BRL

REPORT NO. 1310

ERROR STUDY OF A WORLDWIDE SATELLITE TRIANGULATION NET

by

Edna L. Lortie

January 1966

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U. S. ARMY MATERIEL COMMAND
BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

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Edna L. Lortie

Ballistic Measurements Laboratory

RDT & E Project No. 1L013001A91A

A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

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ABSTRACT

A proposed three-dimensional geodetic reference system, consisting of 36 stations so situated on the surface of the earth as to form a closed system of approximate equilateral triangles surrounding the globe, is studied in an attempt to find out to what degree of accuracy the station positions can be determined when triangulating the corresponding station coordinates by photogrammetric techniques. The polyhedron of the spatially oriented triangles is formed by photographing a high altitude satellite against the star background. The triangulation method depends on applying analytical photogrammetric techniques to satellite observations where the satellite serves as a high altitude auxiliary triangulation target point. Numerical analyses on the whole net and portions of it are presented from which the problem of error propagation and the influence of the number of baselines can be studied.

TABLE OF CONTENTS

I. INTRODUCTION

The use of artificial satellites in scientific research provides geodesy for the first time in its history with a means to create a three-dimensional worldwide reference system without referring to the direction of gravity. Many of the basic difficulties encountered in a classic geodetic triangulation will be eliminated when photogrammetric methods are applied to satellite observations. A satellite triangulation can be used for establishing a worldwide three-dimensional geodetic reference system which, without referring to gravity, is entirely geometrically defined.[1*] The system will consist of stations on the physical surface of the earth, located in such a way as to form a closed system of as near as possible equilateral triangles encompassing the globe. These stations will serve as a reference frame to which can be tied both geometrically-oriented mapping programs and the evaluation of satellite orbits for determining gravitational and related geo-physical parameters.[2]

Coast and Geodetic Survey gave considerable thought to the requirements in planning a worldwide triangulation system. This organization has the technical responsibility for the execution of a worldwide satellite triangulation program. Their conclusions concerning a useful satellite orbit are summarized below.[3-4]

First, the line of sight to the satellite must be at least 30° above the horizon in order to avoid disturbing refraction anomalies.

Second, the maximum angle between any two planes generating the spatial direction between two stations must be at least 60° in order to provide sufficient geometrical strength for the determination of three-dimensional station positions.

Third, the distance to the satellite should be about the same as that between any two stations, so that the scale does not contribute an unproportional amount of error.

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* *Superscript numbers denote references which may be found on page 72.*

A nominal polar orbit is necessary because the satellite must pass over every geographical position. Furthermore, a nominal circular orbit is required in order that changes in altitude are minimized. A near optimum scheme was found which consists of thirty-six stations. Table I is a list of the names and geographical locations of the thirty-six stations, which make up the proposed worldwide satellite triangulation net. Figures 1, 2 and 3 show details of the thirty-six station net arrangement on the globe.

An error analysis of the proposed thirty-six station net is conducted to determine with what accuracy the station positions can be obtained by satellite triangulation. The reference system with respect to which individual satellite directions are determined is the right ascension and declination system. The triangulation is accomplished by photographing a high altitude satellite against the star background simultaneously from at least two ground stations.[5]

TABLE I

PROPOSED WORLDWIDE SATELLITE TRIANGULATION NET

No.	Station Name	Latitude		Longitude	
1	Greenland, Thule A.F.B.	76.5°	N	68.7°	W
2	U.S.A., Aberdeen, Md.	39.5	N	76.1	W
3	U.S.A., Larson A.F.B., Wash.	47.2	N	119.3	W
4	U.S.A., Aleutian Is., Shemya I.	52.7	N	174.1	E
5	U.S.S.R., Tura, Siberia	64.8	N	101.0	E
6	Finland, Kuopio	62.7	N	28.0	E
7	Azores Is., Pico I.	39.0	N	28.5	W
8	Dutch Guiana, Paramaribo	05.5	N	55.2	W
9	Equador, Quito	00.1	S	78.5	W
10	Clipperton Island	10.3	N	109.2	W
11	U.S.A., Hilo, Hawaii	19.8	N	155.0	W
12	Wake Island	19.7	N	166.2	E
13	Japan, Kagoshima	31.7	N	130.6	E
14	India, Gauhati	26.2	N	91.7	E
15	Iran, Sabzevār	36.5	N	57.5	E
16	Libya, Sirte	31.7	N	16.4	E
17	Liberia, Roberts Field	06.8	N	10.2	W
18	Trindade Island	20.5	S	29.4	W
19	Argentina, Villa Dolores	32.0	S	65.1	W
20	Sala-Y-Gomez Island	26.6	S	105.2	W
21	Pukapuka Island	14.7	S	138.8	W
22	Wallis Is., Uveá I.	13.2	S	176.3	W
23	New Guinea, Kikori	07.3	S	144.2	E
24	Sumatra, Palembang	03.0	S	105.0	E
25	Maldive Is., Male	04.2	N	73.3	E
26	Sudan, Juba	04.8	N	31.6	E
27	Southwest Africa, Bogenfels	27.8	S	15.8	E
28	So. Sandwich Is., Saunders I.	58.4	S	26.7	W
29	Antarctica, Peter I.	69.2	S	90.0	W
30	So. Pacific Ocean, Shoal	41.5	S	148.6	W
31	New Zealand, Queenstown	45.0	S	168.2	E
32	Australia, Denmark	35.0	S	117.3	E
33	St. Paul Island	38.7	S	77.0	E
34	Madagascar, Fort Dauphin	25.0	S	47.1	E
35	Antarctica, U.S.S.R. Station	68.0	S	46.4	E
36	Antarctica, France Station	67.0	S	139.0	E

