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ENGINEERING GRAPHICS SEMINAR

FOUR-DIMENSIONAL DESCRIPTIVE GEOMETRY
REGULAR PENTATOPE - ORTHOGRAPHIC PROJECTIONS

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PENTATOPE - DEFINITION

A pentatope consists of five points not belonging to the same three-dimensional space, and the edges, faces and interiors of the five tetrahedrons whose vertices are the same points taken four at a time.

It has 10 edges, 10 faces, and 5 cells (tetrahedrons).

REGULAR PENTATOPE

If all 10 edges of the pentatope are equal, the pentatope is called regular. Therefore, it consists of five regular tetrahedrons two by two having one common face.

By considering the three-dimensional space of a regular tetrahedron, if from the center of the tetrahedron we draw a perpendicular to this 3-D space, every point of the perpendicular is equidistant to the four vertexes of the tetrahedron. The point on this line, whose distances to the four vertexes are equal to the edges of the tetrahedron, form with them a regular pentatope.

ORTHOGRAPHIC PROJECTIONS[1] - Figures 1 and 2.

Let Λ be the 3-D space where we shall construct a regular tetrahedron of edge a. The procedures to follow are similar to those of the three-dimensional descriptive geometry. We selected a plane α, where we draw one of the faces of the tetrahedron. The opposite vertex is obtained by drawing perpendicular through the center of the circumference circumscribed to the triangle face and marking a segment h which length is obtained as shown.

The point at the lower third of this segment h is the center of the tetrahedron (center of the sphere circumscribed to the tetrahedron). From this point draw perpendicular to the 3-D space Λ and mark a segment H which length is obtained as shown.

In the following figures 3, 4, 5, 6, and 7, we show the orthographic projections of each of the five tetrahedrons.

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1) Consult the author's "Descriptive Geometry of Four Dimensions", Engineering Graphics Seminar, Technical Seminar Series, Report No. 9, December 5, 1963, Department of Graphics and Engineering Drawing, Princeton University.

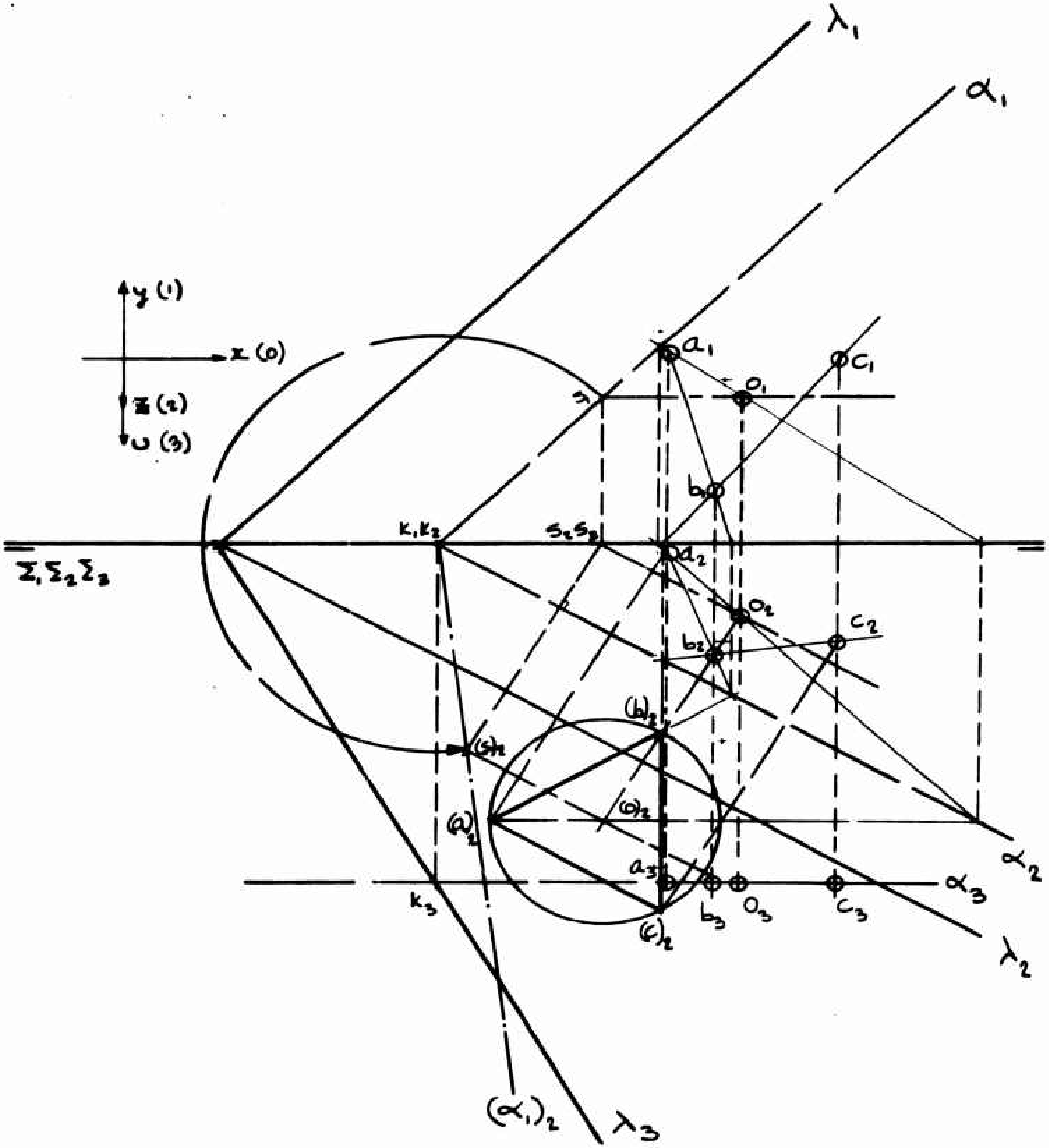
NOTE:

Due to the scale used in the drawing of the orthographic projections, necessarily small to fit within a $8\frac{1}{2}$" x 11" sheet, accuracy could not be attained.

The purpose here is to indicate the step by step procedure for obtaining the orthographic projections of the regular pentatope. We have indicated the method of checking the true length of the edges.

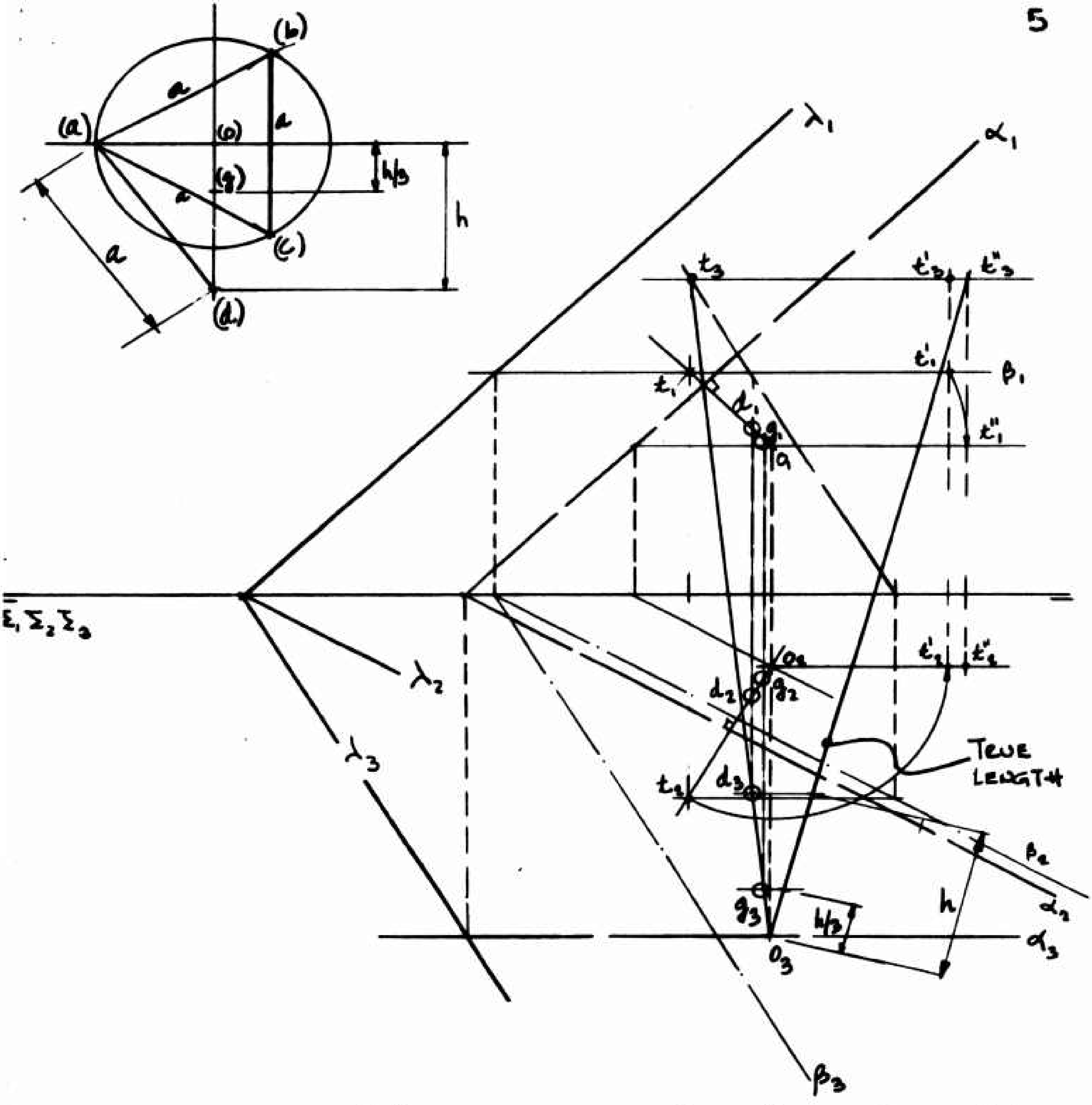
Visibility of elements in the pentatope itself and in the five regular tetrahedrons is not indicated.

