OMEGA
A WORLD-WIDE NAVIGATIONAL SYSTEM

System Specification

And

Implementation

SECOND EDITION

Prepared by the Omega Implementation Committee for the Department of the Navy, and submitted through the Office of Naval Research.

DEFENSE DOCUMENT CENTER NO.
AD-630 900

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Published for the Committee
By
Pickard & Burns Electronics
103 Fourth Avenue
Waltham 54, Massachusetts

Division Of
Gorham Corporation

P&B Pub. No. 886B
P&B Project No. 347

Issued: 1 May 1966
Copy No. 1

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OMEGA

A WORLD-WIDE NAVIGATIONAL SYSTEM

PART I. THE SYSTEM

1.0 Introduction

This report presents a design plan for a radio-locating facility providing world-wide position fixing of moderate (one-mile) accuracy by phase comparison of VLF (10 - 14 kc) continuous wave radio signals from only eight strategically located transmitting stations usable by aircraft, ships, and land vehicles, and by submarines at moderate (40 - 50 feet) antenna depths.

1.1 Purpose of the System

Through all recorded history such proponents of navigation as Prince Henry of Portugal in the Fifteenth Century, the British Board of Longitude in the early Eighteenth Century and the modern Radio Technical Commission for Aeronautics and International Civil Aviation Organization have emphasized the requirement for navigation all over the globe. This need has increased with the expansion of commerce and the advent of aircraft, submarines, and other more exotic vehicles until there is now an urgent requirement for accurate all-weather global navigation on the surface, in the air, and under water.

These requirements have never been satisfied fully - celestial navigation is not all-weather; present electronic systems do not give global coverage; inertial systems are expensive, limited in accuracy, and degrade with time.

The purpose of the Omega system is to fulfill this chronic requirement for global all-weather navigation at an acceptable cost by the establishment of a world-wide radio locating system, making optimum use of recently uncovered facts regarding the propagation of VLF radio signals over substantial distances.

1.2 Background

Extensive and accurate measurements of the phase stability of very low frequency (VLF) radio waves have been made throughout the last decade. These studies show that (in addition to conventional world-wide communications) such signals can provide global position-fixing of good accuracy and high reliability. Such a system can be used by ships, aircraft, and submarines to moderate depths.

Taking advantage of the high phase stability and low attenuation rates of VLF radio signals, Omega can provide this world-wide service with only eight transmitting stations. A new and useful degree of redundancy can be attained in this hyperbolic navigation system, which has many similarities to loran but which achieves vastly greater coverage per station.

Omega is particularly useful for ships because positions can be fixed to within one mile during the day and two or three miles at night, even in the center of the largest expanses of the oceans. For station-keeping or rendezvous, the relative accuracy will be about one-quarter mile. Submarines can obtain the same fix accuracies for antenna depths as great as 40 - 50 feet. Aircraft can use Omega for navigation by employing relatively simple rate control or approximate course and speed compensation circuitry in their receivers.

1.3 History

Omega was developed and has been extensively tested at the U. S. Naval Electronics Laboratory, with assistance from several other organizations. The Naval Research Laboratory has been responsible for the development of an airborne receiver and has played an important role in the observing program. The development at NEL has been in the hands of the group initially responsible for work on Radux, a low-frequency aid to navigation that did not reach implementation. Omega is, therefore, based upon some 15 years experience in the characteristics and navigational utility of the low radio frequencies. Experimental Omega transmitters have been operated in California, Hawaii, New York, the Panama Canal Zone, and recently in Wales. These sites were chosen only because of the availability of existing antennas. The geographic configuration of the experimental system could easily be better, and the radiated powers approach a satisfactory operational level only at the station in Hawaii.

These five transmitting stations have been grouped, at various times, to form six different pairs, and transmissions have been measured at several dozen locations distributed over a wide area.

Many of these monitoring points have been occupied only briefly, but at some, observations have been made for several years. In addition, there have been many months of observations on ships in transit or on station, and scattered observations from aircraft, as well as experimental observations in submarines.

As a result of these efforts, it is now possible to predict readings to be expected; that is, to plot Omega charts -- and to establish the range, accuracy, and reliability to be expected with an operational system.

1.4 General System Description

In Omega, eight VLF Transmitting stations will be distributed around the globe. Each station will transmit for about one second in turn, at precisely the same radio frequency. Position of a receiver will be established by intersection of hyperbolic contours defined by the relative radio-frequency phase of the transmissions of suitably chosen pairs of the eight stations. The stations in the network will have average separations of about 5,000 nautical miles. With minor exceptions, caused by high attenuation in transmission over the polar ice-caps and permafrost, all eight stations will be observable at any receiving point; five or six of the stations will ordinarily supply signals fully satisfying the system standards of fix accuracy and reliability.

The navigator can determine a line of position generated by any convenient pair of these stations, and can cross it with one or more other lines derived from another pair or pairs. Thus the navigator will choose position lines by Omega much as he chooses celestial lines of position -- for good accuracy and for large crossing angles -- and there is a satisfactory degree of redundancy to protect him in case of transmitter failure. He may make readings on four or five lines of position, but usually he will choose the two pairs that jointly give the greatest precision at his location.

The Omega receiver measures the relative phase of the signals from at least two pairs. This receiver may be manual or partly or fully automatic. To obtain a fix, the operator adjusts the receiver to read the lines of position he has chosen. Thereafter an automatic receiver will track the signals until the operator modifies his choice of pairs, or until arrival at his destination. The indication of position lines is explicit and continuous, and may be recorded for the convenience of the navigator.

Charts, tables, or both will be provided that relate the daytime Omega readings to geographic position, as in loran. Because there are minor diurnal and annual changes in the velocity of propagation, the navigator will also be given compensation graphs or tables that permit him to reduce his observed readings to equivalent daytime readings before consulting the main chart or table. All of this information can, of course, be stored in a computer (if desired) or computed as required rather than in advance.

1.5 Differential Omega

Over any more or less restricted area, the accuracy of Omega can be improved by perhaps an order of magnitude by a technique called Differential Omega in which the separation of two or more receivers is determined by the differences of their indications, with all propagation disturbances cancelled to the extent they are correlated at the receivers. One receiver might be a fixed monitor broadcasting or telemetering its readings to vehicles navigating in the vicinity, as in a terminal area or tactical theatre; or two or more vehicles may converge to previously agreed upon Omega coordinates in a rendezvous.

In a rendezvous, where the receivers are brought to the same coordinates, the correlation would be perfect, and the final accuracy would be determined purely by the resolution and accuracy of the receivers under the existing signal and noise environment. In a terminal area, the accuracy would be determined by the extent propagation fluctuations correlate over the area. Correlation would be substantially perfect at distances of a few miles, and useful out to at least several hundred miles.

Another form of Differential Omega is the use of a monitor indication to adjust the diurnal compensation for the fluctuation of the particular moment.

In intercontinental air navigation, for example, it may be feasible to interpolate deviation from the daytime value observed at each end of the path for the diurnal compensation at the immediate position. Other procedures can use the difference between the actual reading of the monitor and its predicted value to correct diurnal compensation at the position of the vehicle.

1.6 Anticipated Performance

As suggested above, distance from the transmitting stations has been substantially eliminated as a major problem in Omega. The use of very long base lines -- several of them actually connecting stations diametrically opposite to each other -- results in position lines that diverge only a little and that cross each other at nearly right angles. This geometrical excellence, together with the range of choices available to the navigator, results in a system whose absolute accuracy varies little with geographical position. This uniformity of accuracy is itself a great convenience and comfort to the navigator.

The accuracy of Omega can conveniently be considered in two parts. First, and most important, is the effect of natural random fluctuations in the times of propagation of the radio signals. This uncertainty can be roughly summarized as producing, in an Omega line of position, a standard deviation of about three-tenths of a mile in the daytime, and about twice that at night. These are average values; they may increase somewhat if operation should be required at extreme distances. This kind of error cannot be reduced by any known method, because it is a fact of nature.

As contrasted with this, the second kind of error is the result of a defect in our present knowledge of the actual average velocity of propagation. This uncertainty is reflected in our inability to draw position lines on a chart in exactly the right place. This charting error is obviously unimportant, if it is small as compared to the natural fluctuation. The velocity is a function of soil conductivity as well as time, and is also affected to a minor degree by the direction of transmission with respect to the earth's magnetic field. At the present time, the velocity is fairly well known, and the charting error seems to average no more than half of the fluctuation error. As more measurements are made, our knowledge of these factors is increasing rapidly. We feel no doubt that the charting error will decrease to relative unimportance as more analyses are made and as more data are taken, especially as future Omega experiments will be designed primarily to that end. By the time the Omega system can become operational, we are confident that the root-mean-square fix error, for all causes combined, will be about one-half mile in the daytime and one mile at night.

Even if two transmitters should fail simultaneously, the system redundancy will be adequate to supply substitute fixes to the navigator, with about a 50 percent increase in the average error.

1.7 System Cost

Transmitting antennas for the Omega frequency are large and expensive. Fortunately, there are two ameliorating circumstances. First, the required radiated power is only ten kilowatts; a very low value by VLF standards. Second, only eight stations are needed for world-wide service, so that the total system costs for both construction and operation are moderate in comparison with other aids to navigation.

We estimate that, in middle latitudes, an Omega transmitting station will cost $8,000,000 to $10,000,000. Some stations will need to be in remote areas, such as the Aleutian Islands, and may cost twice as much, or more. The cost for eight, together with necessary monitoring and communication facilities is expected to be less than $100,000,000.

Operating costs for Omega transmitting stations will be minor in comparison to existing aids to navigation, because of the small number of installations required. It appears that $600,000 per year per station should cover operation, maintenance, and replacement of depreciated or outmoded equipment.

Anticipated receiver costs are very difficult to assess because there may be large differences in the desired complexity and the costs will depend upon the quantities produced. The simplest manual receiver, built in fair quantity, may cost from $3,000 to $5,000. A single frequency automatic-tracking receiver may be estimated at $10,000 to $12,000, and a fully automatic lane-identifying receiver may well cost four times as much.

2.0 System Principles and Geometry

2.1 Introduction

In Omega a network of eight VLF radio stations, strategically located for complete world-wide coverage, provides a radio signal field with which the location of an observer can be determined anywhere on earth within an accuracy of one mile.

Each of the eight stations transmits a 10.2 kilocycle continuous wave signal for one second in turn, every ten seconds, with all transmissions phase-locked to a common Standard Time. Since the transmissions are phase-locked, the signal field phase is everywhere stationary; and the relative phase angle of a particular pair of signals observed at any given point depends solely upon how much further it is to one of the stations supplying the pair of signals from the other. Furthermore, the same phase angle will be observed at all points which have the same difference in the distances from the two stations. The locus of such points is a contour of constant phase (isophase contour) fixed on the surface with respect to the locations of the corresponding pair of transmitters.

Thus, the relative radio frequency phase of every pair of signals observed at any point on earth defines a known isophase contour containing that point, and the intersection of two such contours established by different pairs of stations defines the location of the point.

One great advantage of grid laying navigation devices, including the hyperbolic, is that the necessary computation for position can be done in advance. A "navigator" can visit, in imagination, every point in the service area and calculate the readings that would be observed there. All points having the same reading can be connected by one of many lines on a chart which is given to the actual navigator, thus saving him much effort and avoiding the possibility of computational errors under pressure of time.

The hyperbolic geometry has one advantage over other geometrically possible types of position lines, because the accurate knowledge of relative time (which must appear somewhere) is required only at the ground or shore stations, not at the navigator's position. The great disadvantage of hyperbolic position lines is their divergence. At a distance large compared with the separation between transmitters, the lines are approximately radial. Thus, a measurement with a certain timing accuracy would have a small error in miles near the line connecting the transmitters and an increasingly larger error in miles at increasing distances. It is clear that a more precise time measurement will reduce the errors in distance, but an increase in precision can usually be achieved only by increasing either transmitter power or bandwidth (or both); and there are practical limits which appear all too soon.

In previous hyperbolic aids, the chief design problem has been to provide signals of useful strength out to the distance where divergence has become too severe and then to have enough transmitting pairs to cover the needed area.

Earlier navigation systems have used pulses emitted in the required time relationship (as in loran) or have used continuous harmonically-related carrier frequencies whose phase relationship conveys the needed time information (as in Decca). With the pulses, all the signals in a network can appear at the same radio frequency at different times. With the continuous carriers, frequencies of the various stations must differ so that they can be distinguished and yet have a common basis in time (hence, the required harmonic relationship).

Omega uses a technique which, in a sense, combines both methods and has advantages and disadvantages of its own. The measurements are made on bursts of steady carrier and are of relative phase. They do, however, appear at the same radio frequency at different times. The use of a single frequency is economical in spectrum and has a great advantage, since phase shifts in the receiving equipment need not be known or controlled with great accuracy because they are the same for all signals. On the other hand, we have now raised the problem of measuring the difference in phase between two signals which do not exist at the same time. This can be done most easily by comparing each of the signals with a continuous source of the same frequency in the receiver when it appears. Identifying the signals and storing the information about each received phase now becomes necessary; but this is not difficult to arrange, as some memory is needed in any kind of navigational system.

Aside from the choice of radio frequency, this time-sharing of continuous carrier bursts is the chief distinction between Omega and earlier systems.

Upon the discovery of the good phase stability of transmission at very low radio frequencies, the advantages of the Omega technique appeared quickly. This phase stability is in part a result of the dominant single mode of transmission that is characteristic of VLF. Fortunately, VLF signal propagation is subject to low attenuation; therefore, it is possible to have transmissions that literally cover the world and yet are measurable to one or two microseconds of time.

Interference between two modes of transmissions can cause phase shifts of considerable magnitude. In fact, the resultant phase can shift up to 90 degrees from that of the stronger mode, and alternation between one mode and another can cause even greater difficulties. The domain of single-mode dominance is therefore of great importance. Aside from the ground wave domain near the transmitter, this single-mode dominance may be impossible to achieve at any frequency at distances up to a few hundred or thousand miles. Beyond a few hundred miles, however, it is possible over a somewhat limited range of frequencies. The relative behavior of various low frequencies can be very briefly summarized as follows:

<u>40 Kilocycles</u>

Near this frequency the first mode is very weakly excited so that there may be a second mode dominance up to several thousand miles from the transmitter, and temporal changes can cause serious mode interference at several distances.

20 Kilocycles

Here transmission is primarily by the first-order mode, but the second-mode transmission is so strongly excited at night that the modal interference extends into the edges of the sunlit regions. This frequently causes unpredictable phase progression from nighttime to daytime.

10 Kilocycles

In this region the first mode is well excited, and mode interference effects at sunrise and sunset are much less severe. The wavelength is, however, becoming comparable with the height of the ionosphere, therefore increasing transmission losses somewhat and accentuating an east-west assymetry in the losses, which results in limited useful range of signals traveling toward the west in the region of the geomagnetic equator.

5 Kilocycles

At such a very low frequency, the daytime signal is heavily attenuated, and directional effects are presumably extreme.

Another important fact in the choice of frequency is that the power radiated from a given transmitting antenna varies as the fourth power of frequency. It is possible to radiate a few kilowatts at 10 kilocycles but only at a cost equivalent to radiating 100 kilowatts at 20 kilocycles.

Noise need not be specifically considered at this point, since it is treated in Section 6.2 and is not likely to be a serious factor in the use of Omega. With the exception of a few places on earth, at the most noisy times the effect limiting the useful range of Omega will be interference by its own signals coming the long way around the world rather than interference by natural noise.

Because Omega makes a phase measurement and because one cycle of a continuous signal cannot be distinguished from another, there are lane ambiguities at intervals of one-half a wavelength or more. Various steps can be taken to resolve these ambiguities (as discussed in Section 2.3), but it is advantageous to make the separation between them as large as possible. This separation is eight miles (or more) at ten kilocycles and is inversely proportional to frequency.

This last factor and mode interference combine to indicate the desirability of a frequency near or even below ten kilocycles (but not nearly so low as five). The practical problem of power radiation would be more easily solved at a higher frequency; but the Omega power requirement is reasonably small, and this argument need not be given too much weight. Even at the higher end of the 10 to 14 kilocycle navigation band, the lane width is reduced to 7/10 of the width at 10 kilocycles. In view of this, it is worth the extra power required at 10.2 kilocycles to combat daytime attenuation in order to attain maximum lane width and maximum freedom from mode interference. The primary Omega frequency has, therefore, been taken near the lower end of the 10 to 14 kilocycle band.

2.2 Geometry

The spacial relations which determine the constant of proportionality between timing errors and distance errors are well known; but it is advisable to review them so we may study the special advantages that derive from the use of long base lines.

2.2.1 The Line of Position

The velocity of light is about 162,000 nautical miles per second, or nearly 1,000 feet per microsecond. Moving toward or away from a transmitter will change the time of arrival in this ratio. On the base line between the two stations of a hyperbolic pair, motion toward one station is motion away from the other; and the measured time difference will change by a microsecond for each 500 feet of motion. At a point away from the base line, motion toward one station is not directly away from the other; and more than 500 feet are required to change the reading by one microsecond.

The separation between hyperbolic lines of position is proportional to the cosecant of one-half the angle between the two stations as seen from the navigator's position. If, for example, the stations and the navigator are at the three vertices of a small equilateral triangle, the angle subtended by the stations at the navigator's position is 60 degrees, and the relative separation of the hyperbolic lines is cosecant 30 degrees, or two; that is, the lines are separated by 1,000 feet per microsecond at this point, and measurements are only half as accurate as on the base line. At a distance from the stations equal to three times the length of the short base line, the accuracy of readings is reduced by a factor of a little more than six.

Because timing errors at VLF are in the order of a few microseconds, it is of the greatest importance to avoid this geometrical dilution of accuracy as much as possible. Fortunately, the earth is nearly spherical, and the use of very long base lines can control this effect to a surprising degree. The quantity known as "spherical excess" comes to our aid. On a spherical surface, the sun of the three angles of a triangle is not exactly 180 degrees, as on a plane, but may be considerably larger. When the length of a hyperbolic base line approaches or exceeds the radius of the earth, this effect is tremendously beneficial.

Consider, for simplicity, a base line extending along the equator east and west of the prime meridian. As one moves north or south at zero longitude, the hyperbolic lines diverge but only until the poles are reached. The reason for this limit is the relation cited above -- that the divergence increases as the angle subtended decreases. At the pole, our equatorial base line subtends a minimum angle; at the equator at 180 degrees longitude, the two stations subtend 180 degrees, and the geometrical precision is as good as along the base line itself. Of course, the amount of divergence at the pole depends upon the length of the base line. If it were 60 degrees (or 3,600 nautical miles long) the maximum divergence is two to one. If the base line were 90 degrees (or 5,400 nautical miles in length), the errors at the pole will be only 1.4 times those on the base line. If the two stations of a hyperbolic pair are at opposite ends of a diameter of the earth, all points on earth are "on the base line"; and a geometrical accuracy of 500 feet per microsecond is obtained everywhere.

These remarks concerning geometry are academic unless radio transmission over distances approaching a quadrant of the earth's circumference (5,400 nautical miles) is available. Fortunately, in Omega transmission over an average of 7,000 miles (less toward the west but more toward the east) can be relied upon. Nearly half the area of the earth lies in an equatorial belt within 7,000 miles of both poles; that is, the case of no divergence can be expected to apply nearly half the time, if stations are installed in pairs approximately opposite to each other.

2.2.2 The Fix

When a navigator makes a fix by reading on two pairs of stations, the situation is more complex. His two lines of position may have different divergencies (or miles per microsecond), and the timing errors may be different in the two pairs. Furthermore, the lines cross at some angle that may or may not be advantageous; and there could be significant correlation between the readings on the two pairs.

In general, the figure of minimum area that will contain half (or 90 percent, or some other fraction) of the positions determined at a given fixed point is an ellipse. The shape, size, and orientation of such an ellipse can be calculated; but, for most purposes, the concept and the labor are both too difficult to allow the error ellipse as a convenient measure of error. If very long and narrow ellipses can be avoided, a satisfactory measure of accuracy is the root-mean-square distance error. This is simply the rms value of the distance between a true fix and the apparent fix obtained at the same point without regard for direction of the error. As in all rms values, the true position is expected to be within the cited distance of the apparent position in about two-thirds of the cases.

The formula for computing the rms distance error may be given as:

$$d_{rms} = \frac{1}{\sin\theta} \sqrt{d_1^2 + d_2^2 + 2\, r d_1 d_2 \cos\theta} \qquad (2.2\text{-}1)$$

where:

d_1 and d_2 = standard deviations of the line of position errors in linear units such as miles

θ = crossing angle between the two lines of position

r = correlation coefficient between the readings on the two lines.

d_1 and d_2 include the uncertainty of time measurement and the divergence discussed above. The correlation coefficient is small (assuming small measuring errors in the receiver) for readings on separated pairs, but it is about one-half if readings are made on three stations -- as A against B and A against C -- because the propagational fluctuations on the path between A and the navigator appear in both readings.

One note is required about the crossing angle, which is ordinarily thought of as less than 90 degrees. Consider three stations, A, B, and C, at the vertices of an equilateral triangle with the navigator at the center. Each possible pair subtends 120 degrees, and the half-angle discussed in the prior section is 60 degrees. This half-angle, among other things, defines the direction of the line of position. The crossing angle is the sum of two half-angles, or 120 degrees. It is clear that this interpretation should be used to preserve symmetry, as the crossing angle must be the same whether A be measured against B and C or another common station be used. Because the cosine is negative for angles between 90 degrees and 180 degrees, the cross-product term in Equation 2.2-1 is subtractive in the case of a "surrounding" triplet of stations.

From study of Equation 2.2-1, we find that a circular error pattern is obtained if either (a) a navigator is at the center of a quadrilateral, that is, at a point such that four stations subtend 90 degree intervals, or (b) at the center of a triangle with three stations mutually 120 degrees apart. In the second case, the radius of a circle of constant error probability is a little larger as the position lines diverge by 1/sine 60 degrees or 1.16.

Let us suppose that six stations can be located at the corners of an octahedron (that is, at six points on earth that are mutually 90 degrees apart) and that satisfactory transmission is available in all directions for distances in excess of 90 degrees of arc (5,400 nautical miles). Such a situation is shown in Figure 2.2-1. At any point along the base line, AB, the best geometry is attained by measuring A against B, and C against D. This gives two position lines that have no divergence and which cross at 90 degrees. At a point near a station, such as B, the lines involving that station have large curvature. It is, therefore, geometrically better to neglect B and cross the AF line with CD. Again, perfect geometry results. At a point in the center of one of the great triangles, such as P, the rms fix error (as mentioned above) is 1.16 times greater; but the contour of constant probable error is circular, as along the base lines. It is easy to see that the positional errors in such a perfect octahedron average about 1.05 times those at the intersection of two orthogonal base lines.

Unfortunately, the octahedral pattern is impractical unless reinforced by the addition of extra stations, because transmission to the requisite distance is not satisfactory when the navigator is west of a station in a low latitude and because one or more transmitting stations might be inoperative. It is worthwhile, however, to consider the latter effect.

Suppose that, in Figure 2.2-1, a navigator is too close to station B to make good use of it and that station F is inoperative. Under these circumstances, position must be determined from the triplet DAC. The two position lines bisect the angles at B and accordingly are orthogonal. Also, the angles subtended by each of the pairs are 90 degrees. The relative divergence is, therefore, 1.41; and the error pattern is circular. Thus, even in this abnormal circumstance, the fix error exceeds the optimum by only the square root of two.

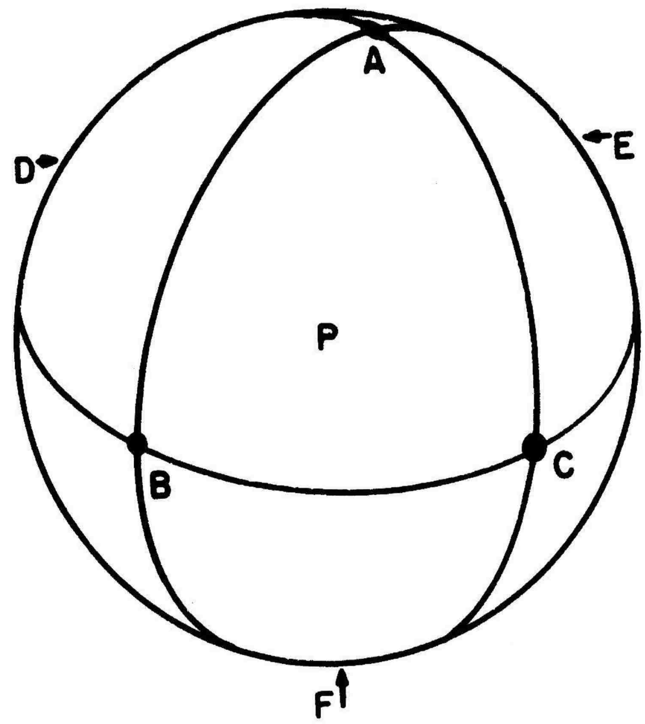

FIGURE 2.2-1. OCTAHEDRAL DISTRIBUTION OF SIX TRANSMITTING STATIONS

Figure 2.2-2 shows some of the relative fix errors (normalized so that unity corresponds to the center of a quadrilateral) for one of the triangles of an octahedron. An assumption is made that no information from other stations outside the triangle is available. These errors are plotted against the distance from the navigator to the outermost of the three stations. The solid line beginning at 5,400 miles shows the errors along the dotted arrow in the diagram on the graph when the arrow bisects the angle at the top. Near the "corner" station the relative error is about 1.23. It reduces slightly until the center of the triangle is reached and increases more rapidly as the navigator moves away from the stations. At an angle θ to the centerline of the diagram, the effect is slightly different from that on the centerline; but the figure shows that the relative errors at all points inside the triangle are between 1.15 and 1.41.

In Figure 2.2-3, we have shown the applicable situation if a navigator had to operate from three stations 90 degrees apart along a great circle, such as D, A, and C in Figure 2.2-1, with no other available stations. In this case, on the centerline (θ = 0), the relative error is always 1.41. As θ increases, the errors become larger but remain less than twice the "perfect" value unless θ exceeds 45 degrees.

The effect of shortening the base lines for a triplet having base lines of 3,284 miles and an angle of 120 degrees is shown in Figure 2.2-4. This is a situation that would result if a station were put in the center of one of the triangles of an octahedron (as at P in Figure 2.2-1). The degradation in accuracy outside the immediate area of the stations is extreme; but within 3,500 miles of all stations, the relative errors remain less than two. A comparison of this figure with Figure 2.2-2 shows how rapidly the fix improves as the base lines are made longer.

These last three diagrams represent abnormal conditions where stations that might greatly improve the fix accuracy are assumed not to be available. These and many other studies can be summarized as follows:

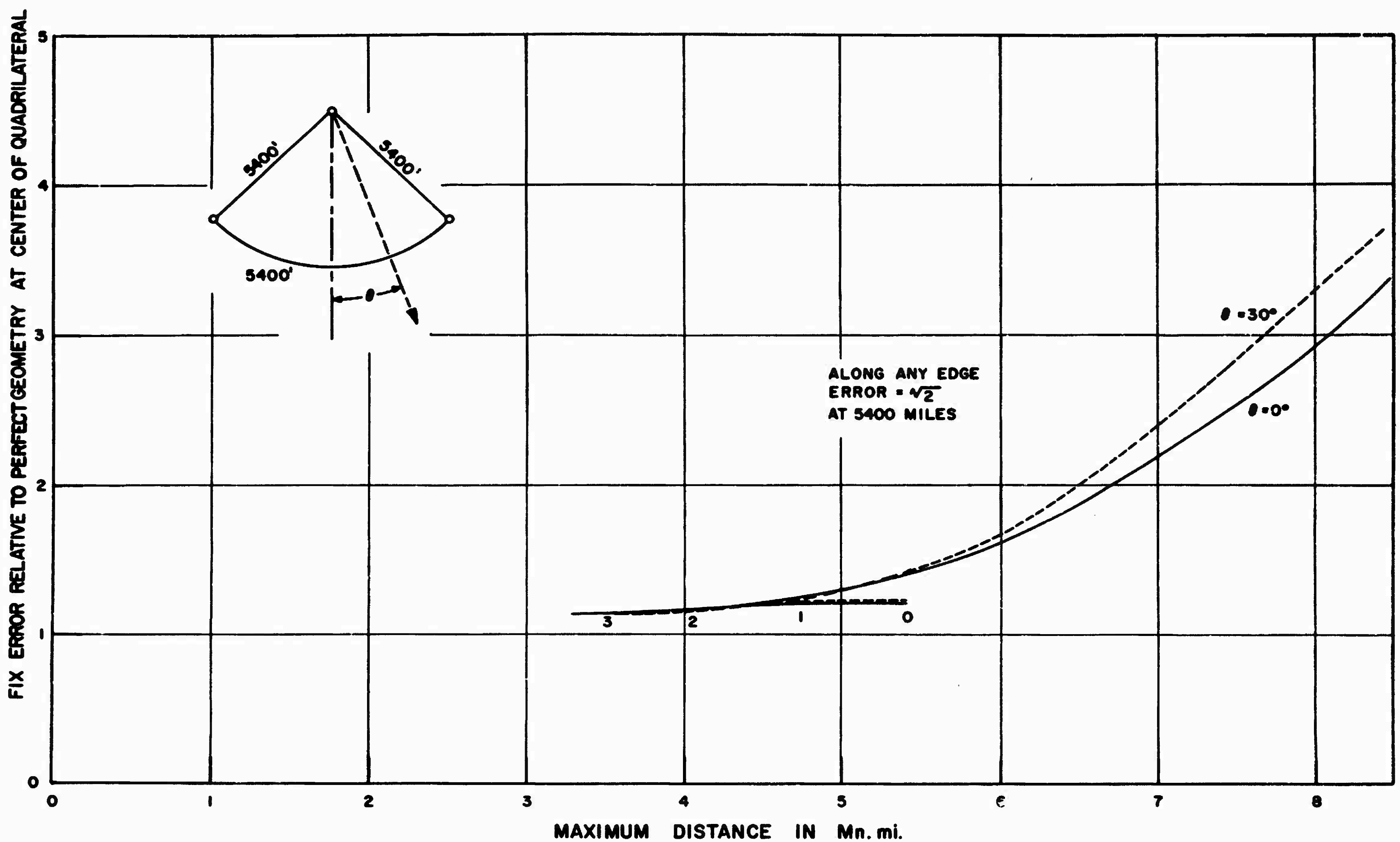

FIGURE 2.2-2. RELATIVE FIX ERROR FOR AN OCTANT OF AN OCTAHEDRON

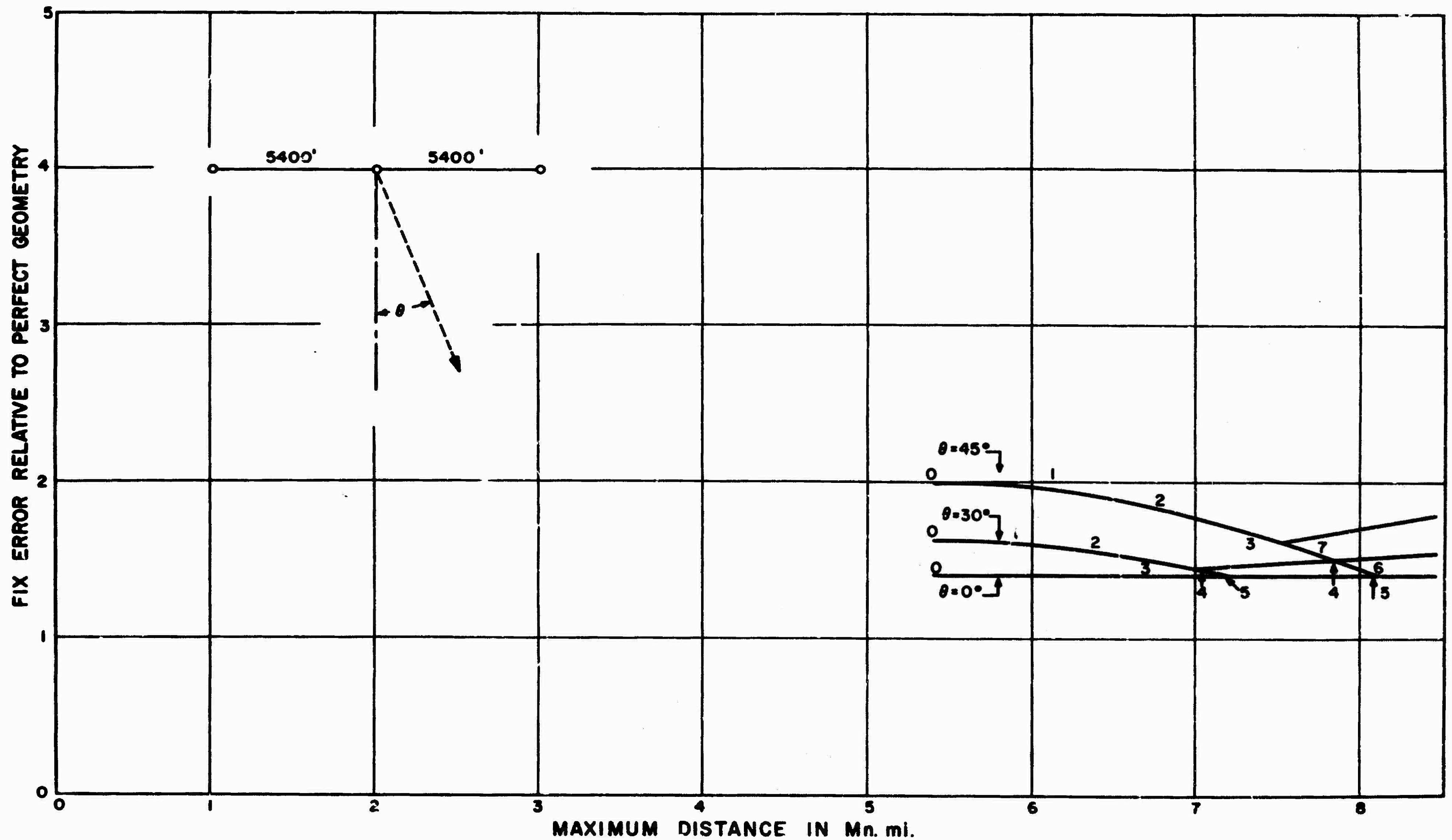

FIGURE 2.2-3. RELATIVE FIX ERROR FOR THREE STATIONS LOCATED 90 DEGREES APART ON A GREAT CIRCLE

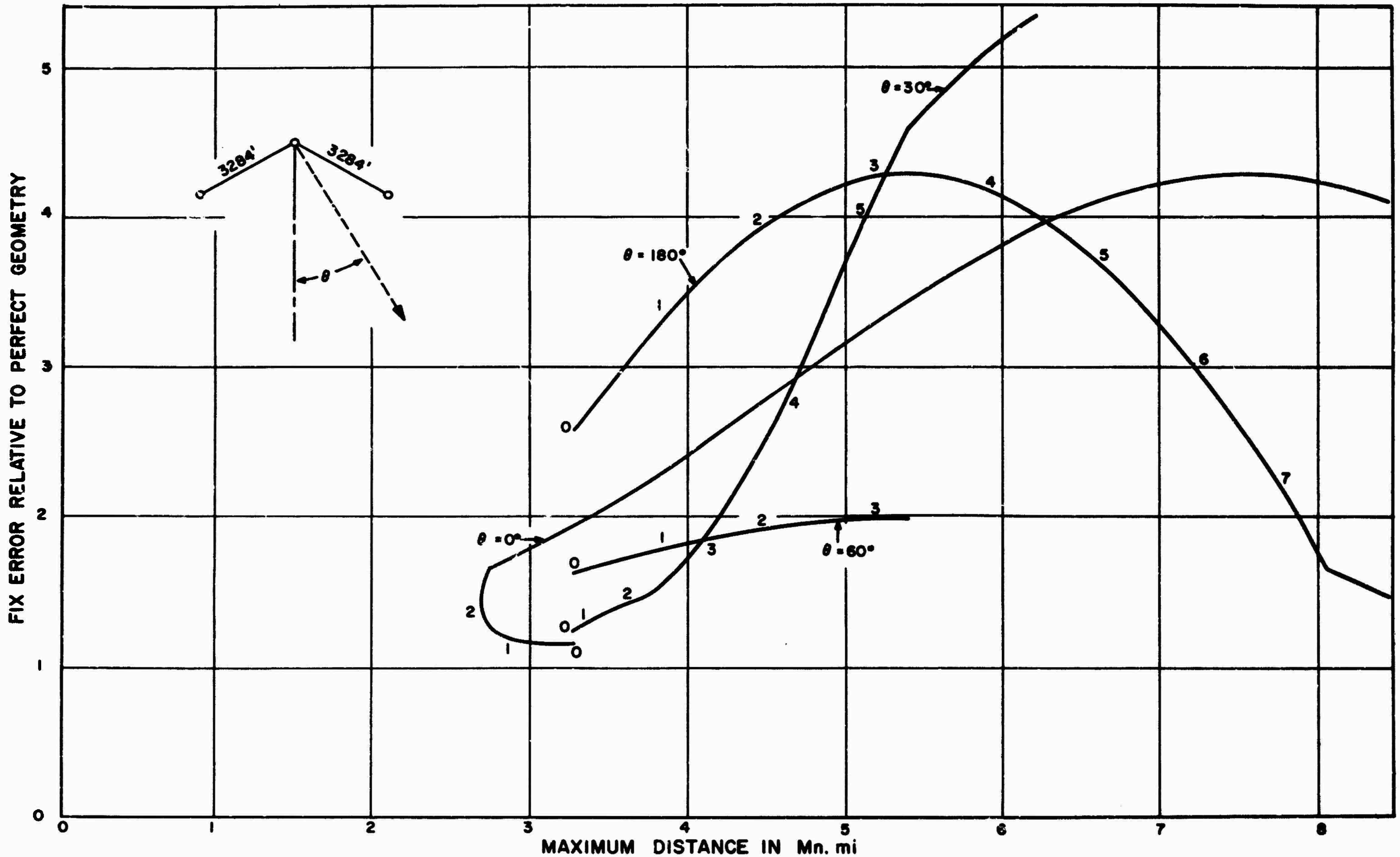

FIGURE 2.2-4. RELATIVE FIX ERROR FOR THREE STATIONS SUBTENDING AN ANGLE OF 120 DEGREES

If eight stations are provided in suitable locations, the average errors over the globe will be not more than one-eighth of a nautical mile per microsecond of timing error. If any one station fails, the average errors will increase by approximately 15 percent; and, even if two stations fail, the average errors will increase less than 50 percent. These estimates testify to the very considerable navigational redundancy available when the fix is moderately overdetermined.

Bearing in mind that a reasonably uniform distribution in azimuth of either three or four transmitters is the requirement for good Omega accuracy, we may examine a few of the situations which may result from a practical distribution of eight stations. In Figure 2.2-5 are shown the directions of eight named stations in a realizable network, as seen from a point near Hawaii. Of these eight, numbers 3 and 8 can be excluded as too remote to provide satisfactory signal strength, considering the nature of the intervening terrain. The remaining six stations can be combined in pairs in any desired way. The possible choices are shown in Figure 2.2-6, where each arrow shows the "positive" direction of a line of position determined from the pair of stations identified by the double number. Beside each arrow is a number giving the relative divergence (or relative error) of the lines. From this figure, a navigator (or the chartmaker) may choose the lines 1-5 and 2-6 to provide a fix. Since these lines have relative divergences of 1.06 and 1.00, and cross at about 102 degrees, the relative fix error would be:

$$\frac{1}{\sin 102^\circ}\sqrt{\frac{(1.06)^2+(1.00)^2}{2}} = \frac{1}{0.978}\sqrt{\frac{2.12}{2}} = 1.05^*$$

If station one were not available, pairs 4 and 5 and 6 and 7 may be chosen with approximately the same resulting accuracy. Should station five

*The 2 in the denominator under the radical accounts for the fact that even with perfect geometry the rms fix error is the square root of twice the square of the line of position error.

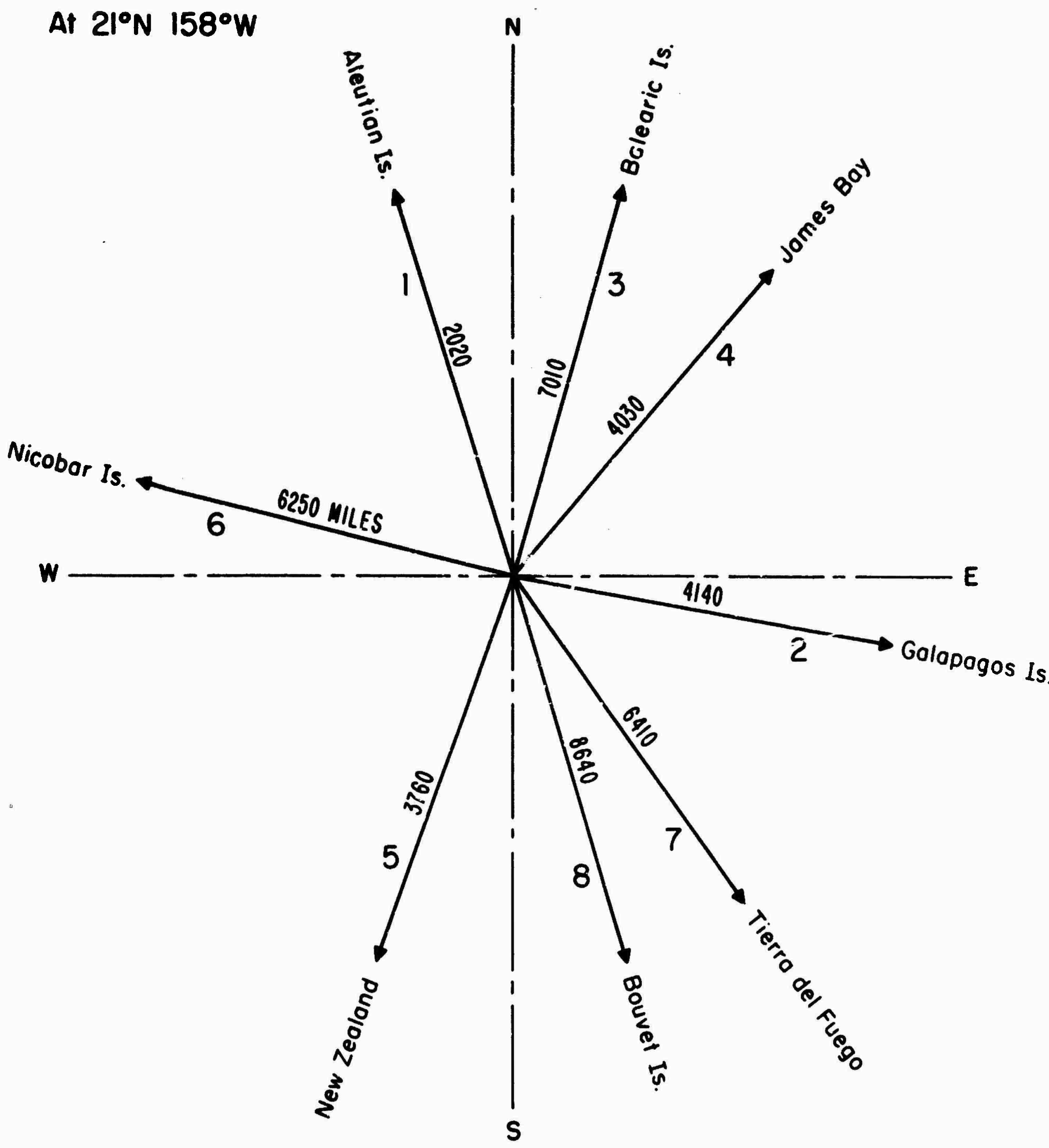

FIGURE 2.2-5. DIRECTIONS AND DISTANCES OF EIGHT STATIONS FROM A POINT NEAR HAWAII

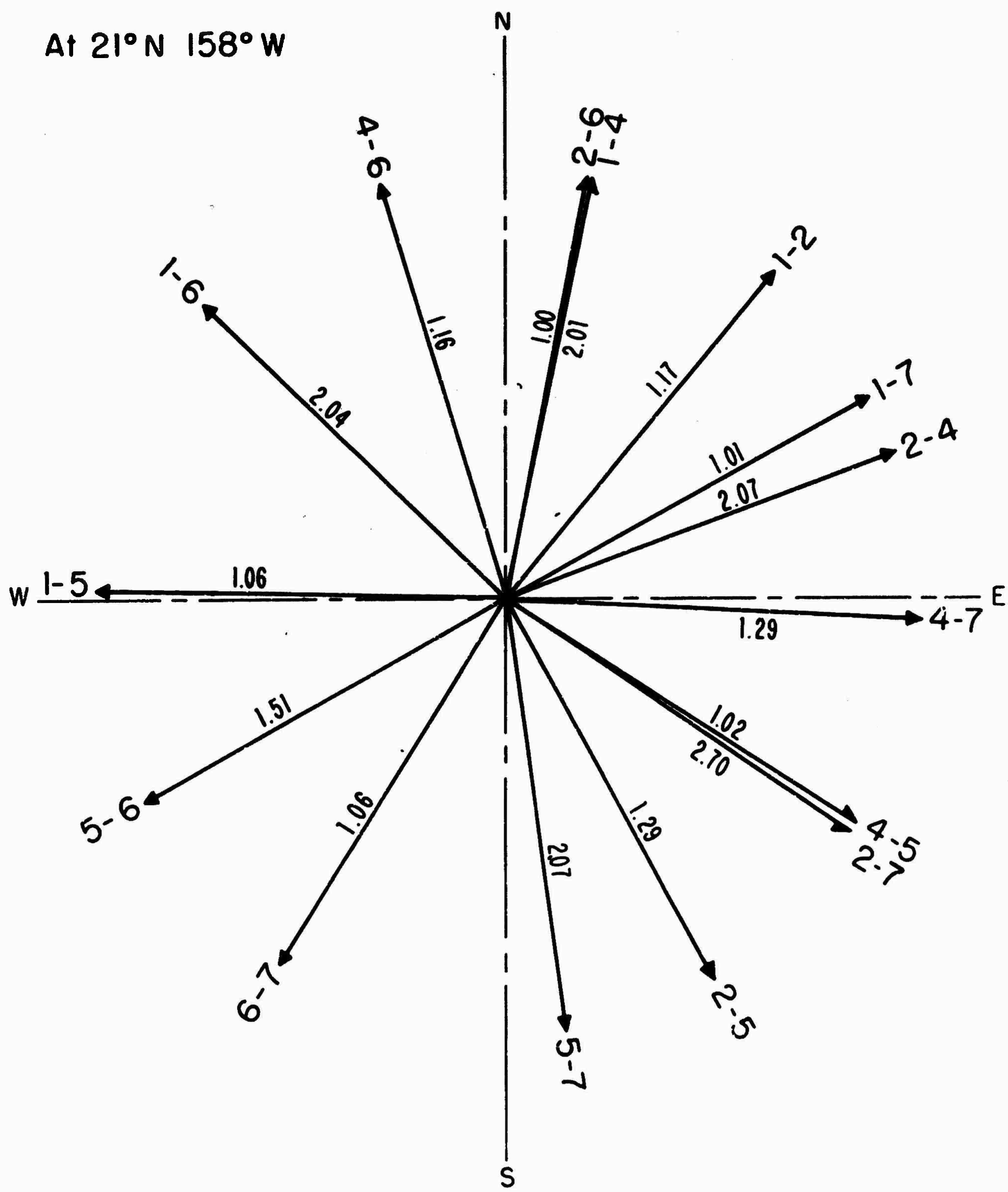

FIGURE 2.2-6. DIRECTIONS AND RELATIVE SEPARATION OF POSITION LINES FROM SIX STATIONS AT A POINT NEAR HAWAII

be temporarily missing, the choice might fall upon pairs 1 and 7 and 4 and 6, with a relative fix accuracy of 1.10. The fix accuracy decreases slowly only if one or two stations are missing, and some type of fix is available as long as three stations can be measured.

Figures 2.2-7, 2.2-8, 2.2-9 and 2.2-10 show the same configuration of transmitting stations as seen from two other points on earth. In each case, two stations are assumed not to be useful (as may be deduced from the numbers not used on the position-line diagrams) and fifteen families of lines of positions are available for choice. In these figures (particularly Figures 2.2-9 and 2.2-10), a common characteristic of the simple geometrical configurations can be seen, that is, the tendency for stations to lie in one of four more or less orthogonal directions so that the position lines lie near one of eight directions. In both cases, Figures 2.2-8 and 2.2-10 show that the first two or three selections of pairs which would be made by a navigator give nearly geometrically perfect fixes (such as those described for Figure 2.2-6).

Diagrams similar to those discussed here have been drawn for a number of places on earth and for several possible sets of transmitter locations. All look superficially similar and lead to the same conslusions--that geometry close to perfection can be expected from a network of eight transmitters. Usually two, and sometimes three, transmitters will be too remote; but the remaining five (or six) will provide ten (or fifteen) lines through the navigator's position. From among these ten (or fifteen) lines, there will always (provided the stations are distributed with sufficient uniformity of separation) be a few pairs of lines of high accuracy and good crossing angles.

The previous discussion has left the suggestion of "too far to be useful" in very nebulous form. It will be shown in Section 6.2 that there are excellent grounds for predicting useful range except, perhaps, over the regions of arctic permafrost or icecap. This range varies greatly with

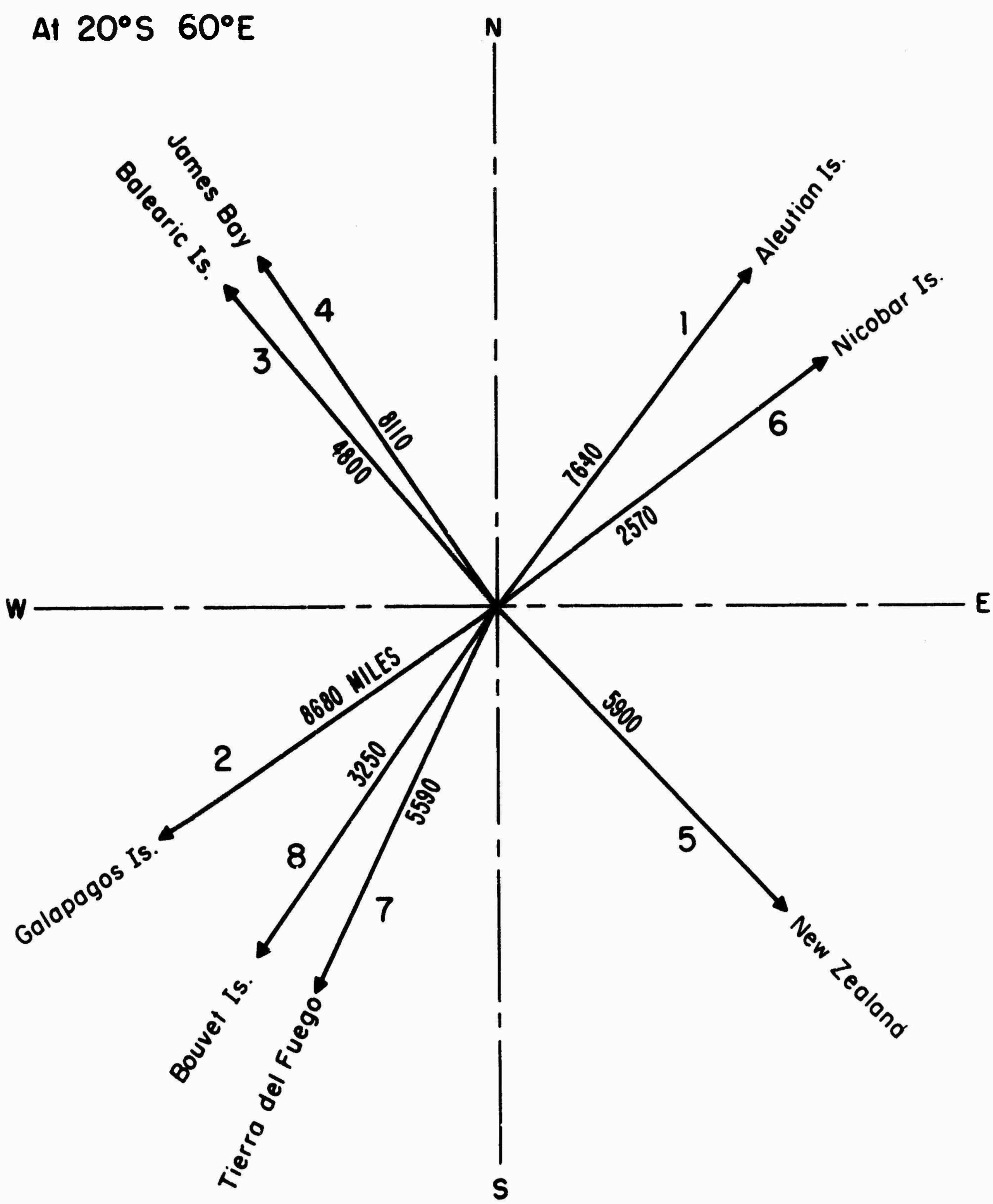

FIGURE 2.2-7. DIRECTIONS AND DISTANCES OF EIGHT STATIONS FROM A POINT NEAR MAURITIUS

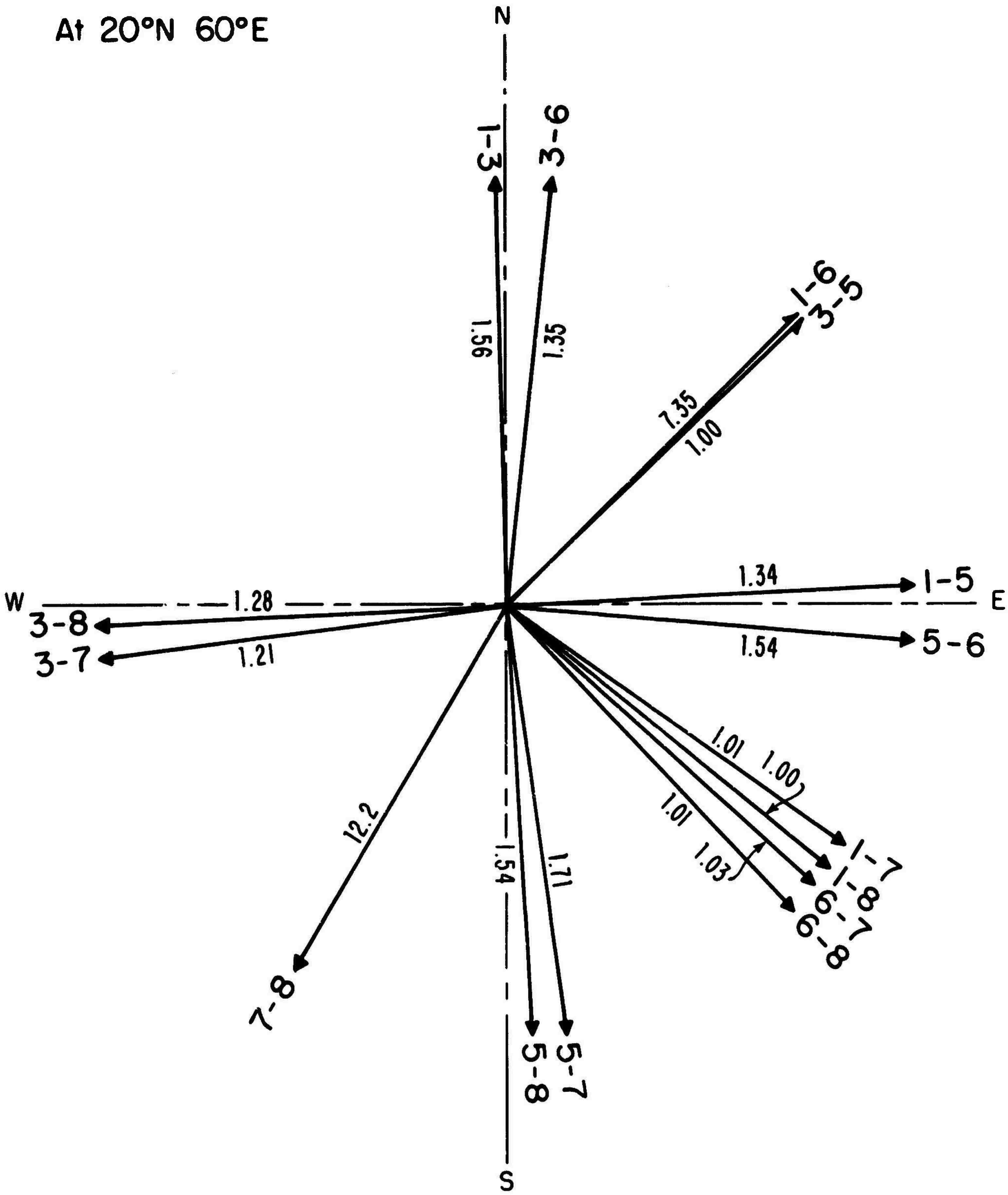

FIGURE 2.2-8. DIRECTIONS AND RELATIVE SEPARATIONS OF POSITION LINES FROM SIX STATIONS AT A POINT NEAR MAURITIUS

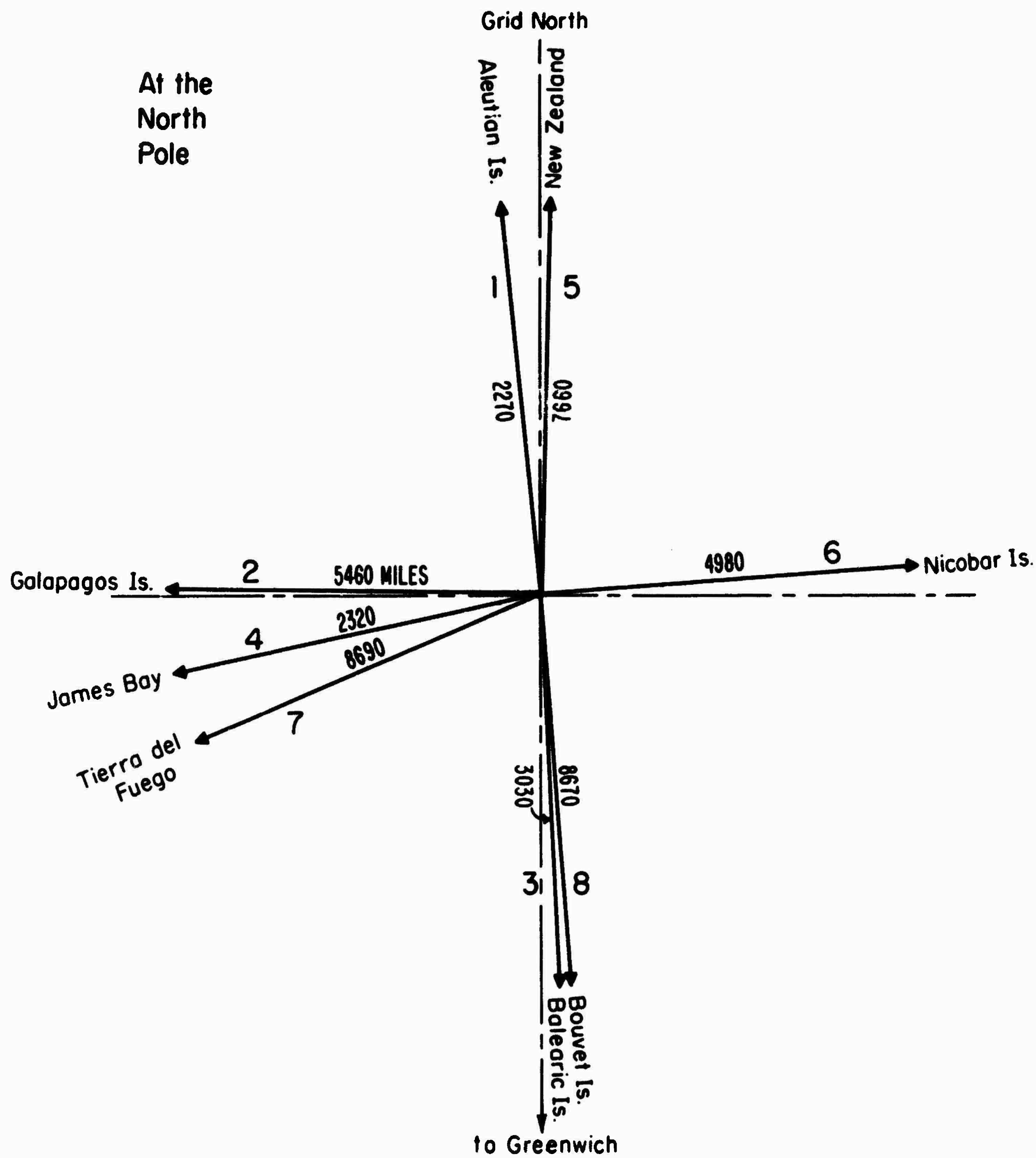

FIGURE 2.2-9. DIRECTIONS AND DISTANCES OF EIGHT STATIONS FROM THE NORTH POLE

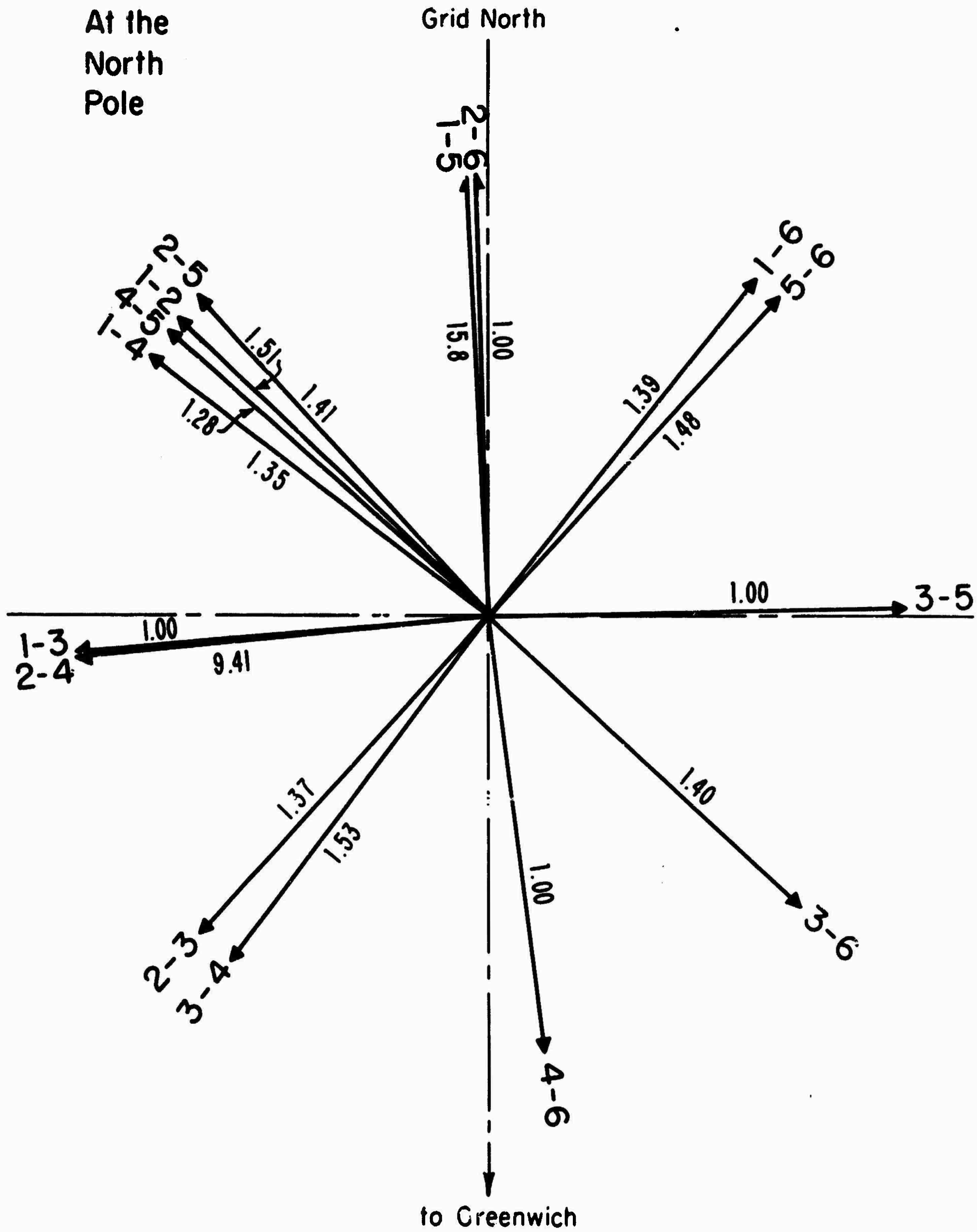

FIGURE 2.2-10. DIRECTIONS AND RELATIVE SEPARATIONS OF POSITION LINES AT THE NORTH POLE

the direction of transmission with respect to the earth's magnetic field.

Diagrams (like those of Figure 2.2-5) can be drawn utilizing directional arrows plotted with lengths corresponding to distance. Such a diagram (as Figure 2.2-11 for example) is a map in azimuthal equidistant projection centered on the navigator's position. We have not attempted, in the diagrams shown, to plot the continents; but we have shown in black the portions of the transmission paths that are over land. Along these segments the losses are a little higher than over sea water. We have also outlined the approximate major areas of excessive arctic attenuation--the Greenland icecap, the nearby permafrost regions of Canada, and the antarctic continent. The radius of these maps (in Figure 2.2-11 and two others that follow) is 10,800 nautical miles. The somewhat oval contours (shown by heavy dotted lines) are the limits within which a transmitter should lie in order to provide a useful signal at the navigator's position. In addition, transmission over much of one of the icecap areas could reduce the signal as much as transmission over another 2,000 to 4,000 miles.

Although two of the postulated receiving points are the same as for Figures 2.2-5 and 2.2-7, the set of transmitting stations assumed for Figures 2.2-11, 2.2-12 and 2.2-13 is different. Study of these latter three diagrams, however, leads to the same conclusions recited above--that usually five or six stations at suitably distributed azimuths can be received at any point on earth, and that fixes of satisfactory accuracy can exist, even in case of occasional transmitter failure.

Thus, the choices of frequency and power for Omega transmissions, to make possible the use of very long base-lines, result in a system of closely optimum geometrical accuracy, with adequate redundancy to protect the navigator against accidental failure of transmission.

2.3 Instrumentation Principles

Omega, as a very long-range, general purpose radio-locating system, is intended for a variety of users, including fixed or very slowly

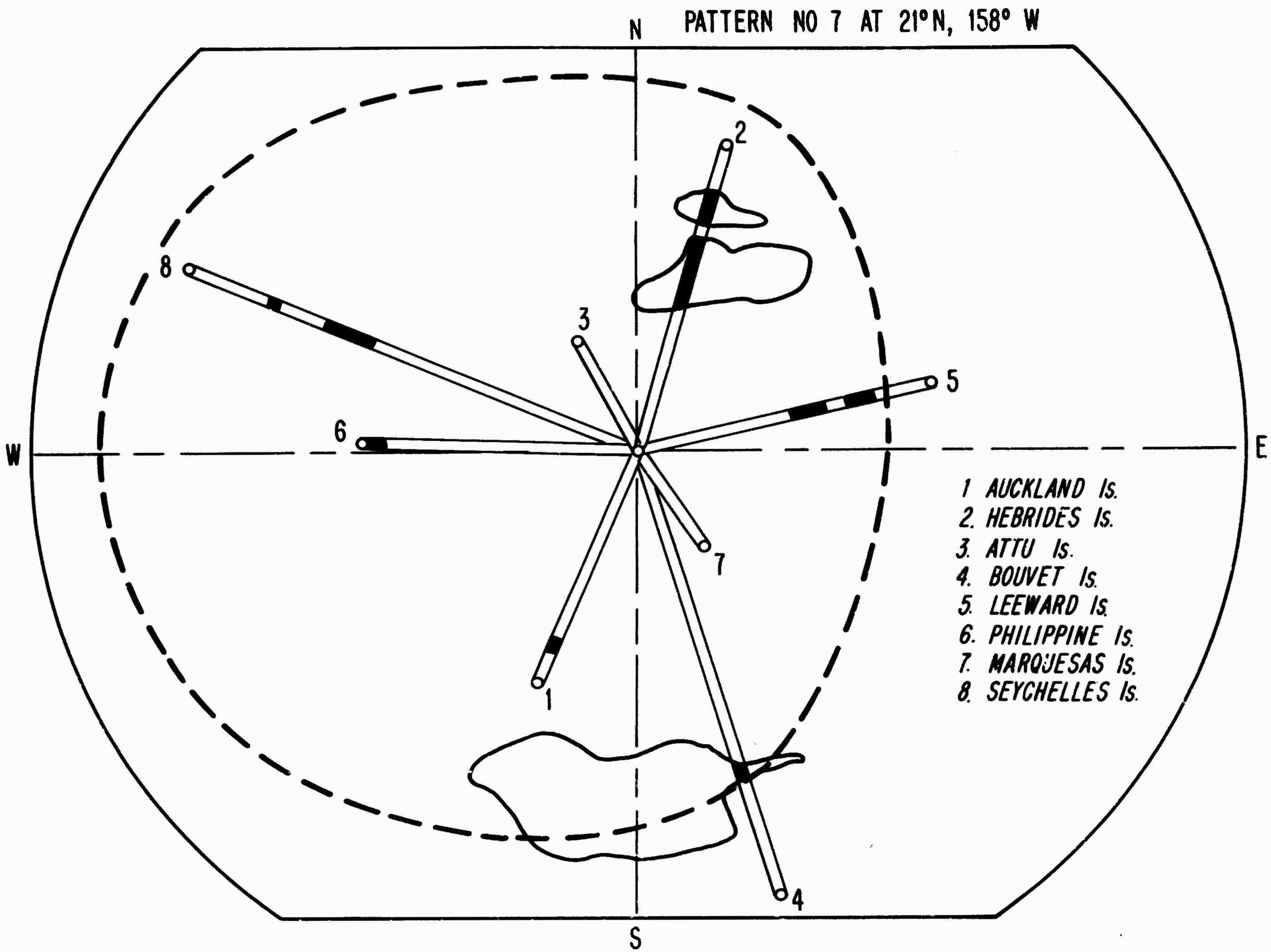

FIGURE 2.2-11. AZIMUTHAL EQUIDISTANT PROJECTION OF POSSIBLE OMEGA STATIONS FROM A POINT NEAR HAWAII

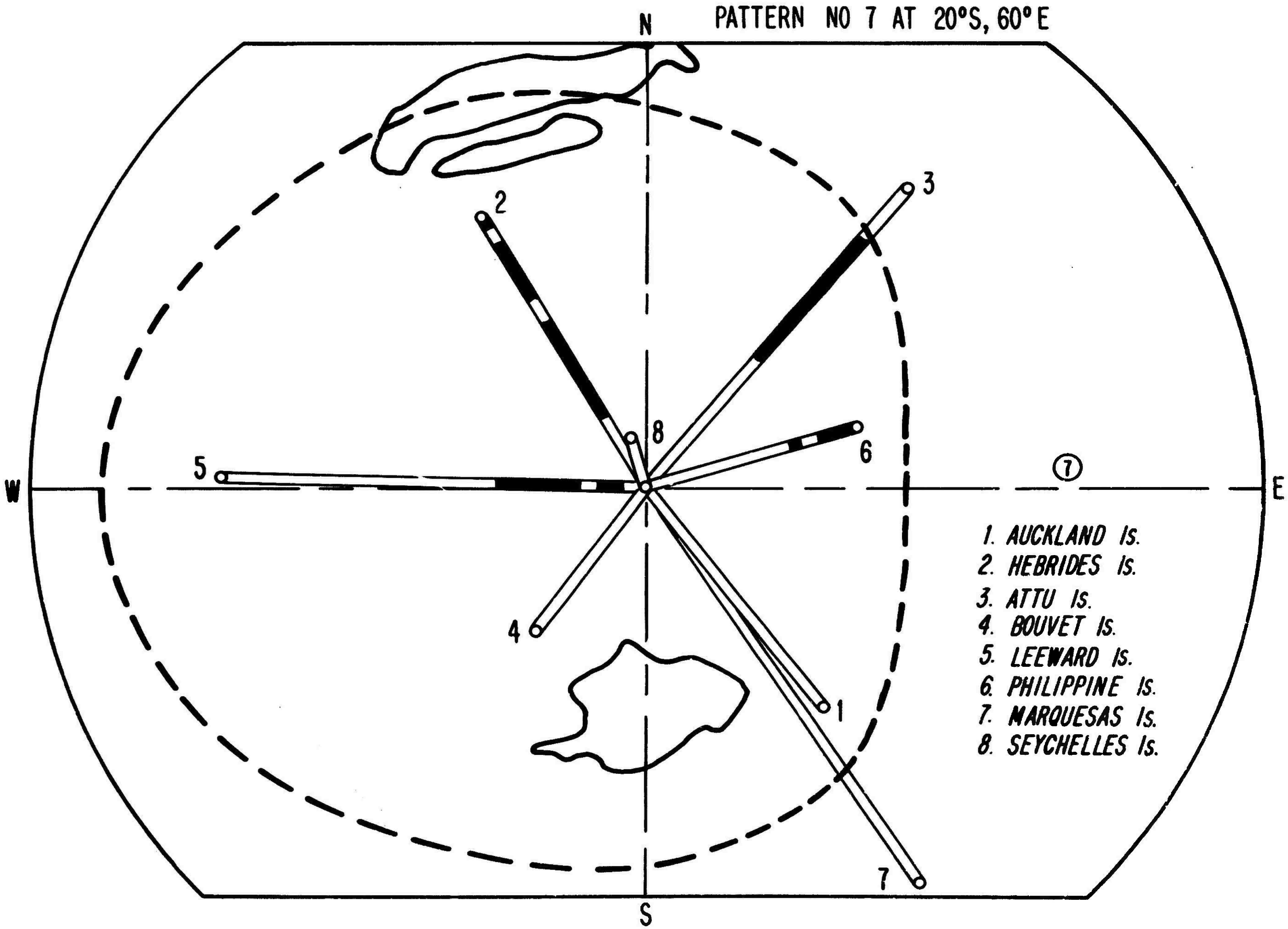

FIGURE 2.2-12. AZIMUTHAL EQUIDISTANT PROJECTION OF POSSIBLE OMEGA STATIONS FROM A POINT NEAR MAURITIUS

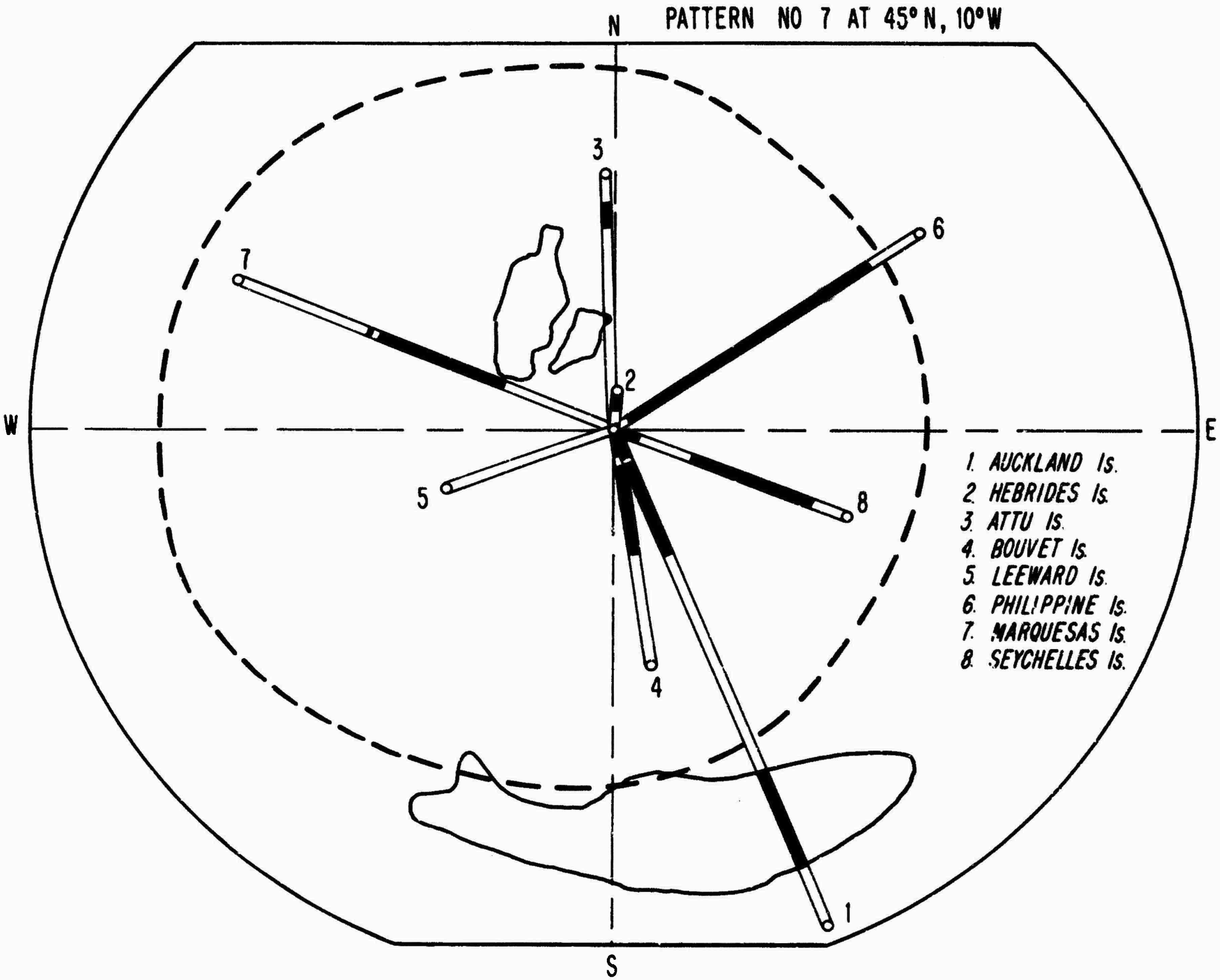

FIGURE 2.2-13. AZIMUTHAL EQUIDISTANT PROJECTION OF POSSIBLE OMEGA STATIONS FROM A POINT NEAR THE BAY OF BISCAY

moving land vehicles, submarines, ships at sea (both large and small), civil and military aircraft of all types (including very high-speed intercontinental jets). The system should be designed to fulfill as nearly as possible the diverse requirements of a variety of users. The position determinations should provide an accuracy, without significant ambiguity, commensurate with the requirements of a particular user. The operation of receiving equipment should be within the user's capability, and the user equipment cost should be commensurate with the value of the missions being performed. The simplicity and usefulness of user equipments are greatly influenced by the characteristics of the signals; and the precise signal format has been developed to provide, as much as possible, a convenient and economical instrumentation for every class of user.

