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# A DESIGN PROCEDURE FOR DETERMINING THE CONTRIBUTION OF DECKHOUSES TO THE LONGITUDINAL STRENGTH OF SHIPS

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A Design Procedure for Determining the Contribution of  
Deckhouses to the Longitudinal Strength of Ships\*

by

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\* The opinions expressed are those of the author and should not be construed as representing the official views of the Navy Department.

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### ABSTRACT

A design procedure developed by the author for determining the stresses in deckhouses is presented. The method includes a tabular procedure for calculating an effective moment of inertia which reflects the effectiveness of the deckhouse in contributing to longitudinal strength. The procedure is based on the theoretical approach by A.J. Johnson of the British Shipbuilding Research Association. The theoretical approach utilizes semi-empirical results of full scale experiments to evaluate the effect of differential deflections between deckhouses and their parent hull girders. This data has been incorporated into an analytical treatment of the problem based on the plane stress theory.

The nature of the problem is discussed and highlights from previously published work on the subject are given. In the conclusions, it is shown that all deckhouses contribute somewhat to the strength of ships and that this fact is useful to the naval architect in his quest for a structurally efficient ship.

## INTRODUCTION

Since about 1890, naval architects have calculated what is known as the longitudinal strength of ships by placing the ship on a static "standard wave" which has a length equal to that of the ship and a height from crest to trough of about 1/20 the ship's length. The combination of buoyancy forces and weight forces results in a load diagram from which shear and bending moment curves are calculated. Assuming that the structural behavior of a ship's main hull girder will be similar to a free-free beam, the Naval Architect calculates the stresses by the well known flexure formula,  $\sigma = Mz/I$ , where  $\sigma$  is the longitudinal stress,  $M$  is the bending moment,  $z$  is the distance from the neutral axis, and  $I$  is the area moment of inertia of the section under consideration. In the inertia calculation, the naval architect has been reluctant to include the deckhouse as a contributing longitudinal strength member.

The reason for this is quite logical. For some time, it has been recognized that the conventional beam theory does not generally apply to the combined deckhouse and hull. In fact, it has been shown that it is possible to have almost any stress distribution in the deckhouse depending on its effectiveness in contributing to the strength of the hull. Vasta [1], in 1949, was the first to demonstrate this with the tests on the S. S. President Wilson.

Realizing that the deckhouse structure may contribute to longitudinal strength, the naval architect, nevertheless, has no way of evaluating its effect. Always conservative in his design, he quite logically omits the deckhouse in his longitudinal strength calculation. In connection with this procedure, there often exists a popular misconception which hypothesizes a dichotomy between "stressed" and "unstressed" deckhouses. If the deckhouse is not considered in the inertia calculation, then it is considered "unstressed." But, in actuality, this would be possible only if the deckhouse were floating on the main deck and completely unattached to the main hull girder. Otherwise, if it is attached in any manner whatsoever, it is experiencing some longitudinal stress, and contributing to the total strength of the ship. This misconception has usually had no ill effects because other considera-

tions (local loads, etc.) have provided adequate structure in the deckhouse to carry the longitudinal stresses. However, it would be beneficial to obtain a clearer picture of the stresses in the deckhouse so that advantage may be taken of its contribution to longitudinal strength. The design procedure presented in the paper is intended to accomplish this objective.

To determine why the flexure formula cannot be applied to the combined deckhouse and hull, one must examine the assumptions upon which the theory is predicated. The beam theory might be referred to as the approach from the strength of materials point of view. One basic tenet of this approach is that the longitudinal strains in both deckhouse and hull vary linearly and are proportional to the distance from the neutral axis. In the main hull of a ship this is approximately true, as numerous experiments have verified. In addition, the beam theory requires that the deckhouse must be constrained to the hull such that the curvature of the two parts are identical during bending. However, the nature of the interaction between deckhouse and hull is complex and the curvature of the two parts may differ radically when subjected to load. As a result, the strains may not remain linear in the deckhouse and therefore, beam theory will not apply. Full scale tests have verified the nonlinearity of strains in the deckhouse.

That the curvature of the deckhouse may differ from that of the main hull may be seen in Figure 1. The system of shear forces which act at the base of the deckhouse, where it is connected to the hull, are eccentric with respect to the neutral axis of the deckhouse and therefore, there is a tendency for the deckhouse to deflect into a curvature of opposite sign to that of the main hull girder. (Curvature of the main hull girder is measured at the top of the side shell whereas curvature of the deckhouse is measured at the base of the deckhouse.) There is another system of vertical forces which tend to cause the deckhouse to follow the curvature of the hull. Depending on the combination of these systems of forces, the deckhouse may have a curvature differing from that of its main hull girder. The effectiveness of the deckhouse and thus its contribution to the longitudinal strength of the ship will depend to a large extent on how closely or how differently the curvature of the deckhouse resembles that of the hull.

Another phenomenon which occurs in deckhouses which must be accounted for in any complete analysis is known as the "shear lag" effect. It usually occurs in thin plating and concerns

the uneven distribution of flexural stress in flange members such as the deckhouse decks. The shear flow, shearing stress, and shearing strain in the flange plate are higher near the web (or deckhouse side) than remote from the web. The unequal shearing deformation causes the section remote from the web to "lag" as the beam is bent. The result is that plane sections do not remain plane which denies a basic tenet of beam theory.

### Historical Review

Concern over the deckhouse problem dates back to 1899 when Bruhn [2], studying discontinuities in ship structures, concluded that deckhouse stresses would not approach the simple beam theory values unless the deckhouse were eight times as long as high.

In 1913, Foster King [3] presented a paper in which he used beam theory to determine the stresses in large deckhouses. His design philosophy was that these stresses should not exceed those in the main hull girder if the deckhouse were omitted. Montgomerie [4], in 1915 extended King's treatment and through an analytical approach attempted to derive rational design formulas, which were later adopted by some of the Classification Societies. Expansion joints were introduced as a solution to the problem of an extremely flexible deckhouse which was unable to take part in the straining action to which a ship is subjected.

Hovgaard [5], considering a vertical plate of limited length attached to a horizontal plate of greater length, was one of the first to recognize the effects of curvature of deck house and hull. He considered shear in the boundary layer and concluded from his analysis that expansion joints might aggravate the stresses rather than relieve them.

The analysis up to and including Hovgaard's work in 1934 appeared to be of a superficial nature. Not until the full scale experiments of Vasta in 1947, on the S.S. Philip Schuyler [6], and in 1949, on the S.S. President Wilson [1], did the problem stimulate comprehensive theoretical attempts. Vasta clarified the existence of the problem, emphasizing the manner in which the stresses vary between the main deck and the deckhouse top. He introduced the concept of deckhouse effectiveness later to be used by Caldwell. At the same time model experiments by Holt [7] and Muckle [8] drew additional attention to the problem. However, there still remained the need of a theory to explain the observed phenomenon.

Although Vasta reported definite shear lag effects in his full scale tests, these were not to be considered in perhaps the first comprehensive theoretical treatment of the problem by Crawford [9] in 1950. Crawford examined the equilibrium of vertical forces between deckhouse and hull and the shear forces at the base. He recognized the possibility of differential curvature between the two parts. His analysis concerned single-level deckhouses extending 35 per cent or more of the length of the ship and he assumed the deckhouse was of such dimensions and scantlings that it would behave as a beam. His solution was cumbersome in that it required the solution of many simultaneous equations.

Bleich [10], in 1953, following Crawford's work and attempting to explain the results found on the S.S. President Wilson, was able to express the stresses in a very simple form. He again used the assumption that Navier's hypothesis (beam theory) applied to deckhouse and hull separately. He used the theorem of stationary potential energy (which states that the deformation of any structure is such that the total potential energy of the system is a minimum) to obtain the general Euler differential equations for the deflections of the deckhouse and hull respectively. He wrote these equations using an average deck flexibility constant  $k$ . Bleich did not take into account the shear lag effect.

Following Bleich, Terasawa and Yagi [11] used the minimum strain energy principle but developed a method to superpose shear lag effects by using Reissner's [12] least work solution of shear lag problems. The Japanese have studied the deckhouse problem quite comprehensively as one may note from their 60th Anniversary Series [13] published by the Society of Naval Architects of Japan. It would be of interest to see a design procedure based on their studies.

In 1957, three papers appeared simultaneously in England on the deckhouse problem. They were by Chapman [14], Caldwell [15], and Johnson [16]. Chapman's approach was to assume the deckhouse and hull acted separately, each as beams. He considered the deckhouse to be a beam on an elastic foundation and solved the applicable differential equations by relaxation theory. Caldwell and Johnson took a different approach from any of their predecessors and used the plane stress theory. Allowing for shear lag effects and nonlinear strains in the deckhouse, they reasoned that nonlinearity of strains was due to the fact that the plating was very thin in comparison with its overall dimen-

sions and that the elastic behavior could only be explained by recourse to the fundamental equations of elasticity. Caldwell's approach was felt to be more complete than Johnson's in that he considered rivet slip at the base of the deckhouse where it was attached to the hull. However, his analysis considered only single level deckhouses. In addition, Caldwell represented the external moment by a Fourier Series expansion, a good representation, but cumbersome to evaluate in the design office. Johnson's approach was considered to be the best with respect to developing a design method. Following his original attempt, he published two other papers [17], [18] with A. W. Ayling, which gave additional impetus to the designer wishing to develop a simplified, quick, design office procedure. Johnson's procedure was followed by the author in developing a design method. Details about his method is contained in the following section of the paper.

Most recently, M.A. Shade [19], of the University of California published a deckhouse theory which is an extension of Bleich's theory differing in that shear lag is included, different structural materials in deckhouse and hull are considered, and different boundary conditions for the deckhouse ends are used. Although design curves are presented and it appears that the procedure could be developed for use in the design office, it is limited to single level deckhouses and depends upon the evaluation of a deck flexibility factor  $k$ .

### JOHNSON'S ANALYSIS

To account for any departure from linearity in the longitudinal strains in the deckhouse, Johnson [16] used the theory of elasticity. His method of analysis is based on the plane stress theory which utilizes the general equations of equilibrium and compatibility of the theory of elasticity. No assumption is made regarding the longitudinal strains. Instead it is required that all forces acting on an elemental particle of the body be in equilibrium and that the displacements be compatible with this requirement.

The approach is to use the Airy Stress function to represent the stress in a rectangular plate, which is attached to the hull and is analogous to the deckhouse side. (Figure 2) The vertical and longitudinal displacements of the plate at the connection to the hull are made compatible with those produced



by the flexure of the hull girder. The longitudinal stress distribution at the midlength of the plate is then obtained.

Consideration is then given to the effect on the stress caused by attaching a plate to the deckhouse side. (Figure 3.) The attached plate represents a deck. The effect of many decks is then considered thus producing an analysis for a multi-level deckhouse. The stress distributions in the decks themselves are considered in the light of effective breadths, taking into account shear lag effects. Finally, empirical data is introduced in order to account for the difference in curvature between deckhouse and hull. The stress components resulting from the above considerations are combined in one equation which gives the stress distribution at the center of the deckhouse.

The basic assumption used throughout Johnson's analysis is that the shearing stress distribution in the deckhouse at the connection to any deck varies linearly along the length of the deckhouse. (Figure 4). In his paper [16], Johnson gives a comprehensive discussion of the rationale of this assumption. Experimentally, the assumption may be supported by the tests on the S.S. Philip Schuyler [5]. In addition to this assumption, Johnson's analysis is based on idealized deckhouse structure but these idealizations are accounted for in the development of the author's design procedure. The idealized deckhouse structure on which Johnson based his analysis was assumed to be (1) symmetrically disposed about amidships, (2) possess decks of equal lengths and widths, and (3) have sides and decks of constant thickness.

#### The Governing Equations

Throughout his analysis Johnson makes use of the following governing equations of the plane stress theory. A more detailed analysis is presented in Appendix I. The treatment given in the appendix is intended for those who wish to know more about the general approach. Anyone desiring greater detail is, of course, referred to Johnson's paper [16].

The state of stress in a thin plate can be represented by Lagrange's equation as :

$$(1) \quad \frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0$$

where  $\phi$  is the Airy Stress function which defines the stresses as follows:

$$(2) \quad \begin{aligned} \sigma_x &= \frac{\partial^2 \phi}{\partial y^2} \\ \sigma_y &= \frac{\partial^2 \phi}{\partial x^2} \\ \tau_{xy} &= -\frac{\partial^2 \phi}{\partial x \partial y} \end{aligned}$$

where  $\sigma_x$  is the longitudinal stress,  $\sigma_y$  is the transverse stress, and  $\tau_{xy}$  is the longitudinal and transverse shearing stress. A solution to Equation [1] may be written as follows:

$$(3) \quad \phi = \sum_n \left[ A_n \cosh \alpha_n y + B_n \sinh \alpha_n y + C_n y \cosh \alpha_n y + D_n y \sinh \alpha_n y \right] \cos \alpha_n x$$

$$\alpha_n = \frac{n\pi}{l} \quad (n = 1, 2, 3, \dots)$$

$A_n$ ,  $B_n$ ,  $C_n$ ,  $D_n$ , are the arbitrary constants obtained by applying the boundary conditions. The stress function  $\phi$  is, of course, different for the side of the deckhouse and the decks, but the governing equations upon which the analysis is based are the same.

As previously stated, one of the most important considerations in any deckhouse study is the differential deflection between deckhouse and hull. If the curvature of the deckhouse differs radically from that of the hull, the deckhouse will be less effective as a longitudinal strength member. However, as the size of a deckhouse is increased in length and beam the deckhouse more closely represents an extension of the hull. As a result, the deckhouse will be constrained to follow the curvature of the hull more closely and will be more effective as a longitudinal strength member.

Many of the theoretical studies on the deckhouse problem have attempted to solve this problem by including a stiffness modulus of the deck on which the deckhouse rests. However, the definition and evaluation of such a parameter has been a drawback with regard to realistic ship structure. In an attempt to find a practical solution to this problem, Johnson adopted a simplified approach which utilizes empirical data from full scale tests. Assuming the deflected forms of the hull girder and deckhouse are mathematically similar, a deflection coefficient,  $C$ , is defined as the ratio of deckhouse deflection to

hull deflection over the length of the deckhouse. To evaluate C analytically is practically impossible, but one may make some logical statements about the choice of C and the factors influencing it. The principal factors influencing C are:

- (1) Width of deckhouse compared to beam of ship.
- (2) Length of deckhouse compared to length of ship.
- (3) Lateral stiffness of deck beams and associated plating.
- (4) Disposition of bulkheads and stanchions under and adjacent to the deckhouse.

