

AD631306

ENGINEERING GRAPHICS SEMINAR

FOUR-DIMENSIONAL DESCRIPTIVE GEOMETRY
SYMMETRY - DESCRIPTIVE SOLUTION

C. Ernesto S. Lindgren
Visiting Research Engineer
United States Steel Corporation

March 1965

CLEARINGHOUSE FOR FEDERAL SCIENTIFIC AND TECHNICAL INFORMATION			
Hardcopy	Microfiche		
\$1.00	\$0.50	19	pp
ARCHIVE COPY			

PROCESSING COPY

Code 1



TECHNICAL SEMINAR SERIES

Special Report No. 1

Department of Graphics and Engineering Drawing
School of Engineering and Applied Science
Princeton University

1965 C. Ernesto S. Lindgren

ABSTRACT

The paper presents the solution by methods of the four-dimensional descriptive geometry of the problem of symmetry of a segment of line in a four-dimensional space.

SYMMETRY-ENUNCIATION OF THE PROBLEM

- To determine the symmetry of a segment of a line (ab), belonging to a given plane α , in relation to a plane β perpendicular to α through a point (p) of α and in relation to the plane γ , absolutely perpendicular to α , through point (p).¹⁾ See figure 1.

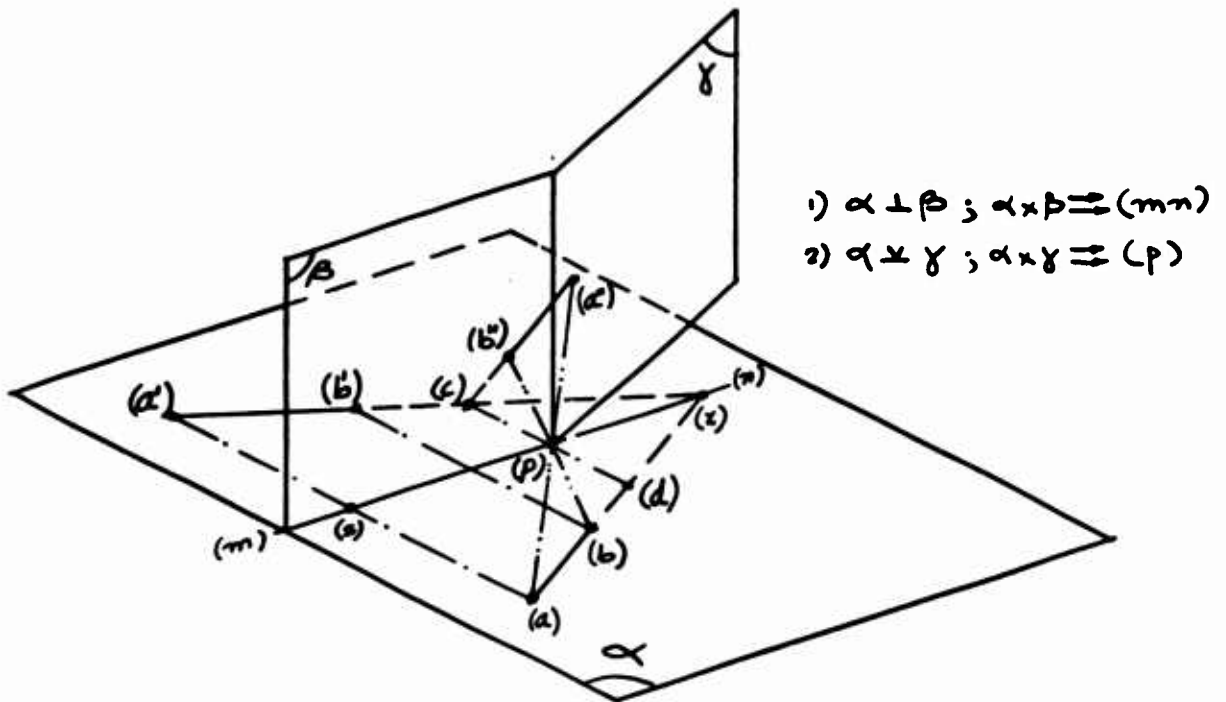


Figure 1

¹⁾ For the discussion of the solution by synthetic method, see the author's "Symmetry in a Four-Dimensional Space", Engineering Graphics Seminar, Technical Seminar Series, Report No. 11, Department of Graphics and Engineering Drawing, Princeton University, October 22, 1964.

2.

Figure 1 shows a plane α and through a point (p) on it, two planes, β and γ , the first perpendicular to α and the second absolutely perpendicular to α . The two symmetries of the segment (ab) are $(a'b')$ and $(a''b'')$.

For the solution by descriptive method we may consider, for example, three of the six planes of four lines perpendicular to each other, belonging to the same point. The determination of four such lines is discussed in our "Geometry of Four Dimensions" ²⁾. According to that solution, four lines, (mn), (mo), (mk), (ml) are two by two perpendicular, and belong to the same point (m). Considering the following planes:

$$(mn - mo) \equiv \alpha$$

$$(mn - mr) \equiv \beta$$

$$(mr - ms) \equiv \gamma$$

We conclude that:

$$\alpha \times \beta \equiv (mn)$$

$$\alpha \times \gamma \equiv (m)$$

where

$$\alpha \perp \beta \quad (\text{perpendicular planes})$$

$$\alpha \times \gamma \quad (\text{absolutely perpendicular planes}).$$

²⁾ Engineering Graphics Seminar, Technical Seminar Series, Report No. 9, Department of Graphics and Engineering Drawing, Princeton University, December 5, 1963.