In developing the signal format, more than one type of operation is envisioned. For example, in a freighter traveling at ten knots, uncertainty of position develops slowly so that loss of lane identification would not ordinarily occur. Also, a reasonable time would be available for its reacquisition. On the other hand, low cost, long-time reliability, and minimum servicing requirement are of considerable importance. These considerations lead to a receiver concept in which only the necessary phase tracking circuits at the basic frequency would be provided, with station identification, multiplex alignment, and gain balance adjustments, etc., performed manually by an operator. The only requirement for automation would be continuous graphic recording to warn of the occurrence of sudden ionospheric disturbances and/or other propagation anomalies affecting the accuracy at the time a fix is taken. Reduction of the hyperbolic Omega data and insertion of diurnal shift compensation would be accomplished manually with the aid of charts and tables. The signal format should permit this type of operation, with simple, inexpensive equipment.

In contrast, a high-speed jet aircraft traveling an intercontinental route, would possess little or no facility for manual operation of equipment controls, nor would the travel time permit a lengthy signal acquisition routine. However, the intrinsic value of such operations justifies the expense of more sophisticated computer-like equipment, capable of acquiring signals, eliminating lane ambiguity, compensating for diurnal shift, and reducing position to off-course and distance-to-go, or other convenient coordinates without intervention by an operator while enroute. The signal format should provide the essential information required for this type of operation in a form reducible to the desired coordinates, without an excessively unwieldy computational program.

A third variation would be, for example, a patrol submarine operating for long periods below radio reception depths. Such an operation requires a different sort of sophistication aimed at maintaining previously acquired information pertaining to position, with updating if and when additional data become available. This leads to a concept of a dead-reckoning computer that is updated from time to time by the Omega data and, in turn, supplies diurnal compensation data and lane identification.

Naturally there are also other types of operation for which different receiver formats would be more suitable. The system should provide for them to the greatest extent possible.

To determine the relative phase of a particular pair of Omega signal components, the desired pair of signals must be identified and separated from each other and the six other signal components of the same radio frequency, all being transmitted via the same transmission medium. Some means must be provided in the signal format whereby the signals of the various stations can be segregated and identified in a navigator's receiver.

2.3.1 Method of Signal Separation

The most straightforward method of separating radio signals of the same frequency is by time multiplexing, in which each station transmits in turn so there is never more than one signal being transmitted at a time

(Figure 2.3-1). The signals can then be separated by synchronously-operating, time-sharing relays (or other commutating action) in the receiver. The phase difference measurement is accomplished by phase-locking a locally generated continuous signal with the transmissions of one station, providing a continuous reference wave, with respect to which the phase of other signals can then be determined.

With eight stations, the use of time multiplex for signal separation imposes a maximum duty ratio of about one to ten on the transmission of each station. Thus, the average available power of each station is approximately 10 decibels less than its continuous cw capability. Alternatively, signal separation may be obtained by FM or other coding methods. These techniques, however, lead to ultrasophistication in the navigator's equipment; and time multiplex appears to be the most practicable method of signal separation.

2.3.2 Multiplex Sequence Parameters

Each station would transmit for a period, T. There are eight stations and, with no gaps between transmissions, the minimum period of a sequence of all eight transmissions would be eight T. The successive transmissions in a hyperbolic system (in which the signals of widely spaced transmitters are separated by time multiplexing) shift in time relative to one another, depending upon the point of observation, up to twice the time of propagation of a signal over the baseline between corresponding stations. If the signal transmissions are not to overlap anywhere in the field, there must be a gap of at least this amount between transmissions.

Figure 2.3-2 shows the duty ratio variation of a signal station versus the ratio of transmission period, T, to gap length, t. As the transmission period is made longer, relative to the gap between transmissions, the duty ratio approaches the limiting value of one to eight, 83 percent of the maximum where the transmission period is five times the gap length and down to one-half the maximum if the transmission period is equal to the gap length.

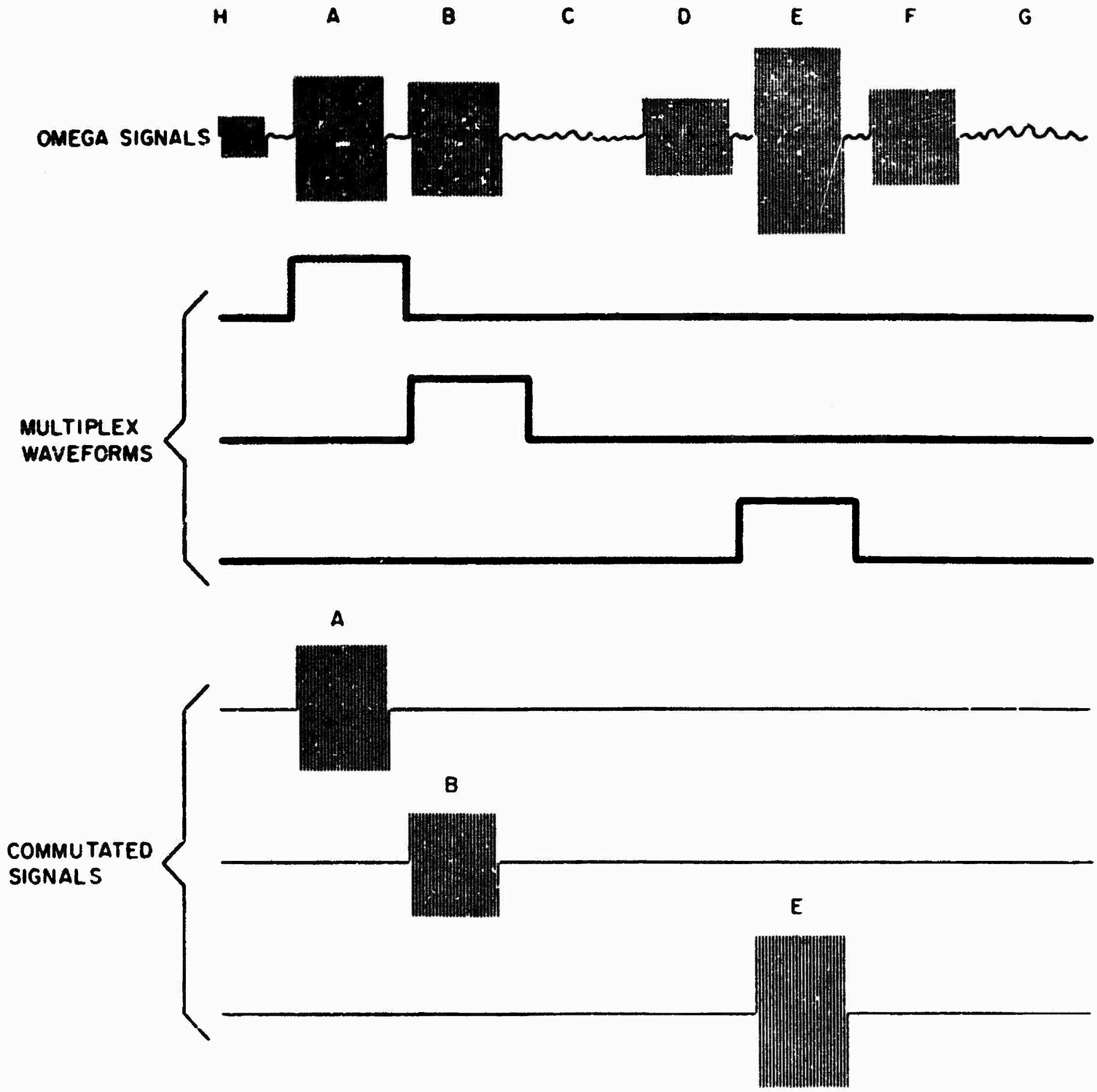

FIGURE 2.3-1. TIME MULTIPLEXING

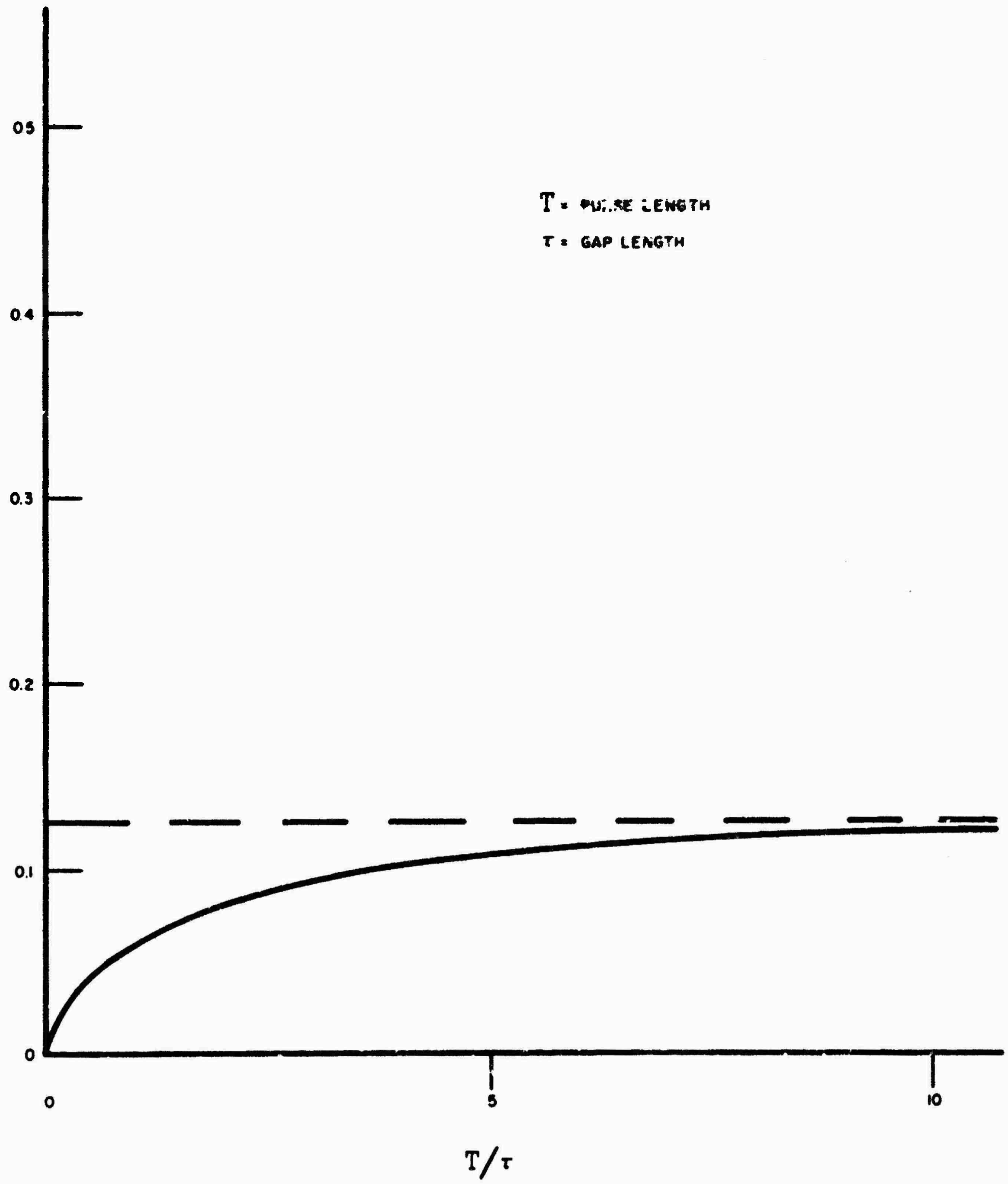

FIGURE 2.3-2. MULTIPLEX DUTY RATIO

In some locations the long path transmission of a remote station may be stronger than the short path transmission, for which the transmission delay would be 0.12 second. Considering allowances for switching time and for the rise and decay of the wave at the beginning and end of each transmission, the mean gap length cannot be less than approximately 0.2 second. Then the average length of transmission of T = 1.05 seconds produces a duty ratio per station of about 85 percent of maximum, and an over-all sequence length of ten seconds, which is commensurate with the scale of Standard Time.

2.3.3 Sequence Phase

In addition to separating the signals, the signal format must also provide some means whereby the signals of a particular station, or pair of stations, can be identified. In time multiplex, this amounts to determining when a particular transmission will be made. The stations always transmit in the same order; thus, if the phase of the transmission sequence can be established, identification of a particular signal component can be obtained by its time of occurrence within the sequence.

The signals form a repeating sequence of eight transmission segments, and the relative amplitude and phase of successive transmissions vary with the location of the observer. The signals of nearby stations are strong, and those of the most remote station or stations are weak or missing entirely (as in Figure 2.3-3). With some judgement and experience, it is feasible to deduce the sequence phase by comparison of the strong and weak signal patterns as observed in a oscilloscope display with the pattern to be expected at the assumed position. However, the logic involved would be complex and not particularly adaptable to automatic recognition circuitry capable of functioning at any and all locations. Hence, if there is to be automatic recognition of the sequence phase, the signals must be coded in some systematic fashion providing sequence phase recognition without ambiguity that can be implemented by an automatic sequence phase recognition

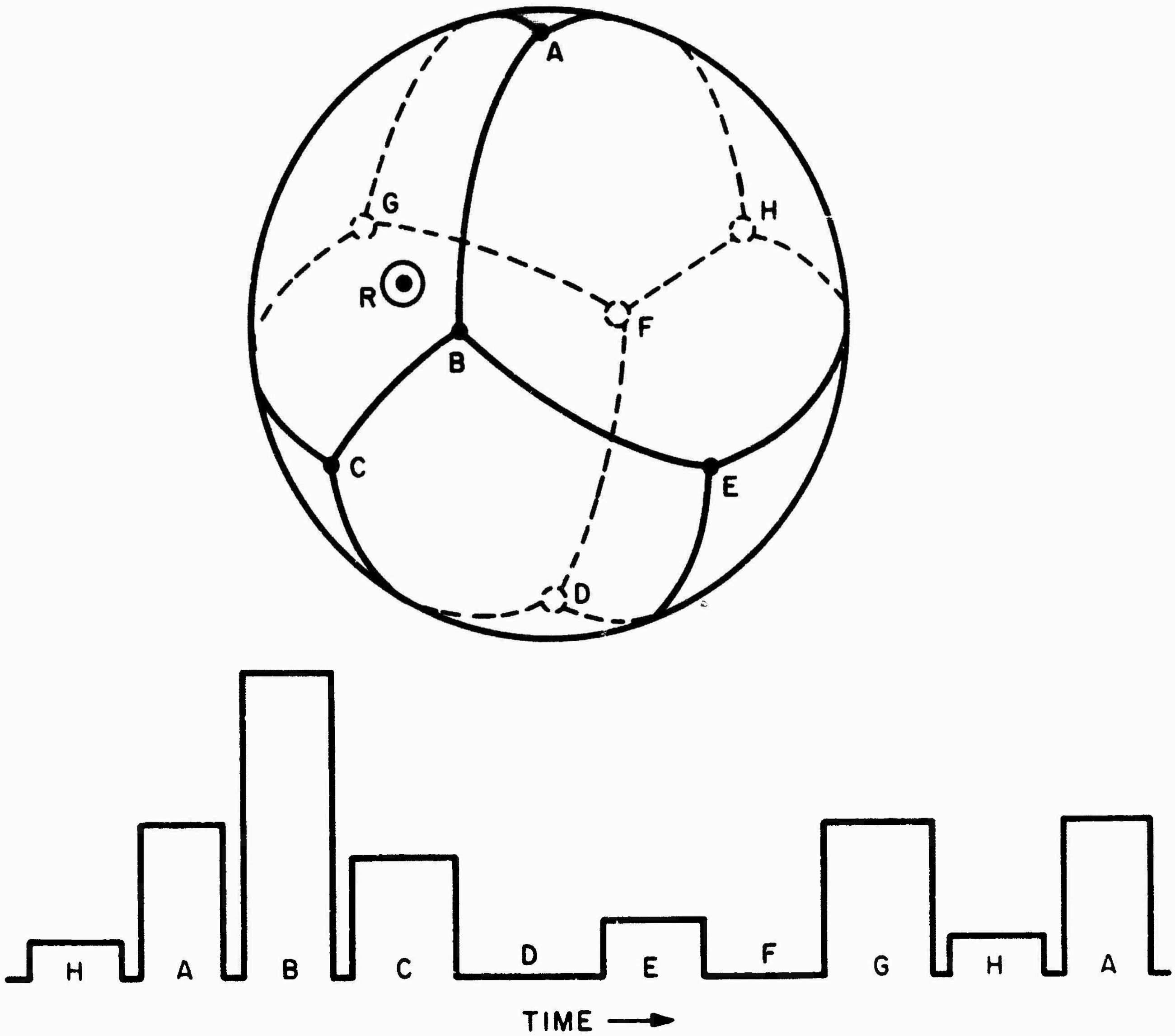

FIGURE 2.3-3. TYPICAL SIGNAL PATTERN

system of reasonable complexity.

The transmitting sequence may be correlated with Standard Time. For example, the transmissions of a particular station may be timed to commence on exact 10-second epochs of Standard Time (UT-2 or atomic time for example) with the other seven transmissions following in known sequence. Coincidence with a Standard Time tick to within one second, as obtained from a chronometer or by reference to radio time signals, would then identify the epoch marking the beginning of a sequence (as in Figure 2.3-4), where the identity of each signal segment would be established. This method, while adequate where Standard Time signals are available, strains the accuracy of chronometric time and requires an external source of information; i.e., a Standard Time receiver, for its accomplishment.

The signals of the various stations can be labeled independently of signal amplitude by coding the lengths of the transmission periods of the different stations. A convenient code has been found (as follows) with a nominal gap between transmissions of 0.2 seconds.

Station	A	B	C	D	E	F	G	H
Length of Transmission (Seconds)	0.9	1.0	1.1	1.2	1.1	0.9	1.2	1.0

This length code has the property that cross-correlation of an equiamplitude signal pattern ("unit correlating function"), with any two or more of the signals as received, has a positive maximum at alignment greater than all other maxima. Hence, cross-correlating the received signals with a "unit correlating function" generated within the receiver indicates correct alignment with any usable combination of incoming signals, with no requirement for additional information or no change in function with location or different grouping of stations.

Since the Omega signals will be separated by time multiplexing and there are eight transmitting stations in the system--with each station

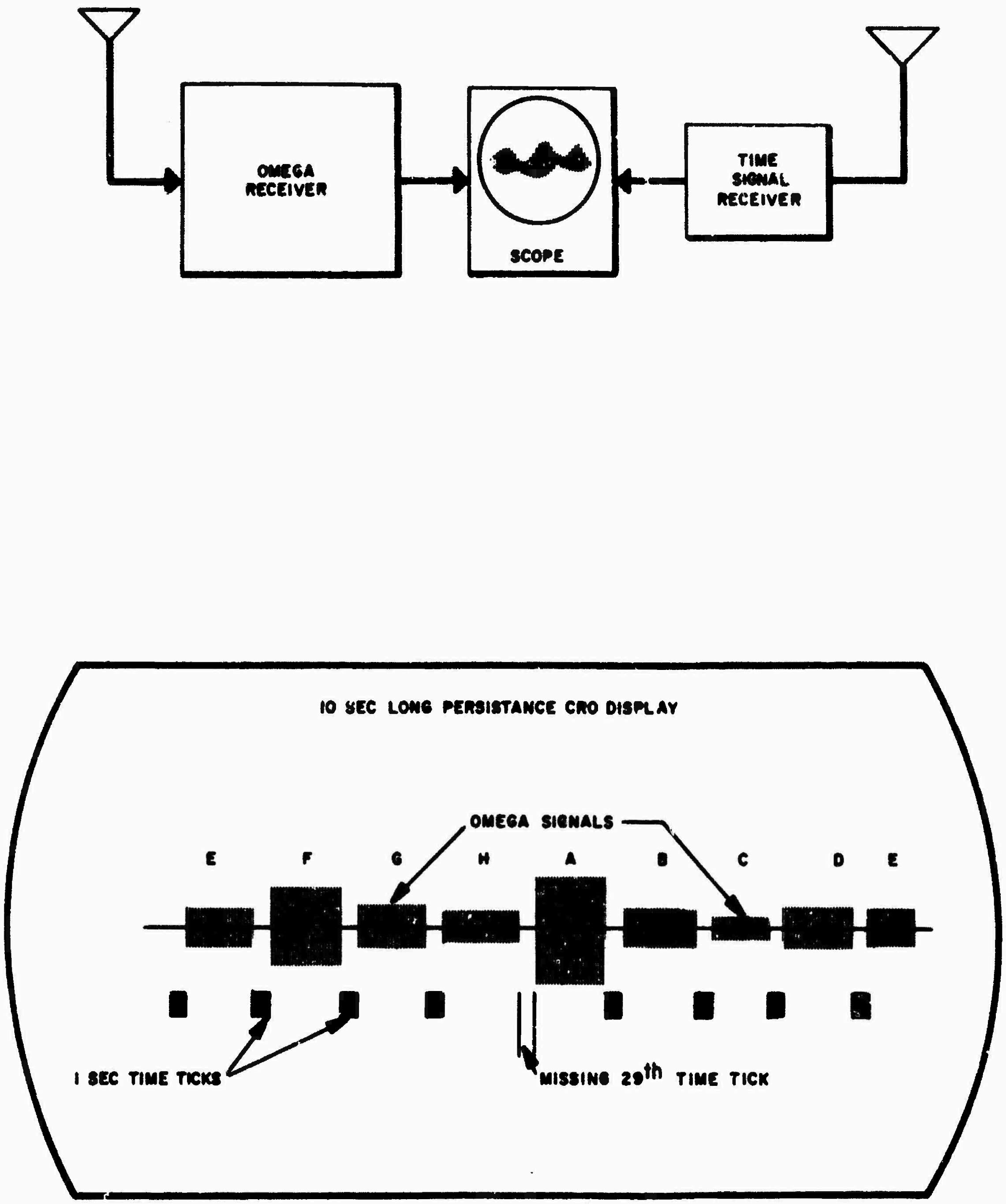

FIGURE 2.3-4. STATION IDENTIFICATION BY STANDARD TIME

transmitting only one-tenth of the time at the basic (10.2 kilocycles) frequency--each station will be available the remaining nine-tenths of the time for transmission of additional signal components at other frequencies. It is proposed that each station will transmit at two other frequencies for lane identification (Section 2.4) during two other segments of the multiplexing sequence. During the remaining seven-tenths of the time, each station can transmit a "side frequency" (Section 2.5) in one of eight adjacent radio frequency channels between 10 and 14 kilocycles when that station is not transmitting one of the three principal frequencies. Only one station would ever transmit in each side channel, and identification of a sequence element would be indicated by absence of signal at the corresponding side frequency.

There are four possible methods of identifying signals in the sequence which have some technical interest but which are not recommended for use in Omega. These methods are excluded from the signal design either because they unnecessarily increase the complexity of the receiving equipment or because they consume time that might otherwise be spent in transmitting the fundamental phase information.

(a) The signals of the various stations could be given identity by orthogonal phase coding. One method would be to divide each signal transmission into a leading and a lagging half and phase coding the lagging half of successive transmissions of each station (with respect to the leading halves) with an eight-element mutually-orthogonal code, as shown in Figure 2.3-5. Phase-code correlating in the receiver would then identify each signal uniquely, even though only the signal of one station could be received. By applying the recognition code pattern to the phase reference with which the signals are compared in the receiver tracking function, no loss in effective signal amplitude need be incurred due to the presence of the phase code modulation.

(b) Station identification could be supplied by offseting the frequency

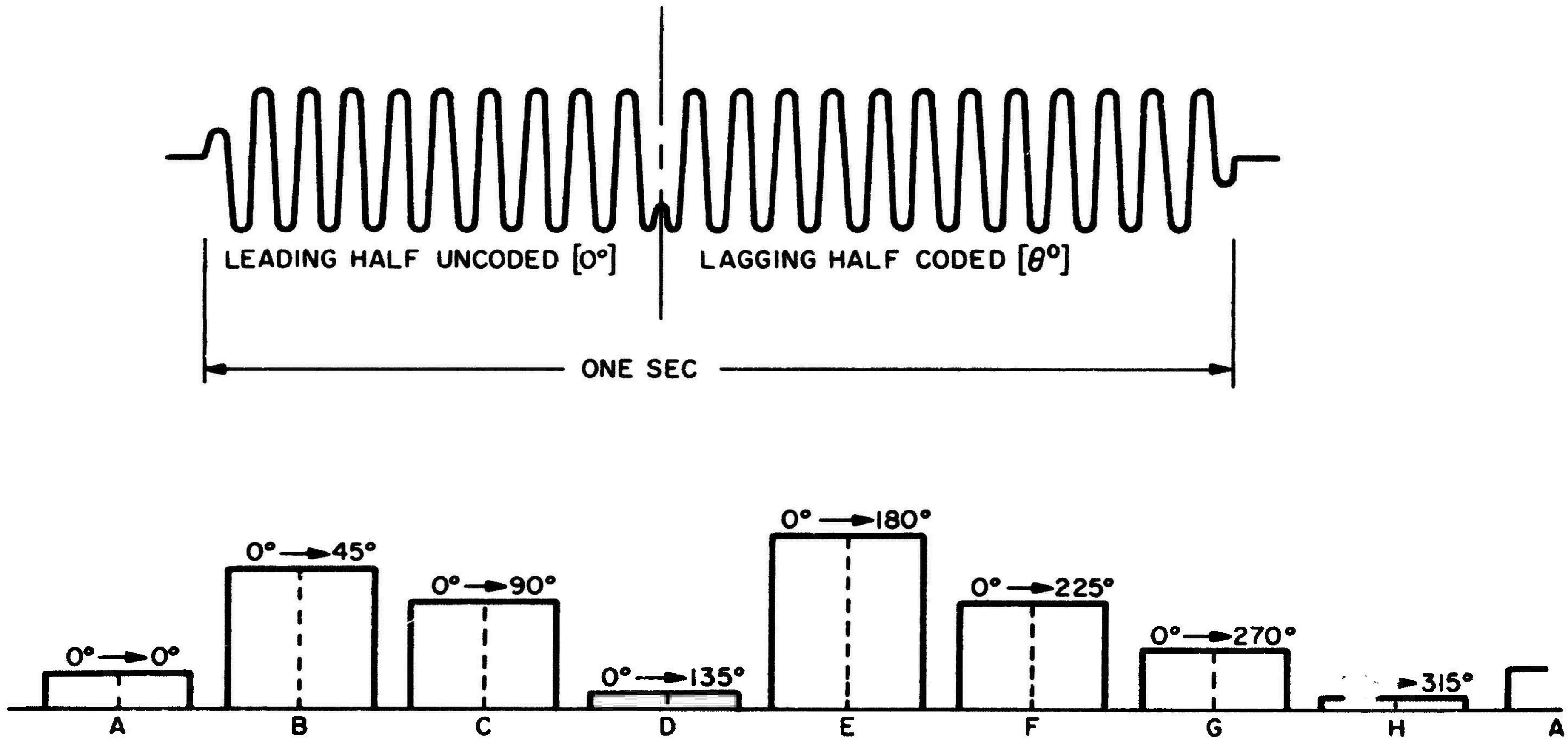

FIGURE 2.3-5. PHASE CODE IDENTIFICATION PATTERN

of each station a characteristic amount from time to time, with an amount and duration that does not interfere with the normal phase-tracking function of the receiver. The offset could be an amount producing an audible one-second beat note with respect to the normally phase-locked reference wave of the receiver, recognizable aurally by those with sufficiently good audio tone discrimination or by a vibrating reed or other simple resonance indicator. All stations may have the same frequency offset, with identification of a particular station being obtained from the absolute time at which the offset occurs. In this case, a single resonance indicator and a clock would serve to identify all stations.

(c) Station identification can be supplied by offseting the phase of each station by a characteristic fraction of a cycle from time to time, to produce a distinctive signature in the phase record, where continuous phase recording is employed in the navigator's equipment. The phase offset signature of the several stations may be programmed with respect to Standard Time with identity established by the occurrence time of the offset signatures, or the signature itself may be coded to provide identity without outside references.

(d) Each station signal could be given a distinctive modulation, either phase or amplitude, which can be recognized by suitable circuitry in the receiver. Because of the one-second commutating pattern, the recognition circuitry bandwidth is limited to approximately one cycle per second, unless a coherent signal method capable of maintaining modulation signal coherence over the nine-second gap between signals of a given station is incorporated in the receiver. Since transmission of a side frequency inherently provides for a narrower bandwidth and the side frequencies have additional uses (over and above station identification), the use of signal modulations for station identification is not recommended.

2.4 Lane Identification

2.4.1 Introduction

The phase of a radio signal is a periodic quantity--a given phase repeating at integral wavelength increments of distance. Consequently, in Omega the relative phase angle of the signals of each pair of stations defines not one contour, but an entire family of contours equal in number to twice the distance between stations in wavelengths, and spaced one-half wavelength apart (eight miles at 10.2 kilocycles) on the baseline. Only one contour of the family contains the position of the observer, and a means must be provided whereby this eight-mile ambiguity can be resolved. In those circumstances where the ambiguity is of operational significance, some means must be provided for its resolution.

The family of isophase contours, for which the two signals are "in phase", divides the coverage of a pair of stations into a set of narrow hyperbolically shaped "lanes". Each lane is the region bounded by adjacent contours of "zero phase" represented by the solid lines of Figure 2.4-1. A given value of phase angle, other than zero, then defines an isophase contour within each and every such lane, as represented by the dotted lines in Figure 2.4-1. The selection of the particular contour passing through R, the point of observation (solid heavy line), thus amounts to selecting the lane (cross-hatching) containing the observer (hence, the term "lane identification").

2.4.2 Information Required for Lane Identification

Resolution of the lane ambiguity is the process of selecting from among all the lanes in which the observer *might* be located, the particular lane that does contain his position. Thus, lane identification essentially consists of establishing the position of the observer by independent means, to within ± one-half lane.

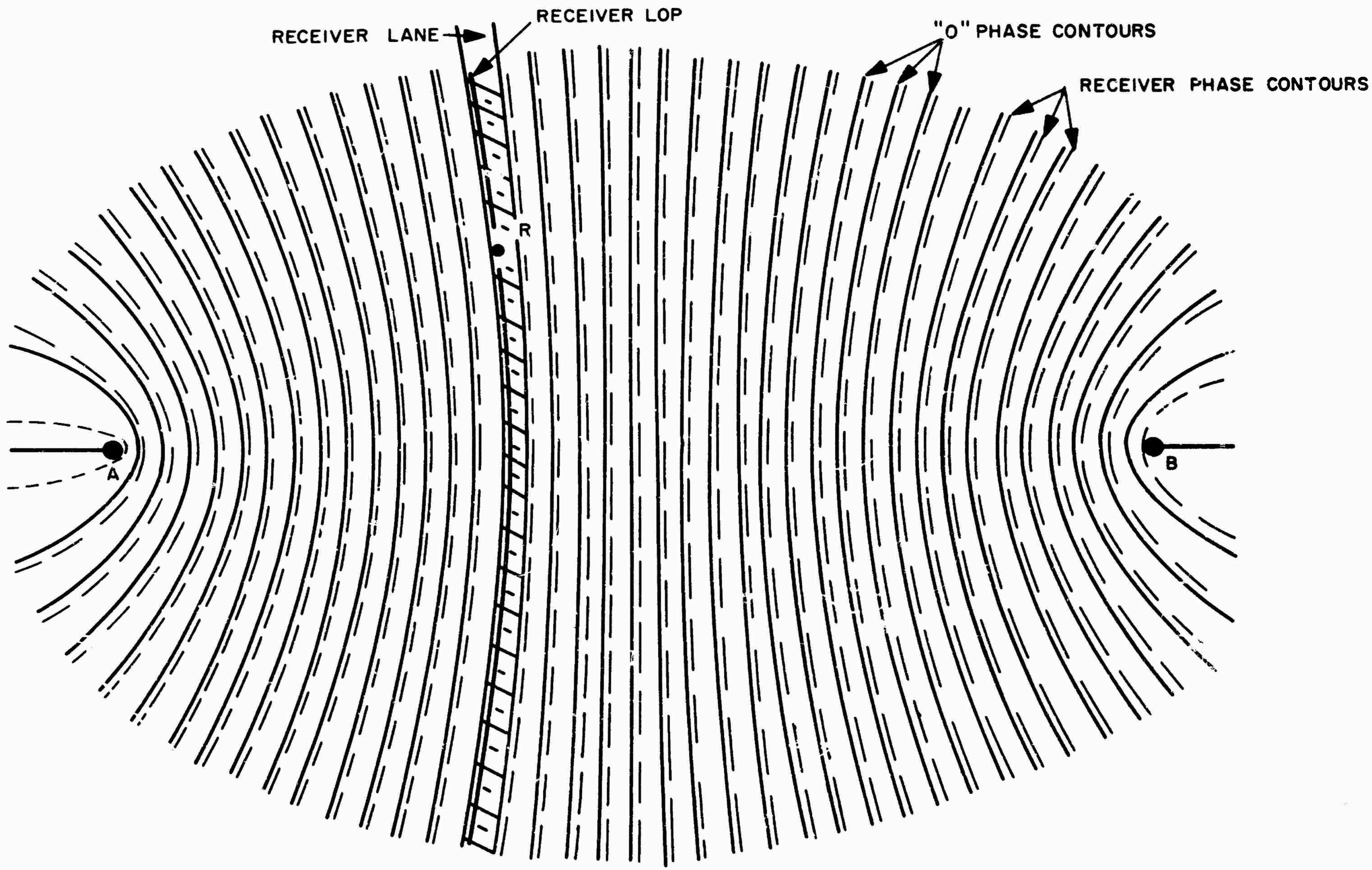

FIGURE 2.4-1. OMEGA LANE PATTERN

With no a priori knowledge of position, the number of lanes that might be occupied would be the total number of lanes defined by the system, i.e., twice the distance between stations in wavelengths. At 10 . kilocycles a wavelength is about 16 miles, and the length of a representative baseline in Omega is some 6,000 miles, or 375 wavelengths. Hence, each pair of stations produces a pattern of 750 lanes; and the lane resolution required is one in 750 for each line of position.

The information required to resolve ambiguity is, by definition, Log_2 of the number of choices. Thus, the information required to resolve one line of position would be

$$W = Log_2 \; 750 = 9.55 \text{ bits.}$$

If a period of ten minutes is available for lane resolution, and one bit per second can be conveyed in a bandwidth of one cycle per second, a bandwidth of ten cycles in ten minutes (or about 1/60 cycle per second) would be adequate for complete resolution of the ambiguity of one line of position. In practice, there is hardly ever a complete lack of a priori knowledge of position; and, where some knowledge of position is available, the amount of information required to resolve the ambiguity would be reduced accordingly. For example, a ship at sea, with a reasonable navigational competence on the part of its officers, ordinarily would not develop an uncertainty in its dead-reckoned position at a rate of more than five to ten miles per day. Since the minimum lane width of Omega is in the order of eight to ten miles, about one day would be required to develop an ambiguity of one lane which, in theory, could be resolved by one bit of information. Thus, the maximum information bandwidth required for Omega lane resolution by a ship at sea is in the order of one cycle per day.

On the other hand, in aircraft navigation it is impractical to utilize navigational equipment requiring extensive manipulation by an operator. Hence, for aircraft operations, a certain amount of automaticity must be assumed. This implies that little or no a priori information about position can be supplied the equipment except initial alignment at takeoff, and the only information available for maintaining lane identification would be that derived from the signal and memory of immediate position held within the equipment.

Automatic insertion of present position from an independent source or sources is feasible in some circumstances. For example, the NAVDAC concept implies the presence of computer keeping a "most probable position" derived from several sources, which the computer would be able to present to equipment such as Omega in suitable coordinates as an aid to lane identification. However, in automating a receiver, it is a distinct advantage to instrument the system so that complete resolution is possible on the Omega signals alone, rather than depending upon integration of generically dissimilar information derived in different coordinate systems.

2.4.3 Lane Identification by Multiple Frequencies

Lane ambiguity may be reduced by repeating the transmissions of the two stations at another frequency integrally related to the base frequency one-third higher, or 13.6 kilocycles for example. The additional signals define another set of zero- or in-phase contours about the same stations as foci (as in Figure 2.4-2), with a contour spacing narrower by the ratio of wavelengths (three-quarters of the 10.2 kilocycle pattern spacing in this instance), as shown by the dotted lines of Figure 2.4-2.

The wavelengths are commensurate in a ratio of four to three; and if the phase synchronization is adjusted so that a contour of the higher frequency family coincides with one contour of lower frequency (as at A in Figure 2.4-2), a Moire pattern is obtained in which every third contour of the lower frequency set will coincide with every fourth contour of the upper frequency set. The coincident contours then define a pattern of broader lanes, each broad lane extending over three lanes of the basic 10.2 kilocycle pattern.

In traversing a broad lane by any path, the relative phase of the

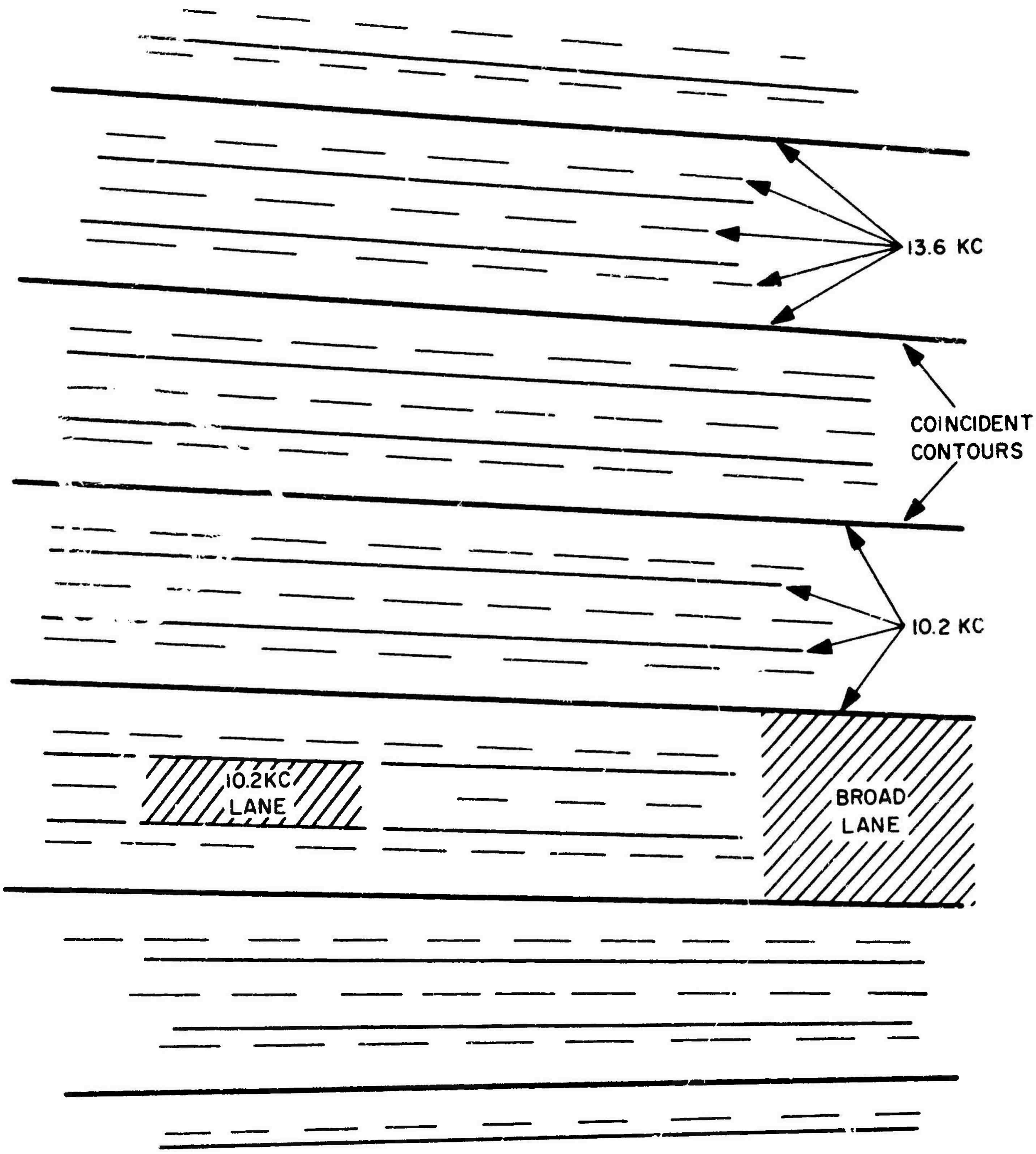

FIGURE 2.4-2. PRINCIPLES OF LANE RESOLUTION

higher frequency signals varies through four cycles and that of the lower frequency through three cycles, so that the difference of the phase indications at the two frequencies changes by one full cycle. Thus, the addition of the second set of signal components at 4/3 the frequency establishes a virtual system with isophase contours defined by the difference of the phase indications at 10.2 kilocycles and 13.6 kilocycles, having everywhere exactly* three times the lane width (and one-third the accuracy).

Thus, it can be determined from the difference of the two phase indications, in which of the three 10.2 kilocycle lanes included in a broad lane the observer is located. If the difference is less than one-third cycle, the observer is in the first lane of the three; if between one-third and two-thirds, the observer is in the middle lane; and if between two-thirds and one cycle, the observer is in the third lane.

The unambiguous lane width on a baseline is expanded from the eight miles of the basic 10.2 kilocycle phase contour pattern to 24 miles. Further expansion of the unambiguous lane width can be obtained by additional transmissions at other related frequencies. For example, a third set of transmissions at a frequency of 11.33 kilocycles defines a pattern of isophase contours with a spacing of nine-tenths the basic 10.2 kilocycle pattern. There is then a triple coincidence every 9 to 10 to 12 lanes (as in Figure 2.4-3) extending the unambiguous lane width on the baseline to 72 miles. **

Under most circumstances, a positional ambiguity of 72 miles would be of little operational significance, since a reasonable competence in dead-reckoning (even in high speed aircraft) should stay well within this limit. In some circumstances, however, it may be desirable to have available the ability to resolve the ambiguity completely from the Omega signal itself.

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* Except for a minor difference in phase velocity at the two frequencies which can be compensated in the charting operation or in the receiver.

** Ibid.

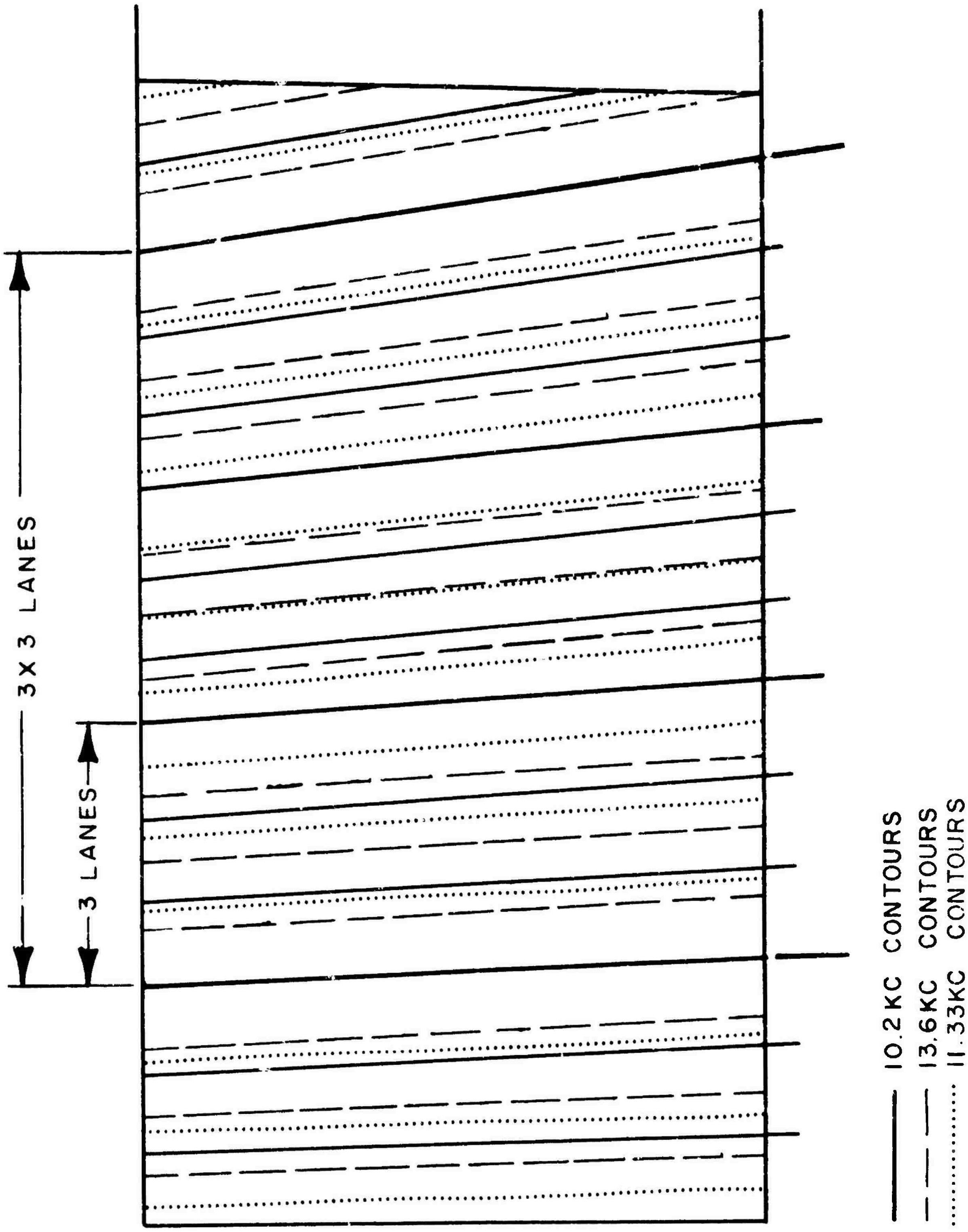


FIG. 2.4-3 DOUBLE MOIRE PATTERN

This is technically feasible through the addition of a small percentage modulation to the above signals. Thus, addition of a 226-2/3 cycles per second modulation to the 13.6 kilocycle signals would extend the unambiguous lane width to approximately 360 miles; a modulation of 45-1/3 cycles on the 11.333 kilocycle transmissions would bring the unambiguous range to 1,800 miles; and a final modulation of 11-1/3 cycles per second on the 10.2 kilocycle transmissions would extend it to 7,200 miles. Since the information required at the lower frequencies is very low, only a small percentage modulation would be required and would result in an insignificant loss of carrier amplitude.

In multi-frequency reduction of lane ambiguity, all phase contours defined by a given pair of stations are members of the same hyperbolic family with the two stations as foci. Hence, no two hyperbolae of any order of ambiguity reduction intersect, and the ambiguity reduction of each line of position contributing to a fix is independent. In contrast, where the ambiguity is removed by reference to an independent position determination, expressed in a different set of coordinations, a coordinate transformation must be performed before the ambiguity can be resolved.

2.4.4 Independent Position Fixing

Where an independent position fix is available -- by celestial observations for example, or by a radar plot, etc., locating an observer to within a lane width of ±4 miles -- the ambiguity would be resolved. Since a lane width is in the order of eight to ten miles, only a relatively low order of accuracy is required of the independent position.

2.4.5 Continuous Tracking from a Known Position

It may be assumed that the departure point of any trip or route would be known to better than the lane width of the Omega pattern at the point of departure. If the Omega navigation receiver is kept in continuous operation (providing a record of the total number of lanes crossed), there is no ambiguity.

Ambiguity, however, can arise from "lane slippage"; i.e., some signal disturbance or receiver failure causing the phase indication to advance or retard one or more full cycles. Furthermore, there would be no way of reacquiring lane identification in the event of equipment shutdown for any reason except through an independent position fix.

2.4.6 Dead Reckoning

In most navigation procedures, a dead-reckoned position, obtained by advancing previous position in accordance with distance and direction traveled, is maintained as a matter of course. Dead-reckoned position is subject to "drift", due to errors in heading, winds, or currents, so that the accuracy of position deteriorates with time but there is no ambiguity. Omega position is always drift free but is subject to ambiguity. Thus, Omega and dead reckoning are complementary. To the extent that Omega fixes are obtained within the interval over which the dead reckoning error does not exceed a half lane width, ambiguity resolution would be maintained; and by updating the dead-reckoned position in accordance with the Omega positions, the drift of the dead-reckoned position would be eliminated. Therefore, combining dead reckoning with Omega, on the one hand, eliminates the necessity for guaranteeing uninterrupted operation of Omega; on the other hand, the availability of Omega fixes, even though ambiguous, eliminates the drift of the dead-reckoned position with time.

It might be noted that keeping the Omega equipment in continuous operation, and maintaining a record of lanes crossed is essentially drift-free dead reckoning in Omega coordinates. However, in this case, there is no protection against failure provided by independent dead reckoning information.

2.4.7 Multiple Intersections

By definition, the family of isophase contours defined by the relative phase of any two signals includes one contour passing through the position of the observer. With three or more signals, there is a multiple intersection with as many contours intersecting at the observer as there are readable pairs of signals.

Omega contemplates a world-wide network of eight Omega stations in continuous operation, with a position contour available from every pair of signals and at least five signals readable at every point on earth. Hence, the position fix will always be overdetermined; and lane identification, as well as some statistical improvement in accuracy, can be obtained from the added information provided by the excess number of lines of position. All contours so determined are not independent, the number of independent lines of position being always one less than the number of readable signals. With five stations within range, there would be four independent contours, giving a sextuple intersection at the location of the observer.

Since the several contour grids overlap, there will, of course, be many other points of intersection between contours of the several families (as in Figure 2.4-4). However, due to the nonrectilinear geometry of the system, the line spacings of the several families of contours will differ and the contour patterns will intersect at different angles. Hence, there can be no multiple intersection of maximum order in the immediate vicinity of the observer, except the one defining his position. All other nearby intersections must be of lower order.

Since the system would not be 100 percent accurate, there will, of necessity, be some dispersion of intersections at the given point. Statistically, however, the most probable point would be that point at which the dispersion of the nearest intersections was the least. It can be shown on statistical grounds that if position lines are accurate to $\pm$ 5 percent, four independent lines should exclude the ambiguities from a region about 25 miles in radius, and five lines would more than double that radius. Reduction of ambiguity by this method is cumbersome, however. A navigator working with chart and pencil would have to plot a large number of possible fixes in order to find the one having the best overall agreement.

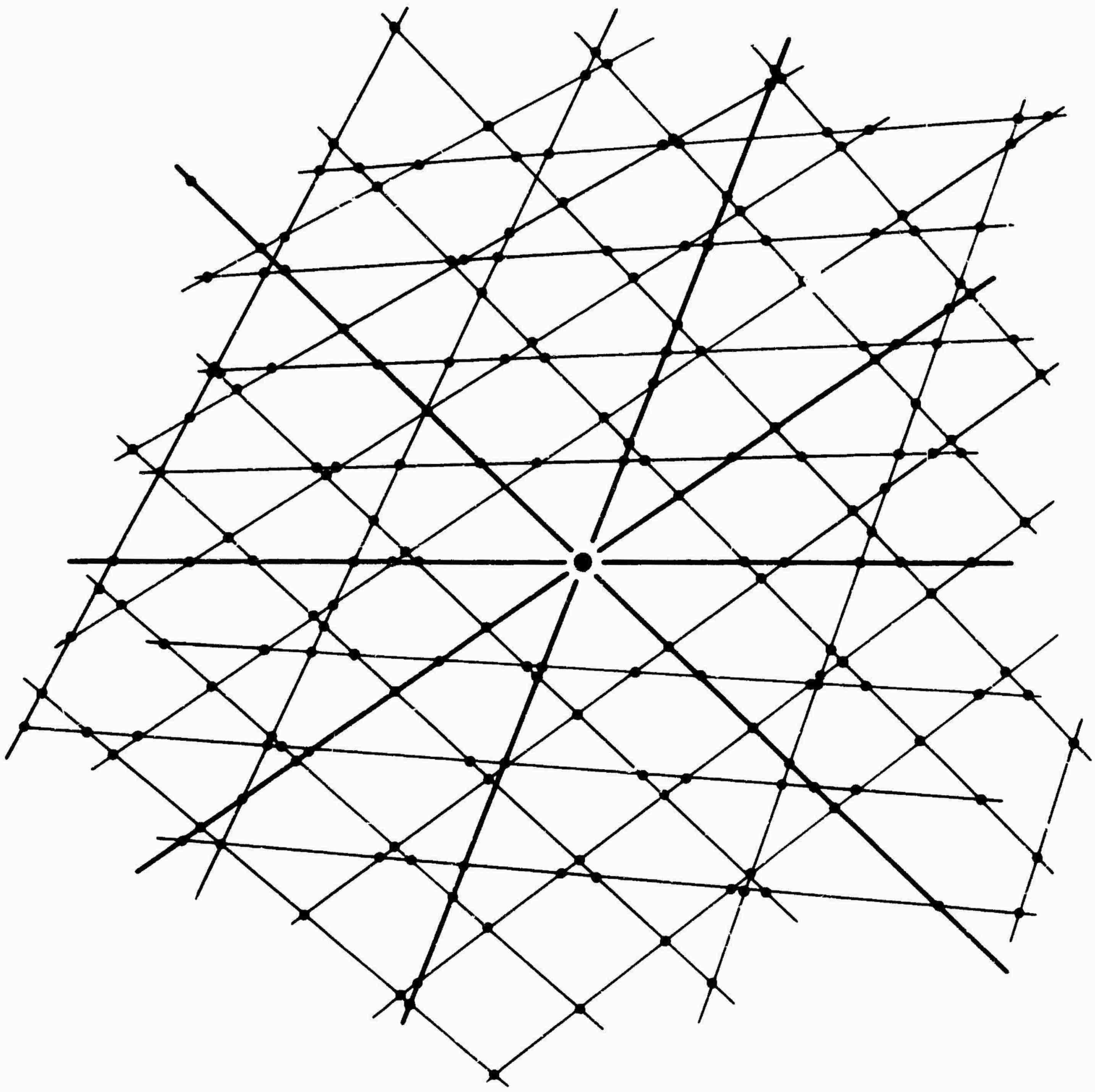


FIGURE 2.4-4. AMBIGUITY RESOLUTION BY MULTIPLE INTERSECTION

A properly programmed computer or an analog device to search for the common intersection would make it easy for the navigator, but would add to the size and cost, unless the computer were already in use for other purposes.

2.4.8 Feasibility of Lane Identification

The successful resolution of lane ambiguities by phase differences at several frequencies depends upon the stability and predictability of ionospheric propagation at the various frequencies needed. Several factors, including velocity dispersion and the ratio between the fine-lane frequency and the difference frequency, determine the reliability. Because little is known about the degree of correlation between fluctuations at various frequencies, it is difficult to arrive at final decisions on theoretical grounds, and it is necessary to base our conclusions on experimental evidence.

It is easy to see that if phase differences at 10.2 kc and 13.6 kc are in agreement along one line of position, the 10.2 kc phase reading will recur every eight miles, while the 13.6 kc phase reading will be repeated each six miles. If the reading at 13.6 kc is to prove that the adjacent lane at 10.2 kc is the wrong one, the entire distribution of errors at the eight-mile point (at 10.2 kc) must not overlap the distribution of errors at the six-mile point (at 13.6 kc). This seems like a difficult condition to satisfy, as we have not been able to postulate navigational errors with a standard deviation small compared with one mile. On the other hand, a reasonably high correlation between observations at the two frequencies comes to our aid, as there is a tendency for the two errors to lean "right" or "left" together.

Perhaps the easiest way to examine this problem is to consider the beat between the 13.6 kc signal and that at 10.2 kc, and to examine the stability of the time of arrival of this 3.4 kc signal. It is easy to see that the entire distribution of times of arrival at 3.4 kc should not exceed 1/2 period of 10.2 kc, or about 50 microseconds. It is possible to make this identification signal-by-signal (although at the cost of additional instrumental complexity), or pair-by-pair, or fix-by-fix, but the latter two possibilities yield progressively poorer reliability than the first.

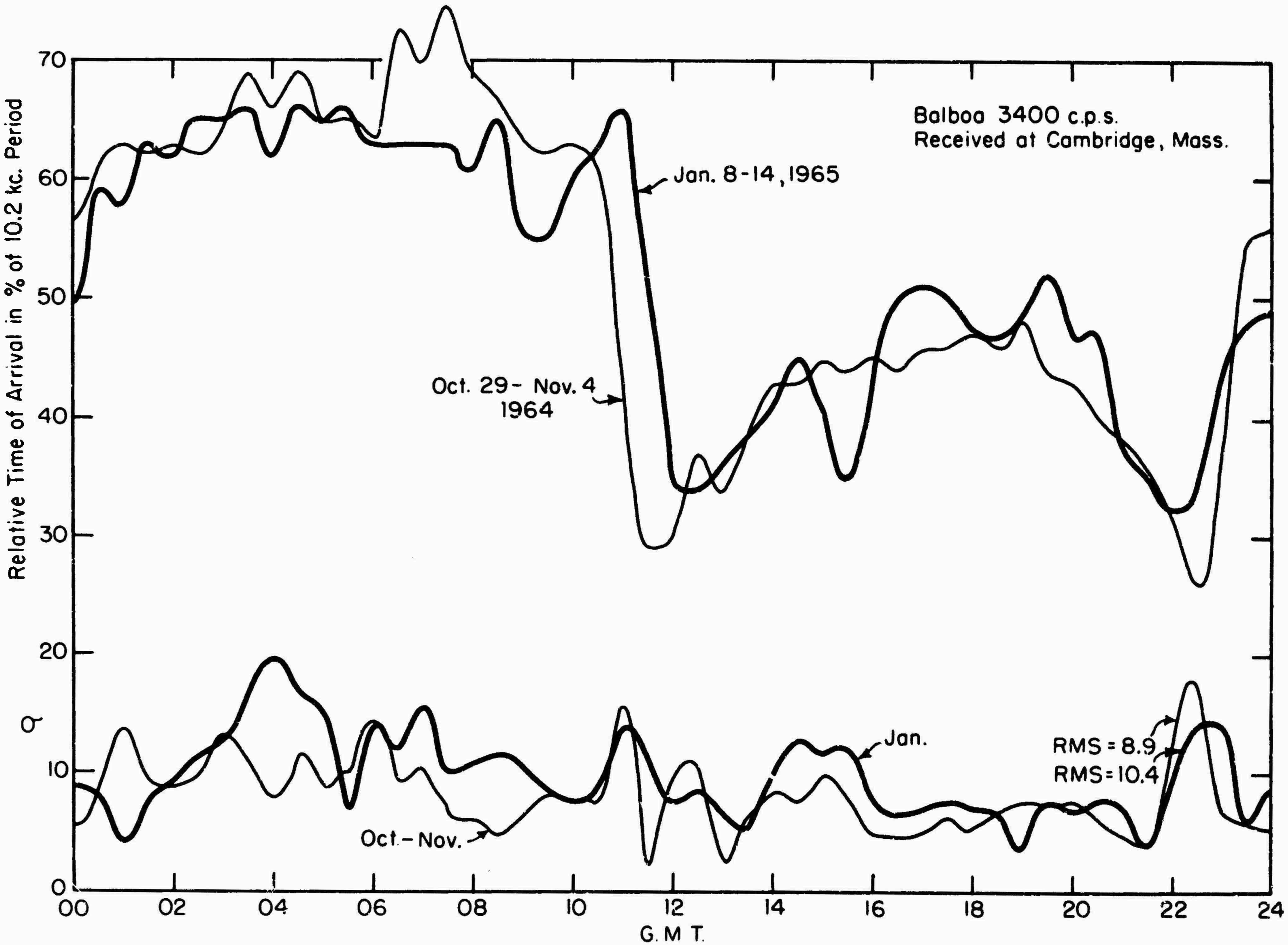


FIG. 2.4-5 DIURNAL PHASE VARIATION OF DIFFERENCE OF 12 KC AND 13 KC SIGNALS RECEIVED AT SAN DIEGO FROM HAIKU

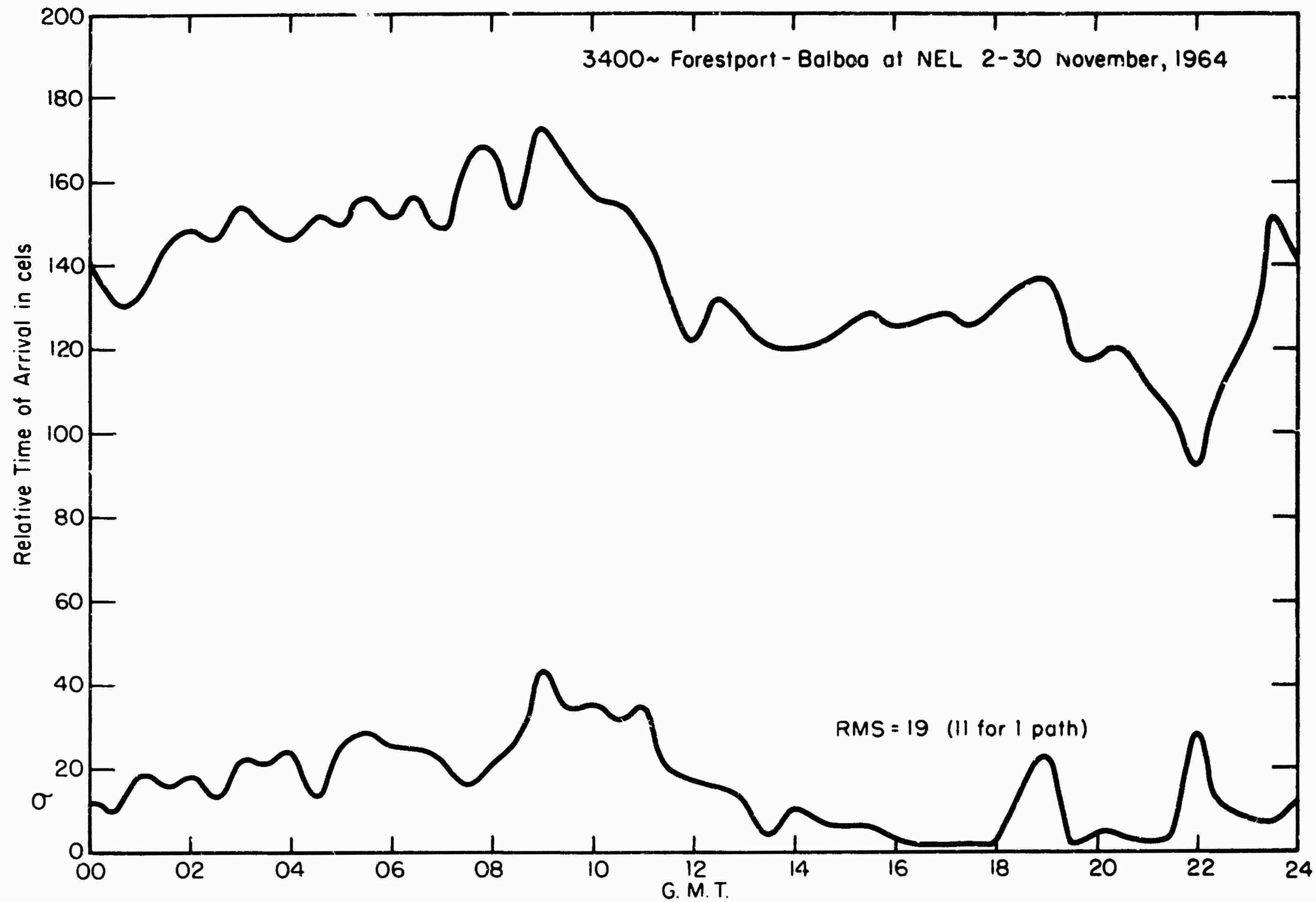


FIG. 2.4-6 DIURNAL PHASE VARIATION OF DIFFERENCE OF 13 KC AND 14 KC SIGNALS RECEIVED AT SAN DIEGO FROM HAIKU

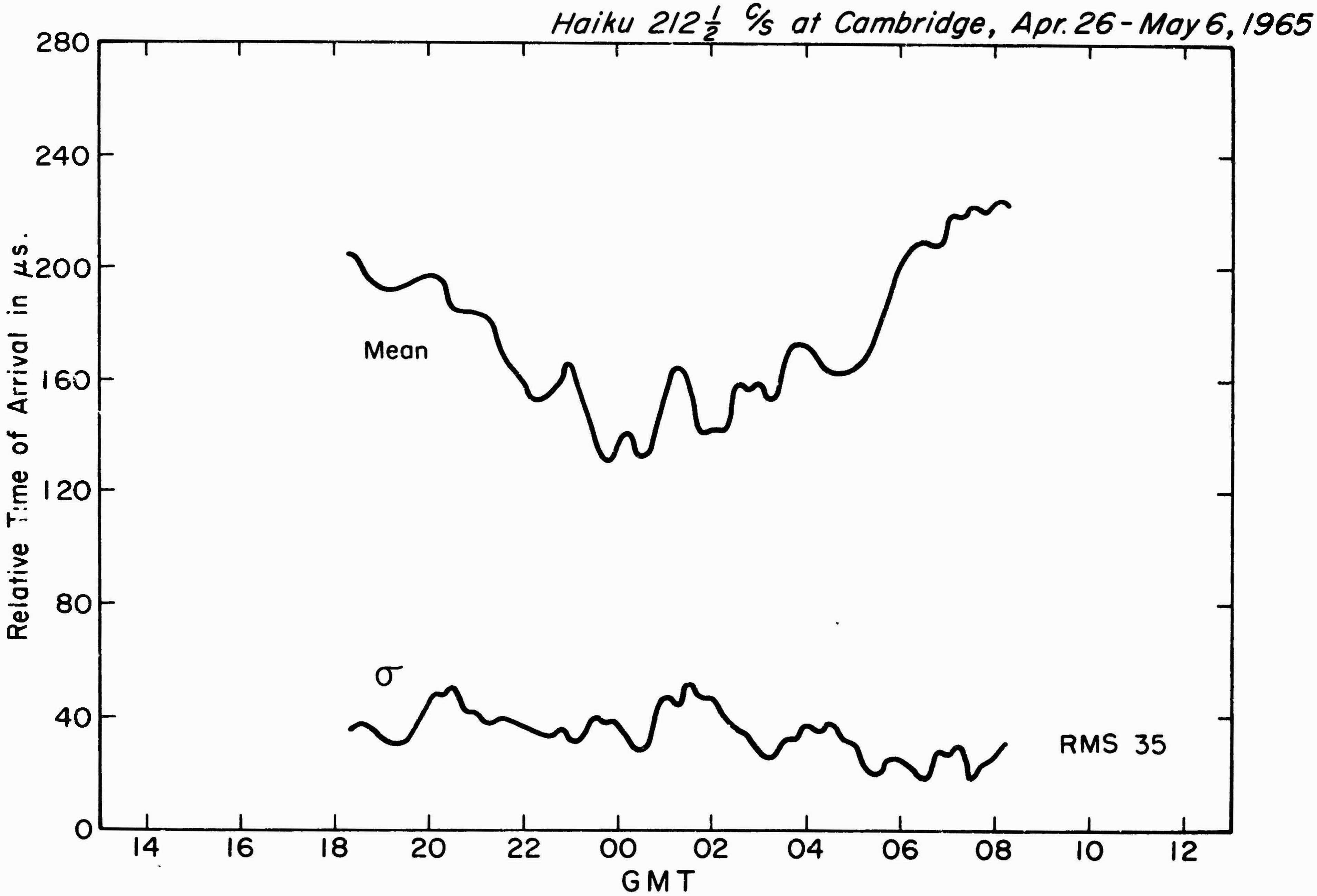


FIG. 2.4-7 DIURNAL PHASE VARIATION OF DIFFERENCE OF 12 KC AND 14 KC SIGNALS RECEIVED AT SAN DIEGO FROM HAIKU

An example of data on the time of arrival of the 3.4 kc difference frequency is shown in Figure 2.4-5. This is an example of one-way transmission over a single path. In this case, probably because of the relatively short distance, there is an unusually well-marked diurnal variation in the average values. Since this total variation is nearly 50 microseconds, or 1/2 period of 10.2 kc, it seems unlikely that satisfactory lane identification can be had without allowance for the expected diurnal changes.

On the other hand, the standard deviations shown at the bottom of Figure 2.4-5 are satisfyingly small in comparison with 50 microseconds, indicating that 10.2 kc lane identification can be made with good reliability if the diurnal variation be allowed for.

Figure 2.4-6 is a more realistic example of three-path data for the Forestport-Balboa pair observed at San Diego for the month of November, 1964. In this case, the root-mean-square errors are somewhat larger than are to be expected for the operational Omega system (because the data were taken with the master-slave relationship that will no longer be used) and even so average less than 20 microseconds. It should be noted that the 11 μs standard deviation deduced for a single path is not greatly different than the values on Figure 2.4-5, and indicates that, if taken signal-by-signal, identification should usually be highly reliable. An exception to this statement may be made for the sunrise period (near 10^h GMT in Figure 2.4-6) when the standard deviation appears high enough to limit the probability of successful lane identification to the 80-90% region.

