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**A METHOD FOR ESTIMATING THE FATIGUE LIFE OF
7075-T6 ALUMINUM ALLOY AIRCRAFT STRUCTURES**

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FOREWORD

This report was prepared by Clarence R. Smith of the Fatigue Laboratory, Engineering Department, General Dynamics/Convair of San Diego, California, under Navy Contract N156-41307 for the Aeronautical Structures Laboratory, Naval Air Engineering Center, Philadelphia, Pennsylvania. This contract was initiated under problem assignment No. RAAD-1-23-60 and was monitored by M. S. Rosenfeld, Superintendent, Structures Research Division, Aeronautical Structures Laboratory.

Other General Dynamics/Convair personnel who made major contributions to this program were: G. G. Green, Chief of Structures, G. D. Lindeneau, Research Test Engineer, and R. B. Alden and E. J. Wehrhan, Technicians.

ABSTRACT

The purpose of this investigation was to assess the validity of the "Smith Cumulative Damage" hypothesis for 7075-T6 aluminum alloy specimens and structures.

It was found that the results of a single-amplitude test (at short life) can be used to estimate the stress at the point of failure, including residual stress. This permits using S-N data for axially loaded un-notched specimens to predict spectrum life.

Excellent agreement was found between calculated and experimental lives of full-scale structures; however, test lives of small specimens were consistently shorter than predicted.

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1 INTRODUCTION

One of the problems related to substantiating fatigue reliability of an aircraft structure is that of obtaining a load spectrum representative of that experienced in service. The main difficulty stems from the fact that service usage for the same airplane differs to such an extent that no given spectrum can possibly be representative of all the airplanes in a given fleet.

This wouldn't be so bad were it not for the wide discrepancies in lives attributed to either adding or deleting one or more high loads early in the lifetime. In some cases, the addition of a single high load will double fatigue life¹ while in others it may cut the life in half.² Whether life is increased or decreased depends largely on the construction of the particular structure. This is particularly true where Taper-Lok bolts are used for fatigue life improvement.

The worst thing about using spectrum test results is that, having passed spectrum test requirements, an illusion of reliability is created which is not always commensurate with fact. Also, once the test is made and it is found that the loads experienced in flight differ substantially from those used in the spectrum, it is extremely difficult to determine which of the loads did what part of the damage.

Inasmuch as airplane reliability is the real objective of any test, the question arises as to whether something other than a spectrum or simulated service (random) load test might be more appropriate for those portions of the structure subjected primarily to some load spectrum.

While constant-amplitude tests are generally used for components whose service loading is constant-amplitude (catapult hooks, catapult holdbacks, mechanical systems, etc.), they are seldom used in qualifying structures which are subjected primarily to spectrum loading. This may be largely due to a greater abundance of test apparatus specifically designed for spectrum loading. Notwithstanding advances in spectrum loading apparatus, the cost of running a spectrum test on a large wing is many times the cost of running a constant-amplitude test at limit loading. In addition, the elapsed time for constant amplitude testing can be 10 percent or less of the time required for spectrum tests. This alone could reflect huge savings in modifications on assembly-line airplanes if modifications can be made before too many airplanes are built.

References 4 and 5 describe a method for using data obtained from unnotched fatigue specimens in combination with results of a constant-amplitude test on a structure for predicting spectrum life of that structure. Constant-amplitude loading of the structure is sufficiently high to insure plastic flow at the point of eventual failure. This permits using ordinary stress-strain and S-N curves for unnotched material to

ascertain stress at the concentration. Knowing the stress at the concentration, it is then possible to predict the life of similar structures for any mixture of loads by using unnotched fatigue data, care being taken to account for the influence of residual stresses remaining from high loads in the spectrum. This method will be discussed further in Section 3; however, a salient feature needing mention at this time is that the Miner relation was used in terms of actual stress at the concentration.

A simple approach might be to bypass having to use $\sum n/N$ at all. This would get away from much controversy provided a direct relation between constant-amplitude and spectrum life exists. While there will always be as many load spectra as airplanes, a definite relation between constant-amplitude life, say at 100 percent limit loading, $R = 0$, and appropriate spectrum life might prove of value, particularly for testing components in early stages of design. Although load spectra and limit load values may subsequently change, the constant-amplitude datum point would make a convenient reference for extrapolating to other load conditions.

Although the method described in References 4 and 5 (the Smith Method) relates constant-amplitude life to spectrum life by assuming $\sum n/N = 1$ for unnotched specimens, the crux of the theory has nothing to do with any particular value of $\sum n/N$. Rather, it is a means for obtaining stress at the point of failure which permits evaluating $\sum n/N$ in terms of unnotched fatigue data. Accordingly, a choice of $\sum n/N$ can be made to suit the particular circumstance. It is anticipated that $\sum n/N < 1$ for stress ratios of less than -0.5, and $\sum n/N > 1$ where the stress ratio (R) is greater than -0.5. This will be discussed further in Part 3 of this report.

Throughout this report, the stress ratio (R) is defined as the ratio of the minimum stress (including residual stress) divided by the maximum stress at the concentration as well as for gross area stress of unnotched specimens. Where (R) implies load ratios, the symbol R_p is used. The following loading conditions were investigated:

1. where constant-amplitude and spectrum loading is at $R_p = 0$
2. where constant-amplitude and spectrum loading is with 1 G minimum load
3. where constant-amplitude loading is at $R_p = 0$ and spectrum loading is with 1 G minimum load.

The purpose of this investigation is to assess the validity of the "Smith Cumulative Damage" hypothesis for 7075-T6 aluminum alloy specimens and structures. While the method should be equally applicable to other materials, the validity would have to be verified by test.

This investigation includes development of experimental data and comparison of theoretical predictions and experimental results as follows:

- a. Development of experimental stress-strain and S-N curves for unnotched 7075-T6 aluminum alloy. These constitute a vital part of the Smith method which likens the stress at a concentration to unnotched specimen equivalents.
- b. Life predictions for full-scale structures by the Smith method and comparisons with actual test results for full-scale structures.
- c. Development of experimental S-N curves for $R = 0$ and spectrum load life data for specimens using open holes, idle NAS 333 bolts, idle Taper-Lok bolts, and butt joint test specimens. These were performed to evaluate the effect of various restraints of stress (different in each specimen type) at the concentration and corresponding effect on spectrum fatigue life.

2 TEST PROGRAM

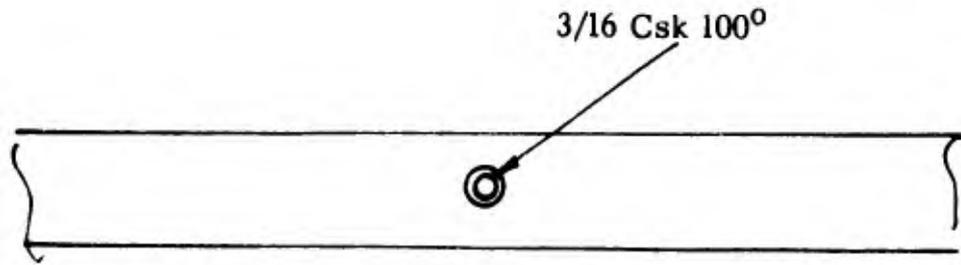
MATERIAL

The material for the specimens used in this program was nominally 0.10 in. thick 7075-T6 aluminum alloy. Average yield strengths were between 74,000 psi and 77,000 psi with 82,500 psi ultimate strength. Some of the material had a yield strength in excess of 77,000 psi and ultimate strength of 84,000 psi. Specimens made from this material were marked with (H) signifying "high yield". Another group having yield strengths lower than 74,000 psi were identified by (L) signifying "low yield".

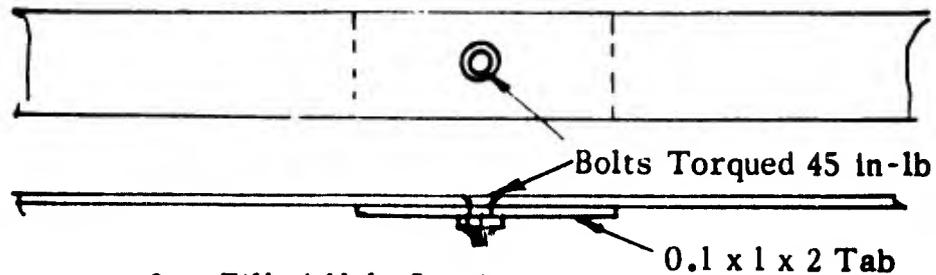
SPECIMENS

Specimens for this program (Figure 1) included the following:

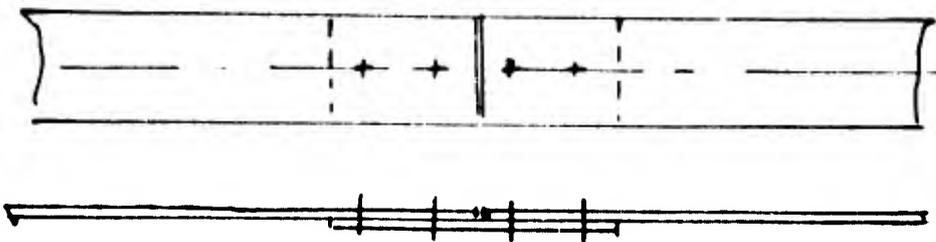
1. One-inch-wide specimens with centrally drilled 3/16 in. dia. 100° countersunk holes
2. One-inch-wide specimens with centrally drilled 100° countersunk holes filled with unloaded fasteners
 - a. NAS-333 screws (0.001 clearance)
 - b. Taper-Lok TL-100-3 bolts (0.003 interference)
3. One-inch-wide single shear butt joints having 3/16 in. dia. fasteners spaced 1 inch apart in tandem and 1/2-inch edge distance. Doubler material was the same thickness as for basic test specimens except as noted in Table XI where special doublers were used to avert failure in doubler material. Fasteners were
 - a. NAS-333 screws (0.001 clearance)
 - b. Taper-Lok TL-100-3 bolts (0.003 interference)
4. One-inch-wide double shear butt joints with AN470DD8 rivets spaced one inch apart in tandem with 1/2-inch edge distance. Splice doublers were same thickness as basic material.
5. One-inch-wide unnotched specimens with 10-inch radius from test section to widened section provided for gripping. Holes along the centroidal axis near the ends served as alignment guides prior to clamping in the fatigue testing machine. A 20-microinch rms finish was accomplished by lengthwise strokes with 00 grit emery paper.



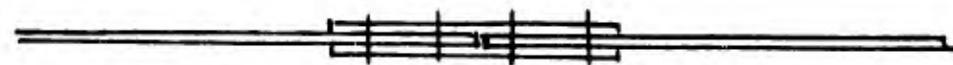
1. Center Hole Specimen



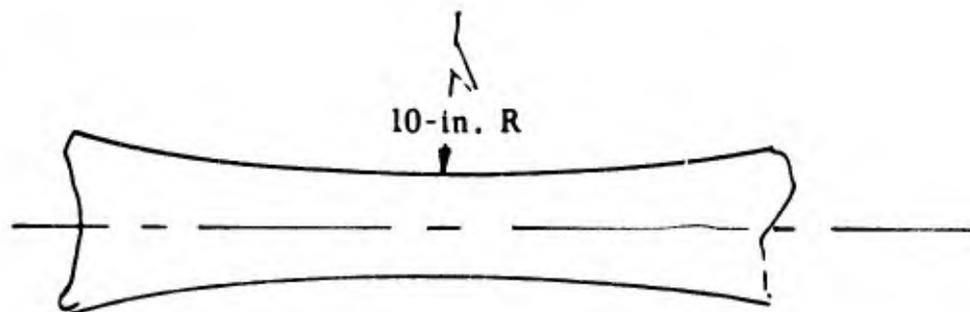
2. Filled Hole Specimen



3. Single Shear Bolted Butt Joint



4. Double Shear Riveted Butt Joint - AN470 DD8 Rivets



5. Unnotched Specimen

NOTE: All specimens were made of 0.1-inch-thick 7075-T6 aluminum alloy having a width of 1 inch at the test section.

Figure 1. Specimen Configuration

FATIGUE TESTING MACHINES

Constant-Amplitude

Constant-amplitude tests for lives in excess of 5000 cycles, except as noted in Table XIII, were made in Sonntag constant-load type fatigue testing machines. A Tatnall-Budd hydraulically operated fatigue testing machine was used for lives below 10,000 cycles. This machine is discussed under Spectrum Loading. The rate of loading for Sonntag machines was 1750 cpm while loading for the Tatnall-Budd machine was held at 5 cps except as noted in Table XIII.

Spectrum Loading

Spectrum tests were made in a Tatnall-Budd hydraulically operated fatigue testing machine having provision for 12 loading steps. Each load is governed by a preset value at any one of the load channels. Load is controlled by a function generator signal to a hydraulic servo valve. A strain gage dynamometer is part of the closed-loop system to maintain the load for all levels. Loading rate was held at 5 cps for steps having 3 or more cycles. See Table I for load schedules. Steps involving 1 cycle were applied at a rate of approximately 1 cps (115% and 125%, Condition IV).

TEST FIXTURES

Clamping

Axial loading was ensured by special three-bolt clamping fixtures which clamped the widened specimen ends. The center bolt (aligned with the centroidal axis of the specimen) was the only one capable of carrying shear. It was used in applying a small preload to the specimen for aligning purposes prior to clamping.

Lateral Support

Lateral buckling was prevented during compressive loading by steel guides attached directly to the end fixtures. A 1/32-inch clearance was provided throughout the entire specimen length to prevent load transfer to the fixture. Buckling within the 1/32-inch gap was not considered significant.

TEST RESULTS

Test data are presented in Tables II through XIII. Tables II through VII present constant-amplitude data for specimen types 1 through 4 (Fig. 1). Spectrum data for similar specimens are presented in Tables VIII through XII. Table XIII presents constant-amplitude data for unnotched specimens. See LIST OF TABLES for contents and page numbers.

A comparison of fatigue lives for constant-amplitude loading of center-hole specimens (Types 1 and 2, Fig. 1) is shown in Figure 2. Of particular significance is the fact that an increase in fatigue strength is experienced where holes are filled with either NAS-333 bolts or Taper-Lok bolts; however, the increase is greater with Taper-Lok.

Single shear bolted butt joints fastened with NAS-333 bolts and Taper-Lok bolts are compared for constant-amplitude loading in Figure 3. Here it will be seen that, while the fatigue strength of joints with Taper-Lok bolts is superior in the short life range, no appreciable difference is found at long lives. This is because failures occurred in the doublers at the washer edges away from fastener holes. Contributing causes for such failures may be :

- a) friction due to clamp-up
- b) fretting
- c) additional stress concentration at the edge of the washer where the washer digs into the doubler due to bending of the specimens

While bending is also a factor at high loading, the stresses at the bolt holes were believed high enough to promote failure at the holes instead of at the edges of the washers, presumably because the ratio of bending stress to stress at the hole is lower at high loading where partial alignment reduces eccentricity. Spectrum lives of similar joints are compared in Figure 4. Testing for positive loading (MIL-A-8866 - Spectrum A), various values of assumed limit load stress are used to demonstrate relation of spectrum life to limit load stress. As might be expected from constant-amplitude results, the superiority of joints with Taper-Lok bolts diminishes where failure takes place in areas away from fastener holes. No explanation is apparent for the crossover below 30,000 psi, other than that if there were one, it would involve washers and not the bolts themselves.

In an effort to avert doubler failures, several experiments were made using various doubler types and lateral stabilization. The results of these tests are given in Table III. Since the object of this program is to develop a method for assessing cumulative damage, the mode of failure was not considered of sufficient importance to continue further testing.

As in the case of single shear butt joints, center-hole specimens (Fig. 1) were tested for positive loading (MIL-A-8866 - Spectrum A) using various values of assumed limit load stress. Results of these tests are shown in Figure 5 (Ref. Tables VII, IX and X).

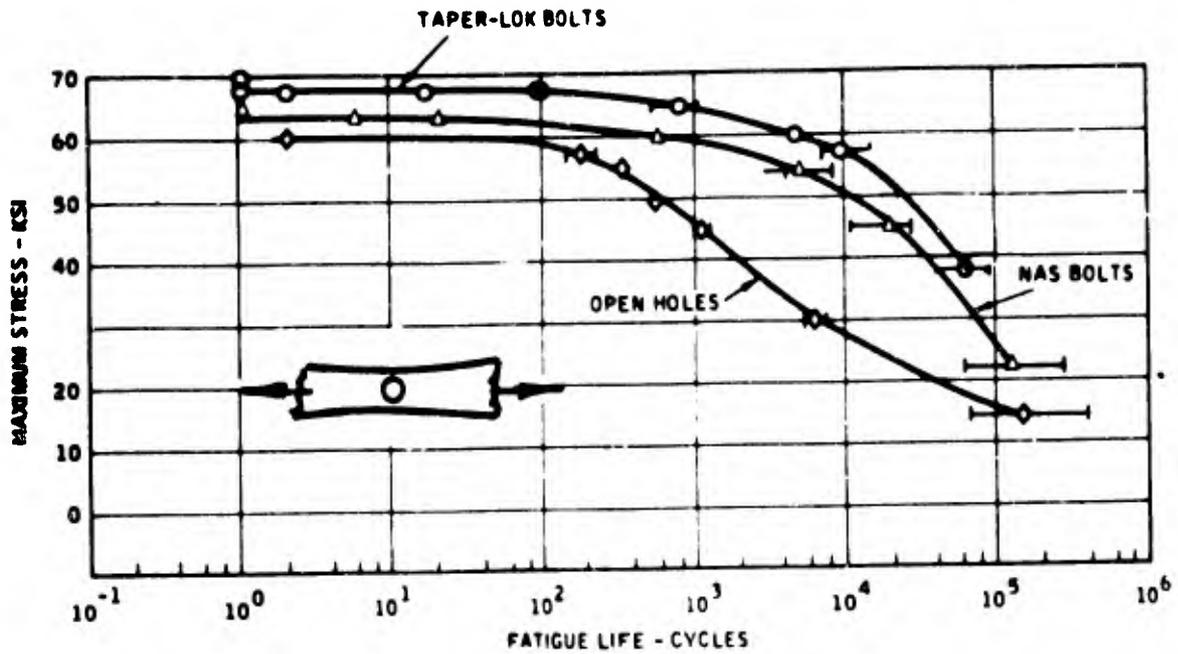


Figure 2. Fatigue Life of 1-in.-Wide Strips of 7075-T6, with 3/16 in. Diameter 100-deg. Countersunk Holes Compared with Lives When Filled with Unloaded NAS-333 Bolts and Taper-Lok Bolts at $R_p=0$