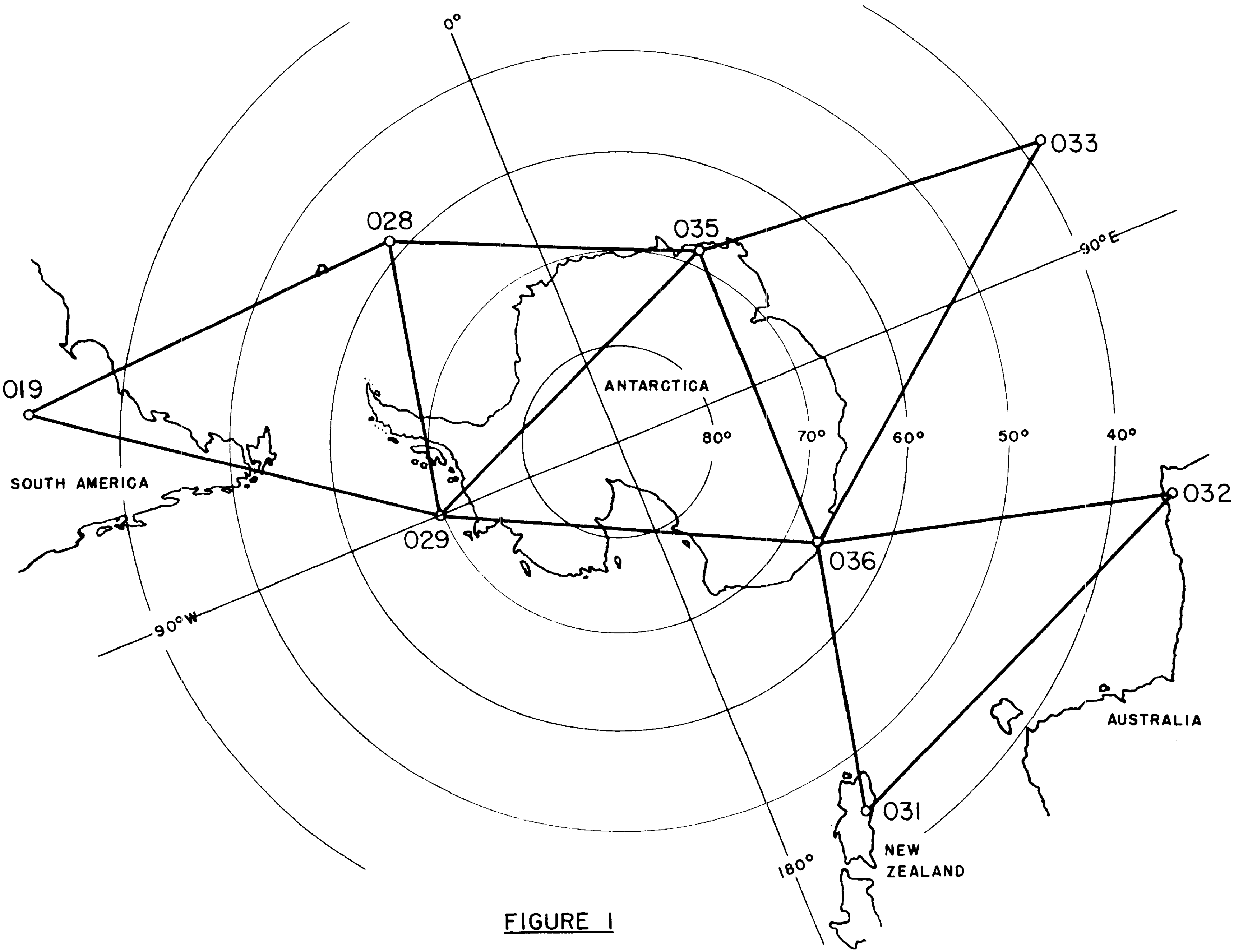

0°
033
028
035
90°E
019
ANTARCTICA
80°
70°
60°
50°
40°
SOUTH AMERICA
032
029
036
90°W
AUSTRALIA
031
NEW
ZEALAND
180°

FIGURE 1

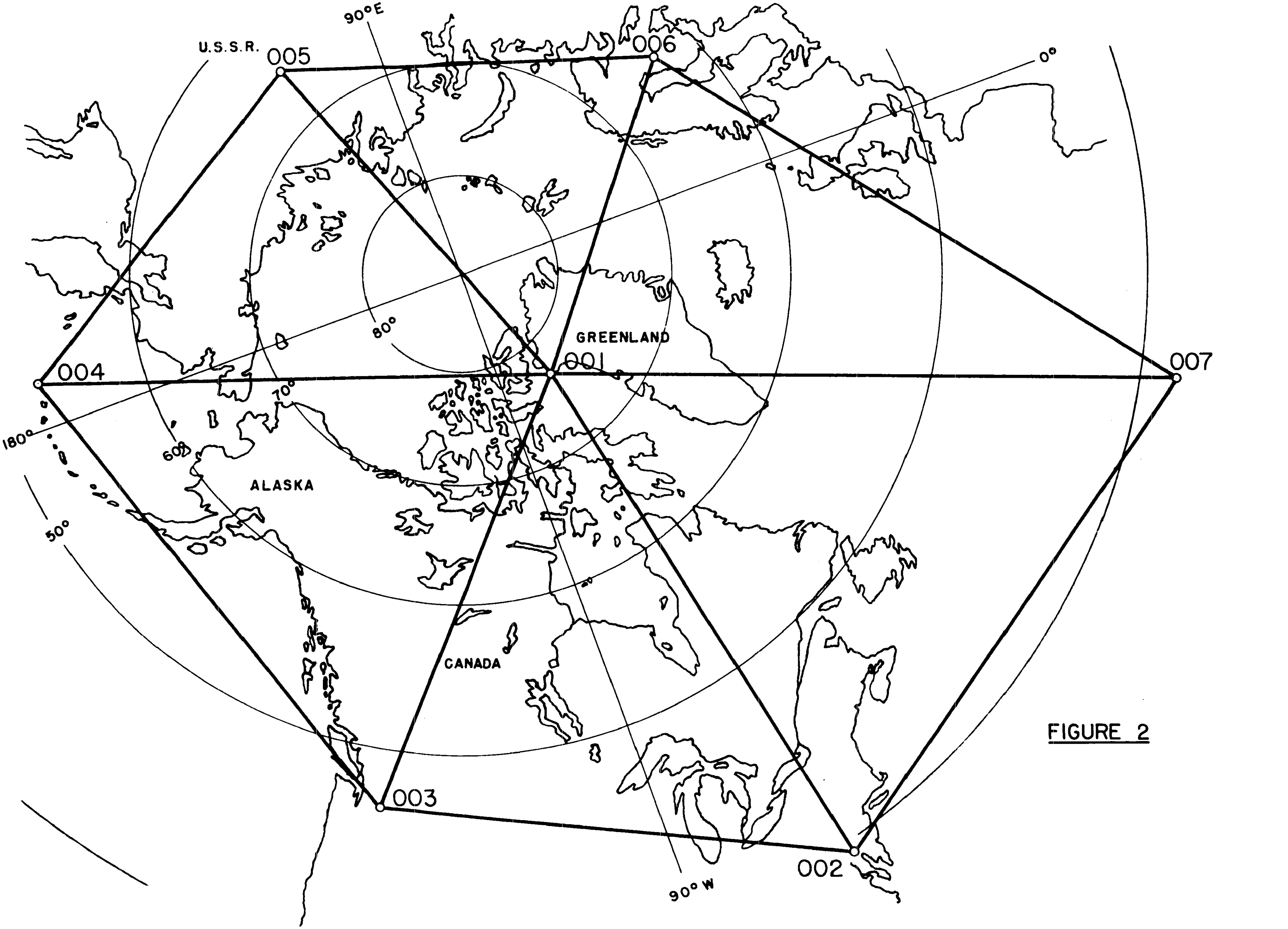

U.S.S.R.
005
90°E
006
0°
80°
GREENLAND
001
007
004
70°