3.

Therefore, following the determination of the projections of these lines, we may follow up with the determination of the symmetries of a segment (ab) of α , in relation to β and γ . Such solution, would require some extensive constructions, such as the determination of the traces of the 3-D spaces defined by two intersecting planes, and the final determination of the symmetries would be obtained in the same manner as discussed in the following approach.

Consider a 3-D space $\mathcal{T}(\tau_1, \tau_2, \tau_3)$. (Figure 2).

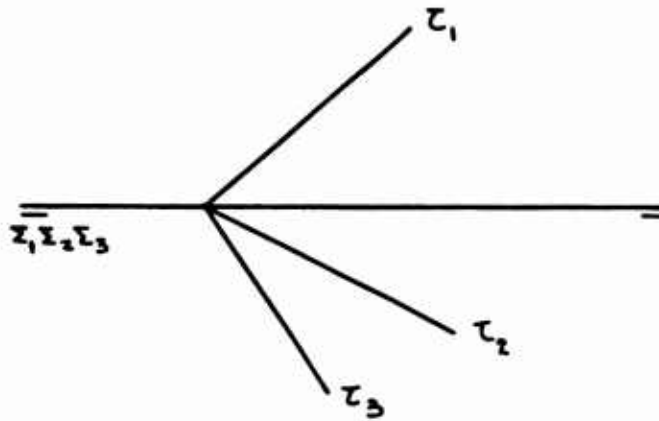


Figure 2

Determine a plane of this 3-D space. We chose it to be a geometric locus of points equidistant to the 3-D space of reference Σ_3 . Let this plane be the given plane α , and (p) the given point of α . (Figure 3). To determine the projections of (p), consider a line (mn) of the plane α . (Figure 3).

4.

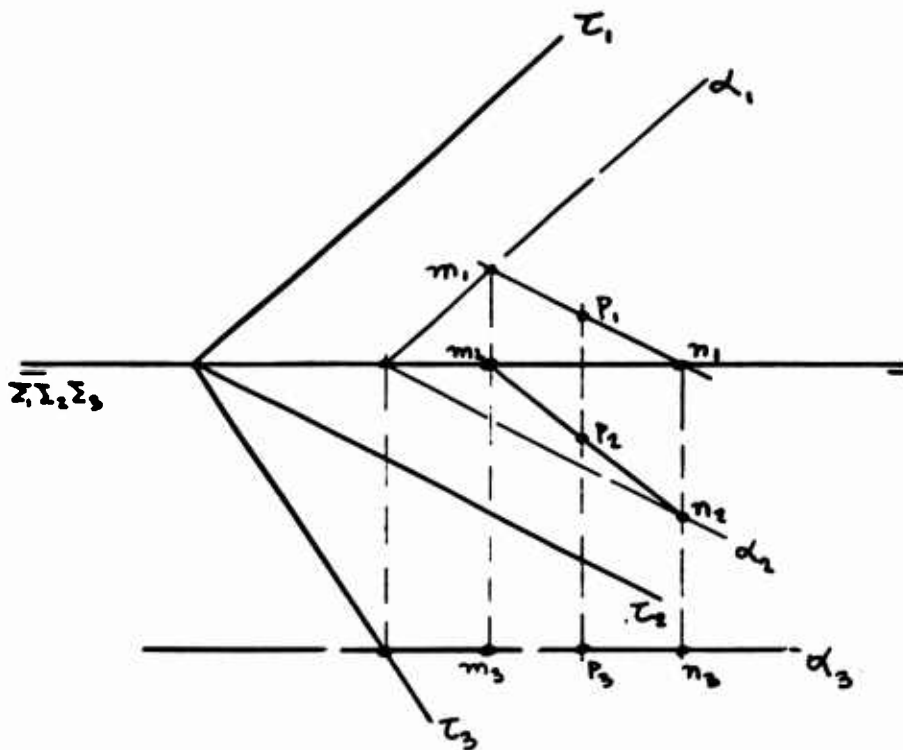


Figure 3

If through (p) we raise a perpendicular (po) to α , (po) and (mn) determine a plane, perpendicular to α . Let this plane be β . (Figure 4).

5.