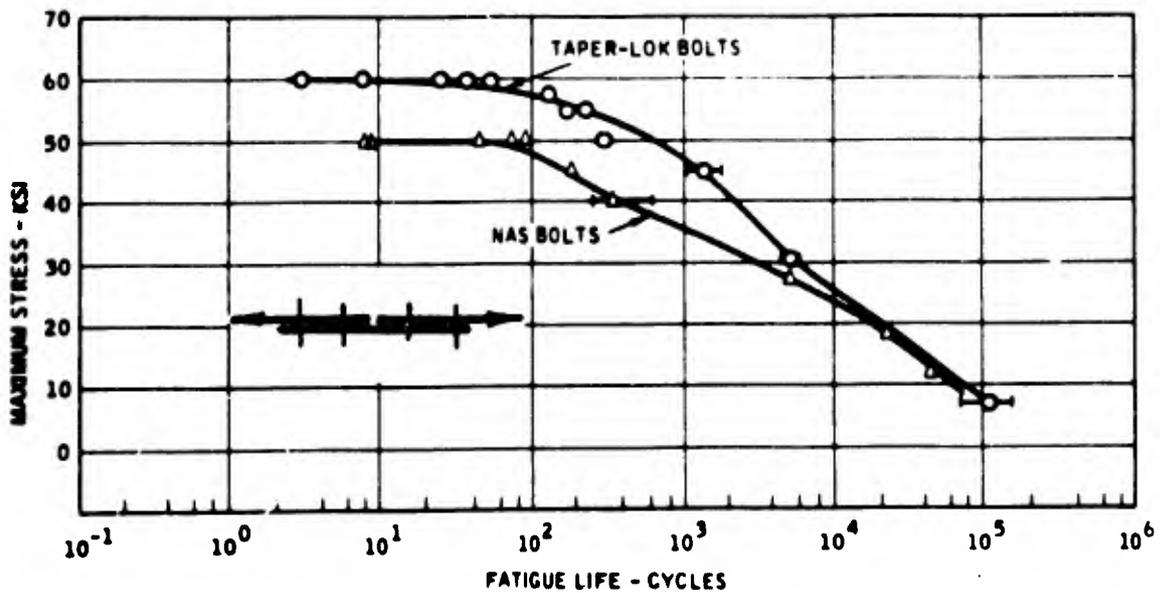


Figure 3. Fatigue Life of Single-Shear Butt Joints Using NAS 333 Bolts Compared with Lives of Joints Using Taper-Lok Bolts at $R_p=0$

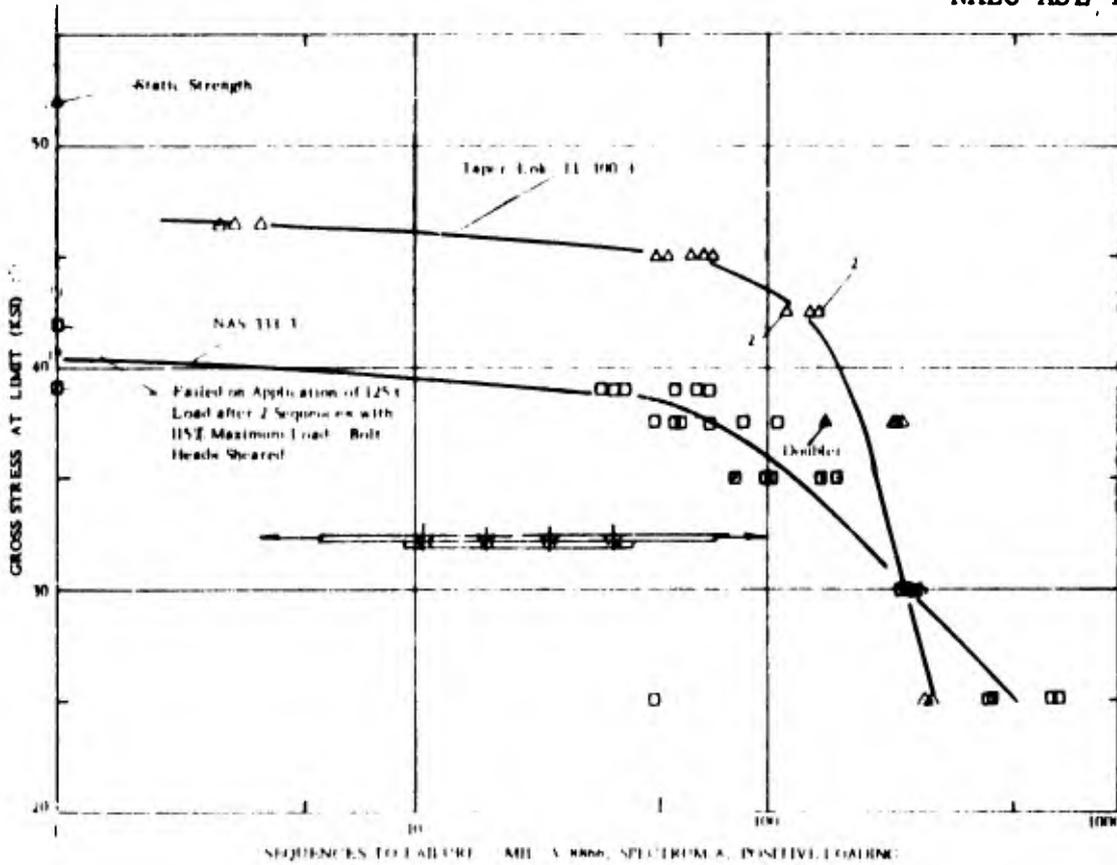


Figure 4. Effect of Varying the Reference Stress Level in a Spectrum on the Fatigue Life of Single-Shear Butt Joints

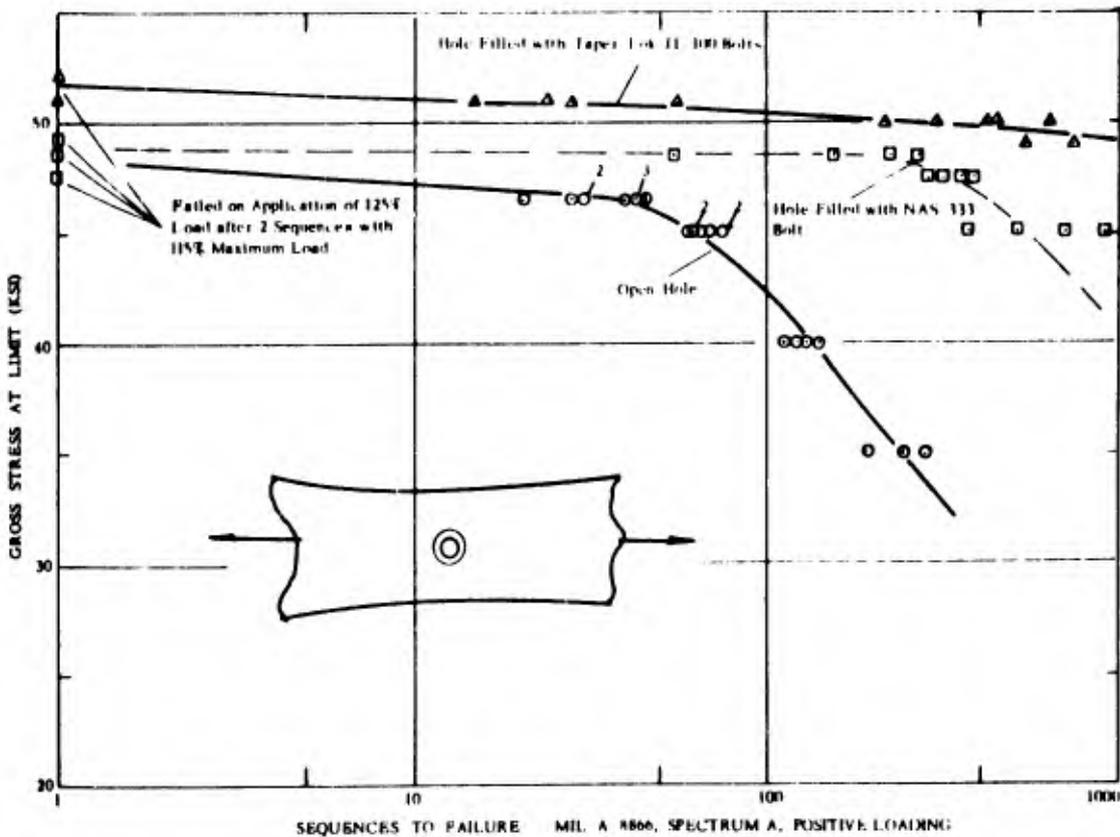


Figure 5. Effect of Varying the Reference Stress Level in a Spectrum on the Fatigue Life of Specimens with Open and Filled Holes

With reference to the data presented in Tables IV and V on double shear butt joints having high, medium, and low yield strengths; average lives for constant-amplitude loading at 45,000 psi gross stress ($R_p = 0$) were 5,248, 5,002, and 3,719 cycles, respectively. Spectrum lives are presented in Table V. Here it will be seen that the medium yield specimens exhibit a slight superiority over high or low yield specimens. Calculations presented in Table VI predict this trend; however, differences are of insufficient magnitude to justify definite conclusions.

Results of constant-amplitude tests on axially loaded unnotched specimens are presented in Figure 6, using the data from Table XIII. Note that some of the apparently wild points appear where standard deviations were fairly low. This would seem to indicate that all of the specimens in the group were run at one machine setting which might have been wrong in the first place. However, no two specimens were run successively at a given load. Rechecks indicated no error in loading. Since the worst discrepancies appear with compressive loading, it seems reasonable to suppose that there might have been some buckling under these high compressive stresses since 1/32 in. clearance was provided between specimen and lateral support guides in the compressive fixture.

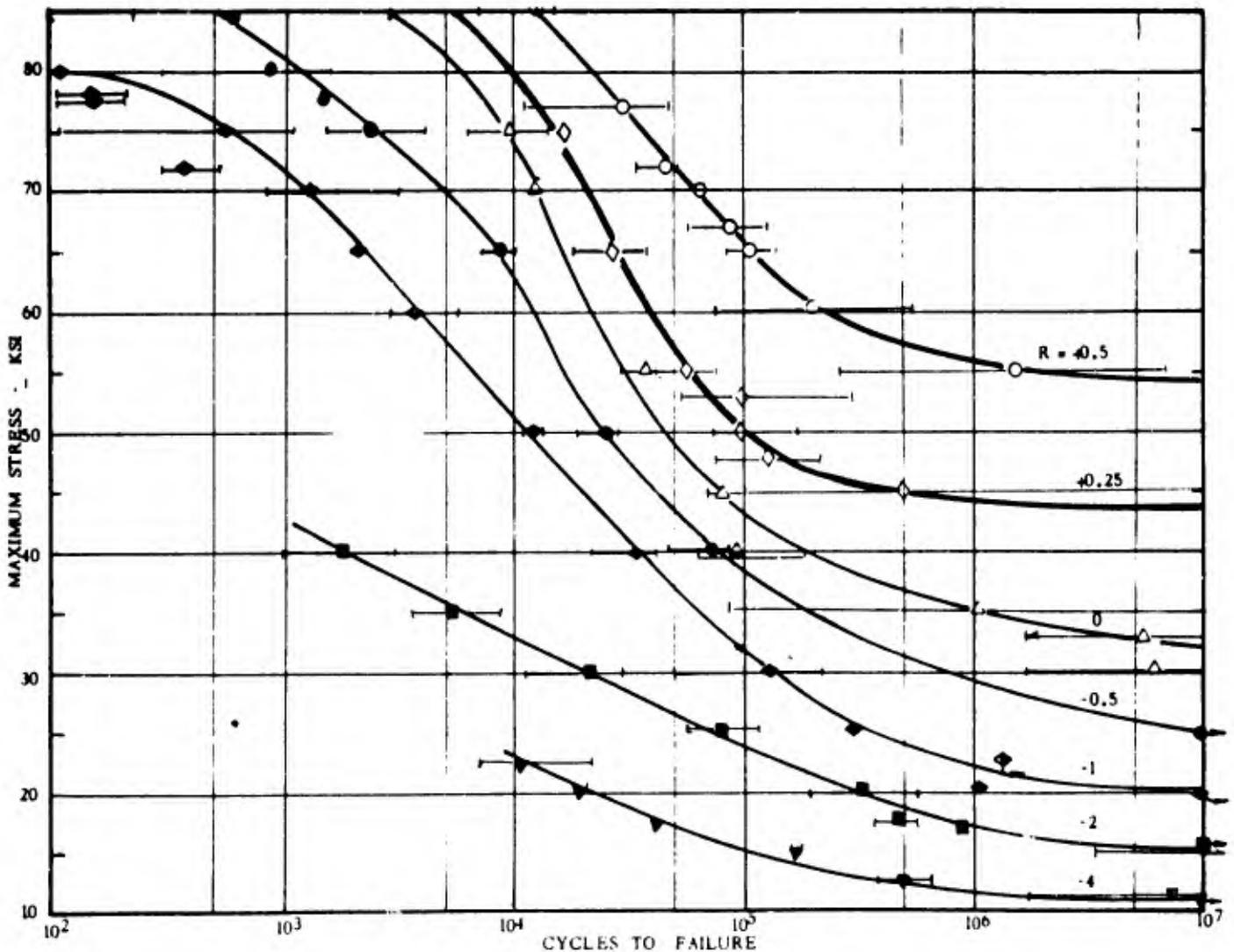


Figure 6. S-N Curves for 7075-T6 Aluminum Alloy Sheet

3 THEORY

If one were asked specifically "what causes fatigue failure in metallic structures?" the logical answer would be "stress cycles." What kind of stress cycles? Can it be a summation of various kinds of cycles? Were it possible to accumulate fatigue damage in accordance with magnitudes and positions of stress cycles, the first task would be to determine what kind of stress cycle exists at the point of fatigue failure. The method about to be described attempts to do just that. By working backward from a fatigue failure caused by a given number of constant-amplitude load cycles, an attempt is made to determine the kind of stress cycle that corresponds with the load cycle causing failure. Were it possible to determine the stress cycle for a given loading, it follows that this particular cycle could be prorated for other loads. In this way, an accumulation of fatigue damage could be based on actual stress rather than on loads.

Since the program involves 7075-T6 aluminum alloy, first considerations will be on the basis that 7075-T6 displays ideal elastoplastic properties having stress-strain characteristics as shown in Figure 7. Allowance will be made later for strain hardening and Bauschinger effects.

Assume that our ideal elastoplastic material is first subjected to a strain ϵ_t which is comprised of elastic and plastic components ϵ_{te} and ϵ_{tp} . The strain ϵ_{tp} will remain as permanent set after load removal (Ref. Figs. 7a & 7b). A compressive strain ϵ_c equal to ϵ_t will completely remove all effects of previous loading in our ideal elastoplastic material, so that repeated strain cycling will exhibit a hysteresis loop as shown in Figure 7c.

Consider next the strain at a stress concentration. With tension loading the tension strain ϵ_t (comprised of its elastic and plastic components ϵ_{te} and ϵ_{tp}) will appear at a time when the strain away from the concentration might amount to a fraction of ϵ_t , depending on the concentration factor. At least, it can be considered sufficiently small that no plastic deformation occurs away from the concentration at a time when considerable plastic deformation occurs at the concentration. On unloading, the material in the elastically deformed region (away from concentration) is reacted by the excess plastically deformed material at the concentration. The net result is that on reaching equilibrium, the material at the concentration goes into compression while that away from the concentration stays slightly in tension, depending on the stress concentration. The strain, however, remains tension for both elastic and plastically deformed material. It is only by virtue of the large strain at the concentration that the material can retain a permanent tensile strain and yet be in compression. Furthermore, very large local tensile strains resulting from the applied load could actually cause the material at the concentration to yield compressively upon unloading because the resulting residual stress exceeds the compression yield stress of the material at the concentration. On reloading in tension, the material at the concentration will behave elastically until the total local strain $\epsilon_{te} + \epsilon_{ce}$ is exceeded. This is shown schematically in Figure 8.

Let us assume that the plastic strain ϵ_{tp} in Figure 8a > 0.2 percent. This means that repeated tension loading ($R_p = 0$) will actually cause the material at the concentration to experience stress cycling from tension yield to compression