180°
60°
ALASKA
50°
CANADA
003
002
90° W

FIGURE 2

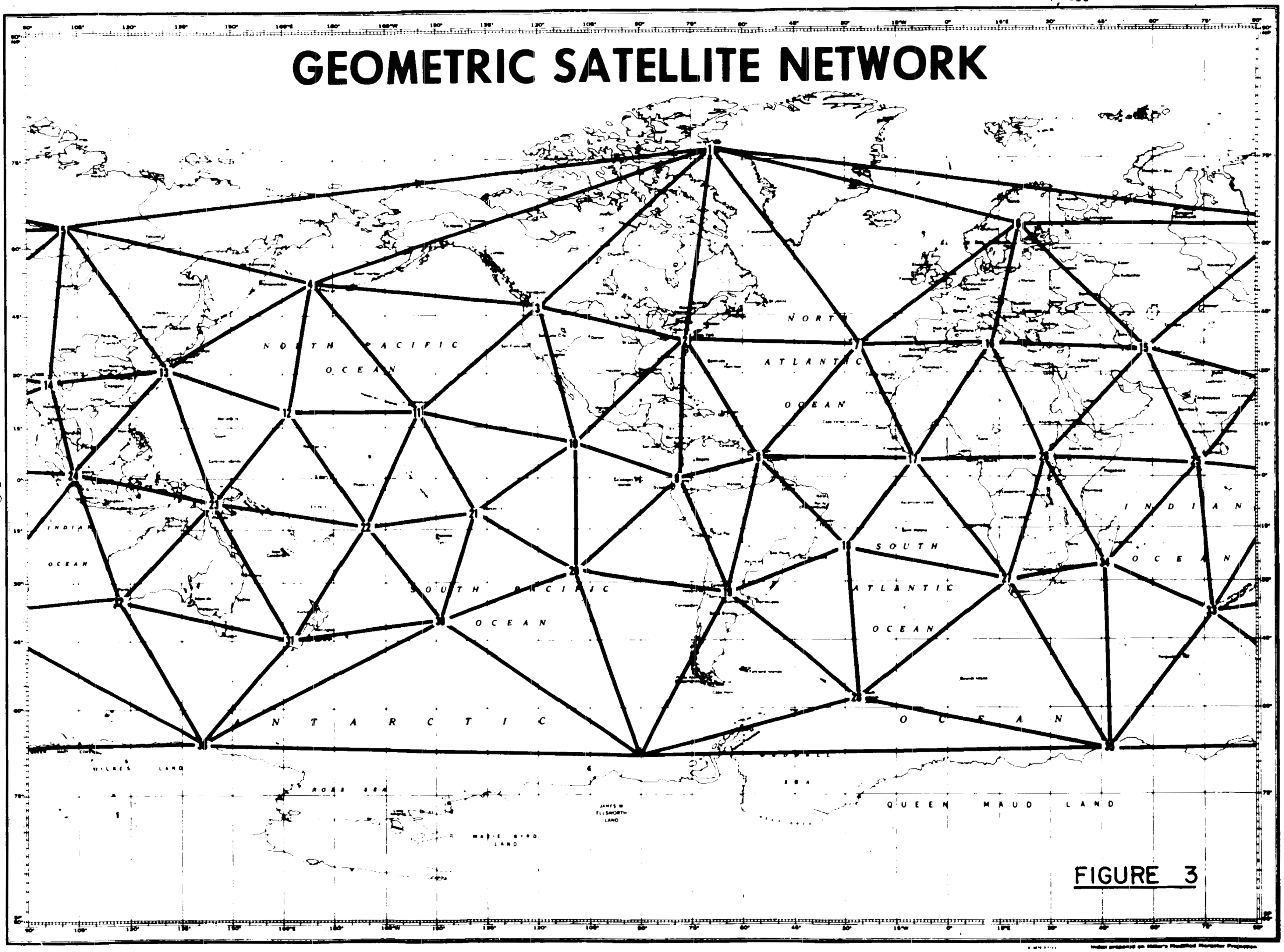

GEOMETRIC SATELLITE NETWORK
NORTH PACIFIC OCEAN
NORTH ATLANTIC OCEAN
SOUTH PACIFIC OCEAN
SOUTH ATLANTIC OCEAN
INDIAN OCEAN
ANTARCTIC OCEAN
WILKES LAND
ROSS SEA
QUEEN MAUD LAND