It can be shown on theoretical grounds that the further stages of lane identification we have proposed are progressively more reliable than the first. A few experiments have been made using very weakly modulated signals radiated from Hawaii and received in Cambridge, Massachusetts. An example of data at 212-1/2 cps, for only 14 hours per day, is given in Figure 2.4-7. No great attention should be paid to the apparent diurnal variation. In this case the requirement for successful identification of the next-higher frequency is that the standard deviation should be small compared with 440 μs, as it certainly is.

Although few lane-identification experiments have been made, it is already clear that the later stages of lane identification will all be more reliable than the first. The first, based on the "beat" between 10.2 kc and 13.6 kc, appears to offer at least a 97-99% probability of successful identification, except that lower probabilities may apply at certain distances during sunrise, and perhaps sunset, hours.

There are indications that instrumental errors may have contributed more than they should have to the few experiments that have been performed. For this reason, and for others, the present evidence should be treated with caution, and additional data are being collected especially for the 10.2/13.6 kc difference.

2.5 Side Frequencies

Unlike the existing very low frequency communication stations, the proposed Omega stations are to radiate sufficient power continuously for long-range reception and are to be properly located for global navigation. Although the time-multiplexed Omega transmissions at three radio frequencies are to provide hyperbolic navigation that is satisfactory for most long-range applications, other transmissions may be more useful for other applications. Since such transmissions can be provided at little additional cost, they merit consideration.

The transmissions of such phase-stabilized VLF stations as GBR in England, NBA in the Canal Zone, and NPM in Hawaii are useful for the measurement of a navigator's distance from these stations. If the phase of a received signal is compared with the phase of a locally generated reference signal of precisely the same frequency (within a few parts in 10^{10}), then the change of the relative phase of the received signal measures the change of distance from the transmitting station. A group of scientists at the Royal Aircraft Establishment in Farnsborough, England, has been investigating the feasibility of such a VLF distance-distance or rho-rho navigation system. While it is not radiating one of the three basic frequencies, each of the Omega stations could transmit a characteristic frequency that would be useful for rho-rho navigation. Such transmissions would be more stable and more continuously available than existing VLF signals, and the geographical locations of the stations would be chosen for navigation purposes.

The same transmissions on station-identifying side frequencies could be used for a hyperbolic navigation system, such as Draco, that would not require a precise frequency standard or a time demultiplexing commutator in the navigator's equipment. In the navigator's receiving set, the changes in the phases of the received signals would be converted to changes of phase of a common frequency. One such phase change would be subtracted from another to yield a phase difference or time difference for a hyperbolic line of position. If the navigator starts from a known position and measures the accumulated change of phase difference during his voyage, he can establish his location on a hyperbolic line of position as in loran or Omega. In this system (as in rho-rho) it is necessary, however, to maintain continuity of the phase-difference accumulation.

The Omega signal sequence is comprised of eight time segments of approximately one second duration each, and, in producing the basic Omega signal pattern with lane identification, each station transmits at the three basic frequencies (10.2, 11-1/3 and 13.6 kilocycles) one after another in three successive segments of the eight and is idle during the remaining five segments. Since the repetition interval is relatively long, it is entirely feasible to switch each station to a different channel and radiate other frequencies when that station is not active in forming its part of the basic pattern.

One use of such side frequencies would be to provide a simple means for station identification by passive frequency-domain filtering, wherein identification of the signal of a particular station is established by lack of signal at the corresponding side frequency.

Transmission of side frequencies solely for station identification would be uneconomical and wasteful of spectrum, since alternate identification methods applicable to the basic pattern are available. However, if the side frequencies are made commensurate, so as to provide stationary phase patterns, an additional mode of phase tracking operation would be made available in addition to station identification.

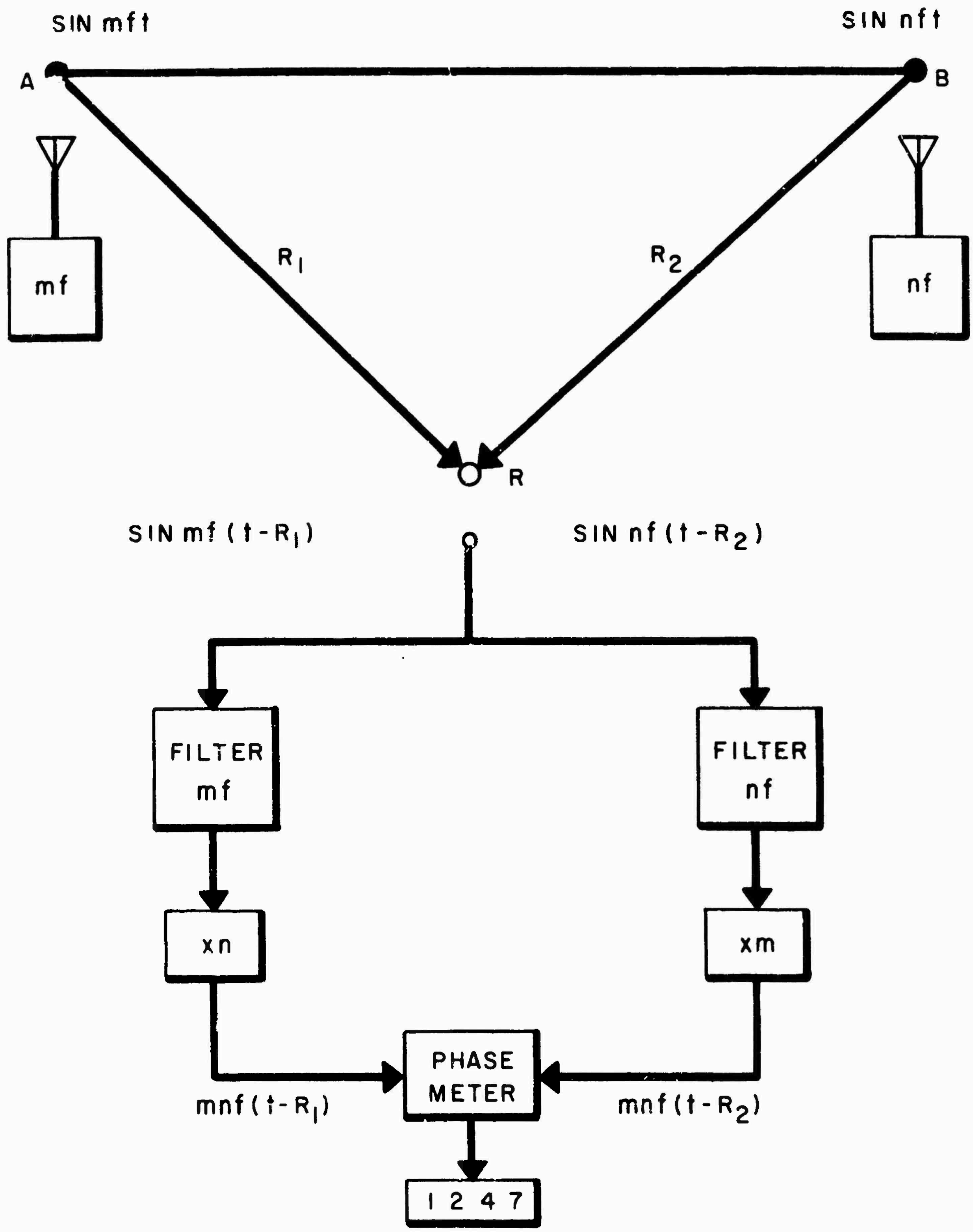
In this alternate mode of operation, the phase pattern would be highly ambiguous (more so than with the basic 10.2 kilocycle pattern). However, the lane widths would be no less than that of the Decca system; the signals would be separable by passive frequency domain filtering, so that the time multiplex search problem would not exist; and the duty ratio would be some six times that of the basic time multiplex pattern, with a corresponding increase in effectiveness of operation in noise and interference.

The principles of operation by this method (as shown in Figure 2.5-1) are essentially those of Decca. Consider A and B to be two stations of the Omega network. When not transmitting components of the basic Omega pattern, station A transmits a cw signal at a frequency, mf, (between 10 and 14 kilocycles) and station B transmits a cw signal at the frequency, nf.

The side frequency, nf, of station B is phase-locked to the frequency, mf, of station A by the ratio of n to m, so that a stationary phase pattern exists between the signals of the two stations.

In a navigators receiver, the signals may be segregated by phase lock or tracking filters, each filter producing an un-interrupted sinusoidal wave phase tracking the discontinuous signal from one of the stations; with a commutating switch operated by signal amplitude disconnecting the tracking circuit between signal transmissions to eliminate excess noise, so that the tracking filter output is phase locked to the incoming signal when it is present and runs freely with the last established phase when the input signal is absent.

The tracking filter outputs are multiplied by m and n respectively, to the frequency that is the least common multiple of the received signal frequencies, or are other wise reduced to a common frequency, and a phase



$$\theta = \sin^{-1} mnf\,(t - R_1) - \sin^{-1} mnf\,(t - R_2)$$

$$\theta = mnf\,(R_1 - R_2)$$

FIGURE 2.5-1. SIDE FREQUENCY PHASE MEASUREMENT meter determines the relative phase of the multiplied signal frequency components.

Since the signals are commensurate, the relative phase observed at any fixed location is stationary. As the receiver moves, the phase varies as the difference of the distances between the two stations, thus producing the typical hyperbolic isophase pattern.

Movement of the receiver perpendicular to the isophase contour pattern produces one cycle of phase change for each wavelength of hyperbolic displacement at the least common multiple frequency. Hence, the basic ambiguity period of the side frequency pattern is a half wavelength of the least common multiple of the signals transmitted.

If the frequencies are selected to minimize the ambiguity, a lane width not significantly different from the basic Decca pattern would be obtained, although the stability of the least common multiple frequency will be less with transmissions at VLF.

The side frequency mode of operation would be useful wherever continuous operation of equipment with a suitable small failure rate might be utilized; and the relative simplicity of signal separation by frequency domain filtering, with its freedom from search problems and simplification of equipment, outweighs the disadvantage of the high order of ambiguity.

2.5.1 Spectrum of Side Frequencies

The basic requirements for the side frequencies are:

(a) The frequencies must be commensurate.

(b) The frequencies must be between 10.0 and 14.0 kilocycles, inclusive.

(c) The frequencies should not lie within the passbands required for the basic system frequencies of 10.2, 11.33 and 13.6 kilocycles.

(d) The separation of the side frequency components should be as large as feasible to maximize the lane width.

The side frequencies can be established as multiples of a common low frequency or as submultiples of a common higher frequency or by heterodying either of the above patterns on a reference frequency or by other relations producing a set of commensurate frequencies.

2.5.2 Division Spectrum

It appears that 408 kilocycles is a common multiple of eleven frequencies lying between 10 and 14 kilocycles with spacings varying from 262 to 439 cycles and including the three basic frequencies (as set forth in Table 2-1).

TABLE 2-1

408 KILOCYCLE DIVISION SPECTRUM

Divisor	Frequency	Difference
40	10.200 kilocycles	
39	10.462	262 cycles
38	10.736	274
37	11.027	291
36	11.333	306
35	11.657	324
34	12.000	343
33	12.363	363
32	12.750	387
31	13.161	411
30	13.600	439

The narrowest ambiguous lane width occurs with side frequencies whose factors are prime (such as 37 and 31) and for which the lane width on the baseline corresponds to a half wavelength of 408 kilocycles (or 1,200 feet). Other pairs of side frequencies produce minimum lane widths of 2,400 feet or 3,600 feet.

In receiving equipment, the necessary frequency ratios can be obtained conveniently by frequency division from the common frequency of 408 kilocycles, with a 40 to 1 counter arranged to reset to counts 0 through 10 (as required for the other division ratios) instead of to 0 only.

A similar commensurate family of frequencies can be found by dividing any other multiple of 204 kilocycles by various integers. The division from 408 kilocycles had the advantage of separating the necessary eleven frequencies most widely, but conversely it requires assignments scattered throughout the whole 10 - 14 kilocycle navigation band, leaving little open space for other possible services.

2.5.3 Product Spectrum

A suitable series can also be derived by multiplication of a common factor. For example, a frequency of 226-2/3 cycles per second multiplied by ratios 45 to 61 inclusive gives the series of Table 2-2.

TABLE 2-2

226-2/3 CYCLE HARMONIC SPECTRUM

Multiplier	Basic Frequency	Side Frequency
45	10,200	
46		10,426-2/3
47		10,657-1/3
48		10,884
49		11,110-2/3
50	11,333-1/3	
51		11,560
52		11,786-2/3
53		12,013-1/3
54		12,240
55		12,466-2/3
56		12,693-1/3
57		12,920

Multiplier	Basic Frequency	Side Frequency
58		13, 146-2/3
59		13, 373-1/3
60	13, 600	
61		13, 826-2/3

When the side frequencies are derived by multiplication of a subharmonic, every pair of side frequencies produces a hyperbolic lane pattern that is a multiple of the common subharmonic lane pattern. The unambiguous lane width of the phase pattern of any pair of side frequencies being the subharmonic lane width divided by the product of the harmonic orders involved, multiplied by any common factors.

For example, in this case the subharmonic lane pattern (corresponding to the common frequency of 226-2/3 cycles) would have a lane width of 360 miles. The maximum unambiguous lane width of the pattern, defined by side frequencies 10.884 kilocycles (48th harmonic) and 12.693 kilocycles (56th harmonic), would be $8/(48 \times 56) = 1/336$ of 360 miles, or 6,500 feet.

The narrowest unambiguous lane width is established by the largest pair of frequencies with no common factor. In this case there would be 13.147 kilocycles (58th harmonic) and 13.373 kilocycles (59th harmonic) for which the lane width is $360/(58 \times 59) \cong 630$ feet.

2.5.4 Other Commensurate Spectra

Other distributions of frequencies providing commensurate frequency ratios can be used to establish the side frequencies. However, division from a common frequency produces the lowest order of ambiguity in the side frequency phase patterns. In addition, with multiplication or division patterns, the frequency synthesizer required in the receivers and in the transmitter synchronizers are simple multipliers or dividers with relatively modest ratios.

3.0 Propagation

In order to obtain reliable world-wide navigation system coverage in an economical manner, it is desirable to employ a relatively small number of transmitting stations. The 10 to 20 kc region of the VLF spectrum has an inherent low attenuation rate that makes this portion of the frequency spectrum attractive. Within this frequency region, and particularly at great distances, most of the energy can be considered as being propagated as a guided wave in the space between the earth and a conducting ionosphere. Observed electromagnetic fields within the earth-ionosphere cavity indicate that the actual physical case can be well approximated with a model such as shown in Figure 3.0-1, where the earth is considered as highly conducting with a reflection coefficient of +1 and the ionosphere is highly reflective with a reflection coefficient of approximately - 1. For these idealized assumptions, the first order transverse magnetic (TM_1) wave will have electrical field lines similar to those shown on the left side of the figure. The manner in which the magnitude of the vertical electric field, E_z, and the horizontal electric field, E_y, vary with height, z, is shown on the lower left side of the figure. The scale chosen for this figure is representative of what is expected with a frequency of 15 kc. On the right of the figure are shown the electric field lines for the TM_2; i.e., the second order transverse magnetic wave. The amplitude-versus-height variations of the vertical electric and horizontal electric fields are again indicated at the bottom of the figure. The electric field lines are only shown within the cavity and no attempt is made to indicate the field lines lying outside the boundaries in the actual physical case where conductivities are not infinite.

There are two important factors relative to the fields produced from a given transmitter as regards its capabilities as a navigation aid. First, the field strength produced for a given amount of radiating power must be great enough to override background noise. This requires that the attenuation rate be relatively small. The second factor is that the phase versus distance pattern must be rather constant with time and also have a sufficiently

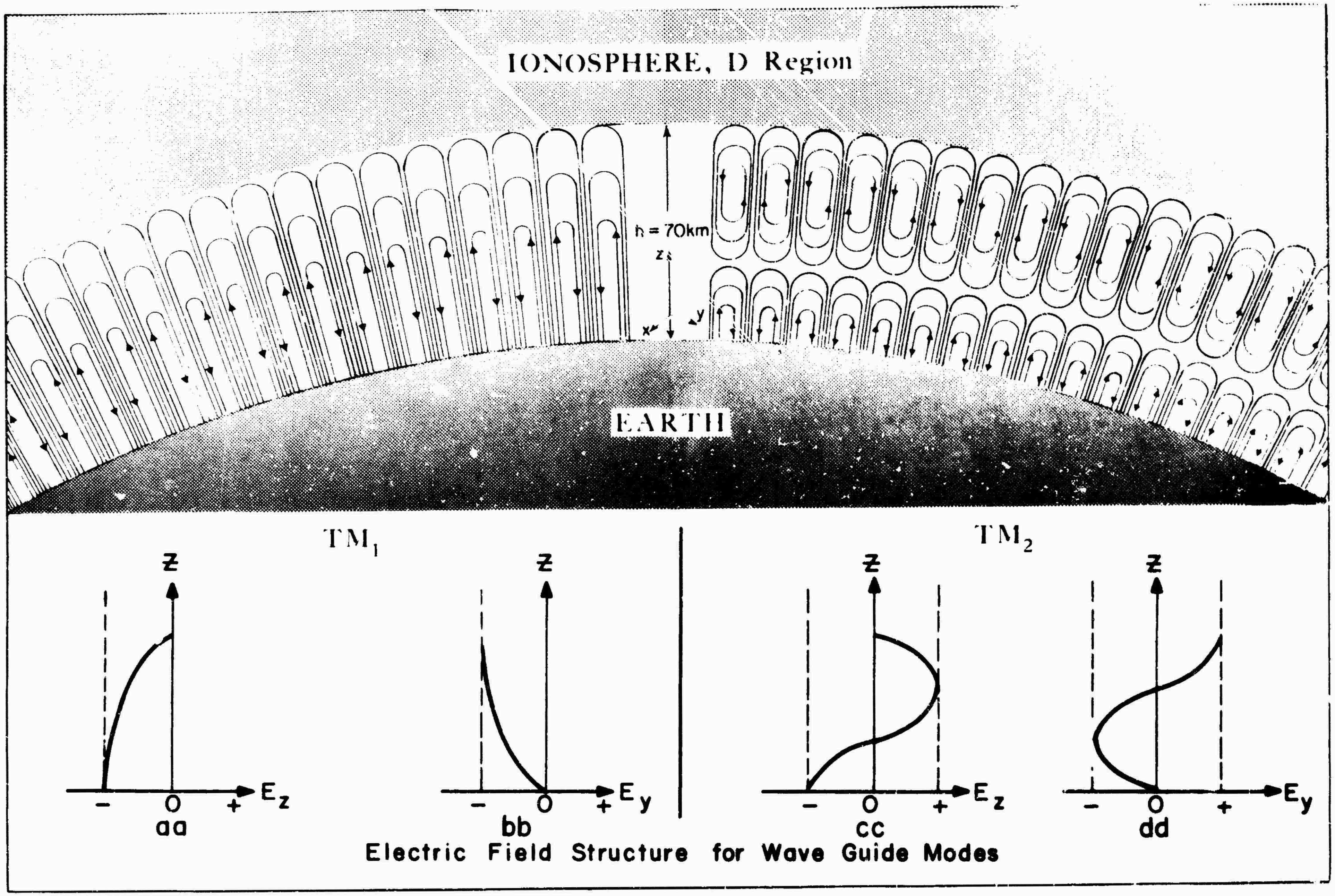


FIGURE 3.0-1. ELECTRIC FIELD STRUCTURE FOR WAVEGUIDE MODES (CURVES FROM WAIT - 1962)

large spacing between 360-degree phase ambiguities so that lane identification is possible. This means that the velocity of propagation must be rather stable and well known, and that large regions must exist where essentially one mode is dominant; i.e., phase interference patterns must not be present over the coverage area. In order to meet the lane resolution problem, the frequency employed must be low enough so that the wavelength is as great as possible. The consideration of all these factors naturally forces one to the lower end of the VLF frequency spectrum, which has resulted in the choice of 10.2 kc nominal carrier frequency for the Omega system.

3.1 Field Strength Versus Distance

For distances where near field effects can be neglected, Wait (1962, p. 199) has shown that the vertical electric field, E_z, can be represented in integral form or as the summation of an infinite number of modes.

For distances greater than one megameter, the vertical electric field can be represented as the sum of a relatively small number of waveguide modes.*

$$E_z \approx \sum_{n=1}^{n=1 \text{ to } 3} E_{z,n}$$

$$1 \text{ Mm} < d \qquad (3.1\text{-}1)$$

The field produced for a given mode can be written in decibel terms as

$$\begin{aligned} E_{z,n} (\text{db, v/m}) = {} & 10 \text{ Log } P_r + K'_n - 10 \text{ Log } f - 10 \text{ Log } h_{i,t} \\ & + 10 \text{ Log } \Lambda_{n,t} - 10 \text{ Log } (a \sin d/a) \\ & - 10 \text{ Log } h_{i,r} + 10 \text{ Log } \Lambda_{n,r} + 20 \text{ Log } G_{h,n} \\ & - L_{d,n} - \alpha d/10^6 \end{aligned} \qquad (3.1\text{-}2)$$

*At VLF the n = 0 mode which is actually a TEM mode is highly attenuated and is not expected to contribute to the observed fields.

where: $E_{z,n}$ (db, v/m) is the vertical electric field produced by the nth mode in db relative to 1 volt/meter, P_r is the power radiated into the half space above the earth in watts, K'_n is a constant which relates field strengths at the surface to power radiated and is equal to 104.3 for modes of n = 1, 2 -- f is the frequency in cps, $h_{i,t}$ and $h_{i,r}$ are the effective ionospheric heights at the transmitter and receiver locations, nominally 7×10^4 meters for day and 9×10^4 meters at night. $\Lambda_{n,t}$ is the magnitude of excitation modification factor which is approximately equal to the ratio of power launched into the concentric spherical shell guide relative to that for a flat guide with perfectly reflecting boundaries, a is the earth's radius $\approx 6.4 \times 10^6$ meters d is the path distance, (a sin d/a) accounts for energy spreading in the spherical guide structure, $\Lambda_{n,r}$ is the mode field modification factor for a curved earth at the ionospheric heights applicable at the receiver, 20 Log $G_{h,n}$ is the height gain factor in decibels relative to the field at the surface; i.e., z = 0, $L_{d,n}$ is a factor in decibels which accounts for losses at discontinuities along the path, * such as abrupt changes in surface conductivity or in guide height, which occur for sunrise or sunset boundary, and α is the effective attenuation rate in db/Mm which is normally different for each mode number.

Details relative to the employment of this equation for calculating field strength over the VLF spectrum are given elsewhere (Watt and Croghan 1964.).

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* A more precise model which allows for conversion of energy from one mode to another at such discontinuities is discussed by Crombie [Crombie, DD., Periodic Facing of VLF Signals Received Over Long Path During Sunrise and Sunset. Radio Sci. J. Res. NBS 68D, No. 1, 27-35.,] Bahar and Wait [Bahar, E., and J.R. Wait, Microwave Model Techniques to Study VLF Radio Propagation in the Earth-Ionosphere Wave Guide, Quasi-Optics, ed. J. Fox, 447-464 (Polytechnic Press, Polytechnic Inst. of Brooklyn, N.Y.)]

There are two important factors to be considered in this relation relative to the choice of frequency. They are the excitation factor Λ as a function of frequency and the attenuation factor α as a function of frequency. Theoretical and experimental values of Λ for the first and second order modes are given in Figure 3.1-1, where it is interesting to observe that the first order mode nominally considered to be dominant becomes less efficiently excited as frequency increases. This is particularly true at night.

The determination and prediction of attenuation rates is a rather complex subject since attenuation rate α is a function of many factors

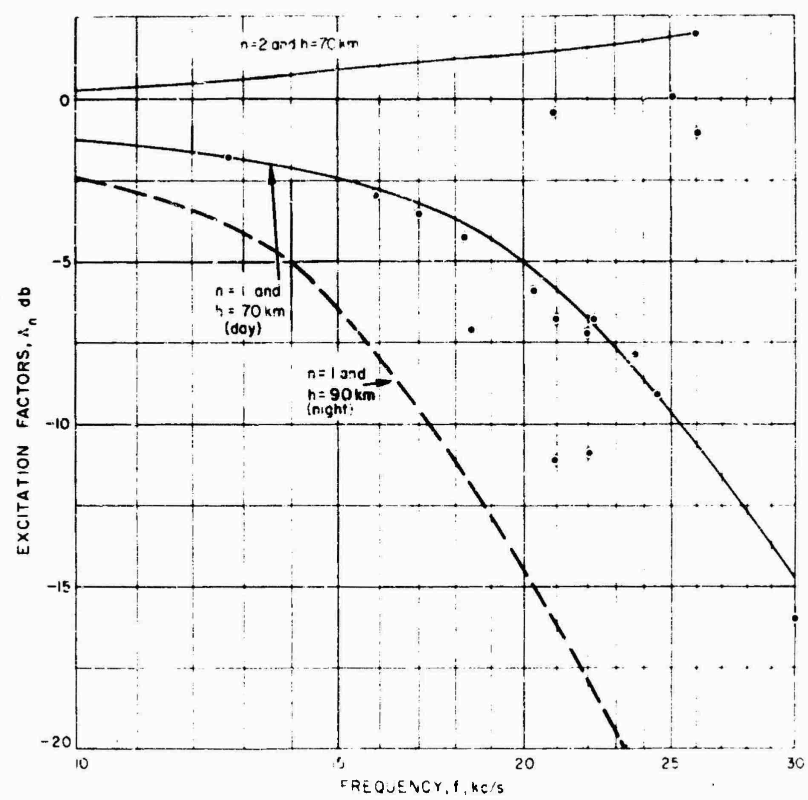


FIG. 3.1-1 EXCITATION FACTORS - THEORETICAL EXPERIMENTAL VALUES FOR DAYTIME CONDITIONS ONLY

including: frequency, diurnal effects, seasonal effects, latitude effects, solar activity, meteoric activity, direction of propagation relative to the earth's magnetic field, and earth's surface conductivity. Based on presently available experimental and theoretical evidence, it appears that typical daytime attenuation rates as a function of frequency will have the values shown in Figure 3.1-2. The rather large second order mode attenuation in the 10 kc region coupled with the approximately equal excitation of both of these modes at this frequency results in the first order mode at 10 kc being dominant for distances much closer to the transmitter than will be true for frequencies in the 20 kc region. This means that we can anticipate less modal interference at the 10 kc frequency chosen for this system than is to be expected for the upper end of the VLF band.

In order to determine the field strengths which will be available for a given amount of radiated power, Figures 3.1-3 and 3.1-4 have been prepared which give calculated field strength versus distance curves for day and night respectively. The various values of attenuation presented are typical of propagation to the east near the geomagnetic equator, propagation north or south, and propagation to the west respectively in order of increasing attenuation.

Although there is some variability in the field strength observed on a day-to-day basis, the fields are expected to lie within the range of values shown and these values of vertical electric field strength will permit a rather accurate calculation of transmitting station power radiation requirements for satisfactory system operation as will be described in Section 6.2.

3.2 Phase Velocity Observations and Calculations

A possible conceptual model for VLF propagation is that the field lines shown in Figure 3.0-1 are moving outward from the transmitter with a phase velocity given by the relation $v_p = f\lambda$. Obviously any physical factors along the path, such as changes in ionospheric height, surface conductivity, etc., which tend to change the velocity, will produce a corresponding change in the effective wavelength λ.

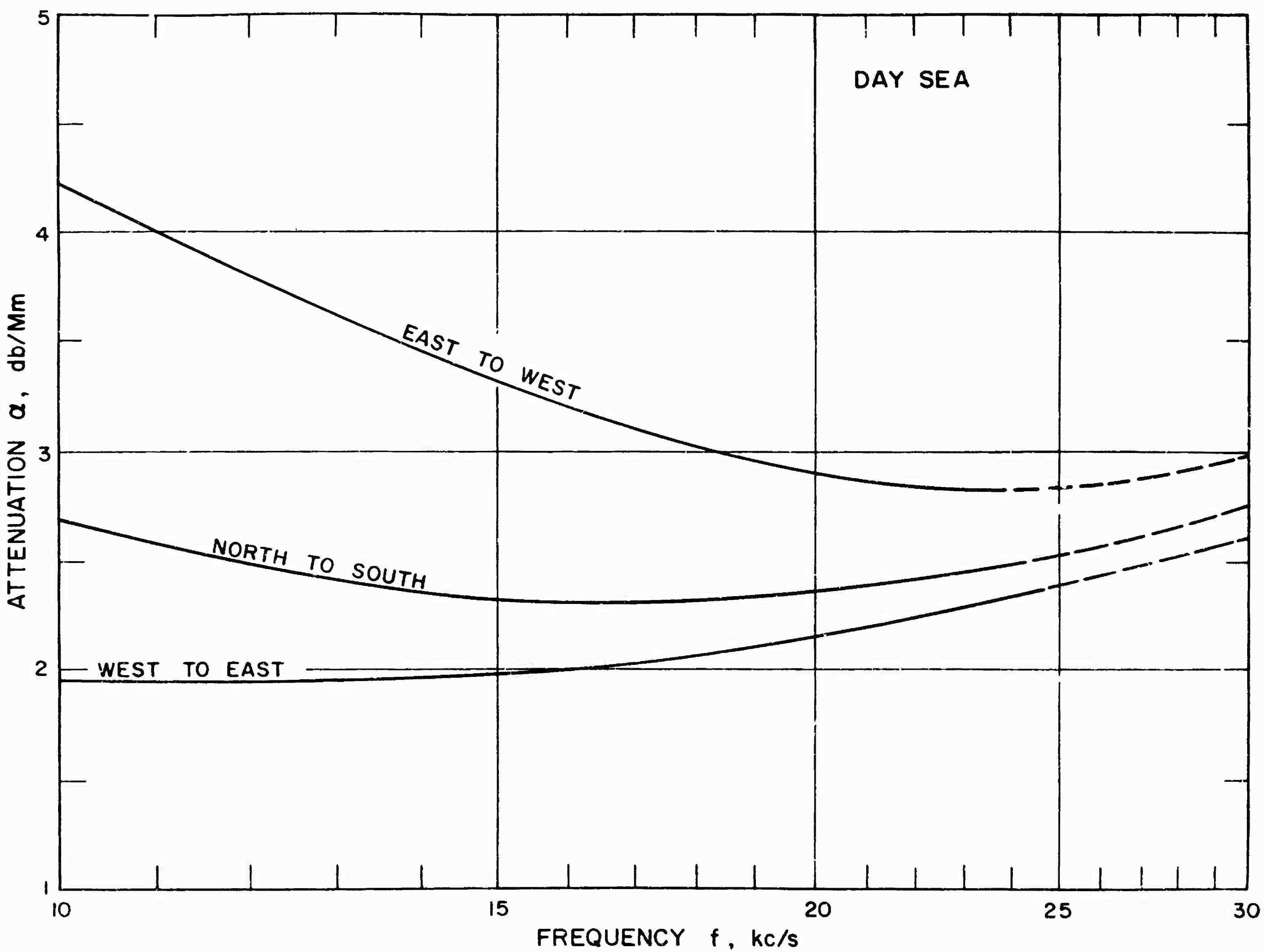


FIGURE 3.1-2. ATTENUATION RATES VERSUS FREQUENCY

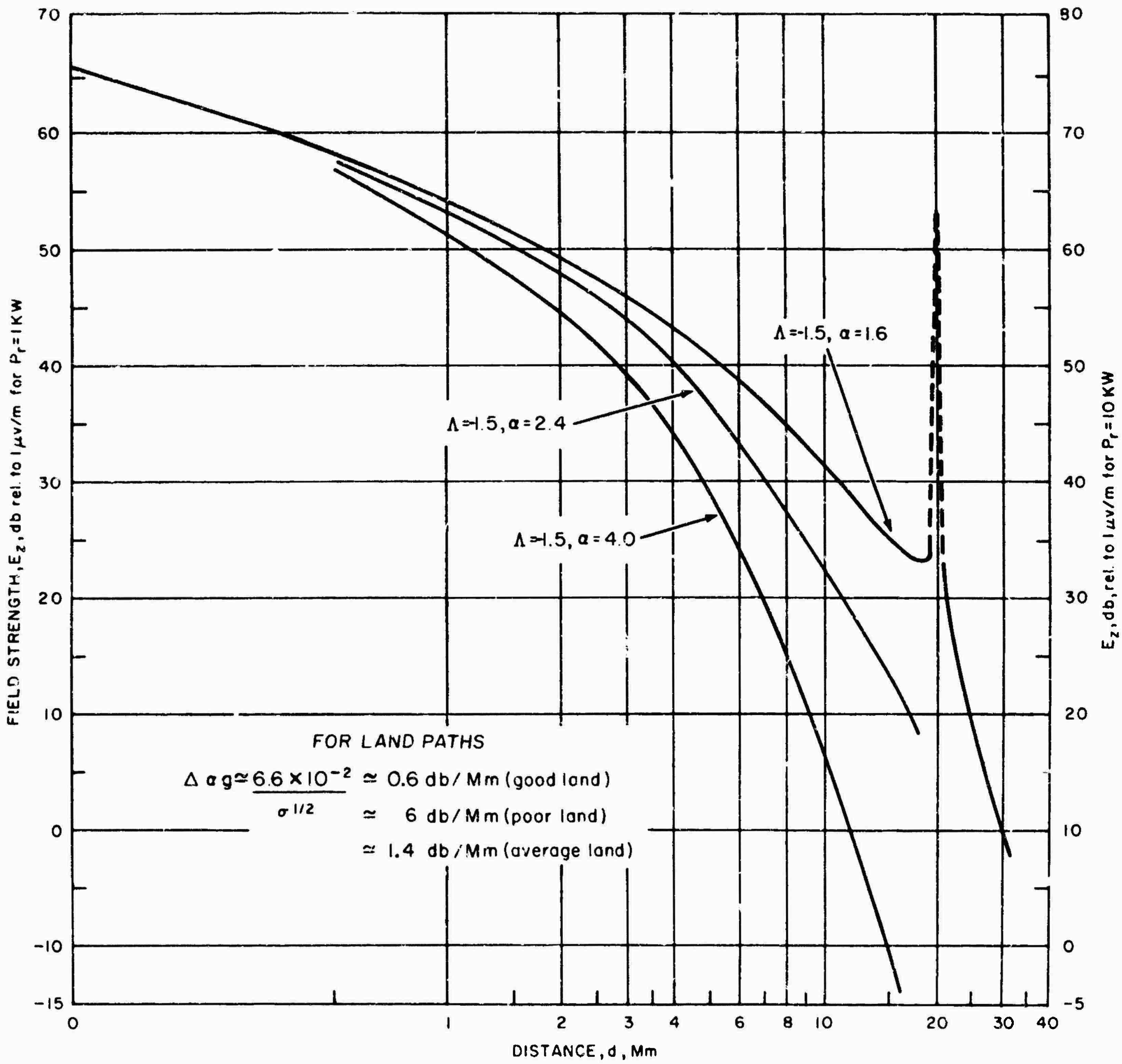


FIGURE 3.1-3. DAYTIME FIELD STRENGTH VERSUS DISTANCE AT 10.2 KC

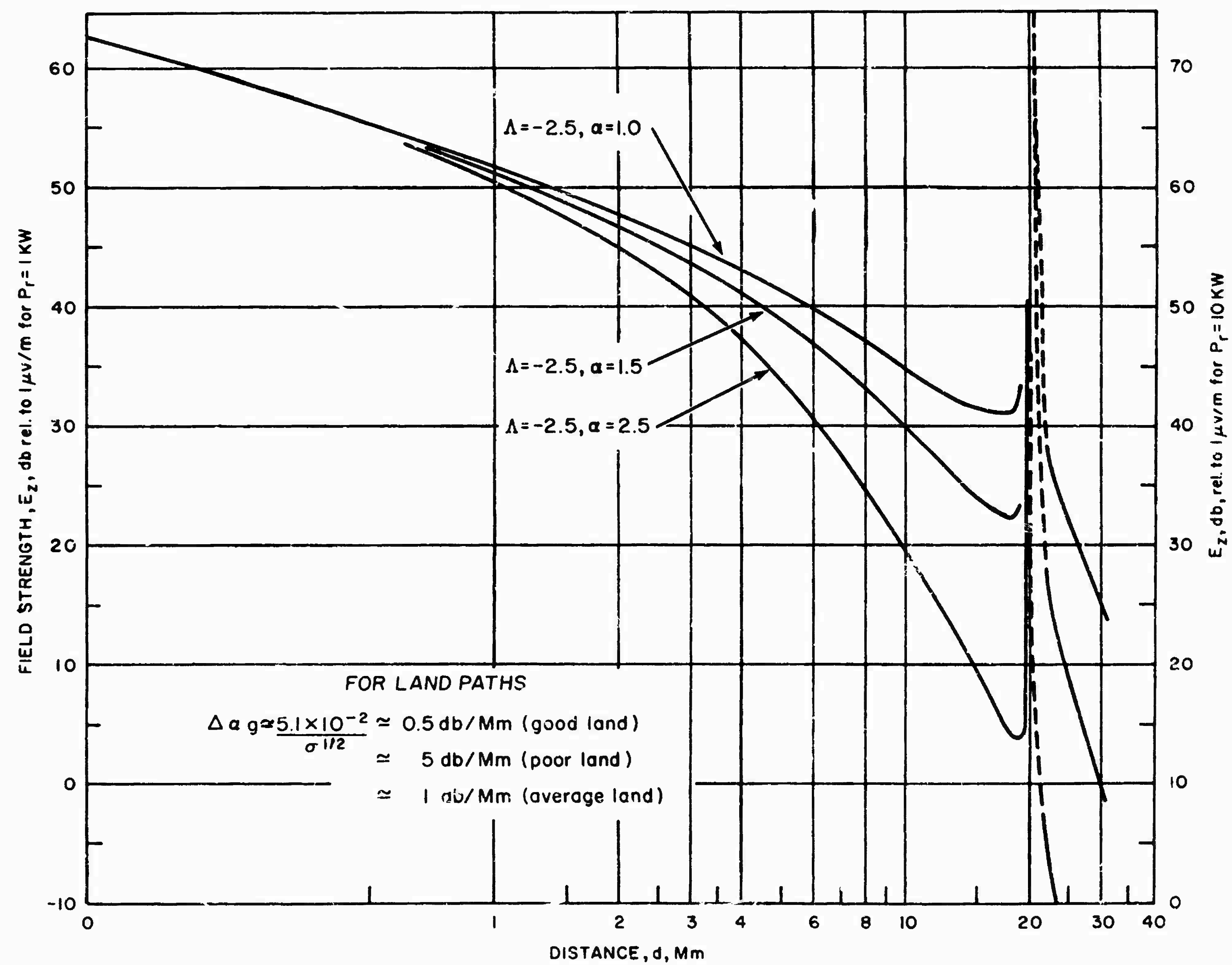


FIGURE 3.1-4. NIGHTTIME FIELD STRENGTH VERSUS DISTANCE AT 10.2 KC

Our present knowledge of path phase velocity at VLF has been attained by both experimental and theoretical investigations. Observations of the propagation path phase variations as a function of time over various length paths lead to the choice of a model in which the velocity is effected by two primary types of influences. The first is the type of effect which tends to be uniform or at least be directly additive over the path. Such variations are caused by changes in ionospheric height which are either uniform or progress uniformly along the path. The second type of effect is one which is not uniform along the path, such as patches of ionization which could result from meteor shower activity or turbulence in the ionosphere. The manner in which each of these types of path variations influences the overall path phase will be considered later.

The phase velocity at any point, y, along the path is

$$v_p(y) = \frac{2\pi f}{d\phi/dy} \qquad (3.2\text{-}1)$$

where $d\phi/dy$ is the phase change in radians per meter along the path. From the preceding equation, it follows that the total phase shift along a path from 0 to d is

$$\Phi = 2\pi f \int_0^d \frac{dy}{v_p(y)} \qquad (3.2\text{-}2)$$

and if $v_p(y)$ is constant over the path

$$\Phi = 2\pi fd/v_p \qquad (3.2\text{-}3)$$

When the total phase delay over the path is known, the average effective velocity along the path is defined as

$$v_{p,ave} = 2\pi df/\Phi \qquad (3.2\text{-}4)$$

It is conceptually possible to solve for the average phase velocity, $v_{p, ave}$ by counting the number of radians of phase difference along the path. This could be accomplished by carrying a stable reference oscillator over a distance d and observing the total phase difference Φ. This experiment has in fact been performed by Reder (1963). Unfortunately, the problem of precise distance measurement and position location of the aircraft coupled with frequency standard problems in the aircraft has made it difficult to draw definite conclusions relative to a precise value of $v_{p, ave}$ from this experiment.

The availability of master-slave stations in an experimental Omega system has provided a means of obtaining round trip total path phase delay with a $2\pi n$ phase ambiguity (Casselman, et al 1959). If data are obtained at enough different distances, it is possible to obtain an average value of v_p provided the velocity can be assumed to be relatively constant with distance. This has been done by Pierce and Nath (1961).

An example of the possible geometry involved is shown in Figure 3.2-1. The solid arrows represent effective directions of propagation when H (Haiku) is the master and F (Forestport) is the slave (i.e., F receives the signal from H and rebroadcasts it with zero phase shift). The phase difference at the observing point, 0, is given by the following equation:

$$\Delta \Phi = 2\pi f \left[\frac{d_1}{v_{p_1}} + \frac{d_2}{v_{p_2}} - \frac{d_3}{v_{p_3}} \right] \qquad (3.2\text{-}5)$$

(slave-master)

If the average phase velocities over the three paths are assumed equal; i.e.,

$$v_{p_1} = v_{p_2} = v_{p_3} = v_p \qquad (3.2\text{-}6)$$

we obtain

$$\Delta \Phi = \frac{2\pi f}{v_p} \left[d_1 + d_2 - d_3 \right] \qquad (3.2\text{-}7)$$

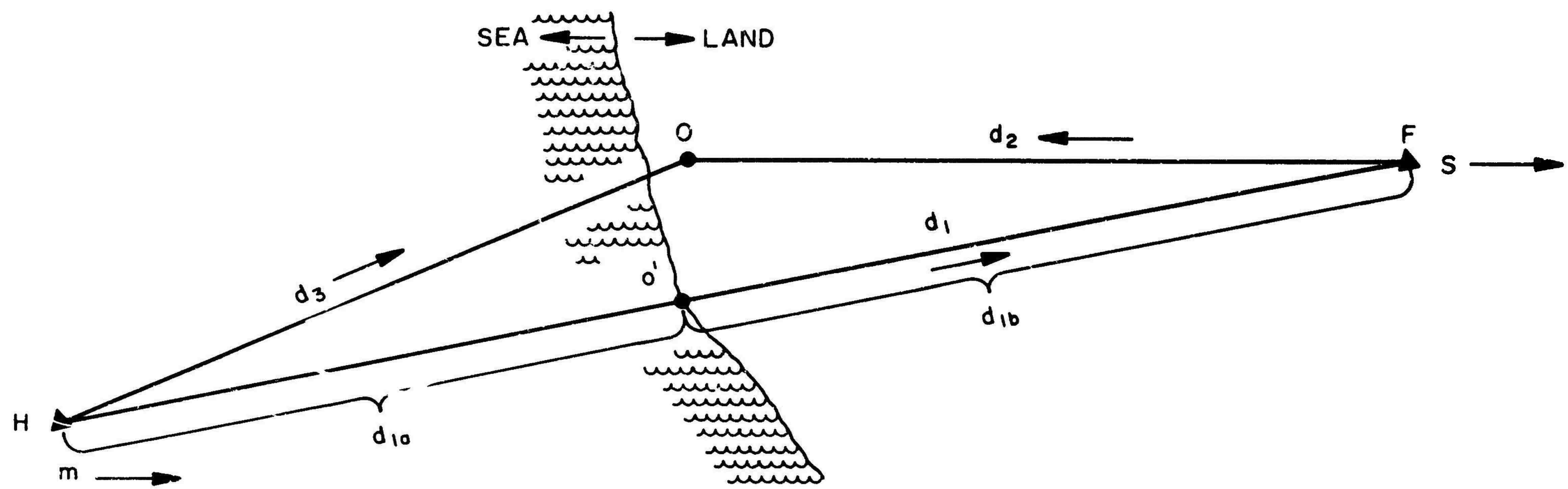


FIGURE 3.2-1. TYPICAL PATH GEOMETRY SHOWING MASTER, SLAVE, AND OBSERVER POSITIONS FOR PROPAGATION INCLUDING BOTH SEA AND LAND PATHS

If the observer is moved to a point 0' which is located on the baseline such that $d_1 = d_{1a} + d_{1b}$ where $d_3 = d_{1a}$ and $d_2 = d_{1b}$, it is apparent that $(d_1 + d_2 - d_3) = 2d_{1b}$. As a result

$$v_p = 2\pi f \frac{d_{1b}}{\Delta \Phi/2}$$

$$= 2 \frac{d_{1b} f}{2\pi n' + \phi} \qquad (3.2\text{-}8)$$

where d_{1b} is the effective path length, f is the frequency in cycles, and ϕ is one-half the actual fractional phase difference observed beyond some unknown number of whole cycles ($2\pi n'$) over the effective path d_{1b}. If n' is chosen as n to yield a nominal $v_p = v_o$ over the path, the possible values of v_p which could result if $n' = n \pm 1$, $n \pm 2$, etc. are shown in Figure 3.2-2.

Each datum point in this experiment is the phase difference between (1) a signal that has passed from a master station to a slave station and thence to an observing point, and (2) a signal from the master station directly to the observing point. In the navigation system, the sum of the transmission times over the first-mentioned paths is taken as positive and the transmission time from the master station is taken as negative.

As there is no identification of cycles, or "lanes," the only experimental observation is, $\Delta\phi$, the fractional part of a carrier period beyond some unknown whole number. The first problem is to deduce the whole number of periods. For convenience, the length of each of the three transmission paths can easily be calculated in free-space velocity-microseconds or, even better, in the number of wavelengths assuming the transmission to be at the velocity of light over the surface of the earth. If this is done, and the algebraic sum is taken, the relative velocity of propagation of phase is simply

$$\frac{v_p}{v_o} = \frac{\text{computed phase difference}}{n + \text{observed phase difference}} \qquad (3.2\text{-}9)$$

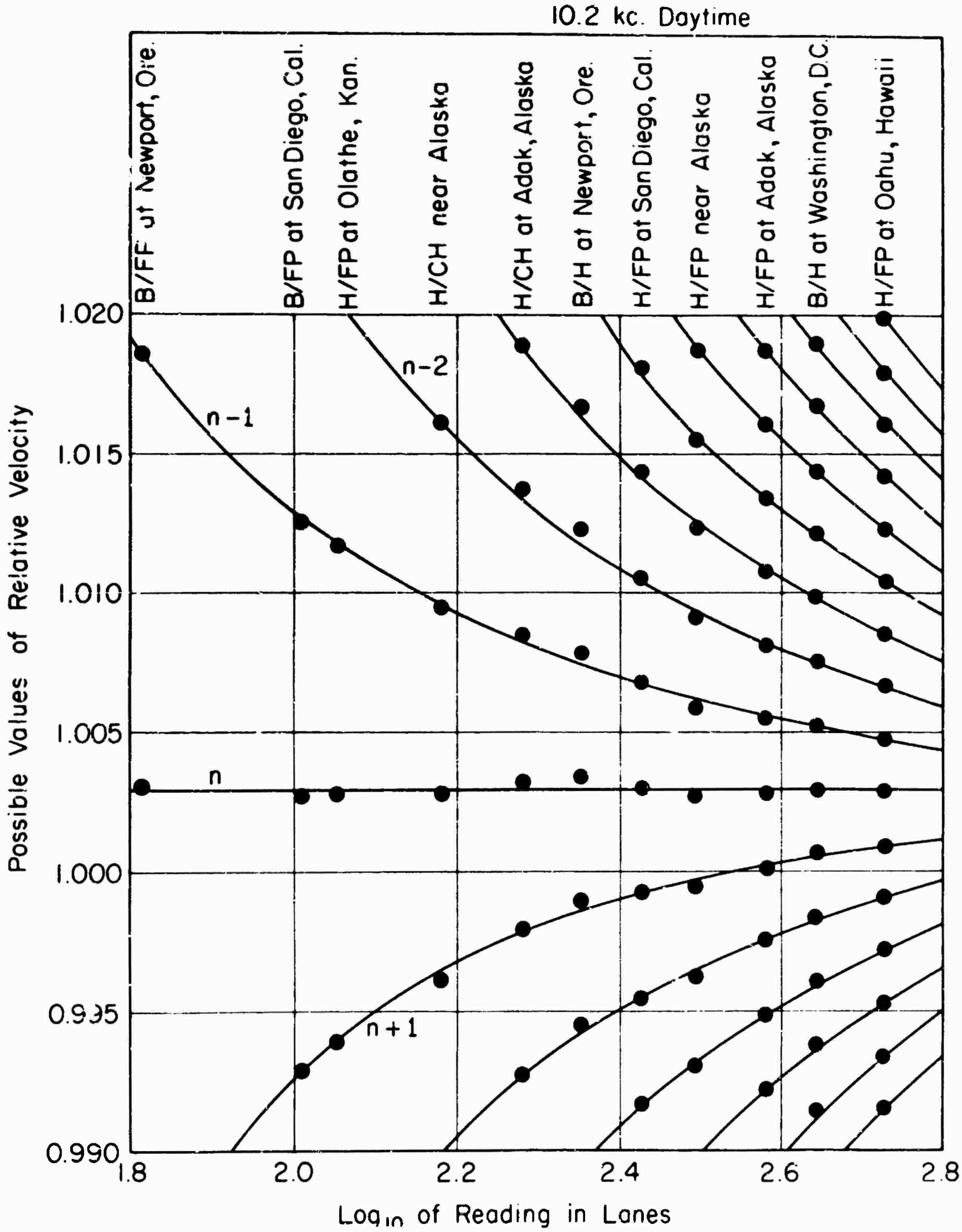


FIGURE 3.2-2. PLOT OF EXPERIMENTAL DATA IN THE FORM OF VARIOUS POSSIBLE VALUES OF RELATIVE VELOCITY IN ORDER TO DETERMINE THE TRUE PHASE VELOCITY FOR DAYTIME PROPAGATION OF 10.2 KC SIGNALS (FROM PIERCE AND NATH)

where v_p is the phase velocity of the signal, v_o is the velocity of light, and n is some integral number. The computed total phase difference in period is, of course, a large number and the observed phase difference is less than unity. In this relation n is not well specified, but it is reasonable to expect that v_p/v_o is of the order of unity and that n must be a whole number nearly equal to the computed phase difference. Repeated observations for a given time of day at a fixed point establish an average value of the observed phase difference and yield a measure of the path phase stability. This alone, however, will not lead to a solution for the velocity.

To solve for the true value of v_p/v_o, we examine a number of possible values of n for each observing site, and compare them with values derived from data taken at other sites. The results are shown in Figure 3.2-2 where each black dot at a given abscissa indicates a value derived for some assumed value of n. At the right side of the figure where the reading is large, there are many non-unreasonable values. As the difference - in -distances grows less, there are fewer and fewer possible solutions.

It is clear from this figure that the family of dots having a nearly constant ordinate at about 1.003 indicates the true value of the relative velocity because only this value is independent of the geometry of the measuring system.

Closer examination of the paths involved in the construction of Figure 3.2-2 reveals the fact that the points lying above the straight line (at $v_p/v_o = 1.003$) are those in which the "effective" transmission is primarily over sea water. The word "effective" in the preceding sentence will require clarification. Assume the example shown in Figure3.2-1 with a master station in Hawaii (H) and a slave station* in the eastern United States (F) at such a position that half of the master-slave path is over water. If now a

*Note, master and slaves, will not be used in this manner in the operational system.

monitor on the west coast at point 0' observes the phase difference. i.e , the slave minus master phase, the reading will be made up as follows:

Path	Water		Land
master to slave:	d_{1a}	+	d_{1b}
+ slave to monitor:		+	d_{1b}
- master to monitor:	$-d_{1a}$		
Total:	0	+	$2d_{1b}$

The effective phase velocity determined from this observation is, therefore, that over land. Had the definitions of the stations been reversed; i.e., F master and H slave, the overland transmissions would have cancelled each other and the observed reading would have been that determined by the average velocity of transmission over sea water.

This concept has been extended in the following way. For each path, the fraction over land is estimated by scaling on a globe, and each computed transmission time (expressed in phase units) is divided into two parts attributable to propagation over land and over water. For each observation (involving three paths) separate quantities, called L, land, and W, water, are derived from the algebraic sum of the individual land and water components. Either L or W may be negative if the negative (master to observing point) part of the hyperbolic sum is the principal land or water path. Thus, a dimensionless quantity L/T, where T is the total algebraic sum, which is the computed reading, becomes a measure of the degree to which the reading is determined by transmission over land. This quantity may be negative or greater than unity.

If the derived velocity ratio for each point of observation (the correct value of n having been determined previously) is now plotted against L/T, we obtain the diagram shown in Figure 3.2-3, where values are given for both daytime and nighttime transmission. The values of v_p/v_o when L/T

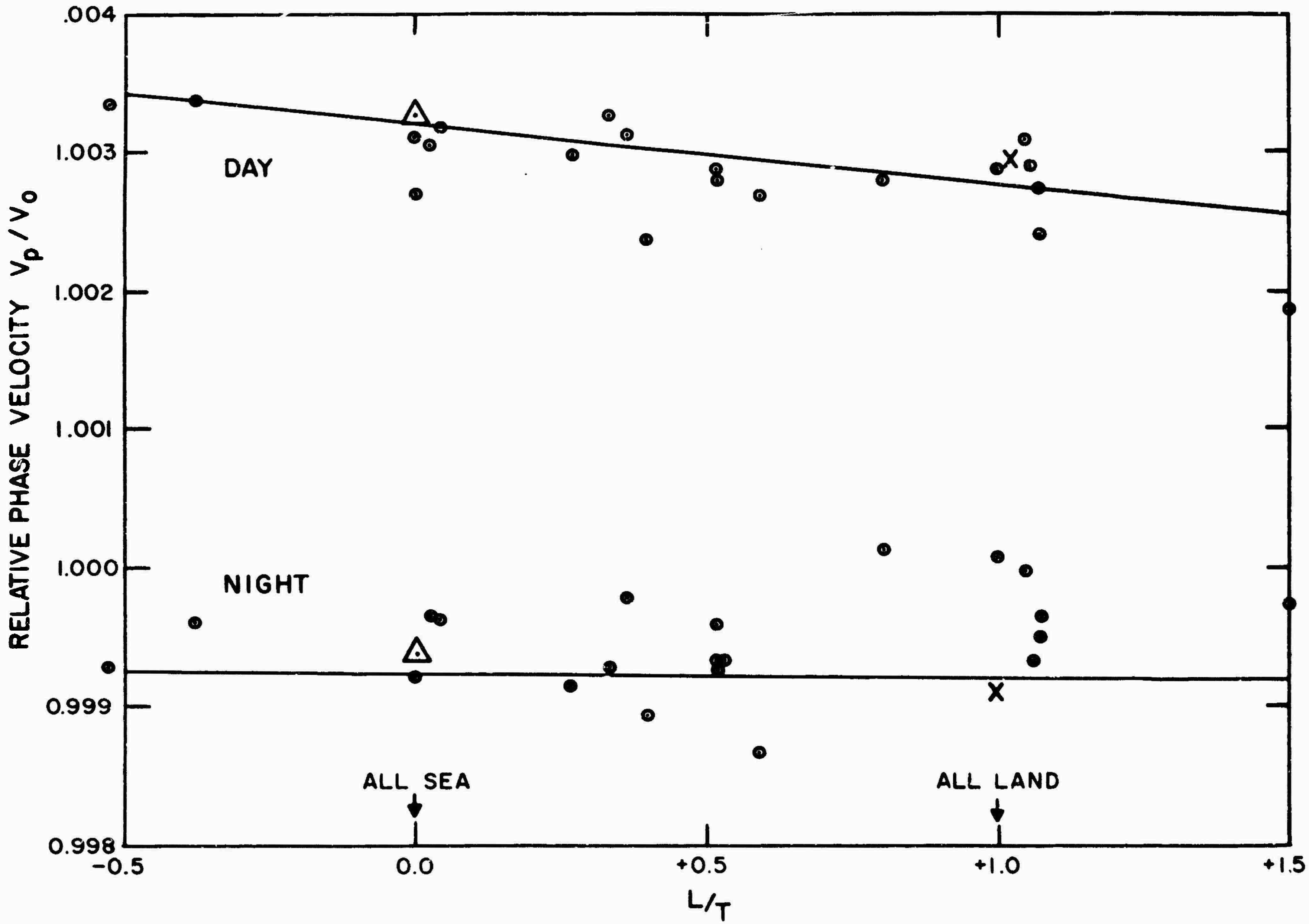


FIGURE 3.2-3. VARIATION OF THE RELATIVE PHASE VELOCITY WITH INCREASING AMOUNTS OF LAND IN THE PROPAGATION PATH (WITHOUT CORRECTIONS FOR DIRECTION OF PROPAGATION)

is zero or unity are the velocities for sea water and land respectively. In the daytime a distinct reduction in velocity over land is evident, while at night the velocity appears to be independent of the amount of land or sea on the path.

Theoretical values of phase velocity that are likely to agree closely with experiment are difficult but not impossible to obtain. If one assumes a flat parallel plate earth with an idealized π radian phase shift at the upper surface and 0 at the lower, the relative phase velocity for the first order mode becomes

$$\frac{v_p}{v_o} \simeq 1 + \frac{v_o^2}{(32 f^2 h^2)} \qquad (3.2\text{-}10)$$

(flat earth approximation)

Since we are primarily interested in the effective phase velocity at the earth's surface, the above relation gives velocities that are too high for an actual spherical earth. A detailed treatment of the whole subject is given by Wait (1962) and also Spies and Wait (1961). To the best of our present knowledge, an exact solution requires approximate consideration of the earth's curvature, effective ionospheric height, and effective phase shifts (at mode resonance) for both the ionosphere and earth surface reflection coefficients. Exact solutions for the effective ionosphere reflection coefficient phase shift have only recently become available (Wait and Walters, 1964). The results of this work have not been included as yet in an exact solution of the spherical earth case. We will show, however, how they indicate an expected faster phase velocity for propagation to the west relative to that for propagation to the east.

Theoretical values of phase velocity are shown in Figure 3.2-4 for a perfectly conducting earth and a sharply bounded isotropic ionosphere. The experimental values quoted previously from Pierce and Nath are plotted on this figure. They are found to fall slightly above the expected theoretical

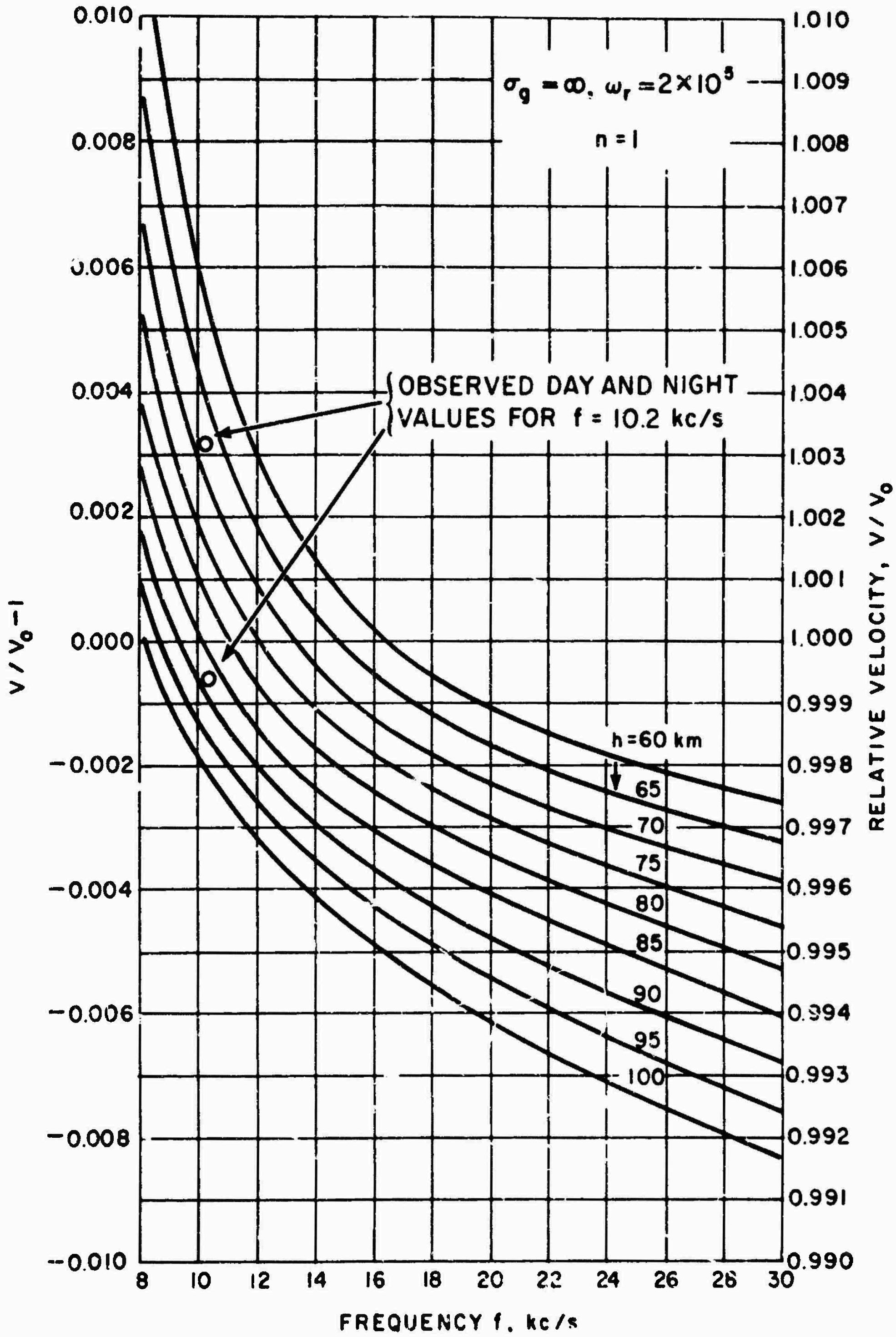


FIGURE 3.2-4. THEORETICAL VALUES OF RELATIVE PHASE VELOCITY FROM WAIT (1962) SHOWING RELATIVE PHASE VELOCITY AS A FUNCTION OF FREQUENCY FOR VARIOUS EFFECTIVE IONOSPHERIC HEIGHTS. EXPERIMENTAL DAY AND NIGHT VALUES FROM THE PRECEDING FIGURE FOR OVER SEA WATER PROPAGATION ARE ALSO SHOWN AT 10.2 KC

curves of h = 70 (day) and h = 90 (night). A rather good fit to both theoretical and experimental phase velocity values can be obtained by modifying the flat earth formula in a semiempirical manner (Watt 1964). The resulting equation is

$$\frac{v_p}{v_o} = 1 - 0.36\, h/a + \left[(2\pi n - \phi_g - \phi_i)\frac{v_o}{4\pi\sqrt{2}\, fh}\right]^2 \qquad (3.2\text{-}11)$$

where h is the effective ionospheric height nominally 7×10^4 meters day, 9×10^4 meters night, a is the earth's radius $\approx 6.4 \times 10^6$ meters, ϕ_g and ϕ_i are the respective ground and ionosphere reflection coefficient phase lags in radians, and f is the operating frequency in cps. The empirical constant 0.36 has been chosen to yield a good fit between the approximate and the exact formulas over the 10-20 kc frequency region. For lower frequencies where the guide cutoff is approached, this constant is expected to approach a value of 0.5. The primary variables in this equation, which cause the phase velocity over a given path to vary as a function of time, are the effective ionospheric height h and the phase lag on reflection ϕ_i, which is a function of the conductivity gradient at the lower boundary of the ionosphere.

The conductivity of an ionized medium is well known to be

$$\sigma = \frac{Ne^2 \nu}{m(\nu^2 + \omega^2)} \qquad (3.2\text{-}12)$$

and if $\nu >> \omega$

$$\sigma \approx 2.78 \times 10^{-8} \frac{N}{\nu} \qquad (3.2\text{-}13)$$

where σ is the effective conductivity in mhos/meter, e is the charge of the electron, ν is the electron collisional frequency, m is the electron mass, and ω is the effective carrier frequency in radians per second. It readily becomes apparent that the manner in which electron density and collisional frequency vary within the ionosphere is the primary governing factor on the time variability of the velocity of propagation. The manner in which the electron density and collisional frequency ν vary with height is shown in Figures 3.2-5 and 3.2-6. These parameters can be employed in determining an effective conductivity versus height which is shown in Figure 3.2-7. It is interesting to observe that, although the electron density increases appreciably during a polar cap anomaly or solar flare, some corresponding increase in collision frequency can partially offset the effects of a variation in N as regards the increase in σ the effective conductivity. Based upon the work of Wait and Walters (1963) we can obtain an effective reflection height, h, which occurs at the point where the effective conductivity is equal to $2\omega\epsilon_o$. As a result, we can write the relation

$$\sigma_{z=0} = 1.11 \times 10^{-10} f \tag{3.2-14}$$

where $\sigma_{z=0}$ is the reference conductivity at the effective ionospheric height, and f is the carrier frequency in cps. This latter relation is based upon an exponential conductivity profile which is seen to be rather well approximated in Figure 3.2-7. If the conductivity profile is not a straight line, a best fit straight line should be drawn over the region of a skin depth, and it is expected the effective height will be determined from this line. Figure 3.2-7 illustrates that the effective gradient at night is such that the conductivity increases by a factor of e in a distance of 0.9 km at night and about 2.5 to 3 during the day. From Wait and Walters (1963) it can be shown that the phase lag on reflection at the ionosphere is not a particularly critical function of ionospheric conductivity gradient for frequencies in the region of 10 kc. This is not true at higher frequencies and in fact in the 20 kc region the effective velocity change with gradient is expected to be much greater than at 10 kc.

It is now possible* to calculate anticipated phase velocities with the aid of equation (3.2-11). From equation (3.2-14), $\sigma \simeq 10^{-6}$ at the reference

*It must be emphasized that available ionospheric data are not highly precise and the following calculations are included to indicate expected trends and orders of magnitude.

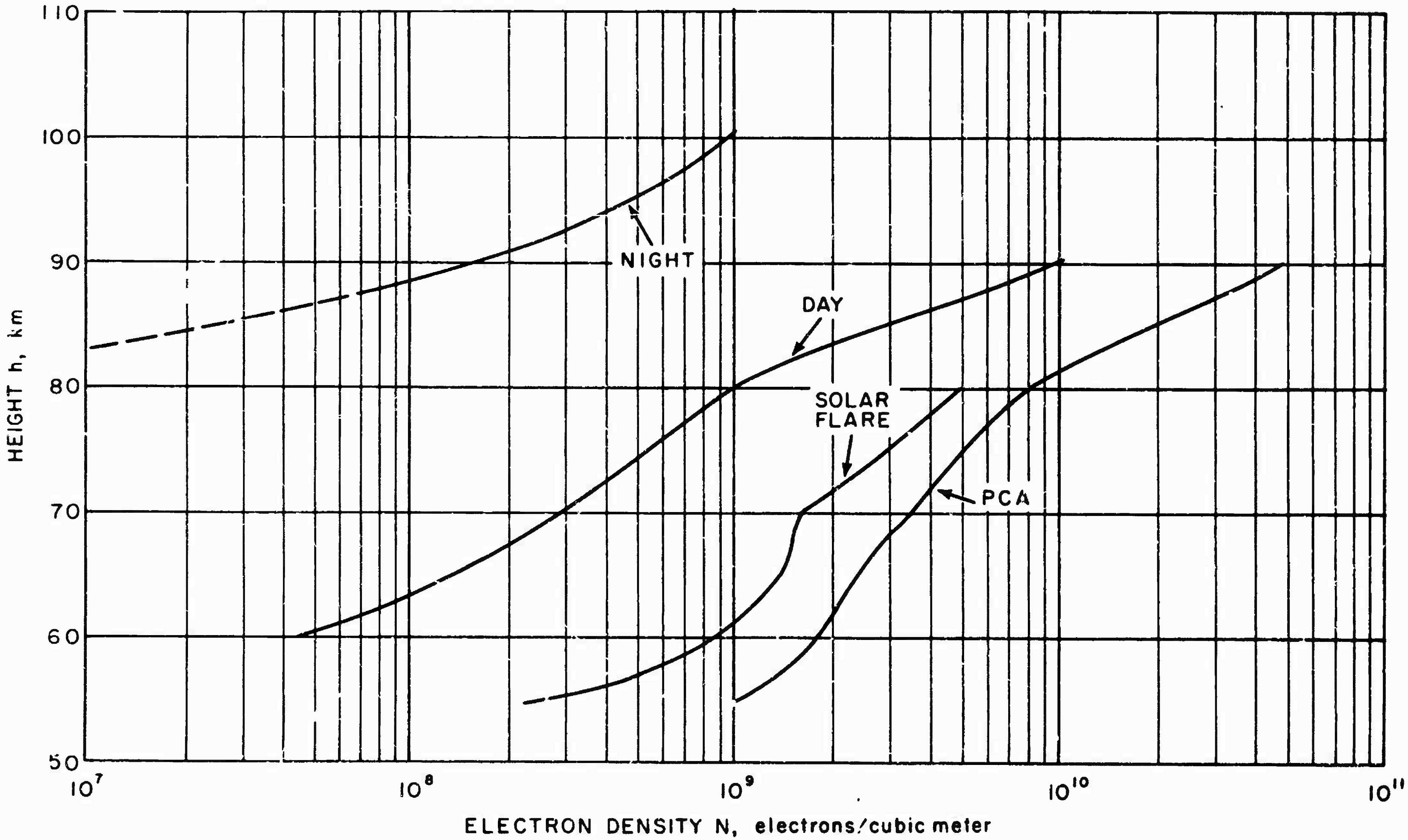


FIGURE 3.2-5. ELECTRON DENSITY VERSUS HEIGHT PROFILES

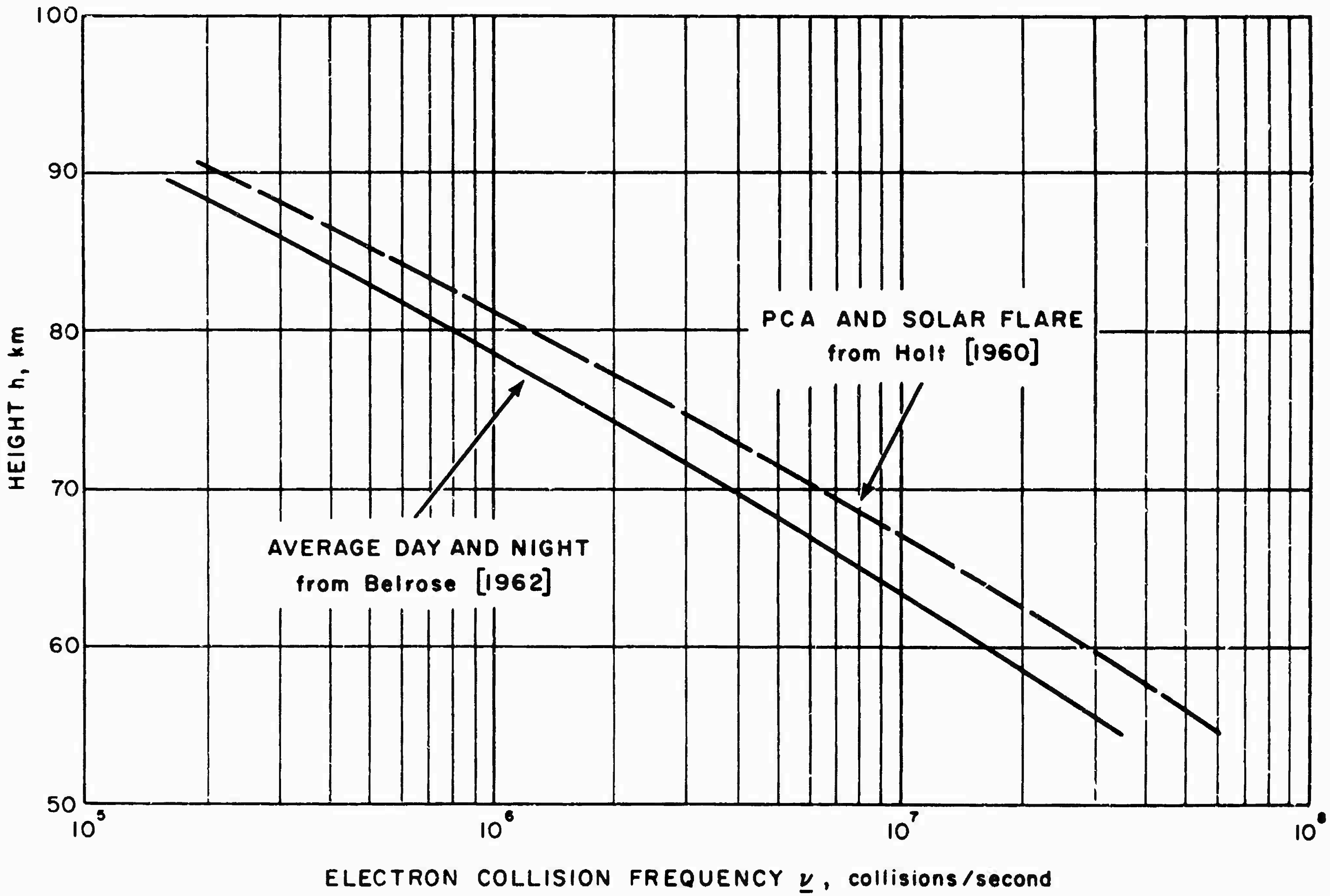


FIGURE 3.2-6. ELECTRON COLLISIONAL FREQUENCY VERSUS HEIGHT PROFILES

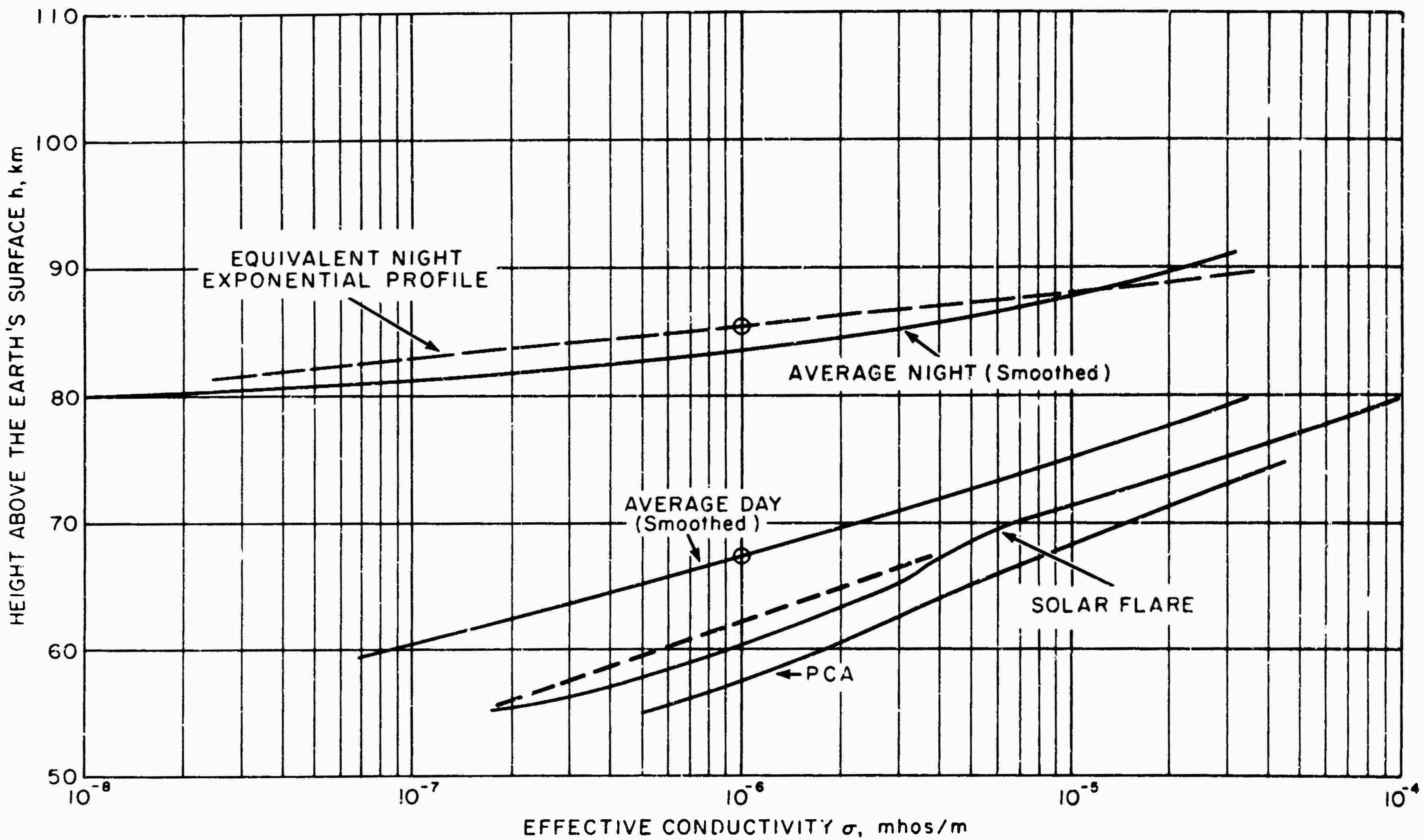


FIGURE 3.2-7. EFFECTIVE IONOSPHERIC CONDUCTIVITY PROFILES

height. From Figure 3.2-7 the height during the day is expected to be about 68 km and 86 at night. The ionospheric phase lag is approximately 160 degrees during the day and may increase to about 165 degrees at night (Wait and Walters 1963). From this information, the anticipated phase velocity assuming $\phi_g = 0$; i.e., sea water conditions, is found to be 1.0034 for day and 0.9994 at night. These values, which are shown as the triangular data points on Figure 3.2-3, are in fairly good agreement with the experimental velocities shown earlier.