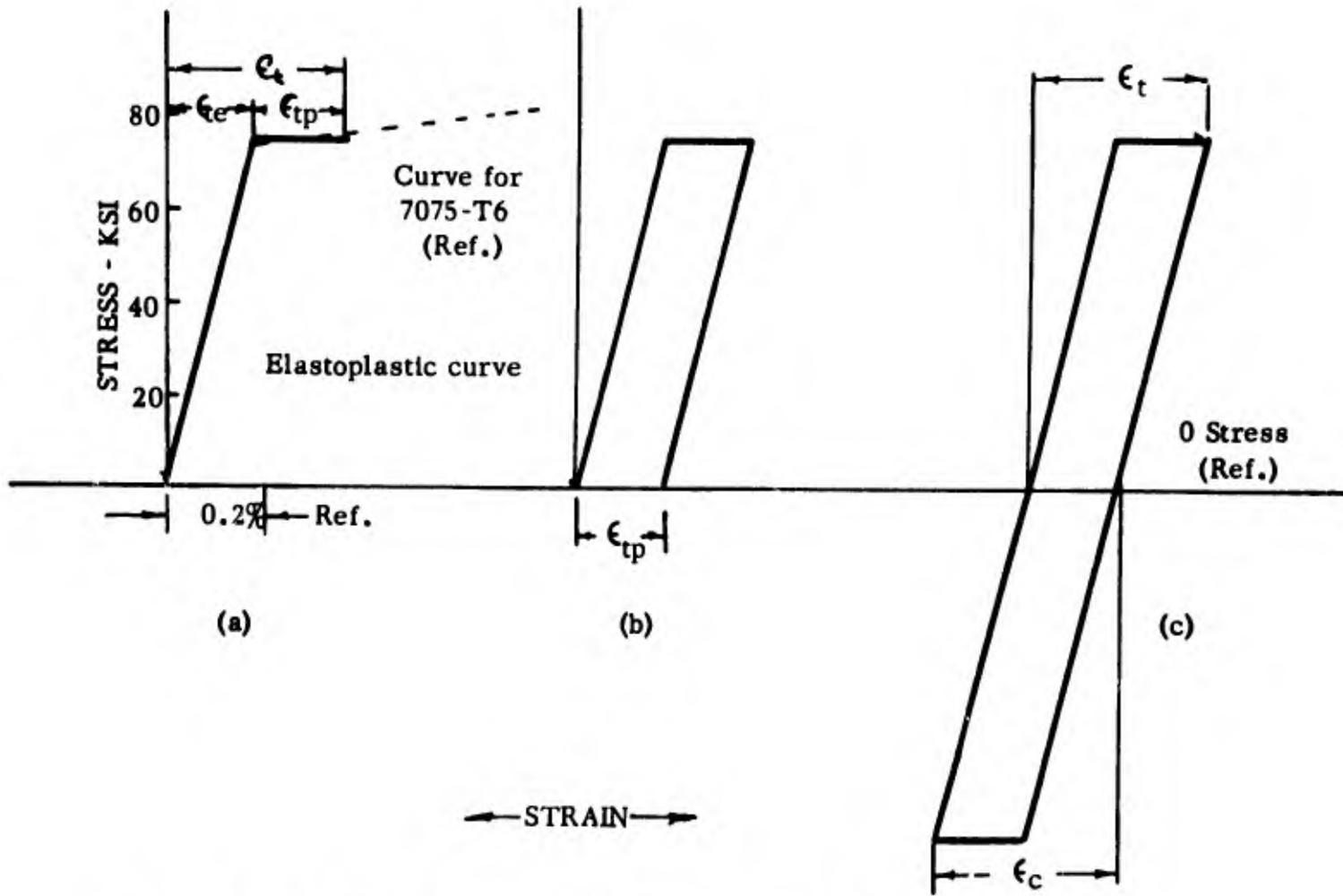


Figure 7. Cyclic Straining for Ideal Elastoplastic Material

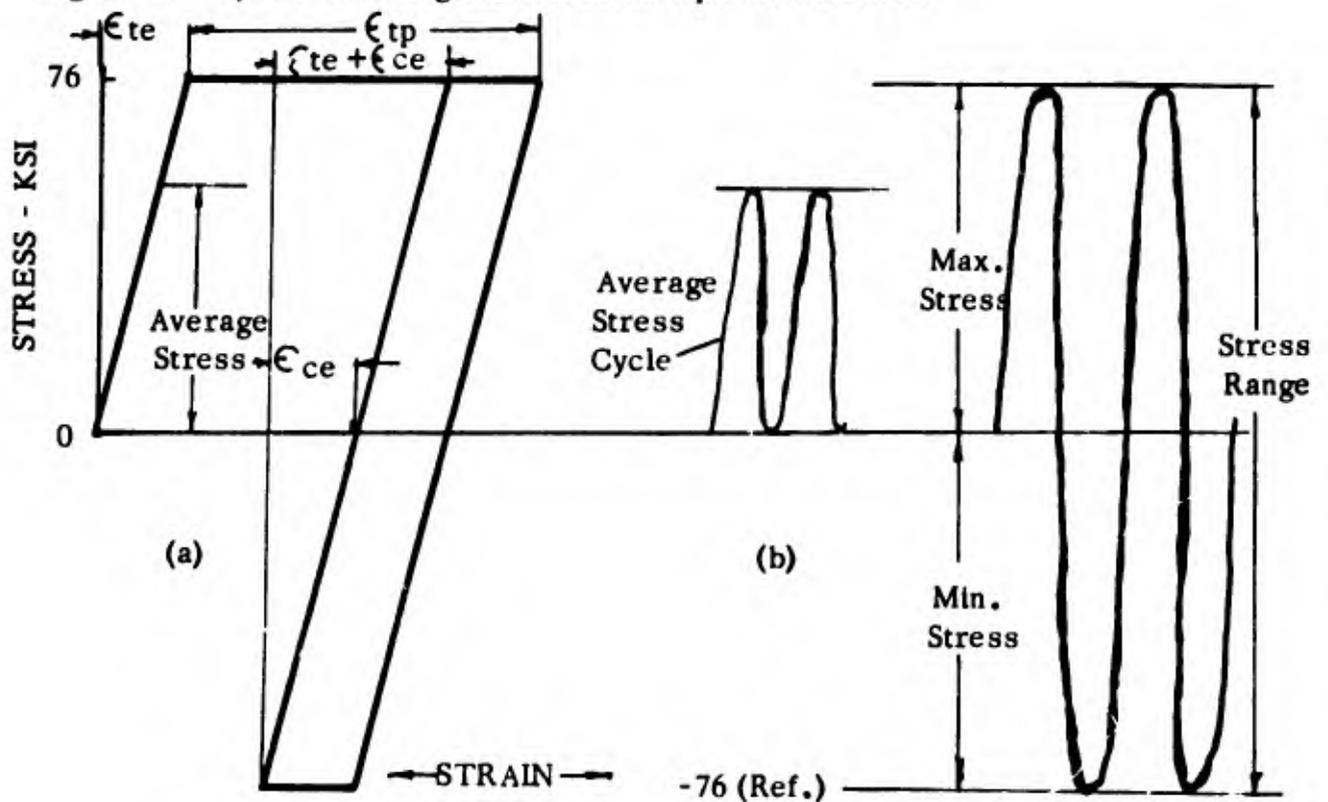


Figure 8. Large Strains in an Ideal Elastoplastic Material at a Concentration when Cycled at $R_f = 0$

yield in our ideal material. This is illustrated in Figure 8b. A value of 76 KSI is assumed for the yield strength of the elastoplastic material to make it compatible with 7075-T6 aluminum alloy. $K_t \cong 2$ is required to cause compressive yielding.

Although 7075-T6 is not an ideal elastoplastic material as considered in our examples so far, let us assume for the time being that strain hardening does not occur below the 76,000 psi stress level. This means that the material at the concentration shown in Figure 8 behaves as an unnotched specimen subjected to reverse stressing of 76,000 psi. Referring to the S-N curves in Figure 6, this means that a structure, cycled as indicated in Figure 8, should fail after about 500 cycles. By reverse logic, if a structure were to fail after 500 cycles of tension-tension loading, we can assume that the actual local stress cycle which caused the failure was $\pm 76,000$ psi, or a total range of 152,000 psi.

It should be noted that one stress range only conforms with a life of 500 cycles and a maximum stress of 76,000 psi. This is clearly shown in Figure 9 where a detailed section of S-N curves near 76,000 psi is presented. If the structure were to last for 2500 cycles instead of 500 cycles, the local stress cycle should be at $R = -0.5$ which indicates a local stress range of $1.5 \times 76,000$ psi = 114,000 psi. Similarly, the range of stress for any lifetime can be found by estimating the local stress ratio intercept at 76,000 psi for constant amplitude lives between 500 and 9,000 cycles. A shorter life would indicate strain hardening, which, according to our assumed ideal elastoplastic material, does not occur below 76,000 psi. According to the stress-strain curve shown in Figure 10, some strain hardening occurs below 76,000 psi; however, for the present our discussion will be confined to the ideal elastoplastic material. Strain hardening will be considered later.

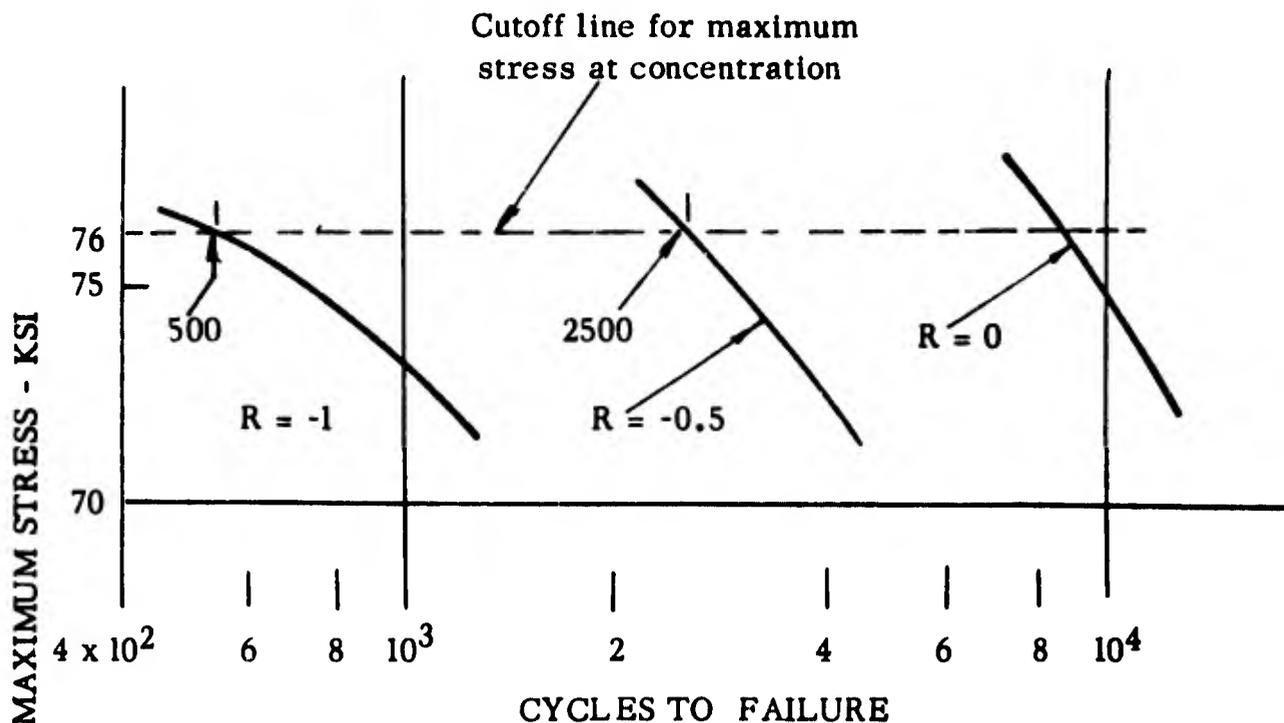


Figure 9. Section of S-N Curves Taken from Figure 6 Showing Intercepts for Illustrative Examples

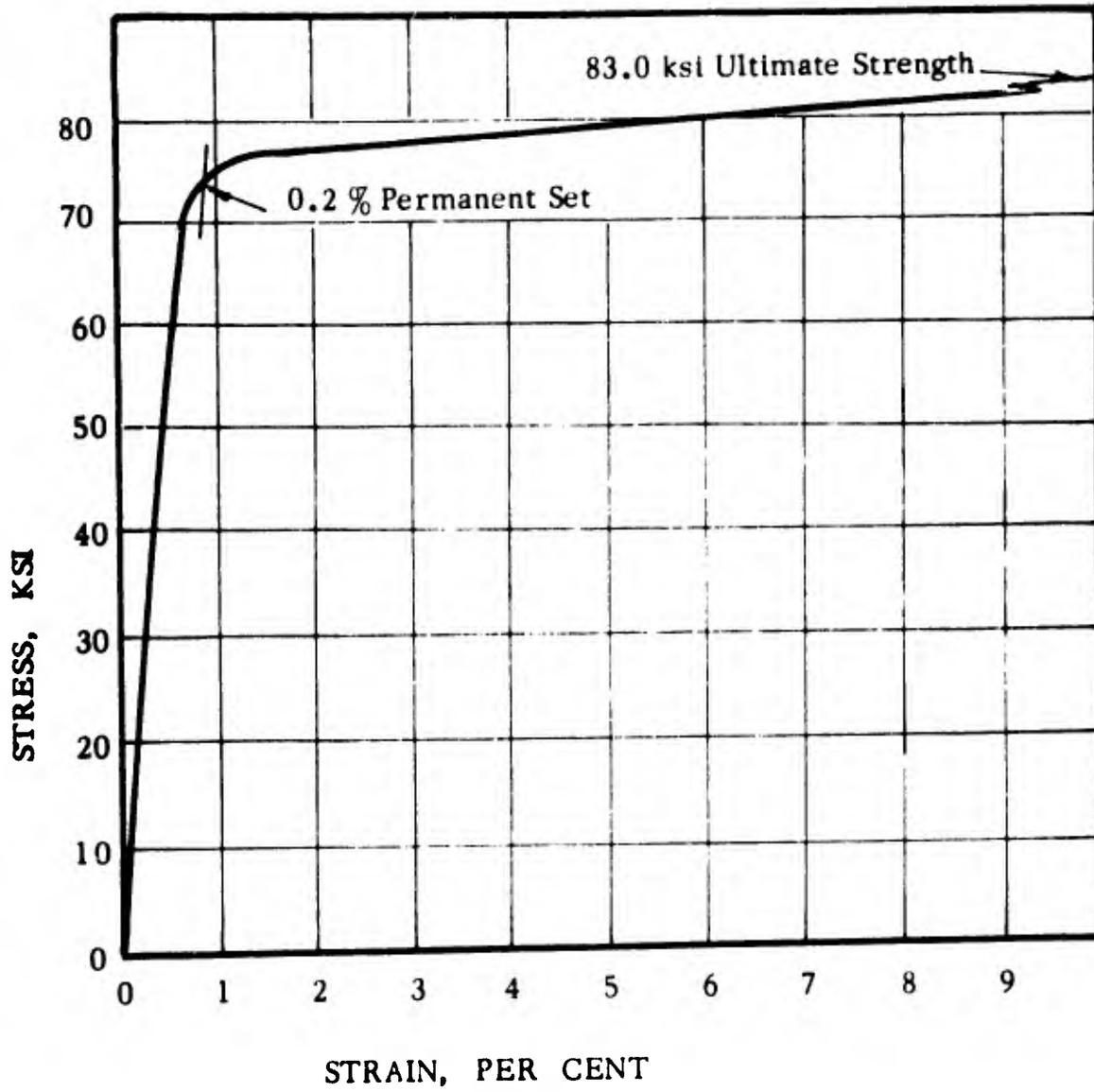


Figure 10. Stress-Strain Curve for 7075-T6 Aluminum Alloy Sheet

Continuing with our example where a structure fails after 2500 cycles, we found that the local stress range for the given load (say 100 percent limit load for convenience) was 114,000 psi. Should we want to know how long the structure would last when loaded to 50 percent limit load for $R_p = 0$, we simply multiply 114,000 x 0.5 obtaining a stress range of 57,000 psi (Fig. 11). The life can then be found on the curve for $R = 0$ and 57,000 psi (Ref. Figure 6) which turns out to be about 25,000 cycles. Had a load of 100 percent limit load been applied prior to cycling at 50 percent, the stress range (Fig. 11) would still be 57,000 psi; however, the stress would now cycle between -38,000 psi and 19,000 psi for a life of 350,000 cycles as obtained from Figure 6 for $R = -2$ and a maximum stress of 19,000 psi. The -38,000 psi represents a residual stress introduced by the 100 percent limit load and is found by subtracting 76,000 psi (assumed yield strength in our ideal elastoplastic material) from the stress range of 114,000 psi.

Similarly, were we desirous of knowing the life for the structure when cycled at 75 percent limit load ($R_p = 0$), the stress range would be $114,000 \times 0.75 = 85,000$ psi. Since 85,000 psi is greater than our assumed yield strength of 76,000 psi, the difference of 9,000 psi would appear as a residual compressive stress resulting in a stress ratio (R) of $-9,000/76,000 = -0.12$. This would predict a life of 6,500 cycles as interpolated between the curves for $R = 0$ and $R = -0.5$ (Fig. 9). Again, had the 100 percent load been applied prior to cycling at 75 percent, the stress cycle would start at -38,000 psi as before and the life would be about 20,000 cycles as found from the curves shown in Figure 6 at an interpolated stress ratio of -0.8. Stress cycling for 75 percent limit loading is schematically shown in Figure 12.

STRAIN HARDENING AND BAUSCHINGER EFFECT

Our discussion so far was based on the premise that we were working with an ideal elastoplastic material which is not subject to strain hardening nor Bauschinger effects. Practically all metals experience strain hardening when subjected to plastic straining - some more than others. Having experienced plastic deformation for a given load, a second application of a similar load will not cause flow until the previous load is exceeded. In effect, this increases the yield strength of the material in the direction of loading. For subsequent loading in the opposite direction, the yield strength is reduced. This is known as the Bauschinger Effect, which says that a structure loaded at $R_p = 0$ could not cause the stress at the concentration to reverse as envisioned in Figure 8. Instead, having yielded in tension, the compressive yield strength should be lessened so that the stress cycle at the concentration would more likely appear as though cycling at $R = -0.9$ or thereabouts.

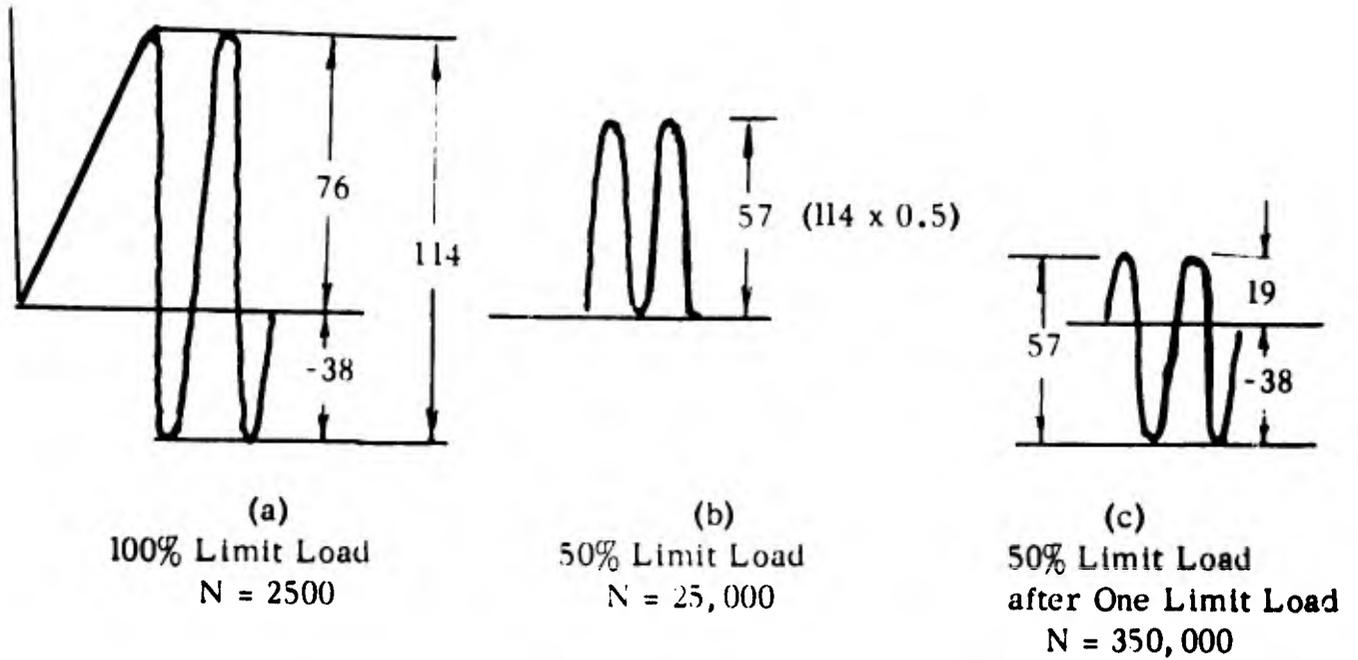


Figure 11. Stress Cycling at Concentration for a Structure Loaded at (a) 100% Limit, (b) 50% Limit, and (c) 50% Limit after One or More Limit Load Applications

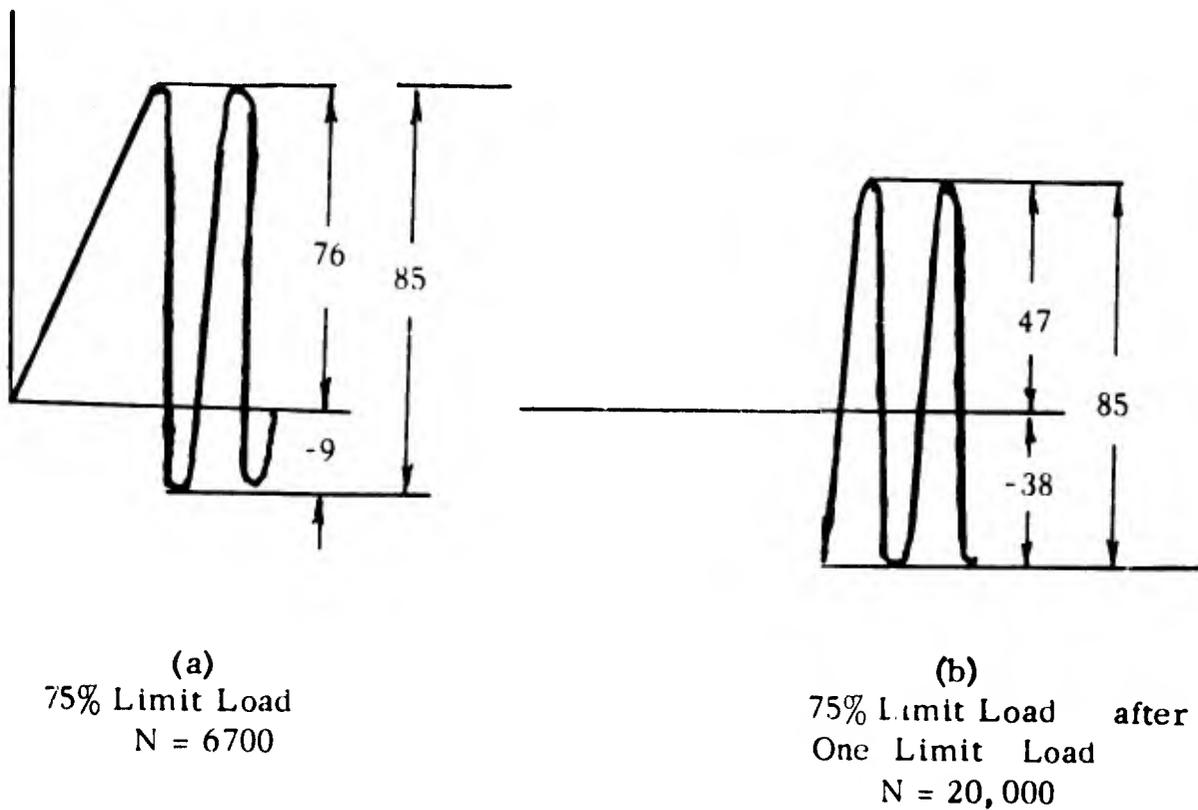


Figure 12. Stress Cycling at Concentration where Structure is (a) Cycled at 75% Limit Load, (b) after One or More Limit Load Applications

A most comprehensive piece of research by Crews and Hardrath of NASA Langley Research Center illustrates the effect of tension cycling on residual stresses at a concentration.⁸ This is shown in Figure 13. Note that for 50 ksi nominal stress, a compressive residual stress of about 41 ksi remains. This indicates a stress ratio (R) of about -0.83 for an original strain at the concentration of more than 1.5%. It is doubtful that a higher original load would result in much more residual stress. Higher loading would simply cause more plastic deformation which would reduce the amount of elastic material available for compressing the plastic material at the concentration.