As a problem derived from this representation we suggest the determination of the traces of the 3-D spaces of each tetrahedron. This additional problem may be considered as another way of checking the graphical constructions, for since the triangle made of the points (a), (b), (c), (d), (e), taken three at a time, is common to two tetrahedrons, its plane is the intersection of the 3-D spaces of those tetrahedrons. The three points, the sides of the triangle, and the plane, should, evidently, satisfy descriptive conditions of belonging to each 3-D space.



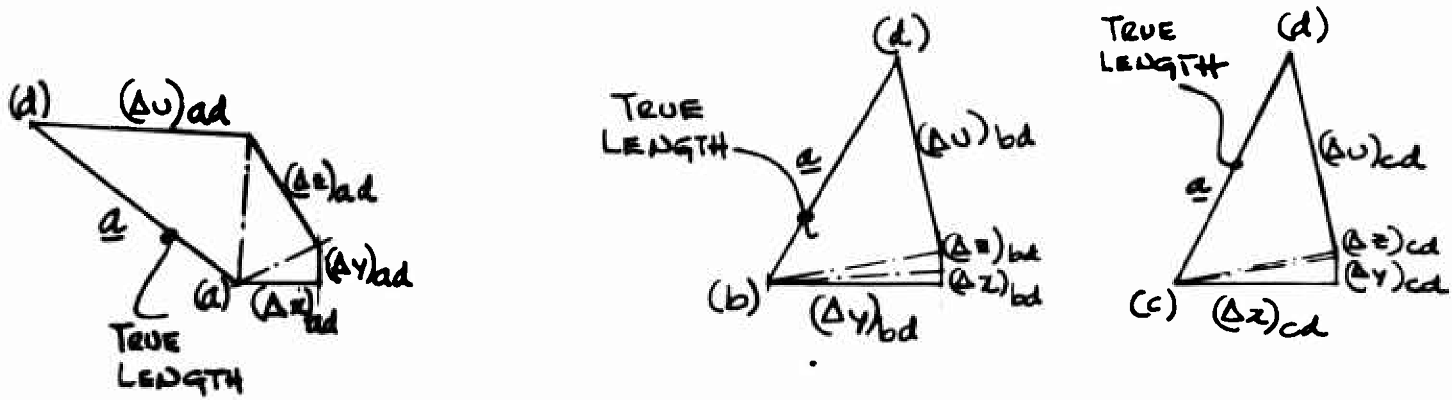
FACE (abc)

FIGURE 1 - a



VERTEX (d) OF TETRAHEDRON (abcd) AND CENTER (g)
[(O d) ⊥ α] BELONGING TO 3-D SPACE Δ

FIGURE 1-b



CHECKING LENGTH OF EDGES (ad), (bd), (cd)