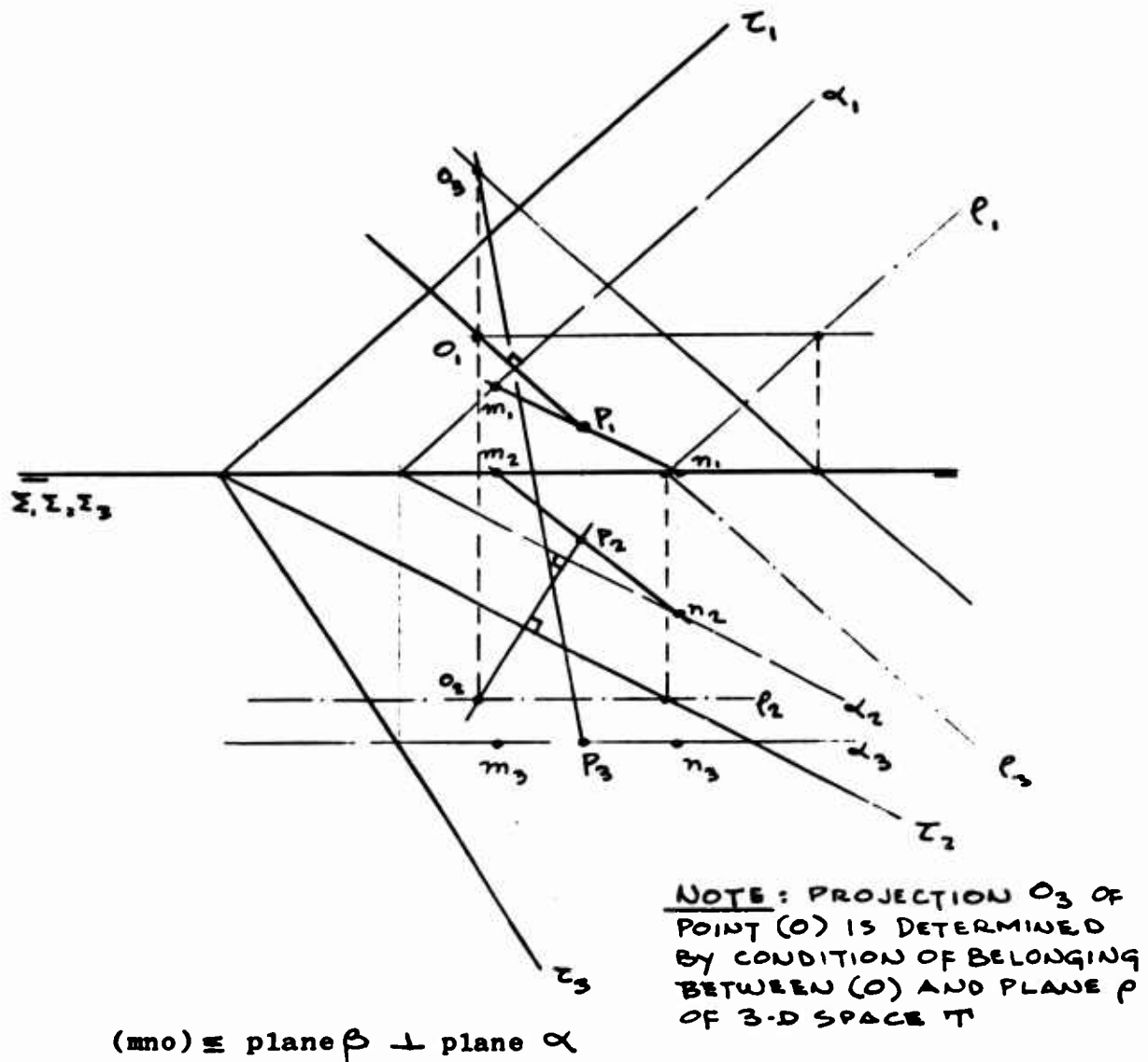


Figure 4

Through (p) raise a perpendicular (pr) to the 3-D space T . This line is perpendicular to α , and with the line (po) , determines a plane which intersection with α is the point (p) .

6.

This new plane, $(\text{por}) \subseteq \gamma$, is absolutely perpendicular to α . (Figure 5).