Several of these factors are inter-related. For example, there is some relationship between the number and spacing of bulkheads to the ratio of length of deckhouse to the length of ship. Since main transverse bulkheads can be considered points of no relative deflection between deckhouse and hull, it seems reasonable that C would approach 1.0 when the length of the deckhouse approaches the length of the ship. Another important parameter is the relation between width of deckhouse (b) and beam (B) of ship. The elastic restraint provided by the transverse frames and deck plating becomes large when b/B approaches 1.0 and therefore C also approaches 1.0.

With these considerations, experimental data showed that the length ratio and beam ratio of deckhouse to hull were the most influencing factors. Also, since these ratios usually increase or decrease proportionately C might safely be related to just one of these ratios. For the purpose of constructing the design curves, the deflection coefficients were taken from the full scale experimental results reported in references [17] and [18] and are:

$l/L$	0.10	0.20	0.40	0.60	0.80
DEFLECTION COEFFICIENT	0.950	0.140	0.670	0.970	1.00

Once the value of C is selected, it is applied directly to the bending component stresses discussed above. The final equation for the stress distribution in the deckhouse is expressed in terms of bending and shearing components in the non-dimensionalized form:

$$(4) \quad \frac{\sigma_x}{\rho_0} = \frac{\sigma_{x_0}}{\rho_0} + \frac{\sigma_{x_1}}{\rho_0}$$

where  $\sigma_x/q_0$  is the total stress divided by the shearing stress  $q_0$  at base and ends of the deckhouse (Figure 4),  $\sigma_{x_0}/q_0$  is the non-dimensionalized shearing stress component, and  $\sigma_{x_1}/q_0$  the non-dimensionalized bending stress component. All are functions of  $y$ , the distance from the strength deck to any point in the deckhouse. The final expression for Equation (4), deduced from simple algebraic considerations is:

$$(5) \quad \frac{\sigma_x}{q_0} = \left[ \left\{ \frac{\sigma_{x_0}}{q_0} \right\}_{y=0} + \left\{ 1 - 2C \frac{C_D}{d} \right\} \left\{ \frac{\sigma_{x_1}}{q_0} \right\}_{y=0} \right] \frac{\sigma_{x_r}}{q_0} \cdot \frac{1}{\left\{ \frac{\sigma_{x_r}}{q_0} \right\}_{y=0}} + C \frac{\sigma_{x_1}}{q_0} + \left\{ 2 \frac{C_D}{d} - 1 \right\} C \left\{ \frac{\sigma_{x_1}}{q_0} \right\}_{y=0}$$

where  $C_D$  is the distance from the strength deck to the neutral axis of the deckhouse,  $d$  is the depth of deckhouse, (Figure 9) and  $\sigma_{x_r}/q_0$  is  $\sigma_{x_0}/q_0$  plus the shearing stress effects due to the various decks. The above expression reduces to the following form for any height at the deckhouse midlength:

$$(6) \quad \frac{\sigma_x}{q_0} = \xi \frac{\sigma_{x_r}}{q_0} + \psi \frac{\sigma_{x_1}}{q_0} + \delta$$

where  $\xi$ ,  $\psi$ , and  $\delta$ , are constants and  $\sigma_{x_r}/q_0$  and  $\sigma_{x_1}/q_0$  vary with  $y$ , the distance above the strength deck.

### THE DESIGN PROCEDURE

#### Development

Soon after Johnson presented his original analysis [16], he published (with A.W. Ayling) a graphical presentation [18] in which he constructed graphs for seven "basic ships" relating such factors as the ratio of inertia of hull to effective inertia of deckhouse and hull, percentage reduction of stress at strength deck and keel, percentage length of deckhouse and the ratio of stress at the top of the deckhouse to that at the strength deck.

In the development of the design procedure presented herein, it was felt that this approach could be extended and generalized. The generalization appeared, at first, to be an almost insurmountable task. There were just too many variables to be considered. These included length of deckhouse, length of hull, beam of deckhouse and variation in beam of deckhouse, beam of hull, thicknesses of all plates in the deckhouse, the height between decks and the variation in heights between decks in the deckhouse, the neutral axis of the deckhouse, the neutral axis of the hull, the number of decks in the deckhouse, the inertia of the hull and deckhouse, and finally the deflection coefficients.

A computer study was made of the various parameters involved. The computerization of Johnson's method and the study of the effect of variation of any one parameter, holding the others constant, made it possible to reach conclusions concerning the construction of design curves.

It was found that some variables had greater effects than others, that some could be neglected for the purpose of constructing design charts and that some could be held constant due to the peculiarities of naval ship design. For example, the height between decks was held constant at eight feet since this is applicable to most U.S. Navy ships, except in the way of helicopter hangars and other special arrangements. Two other parameters, the ratio of deck to side thickness and the distance to the neutral axis of the deckhouse, were found to be relatively unimportant with respect to affecting design scantlings. The ratio of deckhouse side thickness to deckhouse deck thickness was taken as 1.0 for the purpose of constructing the design graphs. The other parameter, the distance from the strength deck to the neutral axis of the deckhouse, was taken as 6.6 feet for one level deckhouses and 10.8 feet for two level deckhouses.

Proceeding with the study of parameters, it was found possible to isolate the most important ones and construct the stress ratio curves shown in Figures 5 to 8. The parameters used in these figures are length of ship, length and beam of deckhouse, and distance from the strength deck to the hull neutral axis. Figures 5 and 8 give the ratio of stress at top of the deckhouse to the stress at the strength deck ( $\sigma_T/\sigma_D$ ). This ratio is used in the calculations to determine the effective neutral axis, the effective moment of inertia, and the stresses in deckhouse and hull. These curves were constructed

for a value of the distance from the strength deck to hull neutral axis of fifteen feet ( $C_M = 15.0$ ). In order to vary this parameter, use is made of Figures 6 and 8, modifying the value of  $\sigma_T/\sigma_D$  by  $K_2$  or  $K_4$  for the correct value of  $C_M$ .

Figures 5 and 7 were based on Johnson's rather involved analysis based on the plane stress theory. Because his analysis had certain limitations with regard to symmetry, it was necessary to obtain design procedures which would not be restricted by the limitations of the theory. Also, it was found necessary to make other assumptions to obtain a quick design office procedure. The assumption regarding the variation in stress from the strength deck to the top of the deckhouse is an example.

In this regard, it was decided important at the outset to choose an analysis based on the plane stress theory in order to accommodate the possibility of nonlinearity of strains including shear lag effects. Now that the curves had been constructed using this analysis, it was found expeditious at this point to rationalize a linear distribution of stress between the possibly nonlinear values of stress at the strength deck and top of deckhouse. The design philosophy in this case purports that if the analysis has a realistic approach, and a solution is obtained based on this approach, then one may make simplifying assumptions based on this solution which give good design results. The approximation of linear stress in the deckhouse based on Johnson's solution for  $\sigma_T/\sigma_D$  was tested in several cases and found to give satisfactory results.

### Derivation of Formulae

#### (a) Equivalent Area

After using the curves to obtain a stress ratio one can readily derive the expressions for Equivalent Area, Effective neutral axis, and Effective Moment of Inertia from the elementary principles of mechanics.

The Equivalent Area of the deckhouse,  $A_{0e}$ , is defined as that area which, if multiplied by the stress intensity at the strength deck, would yield the total longitudinal force in the deckhouse.

From the diagram in Figure 9 and the definition given above, the Equivalent Area of the deckhouse may be written as:

$$(7) \quad A_{DE} = \int_{A_D} \frac{\sigma}{\sigma_D} dA_D$$

where  $\sigma$  is the stress at any height in the deckhouse and  $\sigma_D$  is the stress at the strength deck. It may also be verified from Figure 9 that the stress in the deckhouse may be expressed as:

$$(8) \quad \sigma = \sigma_D \left[ 1 + \frac{y}{d} \left( \frac{\sigma_T}{\sigma_D} - 1 \right) \right]$$

where  $y$  is measured from the strength deck, positive upward and negative downward and  $d$  is the height of the deckhouse. Substituting (8) into (7) and performing the integration over the deckhouse area yields:

$$(9) \quad A_{DE} = A_D \left[ 1 + \frac{c_D}{d} \left( \frac{\sigma_T}{\sigma_D} - 1 \right) \right]$$

where  $A_D$  is the actual area of the deckhouse adjusted by the shear lag factor in Figure 10 and the Modulus of Elasticity ratio if appropriate, and the stress ratio  $\sigma_T / \sigma_D$  may be obtained from the curves in Figures 5 and 7. Notice in the above formulas that  $A_{DE}$  may take on values greater than or less than  $A_D$  depending on whether  $\sigma_T / \sigma_D$  is greater to or less than one. For this reason we designate  $A_{DE}$  as an "equivalent area" rather than "effective area" because effective usually implies a value less than the real value.

(b) Effective Neutral Axis

From the diagram in Figure 9, the stress in the hull,  $\sigma'$ , can be written as follows:

$$(10) \quad \sigma' = \sigma_D \left( 1 + \frac{y}{c_E} \right)$$

where  $c_E$  is the distance from the strength deck to the effective neutral axis of hull area and equivalent deckhouse area.

For equilibrium of longitudinal forces in the deckhouse and hull, one may write:

$$(11) \quad \int_{A_D} \sigma dA_D + \int_{A_H} \sigma' dA_H = 0$$

Where  $A_H$  is area of the hull. Substitution of (10) into (11) and dividing by  $\sigma_D$  gives:

$$(12) \quad \int_{A_D} \frac{\sigma}{\sigma_D} dA_D + \int_{A_H} \left(1 + \frac{y}{c_E}\right) dA_H = 0$$

Performing the integration as indicated and noticing that:

$$(13) \quad \begin{aligned} \int_{A_D} \frac{\sigma}{\sigma_D} dA_D &= A_{DE} \\ \int_{A_H} dA_H &= A_H \\ \frac{1}{c_E} \int_{A_H} y dA_H &= -\frac{c_H}{c_E} A_H \end{aligned}$$

the expression for the distance from the strength deck to the Effective Neutral Axis is obtained:

$$(14) \quad c_E = \frac{A_H \cdot c_H}{A_H + A_{DE}}$$

(c) Effective Moment of Inertia

For equilibrium, the moments of the stresses in hull and deckhouse about the effective neutral axis must be equal to the external moment.

$$(15) \quad M = \int_{A_D} \sigma (y+c_E) dA_D + \int_{A_H} \sigma' (y+c_E) dA_H$$

The effective moment of inertia may be expressed as:

$$(16) \quad I_E = \frac{M c_E}{\sigma_D}$$



Substitution for M from (15) into (16), leads to:

$$I_E = c_E \int_{A_D} \frac{\sigma}{\sigma_D} (y+c_E) dA_D + c_E \int_{A_H} (y+c_E) \left(1 + \frac{y}{c_E}\right) dA_H$$

Performing the integration as indicated and noticing that:

$$(17) \quad \int y^2 dA_H = I_H + c_H^2$$

we obtain:

$$(18) \quad I_E = c_E \left[ \int_{A_D} \frac{\sigma}{\sigma_D} y dA_D + c_E A_{DE} \right] + I_H + A_H (c_H - c_E)^2$$

If we let:

$$(19) \quad m_{DE} = \int_{A_D} \frac{\sigma}{\sigma_D} y dA_D + c_E A_{DE}$$

then (18) becomes:

$$(20) \quad I_E = I_H + A_H (c_H - c_E)^2 + m_{DE} c_E$$

The first two terms of this formula represent the moment of inertia of the main hull about the effective neutral axis. The last term represents the inertia contribution of  $A_{DE}$ , the equivalent deckhouse area.  $m_{DE}$  is the statical moment of the equivalent area of the deckhouse about the effective neutral axis. Carrying out further the integration indicated in (19), we obtain:

$$(21) \quad m_{DE} = c_D A_D + \frac{1}{d} \left[ I_D + A_D c_D^2 \right] \left[ \frac{\sigma_D}{\sigma_D} - 1 \right] + c_E A_{DE}$$

### Stress Distribution

The stresses at the strength deck and keel are calcu-

lated by the flexure formula utilizing the Effective Moment of Inertia and the distance to the Effective Neutral Axis:

$$(23) \quad \sigma_D = \frac{M c_E}{I_E} \quad ; \quad \sigma_K = \frac{M(D-c_E)}{I_E}$$

where  $D$  is the depth of the hull. The stress at the top of the deckhouse is calculated by the following relation:

$$(24) \quad \sigma_T = \left( \frac{\sigma_T}{\sigma_D} \right) \times \sigma_D$$

where  $\sigma_T / \sigma_D$  is obtained from Figures 5 or 7. The stresses  $\sigma_D$  and  $\sigma_K$  are the values of stress amidship since Johnson's analysis is predicated upon the deckhouse being symmetrical about amidship.

For the longitudinal distribution of stresses in the deckhouse, it would be conservative for design of the hull structure to assume that the stress varies linearly from the maximum calculated at amidships to zero at the ends of the deckhouse. The reduction in stress at the strength deck, realized by considering the contribution of the deckhouse, would then be assumed to vary linearly from a maximum at amidships to zero at the ends of the deckhouse.

#### Tabular Method

The Equivalent Area, the Effective Neutral Axis, and the Effective Moment of Inertia may be calculated by a tabular procedure as shown in the example, Appendix II. The procedure is very similar to the ordinary tabular procedure used for calculating moments and thus lends itself readily to design office practice.

The procedure is to use a tabular form with the following headings:

①	②	③	④	⑤	⑥
DECKHOUSE COMPONENTS	ACTUAL AREA	FACTOR(S)	EQUIVALENT AREA	LEVER	MOMENT
			$A_{DE} = \sum ④$	$C_E = \frac{A_H \cdot C_H}{A_H + A_{DE}}$	$A_{DE} \cdot C_E$
					$M_{DE} = \sum ⑥$

The areas of the deckhouse components are multiplied by factors which are described as follows:

- (a) Stress Factor. The stress factor depends on location of the component with respect to the stress diagram. For the house top, the factor is the stress ratio,  $\sigma_T / \sigma_D$ , obtained from Figures 5 or 7. For a two level deckhouse, the factor for the lower level deck is adjusted to suit the straight line variation in stress (see example). For the sides, the factor is the average value of the stress ratio between top and bottom of the deckhouse,  $0.5 (\sigma_T / \sigma_D + 1.0)$ .
- (b) Modulus of Elasticity Factor. For an aluminum alloy deckhouse on a steel hull, the area of the aluminum deckhouse is reduced by the ratio of the moduli,  $10 \times 10^6 / 29.6 \times 10^6 = 0.34$
- (c) Shear Lag Factor. Figure 10 gives the reduction factor for various ratios of breadth to length ( $b/l$ ), applicable to the decks of the deckhouse, including the stiffeners attached to these decks.