From Wait and Walters (1964), the phase shift ϕ_i for daytime propagation is expected to be 165 degrees to the east and 155 degrees to the west. The resultant daytime sea water path relative phase velocities expected are 1.0030 to the east and 1.0037 to the west. It is well known that the electron density versus height profiles can vary some from day to day and particularly from night to night. As a result, there can be an anticipated shift in effective phase velocity for a given path depending upon ionospheric conditions. The magnitude of this effective change in velocity is, of course, one of the crucial limitations to the accuracy of radio navigation systems such as Omega.

The anticipated effects of finite ground conductivity can be determined with the aid of the phase velocity equation. For mode resonance at 10 kc, the effective launch angles are about six degrees day and five degrees night. The resultant phase lags on reflection from the earth's surface when $\sigma \approx 10^{-2}$ mhos/m are from Terman (1943) $\phi_g = 0.09$ radians day and 0.13 radians night. The resultant overland relative phase velocities are 1.0030 day and 0.9991 night. These values are plotted as x's in Figure 3.2-3. Expected phase velocities during SID's (sudden ionospheric disturbances) and PCA's (polar cap anomalies) can be calculated for the conductivity profiles shown. In general, an appreciable increase in phase velocity can be anticipated during such conditions. From Figure 3.2-7, it appears that the effective height during a SID may drop to 62 km. The resulting relative velocity from Figure 3.2-4 is seen to increase to about 1.005 during such conditions.

3.3 Observed Short-Term Phase Variations

Observations of actual one-way path effective phase velocities-versus-time are almost nonexistent. It is possible, however, to examine phase records both from the experimental Omega system as well as those obtained from transmitters with highly stable oscillators in order to come up with an effective stability of phase velocity.

For effective changes in height over the path, which are expected to be correlated over the whole path, the resulting phase change would be proportional to the distance of the path. Such a change occurs when the path goes from all daylight to all night conditions. Under these circumstances, the total phase shift expressed in phase time can be written as

$$\Delta T = (d/v_n - d/v_d) \qquad (3.3\text{-}1)$$

or

$$\Delta T/d = \frac{1}{v_o} \times (v_o/v_n - v_o/v_d) \qquad (3.3\text{-}2)$$

where $\Delta T/d$ is the day-to-night phase shift in seconds per meter, v_n and v_d are the respective night and day phase velocities. At 10 kc this is approximately 12.3 microseconds per megameter. Any ionizing effect such as solar flare, etc. which is felt over the whole propagation path will introduce an effective phase change of this type whose magnitude is proportional to distance.

Random variations occurring along the path are expected to have a somewhat different effect. If the propagation path could be considered as a series of wave guides connected end to end, it would be expected that the phase variations would be proportional to the square root of distance. When the effective transmission path is considered in terms of a Fresnel zone or the spreading around a spherical earth, it can be seen that as the path length increases its width also increases, which is similar to adding a number of parallel paths. The resultant phase fluctuation, for the case where equivalent parallel and independent paths are added, is equal to $1/\sqrt{n}$. Since n increases with path length, it is expected that the two terms will have a somewhat cancelling effect provided the scale size of the ionosphere roughness variations are small compared to both the path length and Freznel zone width.

If fading results from motion of the ionosphere relative to the earth's surface, the scale length, λ_i, can be found from the relation

$$\lambda_i \approx v/f \approx v\,T \qquad (3.3\text{-}3)$$

where v is the velocity of the ionosphere medium, f is the fade rate and T is the period of fading. From Appleman (1964) a velocity of 50 meters per second appears typical, and if T = 2000 seconds, $\lambda_i \approx 100$ km. On the other hand, if the variations are spread as shock waves, $v \approx 300$ meters per second and $\lambda_i \approx 600$ km.

The width of a Freznel zone over a spherical earth is well approximated by the relation

$$\cos\,(W/2a) = \frac{\cos\,(\lambda/4a + d/2a)}{\cos\,(d/2a)} \qquad (3.3\text{-}4)$$

and if $d < a$

$$W \approx (\lambda d)^{1/2} \left[1 + d^2/16a^2\right] \qquad (3.3\text{-}5)$$

For example, if the path lengths are 2, 5, and 10 Mm, the corresponding widths are 244, 400, and 620 km for a frequency of 10 kc.

When the path width is small compared to the ionosphere scale length; i.e., $W < \lambda_i$, the random phase variations are expected to vary as

$$\sigma(\phi) \approx k\,d^{1/2} \qquad (3.3\text{-}6)$$

If $W > \lambda_i$ we can expect

$$\sigma(\phi) \approx k^1 \; d^{1/2} \; W^{-1/2}$$

$$\approx k^1 \; d^{1/4} \qquad (3.3\text{-}7)$$

Observations of received phase compared with a stable local oscillator over numerous paths have been made by a number of individuals, and some of these data from the 10 to 20 kc frequency range (normalized by the relation $\phi = 10/f$, where f is in kilocycles), are shown in Figure 3.3-1. The data show a variation in $\sigma(T_p)$ with distance, which is well approximated over the frequency range from 10 to 30 kc

$$\sigma(T_p) \approx d^{1/4} \, . \, 10^4 . f^{-1} \text{ (day)} \qquad (3.3\text{-}8)$$

$$\sigma(T_p) \approx 2.4 \, d^{1/4} . \, 10^4 . f^{-1} \text{ (night)} \qquad (3.3\text{-}9)$$

where $\sigma(T_p)$ is in microseconds and d is in megameters.

3.4 Effects of Velocity Changes on Expected Fix Accuracy

In a hyperbolic type navigation system the observable phase difference from equation (3.2-7) can be written as

$$\Delta\Phi = \frac{2\pi f}{v_p}(d_1 + d_2 - d_3) + \phi_{cd} \qquad (3.4\text{-}1)$$

where ϕ_{cd} is the coding delay phase at the slave. Through the use of a monitor on the baseline extension from the master, it is possible to adjust ϕ_{cd} such that $\frac{2\pi f \, d_1}{v_p} + \phi_{cd}$ is a constant in spite of changes in v_p. This maintains the hyperbolic grid, Figure 3.4-1, in a fixed position at the baseline path center; i.e., the $\Delta\Phi = 0$ line is fixed. This can also be accomplished by employing very stable oscillators at T_1 and T_2 such that

$$\Delta\Phi = \frac{2\pi f}{v_p} \; (d_2 - d_3) \qquad (3.4\text{-}2)$$

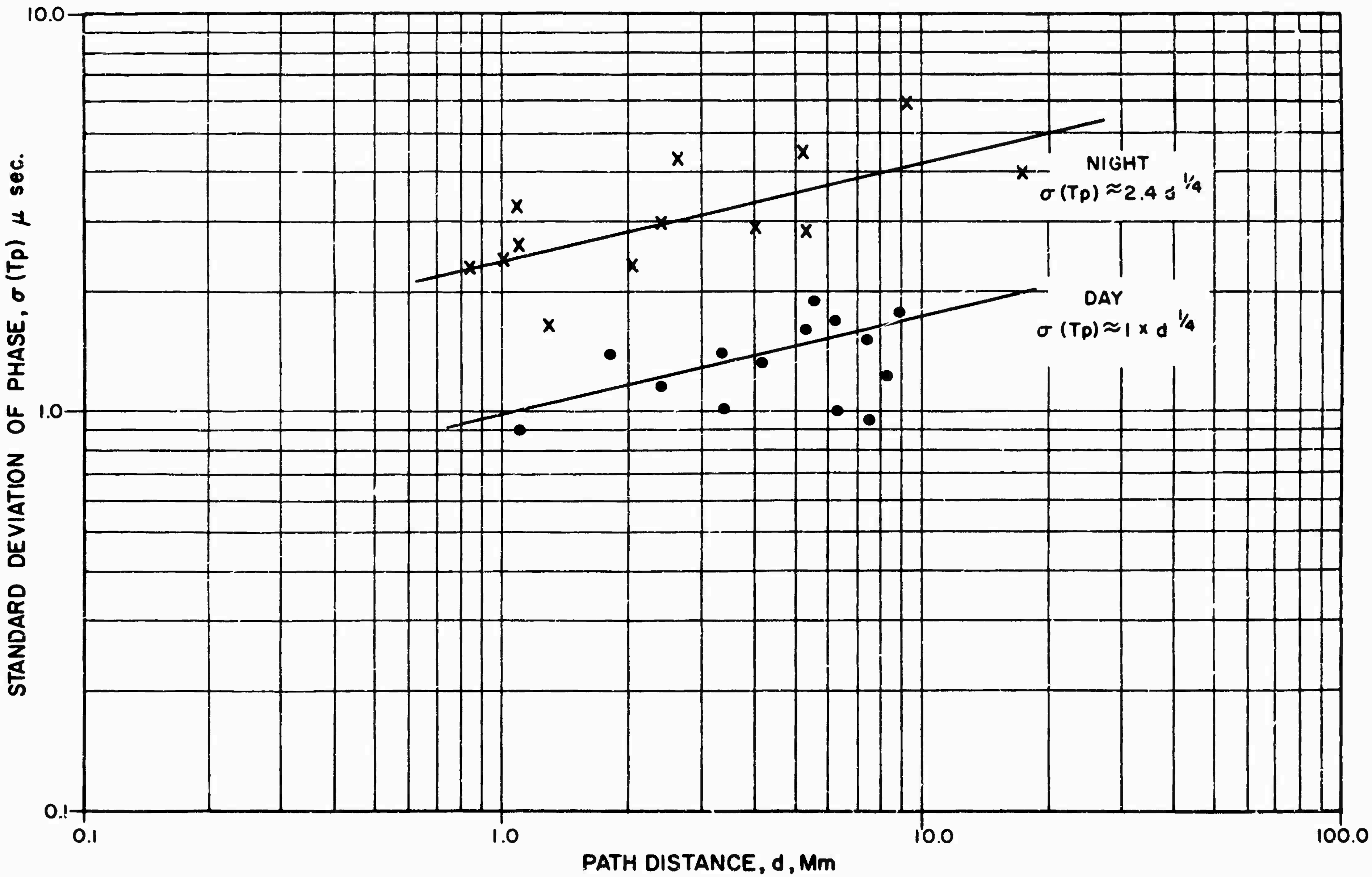


FIGURE 3.3-1. PATH PHASE STABILITY (SHORT-TERM RANDOM FLUCTUATIONS) AS A FUNCTION OF PATH DISTANCE. OBSERVATIONS 10 TO 20 KC NORMALIZED TO 10 KC

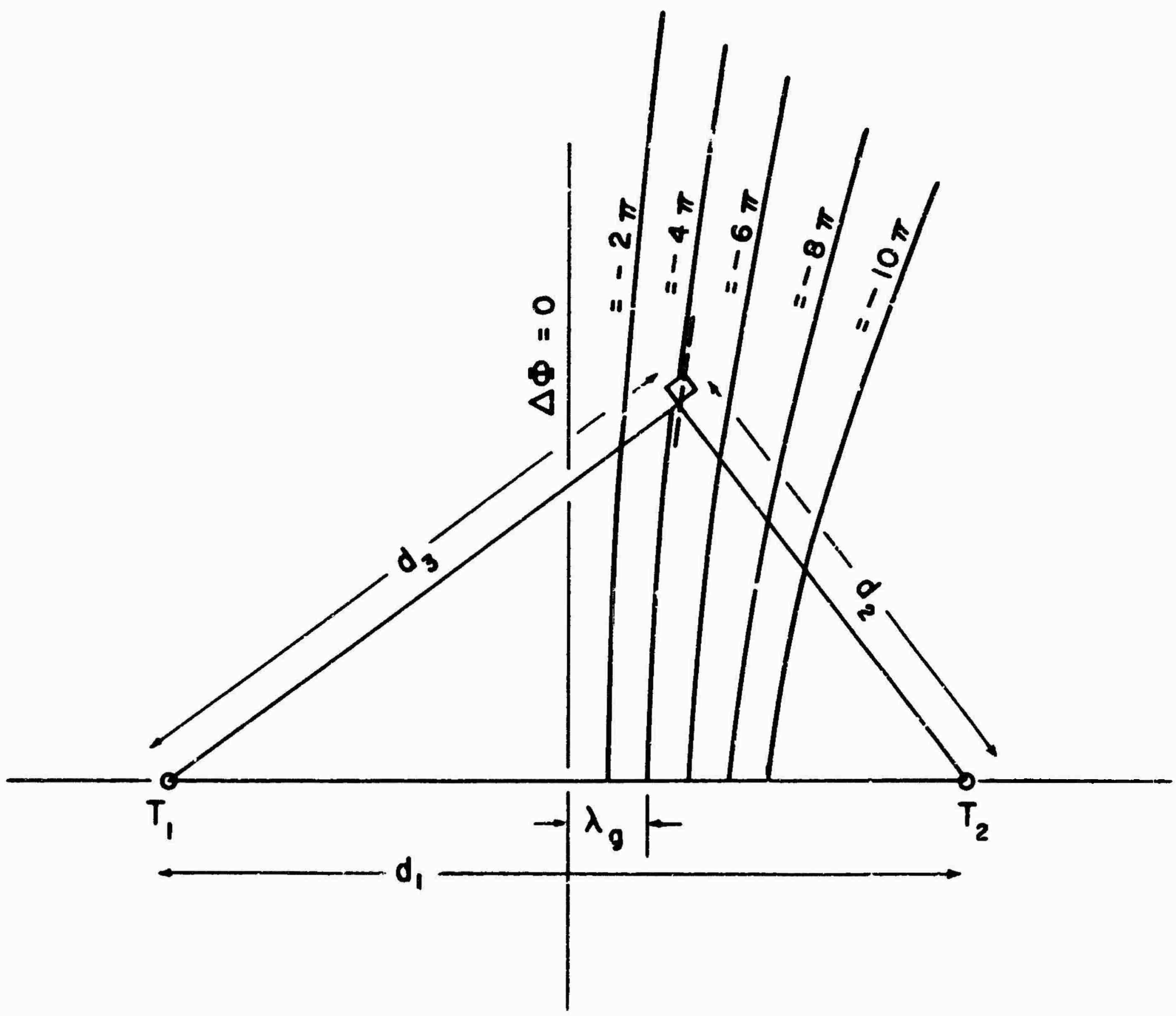


FIGURE 3.4-1. HYPERBOLIC GEOMETRY SHOWING ERROR IN LINE OF POSITION CAUSED BY VELOCITY ERROR

For points equally distant from the stations, it is apparent that a uniform change of velocity over all the paths will produce no error in establishing the $\Delta\Phi = 0$ line of position.

For points not equidistant from the transmitting station, if the velocity of propagation is increased by a factor $(1 + \delta)$, phase reading $\Delta\phi$ decreases in absolute magnitude. The net result is that $\Delta\Phi$ becomes a somewhat greater negative number as shown in the figure and the apparent line of position is moved to the right as shown by the dashed line.

Calibration factors obviously can be applied for the cases where phase velocities change by a known amount such as the normal day to night change. In addition, where some parts of the path are day and others night, a suitable calibration can be applied. Fortunately the residual difference in night to night velocities will have a small influence in fix accuracy for the primary service areas where d_2 and d_3 are comparable in length. The magnitude of error expected due to v_p prediction errors can be determined from the following example. A long baseline of $d_1 = 10,000$ km is assumed and an observing site is chosen near to one transmitter so $d_2 - d_3 = 4,000$ km. From Figure 3.2-3, an estimated daytime standard deviation of average r‹ ‹ive velocity is $\delta = 0.00025$. The resulting standard deviation of the line of position is

$$\Delta d = \delta (d_2 - d_3)$$

$$\approx 2.5 \times 10^{-4} \times 4 \times 10^3$$

$$\approx 1 \text{ km} \qquad (3.4\text{-}3)$$

More recent work which includes corrections for directional effects indicates that standard deviations of $\delta \approx 0.0001$ are possible. In this case $\Delta d \approx 0.4$ km. Night deviations are expected to be about twice the day values. It should be emphasized that in most of the service area $d_2 - d_3$ will be much smaller with a resulting reduction in Δd.

It is important to note that Figure 3.4-1 is the grid system for a flat earth or one where the station spacing is small compared to the earth's radius. Fortunately, in the Omega system, the base lines are usually greater than the earth's radius with a very great reduction in line divergence. It can readily be seen, for example, that, if stations are placed at antipodal locations, the equal phase lines would be parallel throughout. In view of this, distance errors computed for baseline conditions will be applicable over most of the service area.

For a very severe SID where the relative velocity may become as great as $v_p/v_o \approx 1.005$ compared with a normal day value of $v_p/v_o \approx 1.0034$, δ becomes 0.0016 and Δd is about 6 km in the unfavorable part of the service area. On the average, most SID's produce values of v_p/v_o of about 1.0039 with the result that Δd is about 2 km in the worst part of the service area. The expected fix errors will be dependent on position line crossing angles. In general, the fix errors will be about 1.5 times the line of position errors.

3.5 Effects of Random Phase Fluctuations on Expected Fix Accuracy

In this case, the phase variations over the propagation path are independent; as a result the standard deviation of phase difference is $\sigma_{\Delta\phi} = (\sigma_{\phi_1}^2 + \sigma_{\phi_2}^2)^{1/2}$. For locations near the baseline, a change in $\Delta\Phi$ of 2π radians corresponds to a change of $\lambda/2$. From Figure 3.3-1, rather long (10 Mm) paths during the day $\sigma_\phi \approx 2$ microseconds and $\sigma_{\Delta\Phi} \approx 2.8$ microseconds or about three percent of a cycle at 10.2 kc. Since there are two cycles per wavelength, the standard deviation of the line of position is expected to be 0.014λ; i.e., 0.4 km. For locations remote from the baseline, a lane width (Section 2.2.1) increases as the cosecant of half the angle subtended by the stations. For most cases, stations will subtend at least 120 degrees in which case the lanes are spread by about 1.16. Normal crossing angles may introduce a similar increase; i.e., $1/\sin\theta$ equals 1.16 For uncorrelated paths (separate pairs of stations) fix error is

$$d_{rms} = \frac{1}{\sin\theta}\sqrt{d_1^2 + d_2^2} \qquad (3.5\text{-}1)$$

and if $\theta = 120$ degrees, $d_{rms} = 1.16 \times \sqrt{2} \times 1.16 \times 0.4$ km or a fix error of about 0.75 km during the day. During the night when $\sigma_{\phi} \approx 4.2$ micro-seconds the resulting fix uncertainty from random fluctuations is about 1.7 km. At night both average velocity uncertainties and random phase errors can combine to cause fix errors in the order of 1.5 to 2 km.

3.6 References

1. C.J. Casselman, D.P. Heritage, and M.L. Tibbals, "VLF Propagation Measurements for the Radux-Omega Navigation System," Proc. IRE, Vol. 47, No. 5, May 1959, pp. 829-839.

2. J.A. Pierce, Private Communication, 1962.

3. J.A. Pierce, and S.C. Nath, "Very Low Frequency Propagation," Annual Progress Report No. 60, pp. 1-6, Cruft Laboratory, Harvard University, 1961.

4. F. Reder, P. Brown, G. Winkler, and C. Bickart, "Final Results of a World-Wide Clock Synchronization Experiment (Project WOSAC)" (to be published in Jour. of Research, NBS, 1963 VLF Symposium Issue - January-February 1964).

5. K.P. Spies, and J.R. Wait, "Mode Calculations for VLF Propagation in the Earth-Ionosphere Waveguide, "NBS Tech. Note No. 114, July 17, 1961.

6. F.E. Terman, Radio Engineers Handbook, McGraw Hill Book Co., 1943.

7. J.R. Wait, Electromagnetic Waves in Stratified Media, Pergamon Press, 1962, p. 199.

8. J.R. Wait, and L.C. Walters, "Reflection of VLF Radio Waves from an Inhomogemeous Ionosphere." Part I. Exponentially Varying Isotropic Model, Jour. of Research, NBS, D, Radio Propagation, Vol. 67D. No. 3, May-June 1963, pp. 361-367.

9. A.D. Watt, and R.D. Croghan, "Comparison of Observed VLF Attenuation Rates and Excitation Factors with Theory," (to be publishied in Jour. of Research, NBS, 1963 VLF Symposium issue - January - February 1964).

10. D.D. Crombie, "Periodic Fading of VLF Signals Received Over Long Paths During Sunrise and Sunset". Radio Sci., J. Res. NBS 68D No. 27-35.

11. E. Barar, and J.R. Wait, "Microwave Model Techniques to Study VLF Radio Propagation in the Earth-Ionosphere Waveguide," Quasi-Optics, ed. J. Fox, 447-464 (Polytechnic Press, Polytechnic Inst. of Brooklyn, N.Y.)

3.6.1 Additional References

1. J. K. Hargreaves, "Random Fluctuations in Very Low Frequency Signals Reflected Obliquely from the Ionosphere," Jour. Atmospheric and Terrestrial Physics, Vol. 20, pp. 155-166, 1961.

2. J. K. Hargreaves, and R. Roberts, "The Propagation of Very Low Frequency Radio Waves Over Distances Up to 2,000 km," Jour. of Atmospheric and Terrestrial Physics, Vol. 24, 1962, pp. 435-450.

3. B. G. Pressey, G. E. Ashwell, and J. K. Hargreaves, "The Phase Variation of Very Low Frequency Waves Propagated over Long Distances," Proc. I.E.E., B 108, 1961, pp. 214-226.

4. H. S. Appleman, "A Preliminary Analysis of Mean Winds to 67 Kilometers," Jour. of Geophysical Research, Vol. 69, No. 6, March 15, 1964, pp. 1027-1031.

4.0 Signal Format

In accordance with the system design considerations set forth in Chapter 2, the overall signal format provided by the combined transmissions of the eight stations in the Omega network should provide a convenient mode of operation for each class of the user within the limitations of economically feasible transmitter power and available frequency spectrum. To accomplish this, the signal format may be arranged as follows.

4.1 Basic Signal - 10.2 Kilocycles

At ten-second intervals, each station of the eight transmits a one-second signal element in turn at 10.2 kilocycles, so that the combined transmissions of all eight stations (as observed at any given point) form a repeating ten-second cycle of eight one-second segments of 10.2 kilocycle continuous wave signal, with segments of differing amplitudes and phase as shown on line 1 of Figure 4.1-1.

The radio frequency phases of all Omega transmissions are phase locked so that the relative phases of all signal components are everywhere stationary, establishing a fixed pattern of intersecting families of isophase contours defining position in hyperbolic coordinates with an ambiguous hyperbolic lane width of 98 microseconds (eight miles on the baselines).

The stations always transmit in the same order, with the length of transmission varying between 0.9, 1.0, 1.1, and 1.2 seconds from station to station in accordance with the Omega station identification code.

Station	A	B	C	D	E	F	G	H
Length of Transmission	1.1	1.2	1.1	0.9	1.2	1.0	0.9	1.0

as described in Section 2.3, so that the successive sequences of eight transmissions form a cyclic length-coded pattern as shown on line one of Figure 4.1-1.

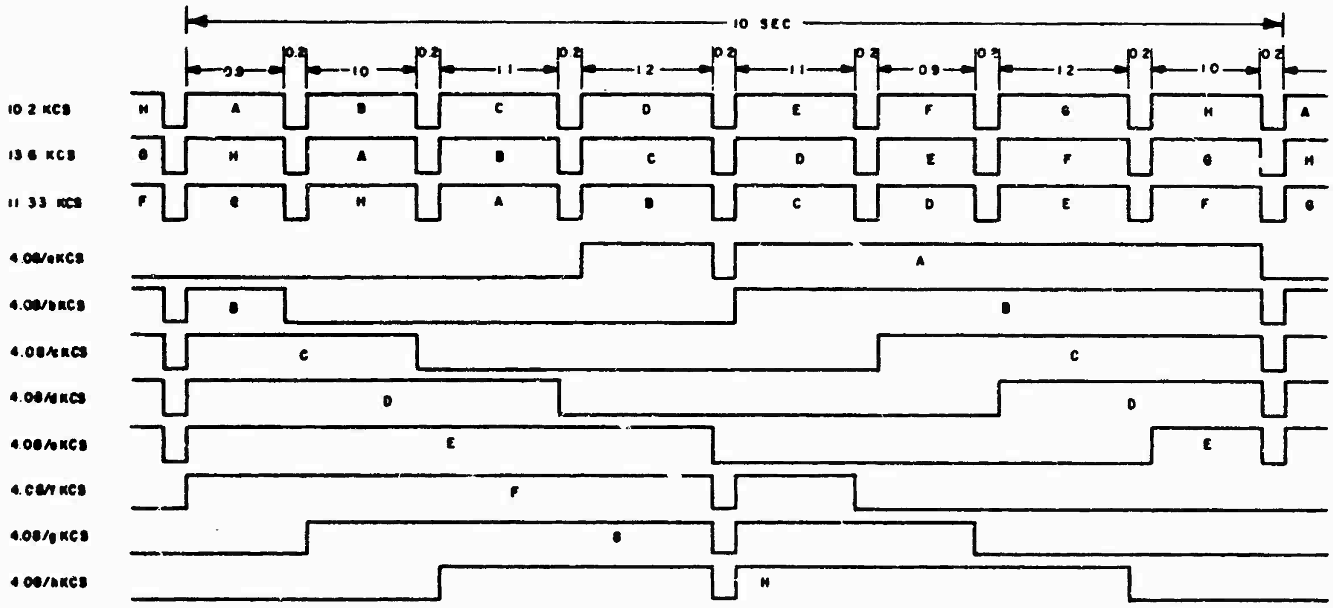


FIG. 4.1-1 OMEGA RECEIVED SIGNAL FORMAT

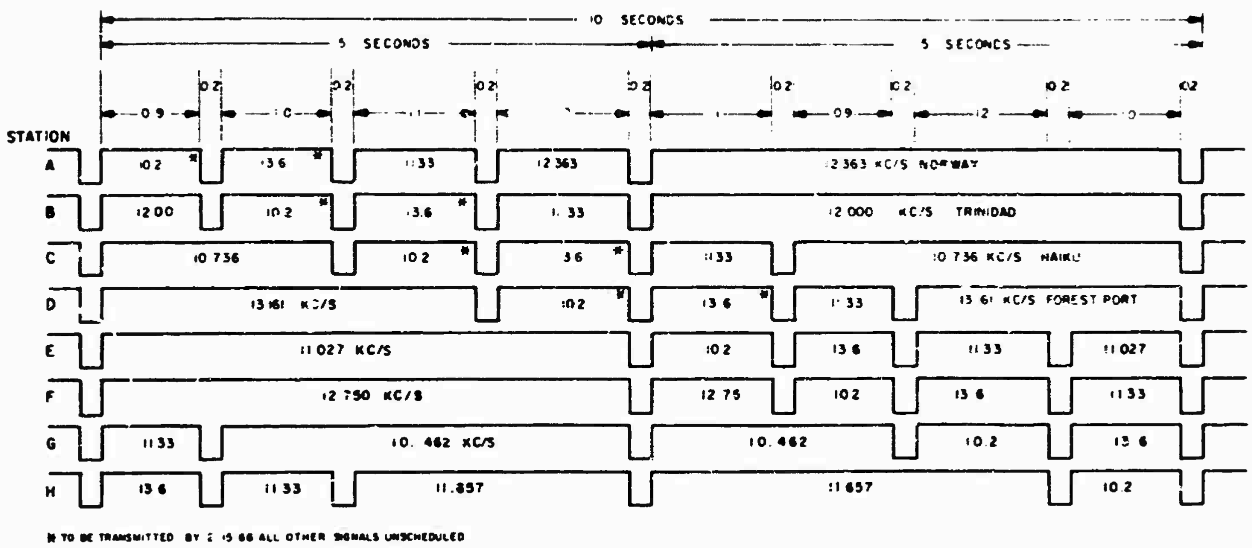


FIG. 4.5-1 OMEGA TRANSMITTED SIGNAL FORMAT

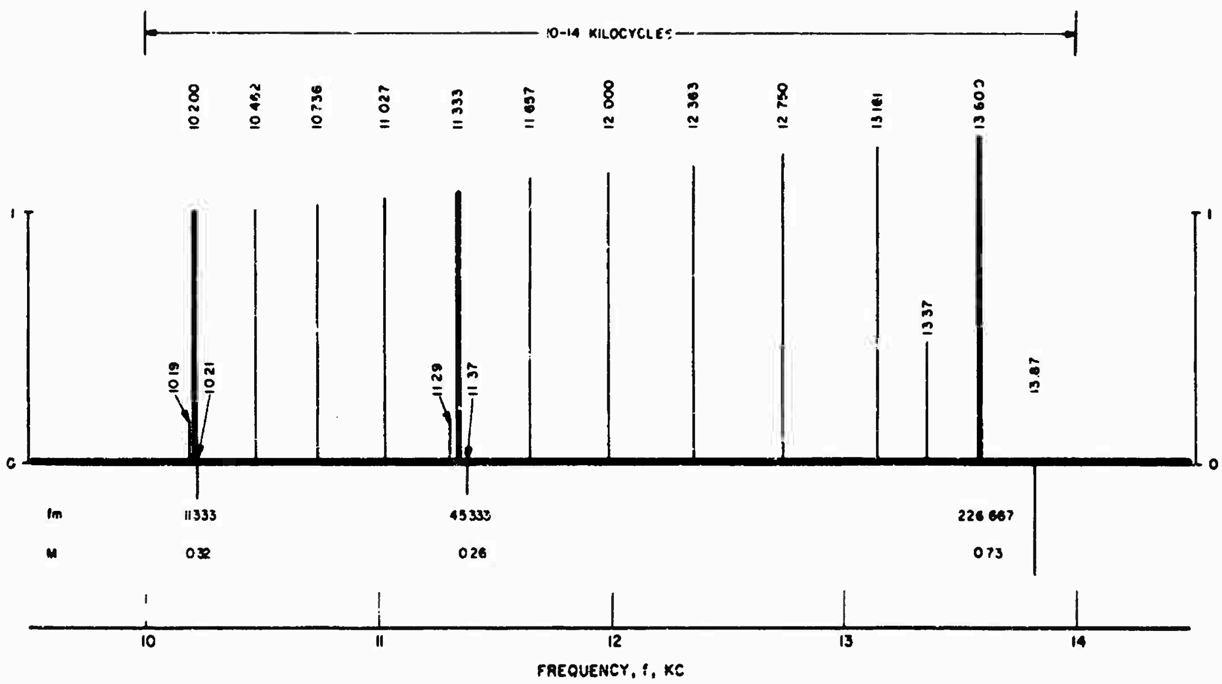


FIG. 4.6-1 OMEGA SYSTEM SIGNAL SPECTRUM

Recognition of the pattern, either visually in an oscilloscope pattern display, or by cross correlating with the related Unit Correlating Function (Section 2.3) establishes the phase of the sequence, from whence the identity of the transmissions of a particular station is established by the time of occurrence in the sequence.

Additionally, the transmissions of a particular one of the stations, Station F, would be timed to begin at integral ten-second epochs of Standard Time, and the transmissions of Station C to begin at odd five-second epochs (5, 15, 25 seconds, etc.) so that the timing of the sequence of transmissions is likewise fixed with respect to Standard Time. Hence, alternatively, station identification could be established or verified by reference to Standard Time to within one second.

This part of the Omega signal format forms a complete radio-locating signal by which position can be defined with respect to the basic Omega position contour pattern useable in:

1. A manually operated receiver with station identification, alignment of the demultiplexing function, and possibly the phase measurement as well, accomplished manually.

2. An Automatic Tracking Receiver, with manual search and multiplex set up, in which the signals are isolated and compared in phase by means of tracking filters providing continuous automatic readout; with manual/visual recognition of the sequence pattern for station identification and multiplex set up, either by recognition of the signal amplitude pattern, recognition of the pulse length coding, or both; or by reference to standard time, in an oscilloscope.

3. An Automatic Search and Tracking Receiver in which an internal correlator recognizes the multiplex sequence phase and aligns the receiver multiplexing functions, from whence tracking filters isolate the desired signals and measure the relative phases of interest, with direct and/or recorded readout as desired.

This part of the signal format, taken alone, provides no means of resolving the lane ambiguity by reference only to the signal.

4.2 Lane Identification - First Stage

In addition to the basic transmissions at 10.2 kilocycles, each station of the eight also transmits a 13.6 kilocycle signal, one segment earlier in the multiplex sequence, as on line two of Figure 4.1-1. The 13.6 kilocycle transmissions are likewise phase locked to Standard Time (and hence to the 10.2 kilocycle signals) so that the combined transmissions of the eight stations at the second frequency establishes a second pattern of isophase contours with three quarters of the lane width of the 10.2 kilocycle pattern. Since both contour patterns are families of hyperbolae about the same points as foci, the two patterns are everywhere parallel, and combine to form a virtual pattern of broad lanes with three times the width of the basic 10.2 kilocycle pattern.

The length coding pattern of the 13.6 kilocycle transmissions is identical to the coding pattern of the 10.2 kilocycle signals, so that the same multiplex timing can be used to separate signals in either channel without realignment with respect to Standard Time. Since each station transmits its two frequencies in adjacent segments of the sequence, the two transmissions of each station are always of different durations.

In a manual/visual receiver, the additional transmissions at 13.6 kilocycles providing the first stage of lane identification would be utilized by retuning the receiver to the second frequency without disturbing the multiplex time alignment and observing the phase relations of the signals occurring one place later in the multiplex sequence than those observed at 10.2 kilocycles.

The differences of the corresponding indications at the two frequencies would then indicate which of the three lanes in a broad lane contained the observer.

In an automatic receiver, reading at the second frequency would be obtained either by switching the phase tracking function of the receiver to the second frequency from time to time as required to maintain lane identification, or by duplicating the phase tracking circuitry so as to provide continuous readout of the phase of the second frequency components, with automatic adjustment of the Lane Count.

4.3 Lane Identification - Second Stage

In addition to the transmissions at 10.2 and 13.6 kilocycles, each station would also transmit a third component at 11.33 kilocycles in the time-multiplex segments preceding its 13.6 kilocycle transmission. These transmissions are likewise phase locked to Standard Time and provide a third stationary isophase contour pattern with a lane width 9/10 of the 10.2 kilocycle pattern, and 6/5 of the 13.6 kilocycle pattern. Coincidence of the three patterns defines a second virtual pattern with a lane width equal to 9 lanes of the 10.2 kilocycle pattern, 10 lanes of the 11.33 kilocycle pattern and 12 lanes of the 13.6 kilocycle pattern.

The differences of the phase indications at the three frequencies, properly combined, then reduce the ambiguity of the basic 10.2 kilocycle pattern by 9, to 72 miles on the base lines.

Utilization of the third frequency signals in a manual/visual receiver would be the same as with second frequency signals, except that observation of the third frequency signals would probably not be made unless there was reason to suspect a gross error in position (greater than ± 12 miles).

In an automatic receiver, additional phase tracking circuitry might be provided for continuous tracking and readout of the third lane identification signal. However, since the probability of error of more than one and a half lanes of the basic lane pattern at 10.2 kilocycles is small, satisfactory observation of the third frequency signals may be obtainable through retuning one of the other tracking systems to the third frequency from time to time, and updating the phase indications to correct the lane count as and if required.

4.4 Lane Identification - Third, Fourth and Fifth Stages

Further expansion of the unambiguous lane width is to be provided by the addition of antisymmetric sideband components to the 13.6 kilocycle transmissions, producing tangential phase modulation of 226-2/3 cycles per second (with an accompanying 453-1/3 cycle amplitude modulation as required to suppress higher order sideband components).

It seems probable that the 13.6 kc carrier with its 226-2/3 cycle side-bands may be taken alone to provide a simple and useful navigational receiver. In this embodiment, the primary information would be the phase of the modulation, which would have ambiguities separated by 360 miles. Present data indicate that the fixes made at the 226-2/3 cycle frequency should have a standard deviation in the neighborhood of five miles, which is an accuracy adequate for many purposes. Users who like this combination of accuracy and freedom from ambiguities need pay no attention to the other components of the Omega spectrum. Measurements of the modulation frequency normally require a long time constant. Because of this it is necessary (except for very slow vehicles) to use carrier phase tracking so that rapid maneuvers can be made without serious delay in following them. The carrier phase in this case does not contribute to the readings. The principles of this implementation are explained in Section 8.5.

The modulation components carried by the 13.6 kilocycle signals are likewise phase locked to Standard Time, establishing a fourth phase contour pattern with a lane width of 45 lanes of the basic 10.2 kilocycle pattern (equivalent to 360 miles on the baseline).

Being phase modulation, the signals can be leveled in the receiver by signal limiting, (without stripping off the modulation) and synchronously demodulated by the same circuitry that performs the phase matching, since the sinusoidal phase modulation will appear as a sinusoidal output at the appropriate frequency in the output of the signal phase detector or signal sampler used in establing the phase of the incoming signal.

Phase modulations similar to that applied to the 13.6 kilocycle signals, but of 45-1/3 and 11-1/3 cycles per second, are also applied respectively to the 11.333 kilocycle and 10.2 kilocycle signals. These modulation components make it possible to obtain further reductions of the lane ambiguity from 45 lanes (360 miles on the baseline) to 225 lanes (1800 miles) at the fourth stage, and 900 lanes (7200 miles) at the fifth stage.

The addition of the more significant modulation components makes it possible to operate a completely unambiguous receiver from the Omega signal alone that resolves the ambiguity on each line of position, independently. Such a receiver would be able to recover unaided from extended loss of signal, or other temporary malfunctioning of the system.

4.5 Side Frequencies

In forming the basic Omega multiplex pattern, each station transmits in three of the eight time segments of the Omega multiplex sequence. In the remaining five segments, each station then transmits a "Side Frequency" component at a different subharmonic of 408 kilocycles between 10 and 14 kilocycles, as shown in Figure 4.5-1 and Table 4-1.

TABLE 4-1. SUBHARMONIC FREQUENCIES

Subharmonic	Frequency-Kc
31	13.161
32	12.750
33	12.363
34	12.000
35	11.651
37	11.027
38	10.736
39	10.462

Being readily separable by frequency domain filtering (adjacent side frequencies differ by 260 to 440 cycles per second), the side frequencies provide an additional method of identifying the transmissions of particular stations, since each transmission is interrupted when the corresponding station is transmitting its portion of the main sequence: (Figure 4.1-1) and synchronizing a receiver multiplex function to the gaps in the side frequency transmission of any one station establishes alignment with the transmissions of all stations in the main sequence.

Being derived from the system reference frequency of 408 kilocycles, the side frequencies are commensurate and provide a stationary phase pattern at the common harmonic frequency. Hence, the side frequencies provide an alternate mode of operation in which the signals are separated by frequency domain filtering instead of time multiplexing, thereby obviating the need for time multiplex synchronization (at the expense of a substantially unresolvable ambiguity). Phase comparison in a frequency multiplex receiver is accomplished by multiplying the frequency of each side frequency component (as received in a separate receiving channel) to the common harmonic frequency of 408 kilocycles and comparing phase (Figure 2.5-1).

One of the side frequencies is 12.000 kilocycles exactly (408/34), a frequency easily divisible to 1000, 60, or 1 cycles per second, etc.; and the timing of the multiplex sequence is to be controlled in such a manner that the start of each 12.000 kilocycle transmission occurs on exact 10-second epochs of Standard Time, thereby providing a world-wide time signal.

In addition to being commensurate so as to provide a stationary phase pattern, the side frequency components are phase modulated by a low data rate balanced code to convey synchronizing information between each station and the Synchronization Control Station and frequency correction data from the Synchronization Control Station to each of the stations.

Thus, the side frequency transmissions provide:

1. An alternate means of identifying the transmissions of a particular station and aligning the multiplexing function by frequency domain filtering instead of time sequence phase alignment

2. An additional mode of system operation, in which the necessity of time multiplex synchronization is avoided at the expense of an essentially unresolvable ambiguity - useful for those operations where a simple automatic receiver is a necessity - and continuous tracking can be assured

3. A convenient set of stable VLF signals giving world-wide coverage with good geometry for one-way ranging with respect to a frequency standard carried on the vehicle (again where continuous tracking of the signal can be assured)

4. Low data rate communication circuits for conveying synchronizing information around the system

5. World-wide time signals

4.6 Signal Spectrum

The over-all spectrum of the Omega signal is thus as shown in Figure 4.6-1, consisting of time-multiplexed transmissions at 13.6, 11.333, and 10.2 kilocycles, respectively phase-modulated with 226-2/3, 45-1/3, and 11-1/3 cycles per second as transmitted in tu from the eight stations (Figure 4.1-1). When not transmitting a component of the basic pattern, each station transmits a side frequency component at a subharmonic of 408 kilocycles (Figures 4.1-1 and 4.5-1).

4.7 System Communication

There will be a continuing need for intercommunication between transmitting stations for mean frequency stabilization and timing calibration purposes. This type of circuit will not need a high information rate. A method that will have high reliability without interfering with the navigational service is required. A satisfactory way will be to phase modulate the station's side frequency at slow speed. Each six-second burst of transmission will have its phase shifted in two halves in balanced amounts, a 45 degree lead followed by a 45 degree lag indicates a mark and a 45 degree lag followed by a 45 degree lead indicates a space. By keeping the phase shifts small compared with 90 degrees and of equal positive and negative duration, their presence or absence would be inconspicuous to any users of the side frequency for navigational purposes, while the signals would be clear and easy to read in the Omega stations.

This simple system has a speed of six bits per minute, so that a few minutes of transmission will convey most necessary messages.

Referring to Section 6.2, it can be seen that, for an average station spacing of 5,000 miles, the median carrier to noise in a one cycle band, C/N_1, will range from 0 to 10 db. Assuming that C/N_1 = 5 db, and an effective bandwidth of about 1/3 cps, the carrier-to-noise ratio will be 3, i.e. about 10 db, under the above conditions, and the resulting standard deviation of phase will be

$$\sigma_\phi \approx \frac{N/C}{\sqrt{2}} \approx \pm \quad 0.21 \text{ radians}$$

An error occurs only if the noise-induced phase deviation ϕ_N exceeds the shift $\Delta\phi$. In the scheme described, the shift employed is ±0.78 radians, which is 3.7 times the standard deviation; the resulting probability of an error ($\phi_N \geq \Delta\phi$) is less than 0.0002. At the stations, errors can be reduced by recognizing that since a long-term reference phase is available, a mark, for example, is signified by the initial 45-degree lead, as well as the differentially coherent lead-lag shift. The integration time in the navigation receiver is much greater than three seconds so that the two phase shifts per signal element will effectively cancel.

4.8 Sudden Ionospheric Disturbance Warning Service

Warning to navigators will be issued if a sudden ionospheric disturbance of large magnitude or some other unexpected natural or man-made effect should be detected.

The method employed will be to temporarily change the 11-1/3 cps modulation frequency on the 10.2 kc carrier to 5-2/3 cps.

The transmitting stations will switch from 11-1/3 to 5-2/3 cycles in avalanche; that is, any station identifying a sudden ionospheric disturbance for itself or receiving a warning signal from outside the Omega system will initiate the Omega warning. All other transmitting stations would honor this action by also changing to the warning frequency whether or not they had any independent knowledge of the event. Thus, the whole transmission sequence will shift and navigators can recognize the warning.

The end of the abnormal event will be announced by the individual stations on the communication channel, and normal 11-1/3 cps Omega operation will be restored when all stations have discovered that the emergency is over.

By placing the warning on the 10.2 kc sidebands, it is possible to have SID warning outputs on even the simplest of receivers. A possible circuit would be to take the phase detector output before it enters through normal low pass filters and feed the alternating current component into a discriminator or pair of mark-space filters tuned to 5-2/3 cycles and 11-1/3 cycles. The change of modulation frequency can be used to activate a warning light on the receiver.

5.0 System Synchronization

For the phase contour pattern, by which position is defined to be stationary, the signal timing of the stations must be synchronized. In operation the signals of each station are independently timed by a stable timer or clock, and synchronism is maintained by having each station report to a computing center signal phases of all other stations within range with respect to its own signal. The computing center determines the deviation of each signal phase from the mean and directs the stations to adjust their timing rates (from time to time as required) to minimize deviations.

In addition, each transmitting station will be equipped with phase difference equipment for recording the relative phase of other Omega signals within range, to which synchronization control may be shifted in the event loss of contact with the central computer should occur.

5.1 Synchronization

Tests of VLF transmissions over long distances have established that the timing stability of such signals will exceed one microsecond for at least some periods of time. Hence, if the accuracy of the system is limited at all times solely by unpredictable propagational disturbances, the relative timing with which the signals are transmitted must be controlled to less than 0.5 microseconds.

In the design and construction of precision frequency and time standards, the state-of-the-art is such that under conditions of controlled temperature and humidity, freedom from shock and vibration, and lengthy periods of undisturbed operation (as will be the case in the control of Omega transmitter timing), a frequency stability of 1 to 10^{11} has been demonstrated; and it is expected that the concensus of several standards may be a factor of two better.

If long-term relative drift between stations is eliminated by synchronizing adjustments, phase drift of one station, controlled by the concensus of three or more timing standards (with respect to the mean of the rest), will be essentially a "random walk" effect with the accuracy multiplied by time as the upper bound of the amount of drift.

Thus, in one day (8.64×10^{10} microseconds) the maximum drift of one station, with respect to the mean of the other seven, will be:

$$\tau = \frac{\Delta f}{f} T$$

$$\tau = 5 \times 10^{-12} \times 8.64 \times 10^{10}$$

$$\tau = 0.43 \text{ microseconds}$$

It appears that adequate synchronization control can be maintained with timing rate corrections applied at intervals of 24 hours or more.

5.2 Theory of Absolute Synchronization

By definition, the common time base of the system is the mean timing of all signals as radiated. To maintain synchronism, a monitor at each station determines the phase of the signals of all other stations with respect to its own signal and, after compensating each observed phase angle for the expected travel time between stations, adjusts the rate of change of the phase of its own signal proportional to its deviation from the compensated mean, so as to reduce deviation from the mean.

It can be shown as follows, that, if the phase of each station is controlled independently according to this procedure, the system will subside to the state of all signals radiating in phase, to within the accuracy with which the mean travel time between stations can be predicted.

The waveform of the signal radiated from each Omega station is a periodic function of period T with recognizeable timing epochs (e.g., points of zero crossing) at times $(nT + t_k)$.

Associated with each station is a monitor determining the difference between times of occurrence of the timing epochs of each of the signals within range, and the adjacent epochs of the local signal. Let d_{jj} be the electrical distance between the jth station and its own monitor, and d_{kj} be the electrical distance between the k_{th} station and the j_{th} monitor, etc.

Each timing epoch of the jth signal arrives at its own monitor at times $nT + t_j + d_{jj}$ and the corresponding timing epoch of the kth signals arrives at times $nT + t_k + d_{kj}$.

For the Omega system to function as a navigation system, the signal travel time, including diurnal variations, over any given path must be predictable with good accuracy. Let d'_{kj} be the predicted travel time from station k to station monitor j, etc. Then the difference in phase of the signals, when compensated for net travel time, would be

$$e_{kj} = t_k - t_j + (d_{kj} - d_{jj}) - (d'_{kj} - d'_{jj})$$

$$= t_k - t_j + e_{kj}$$

where

$$e_{kj} = d_{kj} - d'_{kj} - d_{jj} + d'_{jj}$$

is the error in prediction of the net travel time of the signals from stations j and k to the monitor of station j.

The "free" timing rates or frequencies of the several stations, when uncontrolled, will not be identical. They will differ one from another by small amounts, depending upon manufacturing tolerances and adjustments of the individual timers. Let v_j be the residual frequency offset or drift of the jth station timer with respect to the mean of the uncontrolled rates of all timers in the system.

Then, correcting the timing rate of the jth station proportional to the algebraic sum of its deviation in phase from each of the other signals, including its own, leads to a control function of the form

$$\dot{t}_j = v_j + K \sum_{1}^{N} (t_k - t_j + e_{kj})$$

where t_j is the rate of change of phase of the jth station.
This expression expands into

$$\dot{t}_j + NK\, t_j = K \sum_{1}^{N} t_k + v_j + K \sum_{1}^{N} e_{kj}$$

Let

$$e_j = \sum_{1}^{N} e_{kj}$$

be the sum of the errors in the predictions of net travel time of all signals received at the monitor of station (j). Then taking LaPlace Transforms of the above expression,

$$sT_j(s) - t_j(O) + NKT_j(s) - K \sum_{1}^{N} T_k(s)$$

$$= \frac{v_j + K\, e_j}{s}$$

which can be rearranged into the form

$$(s + K(N-1))\, T_j(s) - K \sum_{\substack{1 \\ k \neq j}}^{N-1} T_k(s) = t_j(O) + \frac{v_j + Ke_j}{s}$$

where the upper limit (N-1) in the summation over T_k implies that the term $T_j(s)$ was subtracted out and combined with the other term in $T_j(s)$

Over the set of N stations, there will be N control functions of the above form. Assuming all stations to have the same control ratio (K) these control functions provide a set of N simultaneous equations of the form

$$-KT_1(s) - KT_2(s) \ldots (s+K(N-1))T_j(s) \ldots -KT_n(s) \ldots = T_j(0) + \frac{v_j + Ke_j}{s}$$

Solving by Determinants, the LaPlace Transform is

$$T_1(s) = \left[t_1(0) + \frac{v_1 + Ke_1}{s} \right] \frac{1}{(s + KN)}$$

$$+ \sum_{1}^{N} \left[t_k(0) + \frac{v_k + Ke_k}{s} \right] \frac{K}{s(s+KN)}$$

The corresponding time function is

$$t_1(t) = t_1(0)\, e^{-KNt} + \bar{t}(0) + \frac{v_1 - \bar{v}}{KN} + \frac{e_1 - \bar{e}}{N} \quad (1 - e^{-KNt})$$

where the barred terms, e.g., $\bar{v}$, $\bar{t}$, $\bar{e}$, etc., signify the mean values of the terms over the set of stations.

By definition, $\bar{v}$, the mean deviation in rate from the mean, is zero. Hence the final value of the phase of station (1)

$$t_1(t) = t_1(0)\, e^{-KNt} + K\bar{e}\, t$$

$$+ \left[\bar{t}(0) + \frac{v_1}{KN} + \frac{e_1 - \bar{e}}{N} \right] (1 - e^{-KNt})$$

The first term of the above expression is the initial phase offset of station 1, which subsides to zero as e^{-KNt}. The second term is a shift of the system timing rate, common to all stations, and arising from the errors in the determination of the net transmission time between stations. Being common to all stations this term introduces NO error in the synchronization.

The term within brackets is a phase offset, initially zero, and rising to a final value as $(1 - e^{-KNt})$ consisting of three parts. The first of these is $\bar{t}(0)$, the average initial phase of all the stations in the net. Since again, this term is common to all stations, its presence is NOT an error in the synchronization.

The second part is the term v_1/KN, which is a phase offset required to pull the initial or uncontrolled rate of Station (1) to the mean rate of the system. This term introduces an error in the synchronization, which is small since the individual frequencies will differ at most by only a few parts in 10^{12}. Furthermore, this term could be eliminated by long term integration of the residual deviation of the particular station from the system mean, although this function is not included in the present control function.

The third part of the fixed offset in phase is the term $(e_1 - \bar{e})/N$ and is the offset arising from the error in determining the net travel times around the system. Again this term is small and would be reduced with improvement in the estimation of signal travel time.

The solutions for the responses of the other stations in the net are formally identical, differing only in the replacing of the subscripts of the terms with the subscript 1 with the subscripts for the particular station. Hence, all stations subside to the same final drift rate with respect to the mean uncontrolled rate of the system, depending upon the mean error in determining the transmission times over all paths included in the system.

The initial phase offset of each station subsides to zero as e^{-KNt}, being replaced by the average initial frequency offset (v_j/K) and the error in determining the travel times associated with that particular station $(e_j - \bar{e})$ with respect to the mean error in determining all travel times in the system.

Thus, the final state of the system is with all stations in phase to within individual errors given by

$$\frac{1}{N}\left(\frac{v_j}{K} + e_j - \bar{e}\right)$$

plus an offset of the timing rate of the <u>system</u> from the mean uncontrolled rate, amounting to $K\bar{e}$.

The mathematical description of the control functions was predicated upon continuous control, wherein the rate of change of the phase of each station was continuously a function of the difference in phase between its signal and the mean of all other signals within range, compensated for propagation time, etc.

In actual operation, however, the control would probably be discontinuous. That is, correction to the phase and rate would be made at intervals of from several minutes to 24 hours or more.

Analysis of the transient response of the system as a Sampled Data System, shows that, even with discontinuous control, the transient response is still absolutely stable, if the control ratio (K) is properly adjusted, and the operation is at least grossly similar to that with continuous control.

It may be noted that, with this type of control, the final mean timing rate is uncontrolled, and the entire system can be steered by reference to some other timing standard, to maintain synchronism with any convenient universal time scale, such as UT-2 or Atomic Time etc.

In normal operation, control of synchronization would be obtained by varying the timing rates only, so that no discontinuities of phase would be introduced.

When phase errors are large, as when bringing up the system after a shutdown for maintenance, etc., a rapid subsidence to the correct phase could be obtained by having each station correct its <u>phase</u> proportional to the deviation of its signal from the mean of all other signals within range, corrected for travel time. With a large number of stations, even gross changes in the phase of the signal from one station would produce only a relatively small change in the mean, so that even with adjustments made independently at arbitrary intervals in each station, the system would still converge to the desired state of all-signals-in-phase. Likewise, with some intelligence on the part of the station operators, an abnormal drift by one station would be immediately apparent, and its phase ignored in the overall system control.

5.2.1 Theory of Central Monitoring

With the synchronizing control system as described, each station phase subsides to a phase error determined by the error in the estimate of the travel time over the various signal paths around the system, and the magnitude of these errors cannot be detected by any measurements within a station. However, if the phase of each signal, as observed at all stations, is reported to a central location, the absolute deviation of the phase of each station signal from the mean of all can be determined substantially independently of distance between stations and changes in velocity of propagation, as can be shown as follows.

As before, the timing of a signal (as observed at any point) is retarded by the

$$\theta_{n'r} = t_n + d_{nn'} - t_r - d_{rn'} \qquad (5.2\text{-}1)$$

where the primed subscripts identify the monitor locations and

t_n = absolute phase of signal radiated from station (n)

t_r = absolute phase of signal radiated from station (r)

$d_{nn'}$ = retardation of signal from station (n) as received at its monitor (n')

$d_{rn'}$ = retardation of signal from station (r) as received at its monitor (n') of station (n).

Similarly, the phase difference of the same two signals as observed at a monitor (r') associated with station (r) would be:

$$\theta_{nr'} = t_n + d_{nr'} - t_r - d_{rr} \qquad (5.2\text{-}2)$$

The mean of the relative signal phases from (n) and (r) as observed at the two monitors is θ_{nr}, where.

$$\theta_{nr} = \frac{\theta_{n'r} + \theta_{nr'}}{2} = t_n - t_r - \frac{d_{rn'} - d_{nr'}}{2} - \frac{d_{nn'} - d_{rr'}}{2} \qquad (5.2\text{-}3)$$

The second term on the right, $(1/2)(d_{rn'} - d_{nr'})$ is one-half the difference of the transmission time from each station to the opposite monitor, i.e., in opposite directions between stations. Assuming this quantity to be D_{nr} and rearranging, an expression for the phase difference between the signal of station (n) and the signal of station (r) as radiated is:

$$t_n - t_r = \theta_{nr} + D_{nr} + \frac{d_{rr'} - d_{nn'}}{2} \qquad (5.2\text{-}4)$$

A similar expression can be obtained for the relative signal phase from station (n) with respect to the signal of each of the other N stations (including the trivial case of observing the phase of the signal of station (n) with respect to itself at both ends of a baseline of zero length, i.e., 0 = 0).

The mean of all N expressions obtained is

$$\frac{1}{N}\sum_{r=1}^{N}(t_n - t_r) = \frac{1}{N}\sum_{r=1}^{N}\left(\theta_{nr} + D_{nr} + \frac{d_{rr'} - d_{nn'}}{2}\right) \qquad (5.2\text{-}5)$$

which reduces to:

$$t - \overline{t} = \theta_n + D_n + d_n \qquad (5.2\text{-}6)$$

where

$$\overline{t} = \frac{1}{N}\sum_{r=1}^{N} t_r$$

Absolute mean phase of all N signals as radiated.

$$\theta_n = \frac{1}{N}\sum_{r=1}^{N} \frac{\theta_{nr'} + \theta_{n'r}}{2}$$

Mean phase difference between signals of station (n) and each of the other stations as observed at all monitors.

$$D_n = \frac{1}{N}\sum_{r=1}^{N} \frac{d_{rn'} - d_{nr'}}{2}$$

Mean of the differences in transmission time in opposite directions over each baseline.

$$d_n = \frac{1}{N}\sum_{r=1}^{N} \frac{d_{rr'} - d_{nn'}}{2}$$

Mean of the differences in transmission time from station (n) to its monitor and from each of the other stations to each of their monitors.

The left side of this expression is the deviation in signal phase from station (n) with respect to the mean of all signals.

The first term on the right (θ_n) is the mean of the relative signal phase from station (n) with respect to the signal of each of the other stations (including itself) as observed at all monitors.

The second term D_n is the average <u>difference</u> in propagation time (in opposite directions over each baseline) and represents the aggregate by which the transmission times in opposite directions over all baselines averaged together do not exactly cancel out. Because each monitor is close to its transmitter and propagation velocities in opposite directions are equal, this quantity will be small or zero, independent of distances between stations and of variations in propagation velocity. In any case, it will consist of a small-fixed or systematic quantity, dependent upon the aggregate of the displacements of the monitors from their parent stations by which the path-lengths are nonreciprocal and upon the systematic part of any nonreciprocal propagation effects, plus a smaller random portion arising from variations in the difference in propagation velocity for opposite directions over the several paths.

The third term d_n is the difference between the propagation time from station (n) to its monitor and the mean of the propagation times of all the other stations to their monitors. To the extent that the distance of the monitors from their parent stations are the same, this term will tend to zero. In any event, the monitors will all be within ground wave range of their parent stations, and this term will be, at most, a small-fixed quantity not subject to diurnal variations.

Thus, the deviation of a particular signal phase from the mean phase of all signals as radiated is given substantially independent of distance between stations and changes in velocity of propagation, etc., by the mean of the phase differences between the signal of the particular station and the signal of each of the other stations (as observed at all monitors of the system).

Thus, system synchronization can be corrected independent of distance between stations and propagation velocity, by having a control center, to which the observed signals phases are reported from time to time, which computes the theoretical absolute deviation of each signal from the mean, and directs each station to adjust the assumed net travel time of all signals to that station so as to cancel out the calculated deviation from the mean.

In theory, if transmission times in opposite directions over all paths are identical and the distances between monitor antennas and corresponding signal radiators are insignificant, this statistical matrix synchronizing method locks the signals of all stations in phase in absolute time independent of the distances between stations, of variations in propagation velocity with time, and of differences in propagation velocity over different paths.

The major portion of the transmission time from station to station is accounted for by noting the phase of the signal at both ends of each baseline so that transmission time is largely cancelled.

There will, of course, be certain systematic biases in the synchronization arising from systematic differences in propagation velocity in opposite directions, from radiation paths of one station to the monitor of another (which are not identical in opposite directions over the same baseline), and from the unequal distances of each station radiator to its station monitor.

These effects give rise to a fixed bias in the synchronization of each station so that each signal phase as radiated will deviate a small fixed amount from the mean of all signals.

The effects also appear as a fixed shift of the signal phase pattern, which will be eliminated by inclusion of compensating factors in the error matrix calculations.

The random portion of the nonreciprocal transmission effects not cancelled by transmission in opposite directions over each baseline is reduced statistically both by the redundant statistical matrix solution for phase deviations and by long period averaging to eliminate diurnal effects and noise.

By using highly stable timers, which allow effective integration periods of more than a diurnal cycle (with data taken at selected observation times), reciprocal cancellation of the major portion of the variations in transmission time between stations, and statistical reduction of random errors in the error matrix calculations, this portion of the synchronizing error can be held to less than 0.5 microsecond.

5.3 Standardization of Station Timing

The exact geometrical locations of the phase contours in space are dependent not only on the locations of the transmitters but also on the particular relative phase with which the signals of the various stations are transmitted. For the phase contours to correspond with the pattern drawn on a navigational chart, the signals of the several stations must be constrained to maintain specified relative phases.

Since charts, tables, and position interpolating formulae can be drawn to fit the locations of the phase contours for any fixed relative signal timing, the particular timing maintained at a given station is arbitrary, even though all equipments are calibrated to read zero phase on a Standard Calibrating Signal.

A convenient specification for relative station timing is to require that the least common multiple period of all components of the signals as transmitted be synchronous. That is, the phase format of the signal of each station will be adjusted so that all signal components as radiated cross zero with positive slope at intervals of the least common multiple period of the signal format; and in synchronizing the system (Section 5.2), the timing of each station will be constrained so these epochs of common zero crossing of all stations coincide in absolute time.

5.3.1 Least Common Multiple Period

By definition, the least common multiple period of the signal is the smallest commensurate period into which all signal component periods divide evenly.

In determining the least common multiple period, not only radiated signals must be accounted for but also the ten-second multiplex sequence, whose phase must be specified in the synchronizing process.

The phase of the side frequency components must also be specified. However, the side frequency components, related to the basic components in frequency by fractional ratios of rather high order (37 : 40, etc.), have very large least common multiple periods with respect to the principle components; it seems advisable to consider the specification of their synchronization separately.

By specific design, the basic Omega frequencies, 10.2, 13.6 and 11.333 kilocycles, and the higher lane resolving modulation components, 226-2/3 and 45-1/3 cycles per second, are all exactly commensurate with the lowest lane resolving component, 11-1/3 cycles per second.

The 11-1/3 cycles per second lane resolving component is likewise commensurate with even seconds, there being exactly 34 cycles of this component in 3 seconds. Hence, it is also commensurate with the ten-second commutation period since there are ten periods of 3 seconds to 3 periods of ten seconds. Thus, both the multiplex period and the lane resolving periods are commensurate over intervals of 30 seconds.

From this it follows that, excluding the side frequencies, a convenient specification of relative station timing may be established by requiring that at all stations all signal components shall cross zero with positive slope simultaneously with respect to absolute time at intervals of 30 seconds. If the overall timing of the system is adjusted so that one of those timing epochs occurs at 0000 Z Standard Time and each 30 seconds thereafter, the multiplex cycle becomes a ten-second Standard Time reference.

5.3.2 Side Frequency Synchronization

In the frequency multiplex mode of operation, the side frequency components of two or more Omega stations at different frequencies between 10 and 14 kilocycles are frequency multiplied to the common harmonic frequency of 408 kilocycles, and the resultant harmonics of equal frequency are compared in phase to establish isophase contours giving position.

For the position contours to be fixed in space, the side frequency components of the several stations must be synchronized so that the relative phases of their 408 kilocycle harmonics are stationary. As developed in Section 2.5.2, all components of the radiated signals from each station are submultiples of 408 kilocycles, and each station derives the elements of its signal by frequency division with the appropriate ratios from a 408 kilocycle timing standard (or by an exactly equivalent frequency synthesis operation from some other standard frequency). The process of synchronizing the transmissions of the overall system (Section 5.2) effectively phase locks the 408 kilocycle timing standards of all stations. Thus, the side frequencies derived by division from these synchronized timers are also inherently synchronized. The multiplication of the side frequency components, as received at any point, to the common harmonic frequency of 408 kilocycles produces components of the same frequency with a stationary relative phase. Separate synchronization of the side frequency components in the transmitters is not required.

However, the phase of each side frequency component as emitted at the corresponding subharmonic of 408 kilocycles may have any of N discreet values with respect to absolute time evenly spaced by whole periods of 408 kilocycles, where N is the division ratio producing the particular side frequency.

Since phase comparison of the side frequencies in a frequency multiplex type of receiver is obtained only after frequency multiplication up to the common 408 kilocycle harmonic frequency (where the phase is independent of the particular timing cycles at which the side frequency dividers reset in the transmitter timers), the particular counts at which the dividers reset in the transmitter timers appear to be immaterial.

However, it is fundamental in side frequency multiplex operation that the receiving equipments shall keep track of the number of lanes crossed, and have sufficient filtering to carry over from one ten-second period to the next at whatever speed the carrying vehicle happens to travel. This amount of filtering may well make a transmitter counter-jump, or series of jumps, appear as a rate of change in signal phase to be tracked, introducing a spurious lane slippage in the receiver indication. Thus, the side frequencies shall maintain phase continuity at the emitted frequencies as well as at the 408 kilocycle harmonics, and a specification of absolute side frequency phase appears necessary.

Since the side frequencies are derived from the same 408 kilocycle timing standards, they are commensurate with the basic Omega signal components, including the basic timing epoch period of 30 seconds (Section 5.3.1). Thus, a specification for side frequency phase can be established by requiring that all side frequency components shall cross zero with positive slope simultaneously with all other components of the signal at some 0000-Z epoch of Universal Time.

At a particular station the side frequency components will again cross zero with positive slope concurrently with the other signal components every 30 x N seconds (maximum).

Each station will have a schedule indicating the beginning and end of each such commensurate period, and the timing equipment will be arranged to indicate the particular timing epochs at which the side frequency counter resets concurrently. Alignment of the side frequency phase to within one count of the 408 kilocycle timing standard would be made when necessary by resetting the side frequency divider so that the concurrent timing epochs match the schedule for that station.

Synchronization of the side frequency within one cycle of the timing frequency would be maintained by the basic system-synchronizing function.

5.4 Phase Continuity

Each station is timed by its own internal timing standard with phase synchronization maintained by correcting the timing-standard rate from time to time. It is important that the timing standards be capable of maintaining continuity of phase within the prescribed system accuracy over minor disturbances of operations, e.g., interruptions of power during switchover, equipment removal from service for routine maintenance, transmitter shutdown by antenna arc-over, momentary to sustained power interruptions due to lightning action, etc. However, it is far more important that phase continuity be maintained through any gaps in transmission, than that there be no interruptions since momentary and even sustained interruption can be suffered with only minor or negligible affect on most operations provided the signals return with the correct phase format and timing when service is restored.

Some of the stations will be at isolated locations with difficult receiving conditions; therefore, it is essential for satisfactory operation of the system that every effort be made to provide equipment capable of maintaining continuity of phase under almost all conditions up to complete destruction of the station.

To establish the transmitted signal format, each transmitting station will be supplied with timing standards and frequency synthesizers deriving the frequency components of the signal format from the stable output of the timing standards, in which continuity of operation must be maintained, in order to have continuity of phase.

At the present state-of-the-art, all timing standards of the requisite accuracy (1 to 10^{11}) are atomic standards operating at gigacycle frequencies with locked oscillator and frequency multiplication techniques utilized to derive an integral frequency in the megacycle range locked to the atomic standard.

In this application, the megacycle frequency will be further subdivided by a complex of frequency dividers, both in parallel and in tandem, as in Figure 5.4-1, to produce the spectrum of frequency components required for the generation of the over-all signal to be transmitted (as described in Section 4.6).

Frequency standards and frequency synthesizers are subject to catastrophic failure of components and result in complete loss of output, momentary failures causing loss of phase lock or phase jumps, and excessive drifts from failure of auxiliary functions such as temperature controls, magnetic field controls, etc.

The probability of such failures occurring cannot be considered negligible. To assure continuity of phase, each transmitter will be supplied with four sets of independent timing standards and frequency synthesizers, complete with emergency battery supplies and automatic switchover, etc., each of which generates all of the frequency components required to establish the absolute phase of the transmitted signal.

Several concensus logic systems will combine the outputs of four timing units, supplying the timing signals to the transmitter from the average of all units that agree in every manner. If any one unit does not agree with the others, the concensus logic systems supply the transmitter from those logic units that do agree, while stepping the phase of the disagreeing unit into coincidence.

In addition to keeping the odd unit in alignment with the others, the

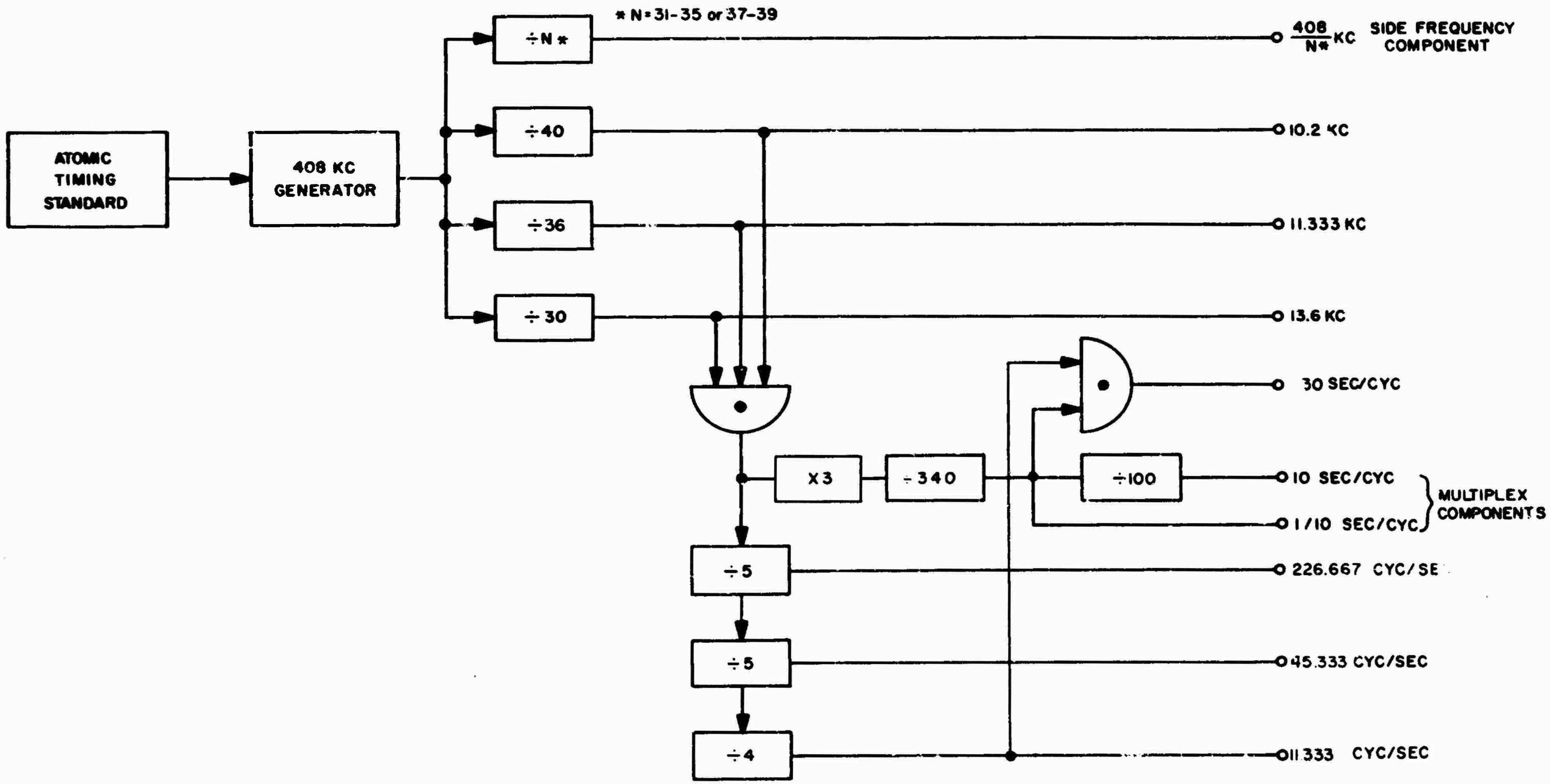


FIG. 5.4-1 OMEGA FREQUENCY SYNTHESIZER

concensus logic systems will sound an alarm whenever a discrepancy occurs and will record the fault and the correction applied to restore the units to operation in unison.