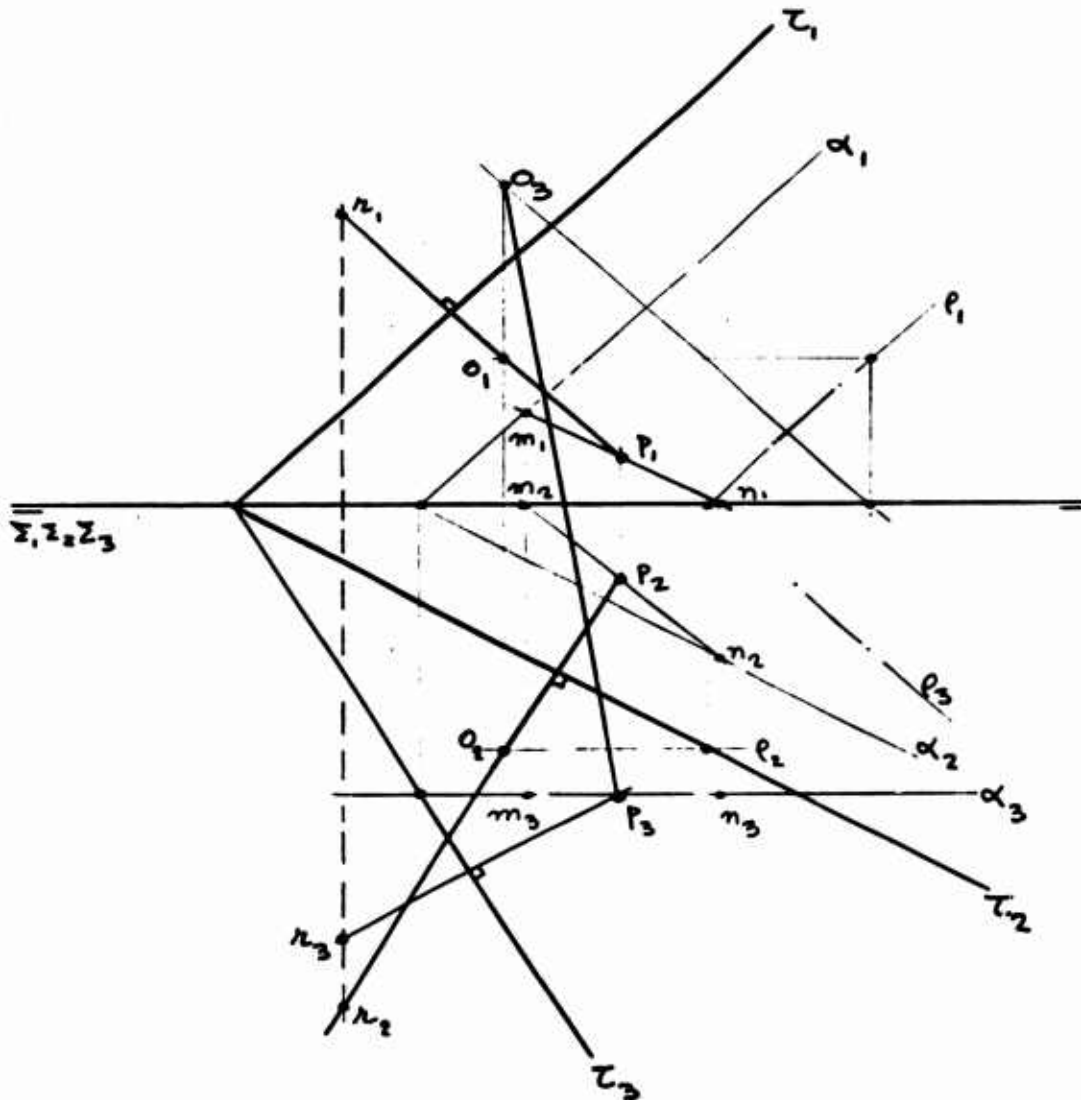


Figure 5

The symmetry of a segment (ab) of plane α in relation to plane β may be obtained by operating with the plane α only, since the feet of the perpendiculars through (a) and (b) to β , are found on the line (mn), intersection of the two planes. This is the solution that we will present.

However, we indicate in the following figures, how to proceed to allow working with plane β and any other plane α_1 , perpendicular to it, by using methods of the three-dimensional descriptive geometry. This procedure consists in making changes of 3-D spaces in the 4-D system of reference²⁾ until both planes, β and α_1 , are superimposed on one of these 3-D spaces. Figures 6-a, b, c, d indicate the necessary changes, until the plane α and the plane (mno) $\equiv \beta$, belong to the 3-D space Σ_{31} of the 4-D system of reference.

2) D.G.F.D., Report No. 9.

8.