Each deckhouse component is multiplied by the appropriate factors to obtain the equivalent area for each component and placed in column ④. The statical moment of the equivalent area,  $m_{DE}$  is then obtained by multiplying the equivalent areas of the deckhouse components by levers which depend upon the centroid of the stress distribution over the component. For example, the side plating lever would be located at the

centroid of the trapezoidal stress diagram:

$$(22) \quad \text{LEVER}_{(\text{SIDE } \#)} = \frac{d}{3} \left[ \frac{1.0 + 2 \left( \frac{\sigma_T}{\sigma_D} \right)}{\frac{\sigma_T}{\sigma_D} + 1.0} \right]$$

For the top of the deckhouse, the lever would be the distance from the strength deck to the deckhouse top.

The sum of column ④ yields the Equivalent Area,  $A_{DE}$ . Once  $A_{DE}$  has been determined,  $C_E$  may be obtained from Equation (14). The value of  $A_{DE}$  is then multiplied by  $C_E$  and included in column ⑥. The sum of column ⑥ yields  $m_{DE}$ . The general use of the tabular procedure is straightforward and its use is best demonstrated by the example shown in Appendix II.

### Design Simplifications

As stated previously, Johnson's analysis had certain limitations with respect to symmetry. His analysis was based on deckhouses symmetrically disposed about amidships. It was necessary to obtain design procedures which would not be restricted by the limitations of the theory.

The basic question to be answered was: If a deckhouse was unsymmetrically disposed about amidships, how should it be treated? In other words, how critical was the midship position? If one examines Johnson's analysis, he finds that the midship section is critical to some extent. The assumption of linear shearing stress with a value of zero at the center of the deckhouse implies symmetrical loading and, as Johnson shows in his second solution, is equivalent to maximum moment occurring at amidships. Since maximum moment nearly always occurs near amidship the assumption is justified. However, if a deckhouse is mostly on one side of amidships then it is considered that the analysis is not applicable.

The solution to this problem was the establishment of an "effective length" of deckhouse to be determined by design procedures which are intended as suggestions. In each case, the design procedures are considered conservative for the main hull structure. The following procedures are suggested to obtain an effective length,  $l_E$ :

- 1) If the deckhouse extends at least  $0.25L$  both forward and aft of amidships, use the actual length of the deckhouse.
- 2) If the deckhouse extends no more than  $0.15L$  either forward or aft of amidships, take the effective length ( $l_e$ ) of the deckhouse to be twice that of the shorter part.
- 3) If the minimum longitudinal extent either forward or aft of the midship section is between  $0.25L$  and  $0.15L$  take the effective length ( $l_e$ ) of the deckhouse to be one half of its actual length plus the shorter part.

As can be seen in the curves, Figures 5 to 8, the design procedure is intended to include one and two level deckhouses. Johnson's analysis was based on deckhouse decks of equal lengths, heights, and widths, so that again it is necessary to propose design procedures to bypass the limitations of the theory. The question arises: When should a second level be considered in the analysis? Or rephrasing, to a more meaningful question: When can a second level be considered as contributing to longitudinal strength? Again, the method must be conservative with a respect to the main hull girder. Figure 11 shows a summary of a study which was made to give guidance for determining a rational design procedure. This study was typical of many such studies made to aid in deciding what design procedures should be used.

From the summary, it may be seen that the consideration of two levels at any particular value of  $l/L$  will provide lower allowable design stresses in both hull and deckhouse. Therefore, since the analysis is based on equal length deckhouses, it is conservative to disregard an upper level thus providing higher design allowable stresses in hull and deckhouse. Because the exact effect of including a second level which is shorter than the first level is not known, it is desirable to be conservative. Thus, the following design procedure is formulated with this objective in mind.

- 1) If the effective length of a second level, considered separately, is at least 80% of the effective length of the first level, then the second level may be included in the effective moment of inertia calculation in which case the mean of the

two effective lengths would be considered the new effective length of deckhouse for use in Figure 7.

It should be recognized that although we may disregard an upper level in the analysis, this is not to say that the level may be assumed to be "unstressed." If an upper level is disregarded in the analysis, it is suggested that the scantlings be designed to the stress at the top of the lower level.

There are numerous little details for which a design procedure must apply. For example, the breadth of the deckhouse may vary over the deckhouse length. In this procedure, it is suggested that the mean breadth be used in Figures 5 and 7. However, in the calculation of  $A_{DG}$  or  $m_{DG}$  the actual breadths at amidship should be used. A similar procedure is suggested for plate thicknesses. Use typical thicknesses of sides and decks, not thicknesses in way of local openings or other special structure. Openings are not considered in this analysis; however a deckhouse with many closely spaced large openings may require some modification to the analysis.

#### Expansion Joints

Although the use of expansion joints has been popular in U.S. Navy ships, the present belief is that they should be avoided whenever possible. It is true that expansion joints, in effect, change the shearing stress distribution in the deckhouse such each span between joints can be regarded as a separate deckhouse. As a result, the effective length is considerably shortened and, as can be verified from Figures 5 and 7 a short deckhouse is able to contribute less to longitudinal strength than a long deckhouse. However, expansion joints cause stress concentrations at the strength deck which may lead to cracking. Also, considerable maintenance problems for these joints have been reported.

Nevertheless, it is conceivable that expansion joints may be warranted in some cases. Consider the example of a long, continuous deckhouse which contributes substantially to longitudinal strength. Perhaps the proportions are such that the stress in this deckhouse approach the beam theory stresses for the combined deckhouse and hull. In order to provide for these stresses, the designer finds he must increase his scantlings in the deckhouse to such an extent that the added topside

weight may become critical. It may then be necessary to add expansion joints to reduce the stresses and thus reduce the scantlings to avoid the added topside weight.

However, if the expansion joints are placed judiciously so as to break the deckhouse into an odd number of equal lengths, then one section will be symmetrical about amidship, come within the scope of this analysis, and possibly still contribute substantially to the longitudinal strength of the ship. If this is the case, it is suggested that the other equal length sections be designed with similar scantlings as the mid-ship deckhouse section.

Therefore, to determine if expansion joints are needed in the deckhouse, it is necessary to calculate the deckhouse stresses without expansion joints. If these stresses exceed the desired level, expansion joints may be inserted and the stresses recalculated.

#### EXAMPLE

In Appendix II, an example of an analysis of a two level aluminum deckhouse is given. The effective length of the 01-level is first calculated. Since the ratio of the shorter part of the first level to the length of ship is  $50/350 = 0.143$ , the effective length,  $l_E$  is twice that of the shorter part ( $l_E = 2 \times 50 = 100$  FEET). Since the effective length of the 02-level is 80 per cent of the effective length of the 01-level, the 02-level is considered in the analysis. The new effective length of both levels is then  $l_E = (80 + 100)/2 = 90$  FT. The stress ratio is obtained from Figure 7 for  $l_E/L = 0.26$ , and is modified by  $K_3$  from Figure 8. The shear lag factor obtained from Figure 10 is 0.93. In the factor(s) column of the tabular procedure are seen the stress distribution factor, the shear lag factor, and the ratio of moduli of elasticity factor. The latter factor transforms the aluminum alloy to an equivalent steel area. Consequently, the calculated deckhouse stresses must be multiplied by this ratio to transfer the stresses back to aluminum alloy stresses. The stress diagram is shown following the calculations and is compared to the stresses with deckhouse omitted. The reduction in stress at the strength deck is not too significant but the example shows that a deck house which has an effective length of only one-quarter the

ship's length and is constructed of light material still contributes somewhat to the strength of the hull. In another calculation, a single level steel deckhouse extending forty-five per cent of the ships length was shown to reduce the strength deck stress from 7 t.s.i. to 4.5 t.s.i. reflecting a substantial contribution to the strength of the hull.

### CONCLUSIONS

In order that a deckhouse contribute nothing to the longitudinal strength of a ship, then the equivalent area,  $A_{DE}$ , must be zero. This may be seen clearly from Equation (14). If  $A_{DE} = 0$ , in this expression, then  $C_E$  will equal  $C_H$ . If  $A_{DE} = 0$ , equation [9] becomes

$$\left[ 1 + \frac{C_D}{d} \left( \frac{\sigma_T}{\sigma_D} - 1 \right) \right] = 0$$

or

$$(25) \quad \frac{\sigma_T}{\sigma_D} = - \frac{d}{C_D} + 1$$

For nominal values of a single level deckhouse,  $d = 8.0$  feet and  $C_D = 6.6$  feet,  $\sigma_T/\sigma_D$  becomes

$$\frac{\sigma_T}{\sigma_D} = \frac{-8.0}{6.6} + 1 = -0.21$$

Since this value of  $\sigma_T/\sigma_D$  is out of range in Figure 5, it may be concluded for practical purposes that all single level deckhouses contribute somewhat to longitudinal strength. For nominal values of a two level deckhouse,  $d = 16$  feet and  $C_D = 10.8$  feet,  $\sigma_T/\sigma_D$  becomes:

$$\frac{\sigma_T}{\sigma_D} = \frac{-16}{10.8} + 1 = -0.48$$

Since this value of  $\sigma_T/\sigma_D$  is out of range in Figure 7, it



may be concluded that all two level deckhouses contribute to longitudinal strength.

### RECAPITULATION

The nature of the problem was discussed dismissing the popular misconception of the dichotomy between "stressed" and "unstressed" deckhouses. A brief review of the literature was given. Johnson's theoretical analysis was chosen because of its approach utilizing the plane stress theory, for its simplified use of empirical data regarding differential deflections between deckhouse and hull, and its adaptability to design office practice. A design method was developed for determining the stresses at the midlength of the deckhouse using Johnson's procedure to construct design curves. Design simplifications were made in order to bypass the limitations of the theory. A first appendix gives the general approach of Johnson's analysis while the second appendix gives a detailed design example of a two level aluminum deckhouse. In the conclusions, it is shown through the derived formulae that all deckhouses contribute somewhat to the longitudinal strength of a ship, and in some cases this contribution may be substantial.

### RECOMMENDATIONS

The theoretical approach used to develop the design procedure has been tested in full scale trials reported by Johnson [17, 18]. Agreement between theory and experiment was shown to exist in these tests. However, the design procedure introduced by the author has not been verified by experimental results. It is recommended, therefore, that full scale tests be conducted not only to substantiate the design procedure but to obtain additional data on deflection coefficients. Such tests on a variety of U.S. Naval ships may lead to a better selection of deflection coefficients based on a wide range of geometric parameters. In addition, neither the theoretical approach nor the design procedure give adequate consideration to the design of structure near the ends of the deckhouse. The shear forces are largest in this area and the

use of "wobble plates" to avoid the cracking of plates is popular, but greater attention should be given to this problem.

A recent paper by Shade [19] shows that deckhouse analysis using the Navier hypothesis on the deckhouse and hull separately is still receiving attention. There are those, however, who claim that one must resort to the more basic tenets of the theory of elasticity, namely the plane stress theory. If one approach were tested against the other in a large number of cases, it may be found that the results of both approaches are in agreement for a majority of cases. However, if the contrary is true, the Navier hypothesis approach would appear to be more suspect.

The final resolution, of course, lies with the experimentalist who will hopefully find agreement not only in the theoretical approach but in the design simplifications proposed herein.

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## APPENDIX I

### JOHNSON'S ANALYSIS

It is the intention of this appendix to give the reader a clear picture of Johnson's approach to the deckhouse problem. It is not intended that the mathematical details be worked out step by step. Those who are inclined to know more about these details are referred to Johnson's paper [16]. It is intended that an overview of the theoretical approach be made available to those who wish to know more about the procedure without plodding through the mathematical details. The governing equations given in the paper are reiterated and the analysis is extended using these equations as a base.

#### Stress Function

Consider first the rectangular plate attached to the main girder as shown in Figure 1. The state of stress in a thin plate can be represented by Lagrange's Equation as follows:

$$(1) \quad \frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0$$

where  $\phi$  is the Airy Stress function which defines the stresses as follows:

$$(2) \quad \begin{aligned} \sigma_x &= \frac{\partial^2 \phi}{\partial y^2} \\ \sigma_y &= \frac{\partial^2 \phi}{\partial x^2} \\ \tau_{xy} &= -\frac{\partial^2 \phi}{\partial x \partial y} \end{aligned}$$

where  $\sigma_x$  is the longitudinal stress,  $\sigma_y$  is the transverse stress, and  $\tau_{xy}$  is the longitudinal and transverse shearing stress. A solution to equation (1) is:

$$(3) \quad \phi = \sum_1^n \left[ A_n \cosh \alpha_n y + B_n \sinh \alpha_n y + C_n y \cosh \alpha_n y + D_n y \sinh \alpha_n y \right] \cos \alpha_n x$$

where

$$\alpha_n = \frac{n\pi}{l} \quad (n = 1, 2, 3 \dots)$$

$A_n, B_n, C_n, D_n$  are the arbitrary constants obtained by applying the boundary conditions. Before stating the boundary conditions, however, it is necessary to write the expressions for longitudinal and transverse displacements of the base of the deckhouse.

### Displacements

The displacements are obtained by integrating the expressions for strain which are obtained from the well known relationships given by the theory of elasticity.

$$(4) \quad \begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{E} \left( \frac{\partial^2 \phi}{\partial y^2} - \nu \frac{\partial^2 \phi}{\partial x^2} \right) \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{1}{\mu} \tau_{xy} = -\frac{1}{\mu} \frac{\partial^2 \phi}{\partial x \partial y} \end{aligned}$$

where  $\epsilon_x$  is the longitudinal strain,  $\gamma_{xy}$  is the shearing strain,  $\nu$  is Poisson's ratio, and

$$\mu = \frac{E}{2(1+\nu)}$$

is the shear modulus of rigidity.