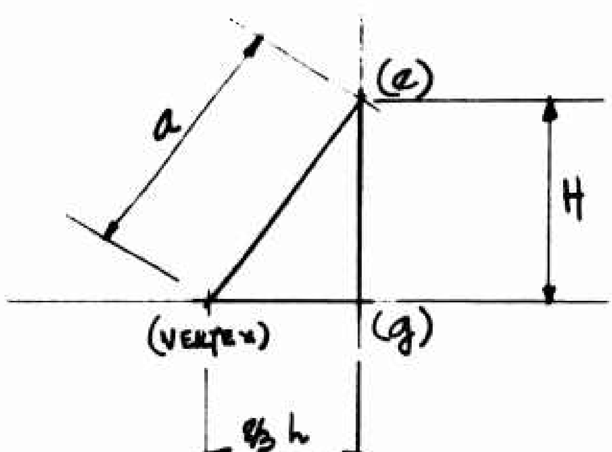
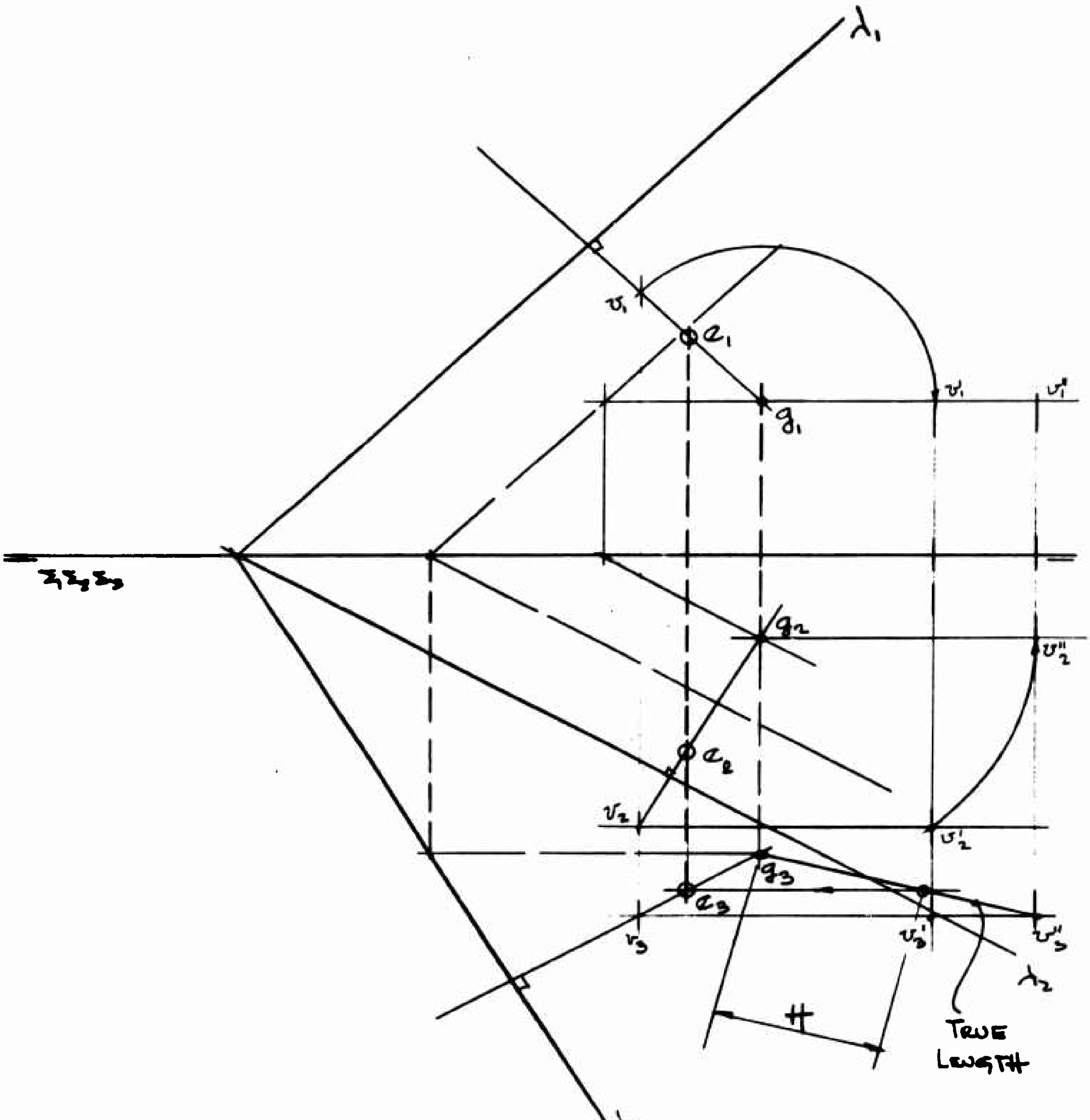