FIGURE 3

II. DISCUSSION OF ANALYTICAL APPROACH

A brief description of the method used to determine the propagation of errors over the net will show how the direction to each satellite point is incorporated in the final least squares adjustment. It is assumed the reader is familiar with the theory developed in Ballistic Research Laboratories Report No. 1065.[5]

Applying this theory the following is assumed.

1. Each line of sight belonging to a specific satellite point is imaged at the center of the corresponding photograph, i.e.,

$$l_x = l_y = 0, \quad x_p = y_p = 0 \quad \text{and} \quad k_1 = k_2 = k_3 = 0$$

2. $\kappa = 0$

Enforcing the orientation parameters α and ω as are determined in practice from the photographed stars, there remains only the station coordinates X_0, Y_0, Z_0 and the satellite coordinates X, Y, Z which must be determined by a least squares adjustment.

Equation(3)[5] which expresses the functional relation that exists for an individual ray can now be expressed by the corresponding observation equation of the form

$$f(V_j) = F(O_i, X_j)$$

where O_i refers to the parameters X_0, Y_0, Z_0 for a certain camera station i and X_j refers to the station location X, Y, Z of any one individual satellite point j.

It is assumed that each satellite point j is observed from either two or three stations, whereby satellites in the center of a triangle are seen from three stations while those on the center of station lines are observed from just two stations.

Since we are interested only in station positions, for computational convenience all satellite positions are eliminated. To accomplish this, it is necessary at the time of elimination that all rays to a specific satellite position have been incorporated. From the geometrical standpoint no one ray has preference over another, therefore each ray may be treated independently. It is necessary to consider only one set of two equations for any individual ray and apply the same computation to all rays one after the other.

The equation which simulates mathematically a single ray is the condition that object point, center of projection and corresponding image point are collinear.

In this case, therefore, we obtain for each ray two equations:

$$v_x = - D_X \Delta X - E_X \Delta Y - F_X \Delta Z + D_X \Delta X_0 + E_X \Delta Y_0 + F_X \Delta Z_0$$

$$v_y = - D_Y \Delta X - E_Y \Delta Y - F_Y \Delta Z + D_Y \Delta X_0 + E_Y \Delta Y_0 + F_Y \Delta Z_0$$

In matrix notation we obtain the B matrices, namely

B_0 matrix for the stations $\begin{vmatrix} -D_X & -E_X & -F_X \\ -D_Y & -E_Y & -F_Y \end{vmatrix}$

and

B_X matrix for satellite positions $\begin{vmatrix} D_X & E_X & F_X \\ D_Y & F_Y & F_Y \end{vmatrix}$

From Figure (4)