Integration of  $\epsilon_x$  leads to the following expression for  $u$  :

$$(5) \quad u = \frac{4}{3E} \sum_1^n (A_n + \frac{3}{2} D_n) \sin \alpha_n x \quad (\nu = 1/3)$$

In this part of the analysis, the deflections at the base of the deckhouse are assumed equal to those of the main hull girder. Later the effect of differential deflections of deckhouse and hull will be brought into the analysis. From the application of the simple beam theory to the hull (Figure 12), we have at the strength deck (where deckhouse meets hull):

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{R}$$

where  $R$  is the radius of curvature. From (4) we have

$$\epsilon_x = \frac{\partial u}{\partial x}$$

and from Figure 12

$$\theta = \frac{1}{R} = \frac{\epsilon_x}{-C_H}$$

so that

$$(6) \quad \frac{\partial u}{\partial x} = \frac{-C_H}{R} = -C_H \frac{\partial^2 v}{\partial x^2}$$

Substitution of [5] into [6] and performing the indicated integration leads to:

$$(7) \quad v = \frac{4}{3E C_H} \sum_1^n \left( A_n + \frac{3}{2} \frac{D_n}{\alpha_n} \right) \cos \alpha_n x$$

Before the boundary conditions can be stated, it is necessary to write the expression for shearing stress, which as stated previously is assumed to be linear. As can be seen in Figure 4, the shearing stress distribution is an odd function (i.e.  $f(x) = -f(-x)$ ) and thus may be represented mathematically by a half-range Fourier expansion:

$$(8) \quad \tau_{xy} = q_0 \sum_1^n \beta \sin \alpha_n x$$

where

$$\beta = \frac{8}{\pi^2} \left( \pm \frac{1}{n^2} \right)$$

In this expression,  $n$  is odd and the terms in the expansion are alternately positive and negative.

#### Boundary Conditions

The boundary conditions needed to obtain the arbitrary constants  $A_n$ ,  $B_n$ ,  $C_n$ , and  $D_n$  are as follows:

$$\begin{aligned}
(a) \quad y = 0 & \quad : \quad \tau_{xy} = \beta_0 \sum_1^M \beta \sin \alpha_n x \\
(b) \quad y = d & \quad : \quad \tau_{xy} = 0 \\
(c) \quad y = d & \quad : \quad \sigma_y = 0 \\
(d) \quad x = \pm \frac{l}{2} & \quad : \quad \sigma_x = 0
\end{aligned}$$

Since (4) may be satisfied simply by making  $n$  odd, an additional boundary condition is needed. This is obtained from the expressions relating strains. By differentiating  $\epsilon_x$  in (4) with respect to  $y$ , and  $\gamma_{xy}$  in (4) with respect to  $x$ , we are able to combine the two expressions to obtain an additional boundary condition, which is:

$$(e) \quad -\frac{1}{\mu} \left( \frac{\partial^2 \phi}{\partial x \partial y} \right) = \frac{1}{E} \left( \frac{\partial^3 \phi}{\partial y^3} - \nu \frac{\partial^3 \phi}{\partial x^2 \partial y} \right) + \frac{\partial^2 v}{\partial x^2}$$

since (7) gives the needed expression for  $v$ , it is possible to apply the four boundary conditions and solve for the arbitrary constants  $A_n$ ,  $B_n$ ,  $C_n$ , and  $D_n$ .

### Expression for Stress

Solving for the longitudinal stress at the midlength of the deckhouse ( $x=0$ ), the following expression is obtained.

$$(10) \quad \sigma_x = \sum_1^n \alpha_n \left[ \left\{ A_n \alpha_n + C_n \alpha_n y + 2D_n \right\} \cosh \alpha_n y + \left\{ B_n \alpha_n + 2C_n + D_n \alpha_n y \right\} \sinh \alpha_n y \right]$$

The above expression conveniently splits into two parts, one for shearing stress components,  $\sigma_{x_0}$ , and the other for bending stress components,  $\sigma_{x_1}$ , which may be expressed non-dimensionally as follows:

$$\begin{aligned}
(11) \quad \frac{\sigma_{x_0}}{\beta_0} &= \sum_1^n \beta \left[ \left\{ A_{(n)_0} + \frac{2}{3} \alpha_n y + 2D_{(n)_0} \right\} \cosh \alpha_n y + \left\{ \frac{5}{3} + D_{(n)_0} \alpha_n y \right\} \sinh \alpha_n y \right] \\
\frac{\sigma_{x_1}}{\beta_0} &= \sum_1^n \alpha_n E V_0 \gamma \left[ \left\{ A_{(n)_1} + \frac{1}{2} \alpha_n y + 2D_{(n)_1} \right\} \cosh \alpha_n y + \left\{ \frac{1}{2} + D_{(n)_1} \alpha_n y \right\} \sinh \alpha_n y \right]
\end{aligned}$$

where the subscripts indicate a simplified expression for the coefficient, e.g.  $A_{(n)_0}$  is a function of  $A_n$  involving the



hyperbolic sine. In his analysis, Johnson expresses the above expression for  $\sigma_x / q_0$  and  $\tau_{xy} / q_0$  in the form of curves for a range of values of  $c_H / \ell$  and  $y / d$ . The resultant longitudinal stress at the midlength of the deckhouse for any combination of  $c_H / \ell$  and  $y / d$  may be expressed as:

$$(12) \quad \frac{\sigma_x}{q_0} = \frac{\sigma_{x1}}{q_0} + \frac{\sigma_{x2}}{q_0}$$

### Restrictive Effect of Deck

Having determined the stress distribution at the midlength of a rectangular plate attached to the hull and analogous to the deckhouse side, Johnson considers the restrictive effect of a deck attached to the side as shown in Figure 2. Assuming linear shearing stress at the connection to the vertical plate, an assumption used previously, Johnson is able to write his solution for stress in the side plate by the use of two different stress functions, one for the side plate above the attachment of the deck and one for the side plate below the attachment.

### Boundary Conditions

The arbitrary constants  $A_n$ ,  $B_n$ ,  $C_n$ , and  $D_n$  previously used correspond to  $G_n$ ,  $H_n$ ,  $J_n$ , and  $K_n$  for the upper section and  $L_n$ ,  $M_n$ ,  $N_n$ , and  $O_n$  for the lower section. The arbitrary constants are found by applying the following boundary conditions:

- (a)  $y = 0$  :  $\tau_{xy} = 0$
- (b)  $y = 0$  :  $\frac{\partial^2 \psi}{\partial x^2} = 0$
- (c)  $y = md$  : LONGITUDINAL DISPLACEMENT  $u_U = u_L$
- (d)  $y = md$  : VERTICAL DISPLACEMENT  $v_U = v_L$
- (e)  $y = md$  :  $\sigma_{y_U} = \sigma_{y_L}$
- (f)  $y = d$  :  $\sigma_y = 0$
- (g)  $y = d$  :  $\tau_{xy} = 0$
- (h)  $x = \pm \ell/2$  :  $\sigma_x = 0$

where the subscripts U and L refer to the upper and lower section respectively and m is the ratio of deck height to deckhouse height. The condition of a longitudinal stress of zero at the free ends is satisfied by making n odd. Therefore a final boundary condition must be obtained from the equilibrium of longitudinal forces as follows:

$$(12) \quad t_s \left[ \int_0^{md} \sigma_{xL} dy + \int_{md}^d \sigma_{xU} dy \right] + t_d q_n \int_x^{\ell} \sum_1^n \beta \sin \alpha_n x = 0$$

where  $t_s$  is the thickness of the side plate and  $t_d$  is the thickness of the deck.

### Expressions for Stress

Applying the boundary conditions, the following expression for longitudinal stress at the midlength of the deckhouse in the upper section is obtained:

$$(13) \quad \frac{\sigma_x}{r_n q_n} = \sum_1^n \frac{2\beta}{3} \left[ \{G_n + y\alpha_n J_n + 2K_n\} \cosh \alpha_n y + \{H_n + y\alpha_n K_n + 2L_n\} \sinh \alpha_n y \right]$$

where  $r_n$  is the ratio  $t_d / t_s$ . And, for the lower section:

$$(14) \quad \frac{\sigma_x}{r_n q_n} = \sum_1^n \frac{2\beta}{3} \left\{ L_n \cosh \alpha_n y + (y\alpha_n \sinh \alpha_n y + 2 \cosh \alpha_n y) O_n \right\}$$

Johnson reduces the above expressions to stress functions which are plotted as curves depending upon  $y/d$ ,  $b/\ell$ , and  $m$ .

### Effective Breadth

Taking account of the shear lag effect in the decks, Johnson expresses the width of the deck as an equivalent width in which the distribution of longitudinal stress can be taken as uniform and equal to that at the deckhouse side. The equivalent width is denoted as  $K_3 b$  where  $K_3$  is the shear lag factor. The uniform effective stress ( $\sigma_{xe}$ ) in the deck is equated to the summation of the actual stress over the deck to obtain an expression for  $K_3$ .

$$K_3 b \bar{\sigma}_x = \int_0^b \sigma_x dz$$

The final result is:

$$(15) \quad K_3 = \frac{\sum_1^n \frac{2\beta}{b\alpha_n}}{4 \sum_1^n \frac{\beta \cosh^2\left(\frac{b\alpha_n}{2}\right)}{\sinh b\alpha_n + b\alpha_n}}$$

The above expression has been evaluated for various values of  $b/l$  and plotted in Figure 10.

#### Superposition of Stresses

Next, by the principle of superposition, Johnson considers the combined restrictive effect of several decks. By equilibrium of longitudinal forces at a deck Johnson obtains the following expression for the resultant stress at any deck:

$$(16) \quad \sigma_{x_N} = - \frac{q_N l}{K_3 b_N}$$

From Equations (13) and (14) and their corresponding curves, the general expression for the longitudinal stress at mid-length due to  $q_N$  can be written:

$$(17) \quad \frac{\sigma_x}{r_N q_N} = - \phi_N$$

Then applying the principle of superposition, letting  $\bar{\sigma}_{x_0}$  be the longitudinal stress due to  $q_0$  when no decks are included and  $\bar{\sigma}_{x_A}$  be the resultant longitudinal stress in the deckhouse at deck A when all decks are included, we obtain:

$$(18) \quad \bar{\sigma}_{x_A} = \bar{\sigma}_{x_{0A}} - \bar{\sigma}_{x_{AA}} - \bar{\sigma}_{x_{BA}} - \bar{\sigma}_{x_{CA}} - \dots$$

The above expression states that the resultant longitudinal stress at A is equal to the stress at A due to  $q_0$  alone minus the stress at A due to deck A, B, C, etc. From equations

(16) and (17) equation (18) may be expressed as follows:

$$(19) \quad \frac{q_A l}{k_3 b_A} = q_0 \phi_{0A} - q_A r_A \phi_{AA} - q_B r_B \phi_{BA} - q_C r_C \phi_{CA}$$

or by rearrangement:

$$(20) \quad \frac{q_A}{q_0} \left[ r_A \phi_{AA} + \frac{l}{k_3 b_A} \right] + \frac{q_B}{q_0} r_B \phi_{BA} + \frac{q_C}{q_0} r_C \phi_{CA} + \dots = \phi_{0A}$$

Similar expressions may be derived for all decks of the deckhouse resulting in a set of simultaneous equations in which  $q_A/q_0$ ,  $q_B/q_0$  etc., are the unknowns. The  $\phi$  terms are obtained from Johnson's curves as explained previously.

#### Final Expression for Stress

Having obtained through superposition the combined effect of several decks on the stress in the deckhouse side, Johnson introduces the deflection coefficient C to reflect the influence of differential deflections between deckhouse and hull. Since considerable discussion was given in the text of the paper concerning the deflection coefficient, it will be sufficient to state that once this coefficient is obtained, the final expression for stress may be written as follows:

$$(21) \quad \frac{\sigma_x}{q_0} = \left[ \left\{ \frac{\sigma_{x_0}}{q_0} \right\}_{y=0} + \left\{ 1 - 2C \frac{C_0}{d} \right\} \left\{ \frac{\sigma_{x_1}}{q_0} \right\}_{y=d} \right] \frac{\sigma_{x_r}}{q_0} \frac{1}{\left\{ \frac{\sigma_{x_r}}{q_0} \right\}_{y=0}} + C \frac{\sigma_{x_1}}{q_0} + \left\{ 2 \frac{C_0}{d} - 1 \right\} C \left\{ \frac{\sigma_{x_1}}{q_0} \right\}_{y=0}$$

where  $\sigma_{x_r}$  is the stress from the combined effect of the various decks, all other terms being defined previously. The above expression was derived from simple algebraic relationships relating all terms to the base of the deckhouse. It may be shown that equation (21) reduces essentially to a shearing component plus a bending component at the base, i.e.,

$$(22) \quad \frac{\sigma_x}{q_0} = \frac{\sigma_{x_1}}{q_0} + \frac{\sigma_{x_0}}{q_0}$$

To do this let:

$$\begin{aligned} \frac{G_{x_0}}{g_0} &= A, & \left(\frac{G_{x_0}}{g_0}\right)_{y=0} &= A' \\ \frac{G_{x_1}}{g_0} &= B, & \left(\frac{G_{x_1}}{g_0}\right)_{y=0} &= B' \\ \frac{G_{x_2}}{g_0} &= D, & \left(\frac{G_{x_2}}{g_0}\right)_{y=0} &= D' \end{aligned}$$

Substitution into Equation (21) yields:

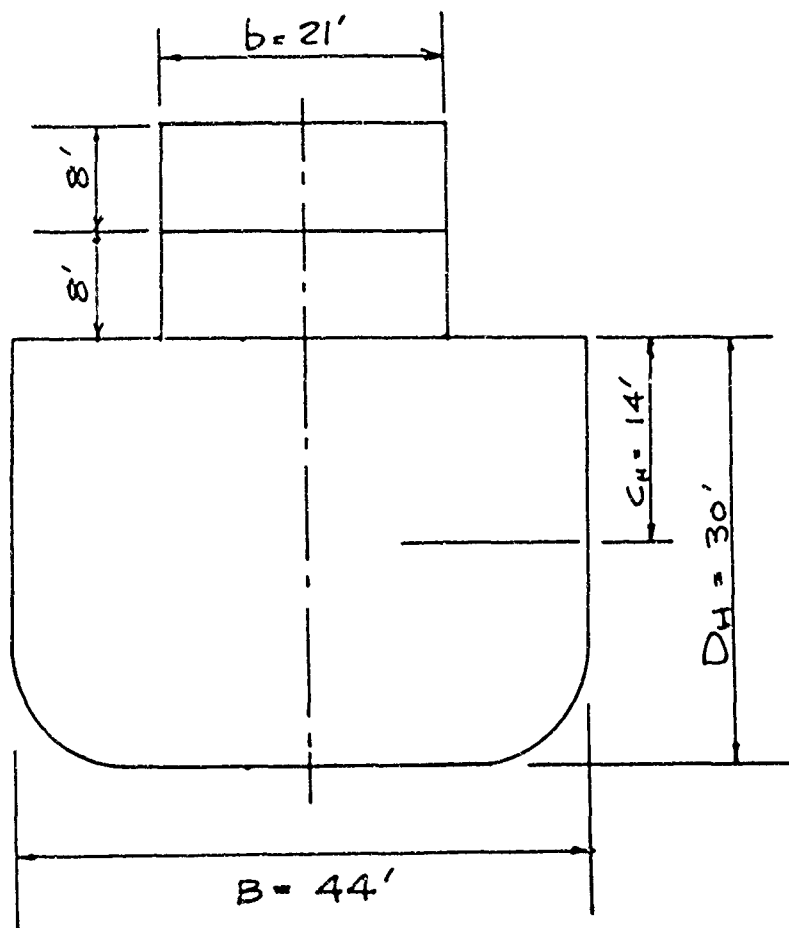
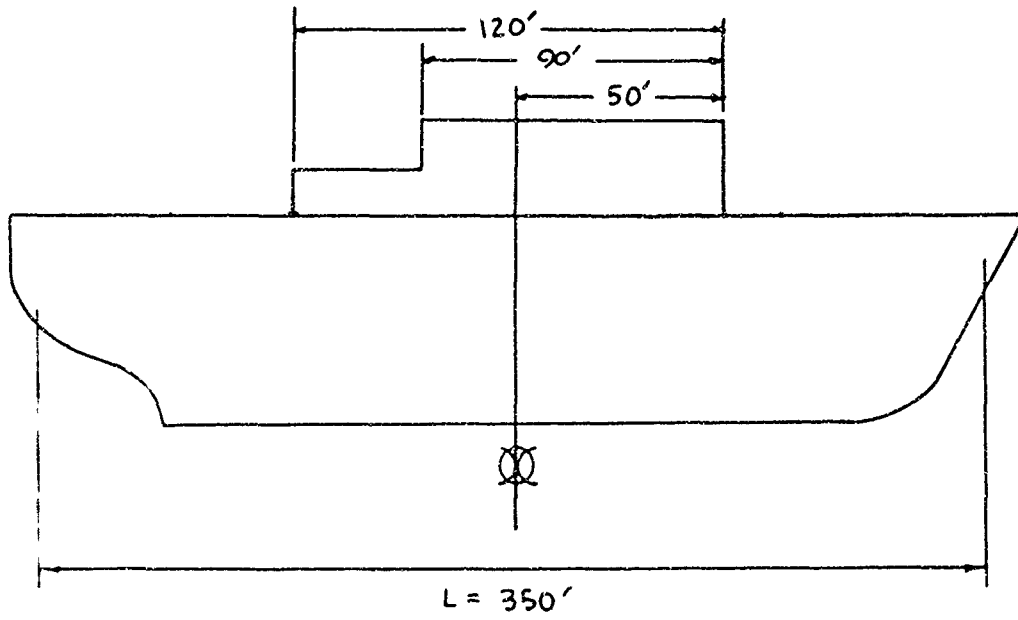
$$\frac{G_x}{g_0} = \left[ A + (1 - 2c \frac{c_0}{d}) B \right] D \frac{1}{D} + cB + (2 \frac{c_0}{d} - 1) c B'$$

Simplifying and noticing that at the base  $A, B, D = A', B', D'$  respectively we obtain:

$$\begin{aligned} \frac{G_x}{g_0} &= A + B - 2Bc \frac{c_0}{d} + cB + 2Bc \frac{c_0}{d} - cB \\ &= A + B \\ &= \frac{G_{x_0}}{g_0} + \frac{G_{x_1}}{g_0} \end{aligned} \quad \text{q.e.d.}$$

APPENDIX II

DESIGN EXAMPLE



Deckhouse Material:	Aluminum
Hull Material:	Steel
Cross Sectional Area of Hull:	$A_H = 1025 \text{ in}^2$
Moment of Inertia of Hull:	$I_H = 141,000 \text{ in}^2\text{-ft}^2$
Maximum Bending Moment:	$M = 58,000 \text{ ft-tons}$

Nonsymmetry Adjustment to Deckhouse Length:

Effective length for O1-level

$50/350 = 0.143$  Therefore, from the Design Simplifications p.19:

$$L_{E_{O1}} = 50 + 50 = 100 \text{ FT.}$$

Effective length for O2-level

$40/350 = 0.114$  Therefore:

$$L_{E_{O2}} = 40 + 40 = 80 \text{ FT.}$$

Since:

$$\frac{L_{E_{O2}}}{L_{E_{O1}}} = \frac{80}{100} = 0.80 ,$$

the second level may be included. Therefore, the effective length for both levels (per the Design Simplifications p. 19) is:

$$L_E = \frac{80 + 100}{2} = 90 \text{ FT.}$$

Stress Ratio,  $\sigma_T/\sigma_D$

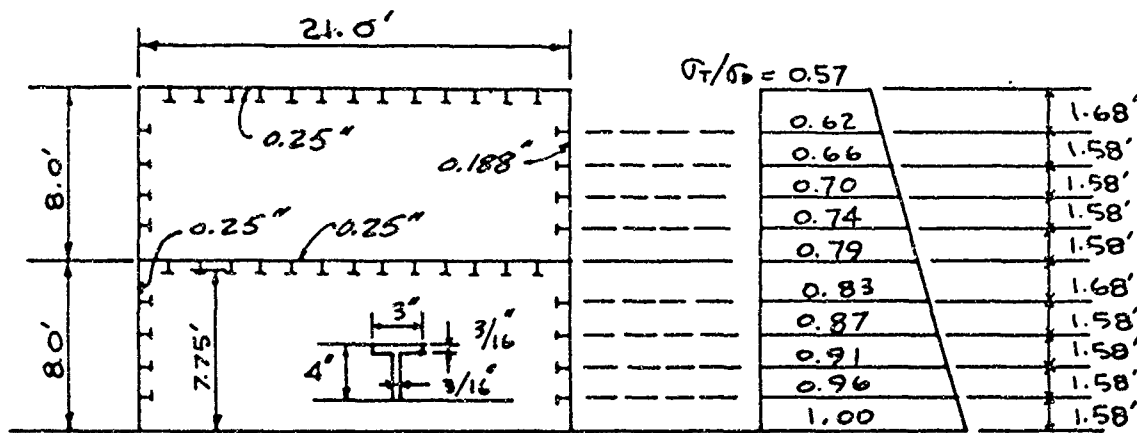
$$L_E/L = 90/350 = 0.26$$

From Figure 7, for  $C_M = 15 \text{ FT.}$ ,  $\sigma_T/\sigma_D = 0.57$

From Figure 8, for  $C_M = 14 \text{ FT.}$ ,  $K_4 = 1.00$

Therefore:

$$\sigma_T/\sigma_D = 0.57 \times 1.00 = 0.57$$



STRUCTURAL ARRANGEMENT  
OF DECKHOUSE

STRESS - RATIO DIAGRAM

STF AREA = 1.3 IN.<sup>2</sup>

SHEAR LAG FACTOR:

$$b/l = 21/90 = 0.23$$

FROM FIGURE 10,  $K_s = 0.93$



ITEM	ACTUAL AREA IN <sup>2</sup>	FACTOR (S)			EQUIV. AREA IN <sup>2</sup>	LEVER FT.	MOMENT IN <sup>2</sup> -FT.
		STRESS	SHEAR	MATERIAL			
02 - R	63.0	0.57	x 0.93	x 0.34	11.3	16.0	180.8
02-DECK STF. (13)	16.9	0.57	x 0.93	x 0.34	3.0	15.75	47.2
02 - SIDES	36.1	0.68	-	x 0.34	8.3	11.8	97.9
02 - SIDE STF (2) (LISTED FROM DECK)	2.6	0.62	-	x 0.34	0.5	14.32	7.2
02 - SIDE STF (2)	2.6	0.66	-	x 0.34	0.6	12.74	7.6
02 - SIDE STF (2)	2.6	0.70	-	x 0.34	0.6	11.16	6.7
02 - SIDE STF (2)	2.6	0.74	-	x 0.34	0.7	9.58	6.7
01 - R	63.0	0.79	x 0.93	x 0.34	15.8	8.0	12.6
01-DECK STF (13)	16.9	0.79	x 0.93	x 0.34	4.2	7.75	32.6
01 - SIDES	48.0	0.89	-	x 0.34	14.4	3.8	54.7
01 - SIDE STF (2) (LISTED FROM DECK)	2.6	0.83	-	x 0.34	0.7	6.32	4.4
01 - SIDE STF (2)	2.6	0.87	-	x 0.34	0.8	4.74	3.8
01 - SIDE STF (2)	2.6	0.91	-	x 0.34	0.8	3.16	2.5
01 - SIDE STF (2)	2.6	0.96	-	x 0.34	0.9	1.58	1.4
A <sub>DE</sub> =					62.6	13.2	826.3
							m <sub>DE</sub> = 1292.4

$$C_E = \frac{A_H C_H}{A_H + A_{DE}} = \frac{(1025)(14)}{1025 + 62.6} = 13.2 \text{ ft.}$$

$$\begin{aligned}
 I_E &= I_H + A_H (C_H - C_E)^2 + m_{DE} C_E \\
 &= 141,000 + 1025 (14.0 - 13.2)^2 + 1292.4 (13.2) \\
 &= 158,716 \text{ IN}^2\text{-FT}^2
 \end{aligned}$$

STRESSES:

AT STRENGTH DECK :  $\sigma_D = \frac{M C_E}{I_E} = \frac{58,000 \times 13.2}{158,716} = 4.82 \text{ TSI}$   
MATERIAL : STEEL

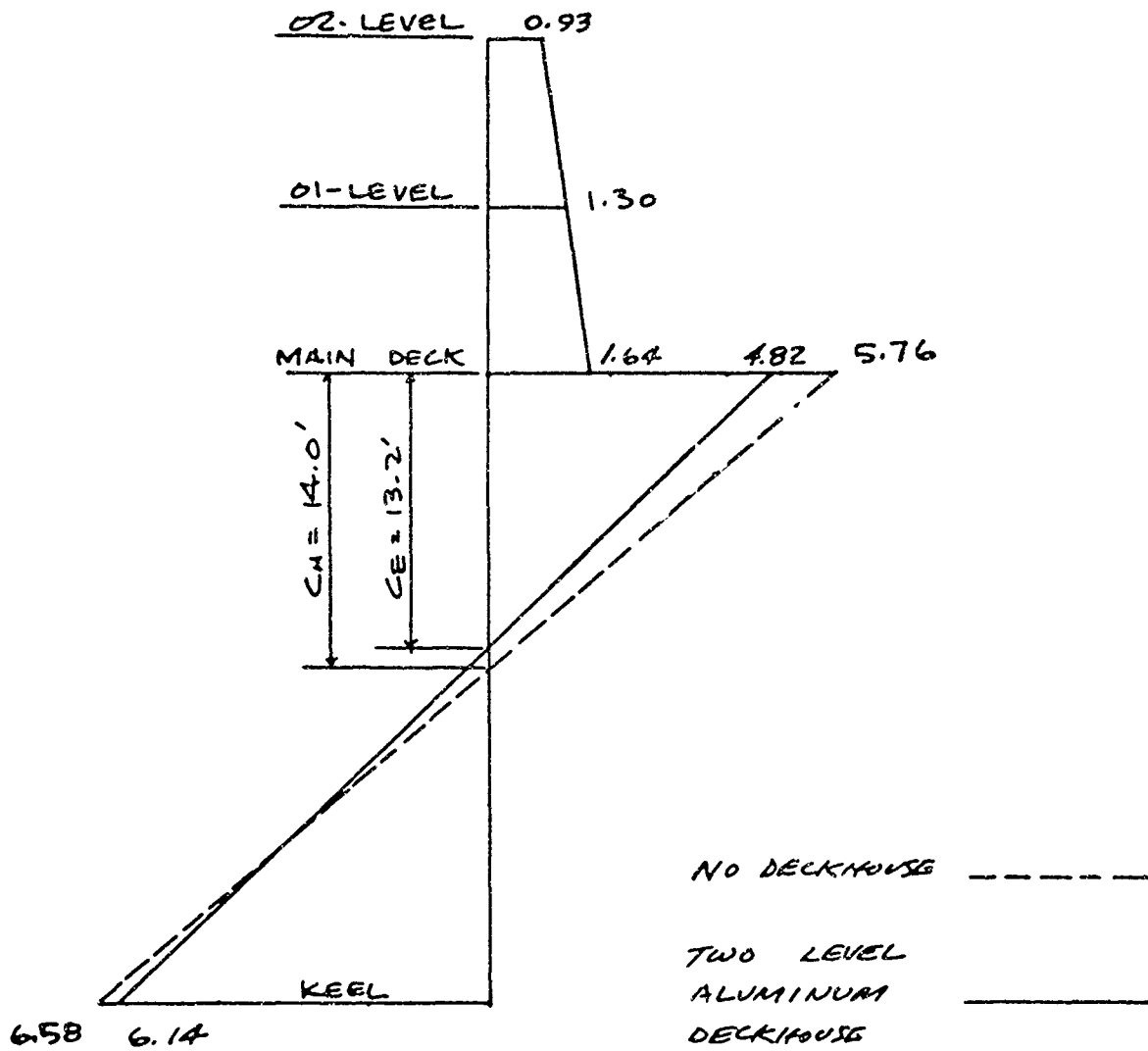
AT BASE OF DECKHOUSE :  $\sigma_D = 4.82 \times 0.34 = 1.64 \text{ TSI}$   
MATERIAL : ALUMINUM

AT DECKHOUSE TOP :  $\sigma_T = \left( \frac{\sigma_T}{\sigma_D} \right) \cdot \sigma_D = 0.57 \times 1.64 = 0.93 \text{ TSI}$   
MATERIAL : ALUMINUM

AT OI-LEVEL :  $\sigma_{oi} = 0.79 \times 1.64 = 1.30 \text{ TSI}$   
MATERIAL : ALUMINUM

AT KEEL :  $\sigma_K = \frac{M (D_H - C_E)}{I_E} = \frac{58,000 (30.0 - 13.2)}{158,716}$   
MATERIAL : STEEL