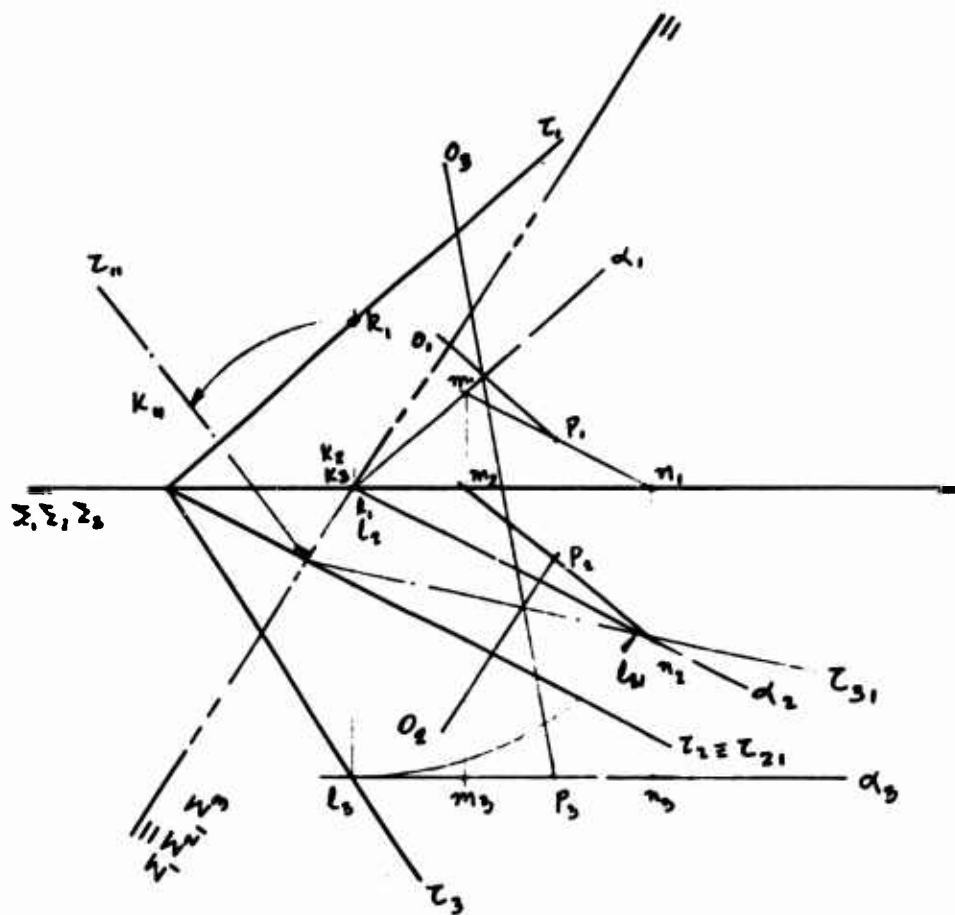


Figure 6-a



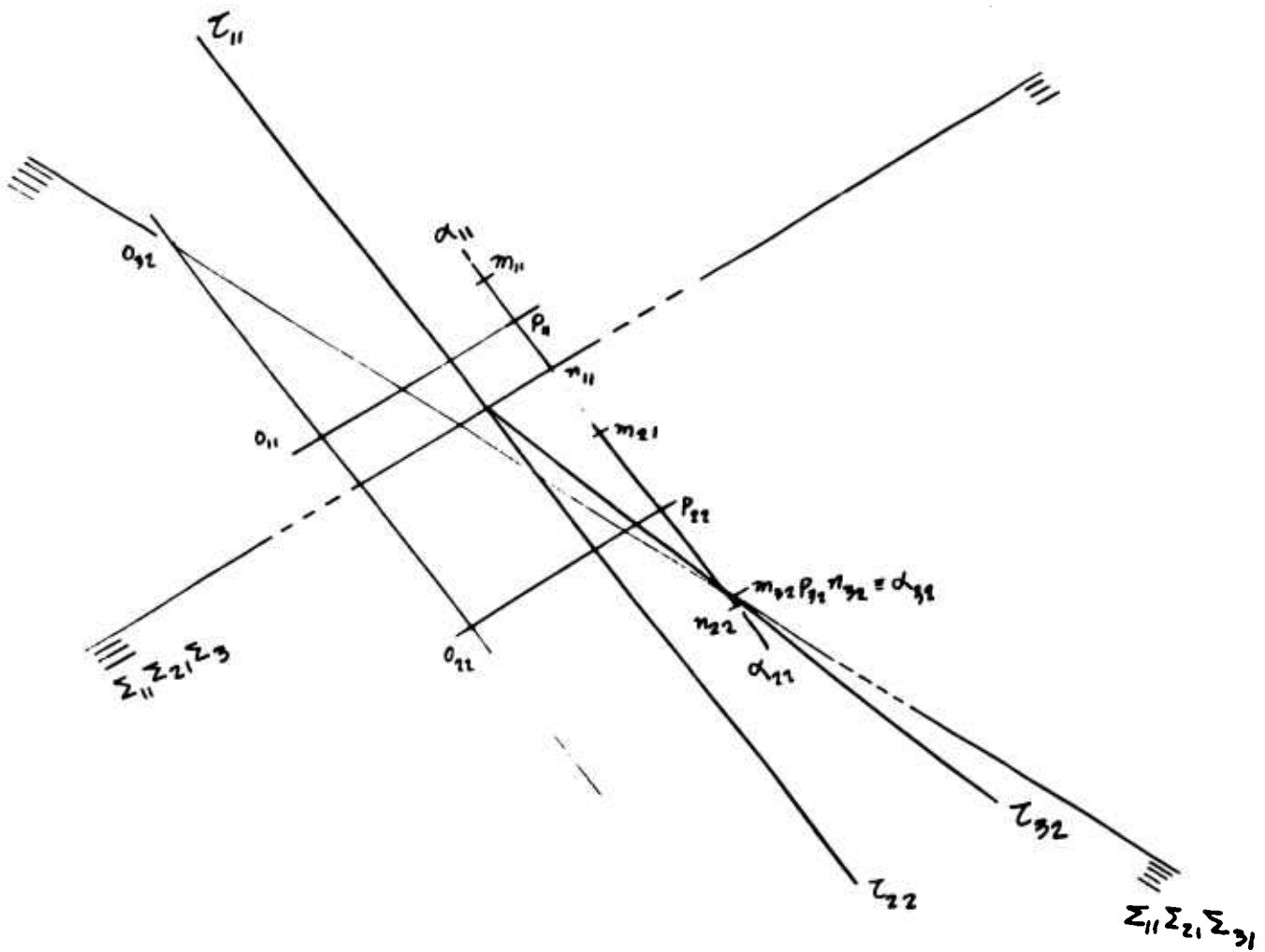


Figure 6-c

11.