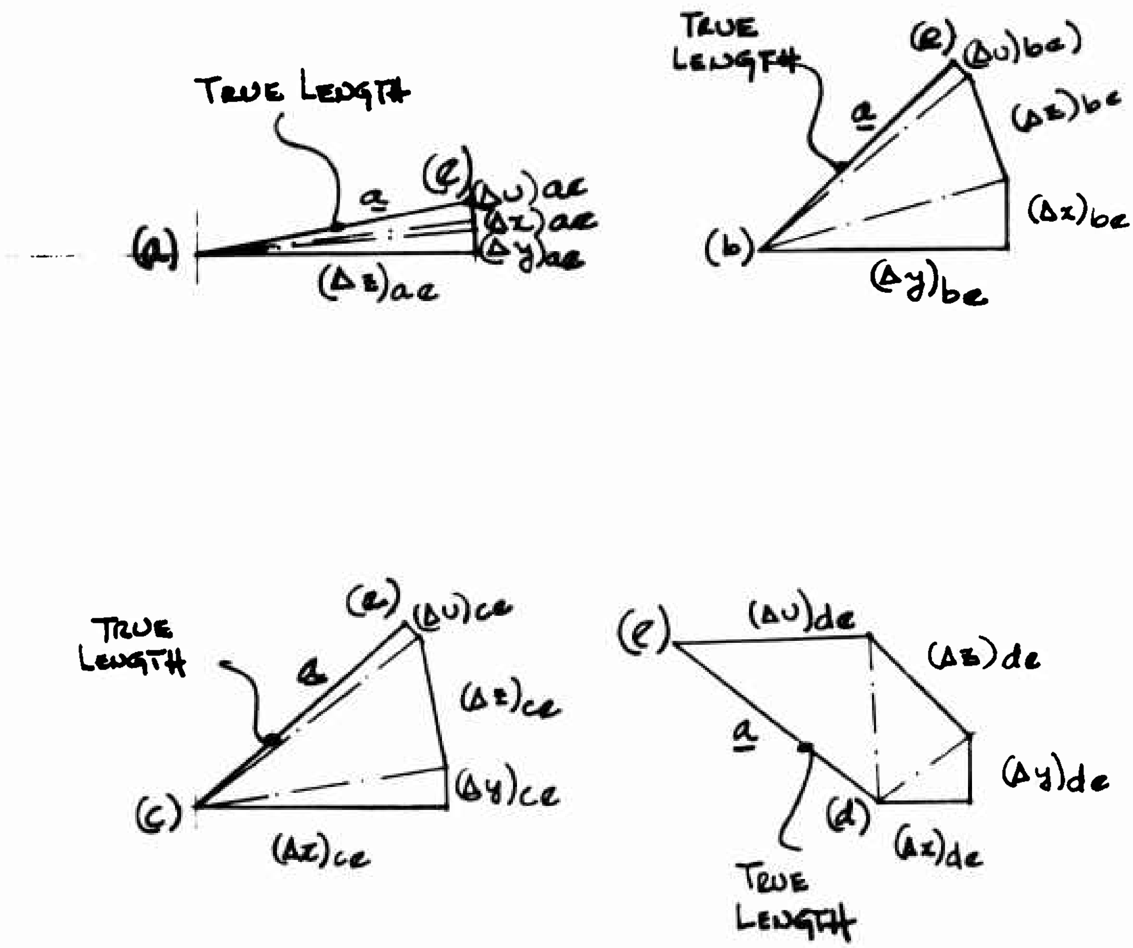