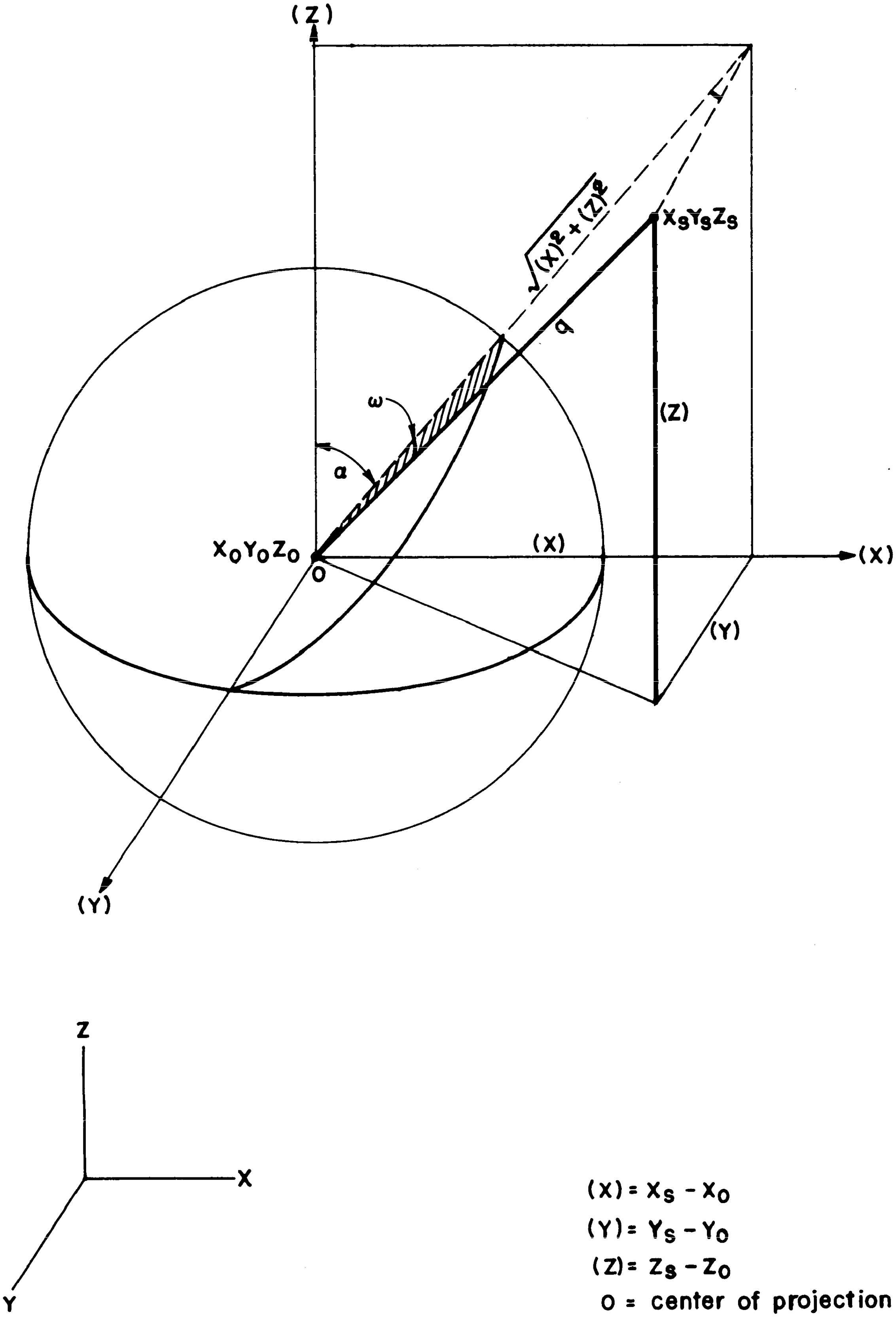
$$(X) = X - X_0$$

$$(Y) = Y - Y_0$$

$$(Z) = Z - Z_0$$

$$q = \sqrt{(X)^2 + (Y)^2 + (Z)^2}$$

c = focal length = constant



$(X) = X_S - X_O$

$(Y) = Y_S - Y_O$

$(Z) = Z_S - Z_O$

O = center of projection

FIGURE 4

$$\sin\alpha = \frac{(X)}{\sqrt{(X)^2 + (Z)^2}} \qquad \sin\omega = \frac{(Y)}{\sqrt{(X)^2 + (Y)^2 + (Z)^2}}$$

$$\cos\alpha = \frac{(Z)}{\sqrt{(X)^2 + (Z)^2}} \qquad \cos\omega = \frac{\sqrt{(X)^2 + (Z)^2}}{\sqrt{(X)^2 + (Y)^2 + (Z)^2}}$$

Since $\kappa = 0$ the direction cosines[5] become:

$A_1 = -\cos\alpha$ $\qquad B_1 = 0$ $\qquad C_1 = \sin\alpha$

$A_2 = -\sin\alpha\cos\omega$ $\qquad B_2 = \cos\omega$ $\qquad C_2 = -\cos\alpha\sin\omega$

$D = \sin\alpha\cos\omega$ $\qquad E = \sin\omega$ $\qquad F = \cos\alpha\cos\omega$

and thus the computational auxiliaries are:

(1) $= 0$ $\qquad$ (4) $= -B_1$ $\qquad$ (7) $= -B_2$

(2) $= 0$ $\qquad$ (5) $= -C_1$ $\qquad$ (8) $= -C_2$

(3) $= -A_1$ $\qquad$ (6) $= -A_2$ $\qquad$ (9) $= 0$

By substitution, the coefficients of the observation equations are found to be:

$$D_X = \frac{c}{q} \frac{(Z)}{\sqrt{(X)^2 + (Z)^2}} \qquad D_Y = \frac{c}{q^2} \frac{(X)(Y)}{\sqrt{(X)^2 + (Z)^2}}$$

$$E_X = 0 \qquad E_Y = \frac{-c}{q^2} \sqrt{(X)^2 + (Z)^2}$$

$$F_X = -\frac{c}{q} \frac{(X)}{\sqrt{(X)^2 + (Z)^2}} \qquad F_Y = \frac{c}{q^2} \frac{(Y)(Z)}{\sqrt{(X)^2 + (Z)^2}}$$

To form the normal equations we must compute for each ray the following expressions:

$$[DD] = D_X^2 + D_Y^2 = \frac{c^2}{q^4} \left[(Y)^2 + (Z)^2 \right]$$

$$[DE] = D_X E_X + D_Y E_Y = -\frac{c^2 (X)(Y)}{q^4}$$

$$[DF] = D_X F_X + D_Y F_Y = -\frac{c^2 (X)(Z)}{q^4}$$

$$[EE] = E_X^2 + E_Y^2 = \frac{c^2}{q^4} \left[(X)^2 + (Z)^2 \right]$$

$$[EF] = E_X F_X + E_Y F_Y = -\frac{c^2}{q^4} (Y)(Z)$$

$$[FF] = F_X^2 + F_Y^2 = \frac{c^2}{q^4} \left[(X)^2 + (Y)^2 \right]$$

Since each ray is treated independently it will contribute to the normal equation system the following submatrix:

$$
\begin{array}{ccc|ccc}
\multicolumn{3}{c}{\text{Satellite}} & \multicolumn{3}{c}{\text{Station}} \\
\Delta X_s & \Delta Y_s & \Delta Z_s & \Delta X_0 & \Delta Y_0 & \Delta Z_0 \\
[DD] & [DE] & [DF] & [-DD] & [-DE] & [-DF] \\
[DE] & [EE] & [EF] & [-DE] & [-EE] & [-EF] \\
[DF] & [EF] & [FF] & [-DF] & [-EF] & [-FF] \\
\hline
[-DD] & [-DE] & [-DF] & [DD] & [DE] & [DF] \\
[-DE] & [-EE] & [-EF] & [DE] & [EE] & [EF] \\
[-DF] & [-EF] & [-FF] & [DF] & [EF] & [FF]
\end{array}
=
\begin{array}{|c|c|}
\hline
B_1 & -B_1 \\
\hline
-B_1 & B_1 \\
\hline
\end{array}
$$

Figure 5

Figure 5 shows the 6x6 submatrix for one ray only. A satellite will never be seen from more than three stations so this 6x6 submatrix will be computed a maximum of three times, once for each satellite ray. If the satellite is observed from only two stations, the third 6x6 submatrix will appear as a null matrix.