While the amount of residual stress shown in Figure 13 results partly from the Bauschinger effect and partly from available elastic strain away from the concentration, no attempt will be made here to separate the two. Since the relative amounts of elastic and plastic material are related to the concentration factor, it follows that a concentration could be sufficiently high to overcome the Bauschinger effect. This, however, would be unlikely in a structure capable of meeting specifications. It would be safe to assume that the concentration factor might be greater than $K_t = 2$ in which case the stress ratio (R), resulting from 1.5% strain, might more nearly be in the neighborhood of -0.9 than -0.8, especially for material having a lower Bauschinger effect than 2024-T3.

In assuming that 7075-T6 remains ideally elastoplastic below 76,000 psi, a maximum stress cutoff line was established at 76,000 psi for stress at a concentration. Since yielding does occur below 76,000 psi (75,000 psi at 0.2% offset - see Figure 10) it is obvious that varying amounts of strain hardening occur between the nominal yield point and rupture. This means that the stress corresponding to 500 cycles (Ref. Figure 9) should have been more than 76,000 psi; however, probably not more than 78,000 psi since the corresponding strain (Ref. Figure 10) would be about 3.5%. This would be highly unlikely in a structure subjected to constant-amplitude fatigue loading.*

Let us assume that the maximum stress corresponding to a life of 500 cycles is 78,000 psi instead of our original 76,000 psi. Also, let us assume that the stress ratio (because of combined Bauschinger effect and equilibrium between elastic and plastic strains) occurs at $R = -0.9$ instead of $R = -1$ as used in the ideal elastoplastic material. The stress range would now amount to $78,000 \times 1.9 = 148,000$ psi instead of 152,000 psi as assumed with the ideal elastoplastic material at $R = -1$.

* A 100% limit loading at $R = 0$ was suggested for practical reasons. While higher loading might also serve as an index, a structure which would not last for 500 cycles of loading up to 125% limit would not be satisfactory in any event.

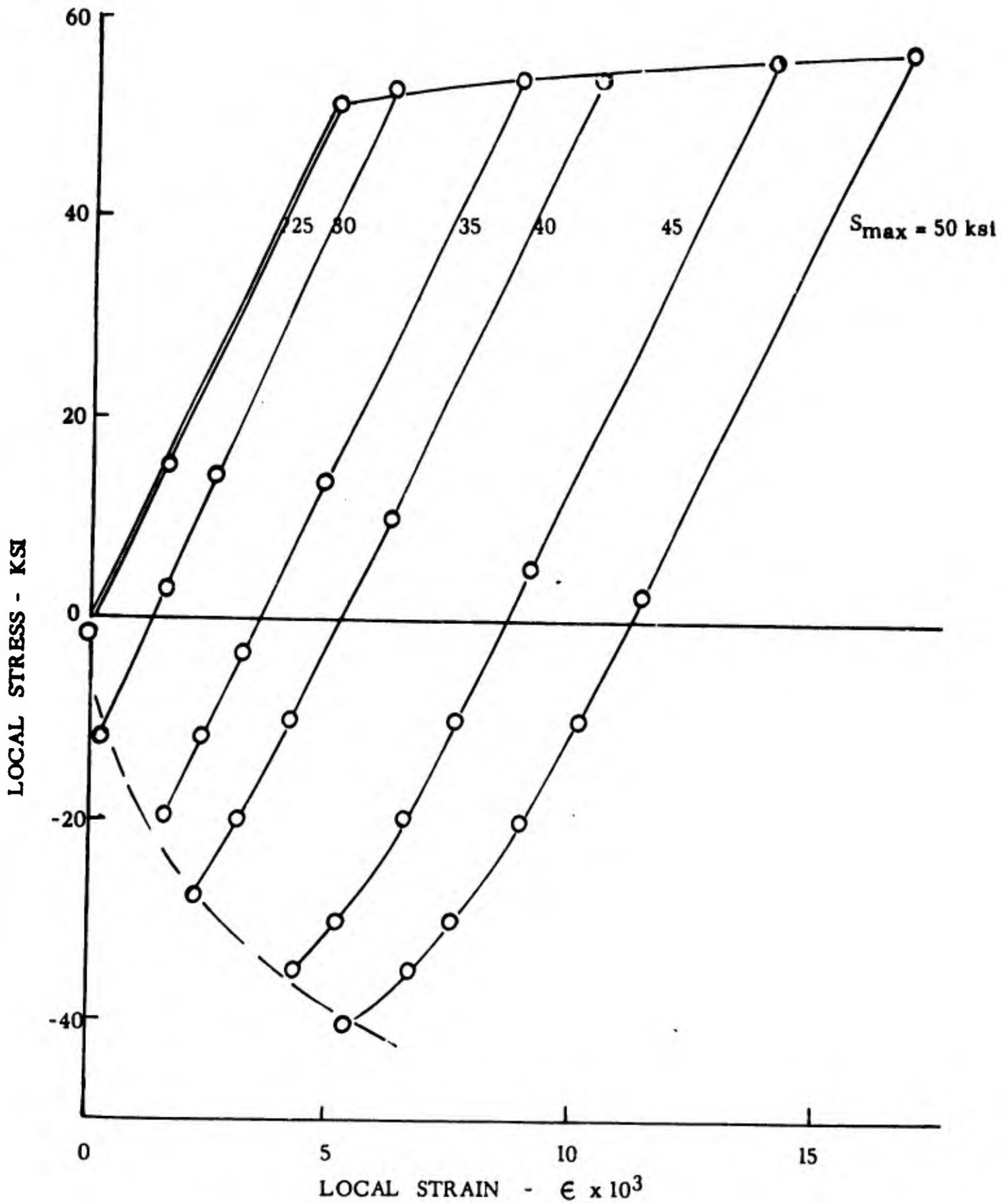


Figure 13. Local Stress-strain Curves for First Cycle Repeated (R = 0) Loading in Notched 2024-T3 ($K_t = 2$) - From Reference 8

While 3 percent would seem to be an acceptable error insofar as fatigue is concerned, considering normal test scatter, it is perhaps best to correct known discrepancies where they occur. Here it is not so much the stress range which is in error, but that the maximum stress cutoff line intercepts the S-N curve for $R = -1$. This infers too much residual compressive stress which is beneficial at lower load cycling.

The intercept at $R = 0$ in our ideal elastoplastic analogy is too high. Since the nominal yield strength (Ref. Figure 10) at 0.2 percent offset already contains some plastic deformation, the intercept for 74,500 psi yield strength should conform with an S-N curve having a negative stress ratio. According to Figure 13, the residual stress corresponding to 0.2% permanent set ($S_{max} = 30 \text{ ksi}$) amounts to about 12,000 psi for 2024-T3 material and a $K_t = 2$. Since the modulus of elasticity (E) for 2024-T3 is about the same as that for 7075-T6, the material used in this study, a residual compressive stress of 12,000 psi for 7075-T6 can be assumed for 0.2% permanent strain. This would indicate a stress ratio of $-12,000/74,500 = -0.16$. Since the proportional limit for 7075-T6 (Ref. Figure 10) is 71,000 psi, the maximum stress cutoff line should intercept the S-N curve for $R = 0$ at 71,000 psi.

To facilitate computation, the S-N curves from Figure 6 are redrawn in Figure 14 with interpolated reference lines between experimental curves. Also, some of the wobbles are smoothed out, using modified Goodman diagrams as aids for estimating positions of curves. The maximum stress cutoff line, as discussed above, is shown to intercept the S-N curve for -0.9 at 78,000 psi, the curve for -0.16 at 74,500 psi, and the curve for $R = 0$ at 71,000 psi. The cutoff line is curved between $R = -0.16$ and 0, more or less in accordance with the shape of the stress-strain curve (Ref. Figure 10) between 71,000 and 74,500 psi.

As it turns out, the predicted lives in our illustrative example on page 12 would be the same for either the ideal elastoplastic material or for 7075-T6 aluminum alloy, since the maximum stress cutoff line intercepts the S-N curve for $R = -0.5$ at 76,000 psi for both. Also, it would appear that very little error would be introduced where the structure (cycled at $R_p = 0$) fails anywhere between 500 cycles and 5000 cycles. However, for lives of fewer than 500 cycles and more than 5000 cycles, the corrected cutoff line shown in Figure 14 should be used.

It should be noted that the maximum stress cutoff line shown in Figure 14 follows the curve for $R = -0.9$ for lives less than 500 cycles. In all probability, because of increased amount of plastically deformed material at the concentration, the maximum stress cutoff might drift more toward $R = -0.8$ or -0.7 for very short lives.

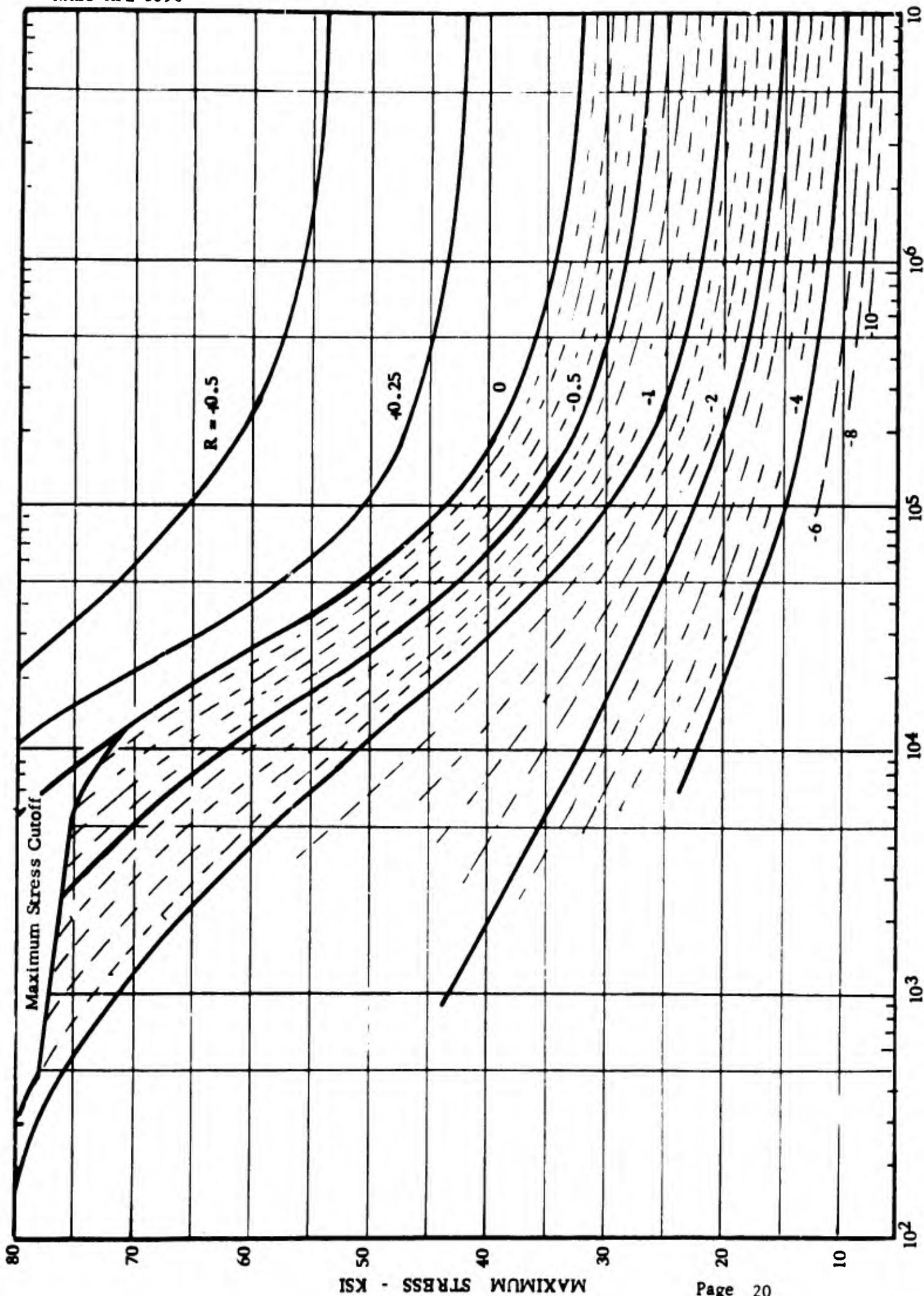


Figure 14

CUMULATIVE DAMAGE

Our discussion so far concerned obtaining the stress at the point of failure on a structure subjected to constant-amplitude loading at $R_p = 0$. This was obtained by establishing a maximum stress cutoff line with relation to experimentally developed S-N curves for unnotched 7075-T6 aluminum alloy. Static stress-strain data plus experimental observations of stress at a notch were used in determining the maximum stress cutoff. Starting with an assumption that 7075-T6 aluminum alloy is an ideal elastoplastic material, a maximum stress cutoff was first assumed to be 76,000 psi. This was later corrected for strain hardening and Bauschinger effects. The corrected maximum stress cutoff, along with interpolated S-N curves, are presented in Figure 14.

Inasmuch as fatigue is not only dependent on maximum stress but minimum stress as well, the minimum stress is found by the intercept of the life of the referenced structure with the maximum stress cutoff. The total stress range is the algebraic difference of the maximum and minimum stresses. Thus, if the intercept should conform with the S-N curve for $R = -0.5$, the range is

$$S_{\max} - S_{\min} = S_{\max} - (-0.5S_{\max}) = 1.5S_{\max}$$

Having obtained the stress range, it was then assumed that the stress range could be prorated for any other load for predicting fatigue life at that load, care being taken to consider residual stresses resulting from previous high loads. An example is worked out on page 23.

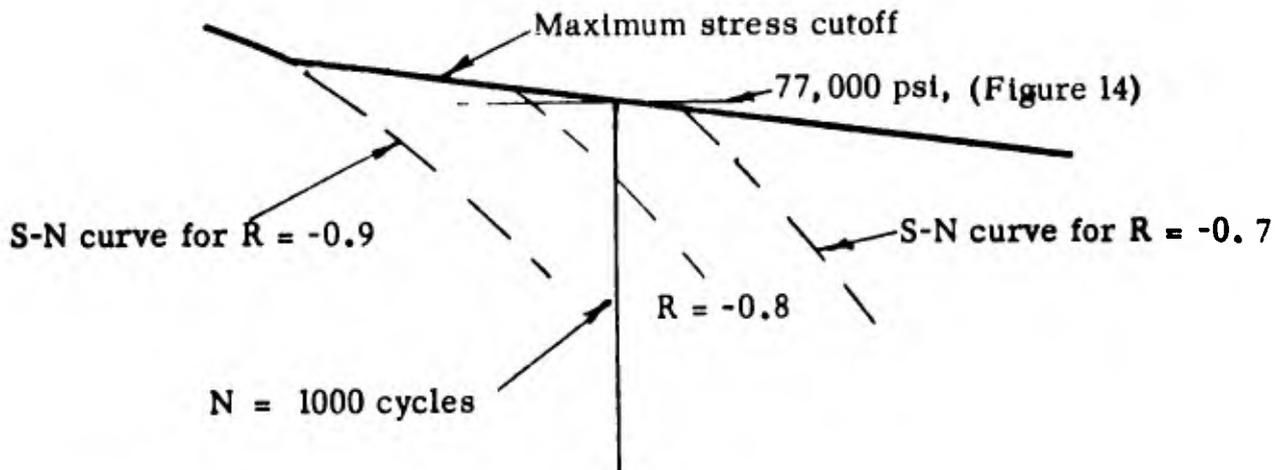
As for the ideal elastoplastic material in the original assumptions, first attempts at predicting cumulative damage will be based on the assumption that the Miner rule would apply if used in conjunction with actual stresses at the concentration, considering effects of residual stresses.

Consider a structure which is cycled at $R_p = 0$ and which fails after 1000 cycles of limit load. How long will a similar structure last when loaded according to the following spectrum?

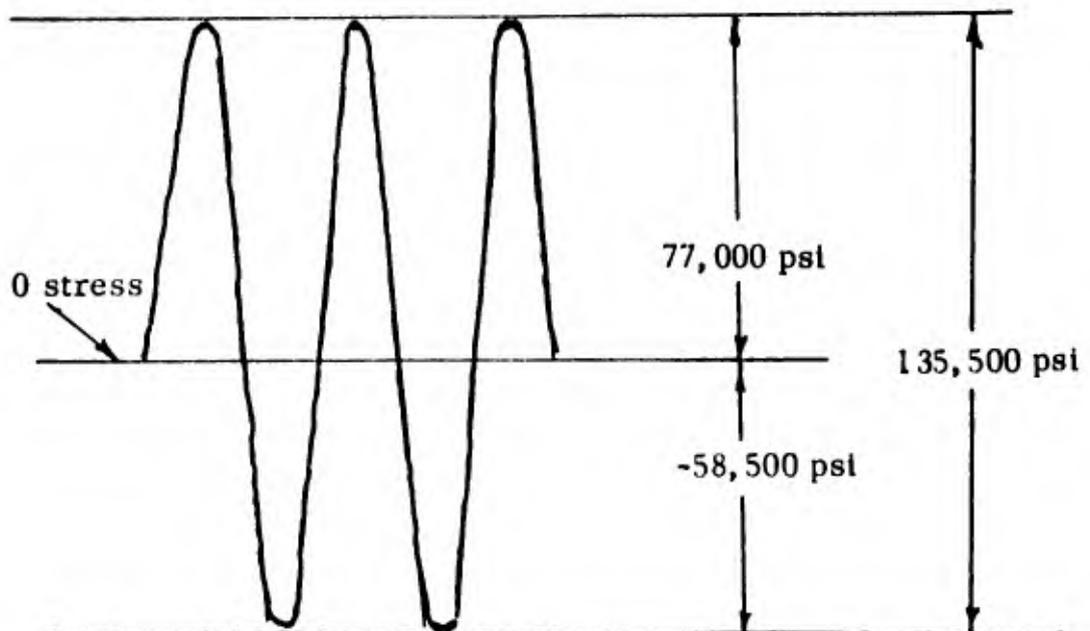
Load - % Limit	n*
100	3
85	17
70	65
55	172
40	283

* Cycles per sequence

Referring to Figure 14, we find that the maximum stress at the cutoff line corresponding with a life of 1000 cycles is 77,000 psi and that the intercept is at $R = -0.76$ (using interpolated values). See sketch below.