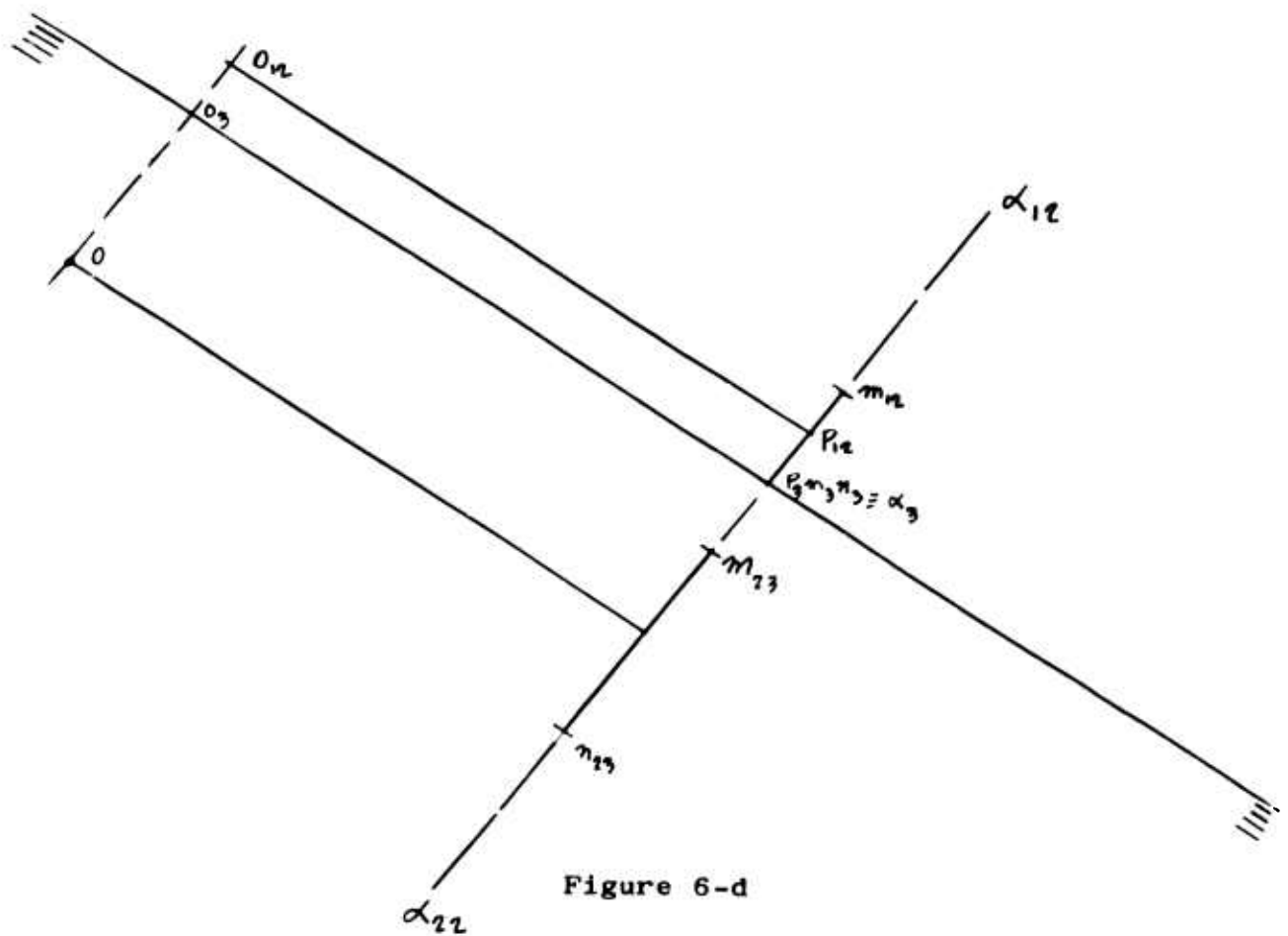


Figure 6-d

12.

The solution here presented is as follows.

Consider the plane α , a segment (ab) in it, and rotate the plane about the line α_2 until it is superimposed on the plane π_2 . Determine the positions of lines (mn) and (ab) and draw the perpendiculars (as) and (bt) to (mn) . Obtain (a') and (b') so that $(as) = (a's)$ and $(bt) = (b't)$ and determine the projections of (a') , (b') and (x) , being (x) the intersection of (ab) , $(a'b')$, and (mn) . See figure 7.