5.5 Synchronization Control

Control of system synchronization will be accomplished by relaying (via phase coding of the side frequency components) the relative phase of the eight signals as observed at each station by timing monitors to a timing control station, which determines the deviation of the phase of each signal from the mean and provides timing rate correcting data to the individual stations (also via side frequency coding) to maintain the signals of all stations in synchronism.

5.5.1 Station Timing Monitors

Each station monitor will contain monitoring equipment consisting substantially of Omega receiving equipments for measuring and recording the phase of each of the other signals with respect to the local signal, but adapted to perform these measurements accurately in the presence of very strong signals.

Associated with this measuring equipment will be data-processing equipment--for smoothing the raw data and deleting, to whatever extent is feasible, data acquired during periods of doubtful reception--and data formating equipment, adapted to encode the system phase data into a form suitable for transmission to the timing control station via phase code modulation of the system side frequencies.

The station monitors function independently of the transmitter timing equipment and may be located at any convenient position sufficiently close to the station so the retardation of the local signal may be considered constant irrespective of weather and state of the ionosphere.

In most locations, positioning the monitor on the station property will be convenient.

5.5.2 Timing Control Station

The timing control station is supplied with communication circuit terminal equipment for receiving and decoding the phase data information relayed from each of the Omega stations via phase coding of their side frequencies, a computer maintaining running integration of the mean phase of the system and deviation of each signal from the mean, communication circuit terminal equipment connecting the system with other sources of Standard Time (Naval Observatory, Bureau of Standards, etc.) for acquisition of intersystem timing information (to maintain the desired Universal Time Scale in the system), and communications circuit sending equipment to encode the rate corrections determined by the computer for transmission via side frequency phase coding of the nearest Omega station to the other stations of the system.

The timing control station can be at any convenient location, since it serves only to process relayed data. Conveniently it could be combined with the timing monitor of one of the Omega transmitting stations and manned by personnel of that station (thus avoiding the necessity of maintaining a separate establishment and eliminating one of the data transmission circuits).

PART II. IMPLEMENTATION

6.0 Transmitting Stations

Suggestions for implementation of the system outlined in the preceding chapters must begin with a study of the eight required transmitting stations. Station location is, perhaps, the subject in which the least guidance can be drawn from past experience with VLF transmitters. The special requirements of Omega will call for discussion of several other matters, especially power capability, antenna size, configuration and provision of suitable timing control for the transmissions. Other subjects, in which Omega requirements are similar to those common to other VLF transmitters, will be treated only in outline.

6.1 Location of Transmitting Stations

The problem of locating transmission stations was outlined in Section 2.2 and especially in Figures 2.2-5 to 2.2-13. From these studies, it is clear that the eight stations must form a pattern of considerable geometrical regularity, which implies that distances between stations must be nearly uniform. We have also considered that in equatorial regions transmission toward the west is far less satisfactory then toward the east. This suggests that a minimum number of stations should be located near the geomagnetic equator. On the other hand, excessive attenuation in arctic regions is to be feared, so that configurations requiring too much transmission across arctic land masses or ice caps should be avoided. These geometrical and propagational requirements constitute a sufficiently difficult problem without the addition of geographical, political, and economic considerations.

Stations of the size required for Omega must be built on land, and land is distributed over the globe in a highly nonuniform manner. Most attempts to conceive an effectual, convenient geometrical pattern fail because one or more of the required points fall far from land.

Political factors are obviously important, but they lie outside the domain of competence of the Omega Implementation Committee. Such a world-wide system must have transmitting stations in several sovereign states, and it will be advantageous if as many of these states as possible have a real interest in helping to solve a navigational problem and are able to contribute energy and enthusiasm (if not money) to the solution.

Unfortunately, Omega stations will be expensive to build. A station in a tropical or arctic location may cost two or three times as much to build as one in a temperate, relatively industrialized area. It is difficult to imagine a pattern of stations that does not involve one or two sites at difficult locations, but the number should be held to a minimum.

When the selection of sites involves so many contradictory factors, an exact solution is impossible. We have collected suggestions on station patterns (where we could find them) and have originated a considerable number, perhaps a total of 30. Of these, eight or ten have been subjected to considerable study of the kind illustrated in the figures of Section 2.2. Only four of the possible patterns have survived this examination, that is, have shown advantages clearly superior to most of the other configurations.

Before suggesting these families of stations, a word about the geometrical pattern types is in order. In Chapter 2, we observed that for pure geometrical excellence, an octahedron of six stations is ideal. Our ideas on proper redundancy and effects of nonuniform service radius, however, force us to conclude that the proper number of stations is eight.

Two ways of grouping eight stations stand out as clearly superior to all others we have examined. An octahedron of six stations with "strengthening" stations at the centers of two of the eight triangles formed by the first six is the first concept and is followed in two of the patterns cited below. In one case (pattern number 1), the strengthening stations are situated to provide signals of extra amplitude in the arctic and antarctic regions where high attenuation will be encountered. In the other (pattern number 6), extra stations are sited to reinforce the American and European-Asian-African land masses.

The second approach, which has proven reasonably satisfactory, is a hexahedron, where the stations occupy the corners of an inscribed cube. In this pattern, there is a smaller probability of finding approximately orthogonal lines of position than there is in the octahedron. Also, realizable hexahedral patterns tend to have four stations near the equator, where transmission toward the west is poor. The two best examples of this pattern that we have examined are identified as number 4 and number 7. Of these, number 4 suffers from having two stations at very high latitudes and would, therefore, be more expensive to implement.

The four patterns, mentioned above and defined in Table 6-1, are arranged as follows in descending order of technical excellence:

Pattern Number 1 - Octahedron

Pattern Number 6 - Octahedron

Pattern Number 7 - Hexahedron

Pattern Number 4 - Hexahedron

We consider that although the advantages of number 1 considerably exceed those of number 7, any of these four -- with or without minor modifications that might be politically desirable -- would be capable of providing satisfactory navigation.

6.2 Power Radiating Capability Required

The radiating power requirements for reliable coverage from a given transmitting installation can be readily determined from the following relation:

$$P_r \text{ (db rel 1 kw)} = -E_z \text{ (db rel } 1\,\mu v/m, 1\text{ kw)} + E_{nm} \text{ (db rel } 1\,\mu v/m)$$

$$+ C/N_1 + T_x \qquad (6.2\text{-}1)$$

TABLE 6-1. POSSIBLE PATTERNS OF EIGHT STATIONS

Pattern Number	Location	Jurisdiction	Approximate Lat. & Long. in Degrees	
1	Central Aleutian Is.	United States	52 N	174 W
	Galapagos Islands	Ecuador	1 S	91 W
	Balearic Islands	Spain	40 N	3 E
	Nicobar Islands	India	7 N	94 E
	Bouvet Island	Norway	54 S	4 E
	East Cape, North Is.	New Zealand	38 S	178 E
	near James Bay	Canada	52 N	79 W
	Tierra del Fuego	Chile	55 S	70 W
4	Midway Island	United States	28 N	177 W
	Spitzbergen	Norway	77 N	15 E
	near Broome	Australia	18 S	122 E
	Socotra Island	United Kingdom	12 N	54 E
	Tristan da Cunha	United Kingdom	38 S	12 W
	McMurdo Sound	Antarctica	77 S	165 E
	Marquesas Islands	France	10 S	140 W
	Leeward Islands	France or U.K.	62 W	16 N
6	Eastern Hawaii	United States	20 N	155 W
	Western Ireland	Eire or U.K.	54 N	10 W
	Chile near 30° S	Chile	30 S	71 W
	Africa near Delagoa Bay	Rep. So. Africa	26 S	33 E
	Northern Luzon	Philippines	18 N	122 E
	Southern South Is.	New Zealand	46 S	167 E
	Southern Texas	United States	27 N	97 W
	near Gulf of Oman	Iran	26 N	60 E
7	Auckland Islands	New Zealand	51 S	166 E
	Hebrides Islands	United Kingdom	58 N	7 W
	Far Aleutians	United States	52 N	178 E
	Bouvet Island	Norway	54 S	4 E
	Windward Islands	United Kingdom	13 N	60 W
	Southwest Mindanao	Philippines	7 N	123 E
	Marquesas Islands	France	10 S	140 W
	Seychelles Islands	United Kingdom	5 S	53 E

where:

P_r = Required radiated power in decibels relative to one kilowatt,

E_z = Vertical electric field strength in decibels relative to one microvolt per meter produced at a given receiving location from a transmitter radiating one kilowatt,

E_{nm} = Median rms noise field in decibels relative to one microvolt per meter in a one cycle per second band,

C/N_1 = Required rms carrier to rms noise in a one cycle effective bandwidth for the type of service involved,

T_x = Factor which assures this type of service for a given percentage of all hours in spite of the time variability of the noise as well as the variation in carrier field strength due to propagation effects.

The field strength produced at various directions from a one kilowatt transmitted has been described in some detail in Chapter 3. Figures 6.2-1, 6.2-2, and 6.2-3 show that the fields produced for propagation to the east are appreciably higher than those for propagation either to the north or south and particularly much higher than those for propagation to the west. Because the transmitting stations are spaced by approximately 10 megameters and propagation conditions within the first megameter of a given transmitter are likely to be contaminated with higher modes, it is desirable for each station to have reliable coverage for a nominal 10 to 12 megameters. Actually, this requirement must be modified to account for nonreciprocal attenuation rates. For propagation to the north or south, Figure 6.2-1 shows that during the day long path signals are likely to interfere with the short path at about 8,000 nautical miles (15 megameters). The one kilowatt signal level, at the point where the short path signal is at least 10 decibels above the long path signal, is about 15 decibels relative to one microvolt per meter. Figure 6.2-2 indicates the east fields of about 25 decibels relative to one microvolt per meter, which are free of long path interference almost all the way to the antipode. Figure 6.2-3 shows that daytime to the west propagation will be useful to distances of about 4,000

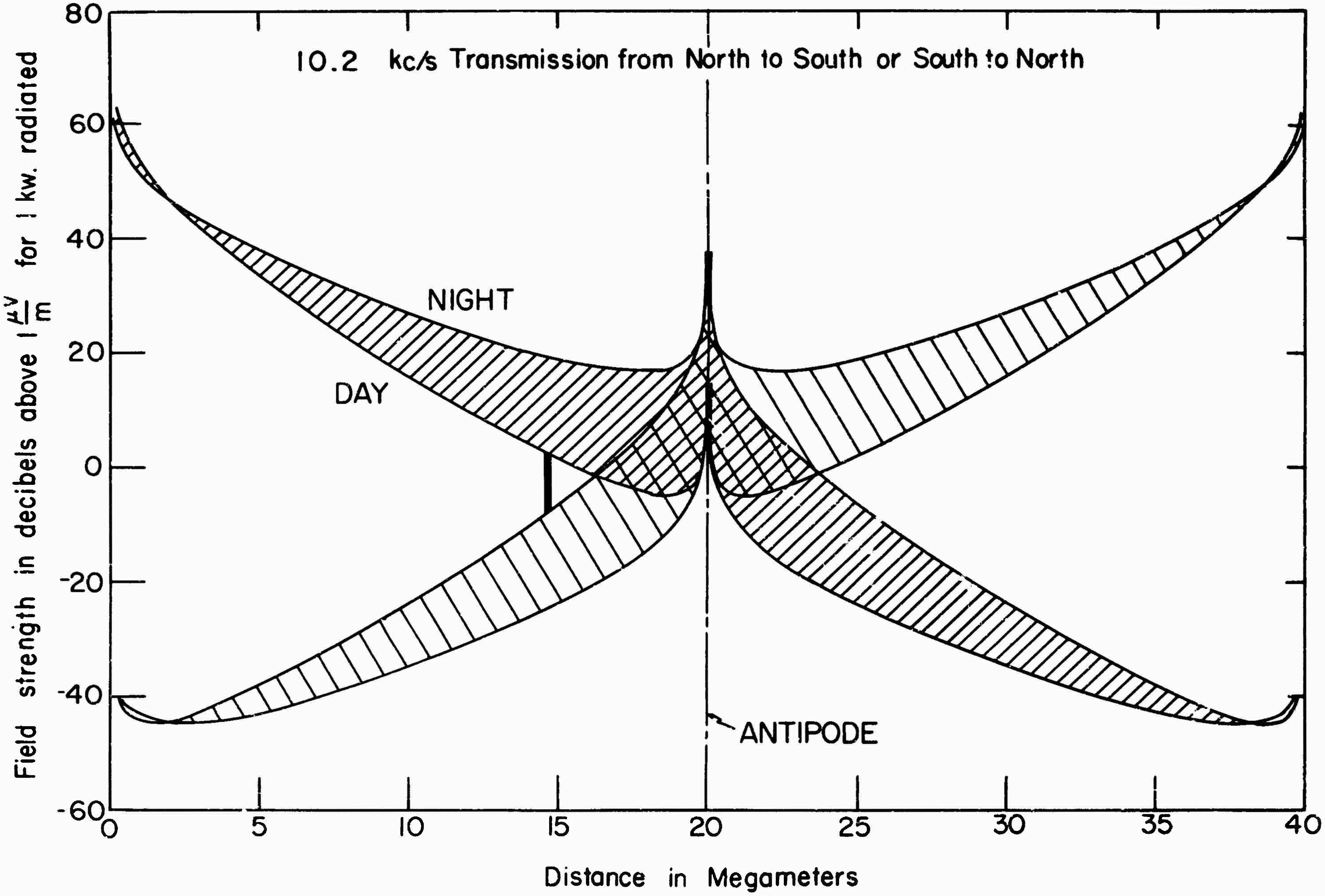


FIG. 6.2-1 10.2 KC TRANSMISSION FROM NORTH TO SOUTH OR SOUTH TO NORTH

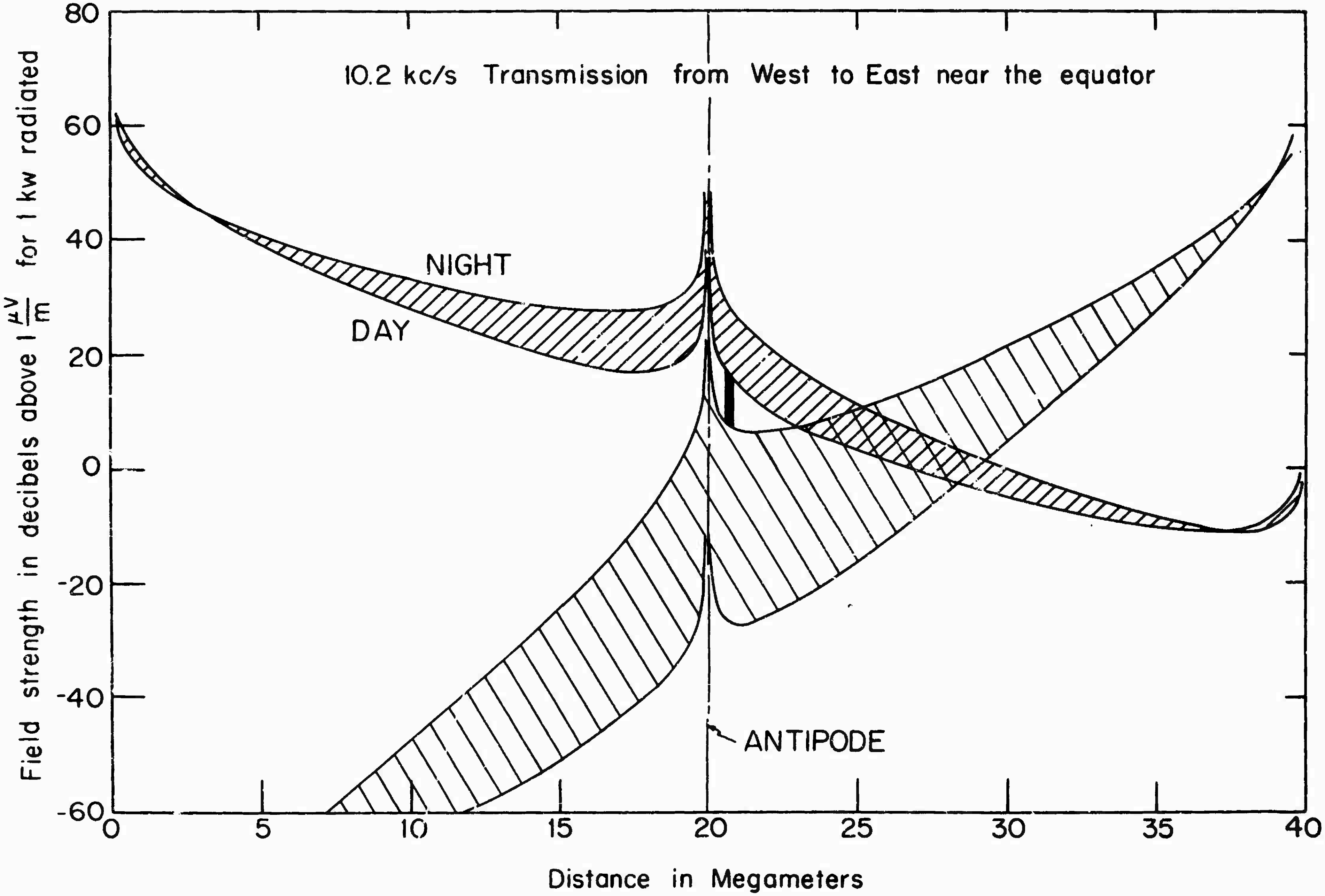


FIG. 6.2-2 10.2 KC TRANSMISSION FROM WEST TOWARD EAST NEAR THE EQUATOR

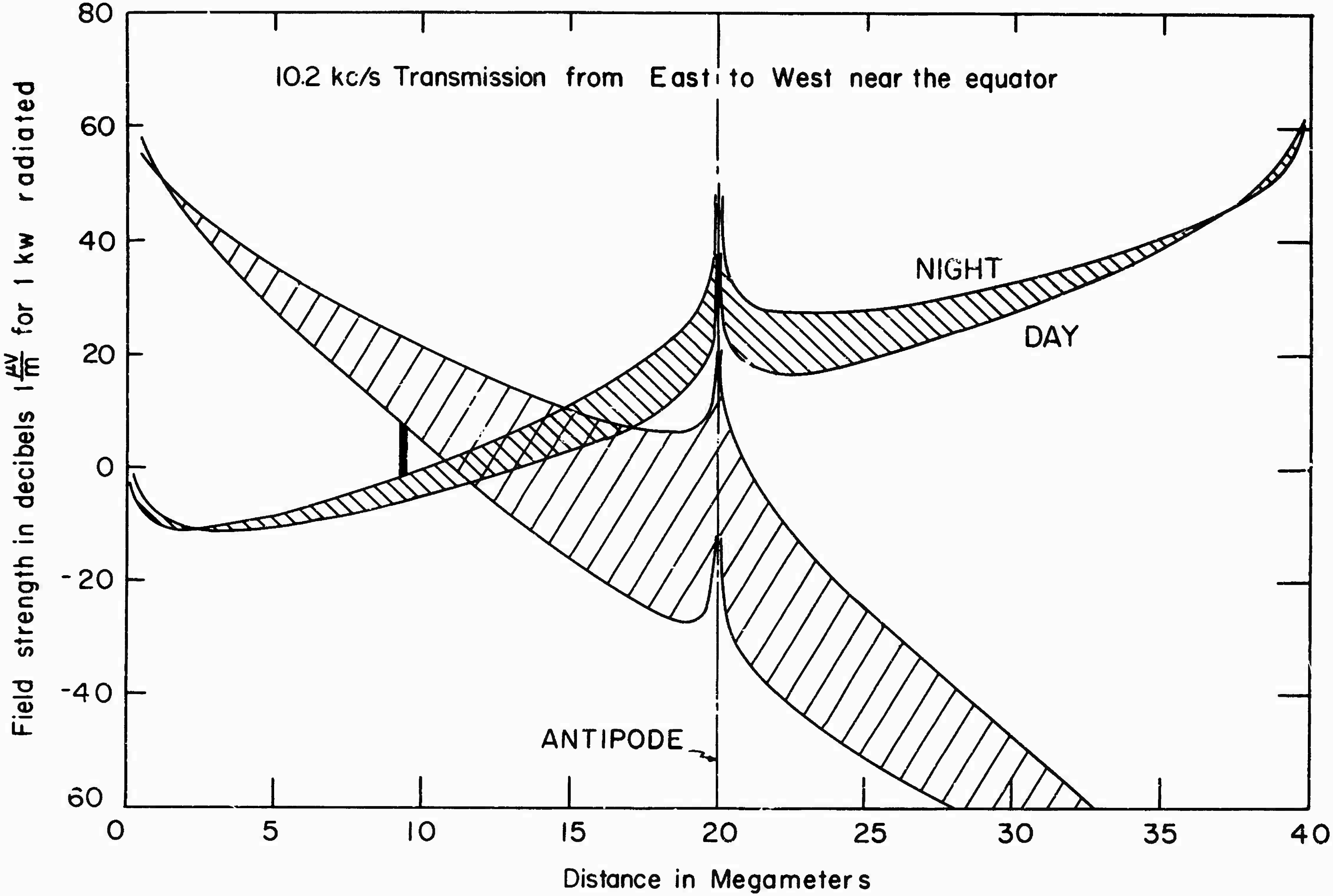


FIG. 6.2-3 10.2 KC TRANSMISSION FROM EAST TOWARD WEST NEAR THE EQUATOR

nautical miles (7.5 megameters). The signal level at this range is approximately 20 decibels relative to one microvolt per meter.

Since the daytime attenuation rates are appreciably higher than nighttime rates and the rather high noise levels are in existence in the late afternoon, we will perform our calculations of required power for daytime propagation conditions and afternoon noise conditions.

The anticipated noise fields (Figure 6.2-4) are approximately 35 db relative to one microvolt per meter in a one cycle per second bandwidth. This is the median noise expected in typical high-noise areas. The majority of system coverage will have much lower noise levels; and, as a result, the reliability of coverage will be much greater.

The carrier-to-noise requirements for an Omega system navigational receiver can be determined by at least two approaches.

1. Operational experience from the NEL group has shown that a receiver with a 60-second time constant requires a -20 decibel carrier-to-noise for satisfactory operation in a white noise background when the noise measured is in a 100-cycle band. As a result, we obtain $C/N_1 = 0$ decibels for a linear receiver in white noise. Experience with clipping receivers in a VLF atmospheric noise environment has shown that phase-type measurements can be made with approximately a 15 decibel advantage over linear receivers. The result is that we can anticipate a C/N_1 of -15 decibels.
2. Assume that an approximate 100-second time constant has an effective bandwidth of 0.01 second. If in the receiver bandwidth a 20 decibel carrier-to-noise is required,* it is evident that the carrier-to-noise in a one cycle band should be equal to 0 decibel

*Note that $\sigma_\phi = \frac{1}{\sqrt{2}} N/C \approx 0.07$ radians if $C/N = 10$. The standard deviation of phase time is $\sigma_T \approx 1.1$ microseconds, while the equivalent distance is $\sigma_d \approx 0.3$ kilometer.

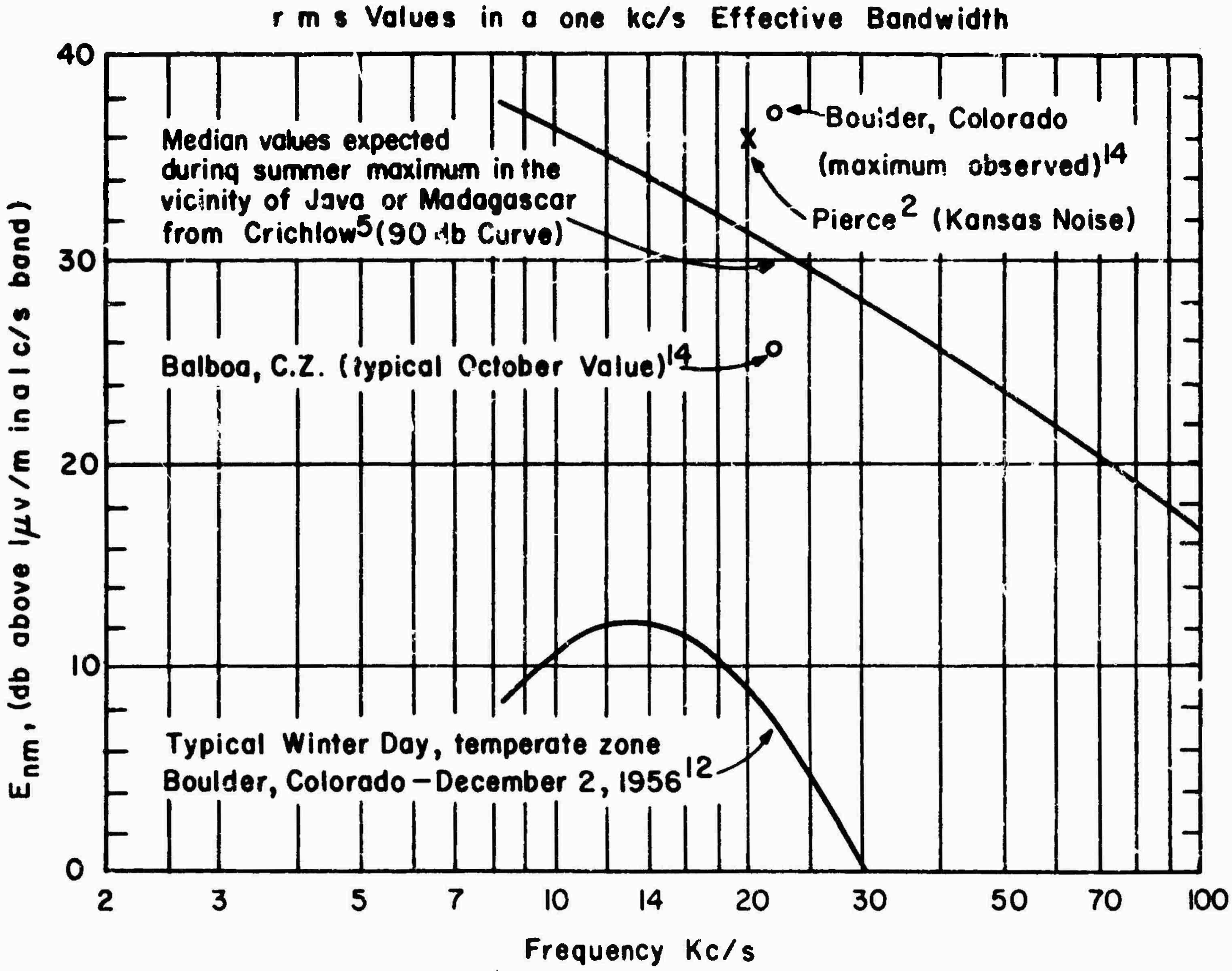


FIGURE 6.2-4. TYPICAL ATMOSPHERIC NOISE FIELD STRENGTHS

for a linear receiver. This is consistent with the previously described NEL experimental results and will lead to the same final value of $C/N_1 \simeq -15$ decibels.

The last factor in the equation for power requirements, T_x, is shown in Figure 6.2-5, where it is apparent that the satisfactory service fading margin for 90% of all hours will be approximately 10 decibels.

We now are in a position to determine the required radiating power capabilities for transmission in each of the different directions. For coverage to the east or west where E_z = 20 decibels for 1 kilowatt, and from Equation 6.2-1:

$$P_r = -20 + 35 - 15 + 10$$

$$= 10 \text{ db relative to 1 kw.} \qquad (6.2\text{-}2)$$

For coverage to the north and south where $E_z \simeq 15$ decibels and assuming the same noise level:

$$P_r = -15 + 35 - 15 + 10$$

$$= 15 \text{ db relative to 1 kw.} \qquad (6.2\text{-}3)$$

For this condition the receivers are likely to be in low noise level areas where E_{nm} is less than 25 decibels. As a result, we will assume a nominal requirement of 10 kilowatts radiated power for the transmitting system.

6.3 Antennas

As shown in the preceding section, the nominal radiating power requirement for the Omega transmitting station is 10 kilowatts. Numerous types of transmitting antennas have been designed and constructed for radiation at VLF, and it is important to first note the requirements of the Omega system before attempting to specify the type of antenna likely to best meet the needs of this system. First, the type of transmission is essentially a fixed frequency keyed on and off at a rather slow rate, with the result that only a small amount of bandwidth is required. This is a very important

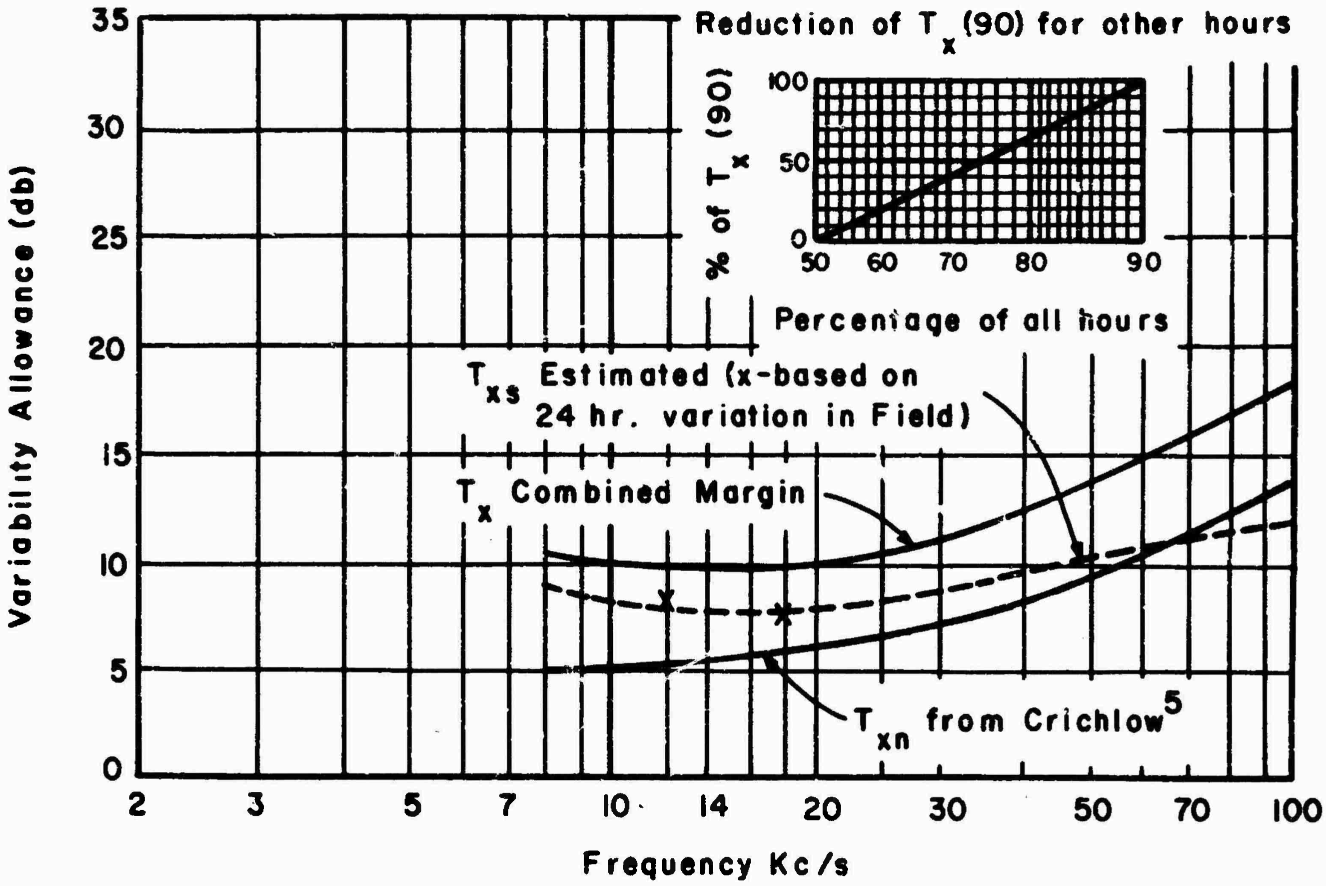


FIGURE 6.2-5. SIGNAL FADING AND NOISE VARIATION MARGIN FOR 90% OF ALL HOURS

consideration since it can materially affect the type of antenna which will give optimum performance for a given cost. In some unpublished work, Watt (1964) shows that the radiating capabilities of a given antenna can be expressed as follows:

$$P_r \text{ (kw)} = 5.44 \times 10^{-20} V_b^2 \text{ (kv)} \; A_{eq}^2 \; f^4 \text{ (kc)} \qquad (6.3\text{-}1)$$

where:

P_r (kw) = Power radiated in kilowatts,

V_b (kw) = Voltage limits of the antenna in kilovolts,

A_{eq} = Equivalent area of the antenna in square meters

f (kc) = Frequency in kilocycles.

For the voltage-limited case in a given frequency, the power limitation is now independent of height and depends only upon area squared. This area is not the actual area of a given solid plate but the effective area which increases with height due to fringe effects.

Since from previous considerations the frequency is fixed at 10.2 kilocycles, we have only to determine the equivalent area and the voltage required to obtain a radiated power capability of at least 10 kilowatts. Operational experience has shown that VLF antennas can be operated from approximately 50 kilovolts to as high as 250 kilovolts and that the large majority operate in the vicinity of 150 kilovolts. Since this voltage range results in fairly straightforward design and insulation problems which are not too difficult, 150 kilovolts will be assumed in the calculations which follow.

In Equation 6.3-1, the equivalent area required is 8.7×10^5 square meters. From studies by W. W. Brown (private communication), the effective area is approximately equal to the actual area plus a fringe area equal to the perimeter of the actual area times the height of the antenna. This means that actual physical area for short antennas must be close to that of the equivalent area, and an antenna radius of about 500 meters is required. As height increases, physical area can be decreased.

The question of optimum height is closely related with the manner in which the cost of supporting structures increases with the height of the mast or tower. Additional studies by Watt (1964) on the cost of multiple unit installations have indicated that the optimum height in terms of power radiating capabilities as a function of cost occurs when the slope of the cost versus height curve reaches 2.0. It is interesting to observe that the cost versus height curve for most vertical support structures reaches an exponent of about 2 in the 600 to 700 foot region. (This height is employed in many of the earlier VLF antenna installations such as NSS Annapolis, etc.) Since slow speed cw operation was employed, the primary concern in these antennas was to radiate power capability with little thought of bandwidth. When bandwidth becomes an important consideration, a different exponent is reached for an optimum condition which actually increases the height of the antenna appreciably.

The actual height which should be employed for a given antenna is not always the 600 to 700 feet stated above. If the power radiation requirements are small, a single vertical mast may satisfy; and the height need only be enough to meet the power requirement for the voltage specified. When the power requirement increases, the height must increase until providing the radiated power with a vertical antenna alone becomes uneconomical. An appreciable increase in power-radiating capabilities of a vertical mast can be obtained by appropriate top loading. As the amount of top loading increases, it is more economical to build an additional element or to elevate the ends of the umbrella wires loading the primary mast. In general, the average cross-section height of the antenna may be somewhere in the 600 foot range, however, in a simple configuration of this type a given amount of power radiation capability can be obtained more economically with the center tower situated higher than the surrounding towers, because the central tower is common to all the top loading sections or panels while each outer tower supports only one end of a top-loading panel.

Since the required Omega power is only about 10 kilowatts, a large enough antenna would be composed of eight radial elements of about 2000 foot length giving a capacitance of about 0.06 microfarads. These radials can be supported from a 1200 or 1400 foot central tower in a single span, if the outer edges of the top-hat are supported by towers at least 300 feet high.

Theoretical studies by Pierce (private communication) and model studies by Woodward (1963) indicate that the effective height of such a structure is about seven tenths of the average height of the center and the edge, or about 150 to 170 meters in this case. This is slightly more than the effective height of the Cutler antenna, although the average physical height is smaller. The power radiating capability of a small electrical antenna is:

$$P_r \approx 6.95 \times 10^{-13} V^2 C^2 h_e^2 f^4 \qquad (6.3\text{-}2)$$

where:

V = Antenna voltage,

C = Capacity in farads,

h_e = Effective height in meters,

f = Frequency in cycles per second.

For the values of h_e and C given above, it appears that 10 kilowatts can be radiated with an antenna voltage of about 120 to 140 kilovolts.

For the power range of 10 kilowatts at 10 kilocycles required here, it appears that the economic advantage lies with the tall center tower as compared with a "flat-top" supported by the same number of towers all of equal height. The optimum ratio of center tower to outer tower heights will depend upon a number of factors. For television-type towers with no lateral load, costs increase as the square of the height. The stresses produced by ice and wind loading on many thousands of feet of antenna cable will require a stronger and more costly center tower. We are not able in this general report to indicate all the details of optimum antenna design, but we wish to call attention to this construction concept. A decision regarding antenna optimization can clearly be made as soon as mechanical, electrical and cost analysis studies have been completed.

Since an appreciable number of transmitting installations are required, it appears desirable to spend appreciable effort in both optimizing the design for a 10 kilocycle transmitting installation and arriving at the necessary requirements for ice loading, etc. which will be required for stations operating in cold climates. Additional factors such as hurricane winds will also have to be considered for some installations.

The last factor which must be determined in the design objective for a VLF radiator is its efficiency. The method of optimization involves a consideration of primary power costs as opposed to amortization of station costs in terms of transmitting antenna, ground system, and transmitters. In general, an increase in antenna efficiency can be brought about by increasing the amount of copper placed in the ground system or by increasing the height of the antenna. As a result, an increase in efficiency will require an increase in antenna system cost. On the other hand, the initial cost of the transmitter and the power bill will be reduced. For remote areas, the power costs can be rather high if primary power is supplied by diesel generating sets.

In view of the fact that investment costs increase while operating costs decrease with efficiency, it is obvious that there is an efficiency which will yield a minimum yearly station cost. This type of analysis has been carried out in Appendix A. The results obtained indicate a minimum in the 30 percent region; however, since the minimum is very broad, bandwidth considerations lead to the choice of a somewhat lower efficiency, probably not more than 20 percent. Varying construction costs or power costs may modify this value in some geographic locations.

6.4 Transmitting Sites

The requirements for an Omega station transmitting site are, in many cases, rather obvious. First, the site must fit into an overall geographic pattern which has been discussed earlier. We will consider here some of the rather local conditions which must be met. There must be sufficient area to permit the erection of the various towers and top loading and sufficient open ground to permit the placement of the radial ground system. The exact length of the ground system required will be resolved after the antenna design is fixed and the specific efficiency is determined. In general, a clear radial distance of one to two kilometers will be required to achieve the design objective of 20 percent efficiency.

The installation must be as far removed as possible from airfields or commercial air routes and at some locality where adequate housing can be provided at a minimum cost. The initial cost of land will, on the other hand, tend to make it desirable to remove the installation from proximity to urban areas. As a result, numerous compromises must be made in each of the specific site selections. Availability of primary power and communication lines will also be important considerations in the exact location of each station.

6.5 Transmitters and Matching Networks

It is important that the transmitters be designed to give highly reliable service. The nominal power requirement is approximately 100 kilowatts; however, consideration should be given to the possibility of obtaining transmitters with a nominal rating of twice this amount for operation in a very conservative manner to prolong the life of various components. In addition, provision must be made for automatic switching which will permit the disconnecting of a transmitter that has undergone failure and the reconnecting of a stand-by unit within a very short time. It is estimated that automatic switching can be provided for this purpose and that a new transmitter can be switched on the line within a matter of a second.

Also, a system of extremely adequate spares must be provided, and the possibility of including a third stand-by transmitter or at least a means of rapidly repairing a disabled unit should be considered.

An alternate method would employ two transmitters, similar to those used in the experimental system, operating in parallel. In the event of unexpected failure, manual or automatic cut-off of the offending unit would result in loss of only half the radiated power. Again, a third unit should be provided in case one is under repair for an extended period.

In the same area of system reliability and spares, extra insulators and antenna parts should be on hand at each of the installations, in addition to a means of repairing damages to the antenna tuning coil. Relative to tuning coil design, it is desirable that each tuning helix have a carefully designed lightning protection system.

The nominal frequency range is from 10 to 14 kilocycles (Chapter 4). The lowest carrier frequency is 10.2 and the highest is 13.6. Provision must be included for rapid switching (within 0.2 seconds) between any of four chosen frequencies in a repeating 10-second sequence.

The possible requirement of a small percentage phase modulation (Chapter 4) must also be considered. Modulation frequencies are expected to range from 11 to 227 cycles (see Section 2.3).

The antenna-transmitter matching network must be carefully designed, since it will be required to operate over the 10 to 14 kilocycle frequency range with rapid switching (0.2 second) between four preset frequencies in a 10-second repeating sequence.

The possible requirement of phase modulation at frequencies from 11 to 227 cycles may increase the complexity of the matching network since the higher frequencies will be much greater than the nominal bandpass of the antenna. In view of the small phase shifts required, toleration of the rather large ratio of input-to output side-band amplitudes that will result from a static, single-tuned antenna circuit may be possible.

6.5.1 Antenna Matching Network

For a transmitter to operate efficiently, the output voltage and current must be in phase and the radio frequency output voltage amplitude must be substantially constant. However, at the Omega signal frequencies, all feasible transmitting antennas are electrically very short, and hence highly reactive, and a complementary reactive coupling network with a response matched to the signal waveform must be provided to store and release reactive energy to the antenna reactance as required to provide an essentially resistive load for the transmitter.

Each transmitter is required to radiate four different types of signals.

(1) A CW signal at some sub-multiple of 408 kc which may be phase modulated at a very low frequency.

(2) A 10.2 kc carrier, tangent modulated at 11-1/3 cycles with an index of 0.32.

(3) An 11-1/3 kc carrier, tangent modulated at 45-1/3 cycles at an index of 0.26.

(4) A 13.6 kc carrier, tangent modulated at 226-2/3 cycles with an index of 0.73.

Tangent modulation is employed, so that the total signal consists of only the carrier and two discrete sideband components.

To radiate the 408/N kc side frequency and the 10.2 kc carrier, the antenna coupling network may employ a simple series inductance to tune the antenna to resonance. This is possible since the impedance at the 11-1/3 cycle sideband frequency is sufficiently close to a pure resistance that acceptable efficiency can be obtained from the transmitter.

With some loss in transmitter efficiency, the antenna may also be series tuned to radiate the 11-1/3 kc component with its 45-1/3 cycle modulation. The information rate required of the 4th step lane identification signal is less than at the finer resolutions, and as a result, lower power can be accepted in these sidebands. By limiting the modulation index to 0.26, the transmitter efficiency can be kept at a level where it is more economical to suffer the resulting energy loss in the transmitter than it is to provide a more effective, multiple tuned, coupling circuitry.

The 226-2/3 cycle modulation of the 13.6 kc carrier must supply signals with power adequate for position fixing with the sideband components alone, as well as to assure the reliability of the third step of lane identification. The spread of these sidebands about the carrier is so large that it is more economical to provide a complex coupling circuit which matches the antenna to the transmitter at each of the three frequencies rather than to suffer the inefficiency entailed in radiating such a signal with simple series tuning.

At those stations where there is a central tower, the transmitter is to be located away from the base of the tower so as to be out of the zone of falling ice and other debris. The transmitter will be coupled to the antenna via a high power cable some 2000 feet long, with transformers at each end which change the impedance level to a satisfactory value. Antenna matching circuitry provides for the storage of reactive energy as required to maintain transmitter efficiency. In such a linear electrical network, each frequency component of the antenna current is the response to a voltage of the same frequency in the output of the transmitter, which is independent of the magnitude and phase of components at any other frequency. The total voltage and current at the transmitter is the linear sum of the partial components giving rise to the carrier and the upper and lower sidebands of the antenna current.

For a reasonable transmitter efficiency, the load impedance at the carrier and both sideband frequencies should be resistive and of the same value, and be transformed to a magnitude that is a proper load for the transmitter.

Since the signal contains only the carrier and the first order sinusoidal components, there is no requirement that the impedance be flat over any finite band of frequencies. The only requirement is that the input impedance and transfer function of the antenna coupling network have the specified values at the carrier and sideband frequencies. The characteristics at other frequencies are immaterial, since, when transmitting the Omega signal, no other frequency components are present.

The transmitter output transformer, the cable, the antenna transformer, the tuning helix and the antenna capacitance, taken together, form a lumped constant electrical network, which, with the addition of a series input capacitance, is equivalent to a series-parallel-series resonant circuit as shown in Figure 6.5-1.

Analysis of this network shows that, if the shunt capacitance is related to the antenna capacitance by the relation

$$C = C_a \frac{2 + q/Q}{y^2 + \frac{1}{Q^2}}$$

where $y = 2f_m/f_c$

= fractional sideband spread relative to the carrier

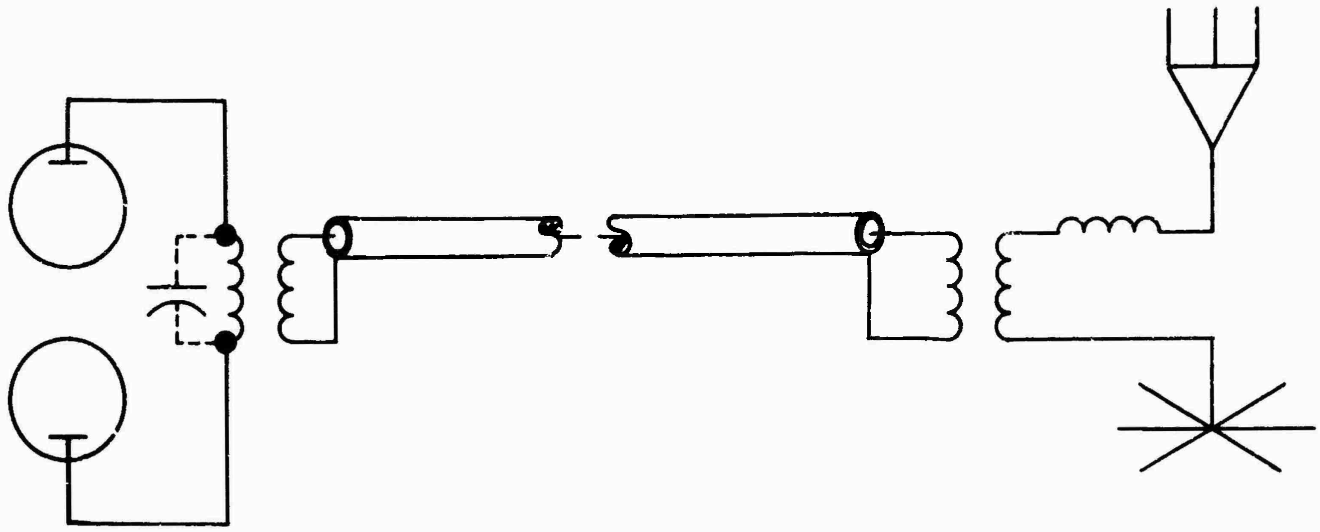
Q = Antenna Circuit Q

q = Cable circuit Q

the real part of the input impedance is reduced to the same value at the sidebands that it is at the carrier. The proper choice of the input series inductance and capacitance makes the input reactance zero at the side band frequencies as well as at the carrier frequency.

Having established the network parameters in the prototype circuit, the network can be transformed to a practical circuit, including the cable capacity as part of the shunt circuit capacitance. The transformation ratios of the transmitter output transformer and the antenna transformers, provide suitable impedance levels in the cable and at the transmitter output.

The resulting coupling network is as shown in Figure 6.5-2.



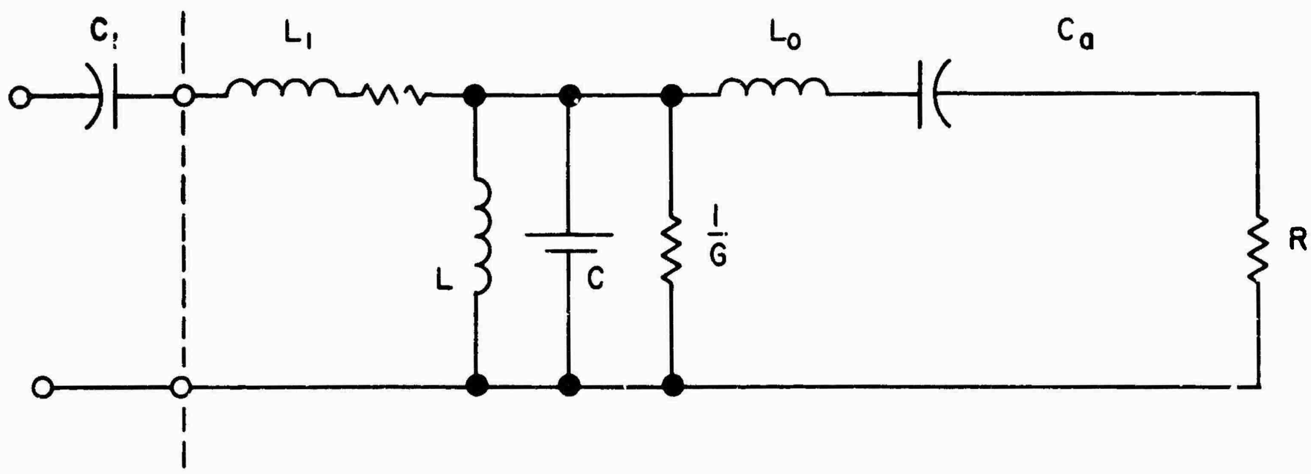


FIG. 6.5-1 ANTENNA COUPLING CIRCUIT AND EQUIVALENT CIRCUIT

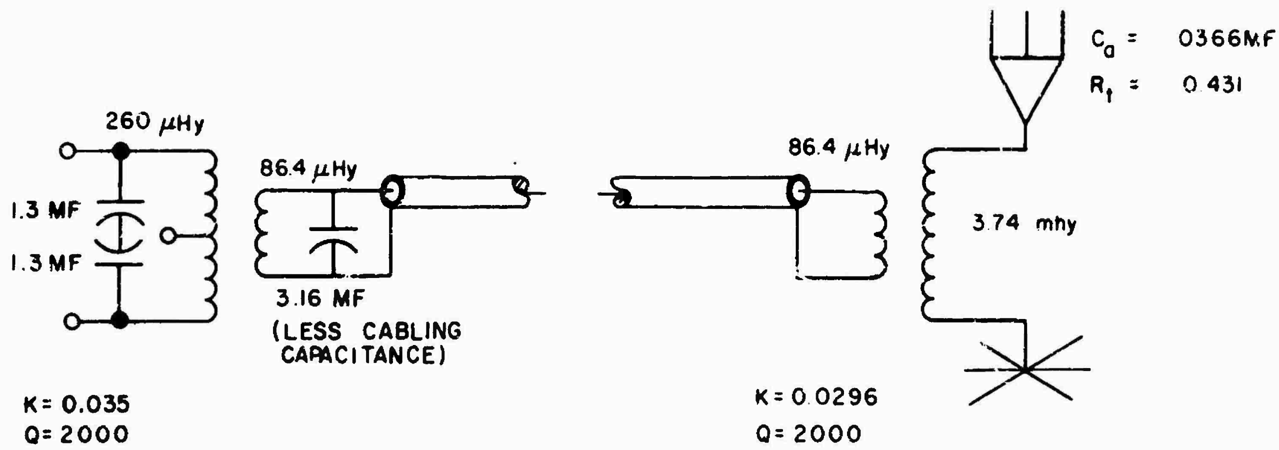


FIG. 6.5-2 REALIZED ANTENNA COUPLING CIRCUIT

6.6 Transmitter Timing

For the phase contour patterns by which position is determined to be stationary, the Omega signals must be synchronized.

Basically, this is accomplished by providing each station with an independent Timing Standard of sufficient stability to maintain phase within a microsecond for upwards of twenty-four hours at a time. Then the relative signal phases, as observed at each of the stations, are relayed to a computer at one of the stations. This determines the deviation of each signal from the mean and instructs the stations to correct their timing rates to maintain synchronism.

The timing system of a station, shown in Figure 6.6-1, consists of a Timing Standard to maintain signal phase with the required accuracy; a Signal Synthesizer to derive the frequency components and multiplexing waveforms required to form the Omega Signal format; a Phase Modulator to generate the phase-modulated signals at the various carrier frequencies in turn; and the transmitter and antenna coupler.

In addition, there are a Communications Circuit Receiving Terminal to receive and decode the modulation on the side frequency component of the Synchronization Control Station, and Phase Data Sending Terminal equipment to encode the phase observations made at the local station for transmission to the Synchronization Control Station.

6.6.1 Phase Continuity

In establishing the Omega transmission, the signal components are derived by a process of frequency synthesis from a 408 kilocycle timing wave; and the relative phase of each derived component may be at any multiple of a period of 4[illegible] kilocycles. Thus, a high order of phase ambiguity exists between the synthesized components, depending upon the particular counts at which the frequency dividers in the synthesizers happen to res[illegible].

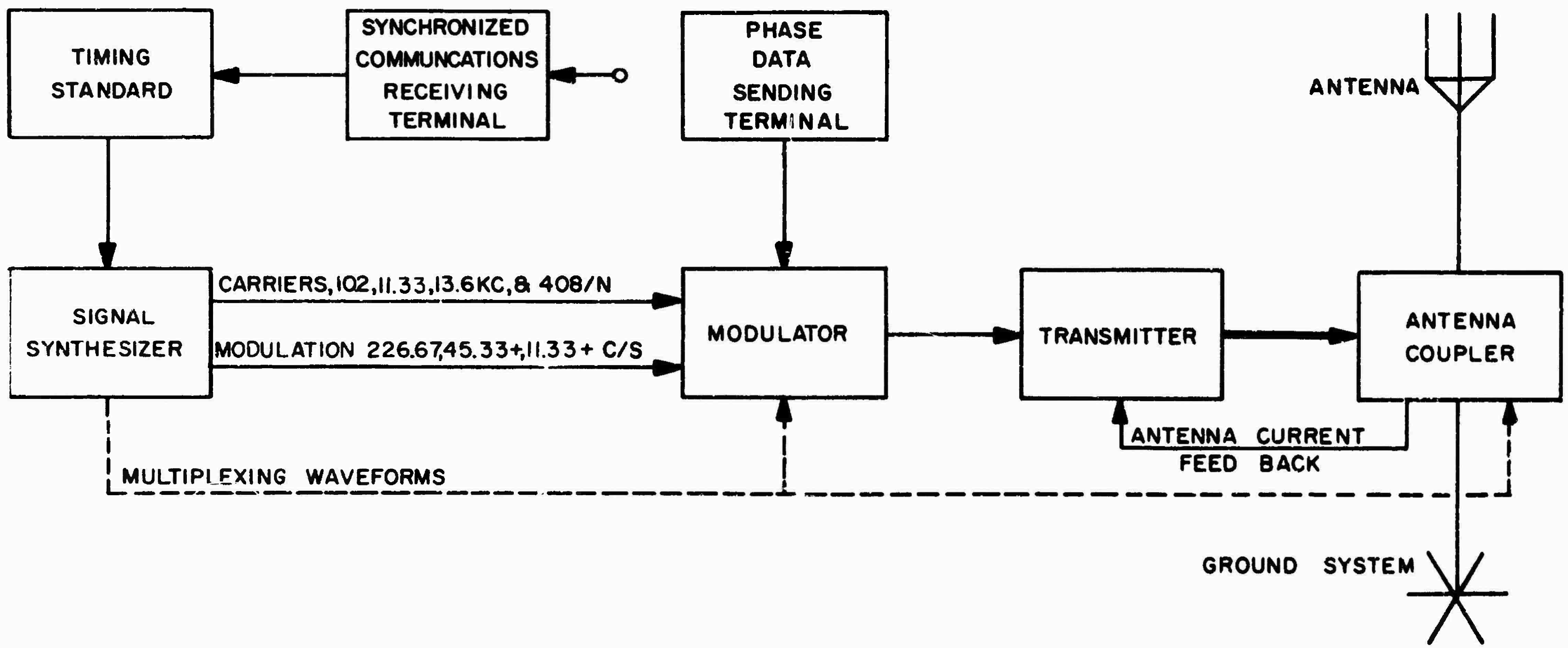


FIG. 6.6-1 TRANSMITTER TIMING SYSTEM

As long as the equipment operates continuously without any internal disturbances, phase continuity of the transmitted signal will be maintained; however, various units of the equipment must be removed from time to time for servicing, and some provision for maintaining phase continuity of the transmitted signal during changeover of equipment is necessary.

The remote probability that equipment disturbances causing a counter or frequency divider to miscount, thereby introducing a phase jump in the emitted signal, also exists.

Continuity of phase is more vital than continuity of signal. A lapse of a few seconds or even a few minutes in signal transmission can be successfully bridged under most conditions provided the signal has the correct phase when it reappears. However, a phase jump, even of a side frequency, causes the loss of lane count in a tracking or dead reckoning receiver without momentary loss of a signal.

Because of the diurnal shift and other propagation anomalies affecting the system, dependence upon the signals received from other stations to resolve internal frequency synthesizer ambiguity is not feasible. Thus, provisions must be made to maintain phase continuity within the station equipment in the event of need to exchange equipments for servicing, the possibilities of counter jumps, etc.

Phase continuity is assured by providing redundant timing standards and frequency synthesizers. A concensus of three or more independently derived signals are taken at all stages of the frequency synthesis operation where phase continuity cannot be recovered within the station equipment to indicate the correct phase or counter reset.

6.6.2 Timing Standard

The Transmitter Timing Standard consists of four Atomic Frequency Sources and combining circuits, shown in Figure 6.6-2, that provide output at the mean ..quency of the four standards. The combiner also makes it possible for any of the four standards to fail, or to be disconnected for maintenance, without any significant change in the phase or frequency of the combined output.

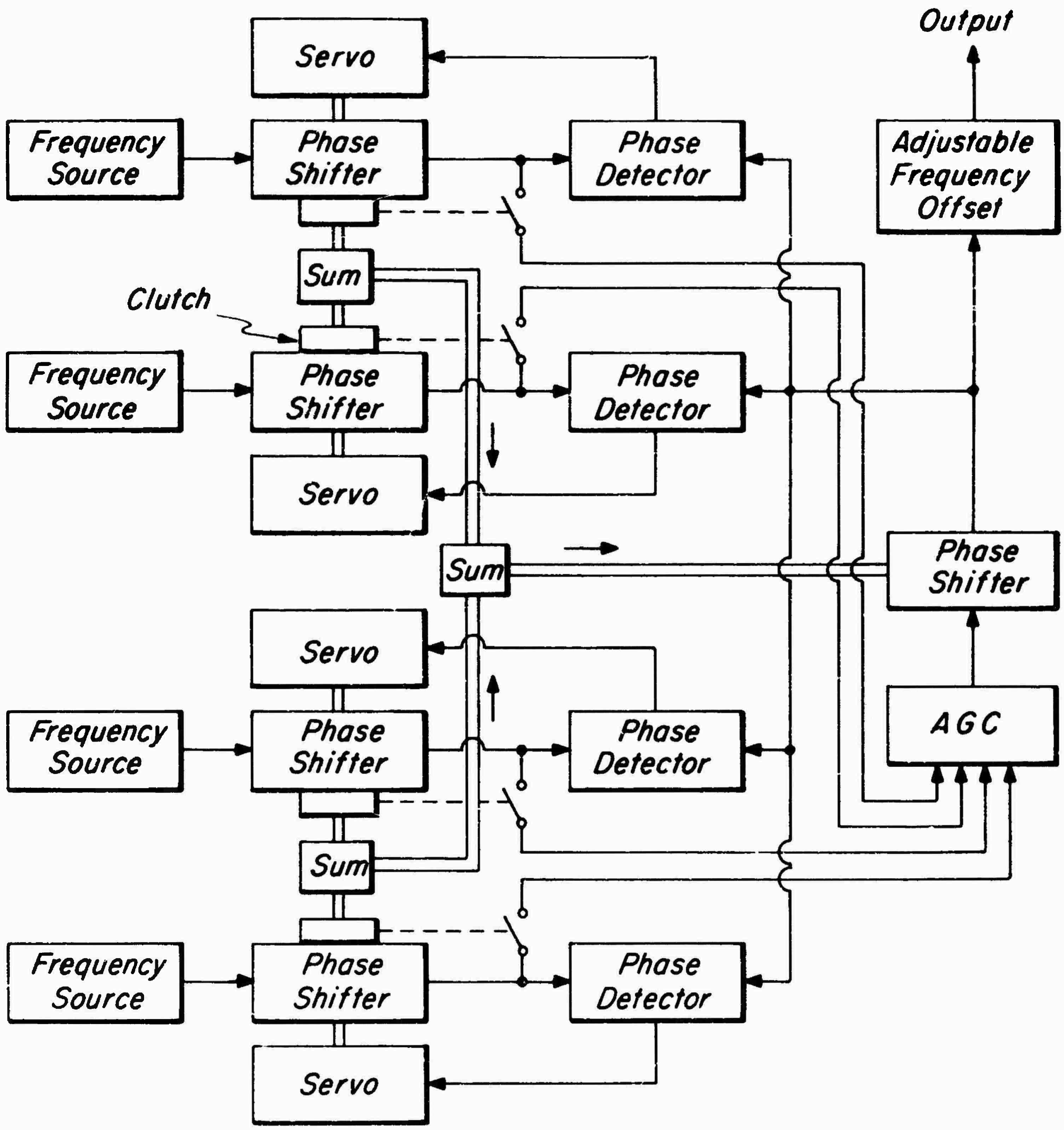


FIG. 6.6-2 TIMING STANDARD

This combination of sources may be made at any convenient frequency. If it be done at one of the conventional output frequencies of the standards, it may make eventual replacement of standards more simple. On the other hand, combination at 408 kc places the derivation of that frequency within the control of the combiner, so that a separate guarantee of the phase of 408 kc is not required.

The combiner works as follows. Each source of frequency is taken through a servo-driven phase shifter and all four outputs are separately compared in phase detectors to a single reference phase. If the servos are operating, the four phase-shifter outputs have substantially identical phases. These four frequencies may therefore be added in a linear network and used, after a 90° phase shift, to supply the reference for the phase detectors. This excitation frequency for the phase detectors is the output frequency of the combiner. It is passed through an automatic gain control circuit so that its amplitude is independent of the number of frequency standards that are contributing to it.

If it happened that the 90° phase shift were not exact, or that (on the average) the four phase detectors did not have zero output at exactly 90°, the phase servos would run indefinitely even if the four frequency sources were identical. This would result in an output frequency differing from the mean frequency of the four standards. To correct this effect, the rotations of the four servos are algebraically added and used to adjust the excitation phase of the phase detectors. This phase will stabilize at the point where the average servo rate is zero, and at which the output frequency is accurately the mean of the four standard frequencies. Since this "fifth" phase shifter need not move far, even if one or more sources be removed from the combiner, the rate at which it is rotated is not critical.

Rate-of-change of each error voltage is to be measured, by devices not shown in Figure 6.6-2, and used to disconnect any standard whose output fails or whose frequency undergoes a change greater than the design limit. In this case, a clutch disconnects the offending servo from the fifth phase shifter and a switch removes the frequency of the offending source from the mean which provides the output frequency. These links may be manually restored by a reset button after the trouble has been corrected and after the disconnected servo has settled into phase again.

A counter should be driven by each servo. This will show the integrated phase difference between that source and the average, and will facilitate keeping records of the behavior of the individual frequency standards.

A single adjustable frequency offset is shown in Figure 6.6-2, although it is probable that both convenience and reliability will be enhanced if four of these devices are used between the frequency sources and the combiner. In its most simple form, this offset may consist of several synchronous motors whose outputs are mixed in differentials and used to drive a single rotary phase shifter. If four of these motors are geared to give the phase shifter individual rotation rates of 1, 3, 9, and 27 units, the motors may be turned on, or off, or reversed to give a total of 81 rates of rotation, 40 in each sense. These rates might most conveniently be from ±0.1 to ±4.0 centicycles of 10.2 kc per day. These small rates would be used to adjust the phase of each station with respect to the others, as discussed elsewhere, while leaving the precision frequency sources alone as much as possible.

Another motor and differential in the frequency offset should be used to adjust the frequency output of the station to keep it in agreement with UT2 time. This is a much greater change (of parts in 10^8 rather than in 10^{12}), but it requires only a modest number of values that can most easily be achieved by change gears, since adjustment will be required not more than once per year.

Frequency offsets can of course be obtained electronically rather than using mechanically driven phase shifters, see for example Barnes and Wainwright in December 1965 issue of the IEEE.

6.6.3 Communication Receiving Terminal Equipment

Associated with the timing standards is the Communications Circuit Receiving Terminal equipment. This equipment is adapted to pick up and decode the modulation of the side frequency component of the Omega Station designated the Synchronization Control Station, which carries the coded messages giving the timing rate corrections required to keep the stations of the system in

The decoded messages may be applied through a digital/analog converter to control the mean rate of all four timing standards, or the correction may be decoded and applied manually.

6.6.4 Frequency Synthesizer

The Frequency Synthesizer is subdivided into three units: the Carrier Component Generator, the Modulation Component Generator, and the Multiplex Timer.

6.6.4.1 Carrier Component Generator (Figure 6.6.3)

The Carrier Component Generator consists of four divider units, each providing the three basic carrier frequencies (10.2, 11.33. and 13.6 kilocycles) and the Side Frequency Component (408/N kilocycles).

The outputs of the four units (16 lines in all) go to a concensus logic combiner, a logic matrix that determines which, if any, of the four units is producing nonsimultaneous or nonsynchronous output and applies aligning reset triggers to the offending unit, whose dividers are brought into step with the remaining units.

Associated with the logic matrix is an alarm unit (not shown) which records each occurrence of out-of-alignment correction.

By having four units with output defined by concensus of the four, any one can be removed for servicing with the phase established by any two out of the three remaining in service, and the fourth can be returned to service and brought into alignment without disturbing the phase continuity of the system.

6.6.4.2 Modulation Component Generator

The Modulation Component Generator, similar to the Carrier Component Generator, consists of four divider units, a concensus logic unit

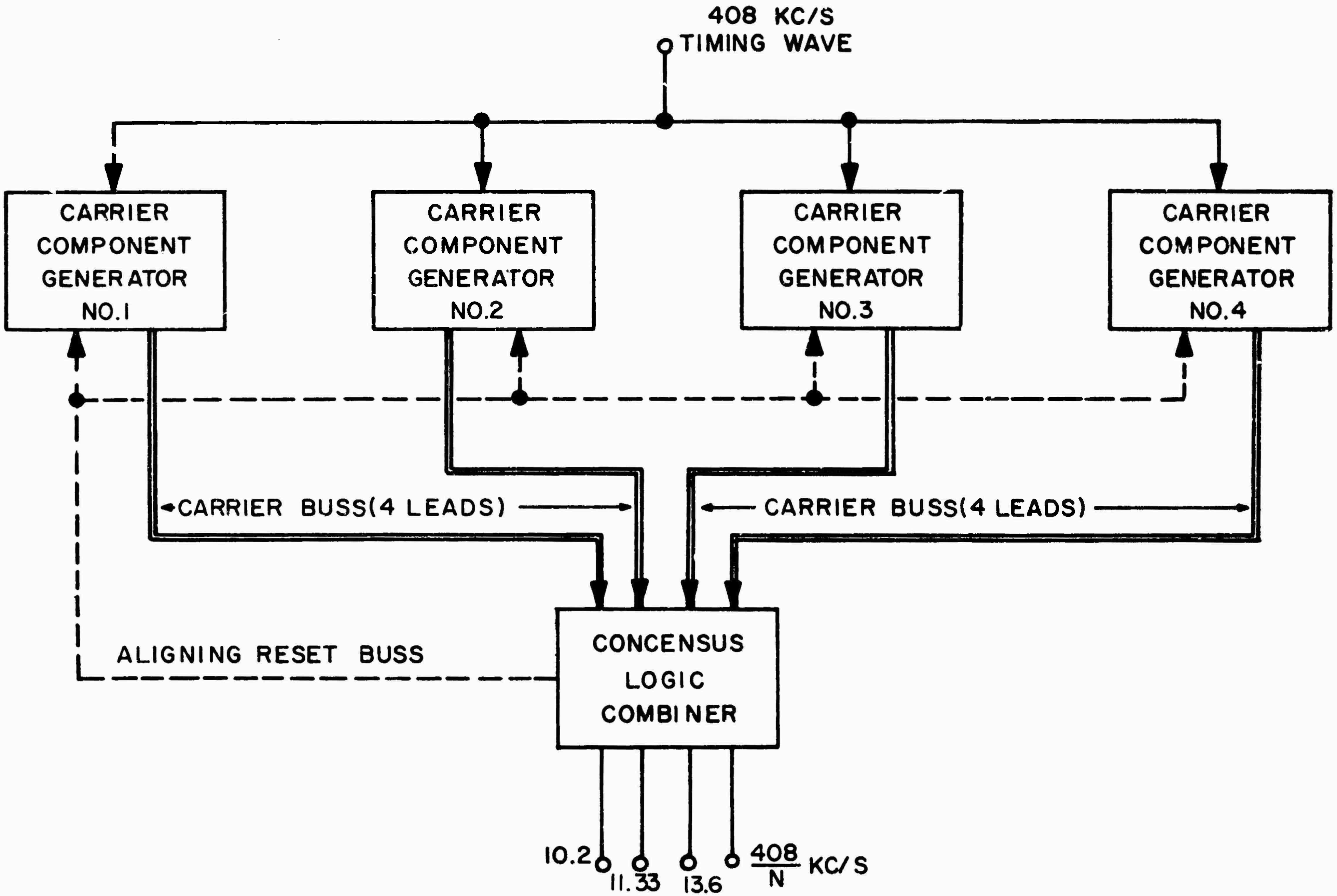


FIG. 6.6-3 CARRIER COMPONENT GENERATOR

producing the three modulation frequency components (226-2/3, 45-1/3 and 11-1/3 cycles), and a series of triggers at 0.1 second intervals to drive the multiplex timer, as shown in Figure 6.6-4.

Again, each divider unit produces all four components. The concensus logic combiner unit determines which, if any, of the dividers in the four units is out of step, provides resetting triggers to bring the offending unit into alignment with the others, and records occurrences of out-of-alignment.

6.6.4.3 Multiplex Waveform Generator

The Multiplex Waveform Generator also contains four dividers, which divide the 0.1 second triggers by 100 and provide the waveforms required to synthesize the Omega multiplex sequence. As before, a concensus logic combiner determines which, if any of the four units is out of alignment and supplies reset triggers to realign the offending unit. (See Figure 6.6-5)

The Multiplex Waveform Matrix is supplied as a single unit since, if the driving waveforms are correct, a failure in the Multiplex Waveform Matrix does not destroy phase continuity in the multiplex sequence, because the phase of the sequence is defined by the multiplex timers and the other dividers and would be re-established when the matrix again became operable.

The Multiplex Waveform Generator also contains a logic matrix combining the modulating (and RF carrier components if necessary) with the multiplexing signals to establish the 30 second timing epochs that define the basic timing period of the station.

6.6.5 Phase Modulator

The actual signal format is established by the Phase Modulator (Figure 6.6-6) consisting of a phase modulator unit and a set of eight gates or relays operated by the multiplexing waveforms, which apply the specified carrier and modulation components to the modulator in turn as required to form the Omega segments transmitted from that station.