FIGURE 1-C



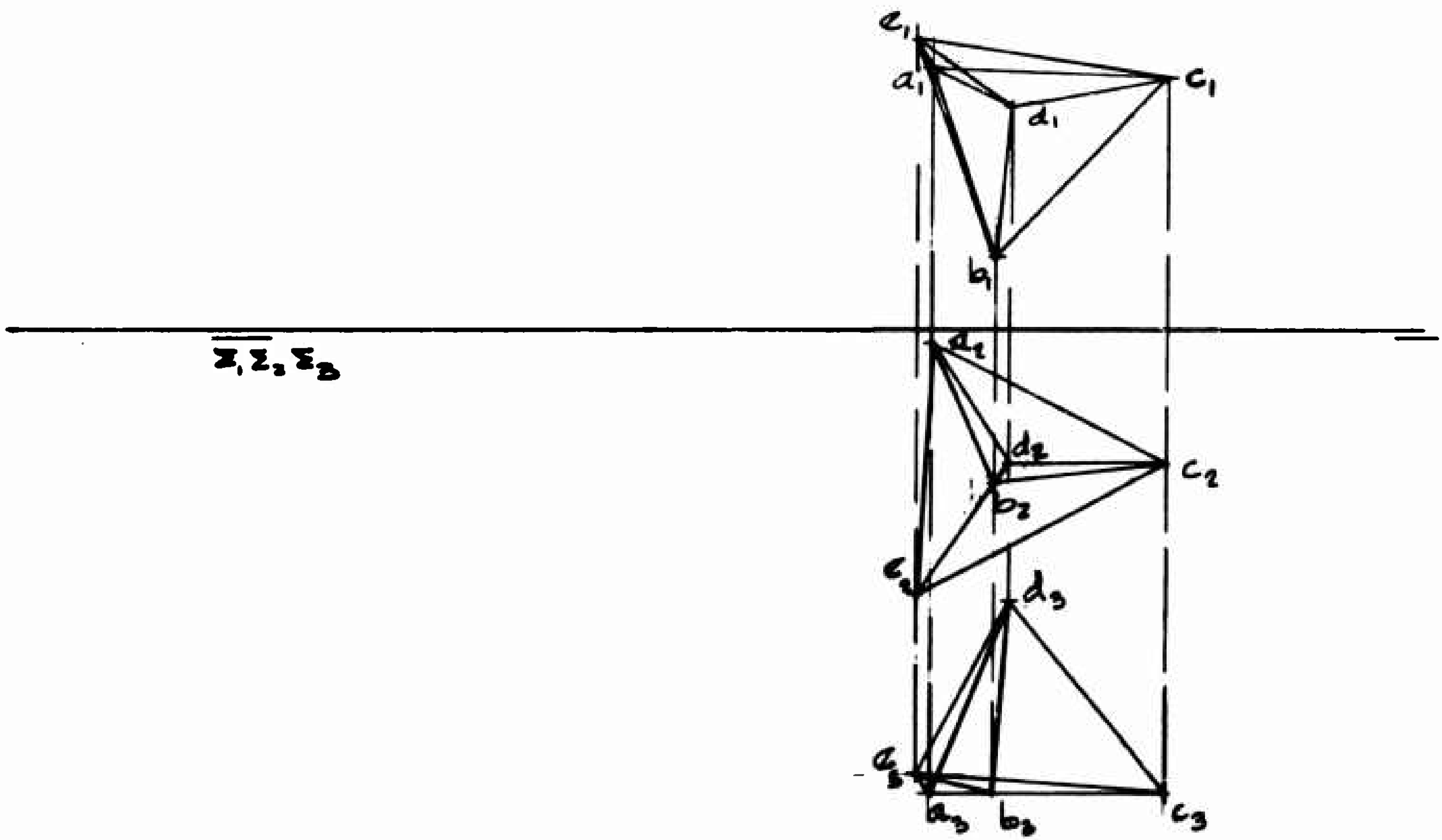
VERTEX (e) ; (ge) ⊥ Δ

FIGURE 1 - d



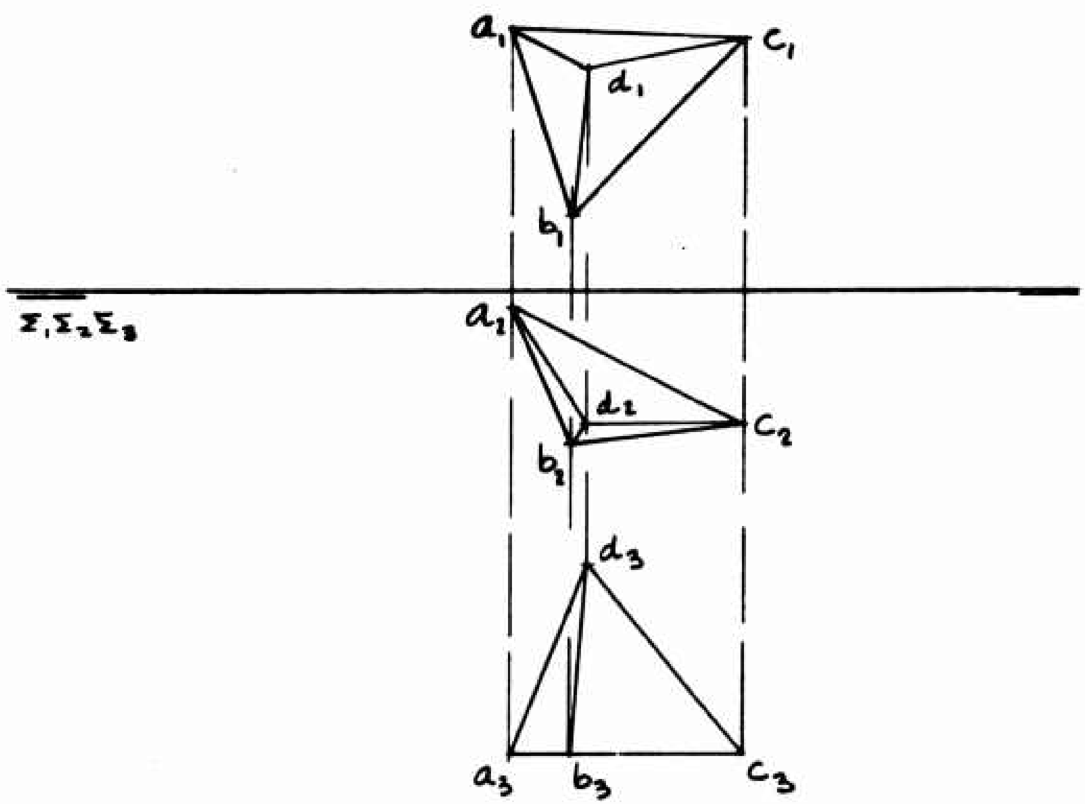
CHECKING LENGTH OF (ae), (be), (ce), (de).

