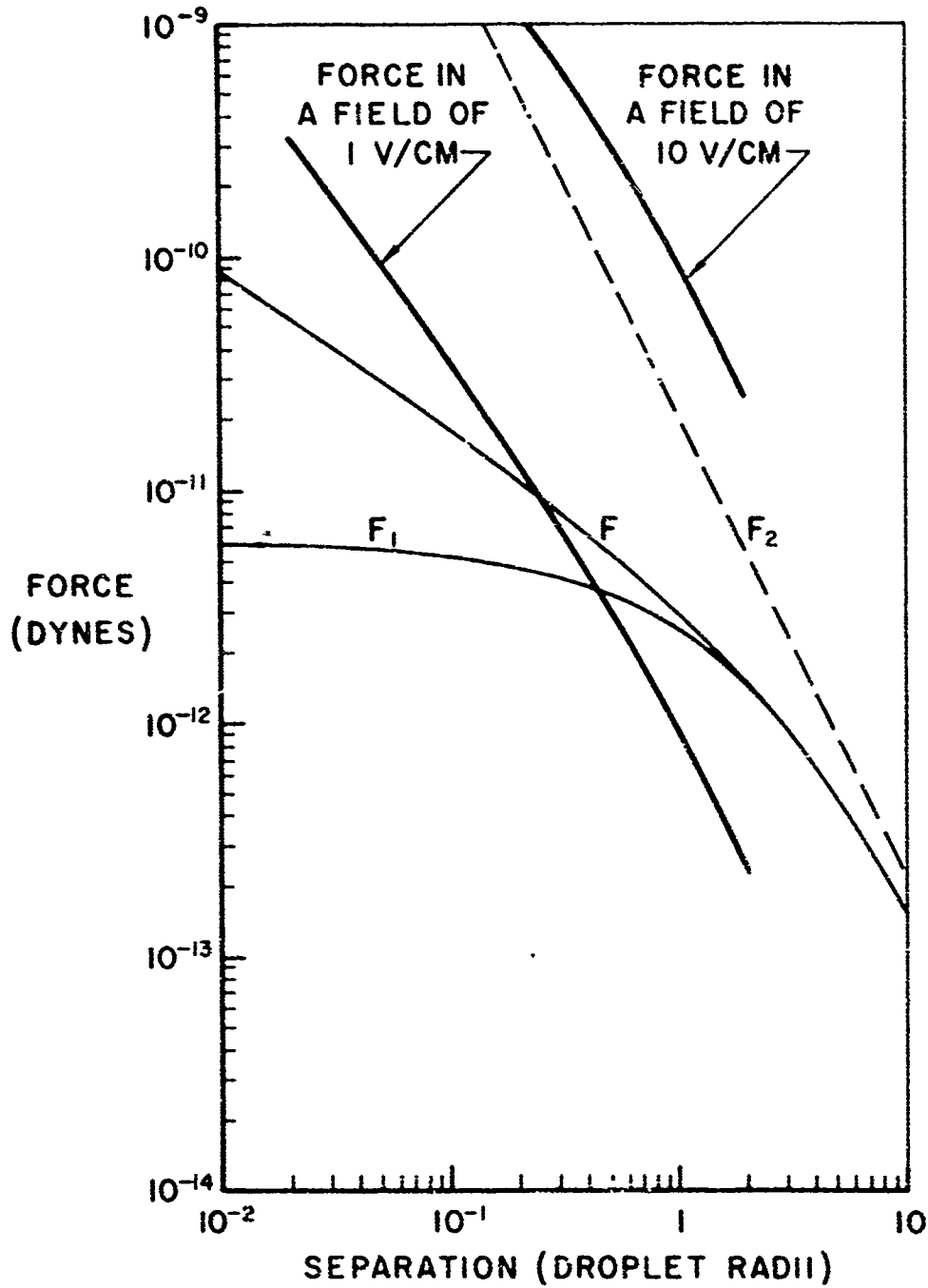


FIG. I

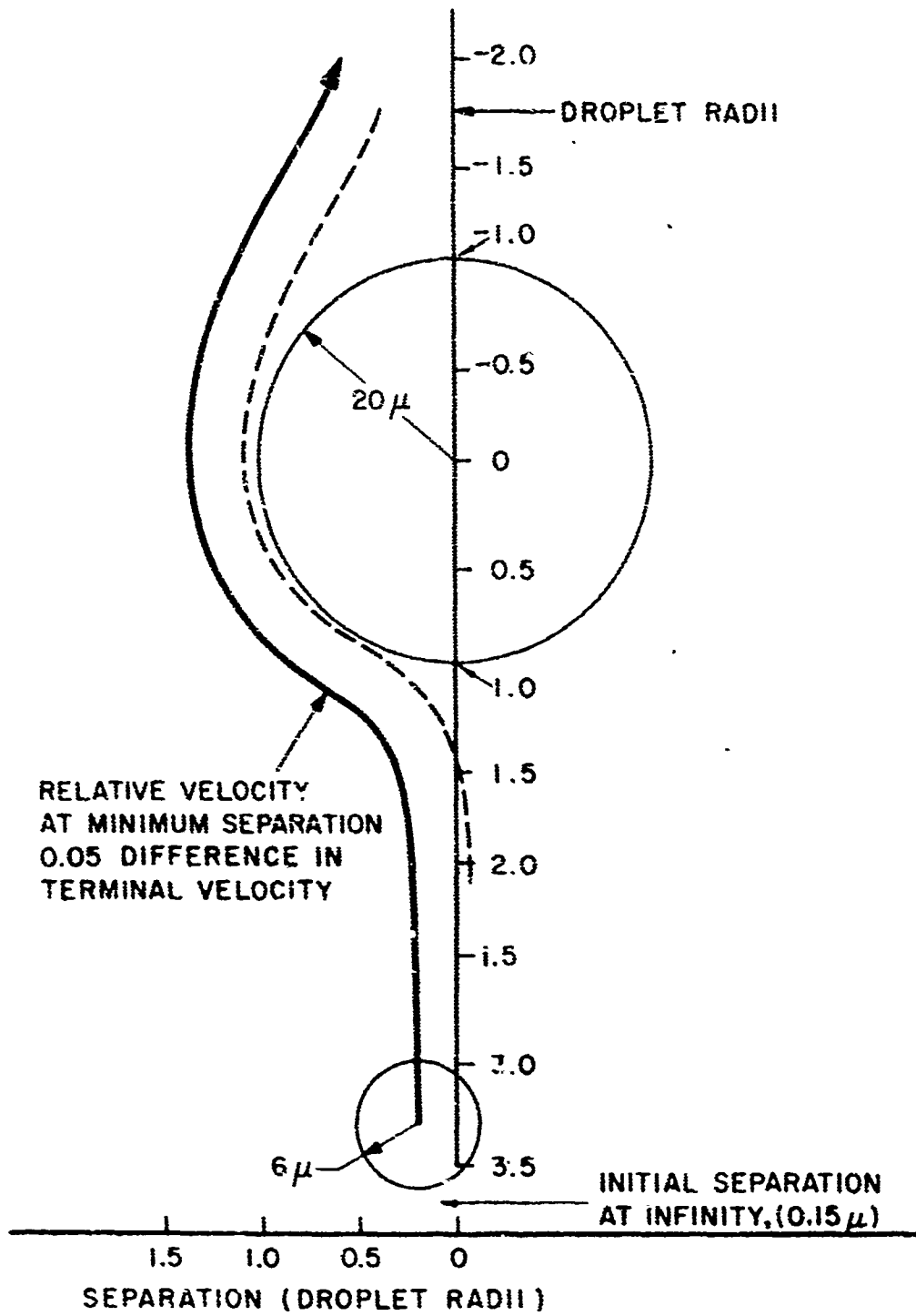


FIG. 2

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