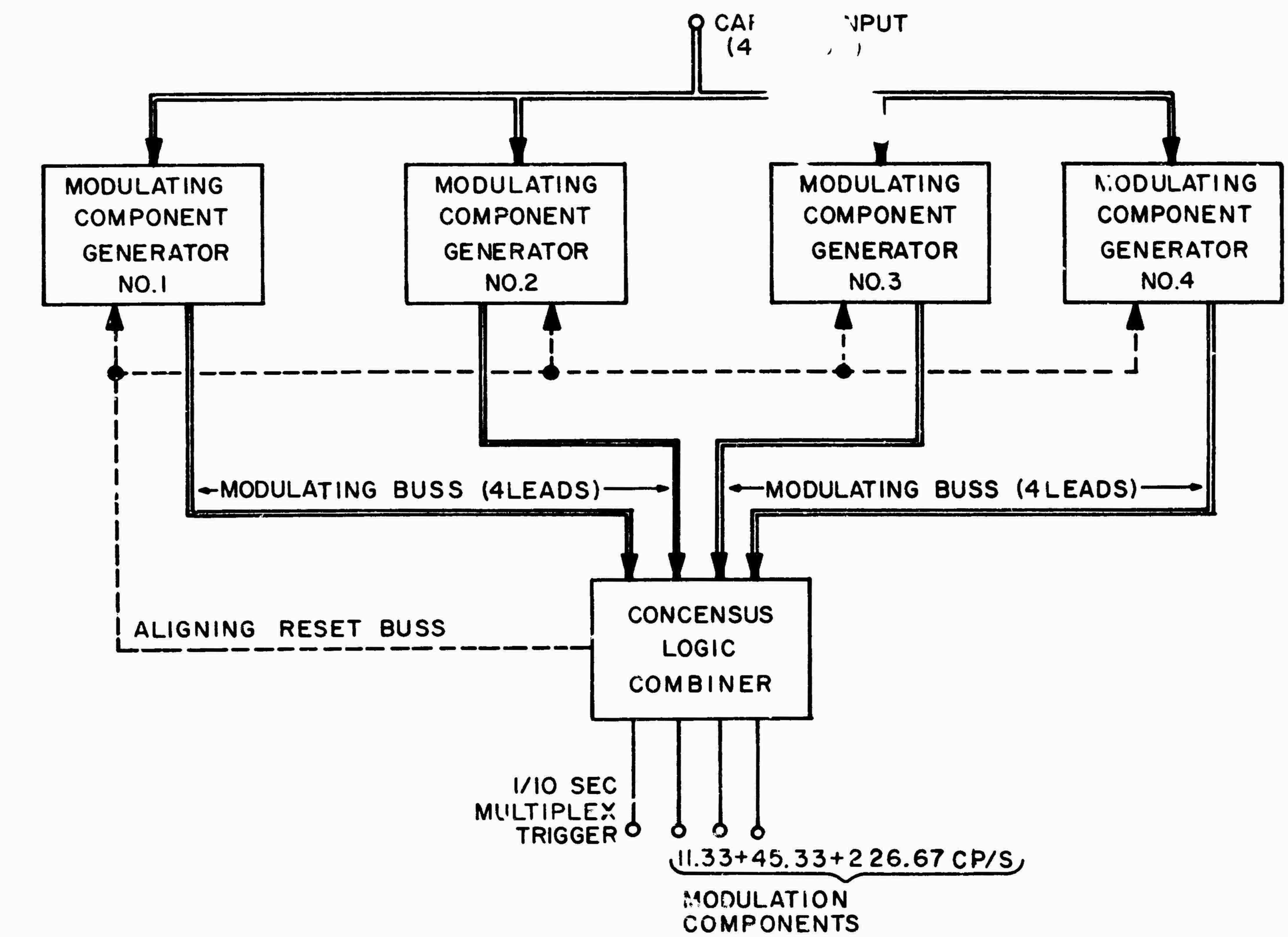


FIG. 6.6-4 MODULATION COMPONENT GENERATOR

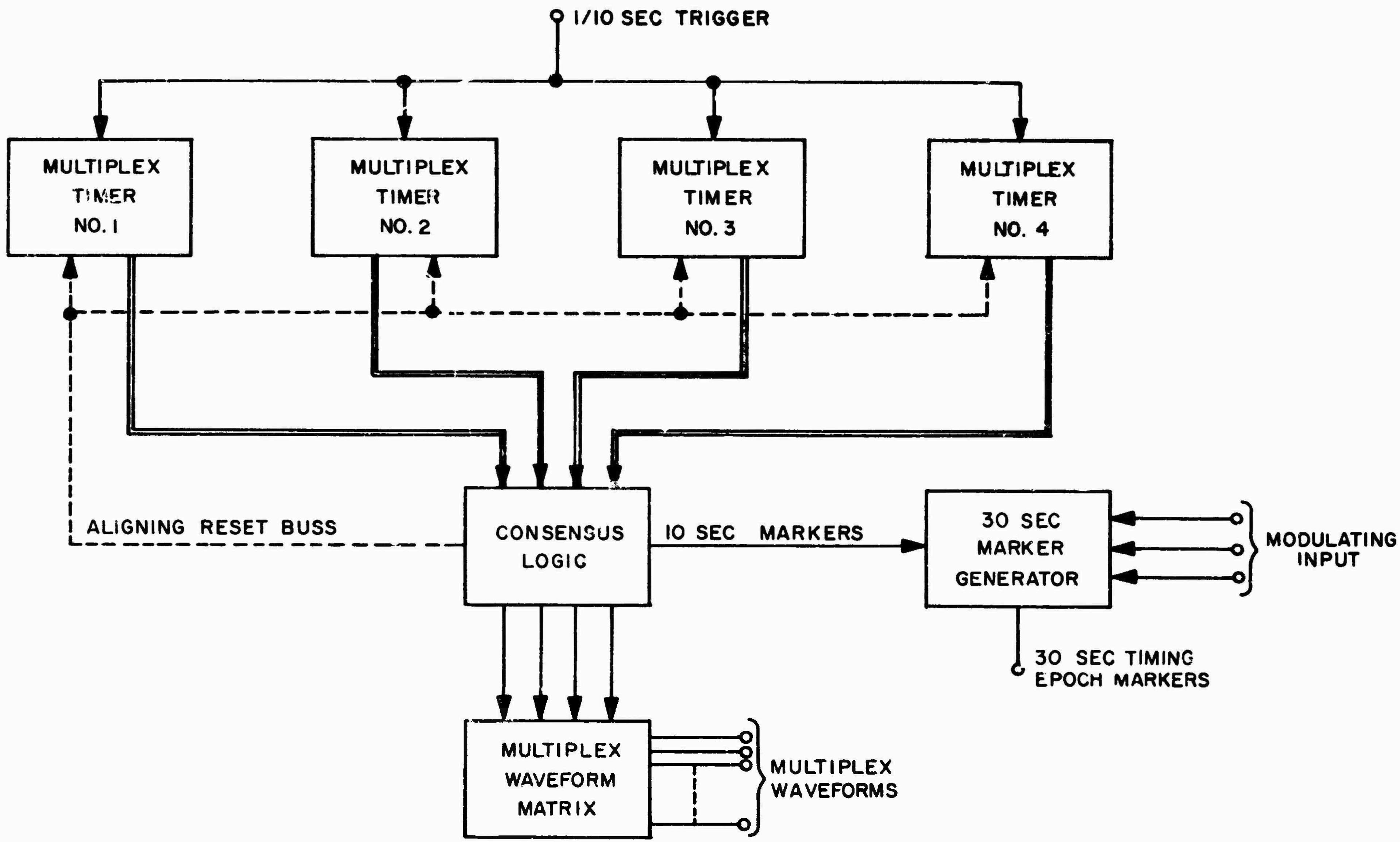


FIG. 6.6-5 MULTIPLEX WAVEFORM GENERATOR

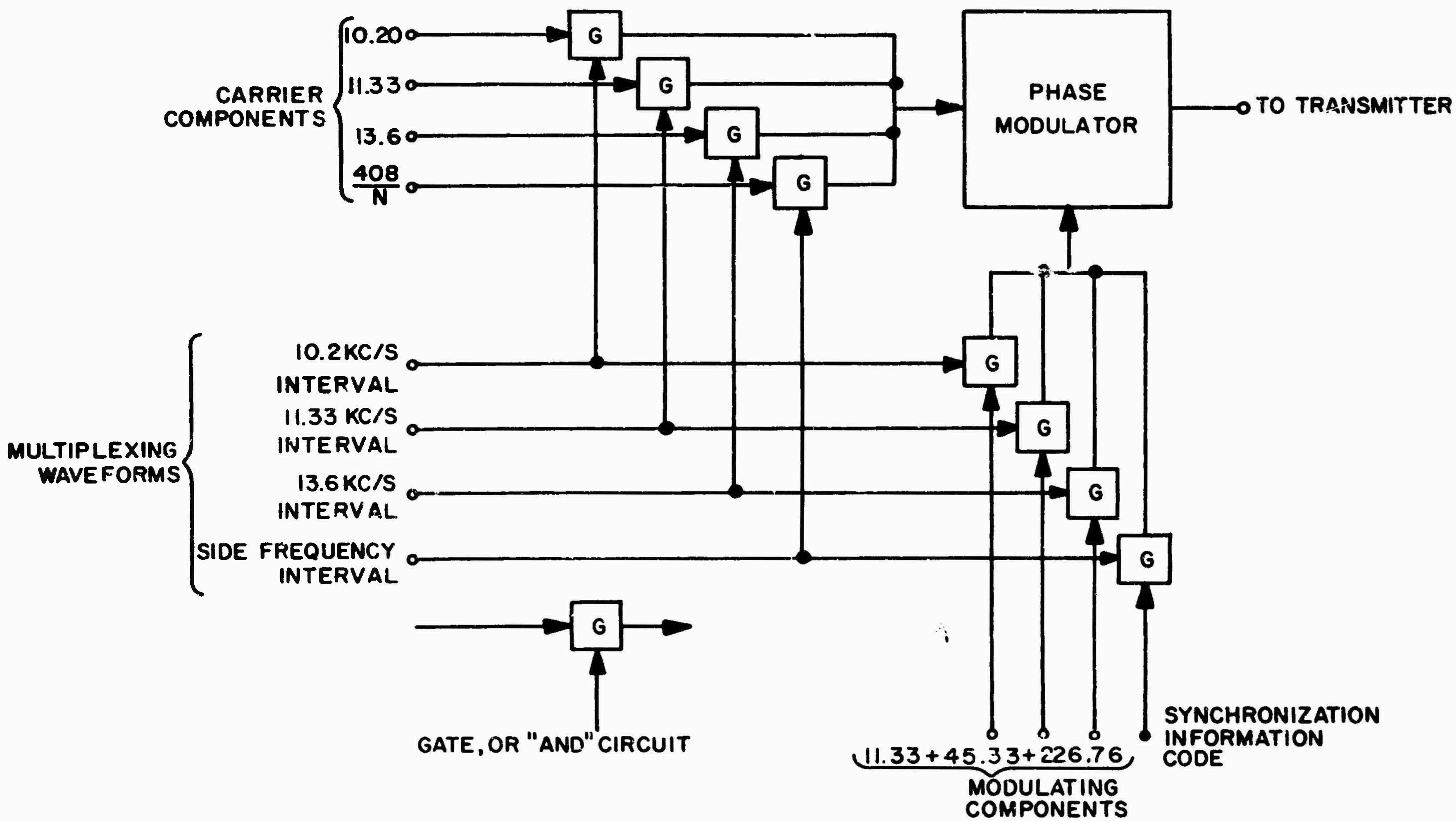


FIG. 6.6-6 MODULATOR

6.6.6 Phase Data Sending Terminal

Associated with the modulator is the Phase Data Sending Terminal. This equipment receives from the local system monitor the data on the relative phase of the other stations of the net with respect to this one, translates it into a phase code suitable for modulation onto the side frequency of the station, and applies this coding to the phase modulator during the side frequency transmission intervals for transmission to the Omega station acting as the Synchronization Control Station.

6.6.7 Transmitter

The Transmitter (also discussed in Section 6.5) consists of an exciter, a power amplifier, and a power supply, which amplifies the output of the phase modulator to the desired output signal level. (See Figure 6.6-7)

Provision is made to combine antenna current feedback with the driving signal to minimize phase shift through the power amplifier and antenna coupler (Section 6.6.9).

6.6.8 Antenna Coupler

A simplified Antenna Coupler (Figure 6.6-8) consists of a servo-controlled antenna tuning unit adapted to keep the antenna resonant to succeeding segments of the Omega signal as radiated by the particular station, in spite of changes in ground impedance with weather, changes in antenna capacity due to varying top loading sag with temperature, etc.

The Antenna Coupler consists of a loading coil in series with the antenna and four tapped variometers, each connected to the transmitter cable output transformer by high-voltage switches operated by the multiplexing waveforms as required to tune the antenna to the various signal segments being radiated.

A current transformer and a voltage divider capacitor deliver replicas of antenna current and antenna input voltage to a phase detector, which generates a signal proportional to the relative phase of antenna current and voltage.

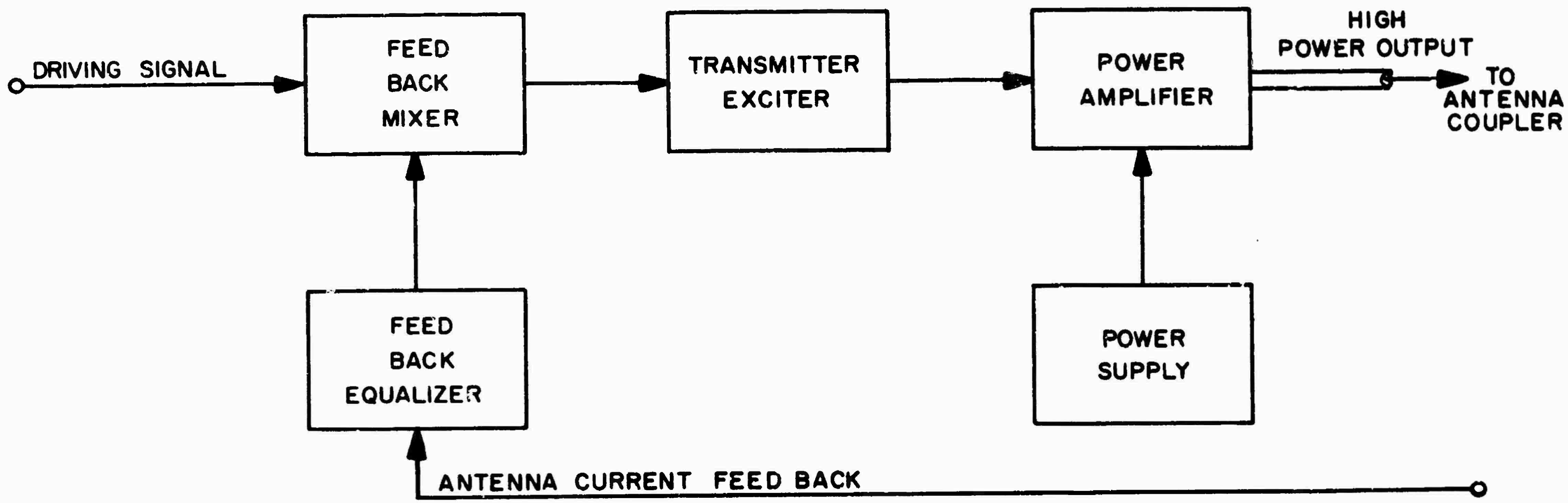


FIGURE 6.6-7. TRANSMITTER

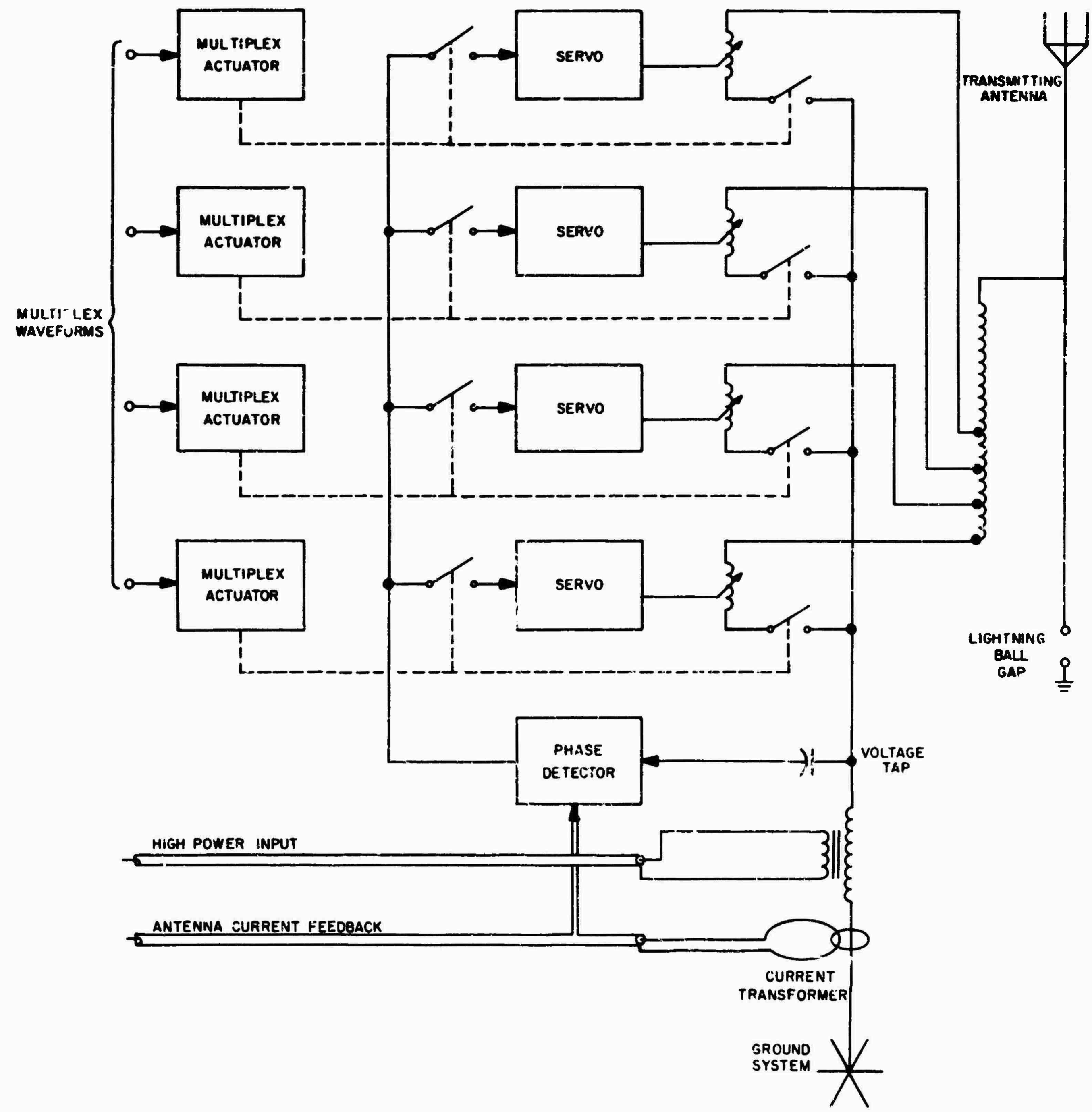


FIGURE 6.6-8. ANTENNA COUPLER

Via switches operated by the multiplexing waveforms, the phase detector output is applied to servos which adjust the variometers to keep the antenna current in phase with the antenna input voltage, i.e., keep the antenna tuned to resonance as each signal segment is transmitted.

For stable mechanical antennas, it is possible to use a single servo driven variometer which will keep the antenna nearly at resonance at each of the frequencies in spite of small changes in antenna capacitance. If wind and weather produce large capacitance changes, individual variometers are necessary. The 13.6 kilocycles tuning switch position will contain additional coupling circuitry to provide resistance loading of the transmitter at the 226-2/3 cycle side-bands as well as the carrier.

6.6.9 Transmitter Stabilization

The signals as radiated must be synchronous to insure stationary phase contour patterns.

Through the use of microsecond logic elements, careful design in the frequency synthesizers and suitable control of the timer environment, signal phase format at the input to the transmitter exciter will be stable with respect to the Timing Standards within the requisite fractional microsecond. However, the antenna impedance may vary with weather and particularly with antenna element sag which is affected by temperature, etc. In addition, there may be variations in effective signal propagation time (through the exciter and transmitter)caused by varying bias levels with resulting changes in operating points as well as changes in the propagation time with temperature and humidity between various units of the transmitter.

Hence, some means must be provided to control with respect to the transmitter excitation the signal actually radiated.

It may be assumed that the actual radiating element is the current moment of the antenna; so that the phase of the antenna current is a faithful representation of the actual radiated field, and stabilization of the antenna signal can be obtained by feedback of the antenna current (through a current transformer in the ground lead) to the transmitter exciter.

Feedback equalization will be required to obtain adequate phase and amplitude margins, since not only the phase shift of the antenna impedance should be considered but also propagation delays of the power cable between transmitter and antenna, of the feedback cable, stray capacities and impedances in interstage coupling, etc.

However, because of the extremely low operating frequencies, all cables will be very short, electrically, so that the principal reactance to be contended with is the single reactance pole introduced by antenna resonance.

With suitable broadband design and attention to circuit details, it is expected that sufficient feedback can be introduced for stabilization of emitted signal phase without introduction of uncontrollable instability in the feedback loop.

7.0 System Monitors

As in any complex aid to navigation, Omega will require a number of monitors. These fall broadly into two categories, (a) control of performance and (b) proof of performance.

The first type of monitoring is considered to be essentially a part of the establishment of synchronism and has been described in part in Section 5.2 and Section 6.6. The second function arises from the wisdom of having always an independent check upon the performance of the system and from the presumed need to explore unchecked areas and areas in which inaccuracies may be reported. Both functions are outlined in the following sections.

7.1 Synchronization Control Monitors

At each of the eight transmitting stations, there must be a receiver adapted to record the phase of every distant signal with respect to the phase of the local signals at the several transmitted frequencies.

Preferably, if such operation is feasible, this receiver or set of receivers should be located at the transmitter and utilize the transmitting antenna for reception of the distant signals, by way of a hybrid or magic tee connection. In any case, it should be located not more than a few miles from its transmitter so that there is no possibility of skywave reflections affecting the phase of the signal from the local transmitter, and so that it may be operated and maintained by personnel of the transmitting station.

The seven phase differences at each of the radio and modulation frequencies used must be continuously recorded. Because this is a specialized function in which seven different stations are observed in terms of one reference and because utilization of considerably more integration than that available in the navigator's receivers is advisable, these synchronization control receivers should be designed and constructed for their purpose.

As explained elsewhere, the phase differences as determined over the best time of day for each signal and compensated for estimated travel time, are processed mathematically to determine a best estimate of the deviation of the local signal phase from the mean of all the signals, and the result applied to control the rate of change of phase, i.e., frequency, of the transmitted signal so as to keep the deviation from the mean to the lowest possible value.

The phase difference measurements will also be reported periodically (perhaps daily) to the analysis and control center. The values reported may be filtered at the monitor so that only a single number is reported at a time for each signal component, or more complete data may be sent, and additional filtering performed at the data analysis center.

Because this monitoring function is a part of the synchronization loop, all equipment should be provided in duplicate at least with adequate spares and maintenance capabilities to insure against failure of this vital information.

7.2 Analysis and Control Center

This center could be located anywhere, but for convenience should be installed at one of the transmitting sites. The site selected should be the one at which the most reliable communication signals can be received from all other stations. The selection, therefore, depends upon the sites chosen and upon the propagation characteristics of the paths from all other stations. If necessary, information could be relayed from a station having a marginal signal; but it is believed that proper design of the slow-speed communication equipment and proper selection of the control center site should make relaying unnecessary.

The control center receives from time to time from each synchronization control monitor the several phase readings observed there. By the analytical process described in Section 5.2, the apparent phase error of each station is determined with respect to the mean of all stations. By a program that is too complex to describe in great detail, the computer at the analysis and control center determines the apparent phase retardation over each path and calculates the appropriate correction. Instructions for making this correction are then sent to each station.

The present difficulty in describing this analysis program in detail lies in the uncertainty of the ratio between probable rates of phase deviations of the sources in the stations to the phase fluctuations in transmission. If there were no "noise" (i. e., if all transmission times were exactly predictable), any phase deviation caused by a sustained frequency error could be measured exactly, and exact instructions for correcting it could be formulated. Unfortunately, the real computer program must account for the fact that an observed phase discrepancy may be no more than an observational error, in which case no action should be taken.

These contradictory probabilities can be reconciled in a program somewhat as follows:

(a) Fit the best straight line through the daily phase errors of each individual station for the latest several (perhaps four) days.

(b) Extrapolate this line to give the most probable phase error at the time a correction is to be made.

(c) Correct the station's frequency for at least two factors:

   (1) the slope of the best straight line determined in (a) above, and

   (2) an additional frequency correction that will reduce the most probable phase error by a significant fraction (say 1/2) during the succeeding day.

These rules, or others similar to them, can be modified by including terms representing the rate of predicted phase change from day to day and the comparison between yesterday's predicted phase and the phase actually observed. These latter refinements probably will somewhat improve the precision of phase-holding, although they are not essential. Final choice of rules of this type must depend upon the outcome of experiments that have only recently been initiated.

For any case, the analysis and control center must be prepared to accept data at several frequencies from eight stations, to make these various intercomparisons, and to deduce the most appropriate frequency correction that each station can make to maintain the integrity of the whole phase grid.

An additional function of the control center may be to keep the entire network operating at standard time and frequency. This is easily done by the addition of inputs to the control center from the U.S. Naval Observatory or the Royal Greenwich Observatory (or both). Corrections common to all stations can then be combined with the Omega corrections to constrain the emissions at standard time, although this is a matter of little importance to Omega itself.

When all these corrections have been determined, the appropriate action information is addressed to each station and sent out over the communication channel described in Section 4.7.

All the necessary analysis can easily be done with a desk computer, provided that the system control monitors filter each day's records into a single phase difference for each station pair. It is not yet clear how much, if any, real advantage might be gained by reporting more information to the analysis and control center and by using a digital computer and automatic signaling system.

The analysis and control center should receive all reports of readings made by both permanent and temporary monitors. It might also with advantage become the repository for operational data and complaints received from the users of Omega. These monitor data should be used, in consultation with administrative and technical authorities, to remove the residual errors of the synchronization system. The same data should be continuously surveyed for any evidence of changing velocity of propagation with time as, for example, over a sunspot cycle. Any corrections needed over such a long time scale should be carefully verified by a competent authority and applied to the system synchronization.

If a minor degradation of synchronization precision is found to be acceptable, it is probable that much of the day-to-day control may be exerted in the stations themselves, without consultation with the control center, as shown in Section 5.2.1.

If this technique be used, the load upon the control center and the amount of inter-communication required will both be reduced. On the other hand, under local control, the average frequency of the system may deviate from the frequency of the station standards. Either the control center or the separate stations (with intercommunication) can correct this falling-off, which could ultimately make synchronization difficult or impossible and which would certainly hamper efforts to use the Omega signals as a primary distribution mechanism for time and frequency.

7.3 Permanent Monitors

For proof-of-performance there should be two monitor stations operating for the lifetime of the Omega system at sites more or less on opposite sides of the world. These sites should be broadly representative of typical navigational situations and may be at any convenient installations for logistic purposes.

Each permanent monitor should be equipped with enough standard navigational receivers so that one monitor or the other has a record of every independent pair of stations at all times. The pairs observed by each monitor should be chosen so that the monitor is in the normal service area of the pair.

Each monitor should immediately report signal failures or serious phase errors to administrative headquarters and to the analysis and control center. Permanent records of the readings should be kept for comparison in the event there are complaints about the system behavior. The data should eventually be made available for studies looking forward to second generation charts and tables.

These two stations should be maintained indefinitely so that, should a serious navigational accident occur, adequate evidence about the state of the Omega system at the time would be available. In addition, the continuous record would furnish a first line of defense against possible degradation of operating standards. To this end, monthly summaries of the standard deviations observed on each pair might well be submitted to the administrative authority.

7.4 Mobile Monitors

For at least two or three years after the implementation of Omega, and perhaps for a decade, a number of mobile monitors should be kept in operation. These should each consist of two or three navigator's receivers, complete with recording equipment, and perhaps supplementary equipment for measuring field strength. Portable power sources and antennas should also be available. It is estimated that from six to ten such mobile monitors, each in the hands of a small but well-trained crew, should be kept in operation, primarily for short periods at each of many places.

The first duty of these monitors would be to observe the actual readings obtained at a large number of different sites. Such readings provide the only possible confirmation of the velocity rules adopted for the system calibration. As shown in Section 5.2, the Omega system can be maintained in constant phase, but the absolute values of the various station phases are subject to small and presumably constant errors. These are best calibrated and reduced to zero by the average of many readings made at various points in each service area.

For this purpose it is suggested that each portable monitor be operated for a week or ten days (or until a good idea of both average readings and standard deviations has been obtained) at each of many well-separated points. At each point all available independent lines of position (usually four or five) should be observed. The data taken, together with accurate position information, should be transmitted to the analysis and control center, where the average errors can be combined with others of the same kind to provide correction constants to be used for the various pairs. By this technique the position lines observed can gradually be brought into the best possible agreement with the charted values.

It is probable that a few months' monitoring activity will provide confirmation on correctional data that will thereafter only gradually be improved. During the same months we may anticipate the first operational reports of difficulties or unusual errors in certain areas. When such reports do not result from a misunderstanding or an operator's error, a mobile monitor should be sent to the area in question to confirm or deny the existence of a charting error or propagational anomaly. By this procedure any consistent errors would be detected, and the general accuracy of the Omega performance would soon be well explored.

In addition to responding to observed difficulties, the authority supervising the mobile monitors should send them, as opportunity may offer, to various potentially questionable areas. It could be that the experimental program, in advance of operation, may never have a chance to search for errors at a point involving transmission across the breadth of the Sahara Desert, for example, or close in the lee of the Greenland icecap. There should be a continuing program searching for any anomalies that may exist.

Obviously, these efforts need not go on forever. We anticipate that the need for the mobile monitors will begin to decline after a year or two, and may decrease to complete unimportance in five years. As faith in Omega navigation spreads and the actual accuracy of the system becomes well known, the mobile monitor program can be permanently discontinued.

Obviously, all data collected in this program should be made available to the analysis and control center, to the charting authority, and to the administrative authority. Should unexpected and serious errors be found in certain areas (a relatively improbable assumption), the users of Omega should be notified through notices to mariners and by any other available method.

8.0 Omega Receivers

In Omega, position is established by the relative timing or phase of the Omega signals at the point being located. To use the system, a user must be equipped with receiving apparatus capable of the requisite measurements in the presence of the usual ambient noise and interference.

The format of the Omega signals allows many different modes of receiver operation, ranging from direct observation of Omega signal timing in an oscilloscopic display to computer type equipments capable of identifying stations, determining signal phase, resolving lane ambiguity, compensating for diurnal effects, and reducing hyperbolic data to steering information or geographical coordinates, etc., all without external aid.

The mode of operation and the form and complexity of the receiving equipment to be used in a given field of operations cannot be specified until the conditions of use are established. It is expected that diverse modifications and adaptations of equipment providing various modes of operation will be developed in the future as the requirements of the users become known.

This section describes what is involved in the Omega receiving equipment; the types of circuitry and typical receiving and phase indicating systems for some of the various possible modes of operation. No attempt is made to establish in detail the precise form of receiving equipment for particular users.

8.1 Receiver System Requirements

The overall Omega signal format consists of transmissions by several Omega stations in turn at various frequencies between 10 and 14 kc in a complex time multiplex pattern.* To establish position by Omega, measurements are made of the relative r-f phase of specific components of the signal format at the point being located.

*Section 3.2

Since only one station transmits at a time at any particular frequency, this measurement of relative r-f phase must be made between signal components that are never present simultaneously, and hence cannot be compared directly. To accomplish this measurement, Omega receivers are fitted with phase measuring circuitry, providing an internally generated continuous time base with respect to which the phase of each signal component of interest can be determined as it occurs, and a time multiplexing function, opening the receiver to the particular components desired for phase comparison with the internal reference.

The differences in times of occurrence, or relative phase, of the selected signal components are then obtained from the differences of the phase of each signal component, with respect to the local time base.

To utilize the Omega signals to determine position, the combination of receiver and operator must be to perform at least the following functions:

a. Recognize the multiplex pattern to identify the transmissions of a particular set of stations.

b. Commutate the signals to isolate the signal components of interest.

c. Determine the relative phases of the segregated signal components with the requisite accuracy.

To accomplish these functions the Omega receiver must perform certain basic operations.

a. Filter the Omega signals from the welter of noise and interference in which they are normally immersed.

b. Provide a time multiplex function (commutator) to isolate the desired signal components.

c. Provide a locally generated stable time base to which the phase of each signal component can be compared as it occurs.

d. Provide phase measuring circuitry capable of determining the phase of each signal component with respect to the local time base with the requisite accuracy.

8.2 Oscilloscopic Phase Indicator

The essential features of the Omega receiver can be visualized by considering an oscilloscopic display of the basic 10.2 kc transmissions from three of the stations, as in Figure 8.2-1. In this illustration, the signals are displayed on a continuous stable 5.1 kc linear sawtooth time base, so that the transmissions of each station appear as a stationary two-cycle sinusoid in the display.

Because the transmissions of the three stations will differ in phase, the resulting sinusoidal traces in the display will be displaced with respect to each other by amounts proportional to their relative phase. In this example, the signal from station A crosses zero with positive slope at the center of the sweep; that of station B leads A by approximately three-quarters of a cycle; the signal of station C leads A by perhaps three-eighths of a cycle.

The distances aa' = bb' = c'c are equivalent to one period of the rf; the distance ab is equivalent to the principal part of the phase lag of signal B over signal A; the distance ac is equivalent to the principal part of the phase lag of signal C over signal A.

The phase angle of signal B with respect to A, in percent of a cycle, is thus given by:

$$\theta_{AB} = \frac{ab}{aa'} \times 100$$

and the phase angle of signal C with respect to A is

$$\theta_{AC} = \frac{ac}{aa'} \times 100$$

Such a phase measurement is periodic, providing only the principal value of the relative phase of the signals, so that an ambiguous indication would be obtained. The ambiguity can be resolved, of course, by retuning the receiver to the second and third Omega frequencies (13.6 and 11.33 kc, with corresponding shifts in the time base frequency to 6.8 and 5.667 kc).

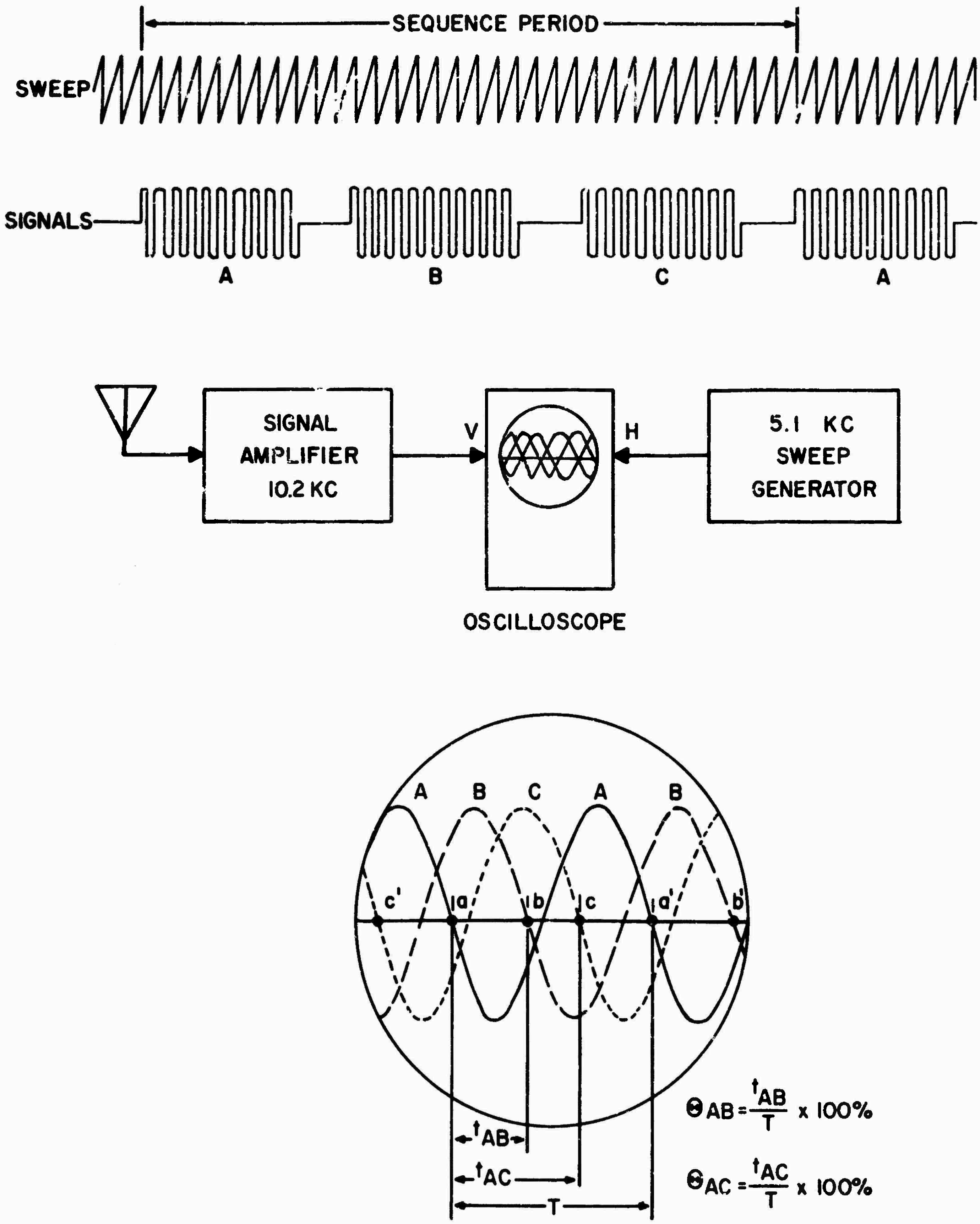


FIGURE 8.2-1. OSCILLOSCOPE PHASE INDICATOR

The relative phase of the components of these frequencies can be determined at the point being located, whence the ambiguity can be resolved as described in Section 2.3.

The following essential functions are implicit in the operation of this rudimentary receiver:

a. The band-pass r-f amplifier provides frequency domain filtering to isolate the Omega signals (to some extent at least) from the noise and interference in which they are immersed; and also provides for selecting the Omega signal components of a particular frequency.

b. The continuous stable linear time base of the oscilloscope display provides an internal timing reference to determine the phase of each signal component as it occurs.

c. Time multiplexing is inherently provided by the oscilloscope display, since each signal component appears as a distinct trace in the display; the traces being separated by the differences in phase.

Thus, this rudimentary embodiment of the Omega receiver provides the basic functions required in the system. However, the effective integrating action of a visual display is of the wrong order of magnitude to match the requirements for signal processing in a navigating system, and the dynamic range of such a presentation is limited. In addition, the commutating period of one second on and nine seconds off for each signal is about the least favorable cycle for a visual presentation. Thus, while in principle, use of an oscilloscopic indicator is operable, it does not appear that a practical equipment can be obtained by this approach.

Although more effective storage can be obtained through the use of a storage cathode ray tube, the intricacies of the circuitry involved are so great that it is probably better to adopt a self tracking system that can be matched more readily to the information requirements of the system.

8.3 Lissajous Pattern Indicator

A more accurate oscilloscopic indication, better adapted to manual operation in view of the one-second-on/nine-seconds-off transmission pattern, would be provided by the system illustrated in Figure 8.3-1.

In this method, an oscilloscopic indicator is used in which the incoming signals provide vertical deflection, and the horizontal time base is a 10.2 kc sine wave so that a Lissajous ellipse is obtained whose aspect indicates the phase of the signal relative to the local 10.2 kc sinusoidal time base.

The accuracy of measurement is then improved by shifting the phase of the local reference to obtain a degenerate ellipse (straight line) in the display. The relative phase of the signal with respect to the time base is indicated by the amount of shift required to produce the degenerate mode.

Equipment for accomplishing this type of measurement on the Omega signals (Figure 8.3-1) includes a 10.2 kc stable oscillator, and frequency dividers dividing by 17, 6 and 125 in tandem to produce timing pulses at 1.25 second intervals.

These triggers then drive three binary dividers in tandem to produce a binary set of waveforms repeating over a ten-second period, from which a station selector switch synthesizes commutating waveforms matching the occurrence sequence of the signals to be measured.

These commutating waveforms are applied to un-blank the display of a cathode ray indicator as a selected two of the eight Omega signals occur.

The output of the 10.2 kc oscillator is also applied to a first phase shifter and a second phase shifter in tandem. A time-sharing relay, switched by an appropriate set of commutating waveforms, alternately selects the output of the first phase shifter and that of the second phase shifter to supply the horizontal deflection of the indicator.

A signal amplifier picks up, amplifies and filters the signals, and applies the resulting sinusoidal waves to the vertical deflection input of the indicator.

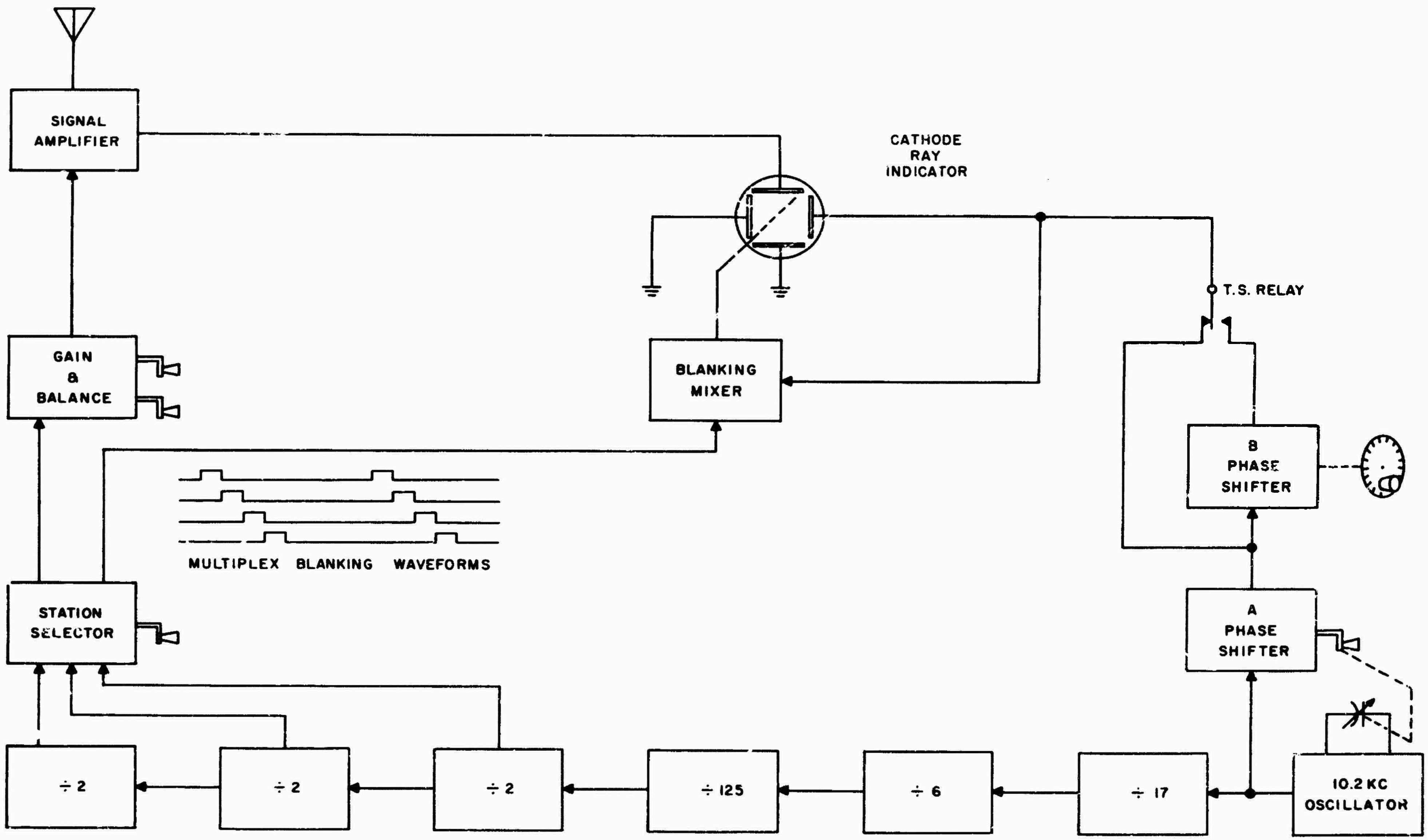


FIGURE 8.3-1. LISSAJOUS PATTERN INDICATOR

In use, the timing of the multiplexing waveforms would be adjusted to occur in synchronism with a desired pair of signals, for example, the signals of Stations B and D. When the B signal is received, the operator would note the width and aspect of the ellipse in the CR display, and would turn phase shifter (1) in a direction to bring the ellipse to the positive degenerate mode; i.e., a straight line slanting upward to the right.

The one-second duration of the signal would not be sufficient to allow the measurement to be made precisely. The operator would remember the aspect of the signal and adjust the phase shifter an amount, determined by experience, to bring the ellipse to the degenerate mode.

A mechanical coupling, or 'aided tracking' connection is provided between the phase shifter (1) and a frequency control of the timing oscillator. Thus, as the phase shifter (1) is rotated to bring the signal and reference into phase alignment, on successive occurrences of the signal, the frequency of the oscillator would be brought into alignment with the frequency of the B signal. The pattern would then automatically tend to subside to the positive degenerate ellipse, and remain stationary between occurrences of that signal.

During occurrences of the other signal being matched, the operator adjusts phase shifter (2) to bring the display of the other signal being observed also to the positive degenerate mode (straight line).

Eventually, a stable state would be reached, in which the degenerate ellipse display recurs with each succeeding signal occurrence without further adjustment. The phase of the D signal relative to the B signal would then be indicated by the dial attached to the shaft of phase shifter (2).

Additional aids to refining the display would be to apply a fraction of the horizontal time base signal to intensify the pattern cyclically to give sense to the rotation of the ellipse. Also to provide gain balance circuitry whereby multiplexing waveforms of adjustable amplitude can be applied to control the gain of the signal amplifier differentially to equalize the amplitudes of the received signals.

An alternative oscilloscope pattern would be with a ten-second linear sweep displaying all of the signals received with respect to the internal multiplexing function of the indicator to aid in preliminary alignment of the multiplexing waveforms with the incoming signals.

Again, the phase measurement would be periodic, providing only the principal value of the relative phase of the signals, so that an ambiguous indication would be obtained. As before, the ambiguity can be resolved by providing for alternative measurements of the second and third frequencies, etc.

8.4 Analog Tracking Receiver - Single Channel

A rudimentary analog tracking receiver is shown in Figure 8.4-1, in which an oscilloscope display is used to permit manual alignment of the multiplexing function, while a continuous measurement of signal phase (phase tracking) is provided by a servo controlled tracking filter.

In this embodiment, a multiplex timer generates a set of switching waveforms, one for each signal component, and a ten-second linear sawtooth time base, which is applied to the horizontal deflection system of an oscilloscope. The Omega signals appearing in one r-f channel (10.2 kc) are isolated and amplified in an amplifier-filter and applied to the vertical deflection system of the oscilloscope, to be displayed on the ten-second time base. Thus, the timing of the multiplexing waveforms can be adjusted manually to match the timing of the incoming signal segments.

The output of the signal amplifier-filter is also applied to the signal input of a phase detector, whose reference input is a sinusoid of the same frequency as the Omega signals obtained from a 10.2 kc timing wave generator, either directly through relay (a), or via a phase shifter through relay (b).

The output of the phase detector is applied to an averaging filter, either directly through relay a', or via an inverter through relay b'.

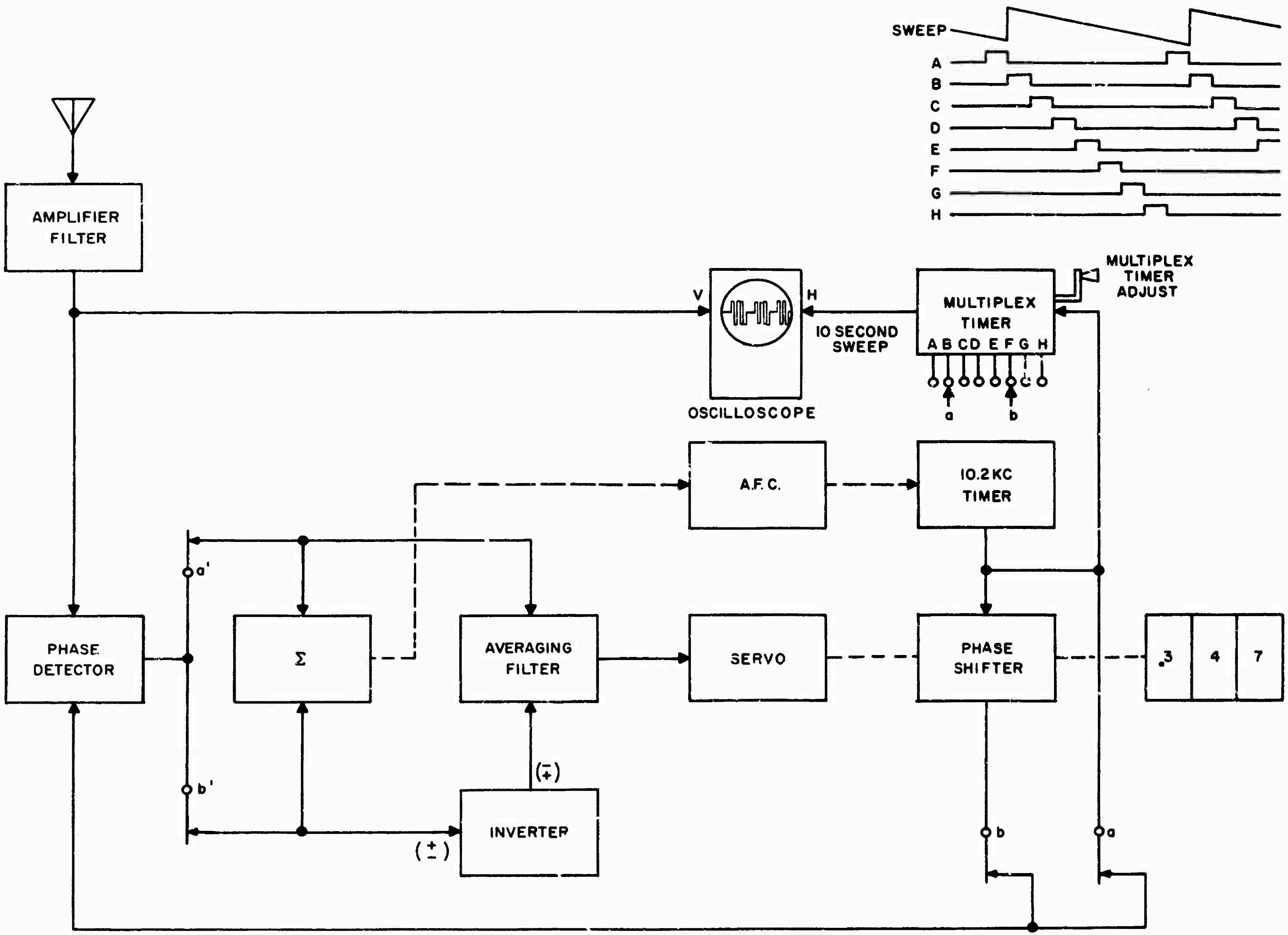


FIGURE 8.4-1. RUDIMENTARY TRACKING RECEIVER

The average amplitude of all inputs applied to the averaging filter excites a servo adjusting the phase shifter that supplies the 10.2 kc reference wave through relay (b).

The relays (a) and (a') are operated by one of the multiplexing waveforms (waveform B in this example) as selected by the operator, to apply the incoming signal and the reference directly from the timer, when the signal from one of the stations (Station B) is being received. Relays (b) and (b') are operated by another multiplexing waveform (waveform F in this example) to open the phase detector output via the inverter and the reference source via the phase shifter during the occurrence of the signals from station F.

In this way, the output of the averaging filter corresponds to the average difference between the phase of signal B with respect to the timer output, and the phase of signal F with respect to the phase shifter output.

The servo adjusts the phase shifter to make this average difference subside to zero, which it does by making the phase of the shifted reference through relay (b) with respect to signal F the same as the phase of the unshifted reference with respect to signal B. In consequence, the shaft angle of the phase shifter, as indicated by the counter, corresponds to the phase of signal F with respect to signal B.

This rudimentary form of receiver tracks the phase of one pair of signals at one frequency. To produce a fix it must be reset to an independent pair of signals to produce the second coordinate required for position determination.

Figure 8.4-2 illustrates a two-channel tracking filter in which a second tracking function has been added to permit phase readout of two pairs of signals simultaneously.

Figure 8.4-3 is a more detailed block diagram of one form of analog tracking receiver constructed in accordance with these principles at the Naval Research Laboratory.

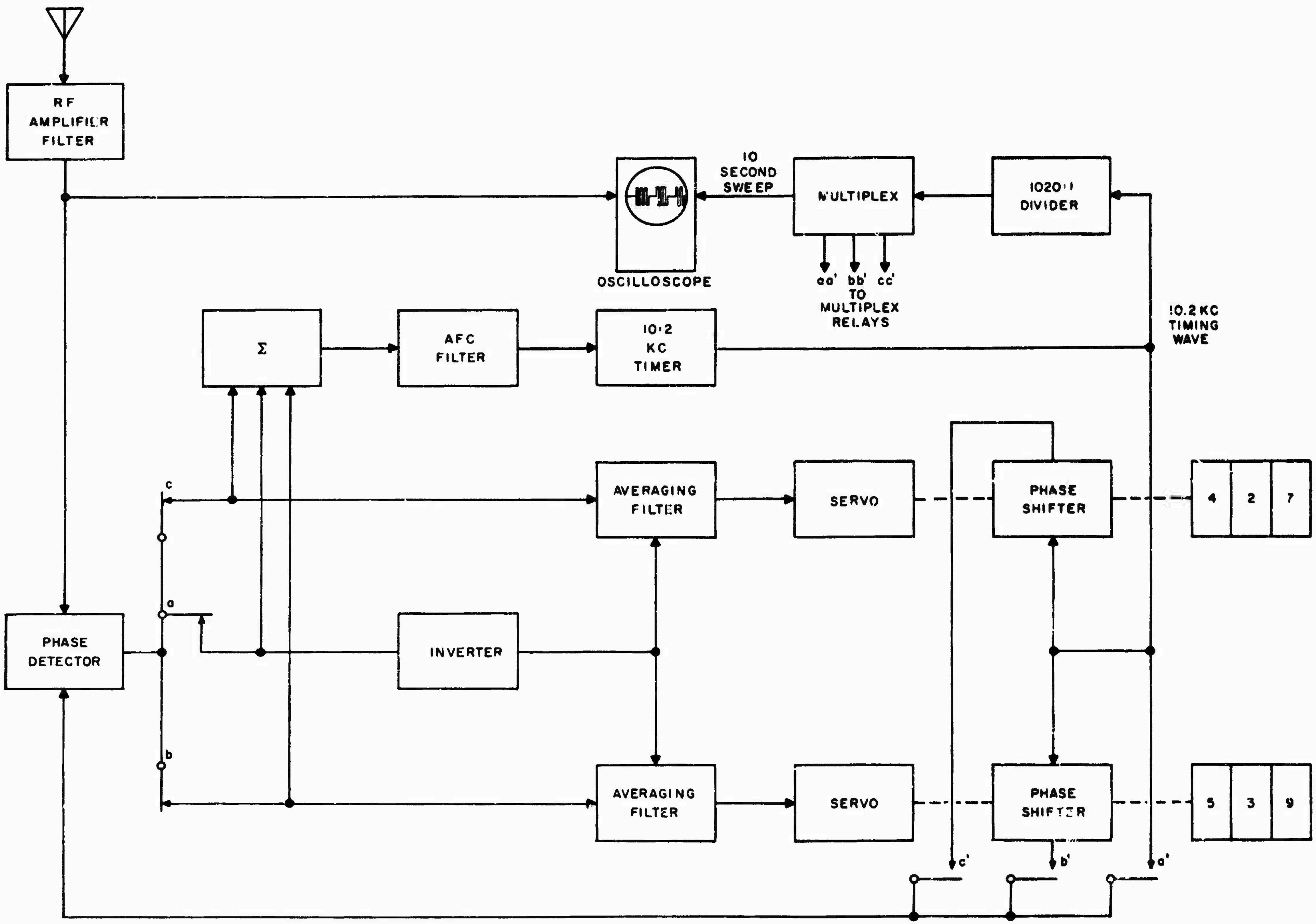


FIGURE 8.4-2. TWO-CHANNEL TRACKING FILTER

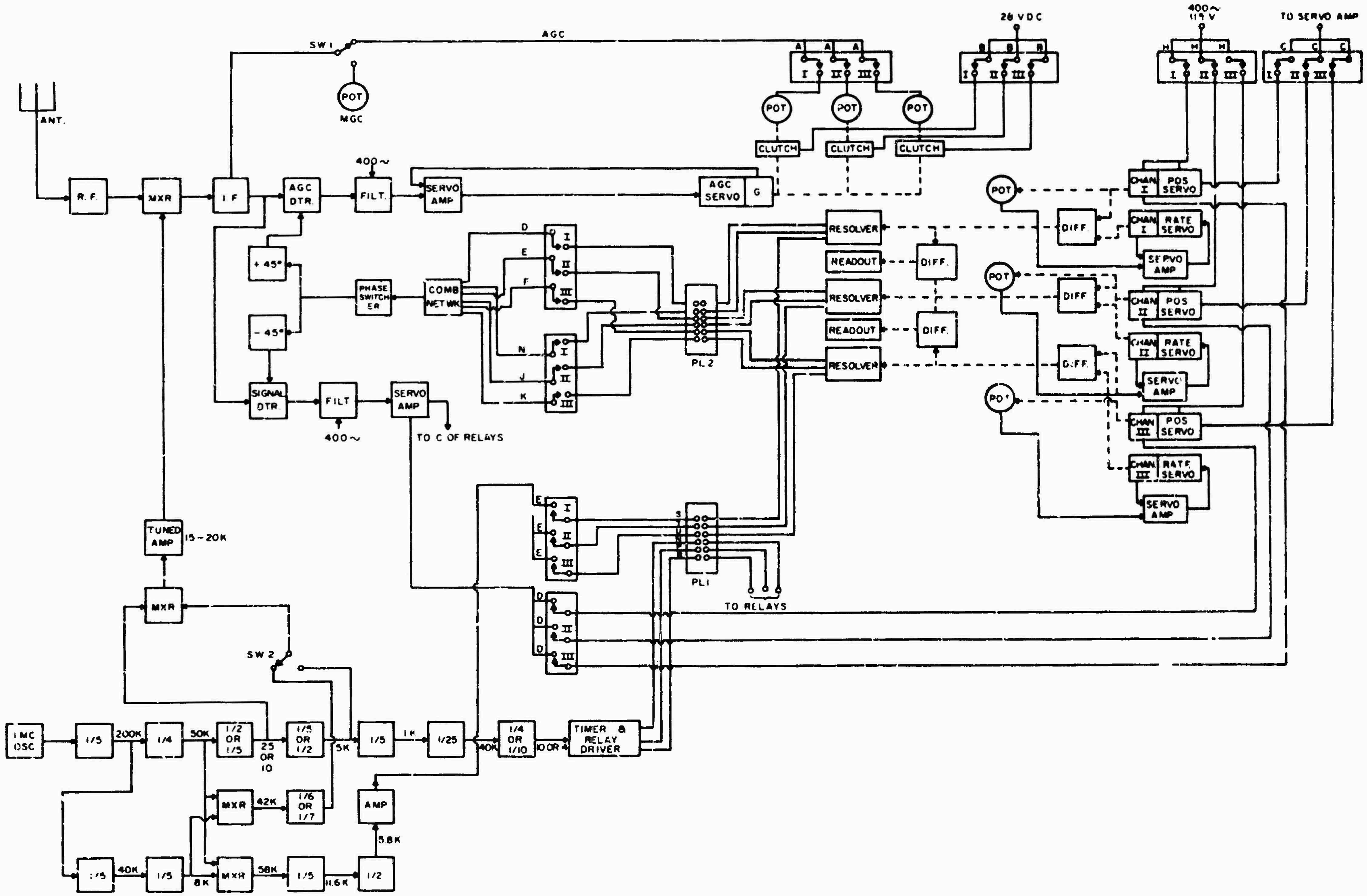


FIGURE 8.4-3. BLOCK DIAGRAM OF RECEIVING SYSTEM

8.5 Phase Modulation (226-2/3-Cycle) Receiver

It may be possible to satisfy the navigational requirements of some operations, (Inter-continental air travel, for example) with limited lane resolution using position data derived from the 226-2/3 cycle modulations only. The size and complexity of receiving equipment is then reduced as compared to that required for full radio frequency phase accuracy with more or less complete resolution of lane ambiguity. Figure 8.5-1 illustrates the structure of a fully automatic analog-type receiver indicating the relative phase of the 226-2/3 cycle modulation components of two pairs of stations.

This receiver would consist of three sections: A multiplexing function with automatic alignment by reference to a side frequency component; an RF phase tracking unit, and a modulation phase-tracking unit.

In this embodiment, alignment of the multiplex switching function is obtained automatically, by cross correlating an appropriate unit multiplex waveform with the envelope response of an auxiliary receiver tuned to the side-frequency component of one station. Then, an RF phase tracking unit essentially identical to the two-channel tracking filter of Section 8.4 and Figure 8.4-2 phase locks an uninterrupted 13.6-kc (CW) reference wave to each of the signals being observed.

Since the received signals are phase modulated, the RF phase detector, in addition to providing tracking error signals to the rf phase tracking function, also provides coherent demodulation of the incoming signals.

The modulation components in the output of the rf phase detector are separated in a 226-2/3-cycle bandpass filter, and applied to a modulation phase tracking unit to indicate the relative phase of the modulation components.

The operation of the modulation phase-tracking unit is essentially identical to that of the rf tracking unit, except that the uninterrupted low frequency cw reference wave is derived from the rf reference wave. The modulation phase-tracking phase-shifters are also mechanically coupled to the rf

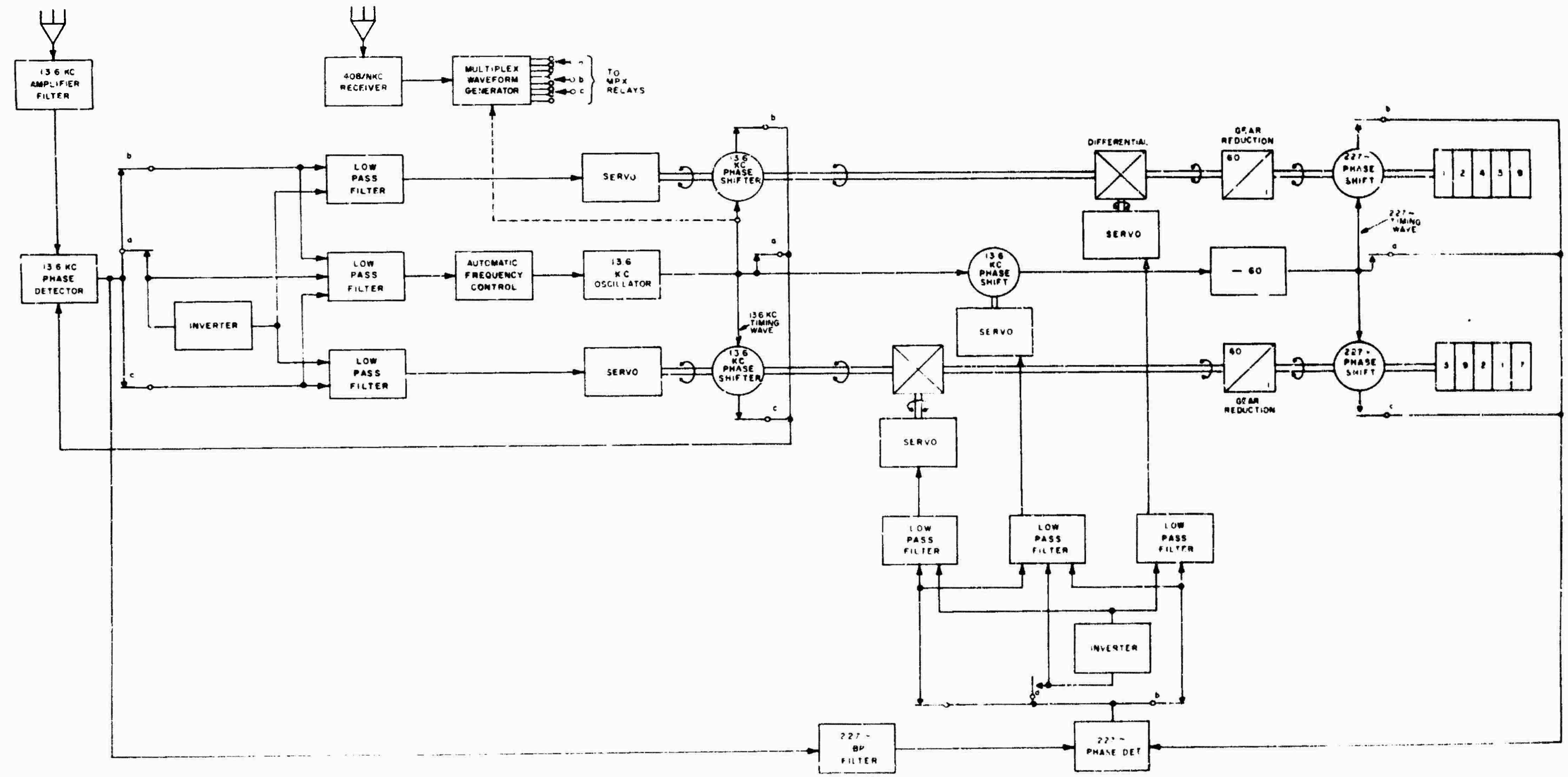


FIG. 8.5-1 AUTOMATIC ANALOG RECEIVER INDICATING RELATIVE PHASES OF 226-2/3 CYCLE MODULATION

phase servos through differentials, with appropriate gear ratios, so that the modulation phase tracking function need only bring the modulation phase indications into alignment. Tracking the signal velocity and acceleration etc., is accomplished by the rf phase tracking function.

For operations that are limited to regions of strong signals, an even simpler receiver could be provided that demodulated the signals with a simple frequency discriminator, thus eliminating need for a phase tracking function.

As with the RF tracking systems, an operationally equivalent structure can be provided with digital circuitry.

8.6 All Electronic Analog Receiver

An all electronic version of the analog tracking receiver with manual set up of the multiplex switching is shown in Figure 8.6-1, in which the electromechanical tracking filters have been replaced with electronic action.

In this version of Omega equipments, the tracking filters consist of voltage controlled oscillators operating at the signal carrier frequency, whose output is compared in phase with the incoming signal. An error signal is derived from the phase difference which, after smoothing in an operational amplifier-integrator, is applied to control the phase and frequency of the oscillator, so that its output phase tracks the incoming signals.

The proper tracking bandwidth and transient behavior is then achieved by suitable adjustments of the integrating factor and the loop gain of the tracking filter.

As illustrated in Figure 8.6-1, a voltage controlled oscillator operating at 1.02 Mc produces a series of clocking or timing pulses with which a whole count or clocking interval corresponds to 0.01 cycle of rf at 10.2 kc. Therefore, the requisite resolution of phase measurement can be obtained from full counts of clocking intervals, and fractional counts are not required.

The clocking pulses operate a 100:1 divider to produce a 10.2 kc reference wave. This is further divided by 1020, down to 10 cps, to provide

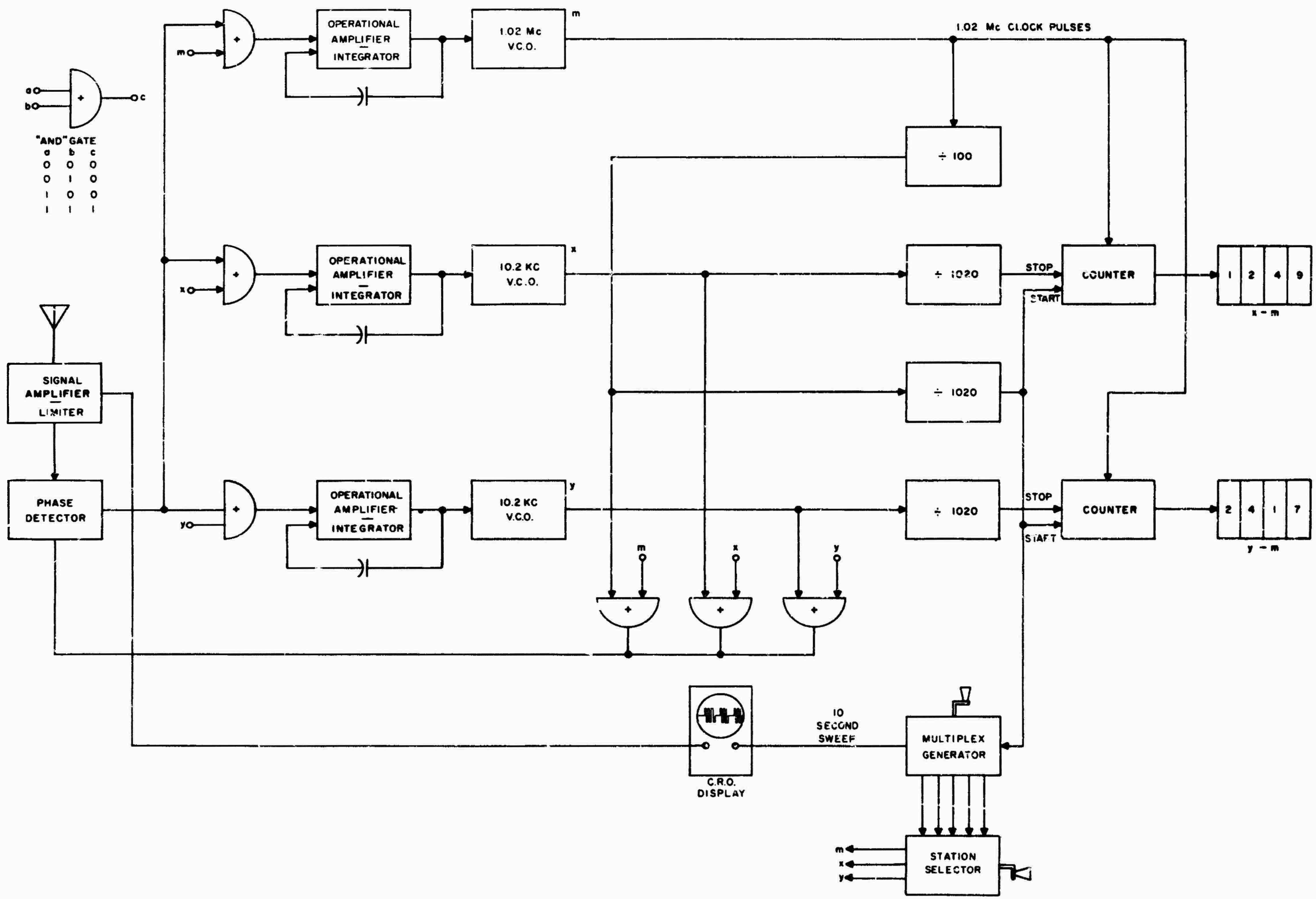


FIG. 8.6-1 ALL-ELECTRONIC ANALOG RECEIVER

a timing signal to operate a multiplex waveform generator producing the multiplex switching waveforms, and a ten-second linear sweep for a CRO display.

The incoming signals are picked up, amplified and leveled in amplitude by a signal amplifier-limiter, and applied to one input of a phase detector.

The station selector switch associated with the multiplex waveform generator determines the multiplexing waveform that opens the receiver to the signals. This switch is set to produce the 'm' waveform, during the occurrences of signals from one of the stations, chosen to be the master.

The multiplex waveform 'm' operates 'AND' gates to supply the 10.2 kc reference wave, obtained by dividing the 1.02 clock pulse rate by 100, to the reference input of the phase detector; and apply the error signal output of the phase detector to the input of the operational amplifier-integrator whose output controls the frequency and phase of the clock pulses.

In the closed loop servo thereby established, the 'm' operational amplifier integrates the phase error between the 'm' reference and the signals of the selected station. The integrated error is applied to control the frequency of the clock pulse generator and the 10.2 kc reference wave is locked in phase to the signal selected to be the 'm' signal.

A second voltage controlled oscillator (vco), designated the 'x' oscillator, operating at 10.2 kc, supplies the reference to the phase detector during a multiplex interval selected to be the 'x' interval by the setting of the station selector switch. Its phase and frequency are controlled by the integrated phase error accumulated during the 'x' intervals.

Similarly, a 'y' oscillator, also operating at 10.2 kc, is locked to the signals occurring during another multiplex interval selected for the 'y' intervals.

The outputs of the 10.2 kc phase locked oscillators, as well as the 1.02 Mc clock pulse generator, are divided by 1020, to 10 cps, so that there are three series of 0.1 second triggers produced. Eash is phase locked to one of the three signals selected for measurement.

At suitable intervals, a counter counts clocking pulses from an 'm' trigger to the next succeeding 'x' trigger, and reads out the time interval; a second pulse counter counts clocking pulses from an 'm' trigger to the next subsequent 'y' trigger.

Each total count so obtained consists of a number of whole counts of 100 clocking pulses each, determined by the particular counts at which the 1020:1 dividers happen to reset, plus a fractional count (i.e., less than 100) equal to the principal part of the corresponding differences in phase of the 'm' and 'x' or the 'm' and 'y' signals.

By resetting the counters arbitrarily to read the whole number of lanes, included in the total Omega phase angles, the readout can be made to correspond to the total count. When so set up, it will track the whole count, without losing count, since the oscillators will track the signals through whole cycles of phase, and the readout triggers are locked to the controlling oscillators.

8.7 Digital Omega Receiver

There may be applications for Omega where it is not feasible to have an operator manipulate receiver controls to reacquire signals in the event of a malfunction anywhere in the system causing the receiver to lose the signals. Possible causes for loss of signal may be due to excessive noise, a servicing changeover of a transmitter, or an interruption of primary power at some place in the system. The Omega signal format has been designed to provide for completely automatic receiver operation, including automatic acquisition of signals, resolution of the lane ambiguity, and readout of the hyperbolic position data.

At the present state-of-the-art, the most practical approach to performing such a multiplicity of functions in a single unit appears to be through digital computer techniques. A description of such a receiver in complete detail is beyond the scope of this report. This section will be limited to an outline of such a computer type receiving equipment indicative of how the essential functions may be accomplished. The elements of a special purpose digital computer adapted to perform all of the operations required to obtain alignment of the multiplex function and an unambiguous indication of two or more lines of position, without external aid except initial selection of the particular hyperbolic data to be displayed are described.

A fully automatic receiver must perform four basic types of functions.

a. Alignment of the receiver commutating function with the signal multiplex sequence so as to identify the particular signals it is desired to measure.

b. Determination of the signal relative phase at all frequencies incorporated in the signal format.

c. Resolution of the lane ambiguity.

d. Presentation of the signal timing in a form suitable for further processing.