FIGURE 1.e



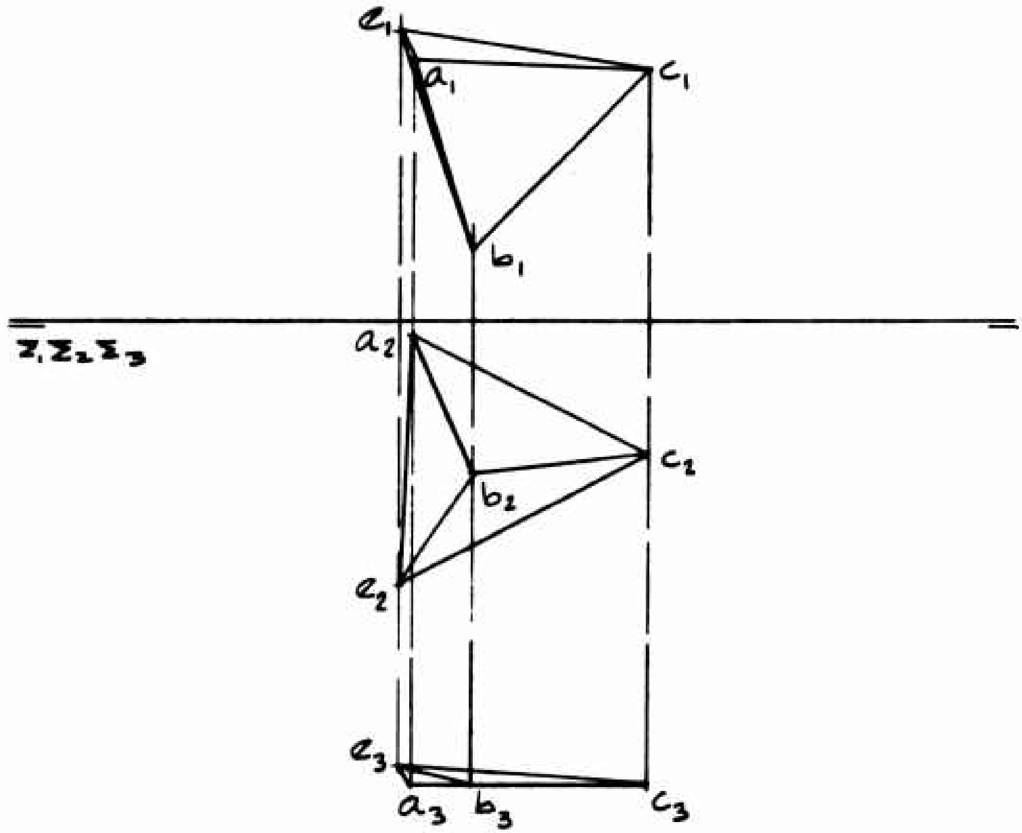
REGULAR PENTAHEDROID (abcde)

FIGURE 2



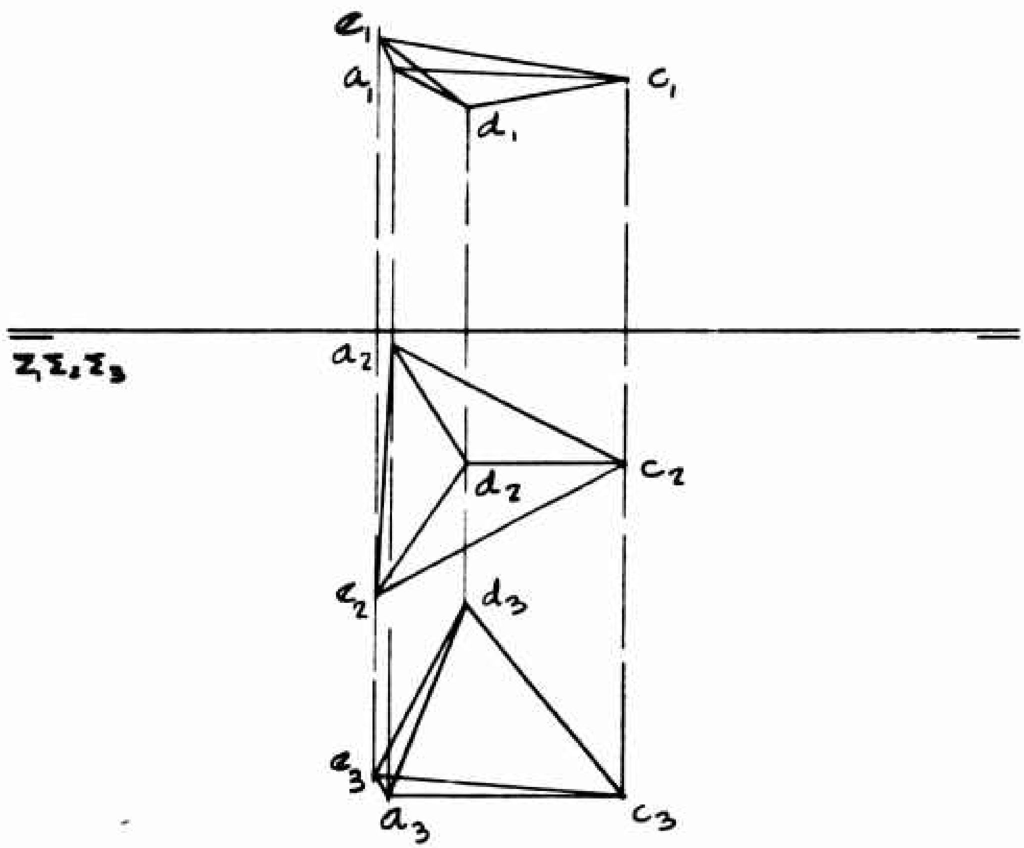
REGULAR TETRAHEDRON (abcd)