The minimum stress (by definition of R) = $77,000 \times -0.76 = -58,500$ psi,
 and the stress range = $1.76 \times 77,000 = 135,500$ psi.



Using the stress range of 135,500 psi, we can now compute the stress range for all other loads in the spectrum.

Load % Limit	Stress Range
100	135,500
85	115,200
70	94,800
55	74,500
40	54,200

Since the order of loading shows that 3 cycles of 100 percent were applied first, the residual stress of 58,500 psi will remain throughout remaining load steps. The residual stress is then to be subtracted from the stress range to find maximum stresses at other loads.

Load % Limit	Stress Range	Minimum Stress	Maximum Stress
100	135,500	-58,500	77,000
85	115,200		56,700
70	94,800		36,300
55	74,500		16,000
40	54,200		-4,300

We now have the maximum and minimum stress for each load step. By dividing the minimum stress by the maximum stress, we can obtain the stress ratio (R) and find corresponding lives (N) from the S-N curves in Figure 14. Knowing the number of cycles per step (n) in each sequence, the proportion of damage is found by dividing n by N. The structure will fail (according to the Miner rule) when $\sum n/N = 1$. The work is simplified by arranging all steps as in the following table:

Load % Limit	n	Stress Range	Minimum Stress	Maximum Stress	R	N	n/N	
100	3	135,500	-58,500	77,000	-0.76	1,000	0.00300	
85	17	115,200		56,700	-1.04	5,400	0.00315	
70	65	94,800		36,300	-1.60	12,500	0.00520	
55	172	74,500		16,000	-3.65	100,000	0.00172	
40	283	54,200		-58,500	-4300	-13.6	∞	0

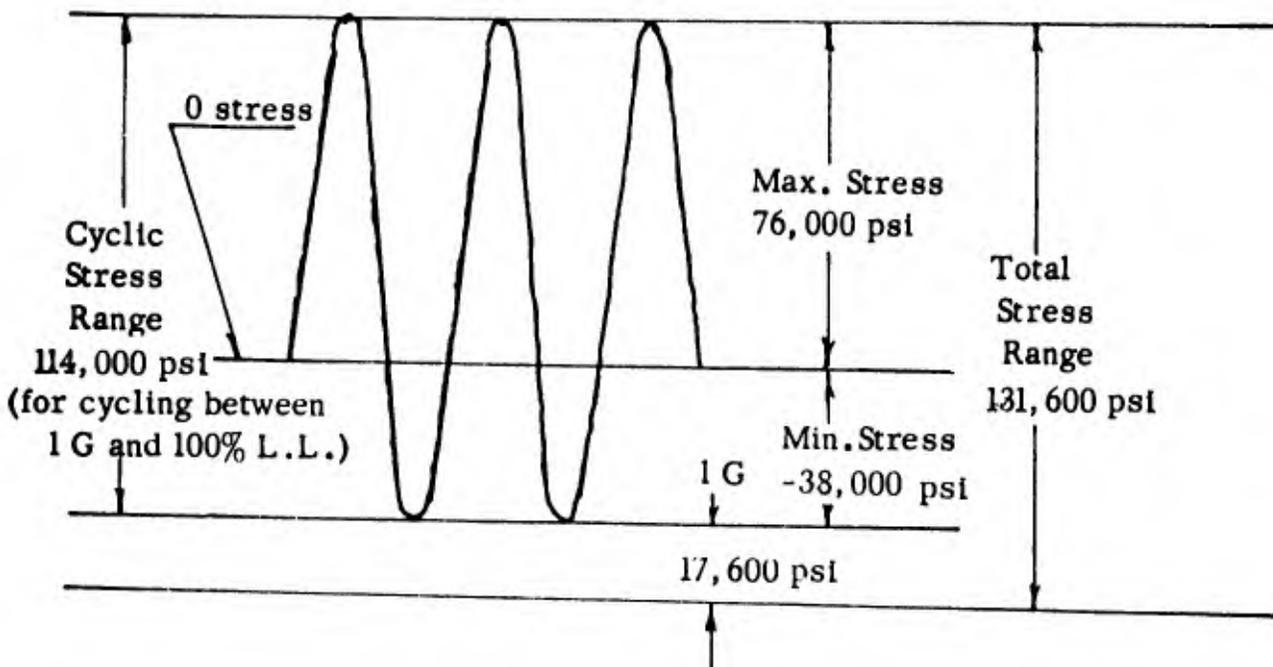
$$\sum n/N = 0.01307$$

$$\text{Life} = 1/0.0131 \quad 76.5 \text{ sequences}$$

1 G Minimum Load

Constant-amplitude tests on full scale structures are usually made with reference to a 1 G minimum load instead of zero load. The effect of this change in base load is dependent on the magnitudes of the local stress at maximum load and of the local residual stress at 1 G. For a constant-amplitude test of an ideal elastoplastic material where the maximum load is sufficiently high to produce a residual stress at the 1 G base load equal to the compressive yield stress (F_{cy}), the cycling will be at $R = -1$ whether the base load is 1 G or zero. Where the residual stress at the 1 G base load is less in magnitude than F_{cy} , the use of a 1 G base load will produce results different from those for zero load. This is illustrated in the following sketch where stress cycles at 100 percent limit load (1 G minimum) and limit load factor of 7.5 G conform with a constant-amplitude life of 2,500 cycles (Fig. 14).

Insofar as the stress at the concentration is concerned, a life of 2,500 cycles corresponds to a maximum stress of 76,000 psi, a minimum stress of -38,000 psi, and a cyclic stress range of 114,000 psi (including residual stress)



However, the 114,000 psi stress range corresponds to a load range of 6.5 G (7.5 G - 1 G) instead of the full load range of 7.5 G. Accordingly, the stress ranges for all other loads in the spectrum must be based on the full 7.5 G limit load range. This is done by multiplying the range for loading from 1 G to 7.5 G by the ratio of 7.5/6.5. This appears as the total stress range in the above sketch.

Computation for cumulative damage on structures having 1 G minimum loading will be the same as used for $R = 0$ with the exception that loading ranges are based on percentages of the total stress range, after which the equivalent 1 G stress plus the residual stress is subtracted. The operations are illustrated in the following table for a structure having a 2,500 cycle constant-amplitude life where 1 G is the minimum load.

Load % Limit	n	Total Stress Range	Cyclic Stress Range*	Minimum Stress	Maximum Stress	R	N	n/N
100	3	131,600	114,000	-38,000	76,000	-0.5	2,500	0.00120
85	17	111,800	94,200	↑	56,200	-0.68	10,500	0.00162
70	65	92,200	74,600		36,600	-1.07	40,000	0.00163
55	172	72,400	54,800		16,800	-2.26	800,000	0.00022
40	283	52,600	35,000	-38,000	-3,000	+12.7		0

* Total stress range - 1G - residual stress
(1 G = 17,600 psi)

$$\Sigma n/N = 0.00467$$

$$\text{Life} = 1/0.00467 = 214$$

Similar computations will reveal the following relation between constant-amplitude test life (1 G minimum load) and sequences to failure:

Constant- Amplitude Life	Sequences to Failure
1,000	105
1,500	145
2,500	214 (from above)
5,000	320
10,000	497
15,000	675

The above points are plotted in Figure 15 (upper graph).

In actual service the airplane structure is subjected to zero load at least once per flight. Similarly a test structure being tested using a 1 G base load is also completely unloaded periodically for inspection, repairs, etc. Consequently it would be unconservative to base calculated life estimates on the use of 1 G base load. Similarly it would be conservative to base life estimate on complete unloading. For practical use both estimates should be made and some average life estimate use. Using the stress ranges above, a structure cycled at $R_p = 0$ would have a life of 147 sequences. This would give an average of 180.5 sequences for both loading conditions.

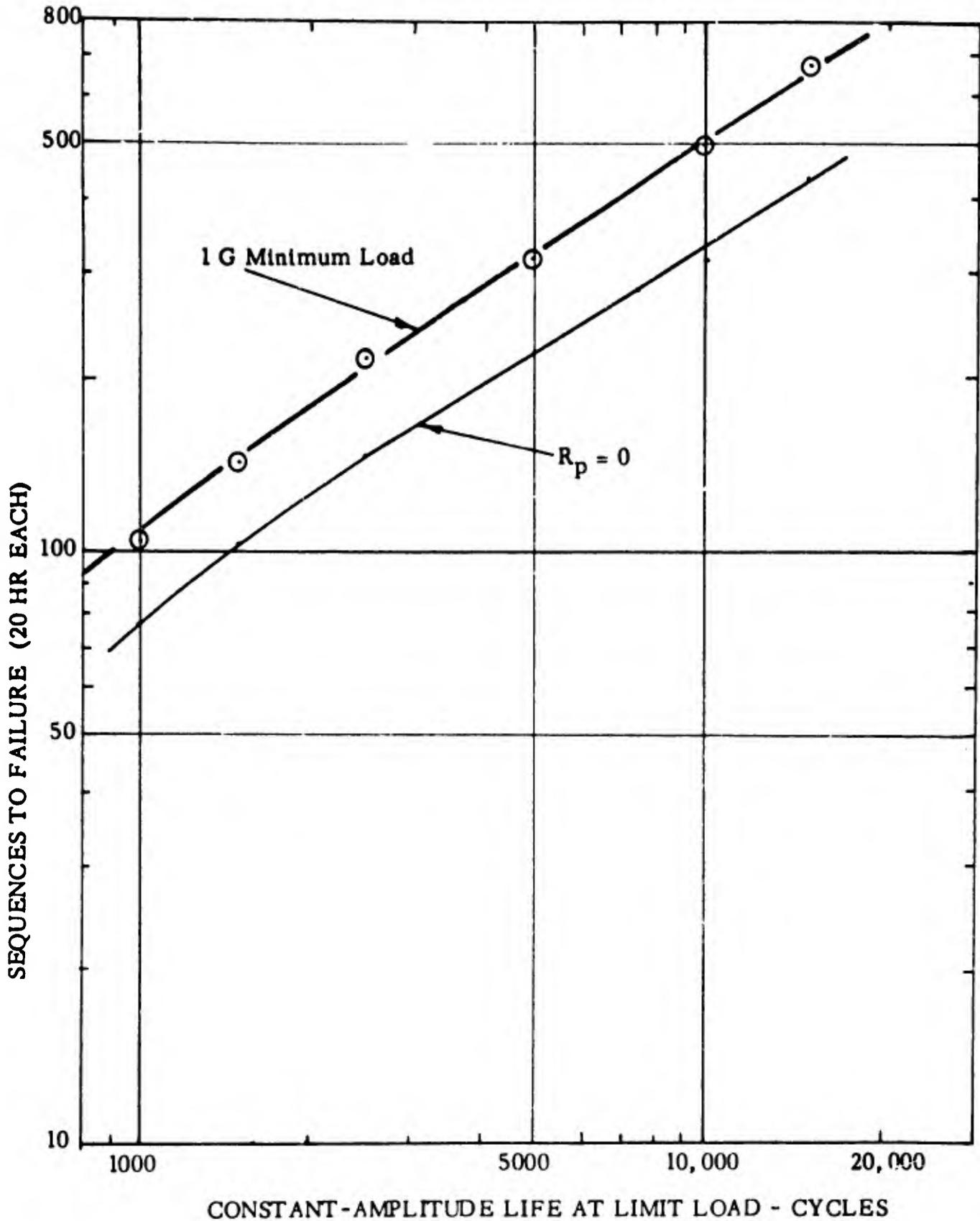


Figure 15. Constant-Amplitude Life at 100 Percent Limit Loading Versus Predicted Sequences to Failure (Table I, Spectrum I)

MIL-A-8866, Spectrum A, Positive Loading ($R_p = 0$)

Consider an airplane structure which has a constant-amplitude life of 5000 cycles when loaded to 100 percent limit load at $R_p = 0$. How long would such a structure last when loaded according to Spectrum A of MIL-A-8866, positive loading, $R_p = 0$? A breakdown of loading is presented in Table I, Spectrum IV. Note that load steps are in an ascending order instead of descending as in the previous example. Because of the ascending order, a series of computations must be performed to account for the residual stress introduced by the highest load in the preceding sequence. The stress range is based on computed values obtained from the 5000 cycle life (Figure 14) as in the previous example. The following steps would then apply:

- Step 1. From Figure 14, note that the maximum stress at the cutoff line and 5000 cycles is 75,500 psi.
- Step 2. The intercept is at $R = -0.26$
- Step 3. so that the minimum stress = $-0.26 \times 75,500 = -19,600$ psi (approx.)
- Step 4. and the stress range is $75,500 + 19,600 = 95,100$ psi
- Step 5. Compute stress ranges and minimum stresses to the nearest 100 psi for all loads.

Load % Limit	Stress Range - psi	Minimum Stress - psi
35	33,300	0
45	42,800	0
55	52,300	0
65	61,800	0
75	71,300	0
85	80,800	-7,400
95	90,400	-15,400
105	99,900	-24,400
115	109,400	-33,600
125	118,900	-42,700

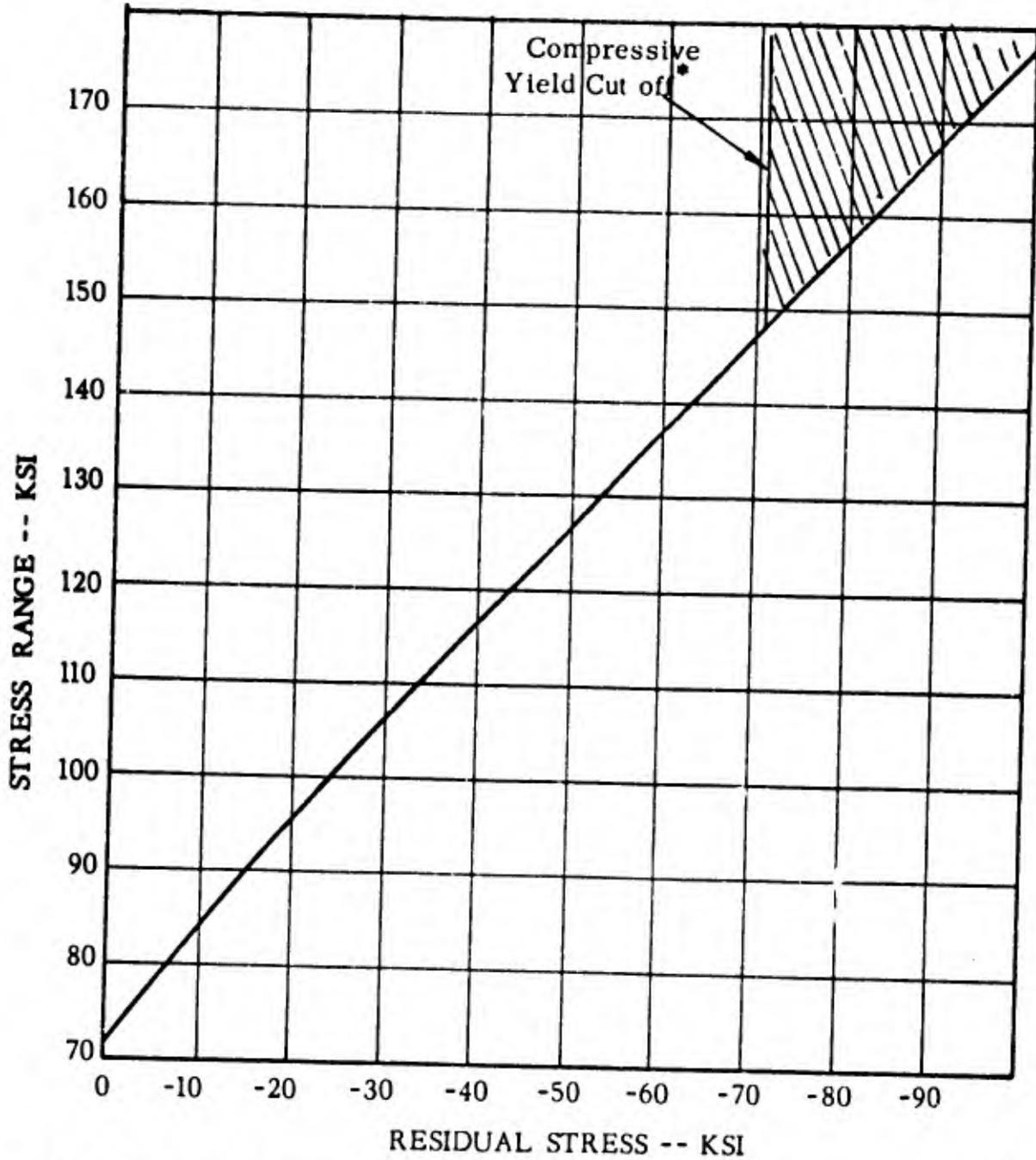


Figure 16. Calculated Stress Range versus Residual Stress for Cycling at $R_p = 0$

Stress ranges are computed by multiplying the 100 percent value times the various percentages. The minimum stresses are assumed to be the difference between the stress range and the stress-strain curve (Figure 10) at the extension of the primary modulus line. A graph of stress range versus residual stress (minimum stress where $R = 0$) is presented in Figure 16. The 1 G stress should be added to the residual stress where the minimum load is 1 G.

*

It is assumed that no more than the compressive yield strength can be retained as a residual stress. Use of the graph beyond this point is permissible only where addition of the stress due to 1 G loading brings the residual stress within the compressive yield limit.

Step 6. The 115 and 125 percent loads are not applied an integral number of times during each sequence. If we assume a sequence simulating 20 flight hours, the loading spectrum is

Load	35	45	55	65	75	85	95	105	115	125
n	340	190	130	90	50	30	6	3	0.8	0.32

Four cycles of 115 percent load must be applied during each five sequences and eight cycles of 125 percent load must be applied during each 25 sequences. Thus the loading sequence will repeat after 25 sequences (500 flight hours).

Let us assume that the maximum load for sequences 1, 6, 11, 16, 21 is 105 percent load; the maximum load for sequences 3, 7, 10, 13, 17, 20, 23, 25 is 125 percent load; and the maximum load for sequences 2, 4, 5, 8, 9, 12, 14, 15, 18, 19, 22, 24 is 115 percent load.

Step 7. Calculate the damage for the first sequence with the maximum load of 105 percent load.

Load (% Limit)	n	Stress Range	Minimum Stress	Maximum Stress	R	N	n/N
35	340	33,300	0	33,300	0	1.9x10 ⁶	0.00018
45	190	42,800	0	42,800	0	115,000	0.00165
55	130	52,300	0	52,300	0	44,000	0.00295
65	90	61,800	0	61,800	0	23,000	0.00391
75	50	71,300	0	71,300	0	11,500	0.00435
85	30	80,800	-7,400	73,400	-0.10	8,200	0.00366
95	6	90,400	-15,400	75,000	-0.21	5,600	0.00120
105	3	99,900	-24,400	75,500	-0.32	4,200	0.00071

$$\sum \frac{n}{N} = 0.01861$$

Step 8. Calculate the damage for the second sequence with maximum load of 115 percent and a residual stress of -24,400 psi from the first sequence.