For each individual observation we are able to form a corresponding partial normal equation system, and by adding each additional ray, the final normal equation system can be accumulated as shown in Figures 6, ABCD.

At this point in the computational procedure the satellite point is eliminated completely.

In general $\quad D = B_0 - B^T_{X/0}\, B_X^{-1}\, B_{X/0}$

12

B_1	$-B_1$		
$-B_1$	B_1		

12

FIGURE 6A

Figure 6A represents the contribution of one ray to the normal equation system.

12

B_1+B_2	$-B_1$	$-B_2$	
$-B_1$	B_1		
$-B_2$		B_2	

12

FIGURE 6B

Figure 6B represents the contribution of two rays to the normal equation system.

12

$B_1 + B_2 + B_3$	$-B_1$	$-B_2$	$-B_3$
$-B_1$	B_1		
$-B_2$		B_2	
$-B_3$			B_3

12

FIGURE 6C

Figure 6C represents the contribution of all three rays to the normal equation system.

12

B_X	$B_{X/O}$
$B_{X/O}^T$	B_O

12

FIGURE 6D

The upper left 3 x 3 B_X-matrix contains coefficients associated with the unknown corrections of the satellite point, ΔX, ΔY, and ΔZ.

The lower right 9 x 9 B_O-matrix is subdivided into three block diagonal matrices, one for each camera station that sees the satellite point, and is associated with the corrections of the unknown positional parameters ΔX_O, ΔY_O, and ΔZ_O for each station.

The $B_{X/O}$-matrix contains the coefficients which represent the geometrical fact that the satellite point was seen from stations shown in the B_O-matrix.

This elimination cycle reduces the 12x12 matrix of Figure 6D to a 9x9 D-matrix as shown by Figure 7, concerned with station data only.

	Sta. i	Sta. j	Sta. k
	ΔX_0 ΔY_0 ΔZ_0	ΔX_0 ΔY_0 ΔZ_0	ΔX_0 ΔY_0 ΔZ_0
Station i	D_{ii}	D_{ij}	D_{ik}
Station j	D_{ij}	D_{jj}	D_{jk}
Station k	D_{ik}	D_{jk}	D_{kk}

Figure 7

In Figure 7 the satellite point has been numerically eliminated, and only station unknowns ΔX_0, ΔY_0, and ΔZ_0 remain for two or three stations depending on how many stations recorded the specific satellite point.

This study is made for a maximum of thirty-six stations where n denotes the number of ground stations included in a specific net. The size of the reduced normal equation system will be a 3nx3n matrix containing unknown ground stations only.

Consequently, each 9x9 D-matrix is partitioned into nine 3x3 submatrices, each of which is added in its proper position within the final 3nx3n matrix system as shown in Figure 8.

Before the resulting system is inverted, the following additional conditions must be incorporated:

1) The geometric condition that sets the sum of all adjusted X_0, Y_0, Z_0 coordinates individually equal to zero in order to obtain a symmetrically arranged error distribution field.

Assume one satellite point is observed from stations 3, 5, and 7.

The 9 x 9 D-matrix is sub-divided and added into the 3n x 3n system (n = 7) as follows:

$$i = 3$$
$$j = 5$$
$$K = 7$$

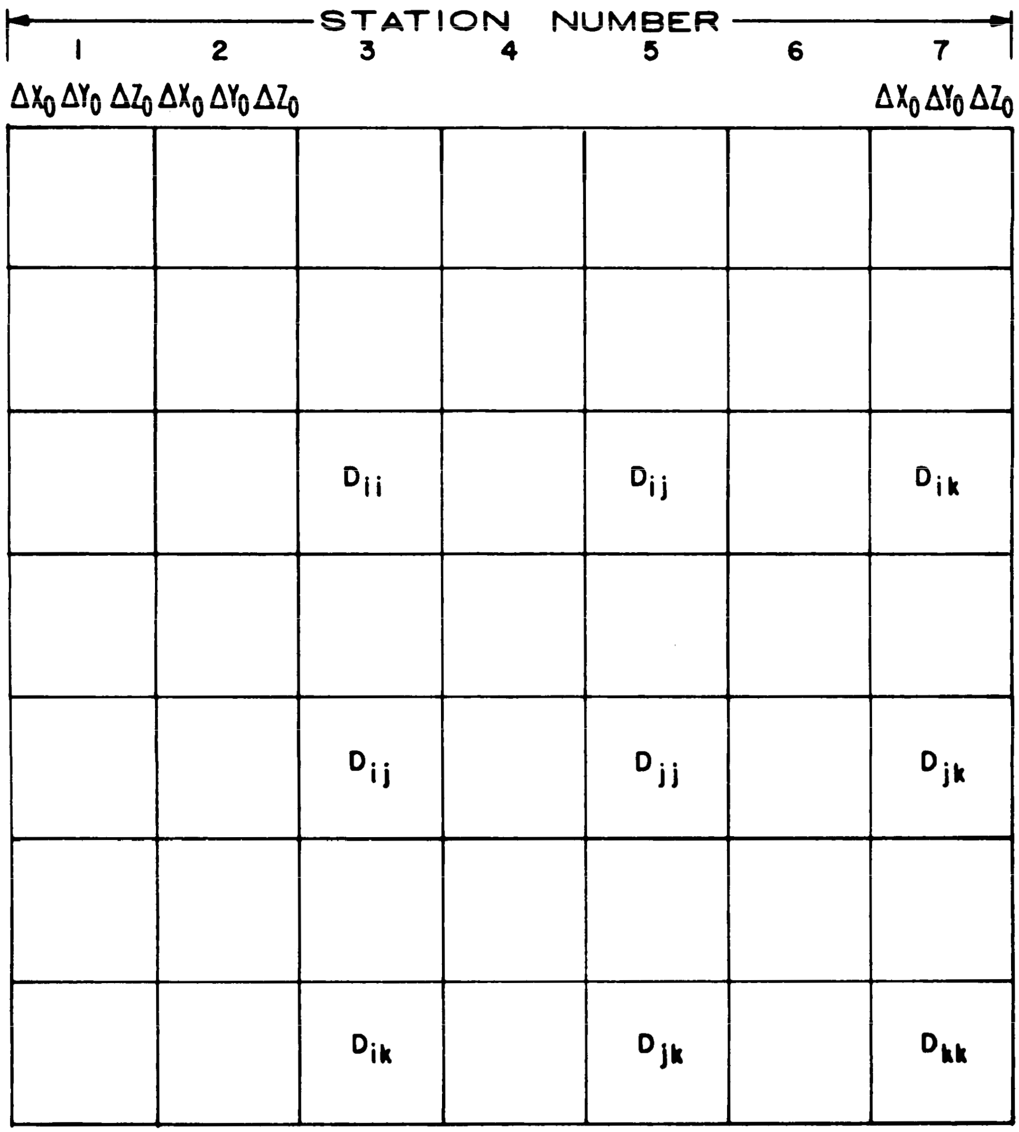


FIGURE 8

$$\sum \Delta X_0 = -\sum X_0$$

$$\sum \Delta Y_0 = -\sum Y_0$$

$$\sum \Delta Z_0 = -\sum Z_0$$

2) Metric condition: At least one scalar must be introduced. Allowance has been made in the computer program to incorporate as many as four scalars with their respective variances.

If the distance between two stations (i and j) is enforced, the corresponding metric conditional equation is introduced:

$$G \cdot \Delta X_{0_i} + H \cdot \Delta Y_{0_i} + J \cdot \Delta Z_{0_i} - G \cdot \Delta X_{0_i} - H \cdot \Delta Y_{0_i} - J \cdot \Delta Z_{0_i} = 0$$

where

$$G = (X_{0_i} - X_{0_j})/d_{ij}$$

$$H = (Y_{0_i} - Y_{0_j})/d_{ij}$$

$$J = (Z_{0_i} - Z_{0_j})/d_{ij}$$

$$d_{ij} = \sqrt{(X_{0_i} - X_{0_j})^2 + (Y_{0_i} - Y_{0_j})^2 + (Z_{0_i} - Z_{0_j})^2}$$

These conditions are incorporated in the final normal equation system by simply adding the specific equations to the normal equation system (3nx3n) and at the same time restoring the system to a quadratic form. (page 33[5]).

Assuming n=36 and the number of scalars introduced to be four, Figure 9 shows the resulting system after the geometric and metric conditions have been incorporated. A system of this size refers to