FIGURE 3



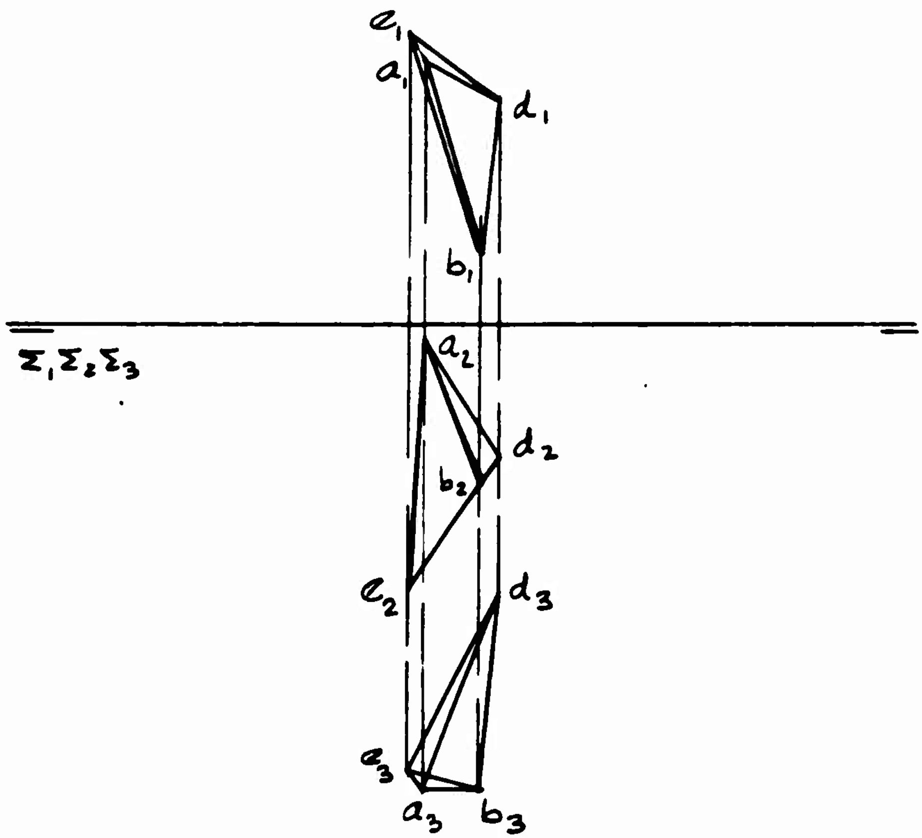
REGULAR TETRAHEDRON (abce)

FIGURE 4



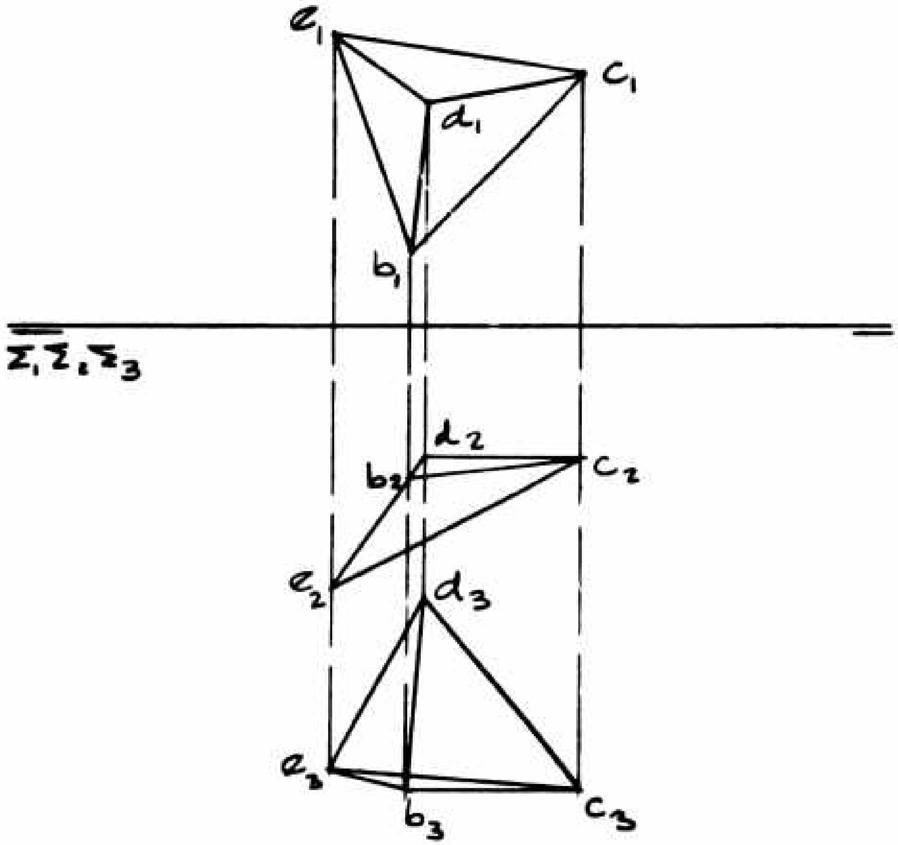
REGULAR TETRAHEDRON (acde)

FIGURE 5



REGULAR TETRAHEDRON (abde)

FIGURE 6



REGULAR TETRAHEDRON (bcde)

FIGURE 7