Load (% Limit)	n	Stress Range	Minimum Stress	Maximum Stress	R	N	n/N
35	340	33,300	-24,400	8,900	-2.74	$>10^7$	-
45	190	42,800	↑	18,400	-1.32	1×10^7	0.00002
55	130	52,300		27,900	-0.88	180,000	0.00072
65	90	61,800		37,400	-0.65	68,000	0.00132
75	50	71,300		46,900	-0.52	33,000	0.00151
85	30	80,800		56,400	-0.43	17,500	0.00172
95	6	90,400	↓	66,000	-0.37	8,800	0.00068
105	3	99,900		-24,400	-0.32	4,200	0.00071
115	1	109,400		-33,600	75,800	-0.44	3,000
							$\sum \frac{n}{N} = 0.00701$

Step 9. Calculate the damage for the third sequence with maximum load of 125 percent and a residual stress of -33,600 psi from the second sequence.

Load (% Limit)	n	Stress Range	Minimum Stress	Maximum Stress	R	N	n/N
35	340	33,300	-33,600	-300	+112.0	∞	-
45	190	42,800	↑	9,200	-3.66	$>10^7$	-
55	130	52,300		18,700	-1.80	580,000	0.00022
65	90	61,800		28,200	-1.19	92,000	0.00098
75	50	71,300		37,700	-0.89	45,000	0.00111
85	30	80,800		47,200	-0.71	23,000	0.00130
95	6	90,400	↓	56,800	-0.59	13,000	0.00046
105	3	99,900		-33,600	-0.51	7,000	0.00043
115	1	109,400		-33,600	75,800	-0.44	3,000
125	1	118,900	-42,700	76,200	-0.56	2,000	0.00050
							$\sum \frac{n}{N} = 0.00533$

Step 10. Calculate the damage for the fourth and fifth sequences with maximum load of 115 percent and a residual stress of -42,700 psi from the third sequence.

Load (% Limit)	n	Stress Range	Minimum Stress	Maximum Stress	R	N	n/N
35	340	33,300	-42,700	-9,400	+4.53	∞	-
45	190	42,800		100	-427.0	∞	-
55	130	52,300		9,600	-4.44	1×10^7	0.00001
65	90	61,800		19,100	-2.23	1×10^7	0.00001
75	50	71,300		28,600	-1.49	60,000	0.00083
85	30	80,800		38,100	-1.12	30,000	0.00100
95	6	90,400		47,700	-0.89	16,500	0.00036
105	3	99,900		57,200	-0.74	8,800	0.00034
115	1	109,400		-42,700	66,700	-0.64	4,900

$$\sum \frac{n}{N} = 0.00275$$

Step 11. Calculate the damage for the sixth sequence with maximum load of 105 percent and a residual stress of -42,700 psi. This is identical to the calculation of step 10 minus the damage due to the 115 percent load.

Load	n/N
35-115	0.00275
115	0.00020

$$\sum \frac{n}{N} = 0.00255$$

Step 12. Calculate the damage for the seventh sequence with maximum load of 125 percent and a residual stress of -42,700 psi. This is identical to the calculation of step 10 with the addition of the damage due to the 125 percent load from step 9.

Load	n/N
35-115	0.00275
125	0.00050

$$\sum \frac{n}{N} = 0.00325$$

Step 13. It is apparent that the residual stress has stabilized at -42,700 psi and that the damage calculations for subsequent sequences will produce results identical to those shown in steps 10, 11, and 12. For the remainder of the first twenty-five sequences the damage is

Max. Spectrum Load	No. Sequences	$\sum \frac{n}{N}$ per Sequence	Damage
105	3	0.00255	0.00765
115	9	0.00275	0.02475
125	6	0.00325	0.01950
			$\sum \frac{n}{N} = 0.05190$

The total damage for the first twenty-five sequences is the sum of the above value and the values calculated in steps 7-12:

$$\sum \frac{n}{N} = 0.01861 + 0.00701 + 0.00533 + 2 \times 0.00275 + 0.00255 + 0.00325 + 0.05190 = 0.09415$$

Step 14. The damage for each subsequent twenty-five sequences is

Max. Spectrum Load	No. Sequences	$\sum \frac{n}{N}$ per Sequence	Damage
105	5	0.00255	0.01275
115	12	0.00275	0.03300
125	8	0.00325	0.02600
			$\sum \frac{n}{N} = 0.07175$

Step 15. Assuming Miner's Linear Damage hypothesis, the total life is now estimated.

$$\sum \frac{n}{N} = 1 = 0.09415 + 0.07175 x$$

$$x = \frac{0.90585}{0.07175} = 12.63 \text{ blocks of 25 sequences each}$$

$$\begin{aligned} \text{Total life} &= (1 + 12.63)25 = 340.6 \text{ 20-hr. sequences} \\ &= 6812 \text{ hrs.} \end{aligned}$$

The above calculations are tedious and time consuming. If a quick estimate is desired, assume hi-lo loading with the 125 percent load applied as the first cycle so that the residual stress of -42,700 psi is present for all cycles. Now it is only necessary to perform the calculations of steps 10, 11, 12, and 14 to determine the damage for each block of twenty-five sequences. The total life is now estimated.

$$\sum \frac{n}{N} = 1 = 0.07175 x$$

$$x = 13.94 \text{ blocks of 25 sequences each}$$

$$\text{Total life} = 13.94 \times 25 = 348.5 \text{ 20-hr. sequences}$$

$$= 6970 \text{ hrs.}$$

The error incurred by this simplified calculation is approximately 2%, which is negligible. This simplified method can be used when the loading is in one direction only. When the direction of loading is reversed, the more precise method involving the actual load history must be used because the inclusion of negative loads will affect the residual stresses.

Throughout this report, the residual stresses are assumed to be the difference between the theoretical stress at a given strain, assuming a linear stress-strain relation, and the actual stress as determined from the monotonic stress-strain curve for the material. This produces the same residual stress as would be obtained by drawing a straight line parallel to the slope of the elastic stress-strain curve from the maximum strain point on the monotonic stress-strain curve and determining its intercept at zero strain. This method is satisfactory for small plastic strains; however, it introduces large errors for high plastic strains as shown in Fig. 13 because of the Bauschinger effect and because shakedown of both the maximum and residual stresses, discussed by Crews and Hardrath⁸, is ignored. Furthermore, it is also assumed that the maximum residual stress that can be retained is equal to 0.9 F_{cy} . Considerable additional work is required to investigate these effects and to incorporate the results of these investigations in a modified theory.

In summarizing the steps required for predicting spectrum life, it is assumed that:

1. the Miner relation is sufficiently accurate for fatigue life predictions provided actual stresses at the concentration are used, including residual stresses.
2. the stress at a concentration can be found from unnotched specimen data for the material involved and the constant-amplitude fatigue life of the structure by

- a. Noting the intercept of life with the maximum stress cutoff line shown in Figure 14
- b. Finding the stress ratio (R) corresponding to the maximum stress and cutoff line. This may have to be interpolated between the lines presented in Figure 14
- c. Compute the minimum stress

$$S_{\min} = R \times S_{\max}$$

and determine the stress range for the constant-amplitude data. The stress range is the algebraic difference between maximum and minimum stresses.

- d. Compute stress ranges for all loads in the spectrum by ratioing each load in the spectrum to the constant-amplitude load.
 - e. Steps c and d were performed assuming the minimum load for the constant-amplitude and spectrum loading are the same. If the minimum load for the spectrum differs from that of the constant-amplitude data, the effect of this minimum load must be considered by the method discussed on page 24.
3. Finally, arrange the data in tabular form so that all computations can be easily checked. It is usually best to plot a graph similar to that shown in Figure 15 for various assumed constant-amplitude lives for the given spectrum. This helps to detect errors in that wide discrepancies are easily seen.

4. COMPARISON OF PREDICTED AND TEST LIVES

SPECTRUM I

1 G Minimum Load--Full-Scale Structures

Full-scale fatigue tests of wings and horizontal tail surfaces were performed by the Naval Air Engineering Center and reported in Reference 1. These data are plotted in Figure 17 along with a theoretical curve (Figure 15) for comparison. The load spectra used for full-scale tests, shown in Figure 17, were applied in an ascending order for wing tests. Order for tails were as follow:

- A. 70, 100, 85, 55, 70, 100, etc.
- B. 40, 70, 100, 85, 55, 40, 70, etc.
- C. 30, 70, 100, 85, 40, 30, 70, etc.

Inasmuch as these spectra were very similar to Spectrum I (Table I), the theoretical graph is taken from Figure 15 which assumed a descending load order in addition to a 40 percent limit loading (not contributing to damage after application of first limit load). This would predict about a 4 percent longer spectrum life for wings having a 1000 cycle limit load constant-amplitude life with virtually no change at 10,000 cycle constant-amplitude life. Such errors are considered negligible considering other discrepancies in fatigue life prediction.

 $R_p = 0$ -- Double-Shear Butt Joints

The only tests in the present program for loading at $R_p = 0$ (both constant-amplitude and spectrum tests) were for double-shear riveted joints. Average results taken from Table VI show the following:

Material 7075-T6	Sequences to Failure		Test/Calculated
	Test	Calculated	
Low yield	269	208	1.29
Medium yield	278	214	1.30
High yield	274	201	1.36

Data taken from Reference 5 for center-hole specimens tested for the same spectrum showed a similar trend. Based on a constant amplitude life of 4500 cycles, the predicted spectrum life was 270 sequences as compared to an average test life of 198 sequences. This gave a ratio of test/predicted life of 1.36.

The above calculated lives are based on $\sum n/N = 1$ for the computed stress at the concentration. Calculations for riveted joints are given in Table VI.

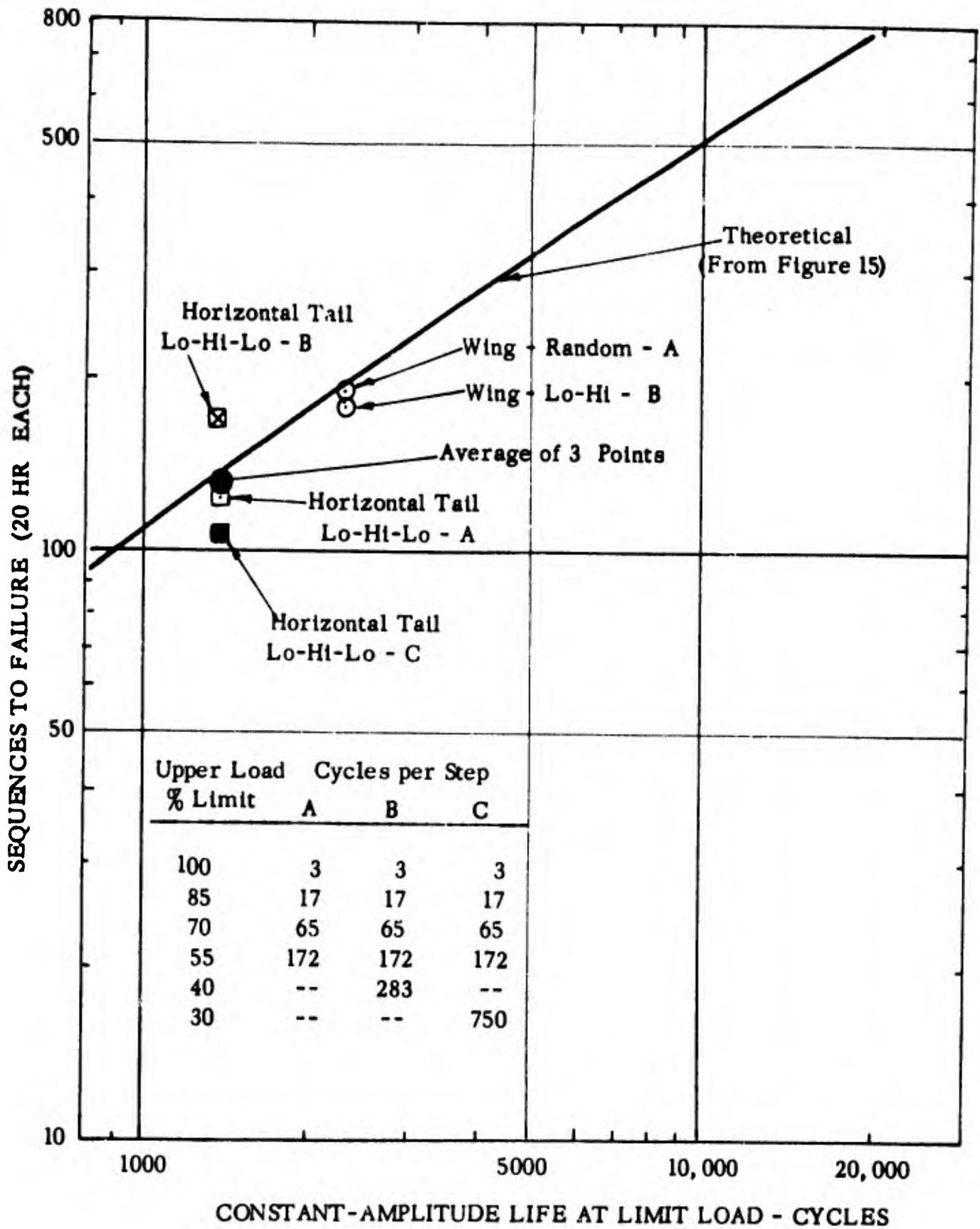


Figure 17. Comparison of Predicted and Test Lives of Full-Scale Structures - Minimum Load = 1 G = 13.3 Percent

SPECTRUM II

1 G Minimum Load - Full-Scale Structures

Referring again to Table I, Spectrum II is the same as Spectrum I (also to Spectrum B shown in Figure 17) except for deletion of the 100 percent load. Horizontal tail surfaces reported in Reference 1 had an average spectrum life of 82 sequences where constant-amplitude loading at 100 percent limit (1 G minimum load) caused failure after 1376 cycles. Calculated life, based on a constant-amplitude life of 1400 cycles of limit loading, amounted to 94 sequences for a test/calculated ratio of $82/94 = 0.87$. Calculations were based on a constant-amplitude life of 1400 cycles instead of 1376 cycles purely for rounding off figures. Had the actual life of 1376 cycles been used, the predicted life would have been about 92 sequences instead of 94 as shown.

 $R_p = 0$ - Double-Shear Butt Joints

Again referring to Table V, riveted butt joints tested for Spectrum II revealed the following:

Material 7075-T6	Sequences to Failure		Test/Calculated
	Test	Calculated	
Low yield	386	123	3.14
Medium yield	415	132	3.15
High yield	304	114	2.67

Here it is seen that tests on full-scale structures and small specimens show opposite trends. This is probably because the 85 percent load (100 percent = 45,000 psi) was insufficient to relieve clampup in the double shear riveted joints.

SPECTRUM III

1 G Minimum Load -- Full-Scale Structures

Spectrum III is the same as Spectrum I except for a 115 percent preload prior to testing. In the case of the horizontal tail surfaces reported in Reference 1, the 40 percent limit load was deleted also. The average life for three horizontal tails from Reference 1 is 142 sequences. The calculated life is 151 sequences for a test/calculated ratio of $142/151 = 0.94$.

SPECTRUM III (Contd)

$R_p = 0$ - Double-Shear Butt Joints

The average spectrum life for double-shear butt joints (Table V) was 280 sequences. The calculated life is 425 sequences for a test/calculated ratio of $280/425 = 0.66$.

Here it is seen that the full-scale structure and double-shear butt joints reflect the same trend insofar as ratio of test/predicted lives is concerned. It should be noted that the predicted life for the full-scale structure is for a 60 percent (228/142) increase in fatigue life over the life for Spectrum I. For double-shear riveted joints, the predicted life is 425 sequences. This is about double the life predicted (Table VI, medium yield) for Spectrum I. As it turned out, the fatigue life for both, full-scale structure and double-shear riveted joints remained unchanged from the life experienced in Spectrum I.

One of the tests reported in Reference 1 included a spectrum similar to Spectrum III, the difference being that the preload was 130 percent of limit instead of 115 percent. The average life for three horizontal tails was 245 sequences. The calculated life is 188 sequences for a test/calculated ratio of $245/188 = 1.30$. Here it was assumed that no more than the compressive yield (71,000 psi) could be retained as a residual stress.

SPECTRUM IV

Constant-Amplitude $R_p = 0$, 1 G Minimum for Spectrum Loading--Center-Hole Specimens

Figure 18 presents results of spectrum tests on center-hole specimens (Tables VIII, IX, and X), relating their constant-amplitude life ($R_p = 0$) to spectrum life wherein a 1 G minimum load of 13.3 percent of limit is assumed. The reason for conducting tests in this manner is twofold:

1. To enable obtaining the full stress range directly from constant-amplitude tests.
2. To demonstrate differences between lives of specimens with open and filled holes - Taper-Lok bolts in particular. The "propping effect" ⁹ of the interference-fit Taper-Lok bolt is known to make the stress at the concentration behave as though cycling were at $R = +0.5$ or more, although actual loading might be at $R_p = 0$ or slightly reversed. A slight positive load would obscure this effect, particularly at low load levels.

A theoretical graph (assuming $\sum n/N = 1$) is also presented. Note that the ratio of the theoretical prediction to test life for open center-hole specimens is approximately 2 at a constant-amplitude life of 3500 cycles. An extension of the test graph indicates that the theoretical and test values would agree at a constant-amplitude life of approximately 60,000 cycles and a spectrum life of approximately 2000 sequences. This indicates a needed correction of somewhere between $\sum n/N = 0.5$ at 250 sequences (spectrum life corresponding to 3500 cycles) and zero at 2000 sequences.

A value of $\sum n/N = 0.5$ is not too far out of line according to reports ⁶ and ⁷ by many researchers on cumulative damage for reverse bending. Since the actual stress at the concentration, considering the residual stress, does behave as though it were completely reversed, a correction of such a magnitude might be in order. The fact that no correction is indicated at 60,000 cycles life points to the need for a variable correction factor.

Before proceeding with any correction whatsoever, it is well to examine data from other tests. The graphs for specimens with filled holes show a marked discrepancy in the short life range. However, indications are that agreement ¹⁰ is attained at a constant-amplitude life of around 60,000 ¹¹ where no plastic deformation is experienced.