8.7.1 Digital Phase Tracking Filter

The phase tracking and filtering functions essential to readout of the signal phase will be described first, to illustrate how such functions can be accomplished in a digital system. The receiver multiplex function must be aligned with the multiplex sequence of the incoming signals, however, before the signal phase can be determined.

Figure 8.7-1A shows, in rudimentary form, a digital phase-tracking filter by which the phase of an r-f signal may be determined by digital techniques. In principle, the operation of this circuit is essentially identical to the familiar analog phase tracking filters utilizing servo adjusted phase shifters or voltage controlled oscillators and phase detectors to phase lock a local reference to the average phase of an incoming signal. Thus, in the digital system, a stable local reference wave of the same frequency as the incoming signals is generated and compared in phase with the incoming

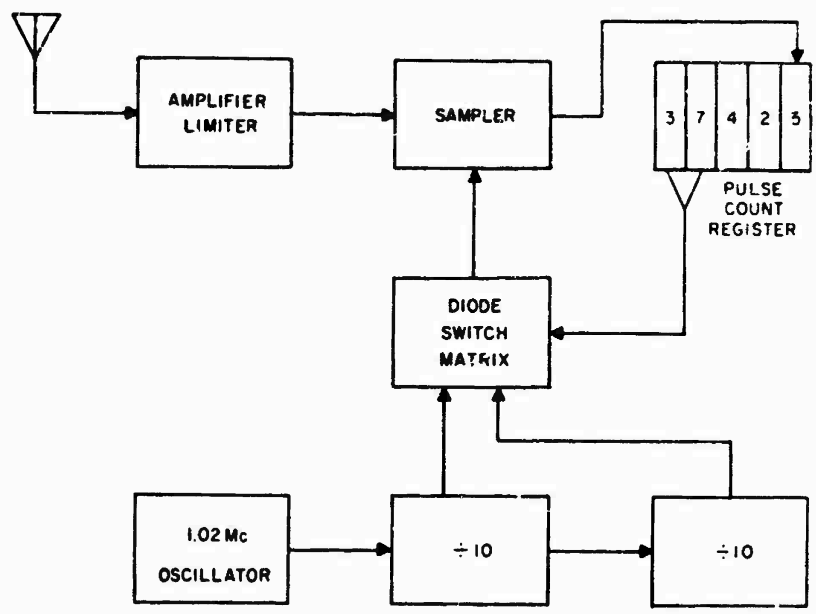


FIG. 8.7-1A RUDIMENTARY TRACKING FILTER

signals. The phase difference, or error, is then averaged, or smoothed, and applied to control the phase of the locally generated reference to maintain alignment with the incoming signals.

In this example, the incoming signals are amplified and clipped to a constant amplitude square wave whose period is essentially that of the received signals, but varies in phase from cycle to cycle with the instantaneous phase and amplitude of the combined signal and noise input, as shown in Figure 8.7-1B.

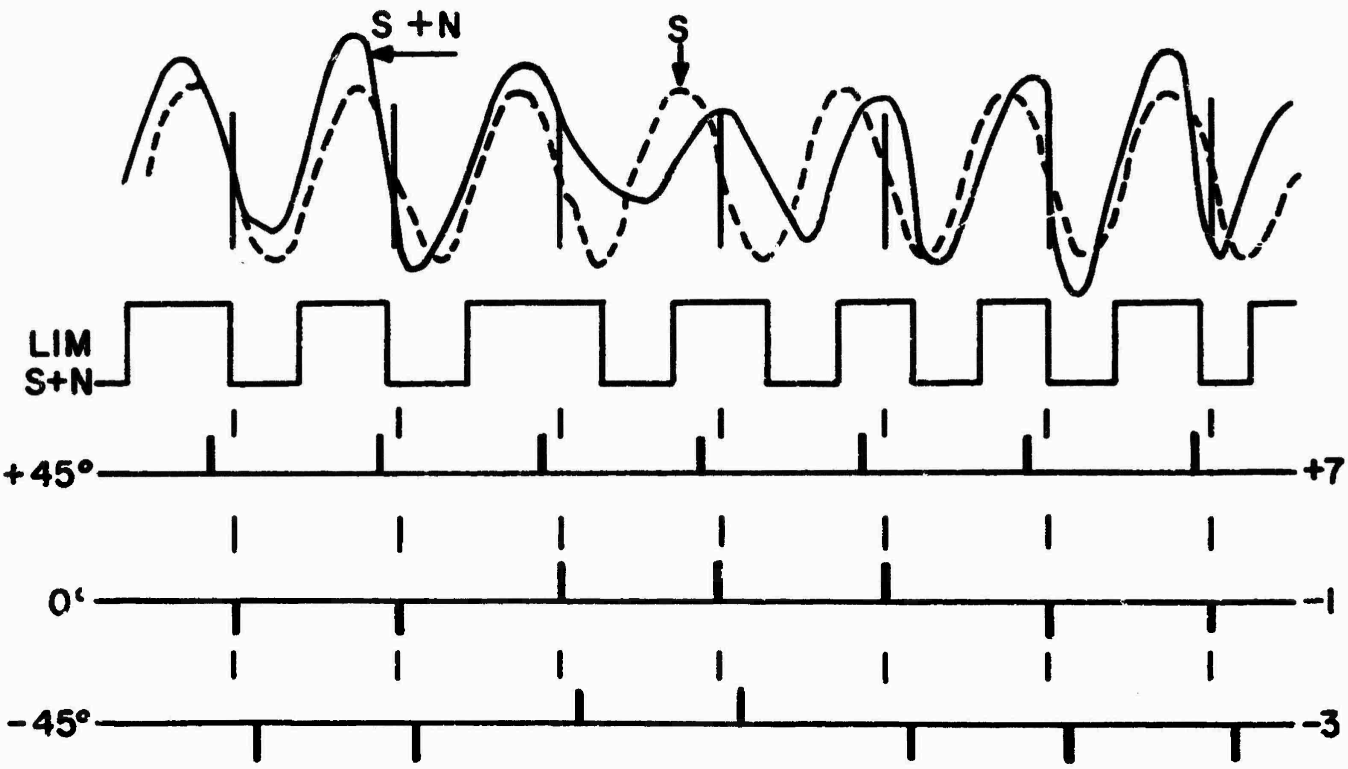


FIG. 8.7-1B WAVE FORMS

The resultant unit amplitude square waves are sampled by a sampling gate operated by locally generated sampling pulses. A positive or a negative unit pulse is produced at each sampling epoch, depending upon the polarity of the incoming signal at that instant. The resulting pulses are counted, or summed, with due regard to sign, in a digital counter or register.

The sampling pulses are provided by a 10.2 kc time code generator, consisting of a 100 : 1 divider driven by a stable 1.02 Mc clock pulse generator; also a diode switching matrix operated by the more significant digits of the pulse count register. This combines the time code divider waveforms to select the clock pulse in each time code cycle corresponding to the count standing in the register, to operate the signal sampler.

If the sampling pulses are coincident with the zeros of the signal in the output of the limiter, equal numbers of positive and negative pulses will be emitted by the sampler, therefore, the total count in the register remains stationary. If the sampling pulses lead the zero crossing epochs of the signal, the net count would be positive, raising the total count standing in the register and causing the switching matrix to select pulses occurring later in the time code cycles to sample the signals; vice versa for lagging samples. Thus, the stable state to which the system subsides is to the count in the register producing sampling pulses coincident with the negative going zeros of the incoming signals.

The rate at which this subsidence to zero occurs is determined by the number of stages in the register. For example, assuming a decimal system with a five place register, of which only the two most significant digits control the time code matrix, a net count of 1,000 samples is required to shift the phase of the sampling triggers by one percent of a cycle. Furthermore, the amount of smoothing so obtained can be extended indefinitely by adding stages to the register.

Since the clock pulse period is one percent of an r-f cycle, the count in the register is directly the percent of a cycle by which the signal lags the time code, and can be read out by an indicator controlled by the register. Additionally, the circuit supplies pulses phase locked within a percent of a cycle to the zeros of the signals.

8.7.2 Type II and Higher Order Tracking Filters

If the phase of the incoming signal shifts at a constant rate with respect to the locally generated sampling pulses, the count held in the register must increase or decrease at a constant rate, to keep the locally generated sampling pulses in alignment with the incoming signals. This requires that the net number of pulses from the sampler be some positive or negative quantity, not zero, so that, to follow an incoming signal with a constant velocity component, there must be a fixed offset in phase between the local reference and the incoming signals; i.e., a tracking error.

A system capable of tracking constant velocity without error can be provided by storing the total pulse count in two registers in tandem as shown in Figure 8.7-2, whereby the net number of pulses emitted by the sampler is totalled in the first or 'velocity register.' The switching matrix, establishing the phase of the sampling pulses, is controlled by a second, or 'phase register,' the controlling quantity being a count obtained by periodically adding the count held in the velocity register to the total standing in the phase register.

This system subsides to a steady state condition when the count in the velocity register is equivalent to the shift in phase of the incoming signals between samples. When added to the total held in the phase register, output of the switching matrix is shifted the right amount to match the change in phase of the incoming signals, so that the net number of pulses emitted by the sampler subsides to zero.

Third and higher order systems capable of tracking constant acceleration, and constant rate of change of acceleration, etc., can be set up by providing an additional count totalling register for each order of integration.

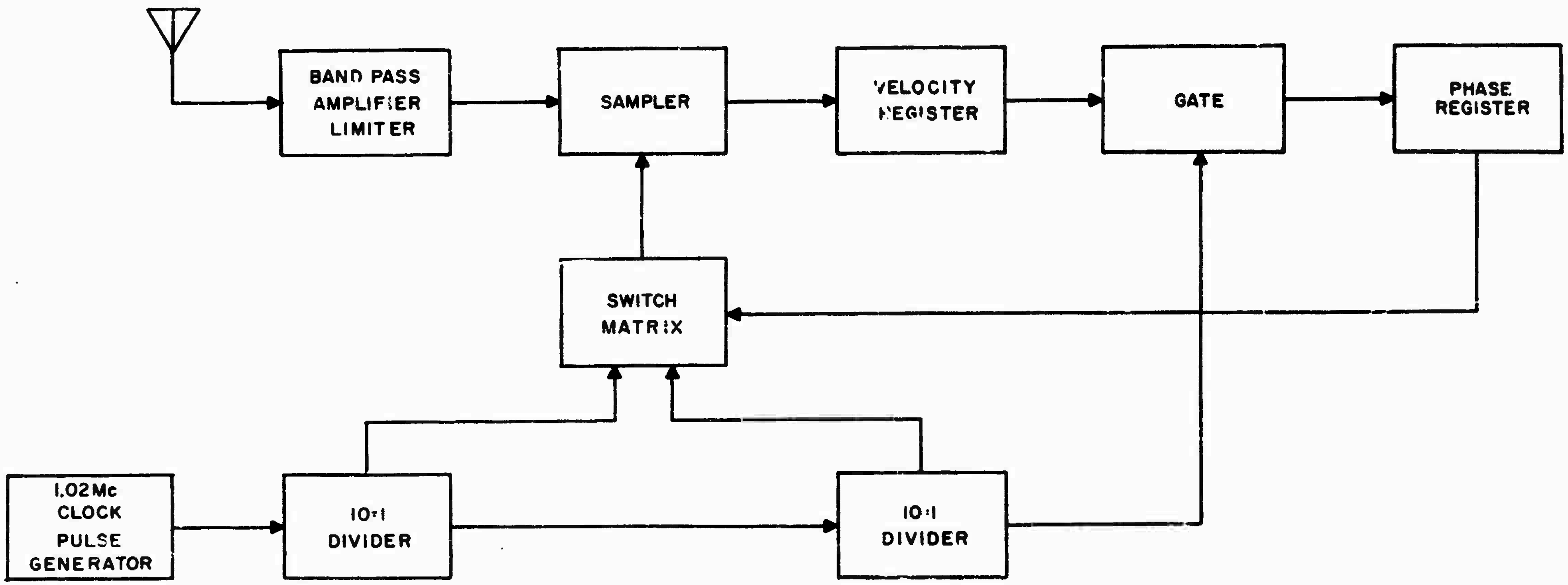


FIG. 8.7-2 TYPE II TRACKING FILTER

8.7.3 Phase Difference Indicator

The digital phase tracking filters of the preceding sections are capable of tracking only one signal. In order to determine the phase difference between time shared signals of the same frequency, separate pulse count registers must be provided for each signal, as shown in Figure 8.7.3

The time code generator, in addition to supplying time code input to the switching matrix, also drives a multiplex waveform generator generating switching or time sharing waveforms matching the time sharing or multiplex pattern of the signals (Section 4.4). These multiplexing waveforms, when aligned (by means not shown) with the incoming signal sequence, operate the gates such that the A register is connected to the sampler and operates the switching matrix during the occurrence of signals from a particular station. The B register is connected during the occurrence of signals from another station. When so connected, the total count in the A register converges to the value causing the sampling triggers to match the zeros of the one signal, and the total in the B register to the count causing the sampling triggers to match the zeros of the other signal.

A third register, called the difference register, is periodically cleared, then the count in the A register is added and the count in the B register is subtracted; the difference being directly the difference in phase of the incoming signals.

8.7.4 Signal Data Processor

Extending the system of Section 8.7.3to one capable of determining the phase of two pairs of signals with Type II servo response for tracking constant velocity without error, would require four phase-registers, four velocity-registers and two difference-registers. Each must be capable of both addition and subtraction. If the system were to be further expanded to read the phase of each pair of signals at each of three radio and three modulation frequencies for complete resolution of lane ambiguity, the number of registers required is multiplied.

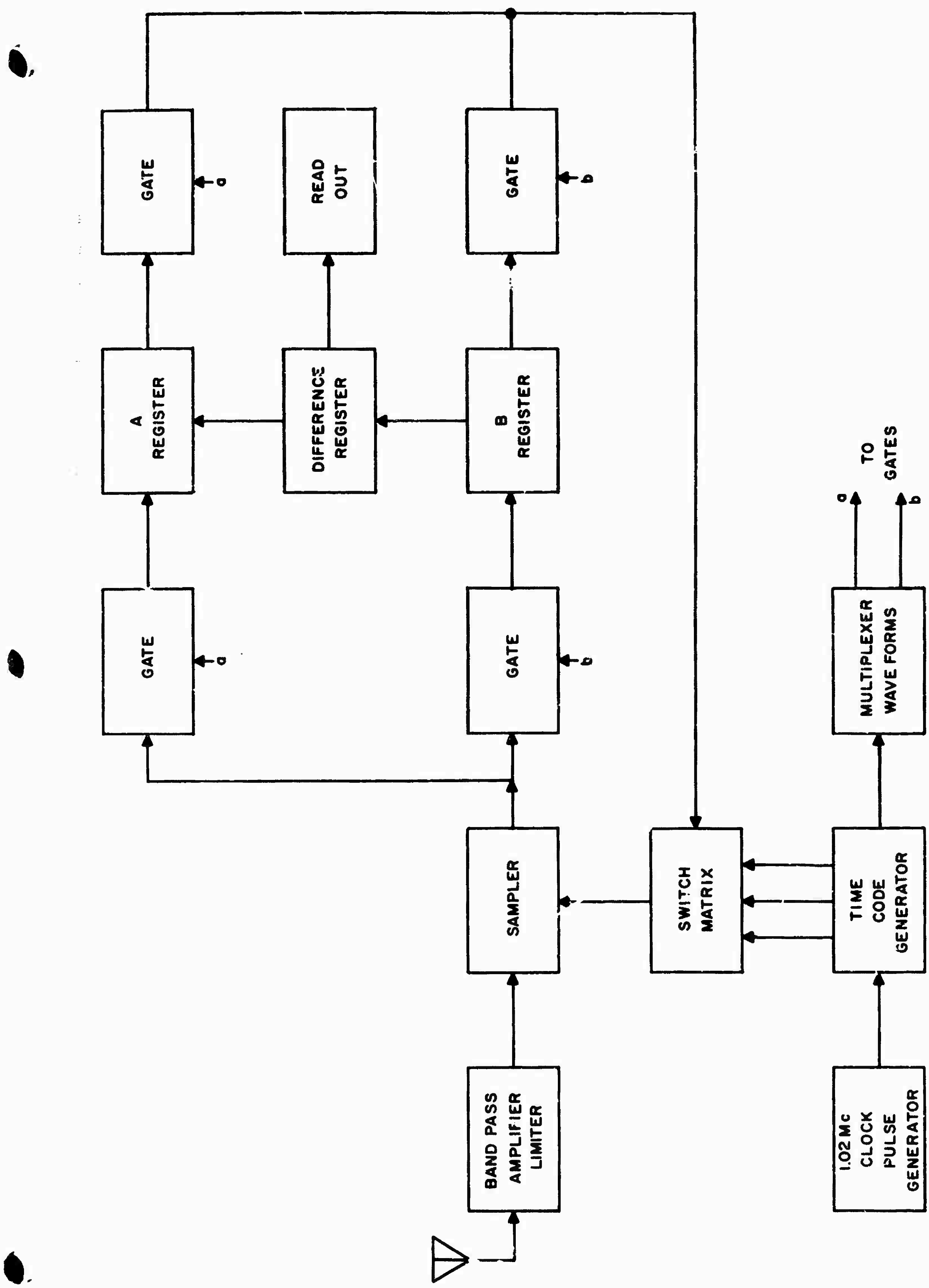


FIG. 8.7-3 DIGITAL DIFFERENCE INDICATOR

A simplification may be achieved by utilizing the structure and logic of the general purpose digital computer, in which data storage is accomplished by a storage unit capable of holding data only, and a single arithmetic unit for performing the logical operations necessary to accomplish the system purposes. Figure 8.7-4 shows such a computer structure.

The basic tracking functions are the same as before: viz. a clock pulse generator driving a time code generator and switching matrix provide sampling triggers at the signal radio frequency. The triggers sample the output of a signal amplifier-limiter to produce signal samples of plus or minus polarity, depending upon whether the sampling triggers lead or lag the zeros of the signal. The net number of such signal samples are totalled and applied to the switching matrix to control the phase of the sampling triggers.

In this case, however, the processing of the signal samples is accomplished by a computer structure shown on the right side of the diagram, Figure 8.7-4.

The computer consists of a data storage unit, holding a set of DATA WORDS in the form of binary symbols, representing the totals and integrals of the signal samples, and the sequence of logical operations required to process the samples of the Omega signals to determine their relative phase and resolve lane ambiguity, etc. An arithmetic unit performs the actual logical operations, and a controller unit directs the arithmetic unit to function accordingly, taking instruction words up in succession from the storage unit. Associated with the controller is a program counter, driven by the time code generator, whose count is the address of the particular operation then to be performed.

The controller operates with a two-phase cycle. In the first phase, it directs the DATA STORAGE to transmit to the controller, the data word stored at the storage unit ADDRESS corresponding to the count standing in the program counter. In the second phase, one or two digits of the word then in the controller cause the arithmetic unit to perform a specific operation, which

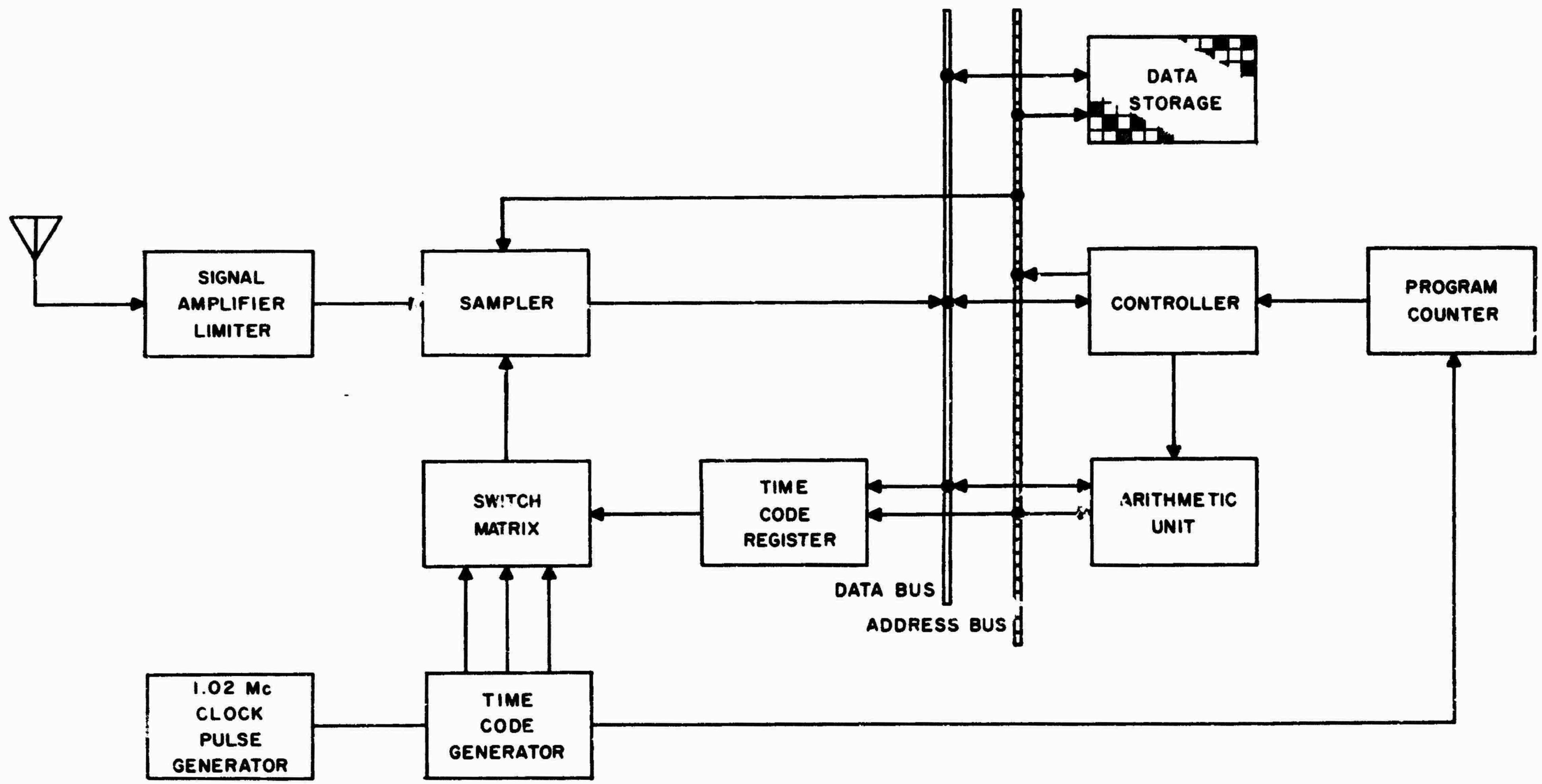


FIG. 8.7-4 COMPUTER DATA PROCESSOR

may be to ADD, to CLEAR and ADD, to SUBTRACT, or simply TRANSMIT. The remainder of the word in the controller register is the address to or from which the data word involved in that particular operation is transmitted. The operation of the entire system is then established by the sequence of orders in the data storage.

The process of tracking the phase of a signal would be accomplished as follows.

The computer program is arranged to go through a complete cycle of operations in exactly 10 seconds (the signal sequence period) so that, when aligned in phase with the incoming signal sequence (by a process to be described in a following section), certain counts in the program counter cycle always correspond to the beginning of each Omega station transmission. Then, certain of the addresses in the storage unit are designated to hold data words corresponding respectively to the phase and phase-velocity of each signal component to be tracked.

At the program count corresponding to the start of a transmission from a particular station, the instruction in the program will direct the storage unit to transmit the phase word for that station to the arithmetic unit and the next subsequent order directs the arithmetic unit to transmit that word to the time code register to control the phase of the sampling pulses being emitted by the switch matrix.

Subsequent orders direct the arithmetic unit to sum algebraically the next 10,000 or so signal samples. At the end of the transmission from that station, the immediately following orders direct the computer to add the total of signal samples to previous data held in storage, form the integral thereof, and store as a new value of the phase word for that station.

These operations repeat for each signal to be tracked, causing phase words for each Omega signal to be developed and stored separately in the storage unit. These phase words are the time code representation of the phase of each signal as observed, relative to the time code cycle of the receiver. At opportune times in the cycle, the program directs the computer to determine the difference between two of the phase words, and transmit the difference to a display unit.

By developing both the sum of the signal samples and the time integral thereof, and combining the results in suitable proportions, a Type II tracking function of any desired time constant is derived that is stable and tracks constant velocity with zero error. Obviously, more complicated tracking functions of higher order can be obtained simply by programming the computer to form the necessary functions to be integrated into the final phase words that control the phase of the sampling triggers. The only equipment requirements are additional storage capacity, and a sufficient number of steps in the program to perform the added computations.

8.7.5 Multiplex Switching Alignment

In the Omega system, all stations transmit at the same r-f frequencies in a repeating ten-second multiplex sequence. The signals are separated in the receivers by multiplex switching functions operating in synchronism with the signal sequence.

In the case of the computer-receivers, the time multiplexing function is incorporated into the computer programming by organizing the program into ten-second cycles with a sequence of orders matching the signal multiplex sequence. Sub-routines for processing particular signals are entered as the signal components occur.

To segregate the signals correctly, the program cycle must be aligned with the incoming signal sequence so that the start of each sub-routine matches the occurrence of the corresponding signal to within about one-tenth of a second. Since the entire sequence period is ten seconds, there are thus at least 100 distinguishable alignments, only one of which is correct.

Automatic alignment of the computer cycle with the incoming signal sequence is provided by a signal envelope correlating sub-program, interleaved into the signal tracking program. Successive samples of the envelopes of the incoming signals, taken at 1/10 second intervals, are each added or subtracted into every one of 100 addresses in the storage unit, so as to develop concurrently the coefficient of correlation for each of the 100 possible phase alignments or displacements between the computer program cycle and the incoming signal sequence. The timing of the signal processing cycle is adjusted to match the correlation function for which the coefficient of correlation with the incoming signal envelopes is a maximum.

8.7.6. Lane Resolution

Information for resolving the lane ambiguity of the Omega phase contour patterns is provided by the multiplicity of frequency components included in the signal format. The hyperbolic phase angles establishing the Omega position contours can be determined in terms of a number of integrally related wavelengths, whose combination into single readings provides complete resolution of the ambiguities.

Thus, each station transmits r-f carriers at 13.6, 11.33 and 10.2 kc; phase modulated with 226-2/3, 45-1/3 and 11-1/3 cps respectively. Determination of the relative phase of the 11-1/3 cps components from two stations establishes a phase contour in terms of a lane width of some 7200 miles, which, being more than the length of a base-line, is without ambiguity, but has a low order of accuracy.

The relative phase of the 45-1/3 cps components from the same stations establishes another phase contour pattern in terms of a lane width of some 1800 miles. The phase of the 226-2/3 cps components, a third pattern in terms of a lane width of 360 miles.

The difference between the relative phase at 13.6 kc and 11.33 kc provides contours with a lane width of 72 miles; the difference between the relative phase at 13.6 kc and 10.2 kc a lane width of 24 miles; and finally the measurement at 10.2 kc provides a lane width of about 8 miles.

The unambiguous indication of the position contour in terms of a whole number of lanes of the 10.2 kc pattern, plus a fraction of a lane from the principal part of the relative phase of the 10.2 kc components, is obtained by combining the measurements at all these frequencies as follows.

The contour pattern at 11-1/3 cps has a lane width four times that of the 45-1/3 cps pattern.* Hence, if there were no dispersion or other error in the measurements, multiplying the 11-1/3 cps phase difference by four produces the same phase difference as that of the 45-1/3 cps components including whole cycles.

However, the 45-1/3 cps indication provides only the principal part of the indication, since the essential ambiguity of such a phase indication suppresses the whole number of cycles. Thus, if the relative phase at 11-1/3 cps were 0.538 cycle, and there were no dispersion or other errors, the 45-1/3 cps indication would be 0.152 cycle (4x0.538 = 2.152 with the digits to the left of the decimal suppressed).

If the lane resolution is certain, a phase indication of lower order, when multiplied by the ratio of frequencies, may deviate by up to ±one-half cycle from the indication of next higher frequency. In this case, the indication of lower order might be anything from 0.413 to 0.663 cycle, and the correct reading would still be 2.152 cycle.

When multiplied by four, these two limiting values produce 1.652 and 2.652 cycle respectively. Hence, in combining readings it is not enough to simply substitute the principal part provided by the reading of higher order for the fractional part of the indication obtained by multiplying the lower order indication by the ratio of frequencies, since a carry into the whole number of cycles may be required.

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*Section 4.5.

In a digital machine, the carry may be generated by the following sequence of operations:

a. Multiply the indication of lower order by the ratio of frequencies.

b. Add the quantity 1/2.

c. Subtract the indication of higher order.

d. Set the principle part of the result (the digits to the right of the decimal) to zero.

e. Add in the indication of higher order.

How this sequence of operations produces the correct whole lane count can be seen from the following table. On the left, the correct (45+) cps total reading is 1.996 and on the right it is 2.004, with the same (11+) cps readings of 0.376 (lower limit), 0.500 (middle) and 0.633 (upper limit) in both cases.

	Case 1			Case 2		
(11+) Reading	0.376	0.500	0.623	0.376	0.500	0.623
(45+) Reading	0.996	0.996	0.996	0.004	0.004	0.004
Four Times (11+) Reading	1.504	2.000	2.492	1.504	2.000	2.492
Same Plus 1/2	2.004	2.500	2.992	2.004	2.500	2.992
Subtract (45+) Reading	1.008	1.504	1.996	2.000	2.496	2.998
Eliminate Principal Part	1.000	1.000	1.000	2.000	2.000	2.000
Add Higher Order Reading	1.996	1.996	1.996	2.004	2.004	2.004

8.8 Complete Computer Type Receiver

The outline of an Omega receiver, capable of reading on any number of signals, while providing automatic multiplex alignment and complete ambiguity resolution, is shown in Figure 8.8.1 It consists of an r-f signal processing

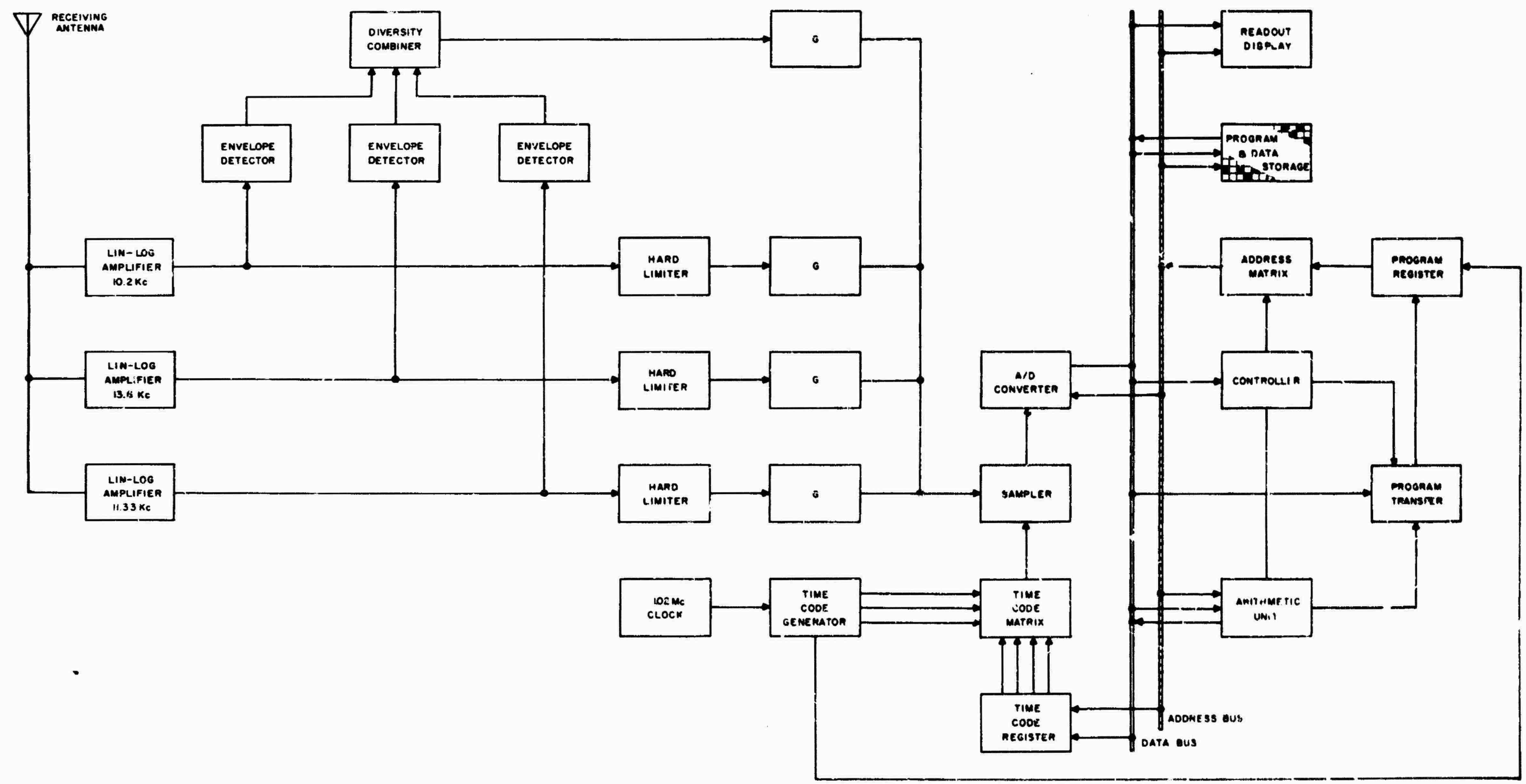

FIG. 8.8-1 COMPUTER TYPE OMEGA RECEIVER

section, which filters the r-f signals from noise and interference in which they are ordinarily immersed, and a digital type data processing section, or computer. This is programmed to perform all the operations involved in processing the Omega signal phase to align the receiver computing cycle with the incoming signal sequence, determine the relative phase of two or more independent pairs of signals at all frequencies incorporated in the signal format, combine the measurements to eliminate ambiguities, and present the resulting position line coordinates either as a numerical readout, or as digital data in a form suitable for further machine processing.

The r-f analog section of the receiver consists of three band-pass signal amplifier-limiters with frequency responses centered respectively on the 10.2, 11.33 and 13.6 kc radio carrier frequencies of the Omega signal format, and bandwidths which maximize the effective signal-to-noise ratio of the low frequency phase modulation of each signal component. The limited and leveled outputs of the three r-f amplifiers, as leveled by the limiting sections, are connected through gates operated by the computer program to a signal sampler and an analog-to-digital converter, producing a digital representation of the signal amplitudes as sampled.

To provide the signal envelope amplitude as a function of time for correlation of the computer program cycle with the signal sequence, linear envelope detectors extract the signal envelopes of each of the three r-f signals in the r-f amplifiers ahead of the limiters. The three envelope signals are combined in a diversity combiner and filtered in a matching filter designed to enhance, as much as possible, the response to the gaps between signal segments, with respect to noise. The composite amplitude function is likewise connected to the signal sampler and analog-to-digital converter through another gate, also operated on demand by the computer program.

The basic timing reference of the system is provided by a 1.02 Mc clock pulse generator, driving a time code generator which in turn operates a program register. The overall timing complex provides a time code with a ten-second cycle (1.02×10^7 clock pulses per period). A time code switching matrix set up by a time code register supplies sampling triggers to the signal sampler at timing epochs determined by the count or Data Word inserted into the time code register.

The computer proper consists of a data storage unit, an arithmetic unit and controller, an addressing unit, and a subsidiary unit labelled the program transfer unit.

The data storage unit is a digital storage device, which may be an acoustic delay line, a magnetic core matrix, a magnetic drum, or any other element or combination, as proves to be convenient and feasible to the overall design of the system. The storage unit is simply a memory, capable of holding, at distinct addresses, any data words transmitted to it, with readout as required by the program, and erasure of stored data only on the insertion of new data at a given address.

The arithmetic unit is an adding register, capable of responding to a limited complement of orders, viz:

TRANSMIT	Transmit the data word in the arithmetic unit register to an indicated address
ADD	Add the word at an indicated address to the total in the register
SUBTRACT	Subtract the word at an indicated address from the total in the register
CLEAR AND ADD	Clear the register and add
ADD LEFT	Add displaced one digit left
ADD RIGHT	Add displaced one digit right
Etc.	

The controller is another register, not capable of addition, into which words from other sources can be inserted. Three of the digits in the controller register are always taken to be instructions for the arithmetic unit; vis,

000	Transmit	100	Add Right
001	Clear and Add	110	Add Left
010	Add	101	Etc.
011	Subtract		

and the remaining digits in the controller register are taken to be an address, in the storage unit, or of the time code register, or the analog-to-digital converter, etc. for the operation demanded of the arithmetic unit. The controller operates in a two-phase cycle. In the first phase, it directs the data storage to transmit the word held at the address corresponding to the count standing in the program counter to the controller register. In the second phase, it then directs the arithmetic unit to perform the operation indicated by the data word so obtained, with respect to the address contained in that word. The timer then cycles the program counter to the next count, upon which the cycle repeats, performing the next operation. In this way, the overall program for processing the data is established and controlled by the sequence of instructions in the storage unit which, when taken up in sequence, cause the computer to process the incoming data to provide the desired operations on or with it.

The program transfer unit is provided to reset the program counter to any predetermined count at any time, so that sub-routines of the program can be iterated; e.g., the signal sampling routine, by resetting the program counter to the sub-routine entry count. This makes a very large reduction in the total storage capacity required to contain the program instructions In addition, this function has the very important application of resetting the phase of the computer cycle to match the incoming signal sequence. Thus, the occurrence in time of the computer sub-routines for processing each signal component matches the occurrences of that signal component, so that the multiplex sequence function essential to Omega signal processing is included in the data processing program.

8.9 Theory of Operation

The computer portion of the equipment is programmed to perform all of the operations required to process the signals to bring the computer operating cycle into alignment with the occurrences of the various signal components. In addition, to determine the relative phase of the specified signal components at all radio and modulation frequencies; combine the measurements of relative phase to resolve ambiguities; and present the results as a digital readout, either in the form of visual signals, or as digital data for further machine processing.

The entire program is designed to operate in a ten-second cycle, or some multiple thereof, matching the period of the incoming signal sequence. One hundred of the addresses in the storage unit are assigned to hold the hundred possible coefficients of correlation between the computer cycle and the signal sequence for all recognizable phase displacements there between.

Other addresses are assigned to hold the time codes corresponding to the phases of various components of the incoming signals, and the totals of the pulse samples, etc., as required to provide the Type II or Type III servo action necessary to follow signals at constant velocity and/or acceleration without error. The remainder of the data storage (which may be non-alterable storage of some form) holds the sequence of instructions and addresses required to specify the processing of the signal data.

The computing program provides for determining concurrently the phase of the incoming signal sequence and resetting the program counter to match, the relative phase of the desired signals at all frequencies incorporated in the signal format, resolution of the ambiguities and readout of the Omega data. Being completely cyclic, the program can be entered at any step, from which it converges automatically to alignment with the signals and readout of the signal phase without attention or external assistance.

At 1/10 second intervals (every 102,000 counts of the time code generator and program counter) the program calls for a sample of the signal envelope amplitude to be transmitted into the arithmetic unit. Subsequent orders then direct that this sample be added or subtracted to the data words at each of the 100 correlation coefficient addresses in the storage unit. The coefficients of correlation between the computer cycle and the incoming signal sequence for all recognizable relative displacements are developed at these addresses. In the process, the computer compares the amplitudes of the correlation coefficients. By means of the program transfer control, the program register is reset to the count corresponding to maximum correlation with the incoming signal sequence, thereby bringing the computer program into alignment with the incoming signal sequence. The sequence correlating action is continuous, independent of the signal tracking functions, and ultimately produces correlation coefficients based upon very long time averages of the signal envelope data. The integration of data from successive envelope sequences into data already acquired produces an even more pronounce reduction of uncertainty, so that, a completely unequivocal indication of sequence phase must be obtained eventually, even under worst conditions of noise and interference.

At program counts corresponding to the beginning of each sector of the signal sequence, the program directs that the data word representing the phase of the corresponding signal component be transmitted to the time code register to control the timing of the next sampling pulse to operate the signal sampler. The resulting signal sample is added to the corresponding velocity word in the data storage unit, and integrated into the associated phase word. The time code generator is set to the phase word of the next signal component and the next sample quantity added to the corresponding velocity word, etc.

To obtain sufficient time for processing samples of each of the three r-f carriers, which may be transmitted from three different stations of interest in the same multiplex sector, use is made of the fact that the noise functions of successive r-f cycles are statistically dependent (because the received signals pass through narrow band filters prior to limiting). Therefore, independent samples of the noise riding on the signals cannot be obtained at sampling intervals less than the reciprocal of the r-f filter bandwidth. The widest bandwidth required is that needed to pass the 226-2/3 cps component modulating the 13.6 kc carrier, or about 600 cycles. Independent samples of noise and interference cannot be obtained at intervals of less than 1600 microseconds or so. All frequency components of the signal are multiples of 1133-1/3 cps, and it is convenient to make this the basic sampling rate, providing some 882 microseconds in which to obtain and store samples of the three r-f carriers.

Recovery of the phase modulation components included in the signal format for lane resolution is obtained by distributing successive samples of a given r-f signal, as obtained at intervals of 1/1133 seconds, cyclically over four addresses in the storage unit, so that each address receives every fourth sample and intercomparing the integrated values. Since all signal components in the Omega signal format are even multiples of factors of 1133 cps, stationary values are obtained at the four addresses, which can be interpolated to provide the relative phase of the phase modulation components.

By the above process, the overall program develops words in storage corresponding to the velocity and the relative phase of each r-f signal component relative to the receiver time code. They are averaged over a large number of samples to provide smoothing of the data, and integrate the velocity components to produce the continuing rates of change of the magnitudes of the phase words, as required to track the incoming signal velocities with zero error. In the 0.2 second intervals between signals, as well as in the sectors of the signal sequence in which no signals of interest are being transmitted, the program carries out the synthesis of the phase difference measurements to resolve ambiguities; transmitting the composite readings to the read-out display, or other digital read-out as desired.

9.0 Navigation Charts and Compensation Graphs

As shown in Section 5.2, it is possible to transmit signals of excellent stability that leave their various transmitting antennas in phase. The navigator observes the phase difference between any two of these signals. Neglecting for the moment ambiguities, this phase difference defines a line of position. The charting problem is to compute in advance the relationship between phase difference and position and to display this relationship in a convenient way for navigational use.

The transmission time determines the phase difference between the signal transmitted and the signal received. Discussion of the problem in terms of transmission time is convenient because this procedure involves no difference between ambiguities, while the transmission times from two stations can immediately be converted into a phase difference. The transmission time is, of course, the distance divided by velocity of propagation. In the VLF navigation band, there is some difference in the phase velocities of various frequencies; and the group velocity, at which a difference frequency or modulation frequency travels, may be as much as one percent slower than the phase velocity which determines the transmission time of a carrier phase. We can neglect these variations for the time being and concentrate on the method of calculation of a received signal carrier phase.

To solve this problem we must know or determine a number of quantities. The distance depends upon the geographical coordinates of the transmitter and receiver and upon the size and shape of the earth. The velocity of propagation is near the velocity of light but varies slightly with the height and electrical state of the ionosphere, the electrical conductivity of the earth, and also with the latitude of the transmission path and its orientation with respect to the earth's magnetic field. As stated below, rules of accounting for these factors have been deduced from Omega measurement studies made at many places and from theoretical considerations.

9.1 Calculation of Distance

Charts for Omega, like those for loran, have been computed on the assumption that the Clarke spheroid of 1866 accurately represents the size and shape of the earth. It is true that recent studies have greatly improved our knowledge of the shape, but they have produced only unimportant changes in our ideas of size. The longest distance attainable on earth differs only by approximately 400 feet when calculated on the Clarke spheroid and on the best modern spheroid. Further, because our rules for determination of velocity were deduced from analysis in terms of the Clarke spheroid, it is better to use the same definition in the inverse computation. There is, therefore, no reason to change from the Clarke spheroid for Omega purposes.

Distances are calculated using the Andoyer-Lambert formula for the length of a geodesic (or path of minimum length) on a spheroid. This is an approximate formula, but there is no evidence that its accuracy does not exceed any potential accuracy of Omega. Here again, there is no justification for a future change in techniques. The Andoyer-Lambert formula calculates the distance on a sphere having a radius equal to the equatorial radius of the earth and then applies a small correction that depends upon the size and shape of the earth, upon the distance and the difference in longitude of the end points, and upon the average latitude and the difference in latitude of the end points. It may be expressed as:

$$d = a\sigma + \delta s \qquad (9.1\text{-}1)$$

where:
- a = equatorial radius of the earth = 6378.206 km
- σ = angular distance in radians
- δs = small correction defined below

σ may be calculated by any convenient trigonometrical formula for the distance on a sphere, such as:

$$\cos\sigma = \sin\phi_1 \sin\phi_2 + \cos\phi_1 \cos\phi_2 \cos(\lambda_2 - \lambda_1) \qquad (9.1\text{-}2)$$

where: ϕ_1 = geographic latitude of transmitter

ϕ_2 = geographic latitude of receiver

λ_1 = geographic longitude of transmitter

λ_2 = geographic longitude of receiver

The small correction is conveniently given (without explicit calculation of the mean latitude) by:

(9.1-3)

$$\delta s = \frac{af}{4}\left[\frac{(3\sin\sigma-\sigma)(1+S+C)(1+S-C)}{1+\cos\sigma} - \frac{(3\sin\sigma+\sigma)(1-S+C)(1-S-C)}{1-\cos\sigma}\right]$$

where: f = flattening of the spheroid = 1/295

$S = \sin\phi_1 \sin\phi_2$

and $C = \cos\phi_1 \cos\phi_2$

It is obvious that two distances, and hence two transmission times and their difference, can be calculated by this method. Unfortunately the problem cannot be solved inversely. It is necessary, therefore, to calculate time differences and times for a large number of conveniently located points in a service area (such as at even degrees of latitude and longitude) and then solve for the geographical positions of lines representing suitable constant values of time difference by inverse interpolation.

Fortunately this extensive computation can be done by machine and need be done only once, because the results can be stored in the form of charts or tables (or in a computer memory) and distributed to all navigators.

9.2 Calculation of Transmission Time

It is convenient to calculate transmission time (like the distance on an oblate spheroid) by finding the time for an assumed signal traveling at the free-space velocity of light along the geodesic at the surface of the earth. This approximate value can then be modified for the various small factors mentioned above.

The nominal value of transmission time is equal to the distance divided by the velocity of light. This can be expressed in any convenient units, such as microseconds or periods of a chosen frequency. We may, for example, take the equatorial radius of the earth, a, as 6378.206 km/299793 km sec^{-1} = 21275.4 microseconds. If we do this for the computations of Equations 9.1-1 to 9.1-3, all geodesics come out directly in microseconds of transmission time for our hypothetical surface wave. If our attention is focused exclusively upon a single frequency, say 10.2 kilocycles, we can take the radius, a, as:

$$\frac{6378.206 \text{ km} \times 10200 \text{ sec}^{-1}}{299793 \text{ km sec}^{-1}} = 217.009$$

This quantity is the number of free-space wavelengths of 10.2 kilocycles in the equatorial radius of the earth. Computations using this constant for the radius, a, will give distances in wavelengths or the numerically identical transmission times in carrier periods.

The rules for correcting this hypothetical transmission time to an actual time (or phase) can be given only approximately at the date of writing, because the values are continuously being improved as observations are made at additional locations. At present, the rules will be given for 10.2 kilocycles only because there has been little experience at other frequencies. At the time when values must be chosen for computation of charts, care should be taken to obtain the latest data and to compare the rules for various frequencies with theoretical values as they may be at that time.

We can predict that computation of transmission time should take account of the following factors:

(a) frequency or frequency difference

(b) time of day

(c) length of path over land

(d) conductivity of land in the path

(e) latitude of the path

(f) orientation of the path with respect to the earth's magnetic field

Several of these factors have unimportant effects and others have not yet been resolved. For example, the changes over land and sea are easy to recognize but are not yet associated with the conductivity of the land. Again, it is not clear that the latitude effect appears except in conjunction with the orientation. Whatever the limitations of our knowledge are, the rules available at present suffice to predict the observed phase (within a few microseconds of time) over considerable geographic areas and follow the variations mentioned in the list above.

9.2.1 Phase as a Function of Distance

Figure 9.2-1 shows two assumed examples of how phase varies with distance. The "phase delay" plotted here is the amount by which the actual phase lags behind the phase calculated for the velocity of light along the surface. The variations shown in Figure 9.2-1 are only a fraction of a period in extent, and are drawn in somewhat exaggerated form for the sake of clarity. The delay curves slope generally downward, which indicates that the phase velocity exceeds the velocity of light.

In Figure 9.2-1 it is assumed that at long distances the variation of phase with distance is linear and is determined by the phase velocity of the first mode of transmission. This behavior is indicated by the constant slope of the dashed lines. At short distances the second mode cannot be neglected and presents a signal with a noticeably different phase velocity but with a lower amplitude. Thus, there are "beats" between the two signals which decrease in separation and increase in size at the shorter distances. At some short distance, control of the phase is taken over by the ground wave (not shown here, but near zero delay). This apparently happens without the second mode having ever assumed control, at least at 10.2 kilocycles.

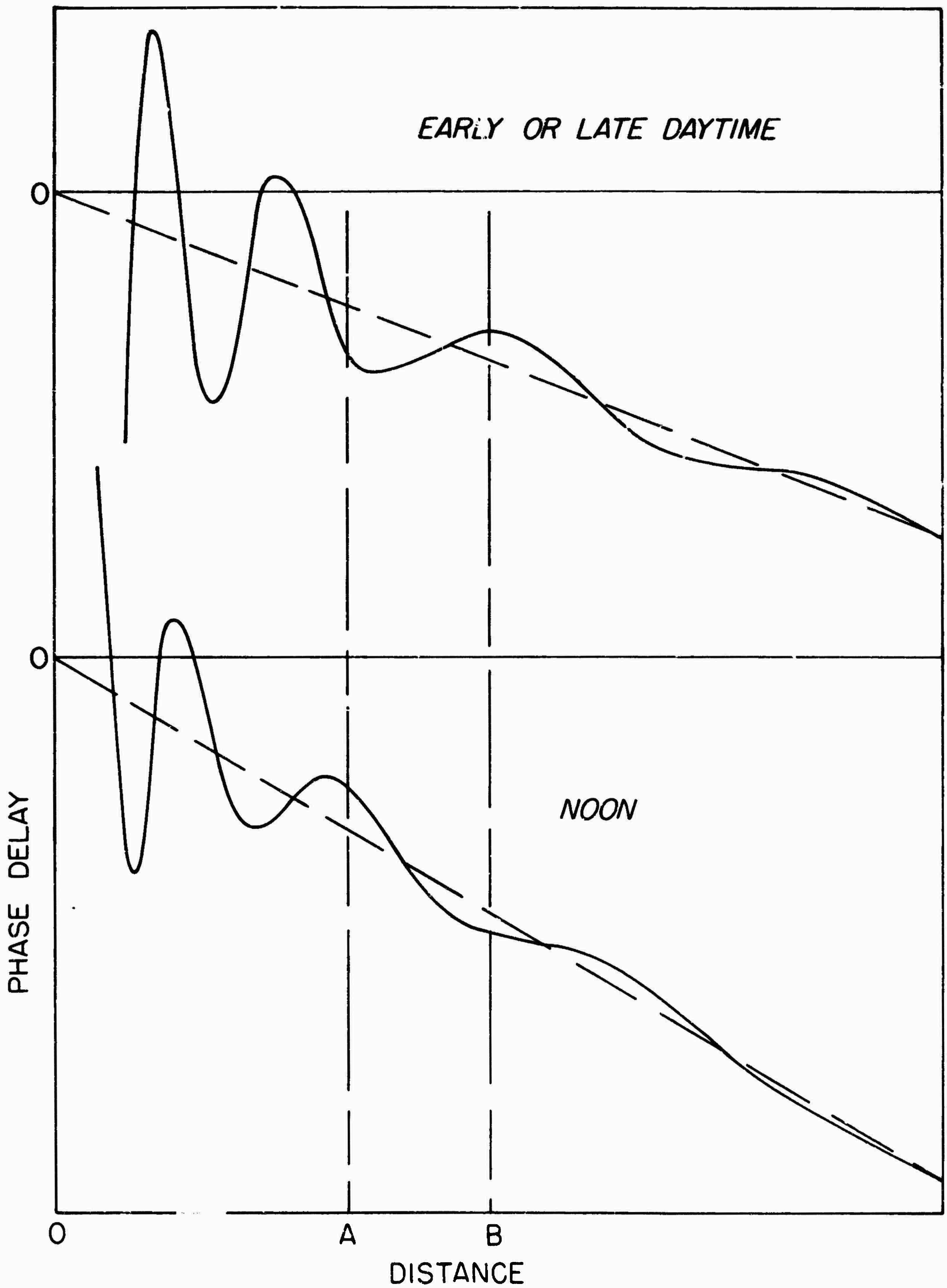

FIGURE 9.2-1. SKETCHES OF VARIATION OF PHASE DELAY AT SHORT DISTANCES

The beats of practical importance can be resolved only in the first few hundred miles from a transmitter, and their effect has not been detected beyond 2,000 miles. Their size, beyond 500 miles or so, seems to be limited to two or three microseconds in time.

Presumed curves are shown in Figure 9.2-1 for two times of day in order to explain a difference that is sometimes observed in diurnal curve form. In the figure, we note that the beats vary somewhat in size and position as a function of time of day. If the phase at noon is later than it is earlier in the day (as in A in Figure 9.2-1), the daytime part of the diurnal phase curve will be convex upward as in A of Figure 9.2-2. At another distance (such as B of Figure 9.2-1) there may be an upward ripple early in the day and a downward ripple at noon. This effect enhances the normal concave-upward character of the daytime curve, as shown in B of Figure 9.2-2.

The terminal slopes of Figure 9.2-1 vary at different times of day. This is equivalent to saying that the phase velocity is a function of time or, more fundamentally, of the sun's elevation above the horizon.

From these figures we see that a separate velocity solution may be formed at each hour of the day for a given frequency. Assumption of a constant velocity takes care of most distances, but a small correction that is an empirical function of distance must be added algebraically in the case of distances less than, say 1,000 miles. This treatment sufficies to cover points (a), (b), and (c) in the list under Section 9.2.

9.2.2 Velocity Over Land and Sea

The phase velocity over land is distinctly less than over sea water in the daytime (see Section 3.2). There seems to be almost zero effect of this kind at night, although we must allow for its introduction into the computations if it is later found to be significant. As in the phase diagrams above, a separate solution may be found from the experimental data for each hour of the day.

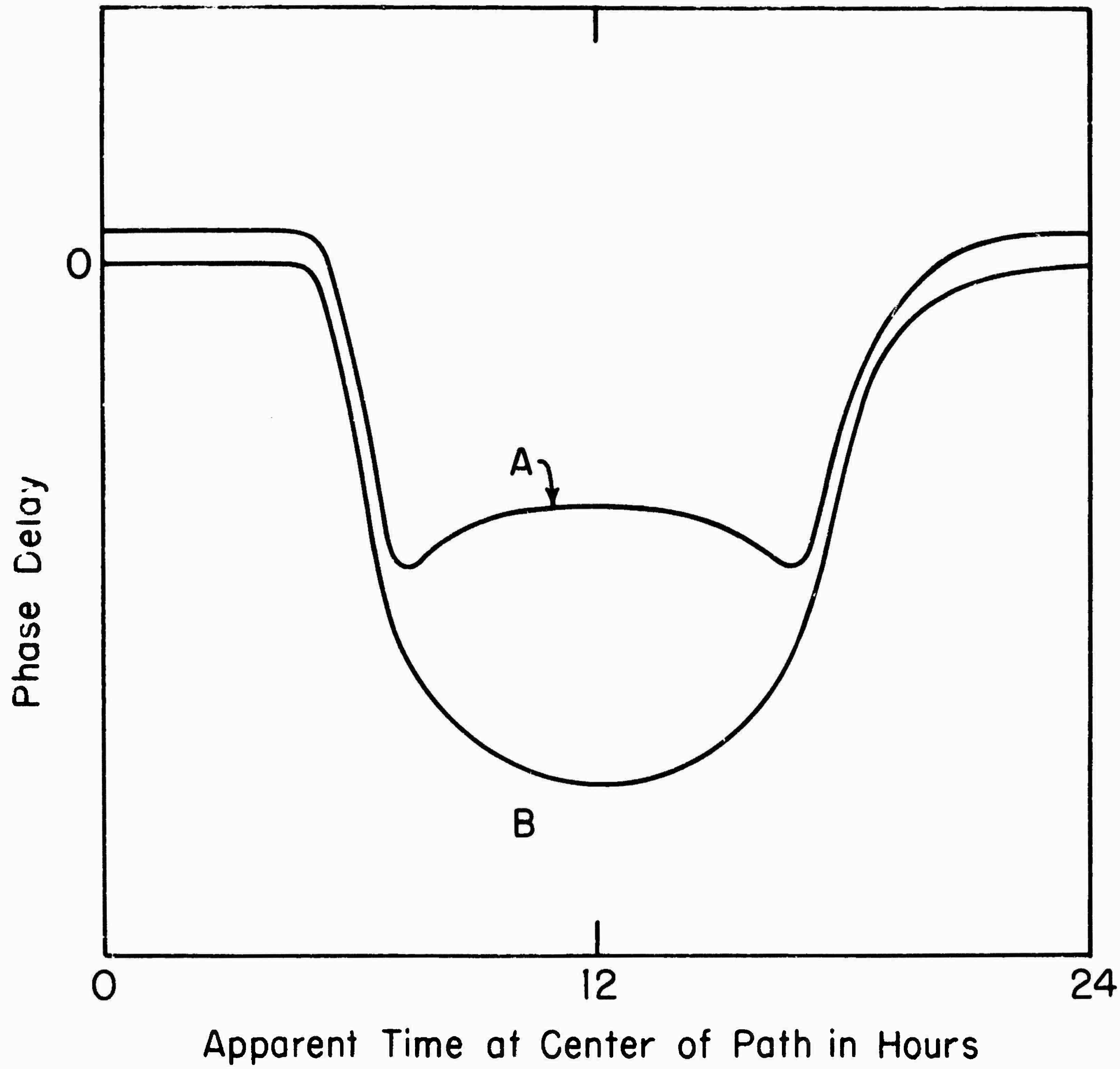

FIGURE 9.2-2. TWO KINDS OF DAYTIME PHASE VARIATION THAT MAY BE FOUND AT DIFFERENT DISTANCES

In principle, the phase delay over land, as compared with that over sea water, should be a function of the electrical conductivity of the land. This quantity is difficult to define for a frequency as low as 10.2 kilocycles, where the depth of penetration into the ground is so great. Also, maps of surface conductivity are very poor for most of the earth. Thus far, the effects of land have been so small that resolution in terms of conductivity has not been possible, although we know that the extremely poor conductivity obtained in the arctic permafrost and icecap areas has a greater effect than that of land in temperate regions.

It appears that a small additive coefficient (eventually a function of conductivity) multiplied by the length of path over land will serve to correct a sea water phase delay to the value for a mixed land-and-sea path. This coefficient, such as others mentioned above, may be evaluated for various times of day.

9.2.3 Velocity as a Function of Direction

Section 3.1 showed that the attenuation rate of a signal depends greatly upon the direction of propagation with respect to the geomagnetic field. There is also a small effect on the phase velocity. These factors have been studied by assuming a law of variation and then testing for the coefficient that brings the best agreement between experiment and calculation.

It is obvious that the velocity cannot vary with magnetic direction at the poles when the magnetic field is vertical. It has, therefore, been assumed as a first approximation that the directional effect is proportional to the horizontal component of the earth's magnetic field and proportional to the sine of the azimuth of a point on the transmission path. Both quantities vary along the path, but their product (if we assume the horizontal field to be proportional to the cosine of the magnetic latitude) is a constant along the path. This constant, K_ϕ, is numerically equal to the cosine of the angle between the great circle containing the transmission path and the geomagnetic equatorial plane and may be found from the following relation after the locations of the end points of the path have been transformed into geomagnetic coordinates:

$$K_{\phi} = \frac{\cos \phi_{1\,Mag.} \cos \phi_{2\,Mag.} \sin (\lambda_2 - \lambda_1)_{Mag.}}{\sin \sigma} \qquad (9.2\text{-}1)$$

where the angular distance, σ, is obviously the same in either geomagnetic or geographic coordinates.

The geomagnetic coordinates, referred to a north pole at 78.5 degrees N, 69 degrees W, may be determined from the relations:

$$\sin \phi_{Mag.} = \sin 78.5^{\circ} \sin\phi + \cos 78.5^{\circ} \cos \phi \cos (\lambda - 69^{\circ}) \qquad (9.2\text{-}2)$$

$$\sin \lambda_{Mag.} = \frac{\cos \phi \sin (\lambda - 69^{\circ})}{\cos \phi_{Mag.}} \qquad (9.2\text{-}3)$$

where $\phi_{Mag.}$ and $\lambda_{Mag.}$ are latitude and longitude in geomagnetic coordinates.

In Equation 9.2-1, the sense of the difference in longitude (receiver minus transmitter with west longitude taken as positive) is such that K_{ϕ} is positive for transmission toward the west. It varies from +1 to -1 and achieves its maximum absolute value only for transmission along the geomagnetic equator.

The most convenient form for use in calculating transmission time is the constant T_{ϕ} for a given transmission path, which is the product of K_{ϕ} times the distance in wavelengths.

9.2.4 Computation of Time of Transmission

The factors discussed above may be summarized in the following relation for the transmission time in periods:

$$T = (c/v)\, T_G + K_L\, T_L + K_M\, T_G + f(T_G) \qquad (9.2\text{-}4)$$

where:

c = velocity of light in vacuum,

v = phase velocity over sea water,

T_G = total transmission time in free-space periods or total distance in free-space wavelengths,

T_L = transmission time in periods over land or distance over land in wavelengths,

K_L = land coefficient = $\left[(c/v)_L - (c/v)_S\right]$,

K_M = magnetic coefficient = $(K_\phi/2)\left[(c/v)_E - (c/v)_W\right]$,

$f(T_G)$ = short distance correction, equal to the difference between the curve and the straight line in a diagram such as Figure 9.2-1.

As a practical example for long distances $\left[\text{neglecting } f(T_G)\right]$, we cite noon and nighttime equations for 10.2 kilocycles.

NOON

$$T = 0.99678\ T_G + 0.0004\ T_L - 0.0002\ T_\phi \qquad (9.2\text{-}5)$$

NIGHT

$$T = 1.00080\ T_G + 0.0000\ T_L + 0.0008\ T_\phi \qquad (9.2\text{-}6)$$

As mentioned above, the land-sea effect seems to disappear at night, while the directional effect becomes much larger and changes sign. It must be emphasized that these coefficients are tentative because they are continually being improved.

Since the coefficients K_L and K_M are so small, it is not necessary to measure the distance over land or to compute T_ϕ to great precision.

The magnitude of the day-night phase variation is given by the difference between Equations 9.2-5 and 9.2-6, or

$$\Delta T = 0.00402\ T_G - 0.0004\ T_L + 0.0010\ T_\phi \qquad (9.2\text{-}7)$$

It is seen that at 10.2 kilocycles the size of this day-night phase shift can vary by a factor of two with changes in direction and in the amount of land in the transmission path.

Considerably larger values of K_L, the land coefficient of Equations 9.2-5, 9.2-6, and 9.2-7, are to be expected in the arctic areas.

9.3 Charts

Because the diurnal variation in transmission time may amount to several miles in position, it is necessary to decide how to present this information to the navigatior. We visualize a two-step system which can best be described in terms of "old-fashioned" navigation with chart and dividers, providing

(a) a chart or charts that show the positions of the most probably useful lines of position on a scale suitable to the navigator's purpose, together with marginal notes that define any needed corrections for secondary frequencies and which are keyed to:

(b) a set of compensation graphs drawn for the different lines of position for various dates and which are given at small enough geographical separations so that interpolation is usually not necessary.

The chart format should follow loran and other precedents in scale and design. For Omega there will be a large number of families available, and some judgement should be exercised so that the chart is not made confusing by too many lines.

Because all stations are equal in Omega, it becomes a matter of definition whether readings are made in the sense A-B or B-A. Because the nomenclature on the charts must be kept simple and yet provide for 28 families of lines of position, it appears best to us to name the eight transmitters for the first eight letters of the alphabet, and then refer to a line in terms of both letters. As an operating convention, we suggest that this reference always be in the order of the alphabet and that the sense of the phase difference shown on the charts be the phase of the signal from the first named station referred to that of the second.

Thus, measurements in a pair consisting of stations B and E will be calculated and read in the sense B-E; and lines will be marked on the chart as BE 237, BE 238, etc., as requisite. The navigator's equipment will be switched, in this case, to measure the phase of B with E as a reference. An arbitrary constant, equal at least to the length of the base line between transmitters, must be added to the computed readings to avoid the occurrence on the chart or table of negative numbers for lines of position. If the chart is calculated in terms of 10.2 kilocycle readings, as seems most reasonable, this constant should be 900 periods. This number is $3^2 \times 4 \times 5^2$, or the ratio between 10,200 cycles and 11.33 cycles (the lowest modulation frequency). Thus 900 would always be the reading of the center-line of every pair. The readings would increase toward the station whose letter identification lay later in the alphabet and would decrease in the opposite direction; and the total range of readings would be twice the length of the baseline in wavelengths.

Position lines on charts should, of course, be keyed by the use of common colors to the compensation graphs to be discussed in the next section. Otherwise, judgements about color, weight of lines, spacing between lines, and other matters should be made by the charting authority. It should be remembered, however, that because of the long base lines in Omega, most position lines are nearly great circles and, consequently, are more nearly straight and parallel than in other systems charted in the past. For this reason and because there are more families of lines to be shown, the interval between lines may well be large, leaving much to interpolation by the navigator should he require maximum accuracy in his plotting.

9.4 Diurnal Compensation Graphs

Calculation of charts in terms of the average daytime phase seems best because daytime is the period of best accuracy and because this is a procedure parallel to the charting of loran. All steps outlined above should be taken with velocity values chosen to represent the average daytime conditions. This will result in charts that are substantially correct when there is daytime over the area containing the two transmitters and the receiver. If the magnitude of the diurnal shift is of importance to the navigator, a compensating quantity must be added to the observed readings before entering the charts. The variation of this compensation with time and space is the subject of this section.

Figure 9.4-1 shows (at the top) two estimated diurnal phase variations observed in Hawaii from hypothetical stations in New Zealand and California, each referred to its daytime value. The difference between these two curves is the diurnal variation of the phase difference to be expected in Hawaii. The negative of the lower curve in Figure 9.4-1 would be the compensation graph for this pair at the date given.

A curve of this kind can be calculated and drawn in all necessary detail, leaving the main chart unencumbered. A family of these compensation graphs can be supplied for various geographical areas and for various dates throughout the year.

An additional advantage is that, upon eventual derivations of more detailed computation methods based upon actual experience with Omega, later editions of the compensation graphs can be prepared without requiring revision of the main library of the more expensive charts. The compensation graphs, because they are drawn for various dates throughout the year, can also be used to absorb the slight annual variation in phase velocity, whose existence is suspected but not yet well evaluated.

In order to visualize the frequency in time and space with which the compensation graphs should be supplied, we have prepared Figures 9.4-2 and 9.4-3. Figure 9.4-2 shows three compensation graphs (for a given pair and date) for the three receiver positions which occupy roughly the corners of an equilateral triangle 1,000 miles on a side. The differences at their most extreme correspond to about two miles in position. It is clear that such diagrams need only be presented every 200 to 500 miles or once for each plotting sheet used by the shipboard navigator.

Figure 9.4-3 was calculated for a pair having a large difference of longitude, to make the sample diurnal patterns shift in time as much as

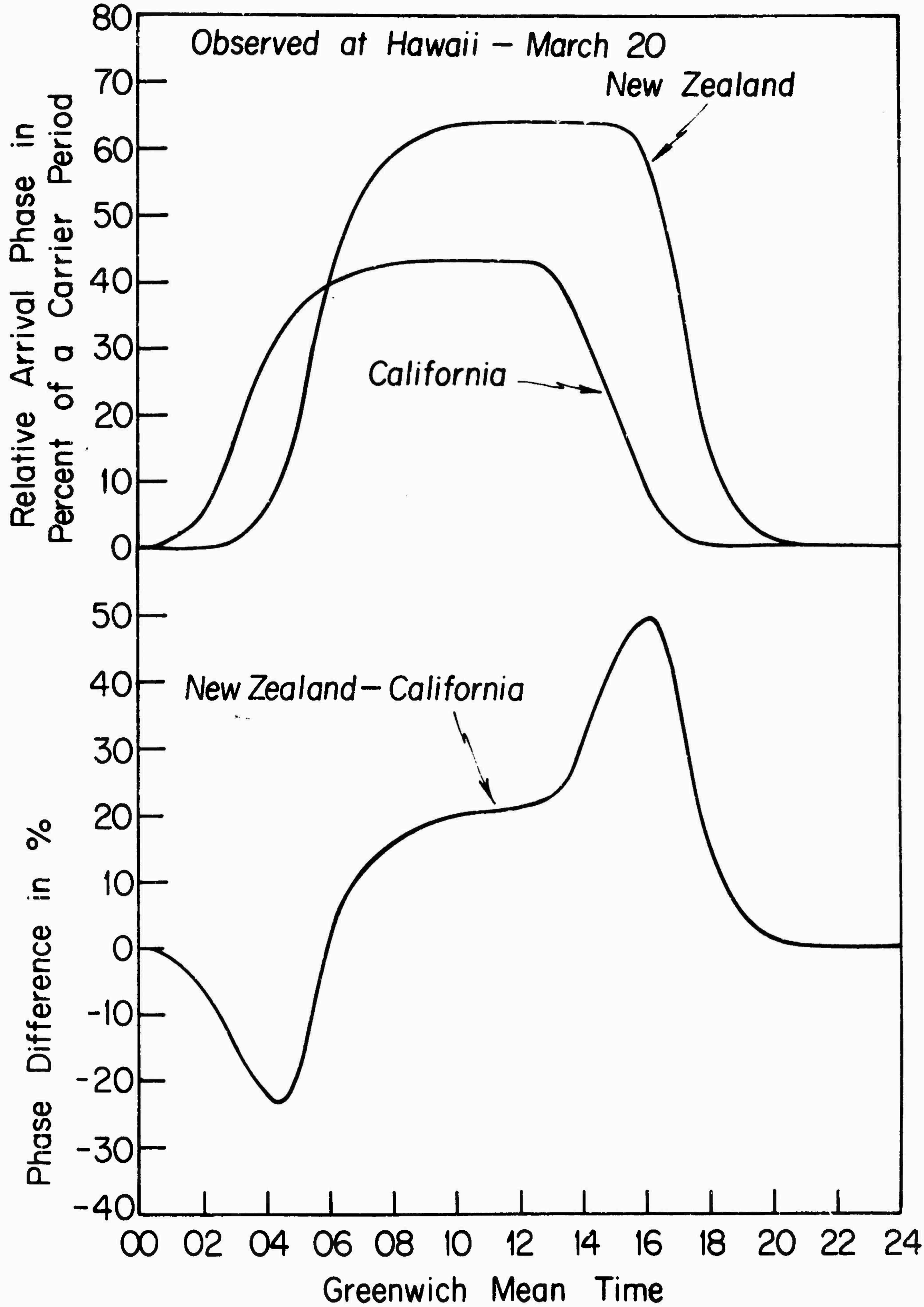

FIGURE 9.4-1. EXAMPLE OF DIURNAL VARIATION OF PHASE DIFFERENCE

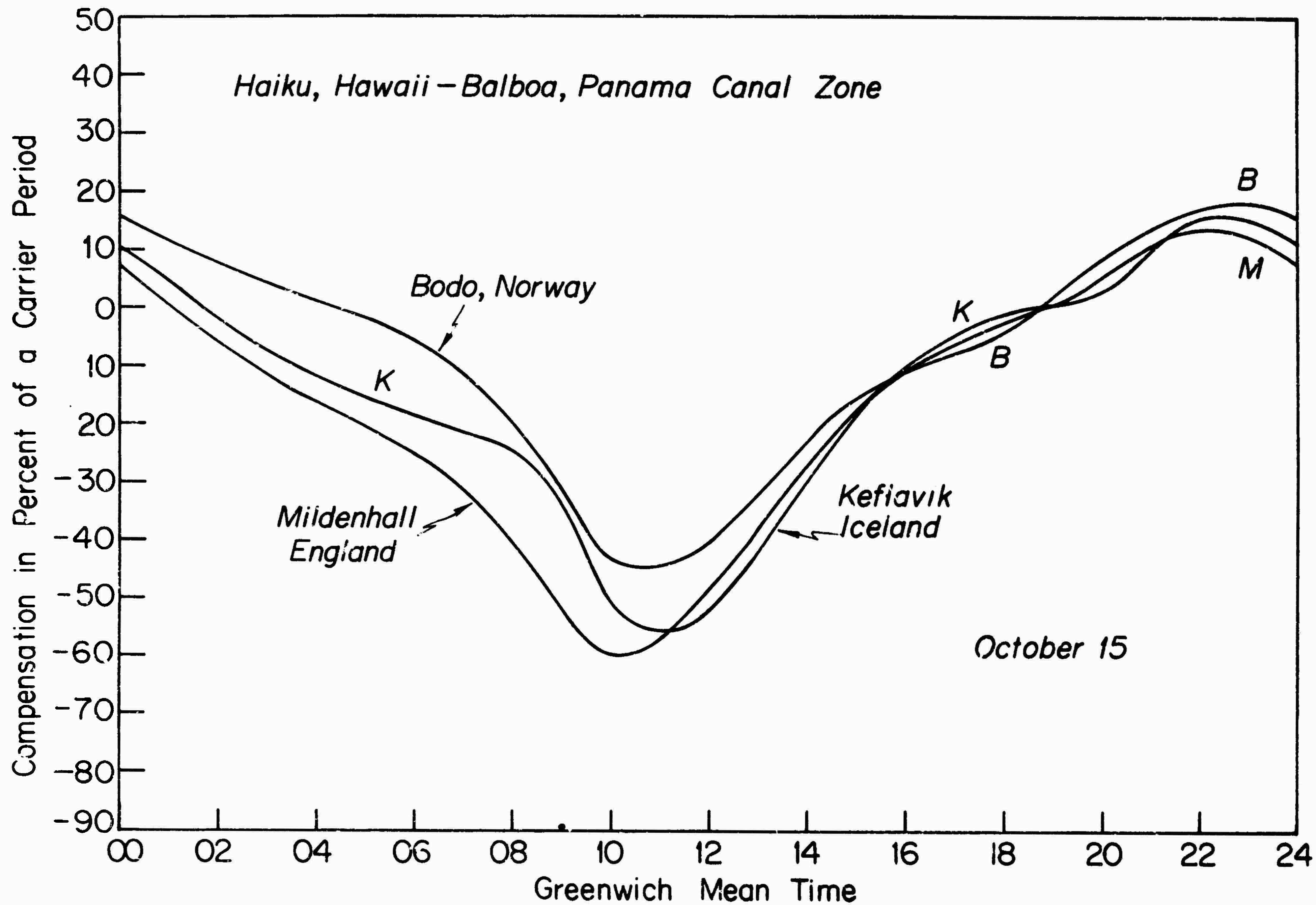

FIGURE 9.4-2. COMPENSATION GRAPHS AT THREE POINTS ROUGHLY A THOUSAND MILES APART

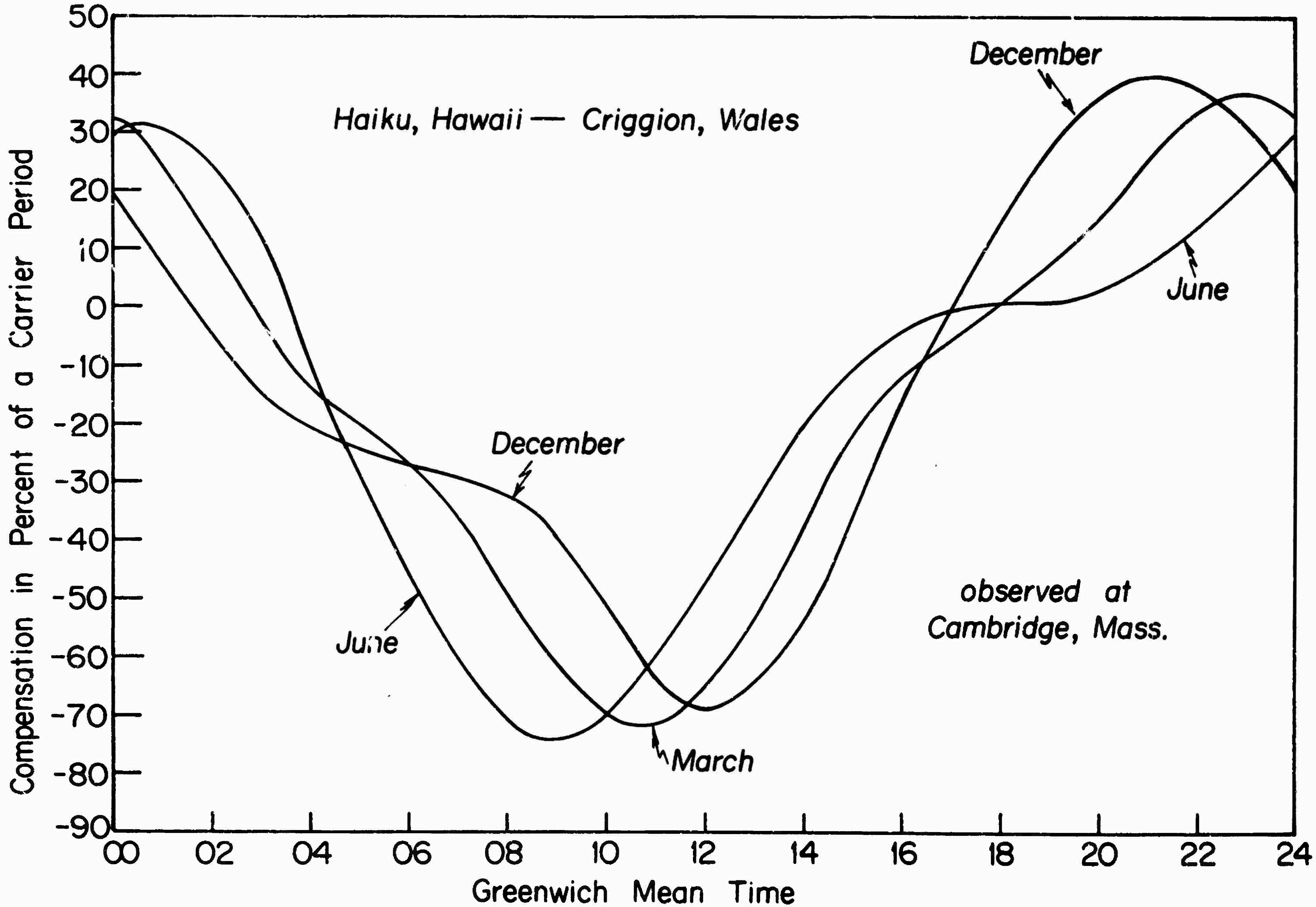

FIGURE 9.4-3. COMPENSATION GRAPHS FOR THREE SEASONS AT A FIXED RECEIVING POINT

possible. For a fixed receiving position and a given time of day, the extreme variation throughout the year is seen to correspond to three or four miles of position. It is, therefore, clear that approximately one diagram per month, or six or seven diagrams for the entire year (since spring and fall diagrams will differ only slightly), will suffice to present the temporal variation in adequate detail.