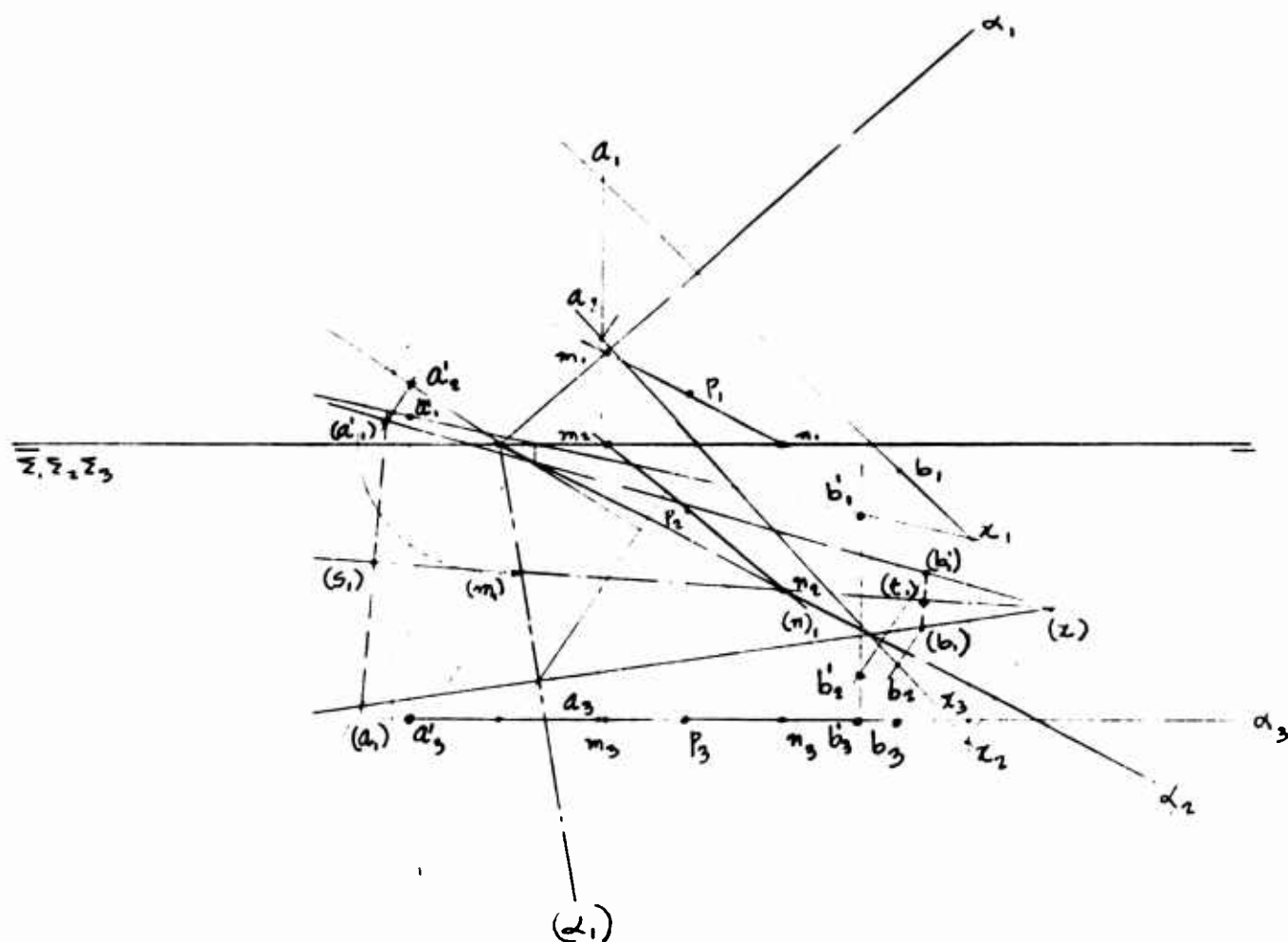
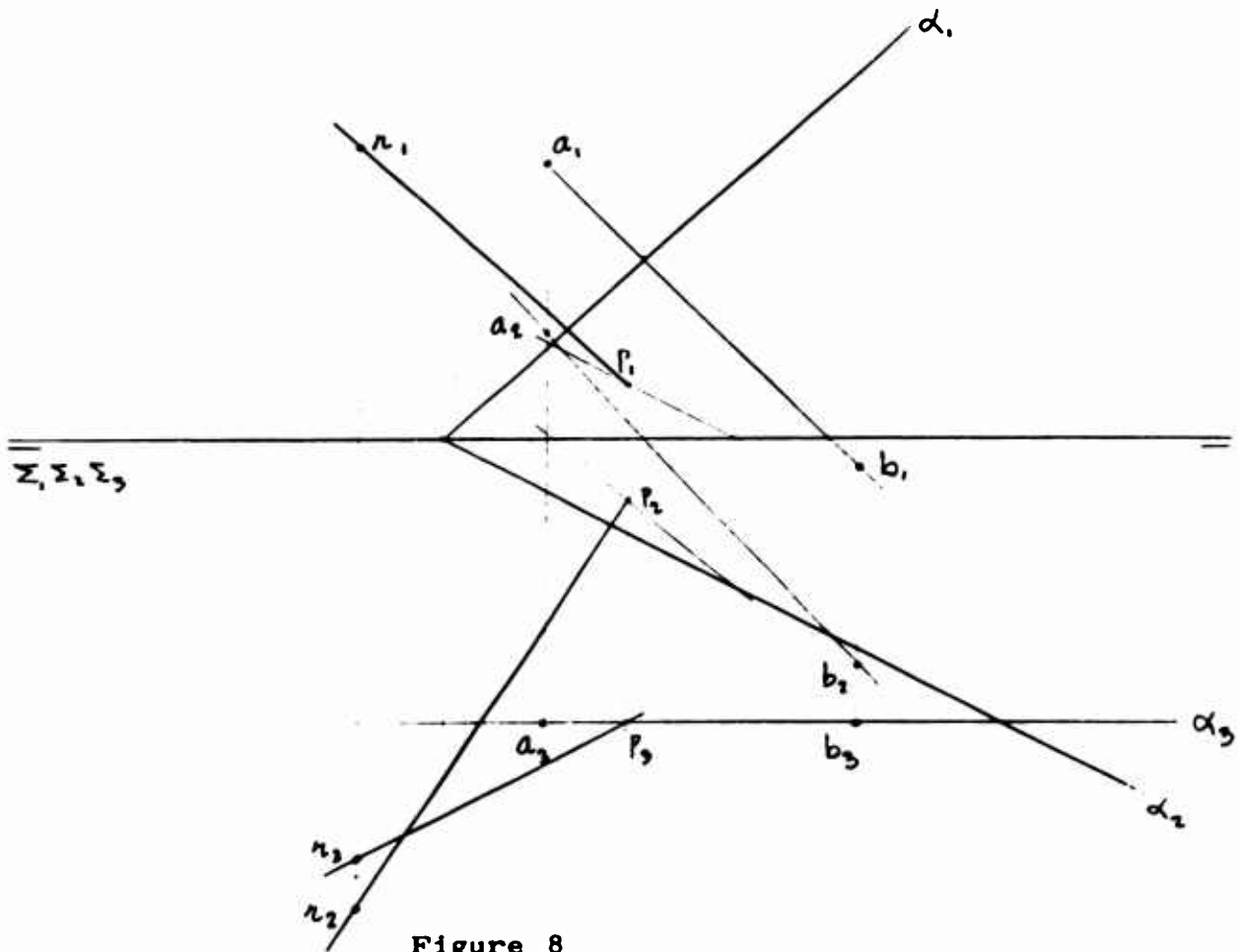


Figure 7

14.