$= 6.14 \text{ TSI}$



STRESS DISTRIBUTION

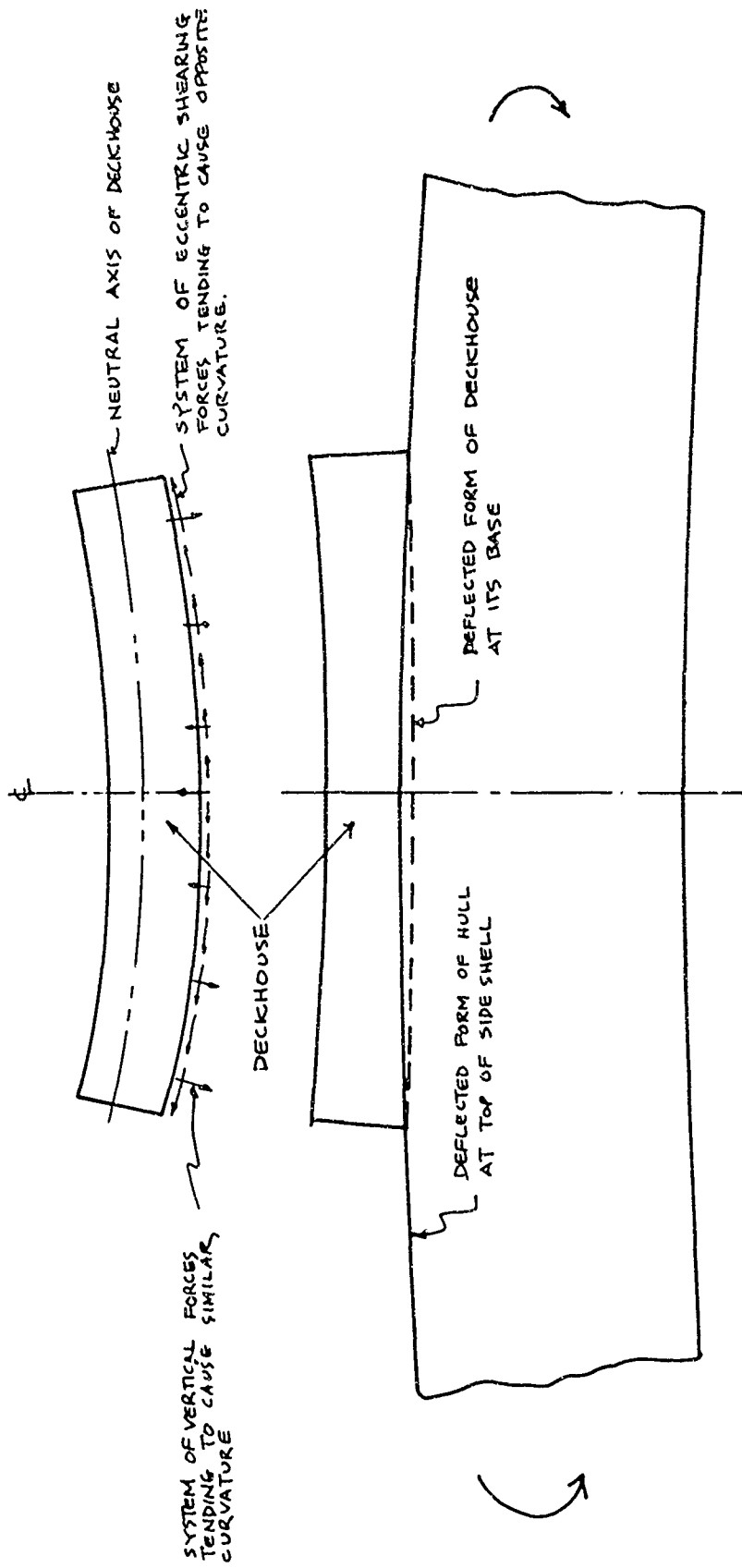
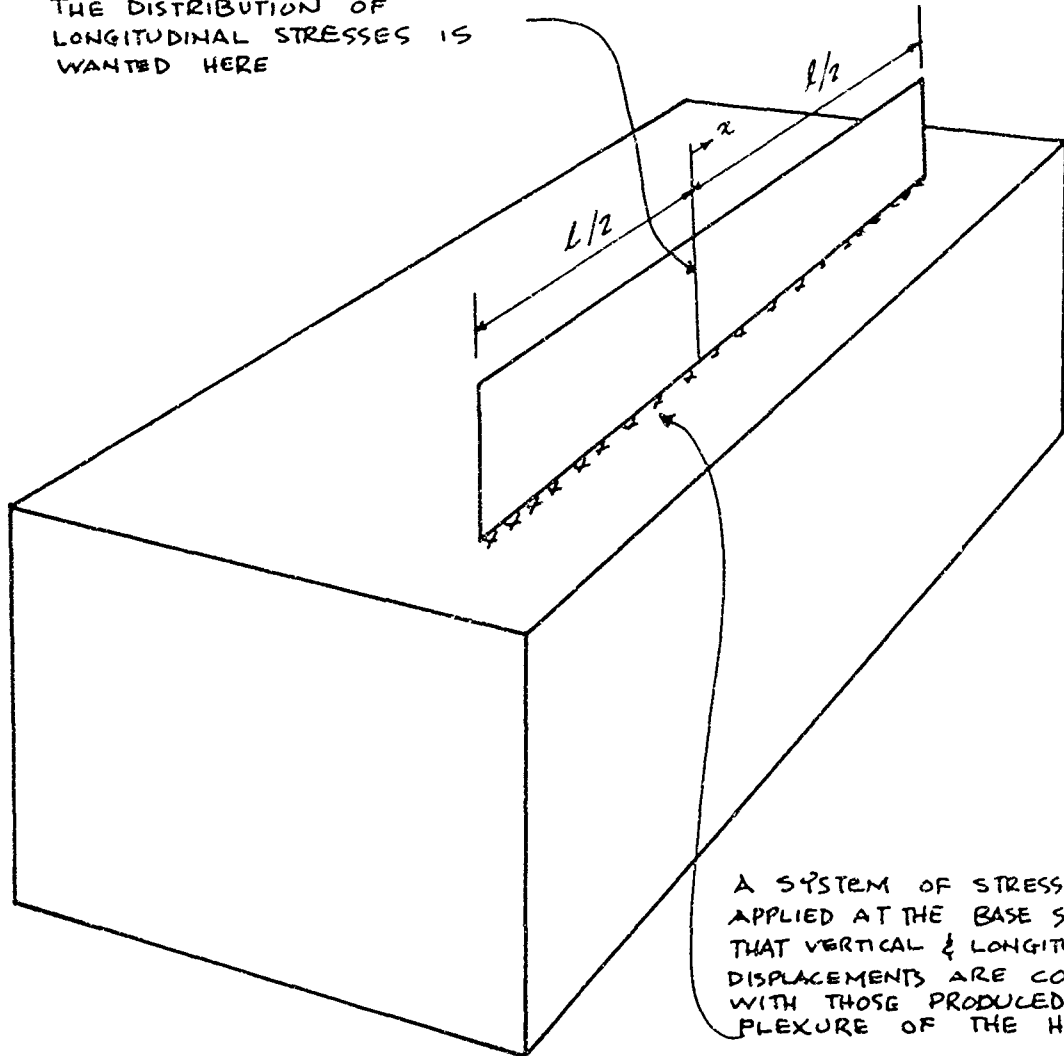


FIGURE 1

The systems of vertical and shearing forces which affect the curvature of the deckhouse.

THE DISTRIBUTION OF  
LONGITUDINAL STRESSES IS  
WANTED HERE



A SYSTEM OF STRESSES IS  
APPLIED AT THE BASE SUCH  
THAT VERTICAL & LONGITUDINAL  
DISPLACEMENTS ARE COMPATIBLE  
WITH THOSE PRODUCED BY  
FLEXURE OF THE HULL

FIGURE 2

"The approach is to use the Airy Stress Function to represent the stress in a rectangular plate, which is attached to the hull and is analogous to the deckhouse side."

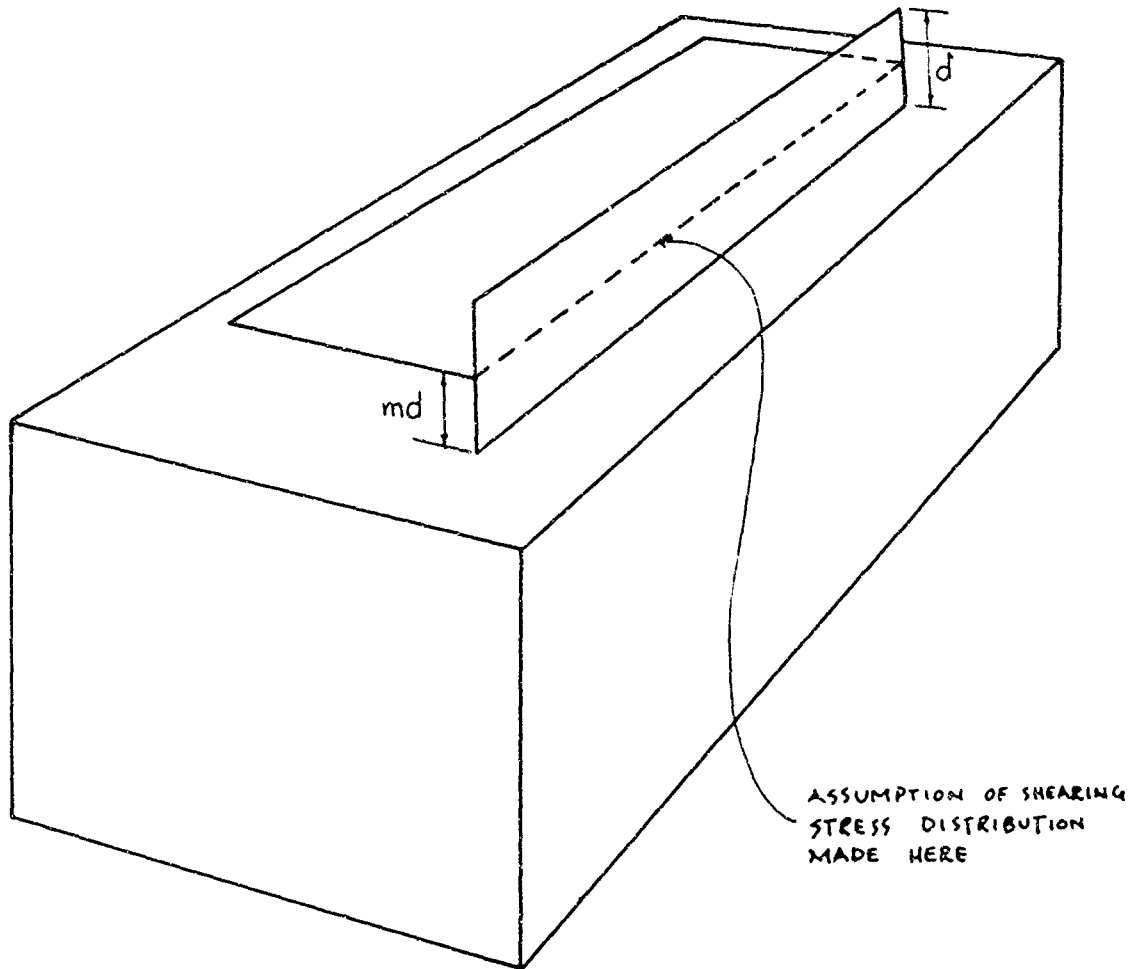
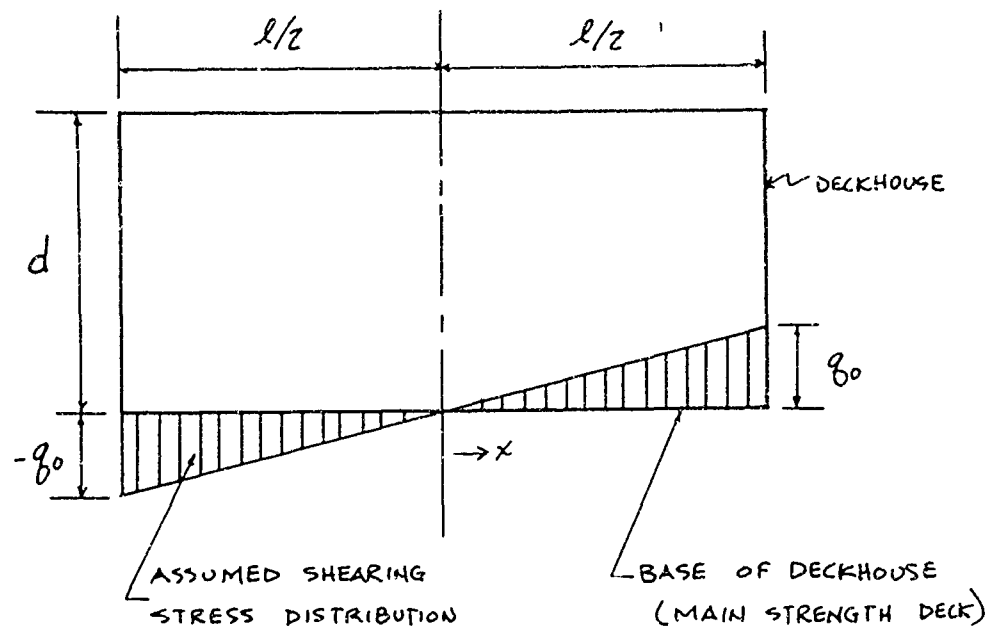


FIGURE 3

"Consideration is then given to the effect on the stress caused by attaching a plate to the deck-house side. The attached plate represents a deck."



$$\tau_{xy} = q_0 \sum_1^n \beta \sin \alpha_n x$$

$$\beta = \frac{8}{\pi^2} \left( \pm \frac{1}{n^2} \right) \quad (n = 1, 3, 5 \dots)$$

FIGURE 4

"The basic assumption used throughout Johnson's analysis is that the shearing stress distribution in the deckhouse at the connection to any deck varies linearly along the length of the deckhouse."