The effort required in preparing charts and compensation diagrams will be large. The magnitude can be judged by considering the number of charts required to cover the world.

It seems likely that map scales used with a long-range aid such as Omega will be in the range from 1, 000, 000 to 5, 000, 000 or, roughly, from 15 to 70 miles per inch. Table 9-1 shows few charts of most general utility.

It is convenient to fix our attention on the VL 15 chart, or plotting sheet. At this scale each map covers some 500 by 700 miles; and, in general, only one set of diurnal compensation graphs should be required per chart. These must be constructed for each of the useful sets of position lines or (as discussed below) for a few sets of lines taken together in pairs. We may estimate that five useful lines of position may be chosen; and, as the table shows, there would be about 500 such charts if it were decided to map the entire world at this scale. There will, therefore, be about 2, 500 sets of compensation graphs, each covering the annual behavior of a position line in a particular area. Fortunately these graphs need not be large or detailed.

In many ways it is more satisfactory to present the compensation information to the navigator in the form of a correction in miles than as a correction in reading (as described above). This implies that the charting authority has estimated which pairs of lines the navigator is most likely to use in a given area. If such a decision has been made, it is easy to compute the diurnal compensation necessary and to plot it for a given date on the same scale as the main chart. This is a most natural and convenient method, but we must remember that there are many more pairs of lines than there are lines. Consequently, selection is necessary, or there will be too many compensation diagrams.

TABLE 9-1. CHART SCALES USEFUL FOR OMEGA

Title	Projection	Scale	Approx. N. Miles per Inch	Size In Inches	Approx. No. of Charts to Cover World
World Aeronautical Charts	Conic	1,000,000	14	22 x 29	1,400
VL 15	Mercator	1,000,000	15	34 x 52	500
3,000 Series Plotting Sheet	Mercator	1,000,000	15	35 x 46	500
1,000 Series C & GS	Mercator	1,000,000	14	36 x 46	500
Jet Navigation Charts	Conic	2,000,000	27	42 x 57	120
VL 30	Mercator	2,000,000	30	35 x 54	120
Global Navigation and Planning Charts	Conic	5,000,000	68	32 x 47	20
VL 70	Mercator	5,000,000	70	33 x 52	20

A diagram drawn for a chosen pair of stations is a continuous curve lying near zero at mid-day and having a highly nonuniform variation of time along the locus. The shape is similar to a parallelogram with extra loops at diagonally opposite ends, and the loops may be as large as the parallelogram. Such diagrams for various times of year overlap considerably, and the times at the extreme vary by many hours if the stations involved are separated (as is usual) by several hours of longitude.

Experimentation with such diagrams has convinced us that they will be useful for many navigational purposes and should be included in the chart margins, but that they will not be convenient to use when the highest accuracy is required.

On a 1,000,000 scale chart, such a diagram might have a maximum departure from the origin of six or eight miles, or about one-half inch on the chart. At 5,000,000 to 1, this maximum dimension would be about one-tenth inch. On a figure with similar dimensions, the time scale cannot be marked with any great precision; and it should be noted that the greatest compensations are transient and are passed through rapidly. Such diagrams should not be used exclusively because they cannot be scaled without errors of a mile or two. For deep-water or aircraft navigation they would be entirely adequate, and they offer the highly attractive feature that the kind of diurnal variation and its general size and importance are clearly apparent to the navigator. On shipboard, for example, they would serve to show the times at which chart compensation is small and unimportant, and, conversely, warn of the hours at which the compensation is large and must be handled with care.

On occasions when maximum accuracy must be achieved, the navigator should have recourse to graphs of the first kind, where reading compensations are displayed pair by pair as functions of time. This procedure calls for more effort on the part of the navigator but makes available the full accuracy of the system.

It should be clearly understood that the difficulty of applying corrections to achieve an accuracy of a fraction of a mile is exactly the problem of charting the world to such a precision. It requires a large number of charts and correction diagrams, if the navigator is not to be forced to work to dimensions in the order of 1/100 of an inch.

Of course, all of this compensation can be represented in other ways, in nomograms, or in a computer memory. It is also true that a navigator will seldom need to draw upon more than a small fraction of the entire library for a given voyage or operation, and that much of the actual compensation will be done by approximation, such as simply remembering a certain reading tends to be two or three miles too far to the northwest at 1100 Z.. Nevertheless, a considerable effort will be required to chart the entire world for Omega and to prepare and disseminate the compensation data.

9.5 Observations of Various Frequencies

If there were no other reason for such a choice, the complexity of the charting problem would suggest that observations made at any of the five lane-identification frequencies should be converted to equivalent 10.2 kilocycle readings. This can easily be done in the receivers, where the difference phase at 13.6 kilocycles, for example, can be indicated at 3/4 its actual value or in periods of 10.2 kilocycles. This would leave a small error caused by the difference in velocity of propagation at 13.6 kilocycles. This error would change very slowly with position, so that a notation such as "add 0.17 lane at 13.6 kilocycle" could be given on the chart at intervals of a hundred miles or so.

A correction of this sort, perhaps the diurnal compensation itself, can be added to the observed reading or may be used by having in the receiver a dial and differential that would advance or retard the indicated reading by small amounts. Adjustment of such a compensation dial from time to time, in accordance with information taken from the chart or from the diurnal compensation diagrams, would keep the indicated readings in agreement with the charts. At 10.2 kilocycles this adjustment would be exactly the magnitude given by the compensation graphs or tables. At other frequencies the adjustment would compensate the difference in phase velocity with respect to 10.2 kilocycles as given by key entries on the main charts.

This discussion has been given in terms of charts and compensation graphs for the sake of simplicity. There are many other ways of storing the computed data for the use of the navigator. The computational effort is large but need be performed only once unless experience over several years should indicate that more accurate calculations can be made. In this case, a second edition of the compensation diagrams can be produced without requiring recomputation and replotting of the charts themselves.

The discussion of charting herein is entirely preliminary and tutorial. Much further study should be given to the twin problems of predicting charted values as a function of time and of presenting them to a navigator in the most concise and easily useful form.

APPENDIX A

STUDY and ANALYSIS
OF THE
OMEGA NAVIGATION SYSTEM

INTERIM REPORT NO. 54-1

Selection of an Optimum
Efficiency for a VLF Transmitting Station
by A. D. Watt

7 February 1964

BUREAU OF SHIPS and OFFICE OF NAVAL RESEARCH

NAVY DEPARTMENT Washington, D.C.

CONTRACT NO. NOnr 4107(00)

Dated 15 January 1963

DECO Electronics, Inc.

Boston Boulder Leesburg Washington

ABSTRACT

At VLF, very low frequency, most antennas are electrically small. Because of the resulting low value of radiation resistance, very large ground systems are required to obtain even nominal radiation efficiency. The structural size requirements of VLF radiation systems are seen to be primarily functions of radiated power capability and bandwidth required. Antenna radiation system costs are found to vary as the square root of power radiating capability. Theoretically determined values of ground system resistances are found to agree well with experimental results for actual VLF antennas.

Since yearly primary power and transmitter investment costs decrease with increasing efficiency while the yearly investment costs of the ground system increase with efficiency, it is obvious that there is a region of transmitting antenna system efficiency that will yield a minimum of total station yearly costs. This minimum cost point is determined for a station operating at 10 kc/s with a power output of 10 kilowatts. It is shown for the example chosen that the optimum region, which is near 30% efficiency, is fairly broad. The location of this optimum region is expected to vary with operating conditions that include frequency, station size, cost of primary power, and interest rates.

1. INTRODUCTION

When specifying the performance of a VLF transmitting station for communication or navigation purposes, there are three fundamental requirements which may be placed upon such a station. These requirements are: (1) power radiating capability; (2) bandwidth; and (3) electrical efficiency. The power radiating capability required is dependent upon data rate, message reliability, range or area coverage required, propagation characteristics, and background noise. The bandwidth required is determined by data rate, signal format, and type of modulation employed. The over-all transmitting station electrical efficiency, which is not as basic and independent a requirement as the first two factors, is a combination of the efficiencies involved by which the transmitting station converts primary electrical power to rf power, and then the rf power to radiated electromagnetic energy. The transmitting antenna efficiency is interrelated with the power levels considered and bandwidth requirements. In addition, it is dependent upon economic factors. In fact, as we shall see, it is possible to arrive at an optimum antenna system efficiency that yields a minimum yearly cost for a given station.

In general, for VLF transmitting antennas, which constitute a large part of the cost of a VLF transmitting station, either the power radiating capability or bandwidth requirement will be dominant in determining the size of an installation to meet specified performance. In attempting to arrive at an optimum electrical efficiency, it soon becomes apparent that one can define an economic rating factor that is the stations operational capability divided by the yearly cost of operation. In some instances, this factor could be defined in terms of satisfactory coverage area divided by the yearly cost of providing this satisfactory coverage. Since the operational requirements frequently can be simply defined in terms of our first two primary station requirements, i. e., power radiating capability and bandwidth, it then becomes a simple matter to determine the yearly costs of a station that will meet these requirements as a function of electrical efficiency.

Yearly costs can be assumed as falling into two primary categories: (1) investment costs, and (2) operational costs. When the power and bandwidth requirements are assumed to be fixed, it is possible to further divide the investment and operational costs into those which are relatively independent of station electrical efficiency and those which are dependent upon electrical efficiency. In general, the efficiency dependent yearly investment costs will increase with increasing efficiency while the efficiency dependent operational costs, i. e., primary power, will decrease with increasing efficiency. In view of this, it is apparent that the overall yearly costs will have a minimum value at some point that will be dependent upon the characteristics of the increasing and decreasing cost functions.

In order to illustrate the manner that an optimum efficiency can be obtained, we have chosen as an example a VLF transmitting station for the Omega world-wide navigation system. Since the data rates required of navigational systems are extremely low, bandwidth requirement will not be dominant in this case and we will be dealing primarily with the power radiating capability as the determining factor in required size of this transmitting installation. Based on previous engineering analysis, it has been determined that the basic frequency for this navigation system will be at about 10 kc/s while the power radiating requirement will be approximately 10 kilowatts per station.

2. RADIATION SYSTEM COST RELATIONSHIPS

For most VLF transmitting stations, the dominant investment cost factor is that of the support towers or masts. In order to determine the manner that these support costs vary with power radiating capability, a number of existing VLF stations have been examined and the expected cost of the support structures determined. A single cost - vs. - height curve for heavy duty masts was used for all structures. The power radiating capabilities expected from each of these stations at a frequency of 10 kc/s with a top hat voltage of 100 kv is shown in Figure 1. It is interesting to observe, as might be expected, that there is some scatter about the empirical line that gives the cost of the towers only as:

$$\underset{\text{(towers only)}}{\text{Cost}} \quad [\$10^6] \approx \frac{1.8 \times 10^4 \, P^{1/2} \, [\text{kw}]}{V \, [\text{kv}] \, f^2 \, [\text{kc/s}]} \, . \qquad (2\text{-}1)$$

This relationship shows that the tower or support structure cost of the transmitting station increases as the square root of the power radiating requirements and decreases as the square of the center frequency. Note also that cost is essentially inversely proportional to the voltage; however, this latter relation is only approximately true over a limited voltage range. As higher voltages are required, the cost of support structures will go up due to increased weight of insulators and wire.

Operating experience, as well as engineering design considerations, lead one to tentatively consider top hat voltages in the 150 kv region as a good compromise between a reduction in tower cost as well as reliable operation and relative freedom from corona or insulator problems. In addition to the tower costs, one must consider the cost of insulators, guys, hoists, and other associated mechanisms. In general, when attempting to determine radiation system costs, these factors can be considered as roughly increasing the tower only cost by about 40%. It should be emphasized that the cost relationships given here are only approximate, and in a detailed design, the exact cost can be determined to a much greater degree of precision. In such detailed design, these exact costs would, of course, be used in the subsequent analysis.

The VLF antennas listed in Figure 1 are all of the type employing masts or towers to support a capacitive "top hat." Several VLF antennas have been built that employ surrounding mountains to support catenary spans for the capacitive top hat. The performance of such antennas, although somewhat more difficult to calculate, has proved the feasibility of these "mountain type" antennas. When suitable terrain is available, it can be shown that the cost of such antennas is much less than that of conventional mast supported structures. In this connection, it is interesting to note that the Haiku station, which has been employed in much of the Omega system

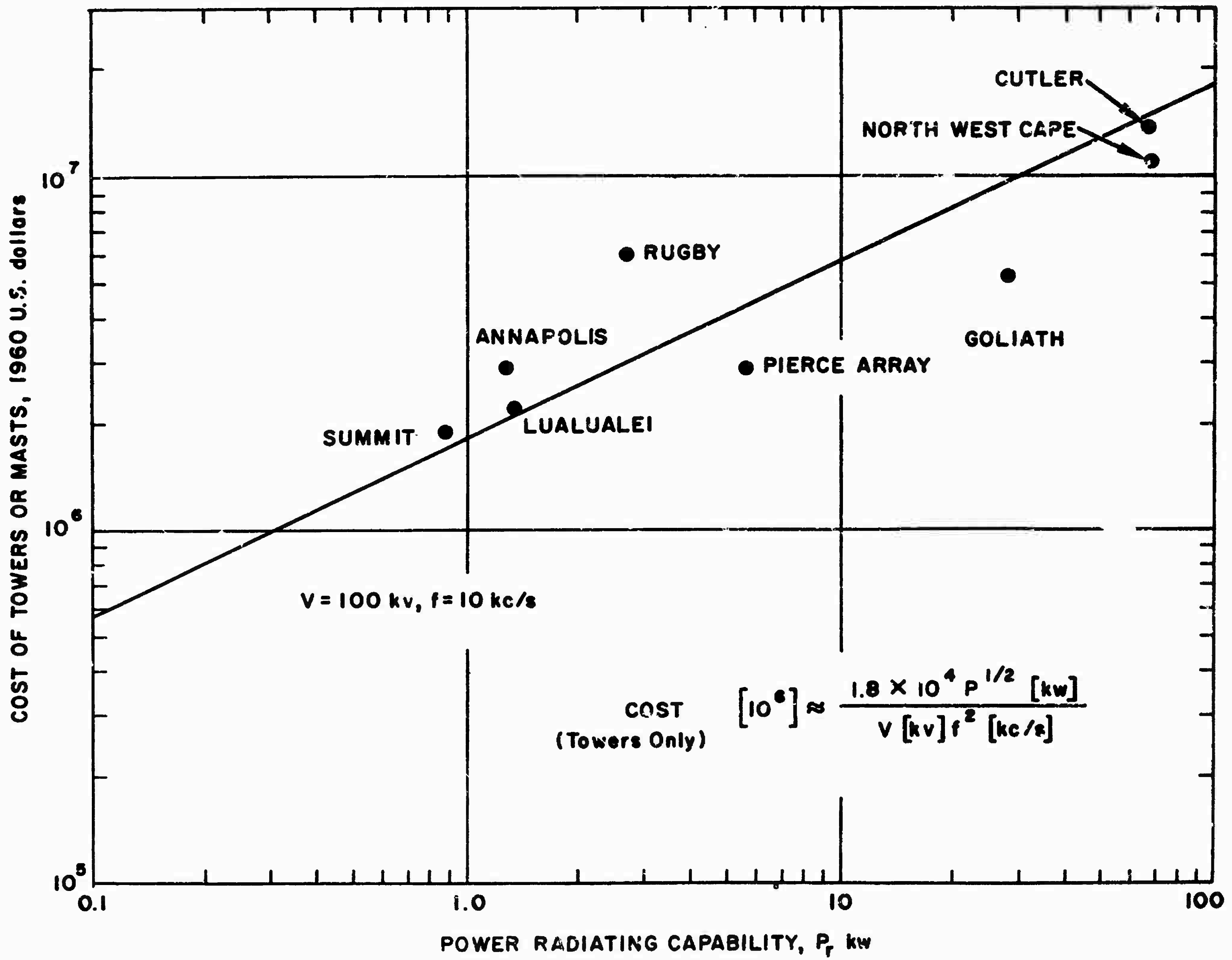

Figure 1. Cost of Support Structures for VLF Antennas as a Function of Power Radiating Capability

3. GROUND SYSTEM RESISTANCE AND COST

The antenna loss resistance, R_ℓ, which is seen at the input to an antenna, can be considered as consisting of at least three components, i. e.,

$$R_\ell = R_c + R_{sd} + R_g. \tag{3-1}$$

where: R_c is the component due to copper losses, R_{sd} is the effective series component due to dielectric losses in the insulators, and R_g is due to losses in the ground system. In most well designed antennas, R_{sd} and R_c should be almost negligible compared to R_g. In view of this, we shall attempt to determine the cost - vs. - R_g from experience as well as theory. Note that the following theoretical analysis will employ the usual method of assuming $R_g = R_E + R_H$, where R_E and R_H are the respective electric and magnetic field loss components of ground resistance.

The total amount of wire required in a ground screen to produce a given ground resistance is dependent upon the actual configuration of the antenna downleads and top loading. In addition, the cost of installing a ground system is not exactly related to the length of wire employed since an appreciable cost is also involved in the cross-bonding that must be done carefully to yield optimum results. In spite of this, experience has shown that it is possible to arrive at a very good approximation to ground costs by means of an average cost per unit length of ground system wire. In order to obtain some idea of the amount of wire that must be employed to obtain a given ground system resistance, R_g, for a typical VLF antenna, values of R_ℓ were obtained for the Cutler, Goliath, Annapolis, and Lualualei VLF antennas. These antenna loss resistance values are shown in Figure 2 where it is interesting to observe the rather complex variation of R_ℓ with frequency. If most of the ground losses are occurring as H field induced loss outside the ground screen area, it is expected that R_g will increase as $f^{1/2}$. As frequency is increased, the losses in the screened area will increase and when they become dominant, R_g should vary as $f^{3/2}$. This behavior is seen for Cutler, while the Lualualei antenna shows the $f^{1/2}$ portion, and the Goliath antenna shows the $f^{3/2}$ portion. The low frequency portion of Goliath is believed to be showing a negative slope because of poor insulators. This means that insulator component, R_{sd}, is not negligible for this antenna.

When attempting to extrapolate ground resistance values expected for the Cutler, Goliath and Lualualei antennas to 10 kc/s, we must disregard any poor insulator influence and realize that in the 15 kc/s region most of the antennas are behaving as though $R_g = k f^{1/2}$. This is the theoretical behavior of the ground resistance component, R_H, frequently referred to as the H field loss resistive component, [Smith and Devaney, 1959]. The $f^{1/2}$ behavior is only expected when ΔR_H, contributed beyond the ends of the ground system screen, exceeds ΔR_H within the screened area. For a given ground system, when frequency is decreased, the so-called electrical field loss component R_E, [Wait, 1958], which is essentially independent of frequency for E losses in the screened area and typical soils, will eventually become dominant. In other words, R_g, is expected to reach a lower limiting value as frequency is decreased. The possibility of this occurring near 10 kc/s has been included in the resulting values of R_g expected at 10 kc/s shown as squares on Figure 3. These R_g values are plotted as a function of total ground screen wire length. The estimated cost of each ground system shown as the right hand ordinate is obtained by multiplying the length of wire in the ground system by \$0. 5 for each meter of installed wire. This value includes wire costs, the cost of plowing in this wire and certain allowance for bonding. This average cost of around 50¢ per meter of installed ground wire yields total ground system costs typical of actual experience. It is well known that R_g is dependent upon the ground conductivity, σ, which can vary appreciably with location. The manner in which the value of σ is likely to influence R_g and required ground system size will be considered in the following analysis.

Theoretical determination of ground system resistance can be very lengthy and complex for the complicated VLF antenna structures shown. When a simple vertical electrical monopole antenna is considered with a radial ground system, values of R_H can be obtained from curves derived by Wait and Pope [1954] and also from a more detailed set of curves and tabulated data by Maley, King, and Branch [1963].

When antennas are electrically very small, which is the usual case at 10 kc/s, R_H becomes a function of the ratio of ground system radius to antenna height, and it is possible to plot the data from the first two pages of tabulated data in Maley, et al in the form shown in Figure 4. The values of R_H shown are for a fixed ground system parameter, which is actually equal to a wave tilt, T_g, of 0.03 (called δ by Maley) where $|T_g| \approx (\omega \epsilon_o/\sigma)^{1/2}$. The H field loss resistance is seen to depend upon ground system radius divided by effective height, a/h_e, and N, the number of radial wires. Values of R_H for other frequencies and ground conductivities can be obtained from the approximate solution for the case where N is large enough so that the dominant H field losses are occurring beyond the edge of the radial ground screen and is shown as the dashed line in this figure. The derivation, to be published elsewhere*, is too lengthy to include here but the result is

$$R_H \approx \frac{R'_H}{2\pi} (h_e/a)^2 \qquad (3\text{-}2)$$

where R'_H is the effective surface resistance of the ground surrounding the antenna. This surface resistance is

$$R'_H \approx 2 \times 10^{-3} f^{1/2} \sigma^{-1/2} \qquad (3\text{-}3)$$

where f is the frequency in cycles/second and σ is ground conductivity in mhos/meter.

---

* VLF Handbook, A. D. Watt, to be published 1964.

The way in which σ influences the total ground system resistance, R_g, and the length of wire required for a given resistance value, is very complex. ΔR_E, is inversely related to σ. ΔR_H due to H field losses outside the ground screen was shown to be inversely related to $\sigma^{1/2}$, and ΔR_H due to losses within the screened area is directly related to $\sigma^{1/2}$.

The length of wire in a simple radial wire ground system is $\ell = Na$ where N is the number of wires, and a is the radial length. The wire spacing, S, at any arbitrary radial distance, ρ, is $S = 2\pi \rho / N$.

The resistive component due to E field losses, R_E, is frequently important for short top loaded antennas. For purposes of comparison with the experimental values of R_g shown in Figure 3, we will assume a top-loaded antenna where $h_e = 200$ meters, and the top loading is assumed to be a disk of radius $a' = 600$ meters.

Under conditions where the ground system radius, a, is equal to or greater than $(a' + h_e)$ and the number of wires, N, in the grid is large enough to keep the wire spacing, S, small compared to a skin depth, δ, in the earth medium, the E field resistance component due to losses in the region $0 < \rho \leq (a' + h_e)$ is very approximately* obtained as

$$\Delta R_{E_1} \approx \frac{4}{3 \sigma N (a' + h_e)} . \tag{3-4}$$

---

* In deriving this expression, the vertical electric field is assumed as constant over the effective area covered by the top hat as shown by Hansen and Larsen [1962]. Actually, near the down lead the field increases above this constant value of $E_z \approx I_o / (2\pi \omega \epsilon_o h^2)$. For our present analysis this increase near the down lead can be considered as being compensated for by a decrease in field when $\rho \rightarrow a' + h_e$.

The E field loss resistance contribution in the region $(a' + h) < \rho < a$ can be approximated as:

$$\Delta R_{E_2} \approx \frac{h^2}{\sigma N} \left[\frac{1}{(a' + h)^3} - \frac{1}{a^3} \right] ; \qquad (3-5)$$

while the contribution due to losses occurring beyond the ground screen where $\rho > a$ is

$$\Delta R_{E_3} \approx \frac{14\, h^2}{f^{1/2}\, \sigma^{3/2}\, a^4} . \qquad (3-6)$$

Resistive values expected at 10 kc/s for the antenna described where $a'/h_e \approx 3$ are given in Table 1 for several values of N and for ground screen radius to antenna effective height ratios, a/h_e of 5 and 10. Values of R_H given are obtained from Figure 4. The resulting values of R_g as a function of total wire length ℓ, and the expected cost using the assumption of 50 cents/meter of wire are plotted on Figure 3 along with the experimental values. It is interesting to observe from the shape of the dashed curves that a simple variation of N with a fixed radius, a, does not yield an economical solution over a wide range of R_g. The solid line with the dotted extrapolation to larger values of R_g with a slope of -1 is believed to be typical of what can be obtained with rather simple types of ground systems. It must be emphasized that this is not necessarily the minimum cost ground system. A detailed optimum ground system design is dependent on many factors including ground conditions at each site in addition to the physical characteristics of the radiation structure.

TABLE 1

Ground System Resistance for a Top Loaded Monopole, where,

h_e = 200 meters, a'/h_e = 3, $\sigma = 3 \times 10^{-4}$ mhos/m, a/h_e = 5, a = 1000 meters

N	$\ell = Na$	R_H	ΔR_{E_1}	ΔR_{E_2}	ΔR_{E_3}	R_g	$
100	10^5	0.0550	0.0400	0.0013	0.0011	0.0974	5×10^4
250	2.5×10^5	0.0400	0.0160	0.0005	0.0011	0.0576	1.25×10^5
1000	10^6	0.0390	0.0040	0.0001	0.0011	0.0442	5×10^5
			a/h_e = 10, i.e., a = 2000 meters				
100	2×10^5	0.0380	0.0400	0.0020	----	0.0800	10^5
250	5×10^5	0.0170	0.0160	0.0008	----	0.0338	2.5×10^5
1000	2×10^6	0.0130	0.0040	0.0001	----	0.0171	10^6

Note: The number of significant figures shown for R values are for computational purposes only. The resulting value of R_g is likely valid only to the order of one significant figure.

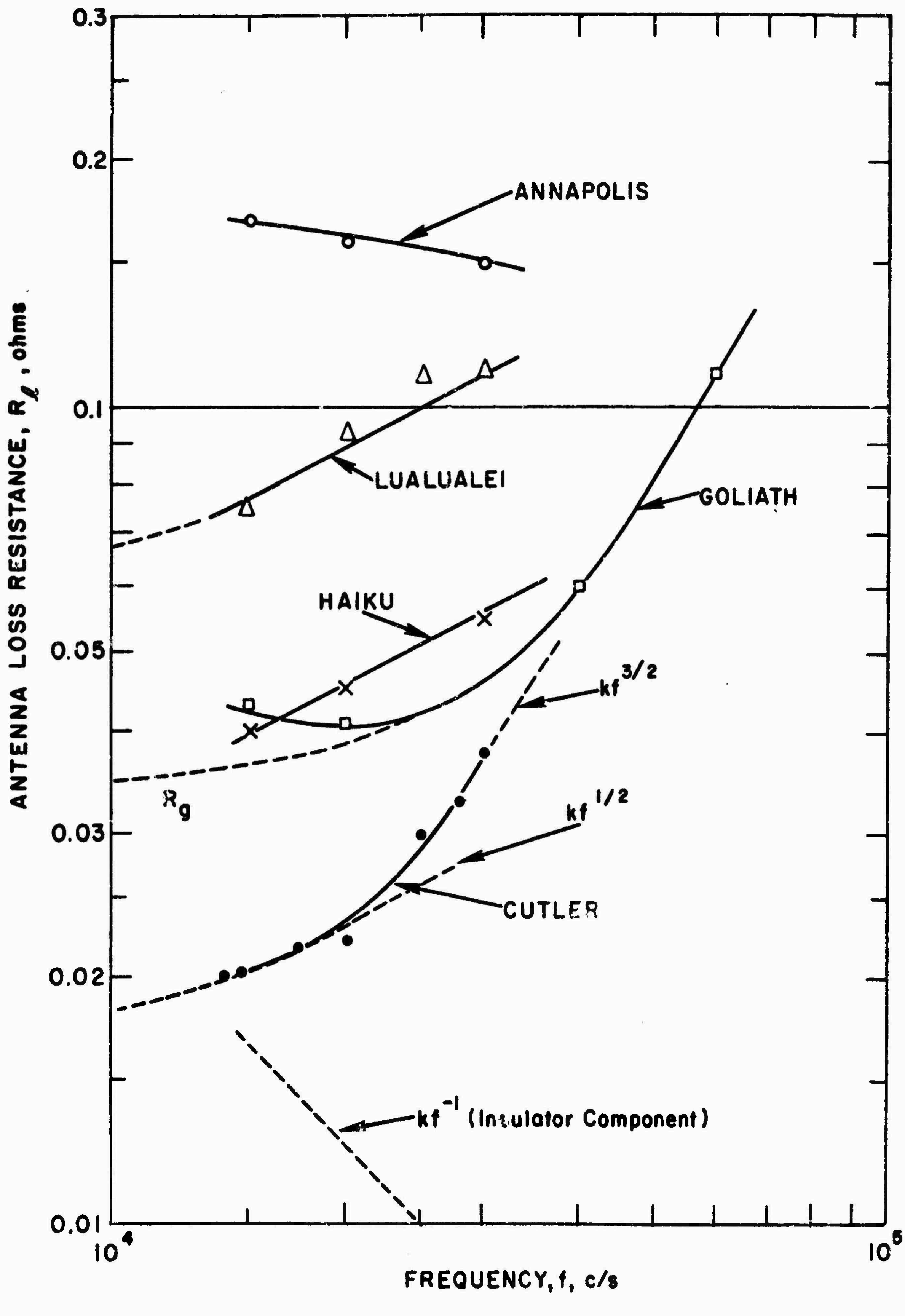

Figure 2. Antenna Loss Resistance as a Function of Frequency for a Number of VLF Transmitting Installations

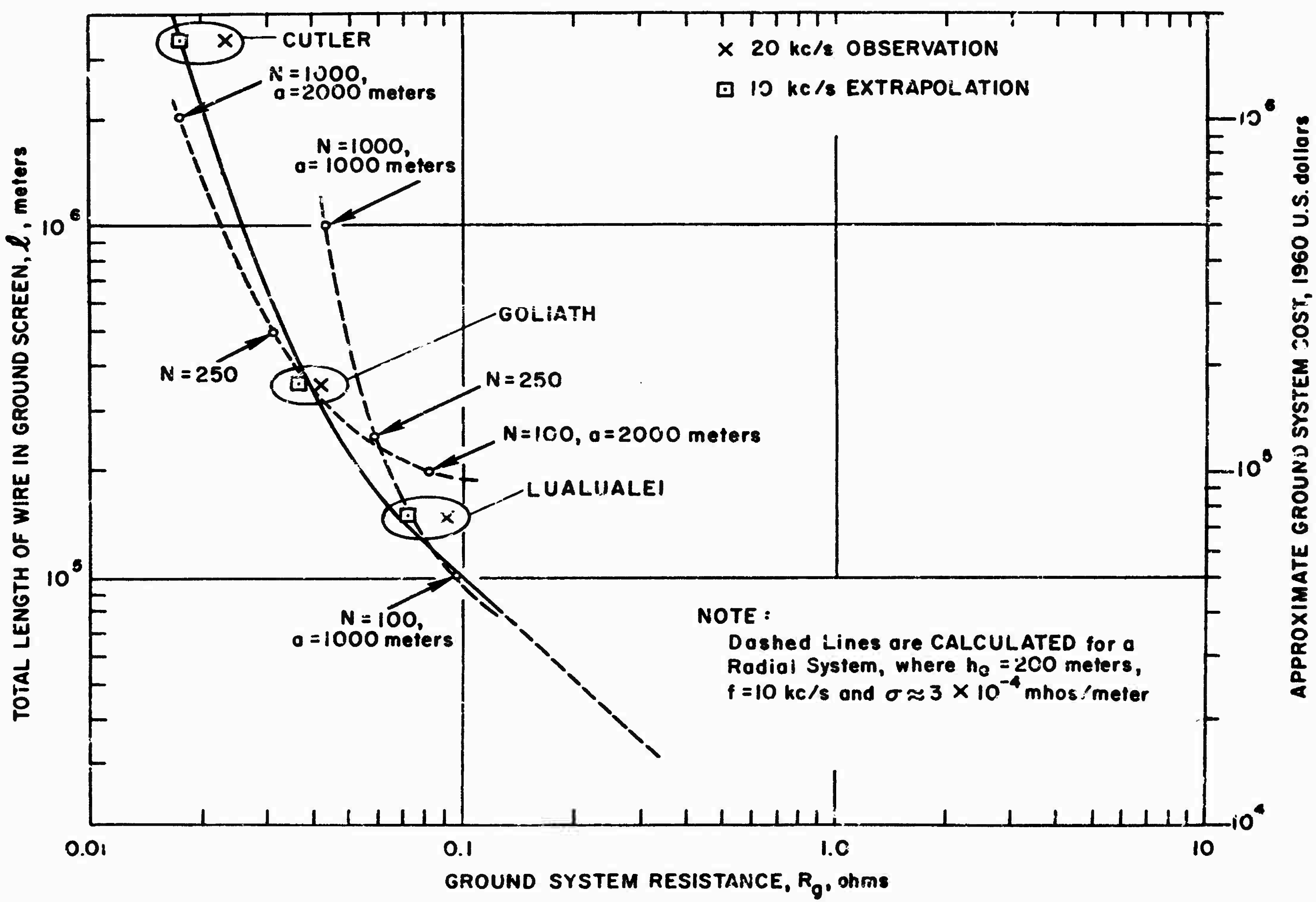

Figure 3. Observed and Calculated Ground System Resistance as a Function of Wire Length and Ground System Cost.

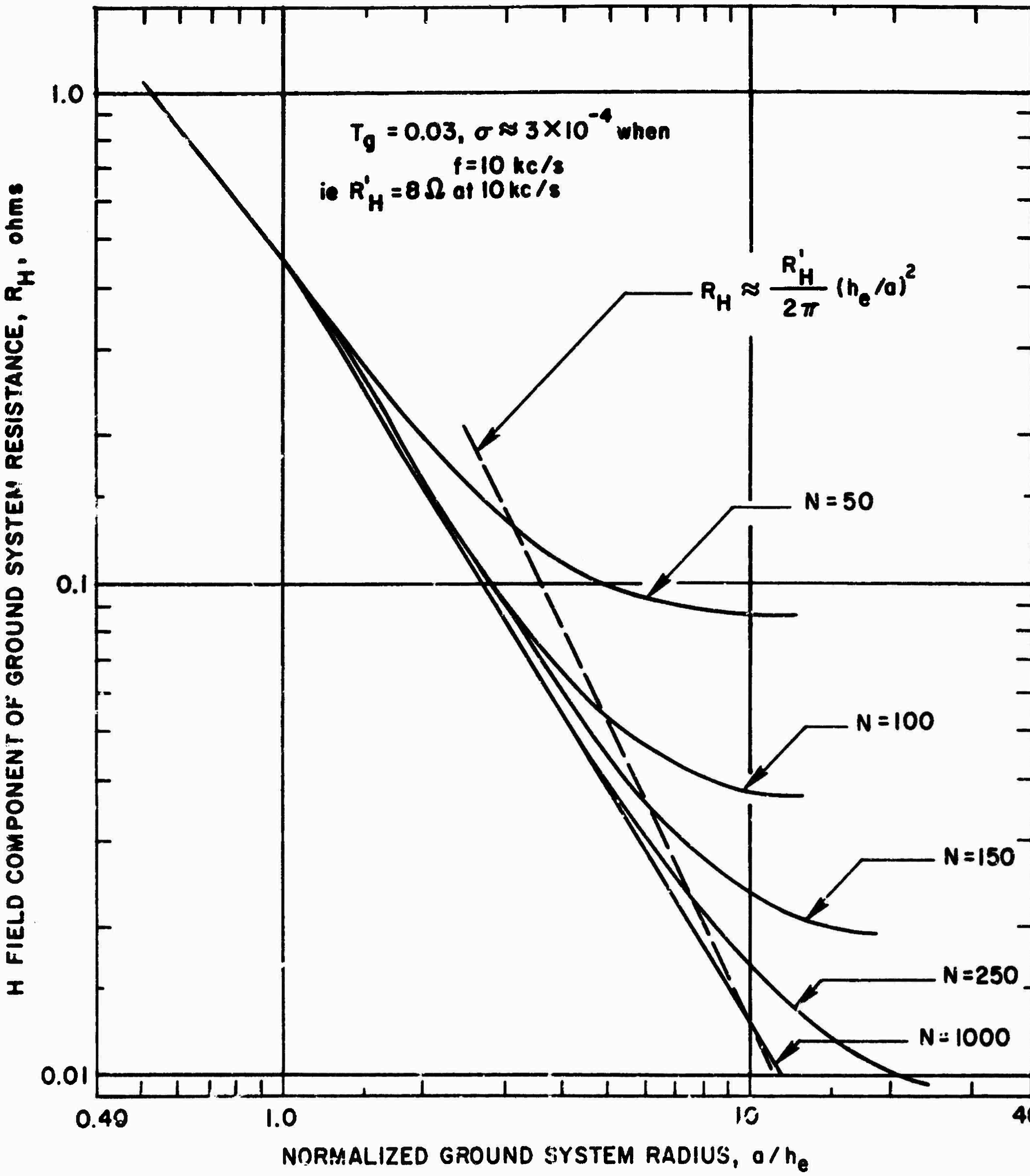

Figure 4. H Field Component of Ground System Resistance as a Function of Ground System Radius and Number of Radials. (Maley, et al, 1963)

4. RESISTANCE EFFICIENCY RELATIONSHIPS

When the effective height of the antenna is 200 meters, the radiation resistance is obtained from the relation

$$
\begin{aligned}
R_r &= 1.76 \times 10^{-8} h_e^2 f^2 \ [\mathrm{kc/s}] \\
&= 1.76 \times 10^{-8} \times 4 \times 10^4 \times 10^2 \\
&\approx 0.07 \ \Omega.
\end{aligned}
$$

The antenna system efficiency, η_{as}, can be written as

$$\eta_{as} = \frac{R_r}{R_r + R_g + R_i}$$

where R_r is the radiation resistance, R_g is the ground system resistance, and R_i is the resistance of the tuning inductor. As a first approximation we can assume that we will divide the losses equally between the ground system and the inductor, in which case, $\eta_{as} = R_r/(R_r + 2 R_g)$ and as a result we obtain

$$R_g = \frac{R_r (1 - \eta)}{2\eta} .$$

It is now possible to show the relationship between antenna efficiency and required ground system resistance and resulting costs. These are shown in Table 2.

TABLE 2

η	$1 - \eta$	$(1 - \eta)/2\eta$	R_g	Cost $\approx$
0.1	0.9	4.5	0.31	1.8×10^4
0.2	0.8	2	0.14	3.7×10^4
0.3	0.7	1.17	0.082	6×10^4
0.4	0.6	0.75	0.052	10^5
0.5	0.5	0.5	0.035	2.2×10^5
0.6	0.4	0.33	0.023	7×10^5

5. STATION COSTS

An exact determination of the investment and operational costs of a given installation is an extremely complex problem and we shall only attempt to roughly indicate some of the important factors which can be divided into three main categories: (1), those that are relatively independent of station power; (2), those that are dependent upon station power, and or bandwidth; and (3), those that are dependent upon power and efficiency.

5.1 Fixed Costs

In actuality, completely fixed costs do not exist for transmitting facilities, since usually as the power increases, the complexities and number of personnel increases so that housing site size and everything increases as the size of the facility increases. For our analysis, particularly if we are dealing with a rather small range of powers, or as in this case fixed power, the following factors are essentially fixed. These costs, which are all in the investment category, include site purchase* and preparation cost, general site improvement, including roads, buildings, communications, etc. The exact amount of these fixed costs will depend a great deal upon the location of the station and for purposes of illustration only, we will assume that for a 10 kilowatt Omega station the above relatively fixed costs will total approximately 3×10^6 dollars.

5.2 Power and Bandwidth Dependent Costs

This category includes factors that are determined by the size of the transmitting antenna; e.g., towers, top loading, insulators, hoists, and other associated equipment, which increase as the size of the antenna is increased.

For the 10 kw power radiating capability considered, if the trans-

* At some locations, the ground area necessary for installing a large ground screen may place this factor in Section 5.3 with the efficiency dependent cost factors.

mitting antenna is operated at 150 kv, the tower only costs from equation (2-1) will amount to about 3.8×10^6 dollars. If the towers, insulators, guys, hoists, etc. are included, the above must be increased by about 40 percent, with the result that the antenna minus the ground system and load coil is expected to cost approximately 5.2×10^6 dollars.

5.3 Efficiency Bandwidth and Power Dependent Costs

This category, which consists of both investment and operational costs, includes such items as standby power, transmitter, transmitter to antenna matching networks, and a special category, the ground system. Note that this latter system actually is primarily dependent upon efficiency; although to some extent, on the other two factors. The various efficiency dependent costs as a function of efficiency are shown in Table 3, where the ground system plus tuning costs are assumed to be twice the ground system only costs in Table 2. The required transmitter power shown in Table 3 is seen to decrease with efficiency and the column showing the cost of the transmitter and ground plus tuning systems is based upon an assumed transmitter cost of \$1.00 per watt. As we will see in the next section, it is important to consider the cost of station operation on a prorated yearly basis. In order to do this, we have taken the efficiency dependent costs and arrived at a prorated cost per year. In this case, the station life is assumed to be 20 years and interest rates at 5 percent per annum. The yearly costs in this case are approximately* $x/20 + 0.05x/2 \approx 0.075x$, where x is the cost of an item. We have not included the investment costs of the variation in primary standby power required as a function of efficiency since it will be essentially 10% of the transmitter cost.

---

* The x/20 term is payment on principle, the next term will depend upon the type of interest computation involved. A straight line average approximation to simple interest is $0.05x/2$. A somewhat more precise relation for 20 payments is an average of $0.05x/1.905$. Since interest rates and station life can vary, it is believed that $0.075x$ is a good approximation to use in computing yearly investment type costs.

TABLE 3

Transmitter, Ground System, and Antenna Tuning System
Costs - versus - Efficiency

η	Ground + Tuning	$P_{trans.}$ watts	Cost of Trans. + Ground + Tuning	Cost/Year
0.1	3.6×10^4	10^5	1.36×10^5	1.0×10^4
0.2	7.4×10^4	5×10^4	1.2×10^5	0.9×10^4
0.3	1.2×10^5	3.3×10^4	1.5×10^5	1.1×10^4
0.4	2×10^5	2.5×10^4	2.3×10^5	1.7×10^4
0.5	4.4×10^5	2×10^4	4.6×10^5	3.5×10^4
0.6	1.4×10^6	1.67×10^4	1.4×10^6	10.5×10^4

Figure 5 shows the yearly cost of primary power as a function of antenna efficiency. Also included in Figure 5 is the prorated investment cost for the other efficiency dependent parts of the transmitting system that include the ground system, transmitter, and tuning inductor. The sum of these is indicated as the yearly costs that are a function of η. It is important to note that if the cost of primary power is greater, as may be the case in remote locations, the point of minimum yearly costs will shift to the right. Conversely, if the power is cheaper, the point of optimum efficiency will be lower. In general, it appears for the conditions assumed in this example, that an antenna system efficiency of 25 to 35 percent is a good design objective. It should be emphasized that this design objective will not be the same for all VLF transmitting facilities since the optimum efficiency will vary with: power radiating capabilities of the system, frequency range employed, station location and antenna configuration.

In order to give some idea of the actual variation in total station operating costs as a function of antenna system efficiency, the upper curve in Figure 3 has been shown, which is based on η independent costs of 8×10^6 dollars, that includes all site costs and η independent antenna costs. This fixed cost, when prorated in the same manner as was done for the efficiency dependent costs, is 6×10^5 dollars per year. When this is added to the yearly cost of the efficiency dependent functions, one obtains the upper curve, which does not include any allowance for station personnel salaries.

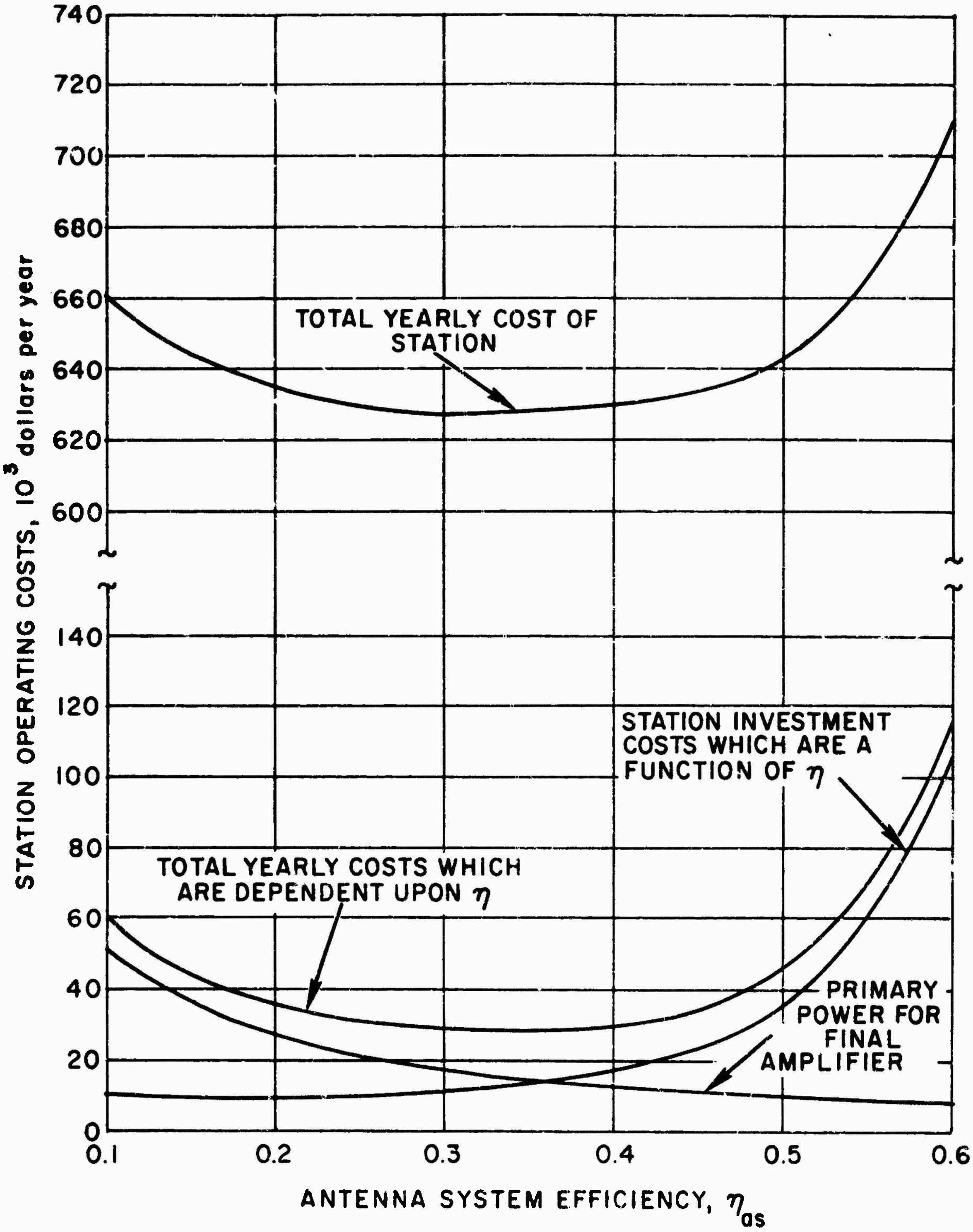

Figure 5. Comparison of Station Costs as a Function of Antenna System Efficiency Showing the Region of Minimum Station Yearly Cost Resulting from those Investment and Operational Costs which are Dependent upon η.

6. DETERMINATION OF YEARLY OPERATING COSTS - VS - ANTENNA SYSTEM EFFICIENCY

The yearly cost of operating a transmitting station can be determined as the sum of all fixed costs, which can be expressed in a prorated yearly basis, depending upon the life expectancy of the station and the current interest rate. Added to this prorated investment cost must be operating costs that include primary power and station operating personnel salaries, etc. Since this item of salaries is difficult to assess, and not likely to be dependent upon transmitting system efficiency, it has not been included.

The point of optimum efficiency can readily be obtained by considering only those prorated yearly costs that are a function of antenna system efficiency. The primary power cost for the final amplifier is essentially the only variable that will be dependent upon antenna efficiency where the variable station operating costs are concerned. Table 4 shows this cost, based on the assumption that the power required from the transmitter is equal to P_r/η_{as}, where η_{as} is the antenna system efficiency, and the final amplifier efficiency is $\eta_{pa} \approx 0.5$. It is also assumed that the station will be operating essentially full time; i.e., $\approx 8.7 \times 10^3$ hours per year and that the cost of 60 cycle power is approximately 0.03 dollars per kilowatt hour.

TABLE 4

Cost of Primary Power - versus - Efficiency

η	Transmitter final Primary Power, watts	KWH/Year	Transmitter final Primary Power. Cost/Year
0.1	2×10^5	1.7×10^6	5.1×10^4
0.2	10^5	8.7×10^5	2.6×10^4
0.3	6.6×10^4	5.7×10^5	1.7×10^4
0.4	5×10^4	4.4×10^5	1.3×10^4
0.5	4×10^4	3.5×10^5	1.05×10^4
0.6	3.3×10^4	2.9×10^5	8.7×10^3

7. CONCLUSIONS

The results of the analysis for the particular VLF antenna considered show a very broad minimum in yearly costs located in the region between 20 and 40% efficiency. It should be emphasized that a number of approximations have of necessity been made in this analysis which, although justifiable for planning purposes, should not be employed when solving for the final design of a VLF transmitting facility. When a final design optimization is carried out, the exact properties of the soil on the site should be employed in the calculations and, of course, the detailed variations in electric and magnetic fields considered when arriving at ground system losses for a given arrangement of ground system wires.

If an antenna is located in an area where real estate is expensive and the total area employed for the ground screen system must be purchased, the optimum efficiency may be a few percent lower than given above. As pointed out earlier, the point of optimum efficiency is expected to vary with frequency as well as other factors that include primary power costs, ground conductivity, and radiating power level.

8. ACKNOWLEDGMENTS

Helpful discussions relative to ground system characteristics and design with the following individuals are gratefully acknowledged: A. N. Smith, W. W. Brown, J. R. Wait, W. E. Gustafson, T. E. Devaney, W. S. Alberts, and G. F. Leydorf. The need for such an analysis was pointed out by J. A. Pierce. The assistance of Mrs. W. Mau in preparing the manuscript, Mr. N. Kline and Mrs. E. Oberteuffer in preparing the illustrations is also gratefully acknowledged.

REFERENCES

Hansen, J. and T. Larsen, The Electri Field at the Ground Plane near a Disk-Loaded Monopole, NBS, Jour. of Research, Vol. 66D, No. 2, March-April 1962.

Maley, S. W., R. J. King, and L. R. Branch, Theoretical Calculations of the Impedance of a Monopole Antenna with a Radial-Wire Ground System on an Imperfectly Conducting Half-Space, Scientific Report No. 26, December 13, 1963, prepared for Air Force Cambridge Research Laboratories.

Smith, A. N. and T. E. Devaney, Fields in Electrically Short Ground Systems: An Experimental Study, NBS Jour. of Research, Vol. 63D, No. 2, September-October 1959, pp. 175-180.

Wait, J. R. and W. A. Pope, Characteristics of a Vertical Antenna with a Radial Conductor Ground System, Applied Scientific Research, B, Vol. 4, The Hague, Martinus Nijhoff, 1954.

Wait, J. R., On the Calculation of Transverse Current Loss in Buried Wire Ground Systems, Applied Scientific Research, B, Vol. 7, pp. 81-86, 1958.

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Smith, A. N. and T. E. Devaney, Fields in Electrically Short Ground Systems: An Experimental Study, NBS Jour. of Research, Vol. 63D, No. 2, September-October 1959, pp. 175-180.

Wait, J. R. and W. A. Pope, Characteristics of a Vertical Antenna with a Radial Conductor Ground System, Applied Scientific Research, B, Vol. 4, The Hague, Martinus Nijhoff, 1954.

Wait, J. R., On the Calculation of Transverse Current Loss in Buried Wire Ground Systems, Applied Scientific Research, B, Vol. 7, pp. 81-86, 1958.

are three series of 0.1 second triggers produced. Each is phase locked to one of the three signals selected for measurement

At suitable intervals, a counter counts clocking pulses from an 'm' trigger to the next succeeding 'x' trigger, and reads out the time interval; a second pulse counter counts clocking pulses from an 'm' trigger to the next subsequent 'y' trigger.

Each total count so obtained consists of a number of whole counts of 100 clocking pulses each, determined by the particular counts at which the 1020:1 dividers happen to reset, plus a fractional count (i. e., less than 100) equal to the principal part of the corresponding differences in phase of the 'm' and 'x' or the 'm' and 'y' signals.

By resetting the counters arbitrarily to read the whole number of lanes, included in the total Omega phase angles, the readout can be made to correspond to the total count. When so set up, it will track the whole count, without losing count, since the oscillators will track the signals through whole cycles of phase, and the readout triggers are locked to the controlling oscillators.

8.7 Digital Omega Receiver

There may be applications for Omega where it is not feasible to have an operator manipulate receiver controls to reacquire signals in the event of a malfunction anywhere in the system causing the receiver to lose the signals. Possible causes for loss of signal may be due to excessive noise, a servicing changeover of a transmitter, or an interruption of primary power at some place in the system. The Omega signal format has been designed to provide for completely automatic receiver operation, including automatic acquisition of signals, resolution of the lane ambiguity, and readout of the hyperbolic position data.

At the present state-of-the-art, the most practical approach to performing such a multiplicity of functions in a single unit appears to be through digital computer techniques. A description of such a receiver in complete detail is beyond the scope of this report. This section will be limited to an outline of such a computer type receiving equipment indicative of how the essential functions may be accomplished. The elements of a special purpose digital computer adapted to perform all of the operations required to obtain alignment of the multiplex function and an unambiguous indication of two or more lines of position, without external aid except initial selection of the particular hyperbolic data to be displayed are described.

A fully automatic receiver must perform four basic types of functions.

a. Alignment of the receiver commutating function with the signal multiplex sequence so as to identify the particular signals it is desired to measure.

b. Determination of the signal relative phase at all frequencies incorporated in the signal format.

c. Resolution of the lane ambiguity.

d. Presentation of the signal timing in a form suitable for further processing.

8.7.1 Digital Phase Tracking Filter

The phase tracking and filtering functions essential to readout of the signal phase will be described first, to illustrate how such functions can be accomplished in a digital system. The receiver multiplex function must be aligned with the multiplex sequence of the incoming signals, however, before the signal phase can be determined.

Figure 8.7-1A shows, in rudimentary form, a digital phase-tracking filter by which the phase of an r-f signal may be determined by digital techniques. In principle, the operation of this circuit is essentially identical to the familiar analog phase tracking filters utilizing servo adjusted phase shifters or voltage controlled oscillators and phase detectors to phase lock a local reference to the average phase of an incoming signal. Thus, in the digital system, a stable local reference wave of the same frequency as the incoming signals is generated and compared in phase with the incoming

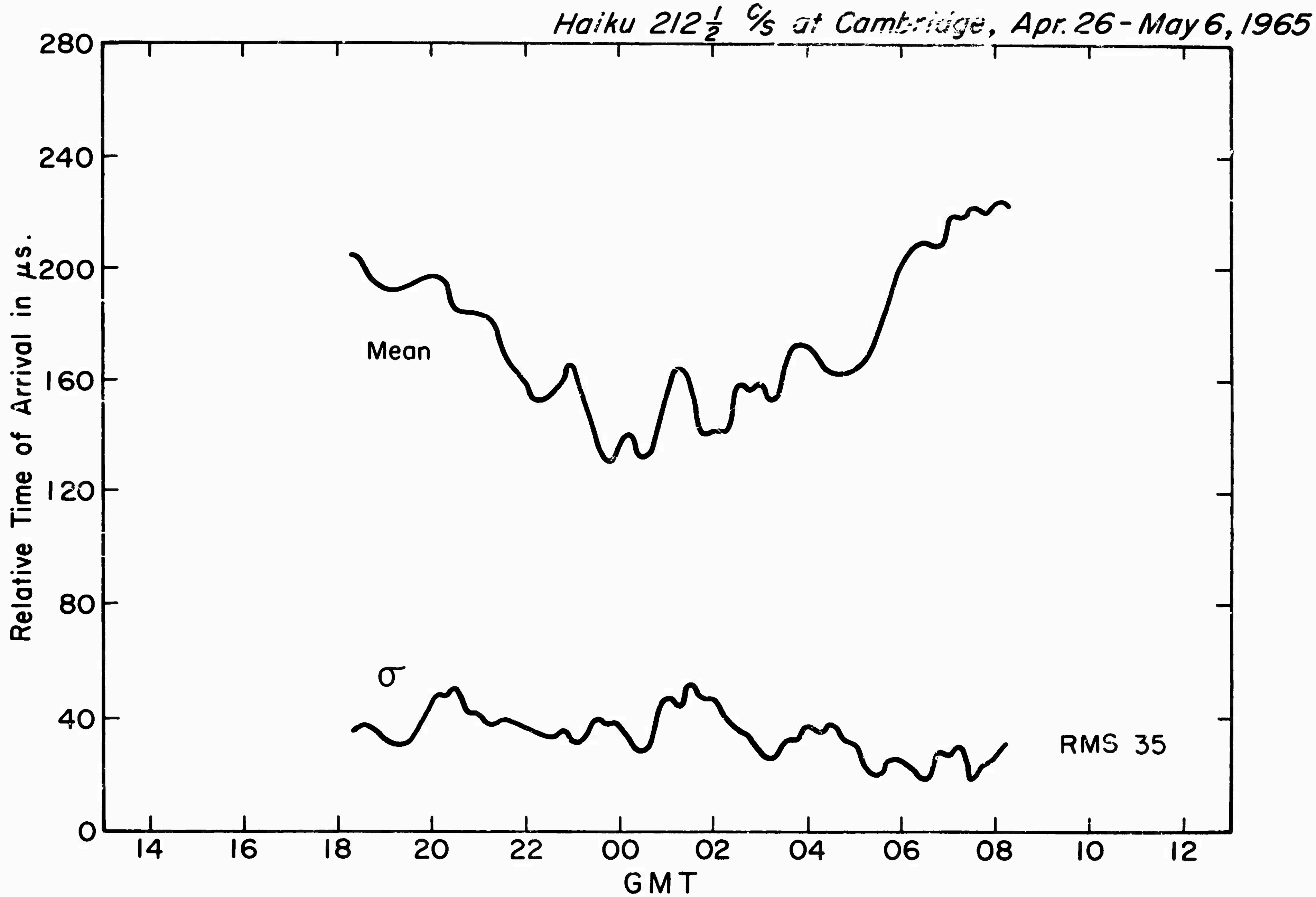

FIG. 2.4-7 DIURNAL PHASE VARIATION OF 212-1/2 CPS MODULATIONS OF SIGNALS RECEIVED AT CAMBRIDGE FROM HAIKO

An example of data on the time of arrival of the 3.4 kc difference frequency is shown in Figure 2.4-5. This is an example of one-way transmission over a single path. In this case, probably because of the relatively short distance, there is an unusually well-marked diurnal variation in the average values. Since this total variation is nearly 50 microseconds, or 1/2 period of 10.2 kc, it seems unlikely that satisfactory lane identification can be had without allowance for the expected diurnal changes.

On the other hand, the standard deviations shown at the bottom of Figure 2.4-5 are satisfyingly small in comparison with 50 microseconds, indicating that 10.2 kc lane identification can be made with good reliability if the diurnal variation be allowed for.

Figure 2.4-6 is a more realistic example of three-path data for the Forestport-Balboa pair observed at San Diego for the month of November, 1964. In this case, the root-mean-square errors are somewhat larger than are to be expected for the operational Omega system (because the data were taken with the master-slave relationship that will no longer be used) and even so average less than 20 microseconds. It should be noted that the 11 μs standard deviation deduced for a single path is not greatly different than the values on Figure 2.4-5, and indicates that, if taken signal-by-signal, identification should usually be highly reliable. An exception to this statement may be made for the sunrise period (near 10^h GMT in Figure 2.4-6) when the standard deviation appears high enough to limit the probability of successful lane identification to the 80-90% region.

It can be shown on theoretical grounds that the further stages of lane identification we have proposed are progressively more reliable than the first. A few experiments have been made using very weakly modulated signals radiated from Hawaii and received in Cambridge, Massachusetts. An example of data at 212-1/2 cps, for only 14 hours per day, is given in Figure 2.4-7. No great attention should be paid to the apparent diurnal variation. In this case the requirement for successful identification of the next-higher frequency is that the standard deviation should be small compared with 440 μs, as it certainly is.

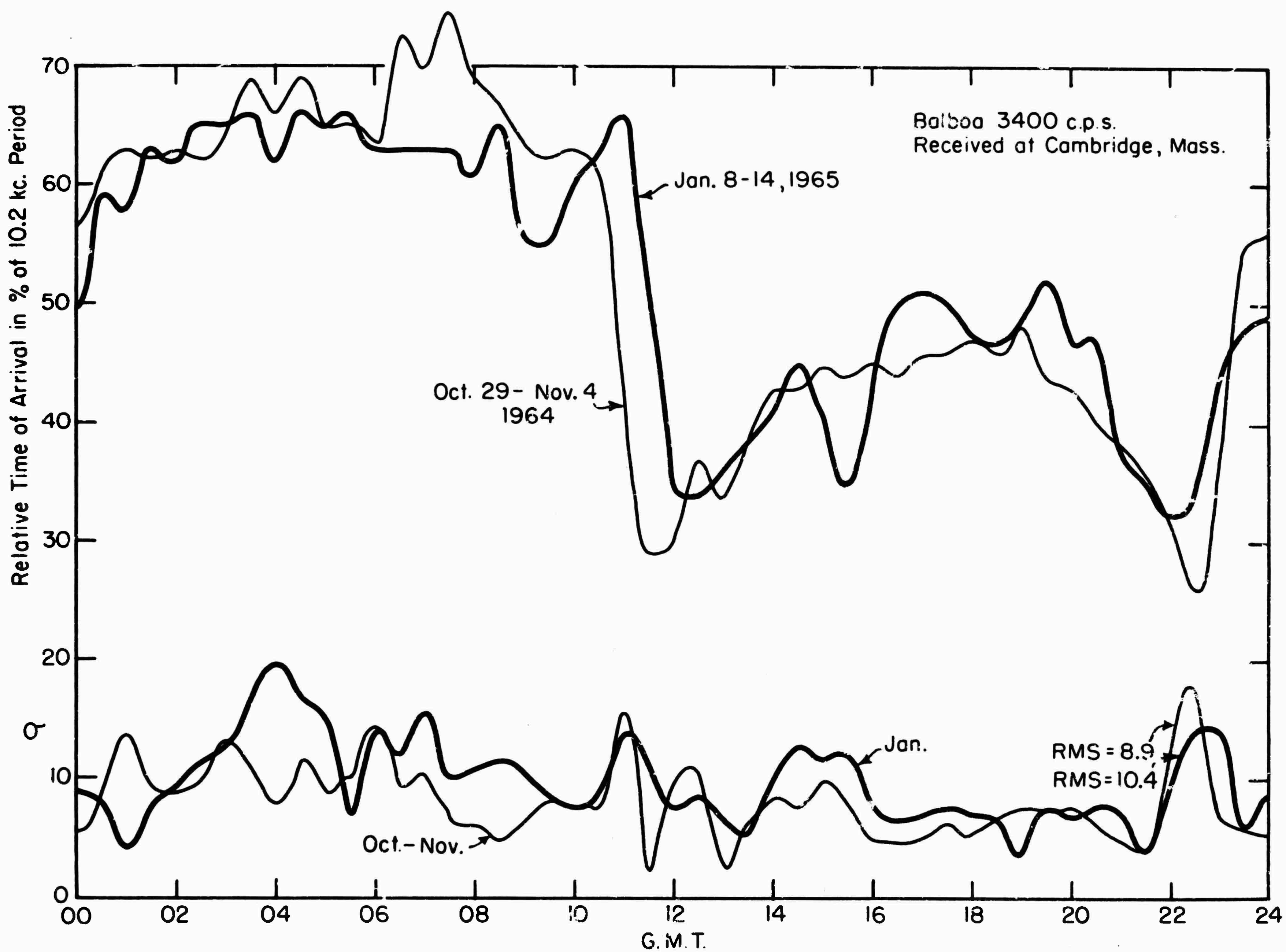

FIG. 2.4-5 DIURNAL PHASE VARIATION OF DIFFERENCE AT 10.2 KC AND 13.6 KC SIGNALS RECEIVED AT CAMBRIDGE FROM BALBOA

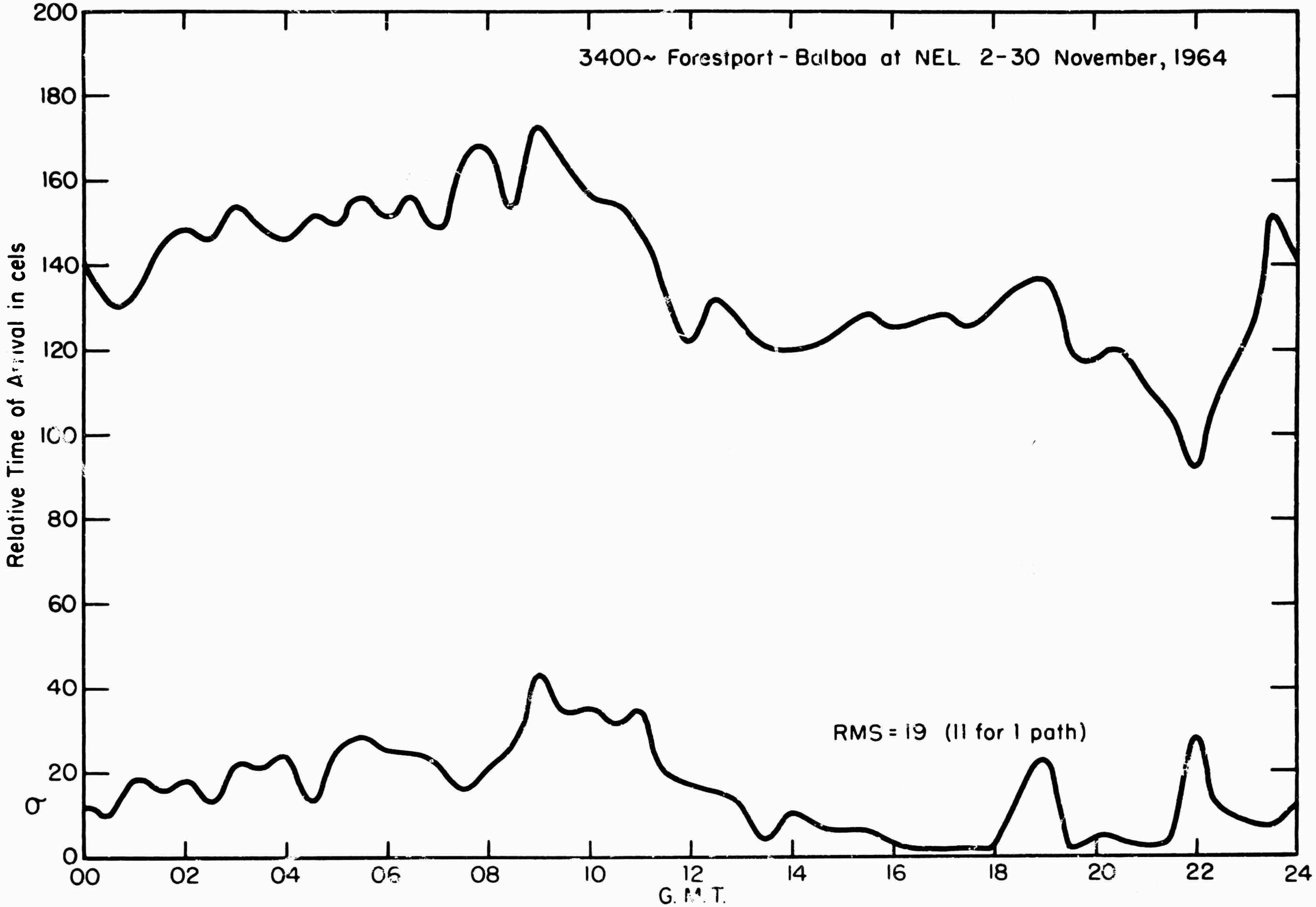

FIG. 2.4-6 DIURNAL PHASE VARIATION OF DIFFERENCE AT 10.2 KC AND 13.6 KC SIGNALS RECEIVED AT SAN DIEGO FROM FORESTPORT AND BALBOA

LIST OF ILLUSTRATIONS (Continued)

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OMEGA, A WORLD-WIDE NAVIGATIONAL SYSTEM, SYSTEM SPECIFICATION AND IMPLEMENTATION, SECOND EDITION, by
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