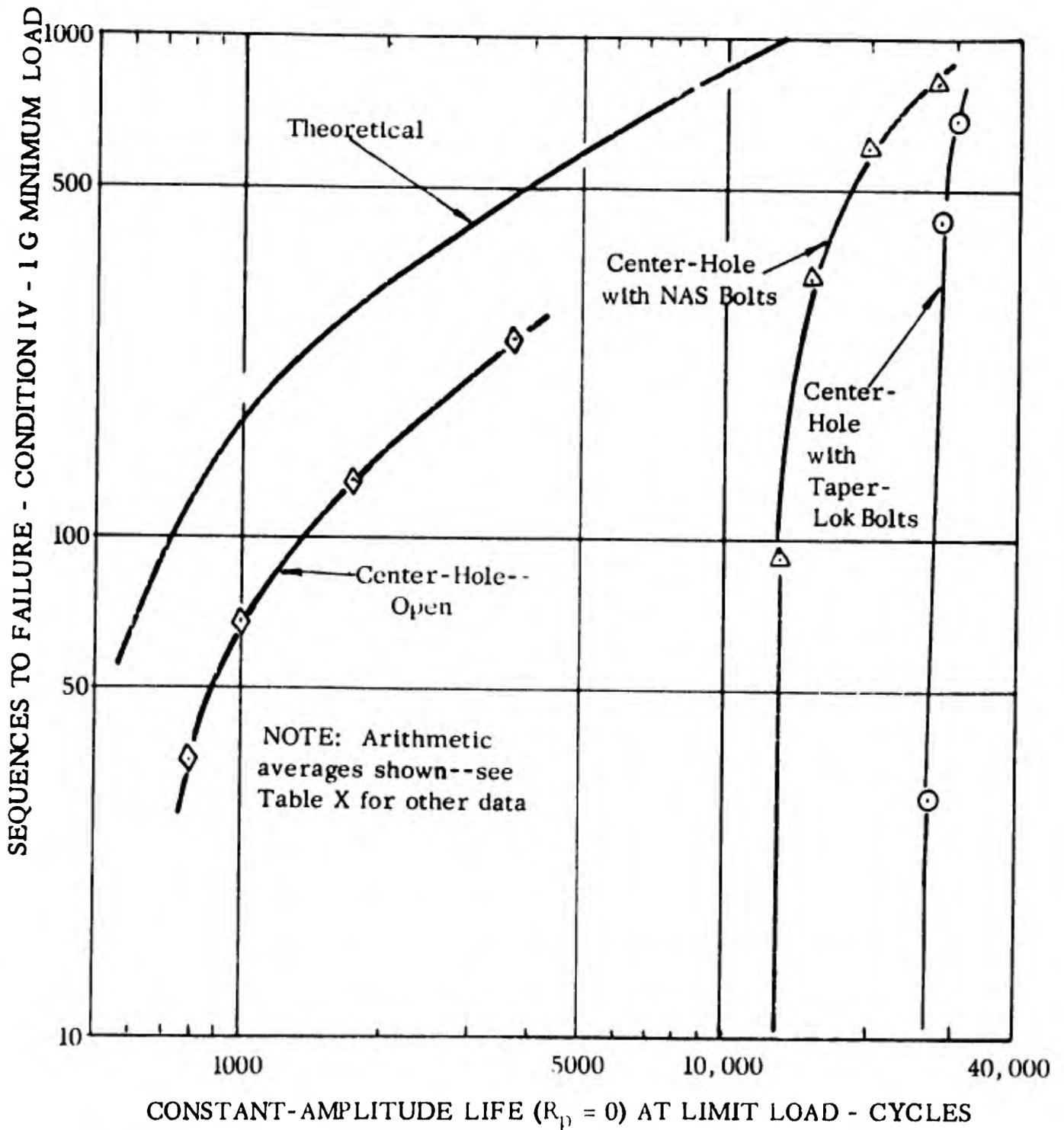


Figure 18. Constant-Amplitude Life at 100 Percent Limit Loading ($R_p = 0$) versus Predicted Sequences to Failure (1 G Minimum Load - Spectrum IV, Table I)

Single-Shear Butt Joints

Figure 19 presents spectrum life versus constant-amplitude life for single-shear butt joints. Again, the theoretical graph (same as in Figure 18) is shown for comparison. Here it will be noted that single-shear butt joints fastened with NAS Bolts agree more closely with theoretical values than the open-hole specimens in Figure 18. Had $\Sigma n/N = 0.6$ been used, the theoretical and experimental graph for joints fastened with NAS bolts would coincide, except at very short life where the downward hook on the theoretical curve agrees without correction.

The downward hook on the theoretical curve results from assuming that no more than 71,000 psi residual stress can be sustained. This is of academic interest only, since such a lifetime is nowhere near the life of a satisfactory structure. The fact that limiting the residual stress causes the theoretical to approach test values indicates that a reasonable correction would be to limit residual stress to some percentage of that introduced by the 125 percent load. Perhaps a combination of both, relaxation of residual stress and varying values for $\Sigma n/N$ would be in order.¹⁰ This is beyond the scope of the present investigation.

Single-shear butt joints fastened with Taper-Lok bolts showed a non-linearity throughout. This stems from the fact that failures occurred through bolt holes (nearest load) at constant-amplitude lives of fewer than 10,000 cycles, and by breaking splice plates at the edge of the washer (nearest splice center) for lives of more than 10,000 cycles.* However, for spectrum tests, all but two failed through bolt holes. While similar failures (Table III) prevailed for joints fastened with NAS-333 bolts, the lower fatigue strength of joints with NAS bolts in the short life ranges might account for the more nearly linear relation between short and long life test results.

Although the spectrum lives of joints with Taper-Lok bolts (also center-hole specimens) appears to be lower than of similar specimens with NAS bolts, it should be noted that the spectrum life in each case is with reference to its own constant-amplitude life. Refer to Figures 2, 3, 4, and 5 for relative fatigue strengths of specimens for constant-amplitude and spectrum loading.

* Considering that the nominal bending stress in the doubler amounts to about three times P/A at the time the load (P) approaches zero, it is not surprising that failures of single-shear butt joints occurred in this manner at low stress levels. The value of 3 (P/A) is derived from letting the bending moment equal P times half the thickness of the material on initial loading. This is reduced with higher loading as the material aligns itself, approaching zero at the yield strength of the material for heavy sheet gages and at somewhat lower stresses for thin material.

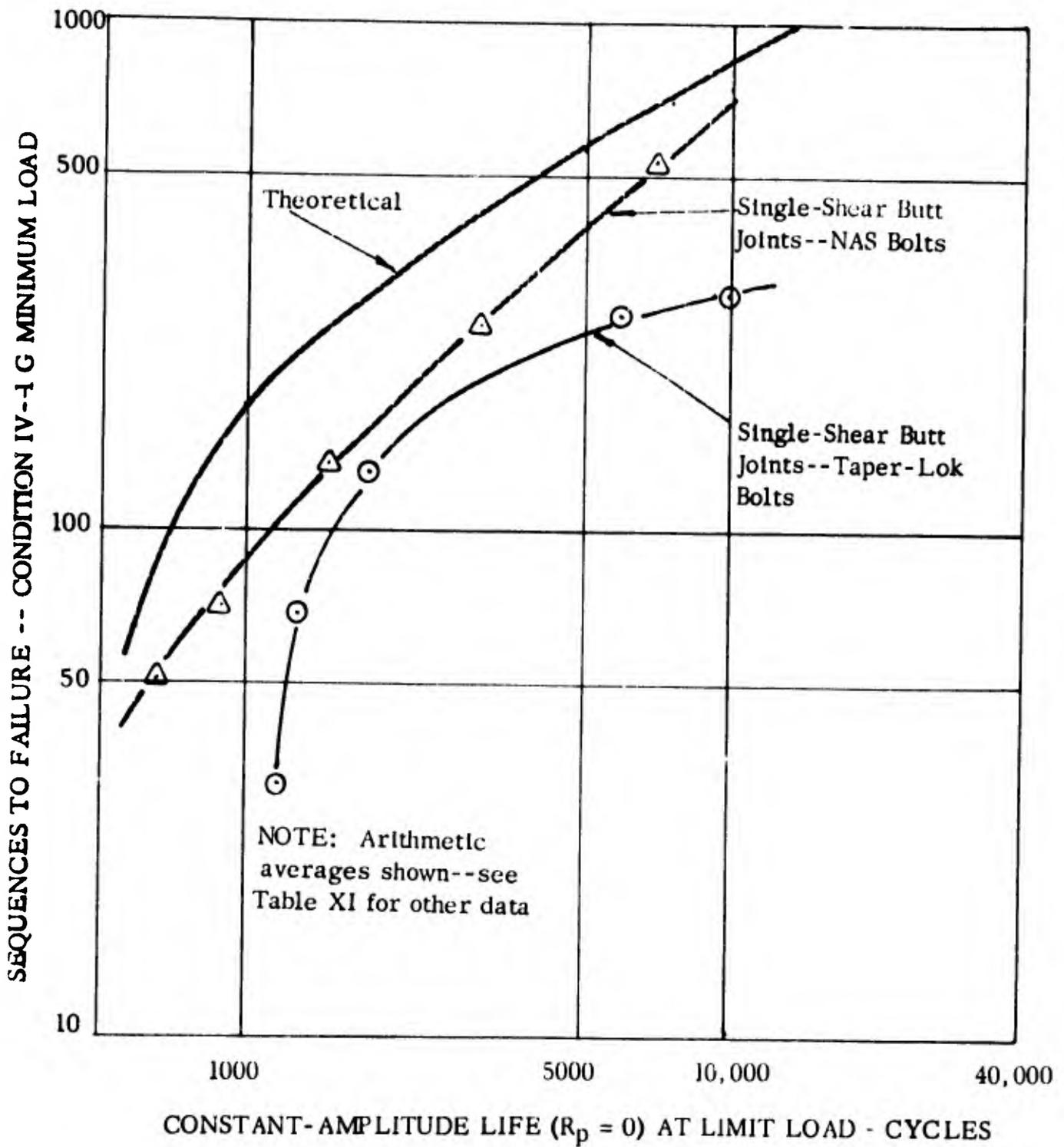


Figure 19. Constant-Amplitude Life at 100 Percent Limit Loading ($R_p = 0$) versus Sequences to Failure (1 G Minimum Load - Spectrum IV, Table I) for Single-Shear Butt Joints

Constant-Amplitude and Spectrum Loading - 1 G Minimum Load - Full-Scale Tests

Theoretical predictions along with test data from full-scale test wings are presented in Figure 20. In comparing the predicted curves of Figures 19 and 20, it is seen that the characteristic downward hook in Figure 20 occurs at longer constant-amplitude life than in Figure 19. This is because the constant-amplitude life at 1 G minimum load represents a higher stress ratio which inherently results in longer life as seen in Figure 14.

5 DISCUSSION

In relating constant-amplitude fatigue life to spectrum life, there is a possibility that some load other than limit would be more appropriate. Theoretically, this should make no difference in predictions as long as constant amplitude life were sufficiently short to ensure plastic-deformation at the concentration. That is to say, the predicted flight hours would not change; however, the slope of the constant-amplitude versus sequences to failure would vary in accordance with constant-amplitude lives which would differ. This is illustrated in the following data taken from Reference 1 (also Figure 17 of this report):

Load % Limit	Sequences to Failure/Constant-Amplitude Life	
	Horizontal Tail	Wing
100	0.0902	0.0757
80	0.0315	0.0325
60	0.0121	0.0063

While the above represent averages of only a few tests, the difference at 60 percent limit loading appears significant. This logically follows from the fact that the constant-amplitude life for the wing at 60 percent of limit loading was 26,930 cycles where no plastic deformation occurs. Life for horizontal tails was 10,094 cycles which involves some plastic deformation as indicated in Figure 14.

Theoretical values throughout this report were uncorrected for values of $\Sigma n/N$ equaling anything other than unity. While there is no proof that $\Sigma n/N$ will equal any constant, the convergence of the graphs shown in Figure 18 indicates that unity is reasonable where no plastic deformation is involved. Perhaps varying amounts from 1 to 0.5 or less can be directly related to stress ratio at the concentration. This is beyond the scope of the present investigation.

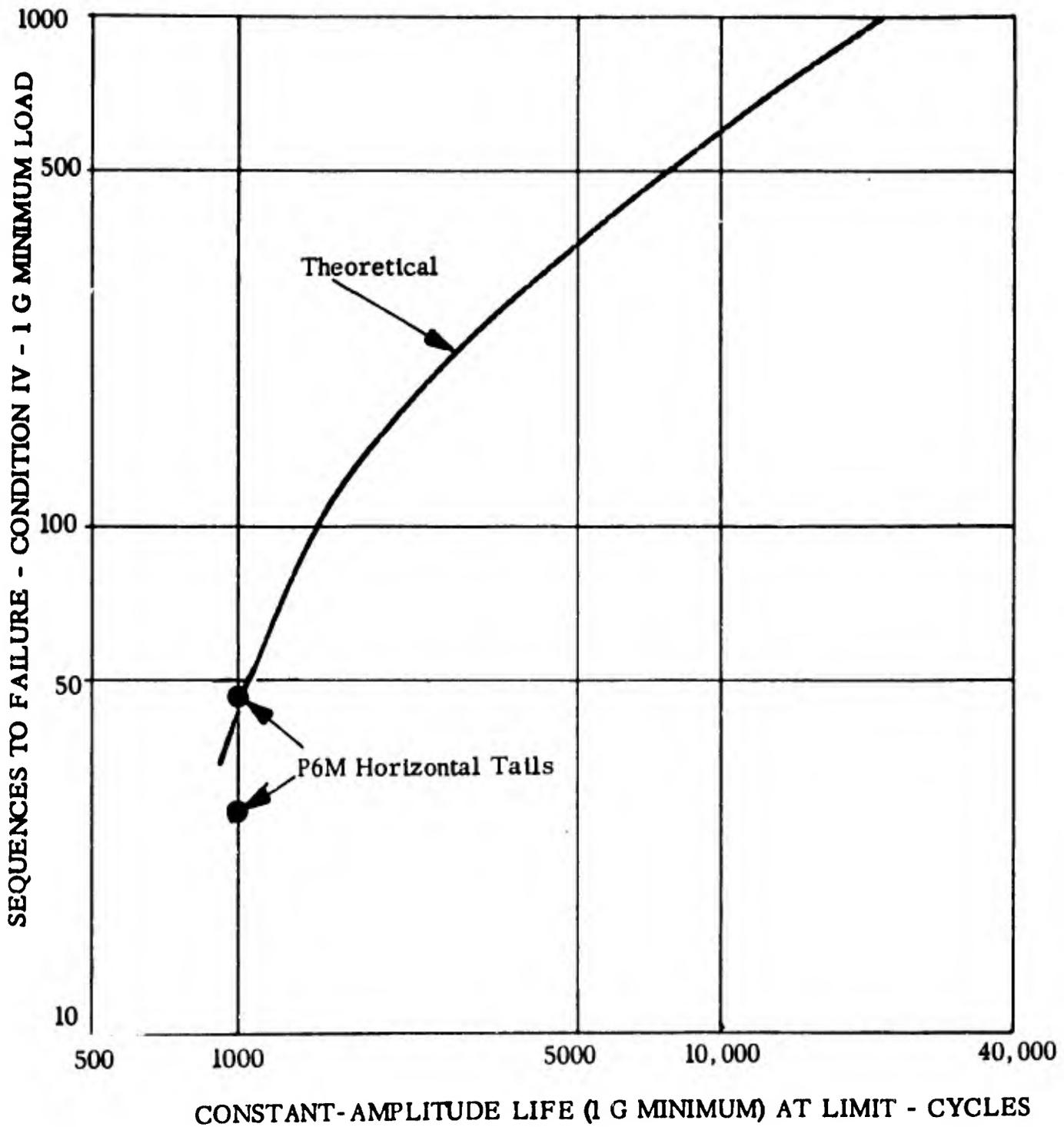


Figure 20. Constant-Amplitude Life at 100 Percent Limit Loading (1 G Minimum) versus Predicted Sequences to Failure (1 G Minimum Load - Spectrum IV, Table I)

6. CONCLUSIONS

The results of the present investigation indicate the following:

1. That a relation between constant-amplitude and spectrum fatigue life exists for a given structure.
2. That because of this relation, it is possible to determine the stress cycle at the concentration. Qualification is that fatigue failure for the constant-amplitude test occurs at fewer than 10^4 cycles in a structure built from 7075-T6 aluminum alloy.
3. Knowing the stress cycle, it is then possible to prorate the stress at the concentration to estimate cumulative fatigue damage, using values of $\Sigma n/N$ appropriate for the stress ratio at each of the steps in the load program.

TABLE I

Load Spectra for 20 Hour Block of Positive Flight Loads

Spectrum No. Upper Load - % Limit	Cycles per Block				
	I	II	III	IV	
				Upper Load	Cycles per Block
115			1*	35	340
100	3		3	45	190
85	17	17	17	55	130
70	65	65	65	65	90
55	172	172	172	75	50
40	283	283	283	85	30
				95	6
				105	3
				115	0.8
				125	0.32

* Applied once prior to testing

TABLE II

Constant-Amplitude Data for Center-Hole Specimens

Unfilled 0.191 Hole		Hole Filled with NAS-333 Bolt		Hole Filled with Taper-Lok TL-100 Bolt	
Max. Stress*	Cycles	Max. Stress*	Cycles*	Max. Stress*	Cycles*
60.0	2	65.0	1	70.0	1
57.5	181	63.0	6	67.5	1
57.5	235	63.0	44	67.5	2
57.5	242	63.0	21	67.5	18
		63.0	45	67.5	92
55.0	346			67.5	109
		55.0	6,822		
50.0	548	55.0	4,365	63.0	663
				63.0	632
45.0	967	45.0	11,000	63.0	900
45.0	1,036	45.0	15,000	63.0	1,039
		45.0	21,000		
30.0	6,000	45.0	22,000	60.0	5,000
30.0	6,000	45.0	27,000		
30.0	7,000			56.0	14,000
30.0	7,000	22.5	68,000	56.0	9,000
30.0	7,000	22.5	132,000	56.0	14,000
		22.5	133,000	56.0	13,000
15.0	70,000	22.5	195,000	56.0	15,000
15.0	85,000	22.5	252,000		
15.0	150,000			50.0	27,000
15.0	270,000			37.5	48,000
15.0	400,000			37.5	67,000
				37.5	77,000
				37.5	84,000
				37.5	87,000

*

Stress based on gross area through test section - (R = O) in ksi.
Cycles to failure

TABLE III

Constant Amplitude Data for Single Shear Butt Joints

NAS-333 Bolt		Taper-Lok Bolt TL-100	
Max. Stress	Cycles	Max. Stress	Cycles
50.0	9	60.0	39
50.0	8	60.0	39
50.0	44	60.0	8
50.0	76	60.0	54
50.0	264		
		57.5	147
45.0	198		
		55.0	188
40.0	264	55.0	244
40.0	301		
40.0	313	50.0	1,305
40.0	391		
40.0	622	45.0	1,142
		45.0	1,578
27.5	5,000	45.0	1,622
27.5	8,000	45.0	1,779
27.5	9,000		
27.5	9,000	30.0	5,000
27.5	10,000	30.0	5,000
		30.0	5,000
18.0	21,000	30.0	6,000
		30.0	6,000
12.0	41,000		
		7.0	71,000
9.0	67,000	7.0	143,000
9.0	94,000	7.0	146,000
		7.0	146,000
7.0	98,000	7.0	147,000
7.0	100,000	7.0	165,000
7.0	100,000		
7.0	106,000		
7.0	111,000		
7.0	122,000		
7.0	123,000		

NOTE: All joints lasting more than 10,000 cycles failed in doubler away from bolt hole. NAS 333 bolts loaded at 50.0 ksi failed by popping off heads at bolt recess.

TABLE III Contd.