Thus, we have obtained the first symmetry of (ab), in relation to plane β . That is the segment (a'b'). Let us proceed with the determination of the second symmetry, (a''b'') in relation to plane γ .

Referring to figure 5, consider the plane $(mn - pr) \equiv \gamma$, plane α and the same segment (ab) of α , as shown in figure 7. See figure 8..



•

$$z_1, z_2, z_3$$


Figure 9

RELATION BETWEEN THE TWO SYMMETRIES

Evidently, the two segments, $(a'b')$ and $(a''b'')$, because they belong to the same plane α , are concurrent in a point (c).

This point we will call a double point in the sense that it corresponds to a point (d) of (ab) which symmetries in relation to α and γ , coincide on (c). (See figure 1). Therefore, the line (cd) is perpendicular to α and γ . Due to this last condition it belongs to the point (p).

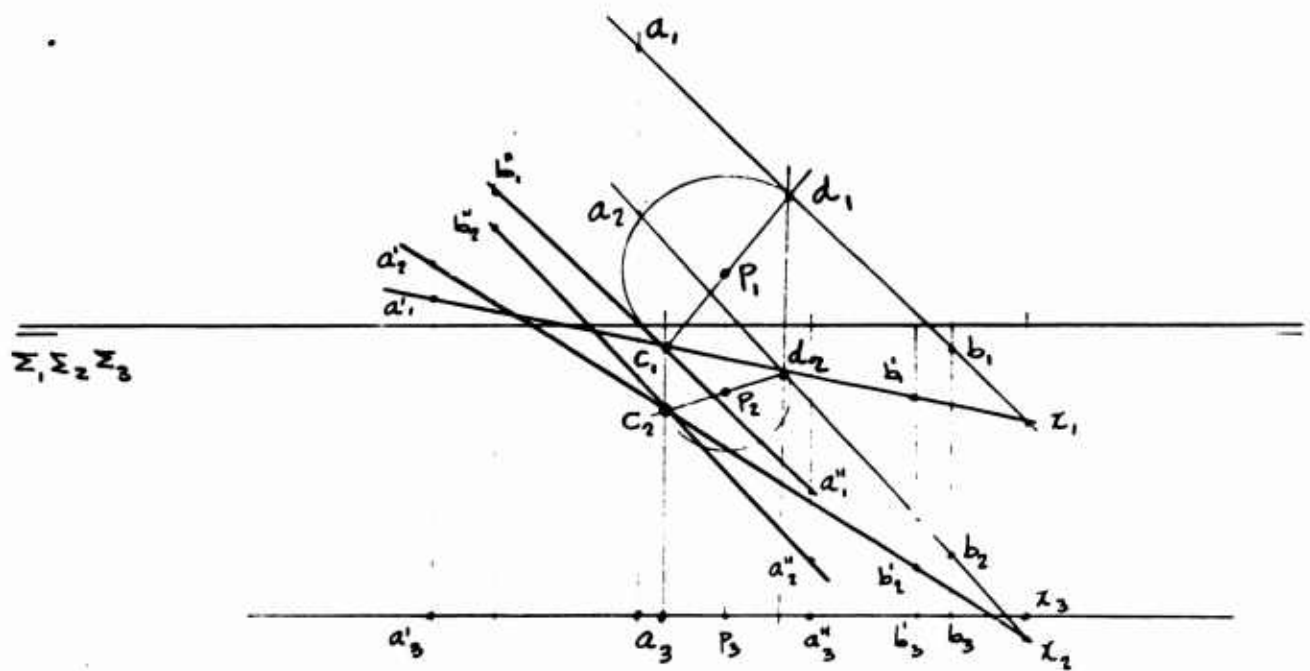
We can make the following statement regarding this point:

"The segment of a line belongs to one and only one point whose symmetries - in relation to a plane β and a plane γ , the first perpendicular and the second absolutely perpendicular to a plane α of the line - coincide." Figure 10.

Furthermore, we can consider the following lines: (pd), (pm), (po), and (pr). These four lines, two by two are perpendicular, determine six planes, two by two perpendicular, three by three belonging to the same line and so related that there are three pairs of planes which intersection is the point (p). In addition, the four lines determine four 3-D spaces, two by two perpendicular. In other words, all the relations among the geometric elements, determined by

17.

the four lines, are identical to that shown to exist among the elements of the system of reference.³⁾



CHECK : $c_1 P_1 = d_1 P_1$; $c_2 P_2 = d_2 P_2$

Figure 10

³⁾ See Report No. 9.