FIGURE 5

Stress ratio for single level deckhouse, with neutral axis of hull 15 feet below strength deck. Erect ordinate AB from the given value of  $l_f/L$  to the curve corresponding to ship length, L (interpolating as necessary). Draw a horizontal line BC to the curve for the given breadth of deckhouse, b. Project CD vertically upward to obtain the value of  $\sigma/\sigma_s$  corresponding to  $C_H = 15$  FT

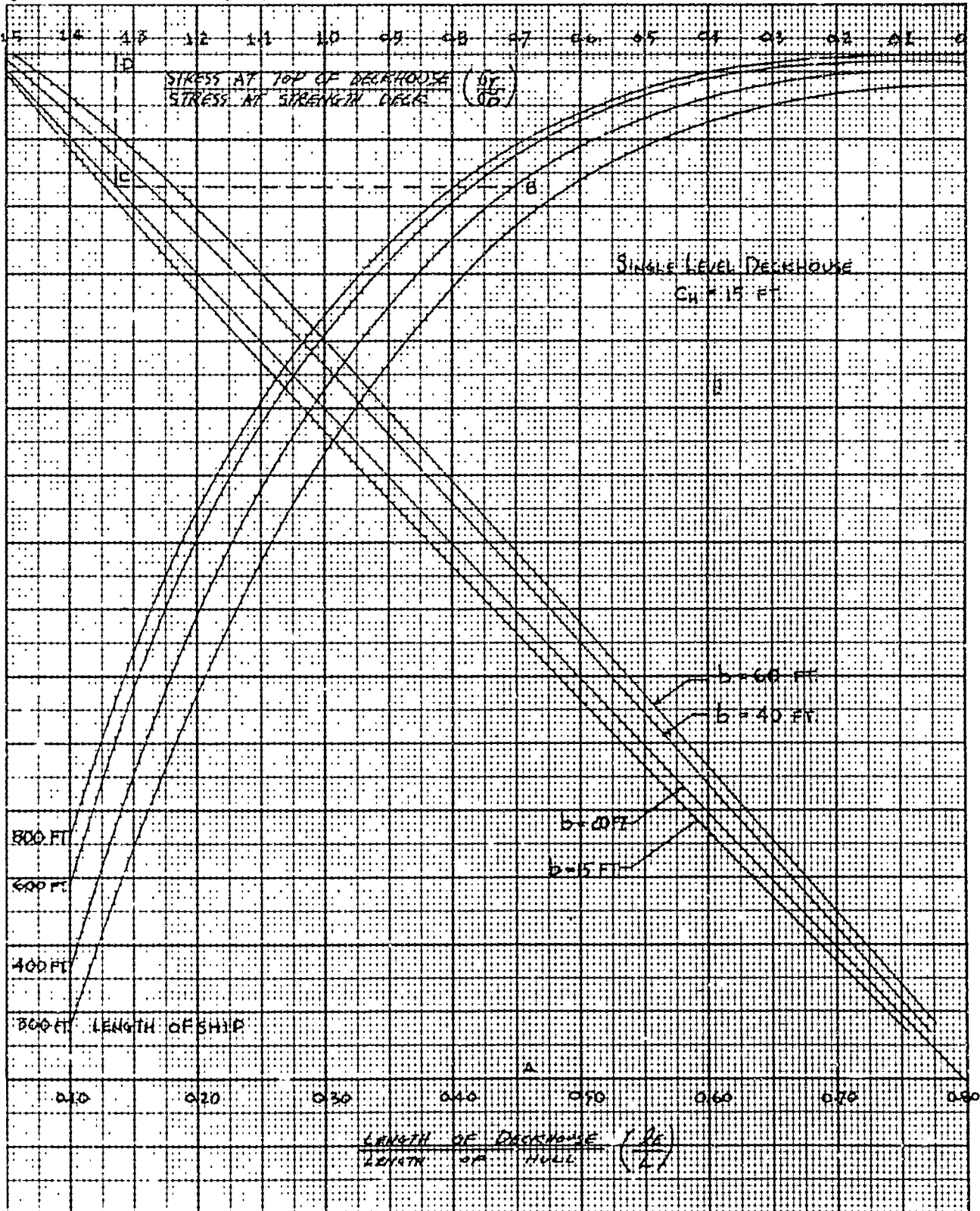




FIGURE 6

Correction to Stress Ratio for Single Level Deckhouse for location of neutral axis of hull. Enter diagram at given value of  $L/\lambda$ , interpolating to suit the given length ratio,  $L/\lambda$ . Multiply the correction factor  $K_2$  by the value of  $\sigma_T/\sigma_b$  obtained from Figure 5.

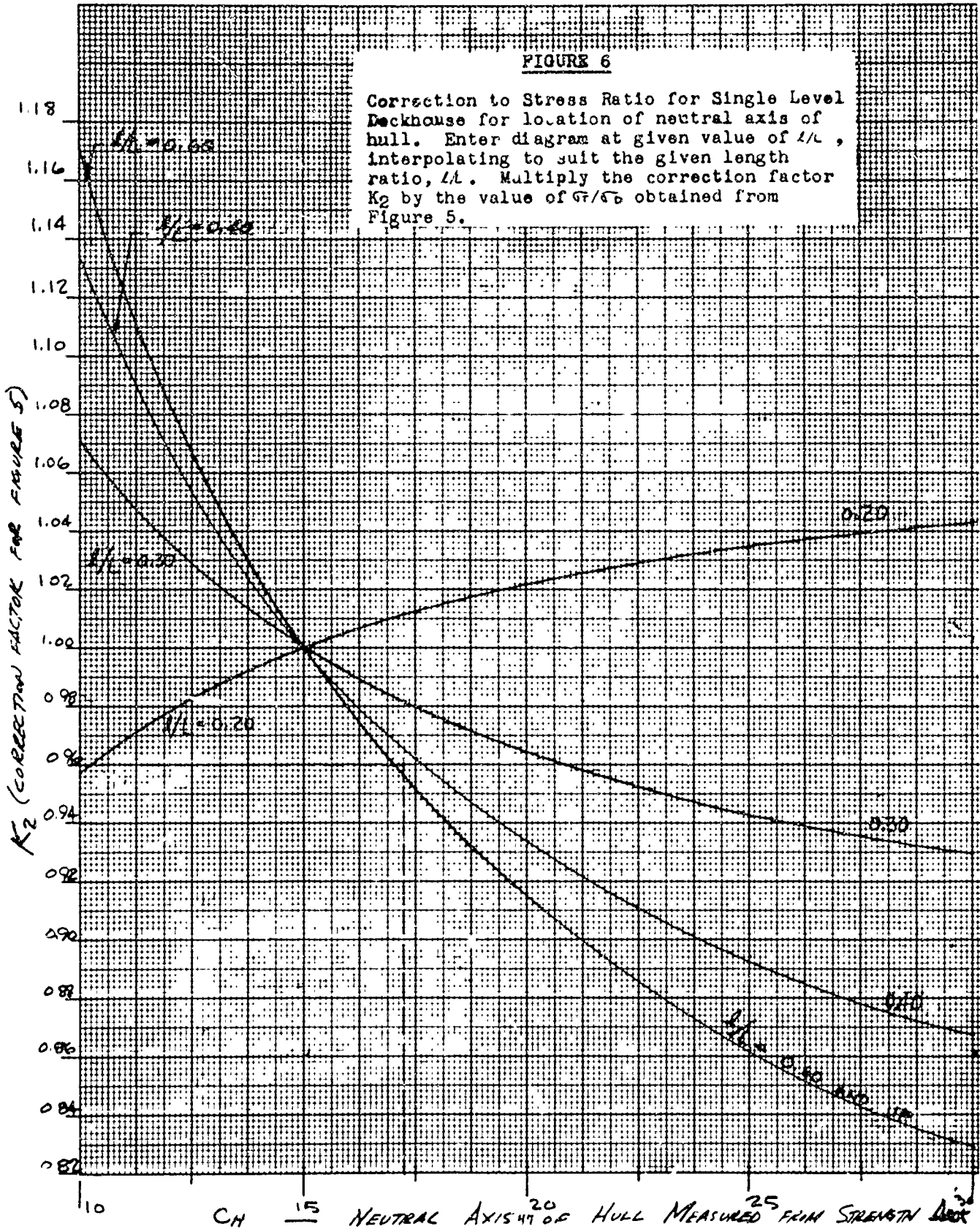
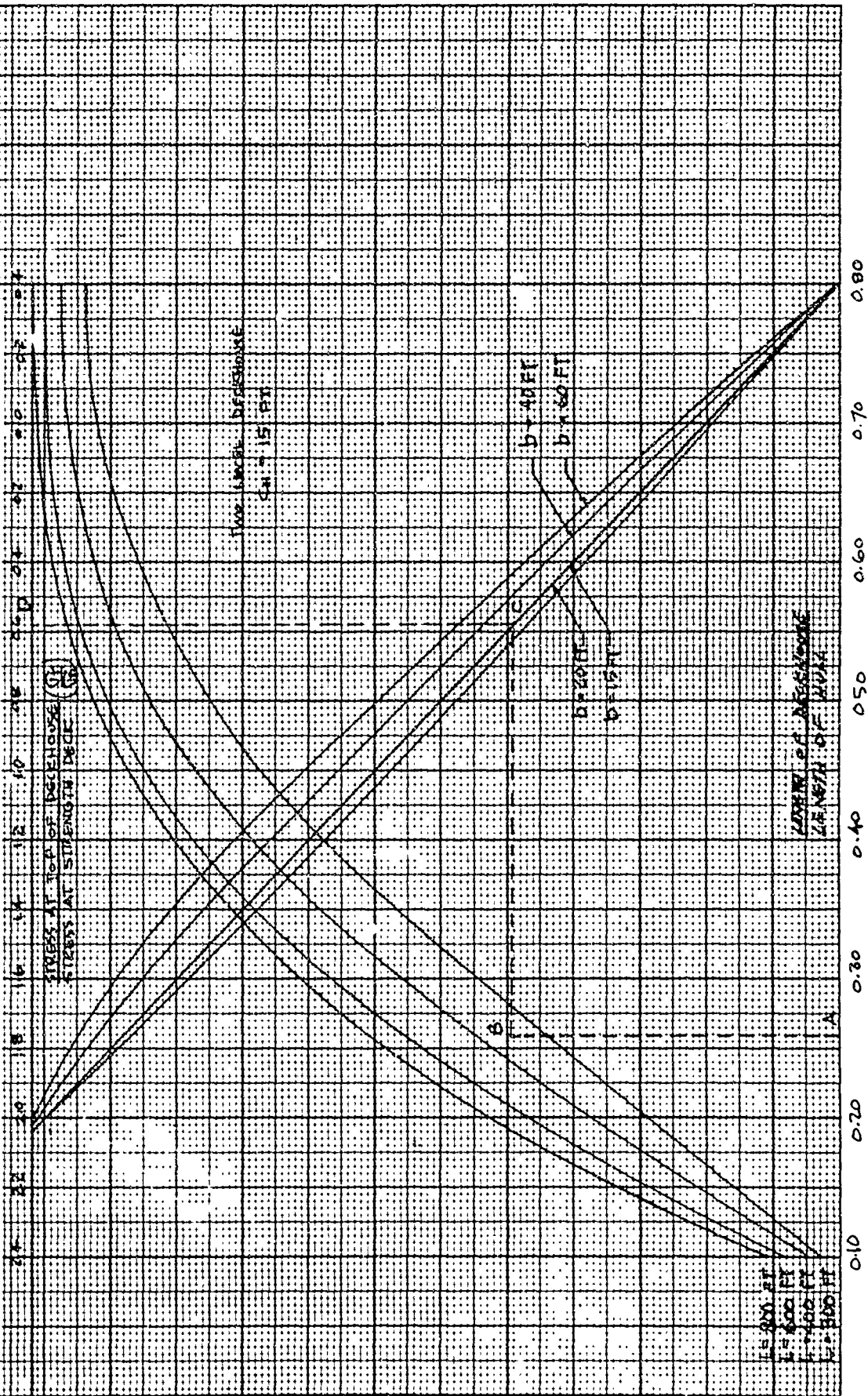
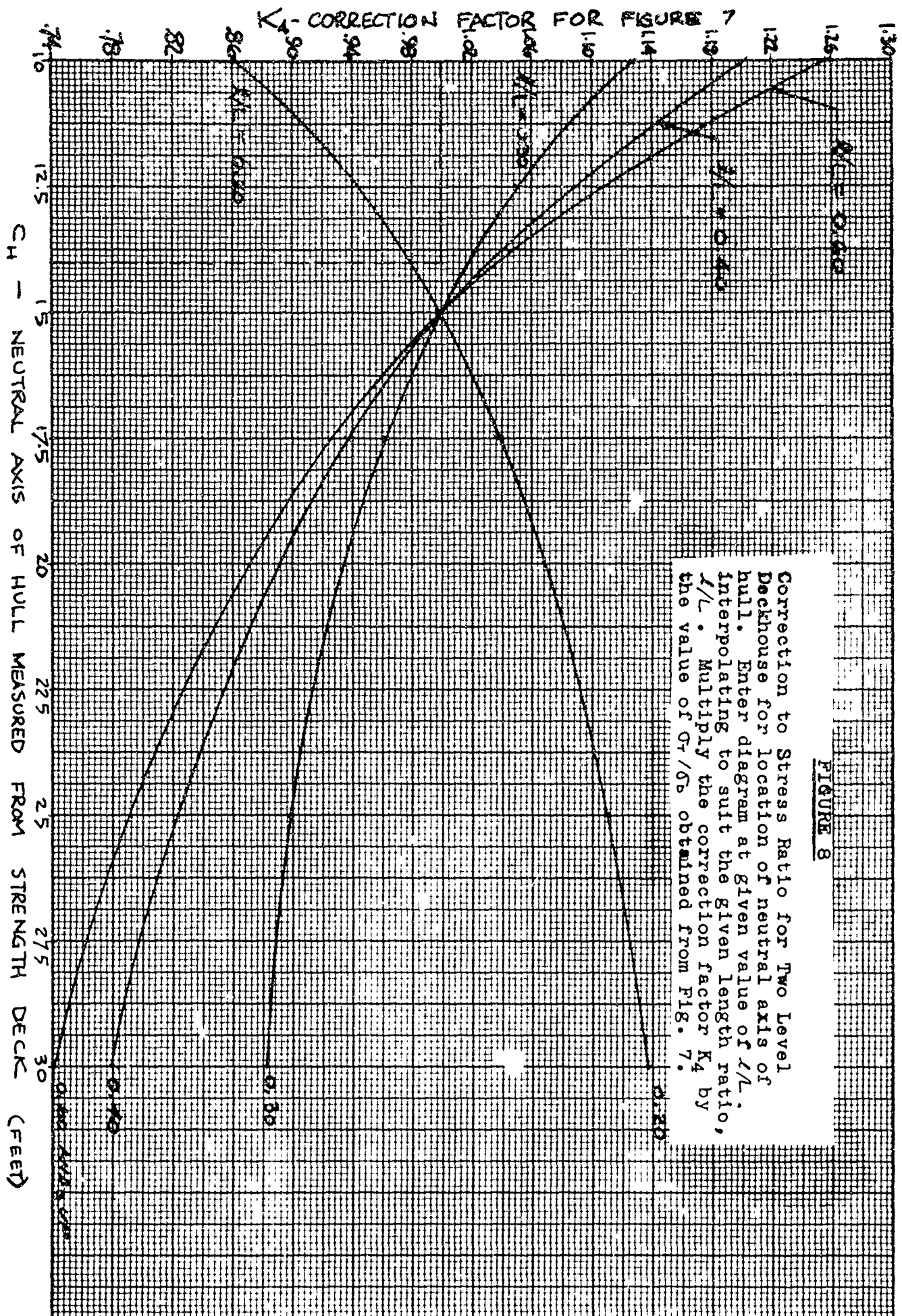


FIGURE 7

Stress Ratio for Two Level Deckhouse. Erect ordinate AB from the given value of  $l/L$  to the curve corresponding to ship length, L (interpolating as necessary). Draw a horizontal line BC to the curve for the given breadth of deckhouse, b. Project CD vertically to obtain the value of  $\frac{\sigma}{\sigma_0}$ .