Constant Amplitude Data for Single-Shear Butt Joints - Modified Doublers

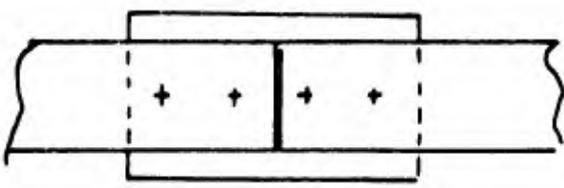
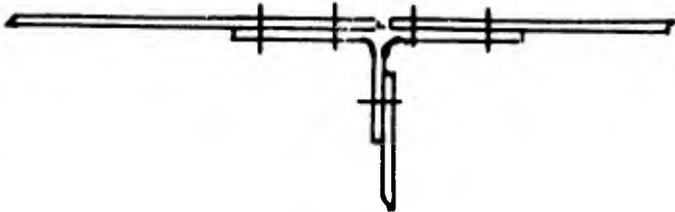
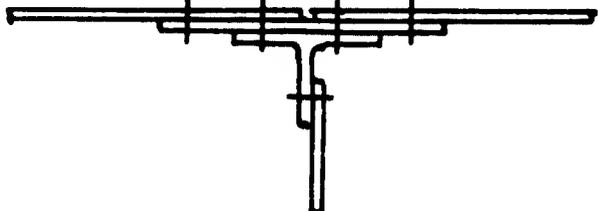
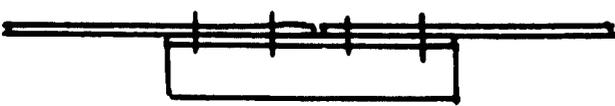
Specimen Configuration	Max. Stress (KSI)	Cycles (R = 0)	Remarks
 <p>WIDE DOUBLERS</p>	7,000	273,000	0.1 thick x 1.5 wide doubler
	7,000	193,000	same as above with 0.016 x 1 x 1 tabs under washer
	7,000	559,000	0.160 thick x 1.5 wide doubler
<p>VARYING DOUBLER THICKNESS</p> <p>See Figure 1</p>	7,000	311,000	0.160 thick x 1.0 wide
	7,000	202,000	0.125 " doubler
	7,000	104,000	0.125 " " (Beveled washer)
	7,000	68,000	0.0767 thick doubler
	7,000	147,000	0.050 two thickness doubler
 <p>TEE DOUBLER -- LATERALLY SUPPORTED</p>	7,000	400,000	0.160 thick Tee
	12,000	19,000	" " "
	12,000	63,000	0.100 thick Tee
	12,000	40,000	" " "
	7,000	113,000	" " " (10 in-lb torque)
 <p>AUXILIARY TEE DOUBLER -- LATERALLY SUPPORTED</p>	12,000	184,000	45 in-lb torque
	12,000	189,000	" " "
	12,000	158,000	30 in-lb torque
	12,000	151,000*	20 in-lb torque
	12,000	206,000	" " "
 <p>ANGLE DOUBLER (0.10 x 1 x 1)</p>	12,000	248,000	Taper-Lok bolts - 45 in-lb torque

TABLE IV

Constant Amplitude Data for Double-Shear Riveted Butt Joints

Maximum Stress (R = 0)	Cycles to Failure		
	HIGH YIELD Over 77.0 ksi	MEDIUM YIELD 74.0 - 77.0 ksi	LOW YIELD Below 74.0 ksi
45,000 psi (gross)	3,960	4,120	3,226
	3,151	4,999	3,606
	6,074	4,715	3,636
	6,345	5,329	3,867
45,000 psi (gross)	6,711	5,845	4,258
Arithmetic average	5,248	5,002	3,719

TABLE V

Spectrum Data for Double-Shear Riveted Butt Joints

Load Spectrum (Table I)	Sequences to Failure (20 hr blocks)		
	HIGH YIELD	MEDIUM YIELD	LOW YIELD
Spectrum I	148	222	180
	160	222	180
Limit Stress = 45,000 psi (gross)	308	254	194
	328	276	248
	424	414	542
Arithmetic average	274	278	269
Spectrum II	98	346	200
	266	368	362
Limit Stress = 45,000 psi (gross)	298	388	362
	358	670	264
	504	404	716
Arithmetic average	304	415	386
Spectrum III		240	
		284	
Limit Stress = 45,000 psi (gross)		284	
		294	
		296	
Arithmetic average		280	

TABLE VI

Calculations for Spectrum Lives of Double-Shear Riveted Butt Joints

HIGH YIELD Load	n	Range	Residual (minimum) Stress	Maximum Stress	R	N	n/N
100	3	89,300	-12,300	77,000	-0.16	5,248	0.00057
85	17	75,900	↑ ↓	63,600	-0.19	15,000	0.00113
70	65	62,600		50,300	-0.25	35,000	0.00186
55	172	49,100		36,800	-0.33	140,000	0.00123
40	283	35,700	-12,300	23,400	-0.53	10 ⁷ +	0.00003

$\Sigma n/N = 0.00482$
Sequences to failure = 208

MEDIUM YIELD

100	3	94,200	-18,800	75,400	-0.25	5,002	0.00060
85	17	80,100	↑ ↓	61,300	-0.31	14,000	0.00121
70	65	66,000		47,200	-0.40	37,000	0.00175
55	172	51,800		33,000	-0.57	160,000	0.00108
40	283	37,700	-18,800	18,800	-1.00	10 ⁷ +	0.00003

$\Sigma n/N = 0.00467$
Sequences to failure = 214

LOW YIELD

100	3	104,700	-31,500	73,200	-0.43	3,719	0.00081
85	17	89,000	↑ ↓	57,500	-0.55	13,000	0.00131
70	65	73,300		41,800	-0.75	37,000	0.00176
55	172	57,600		26,100	-1.20	160,000	0.00107
40	283	41,900	-31,500	10,400	-3.03	10 ⁷ +	0.00003

$\Sigma n/N = 0.00498$
Sequences to failure = 201

NOTES:

1. All load cycles at $R_p = 0$ - loads in percent limit
2. N for 100 percent loading taken from Table IV
3. Material properties as follow: F_{ty} F_{tu}

HIGH YIELD	77,000 psi or over	84,000 psi
MEDIUM YIELD	74,000 psi - 77,000	82,500 psi
LOW YIELD	below 74,000 psi	80,200 psi

4. The above calculated lives are based on $\Sigma n/N = 1$ for stress at the concentration, using cutoff lines parallel to that shown in Figure 14 (medium yield) and separated by differences in yield strengths. Basic S-N curves below the cutoff line were the same in all cases.

TABLE VII

Calculations for Spectrum Life of Preloaded* Double-Shear Butt Joints

Load % Limit	n	Range	Residual (minimum) Stress	Maximum Stress	R	N	n/N
100	3	94,200	-32,000	62,200	-0.51	9,600	0.00031
85	17	80,100	↑	48,100	-0.67	22,000	0.00078
70	65	65,900	↓	33,900	-0.94	66,000	0.00098
55	172	51,800	↓	19,800	-1.61	620,000	0.00028
40	283	37,700	-32,000	5,700	-5.62	--	--

$$\sum n/N = 0.00235$$

Life = 425 sequences

* See Spectrum III, Table I. The residual stress is that left from the 115 percent load where the stress range is $94,200 \times 1.15 = 108,300$ psi. Since this stress range is comprised of a combination of maximum and minimum stresses that must agree with Figure 14, a try and fit process will indicate a reasonable agreement at a maximum stress of 76,000 psi and $R = -0.42$.

TABLE VIII

Spectrum Test Data for Center-Hole (open) Specimens					
Limit Stress KSI	48.5	46.5	45.0	40.0	35.0
Spectrum IV --	3*	21	62	115	199
20 Hour Blocks	3*	28	65	128	256
to Failure		31	68	131	259
		31	68	143	287
MIL-A-8866		40	71	146	
Spectrum A,		43	78		
Positive Loading		43	78		
		46			
		46			
Arithmetic Average 3*		36.6	69.2	132.6	250.0

TABLE IX

Spectrum Test Data for Specimens with Unloaded NAS-333 Bolts in Center Holes

Limit Stress KSI	49.0	48.5	47.5	45.0	43.0
Spectrum IV--	3*	3*	286	376	534
20 Hour Blocks	3*	3*	296	520	818
to Failure	3*	4	300	640	1149
	3*	56	324	712	
	3*	159	364	924	
	74	231	399		
		180			
Arithmetic Average			334	604	833

TABLE X

Spectrum Test Data for Specimens with Unloaded Taper-Lok Bolts in Center Holes

Limit Stress KSI	52.0	51.0	50.0	49.0	48.5
Spectrum IV--	3*	3*	226	468	328
20 Hour Blocks	3*	15	306	603	
to Failure		24	438	753	
		28	459	943	
		31	657		
		57			
Arithmetic Average		31	438	692	328

* Failed on application of 125 percent limit load at end of third sequence.

TABLE XI

Spectrum Test Data for Single-Shear Butt Joints with NAS-333 Bolts						
Limit Stress						
KSI	42.0	39.0	37.5	34.0	30.0	25.0
Spectrum IV-- 20 Hour Blocks to Failure	Bolt					
	Head	3*	49	81	243	48**
	popped	37	56	99	254	414
	off	34	59	103	265	421
	at 105%	40	71	143		642
	limit	56	71	159		686
	load	65	87			
		71	109			
Arithmetic Average	0	51	71	117	254	542

TABLE XII

Spectrum Test Data for Single-Shear Butt Joints with Taper-Lok TL-100 Bolts						
Limit Stress						
KSI	46.5	45.0	42.5	37.5	30.0	25.0
Spectrum IV-- 20 Hour Blocks to Failure	31	49	121	149	253	283
	28	53	124	178	271	257
	37	62	124	181		
		68	134	189		
		71	140			
		87	140			
	93					
Arithmetic Average	32	69	130	174	262	270

* Failed on application of 125 percent limit load at end of third sequence - not included in arithmetic average. Bolt head popped off.

** Not included in arithmetic average.

TABLE XIII

Constant-Amplitude Data for Unnotched 7075-T6 - R = +0.5

Stress			Stress		
KSI	N	$\sigma_{\log N}$	KSI	N	$\sigma_{\log N}$
88.0	4		77.0	26,000	
87.0	6		77.0	40,000	
87.0	6	0.225	77.0	44,000	0.191
87.0	9		77.0	46,000	
87.0	14		77.0	46,000	
87.0	23		72.0	33,000	
86.5	8		72.0	47,000	
86.5	9		72.0	49,000	0.074
86.5	10		72.0	50,000	
86.5	15		70.0	63,000	
86.5	30	1.417	70.0	63,000	
86.5	7,580				
86.5	8,280		67.0	57,000	
86.5	9,250		67.0	64,000	
86.5	9,640		67.0	90,000	0.039
86.5	9,850		67.0	95,000	
86.5	10,550		76.0	126,000	
86.5	12,070				
86.0	5		65.0	83,000	
86.0	90	1.410	65.0	75,000	
86.0	13,030		65.0	92,000	0.092
			65.0	103,000	
85.5	15	0.039	65.0	134,000	
85.5	18		65.0	114,000	
85.0	6,950		65.0	135,000	
85.0	9,200				
85.0	11,000		60.0	51,000	
85.0	12,000		60.0	75,000	
85.0	11,600		60.0	99,000	
85.0	13,000	0.094	60.0	98,000	0.350
85.0	13,520		60.0	282,000	
85.0	13,650		60.0	336,000	
85.0	13,950		60.0	511,000	
85.0	14,000				
85.0	14,640		55.0	252,000	
85.0	16,000		55.0	735,000	
			55.0	2,132,000	0.470
77.0	11,000		55.0	2,820,000	
77.0	21,000		55.0	6,865,000	
77.0	25,000				
77.0	26,000		50.0	10 ⁷ +	
			50.0	10 ⁷ +	

TABLE XIII, Contd

Constant-Amplitude Data for Unnotched 7075-T6 - R = +0.25					
Stress KSI	N	$\sigma_{\text{Log N}}$	Stress KSI	N	$\sigma_{\text{Log N}}$
88.0	7		50.0	58,000	
87.0	10		50.0	62,000	
			50.0	62,000	
86.5	5		50.0	117,000	0.182
86.5	2,750	1.162	50.0	125,000	
86.5	544		50.0	147,000	
			50.0	167,000	
86.0	6				
86.0	3,490	1.285	47.5	56,000	
86.0	2,980		47.5	74,000	
			47.5	144,000	0.235
84.0	3,590		47.5	195,000	
			47.5	221,000	
82.5	5,830				
			45.0	68,000	
75.0	10,000		45.0	72,000	
75.0	15,000		45.0	92,000	
75.0	16,000		45.0	93,000	
75.0	17,000		45.0	236,000	0.838
75.0	16,000		45.0	237,000	
75.0	16,000	0.022	45.0	7,355,000+	
75.0	17,000		45.0	10,360,000+	
75.0	17,000				
75.0	17,000		35.0	10,000,000+	
75.0	18,000		35.0	10,000,000+	
75.0	18,000				
75.0	19,000				
65.0	17,000				
65.0	21,000				
65.0	21,000				
65.0	22,000				
65.0	23,000	0.111			
65.0	27,000				
65.0	33,000				
65.0	34,000				
65.0	38,000				
53.0	53,000				
53.0	64,000				
53.0	87,000	0.176			
53.0	294,000				

TABLE XIII, Contd

Constant-Amplitude Data for Unnotched 7075-T6 - R = 0

Stress KSI	N	$\sigma \log N$	Stress	N	$\sigma \log N$
87.0	12		70.0	12,000	
87.0	15		70.0	14,000H	0.134
87.0	55	0.856	70.0	16,000	
87.0	1,660		55.0	28,000H	
86.5	1,215		55.0	36,000H	0.116
86.5	1,960		55.0	37,000	
86.5	2,440	0.136	55.0	38,000	
86.5	2,680		55.0	39,000H	
86.5	2,720		45.0	60,000H	
86.5	3,150		45.0	80,000	
86.0	1,935		45.0	81,000H	0.053
86.0	2,105		45.0	88,000	
86.0	2,460	0.055	45.0	99,000	
86.0	2,650		40.0	51,000	
85.0	1,410		40.0	52,000	
85.0	1,710		40.0	100,000	0.215
85.0	2,110		40.0	130,000	
85.0	2,470	0.119	40.0	178,000	
85.0	2,850		35.0	84,000	
85.0	3,060		35.0	88,000	
84.0	3,290		35.0	201,000	0.724
83.0	1,800		35.0	212,000H	
83.0	2,980	0.532	35.0	678,000	
83.0	3,200		35.0	1,591,000H	
83.0	3,610		35.0	2,230,000	
83.0	190L		35.0	2,239,000H	
83.0	250L		35.0	2,230,000	
			35.0	4,423,000	
82.0	1,095L		35.0	7,684,000	
82.0	4,155L	0.289	35.0	15,320,000	
81.0	3,030L	0.088	32.5	1,658,000	
81.0	4,540L		32.5	4,616,000	
75.0	6,000L		32.5	$10^7 +$	
75.0	9,000L		32.5	$10^7 +$	
75.0	10,000	0.131	25.0	1,571,000+	
75.0	14,000H		25.0	4,455,000+	
70.0	9,000		25.0	6,911,000+	
70.0	11,000		25.0	$10^7 +$	
70.0	11,000	0.134	25.0	$10^7 +$	
			25.0	$10^7 +$	

TABLE XIII, Contd

Constant Amplitude Data for Unnotched 7075-T6 - R = -0.5

Stress KSI	N	$\sigma \log N$	Stress	N	$\sigma \log N$		
85.0	12L		65.0	7,000H			
85.0	214H	0.628	65.0	8,000H	0.059		
85.0	216H		65.0	9,000			
85.0	604H		65.0	10,000			
85.0	641H		65.0	10,000			
86.0	85			50.0		18,000	
86.0	96	0.401	50.0	25,000	0.065		
86.0	146		50.0	25,000L			
86.0	859		50.0	27,000H			
86.0			50.0	27,000H			
84.0	410	0.151	40.0	35,000	0.145		
84.0	478		40.0	51,000			
84.0	640		40.0	52,000H			
84.0	1025		40.0	72,000H			
84.0			40.0	83,000			
83.0	850	0.058	40.0	83,000			
83.0	1,108		25.0	152,000			
81.5	525L	0.142	25.0	209,000H	0.097		
81.5	874L		25.0	241,000			
81.5	906H		25.0	271,000L			
81.5	996L		25.0	271,000H			
81.5	1,025H		22.5	405,000 H			
81.5	1,427H		22.5	1,200,000+ L		0.223	
81.5	1,533H		22.5	1,200,000+ H			
80.0	288L			22.5		1,200,000+L	
80.0	851L	0.254	22.5	1,362,000+L	0.108		
80.0	312L		19.0	1,200,000+H			
80.0	1,000L		19.0	1,400,000			
80.0	1,073L		19.0	1,701,000+H			
80.0	1,195L		19.0	1,780,000+H			
80.0	1,270		19.0	1,998,000*			
80.0	1,248L		19.0	1,600,000*			
80.0	1,635L		19.0	2,785,000*			
77.5	1,698		0.064				
77.5	1,266						
75.0	1,483		0.129				
75.0	1,887L						
75.0	1,920H						
75.0	2,275						
75.0	2,649L						
75.0	2,750H						
75.0	3,049L						
75.0	3,999L						

TABLE XIII, Contd

Constant-Amplitude Data for Unnotched 7075-T6 - R = -1

Stress KSI	N	σ Log N	Stress KSI	N	σ Log N
80.0	41H		65.0	2,043L	
80.0	74H		65.0	2,160L	0.082
80.0	85				
80.0	101		60.0	2,850H	
80.0	135	0.215	60.0	3,060 H	
80.0	140		60.0	3,230L	
80.0	184 H		60.0	3,280L	
80.0	207H		60.0	3,420 H	0.085
			60.0	3,600 L	
77.5	101		60.0	3,630L	
77.5	140		60.0	4,350H	
77.5	155	0.077	60.0	5,510 L	
77.5	197				
			50.0	11,000 L	
78.0	55		50.0	11,000 L	
78.0	120	0.232	50.0	13,000 H	0.035
78.0	196		50.0	13,000 H	
78.0	209				
			40.0	21,000H	
75.0	285		40.0	29,000H	
75.0	312		40.0	35,000H	0.116
75.0	360		40.0	40,000H	
75.0	600L	0.222			
75.0	885		30.0	47,000H	
75.0	1,080		30.0	95,000L	
			30.0	226,000L	0.290
72.0	298		30.0	235,000H	
72.0	302				
72.0	320	0.179			
72.0	530		25.0	291,000H	
			20.0	460,000H	
70.0	777L		20.0	629,000L	
70.0	825L		20.0	1,220,000+H	0.177
70.0	984H	0.182	20.0	1,200,000+H	
70.0	1,042L		20.0	1,200,000+H	
70.0	1,118L				
70.0	1,169				
70.0	1,220H				
70.0	1,275H				
70.0	2,230				
70.0	3,300				

TABLE XIII , Contd

Constant Amplitude Data for Unnotched 7075-T6

R = -2			R = -4		
Stress KSI	N	σ Log N	Stress KSI	N	σ Log N
40.0	940L		22.5	5,730L	
40.0	1,620L	0.195	22.5	8,980L	0.244
40.0	2,560L		22.5	22,100L	
40.0	2,985L				
			20.0	17,300L	
35.0	3,431L		20.0	19,200L	0.023
35.0	4,520L	0.170			
35.0	8,750L		17.5	30,000	
			17.5	30,000	0.000
30.0	11,200L				
30.0	22,700L		15.0	159,200	
30.0	28,320L	0.165	15.0	162,000	0.104
30.0	28,490L		15.0	168,000	
			15.0	169,000	
25.0	55,000				
25.0	71,000		12.5	382,000	
25.0	94,000	0.125	12.5	628,000	0.342
25.0	111,000				
			11.5	1,680,000	
20.0	184,000		11.25	4,348,000	
20.0	272,000		11.25	8,867,000+	0.307
20.0	404,000	0.160	11.0	10 ⁷ +	
20.0	412,000				

LEGEND:

- + Specimen did not fail -- test discontinued
- * Failed away from test section
- N Cycles to failure

σ Log N Standard deviation based on logarithmic mean. At very short lives and at long lives (particularly where runouts are involved), the standard deviation in terms of life has but little meaning; however, are presented for whatever they are worth.

The above table is comprised of data obtained in this program plus data taken from Reference 5.

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13. ABSTRACT

The purpose of this investigation was to assess the validity of the "Smith Cumulative Damage" hypothesis for 7075-T6 aluminum alloy specimens and structures.

It was found that the results of a single-amplitude test (at short life) can be used to estimate the stress at the point of failure, including residual stress. This permits using S-N data for axially loaded unnotched specimens to predict spectrum life.

Excellent agreement was found between calculated and experimental lives of full-scale structures; however, test lives of small specimens were consistently shorter than predicted.

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