Correction to Stress Ratio for Two Level Deckhouse for location of neutral axis of hull. Enter diagram at given value of  $l/L$ . Interpolating to suit the given length ratio,  $l/L$ . Multiply the correction factor  $K_4$  by the value of  $\sigma_r/\sigma_b$  obtained from Fig. 7.

**FIGURE 8**

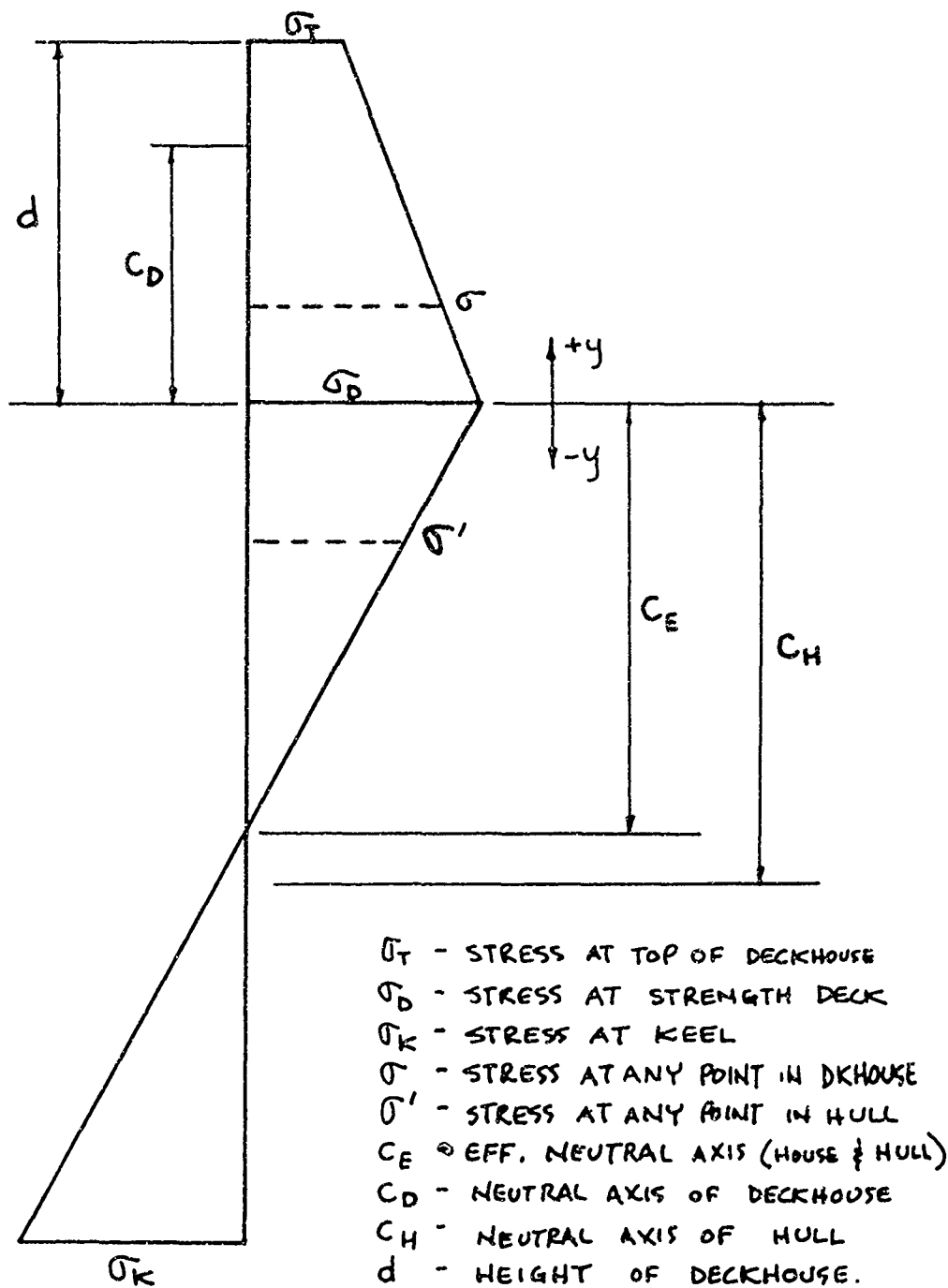


FIGURE 9

"The approximation of linear stress in the deckhouse based on Johnson's solution for  $\sigma_T/\sigma_D$ . It was found expeditious at this point to rationalize a linear distribution of stress between the possibly nonlinear values of stress at the strength deck and top of deckhouse."

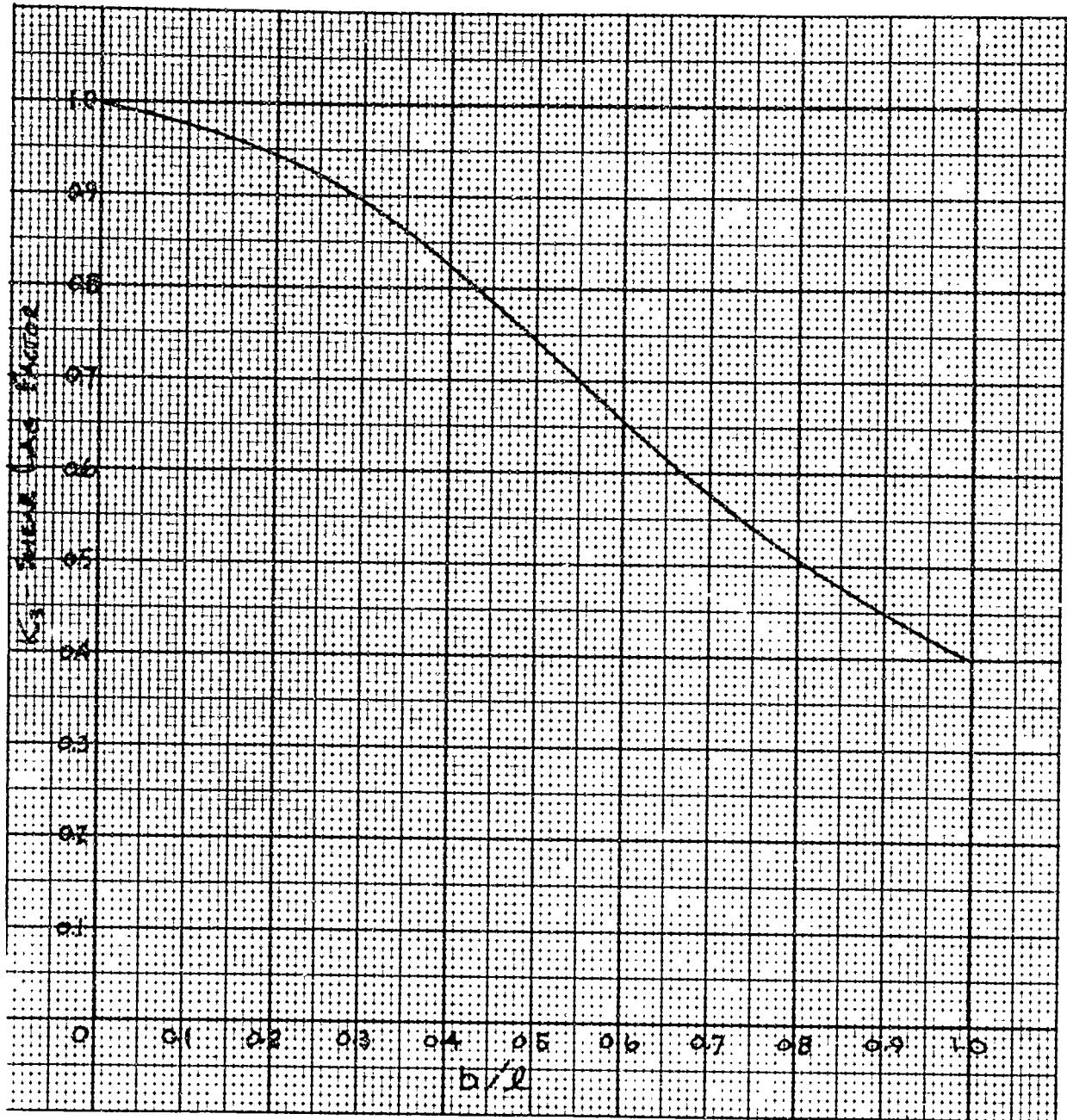


FIGURE 10

Shear Lag Factor. Multiply the actual areas of plating and stiffeners by the factor corresponding to the given ratio of  $b/l$ . This allows for the variation in stress across the breadth of the deckhouse due to "shear lag".

SUMMARY

l/L	C <sub>E</sub>		I <sub>E</sub>		σ <sub>T</sub> /σ <sub>D</sub>		σ <sub>D</sub>		σ <sub>01-LEV</sub>		σ <sub>T</sub>	
	1-LEVEL	2-LEVELS	1-LEVEL	2-LEVELS	1-LEVEL	2-LEVELS	1-LEVEL	2-LEVELS	2-LEVELS	1-LEVEL	2-LEVELS	1-LEVEL
0.10	14.0	13.9	170,800	173,410	0.4	0	4.9	4.8	2.4	2.0	2.0	0
0.35	13.3	12.0	185,590	224,300	1.2	1.2	4.3	3.2	3.5	5.2	3.85	
0.80	12.9	11.1	194,416	252,245	1.5	2.0	4.0	2.6	3.9	6.0	5.2	
BEAM THEORY RESULTS	12.9	10.5	195,035	268,046	1.6	2.6	4.0	2.3	4.1	6.4	5.9	

FOR NO DECKHOUSE MATERIAL INCLUDED IN STRENGTH CALCULATION:

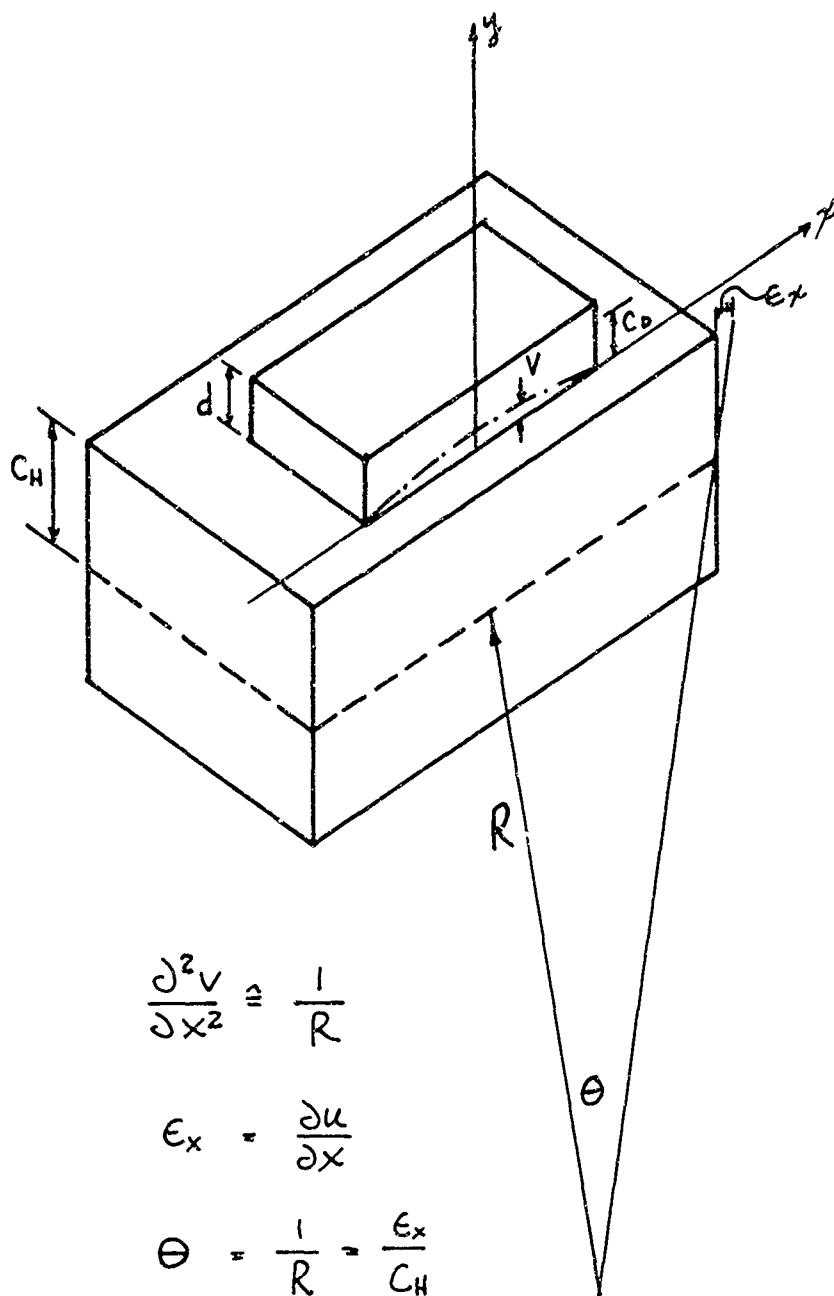
ASSUMED:

$C_E = C_H = 15 \text{ FT.}$   
 $I_E = I_H = 150,000 \text{ IN}^2\text{-FT}^2$   
 $\sigma_D = 6.0 \text{ T.S.I.}$

$A_H = 1000 \text{ IN}^2$   
 $M = 60,000 \text{ FT-TONS}$   
 $L = 400 \text{ FT.}$   
 $b = 20 \text{ FT.}$   
 $d = 8 \text{ FT., } 16 \text{ FT.}$

FIGURE 11

From the summary, it may be seen that the consideration of two levels at any particular value of l/L will provide lower allowable design stresses in both hull and deckhouse. Therefore it is conservative to disregard an upper level." (Last row reflects the results of beam theory applied to deckhouse and hull as a single unit.)



$$\frac{\partial^2 v}{\partial x^2} \cong \frac{1}{R}$$

$$\epsilon_x = \frac{\partial u}{\partial x}$$

$$\theta = \frac{1}{R} = \frac{\epsilon_x}{C_H}$$

FIGURE 12

"Simple beam relations. In this part of the analysis, the deflections at the base of the deckhouse are assumed equal to those of the main hull girder. Later the effect of differential deflections between deckhouse and hull will be brought into the analysis."