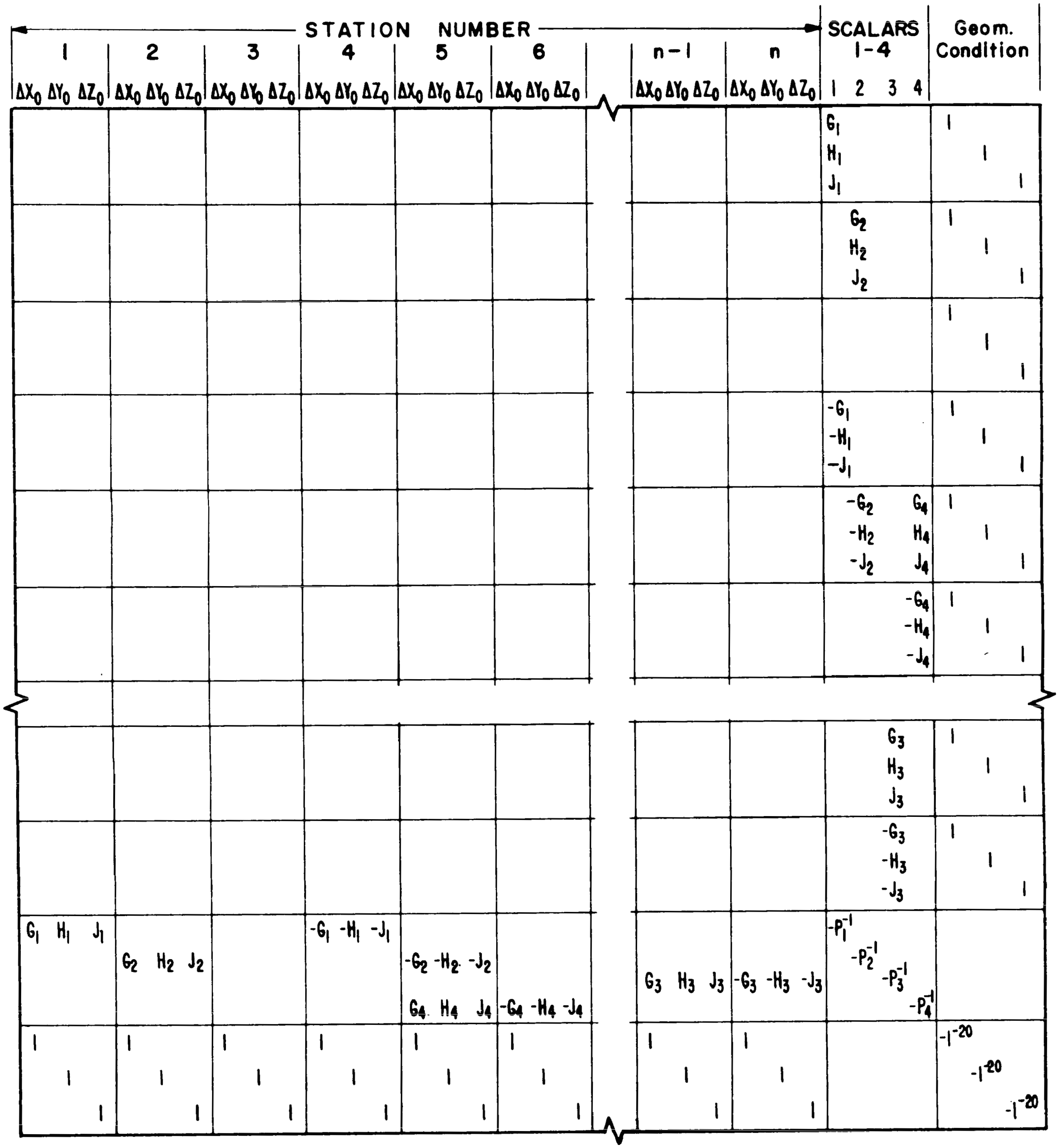


FIGURE 9

FINAL MATRIX TO BE INVERTED

108 station coordinates and requires the inversion of a 115x115 matrix. The inverted Q-matrix refers to the Cartesian coordinate system in which the triangulation was performed with its origin at the center of gravity of the whole system.

$$Q = \begin{vmatrix} Q_{X_{O_1}} & & & & & & & \\ & Q_{Y_{O_1}} & & & & & & \\ & & Q_{Z_{O_1}} & & & & & \\ & & & Q_{X_{O_2}} & & & & \\ & & & & \cdot & & & \\ & & & & & \cdot & & \\ & & & & & & \cdot & \\ & & & & & & & Q_{Z_{O_n}} \end{vmatrix}$$

The diagonal terms can be used to compute the mean errors of the station coordinates $m_{X_{O_1}}$, $m_{Y_{O_1}}$, $m_{Z_{O_1}}$ . . . $m_{Z_{O_n}}$ with

$$m_{X_{O_i}} = \mu \cdot 10^7 \cdot \sqrt{Q_{X_{O_i}}}$$

For each station a transformation matrix T_{ϕ}, obtained from a coordinate transformation subprogram, is used to transform the diagonal terms of the Q-matrix from the Cartesian coordinate system to a geodetic ellipsoidal system, in terms of latitude (ϕ), longitude (λ) and elevation (H).[6]

Appropriate constants are applied to the diagonal terms to give results in meters.

$$m_{\phi} = K_{\phi} \cdot \mu \cdot 10^7 \qquad \text{where} \qquad K_{\phi} = 30.85 \sqrt{Q_{\phi}}$$

$$m_{\lambda} = K_{\lambda} \cdot \mu \cdot 10^7 \qquad \text{where} \qquad K_{\lambda} = 30.85 \cos \phi \sqrt{Q_{\lambda}}$$

$$m_H = K_H \cdot \mu \cdot 10^7 \qquad \text{where} \qquad K_H = \sqrt{Q_H}$$

A computer program was written at BRL in the Forast programming language which enables the investigation of error propagation for various size nets up to a maximum of 36 stations, allowing the number of scalars to vary from one to four.

An error analysis of various station schemes using hypothetical data is presented. The scheme which is thought to be most nearly ideal consists of 36 stations forming 68 triangles with 102 sides, 4000 to 4500 km in length, using a satellite altitude of 3600 km. In this scheme it was assumed that all satellite positions over the center of each of the 68 triangles were observed simultaneously from three ground stations and satellite positions over lines were observed from two stations. A total of 170 satellite positions and 408 rays were incorporated in the least squares adjustment of the 36 station net. Four specific nets are considered:

No. of stations in net	No. of triangles formed	No. of lines	No. of satellite positions observed	No. of rays incorporated in least square solution
36	68	102	170	408
35	62	96	164	390
34	56	90	158	372
28	38	66	122	282

1. In the 36 station net the stations are arranged to cover the entire earth.

2. In the 35 station net Russia is omitted.

3. In the 34 station net Russia and the South Pacific are omitted.

4. In the 28 station net the stations in the polar regions are omitted leaving a 28 station equatorial net with two rows of triangles straddling the equator.

The propagation of the errors in the station coordinates of the aforementioned triangulation nets are presented in graphical form. These results have been obtained by inverting the final normal equation system. The diagonal terms of the inverted matrix give quantitative information about the propagation of errors in the unknowns of the solution and the non-diagonal terms furnish the corresponding covariances.[5]

In all schemes the effect of a specific number (one to four) of scalars and various accuracies of these ground established baselines on the propagation of errors over the net were investigated.

The stations between which distance measurements are assumed are listed below and are indicated by scalar number on the vertical scale of Figures 10-49.

Scalar No.	Baseline	Stations between which distance measurements are assumed
1	North America	2 - 3
2	Australia	23 - 32
3	Europe	6 - 16
3*	Iran	14 - 15
4	South America	8 - 19

* For the twenty-eight station net the third scalar between stations 6 and 16 was replaced by a scalar in Iran between stations 14 and 15.

III. DISCUSSION OF RESULTS

In order to introduce the various scalars with different variances it is necessary to compute the corresponding weighting factors.[7]

Assuming a standard error of unit weight on the plate measurements of $\pm\ \mu$ microns we have

$$P_{\ell} = 1 = \frac{K}{\mu^2 \cdot 10^{-12}} \qquad \text{or} \qquad K = \mu^2 \cdot 10^{-12}$$

Correspondingly the weight for any one introduced scalar, if its specific standard error is denoted by $\pm$ m (meters), is

$$P_X = \frac{K}{m^2} = \frac{\mu^2 \cdot 10^{-12}}{m^2}$$

Stipulating a realistic value for μ of $\pm$ 0.75 microns, $K = 0.5 \cdot 10^{-12}$ and consequently we obtain the following values for m.

P_X	m
$10^4 \cdot 10^{-14}$	assumed to be zero
$1 \cdot 10^{-14}$	$\pm$ 7 m
$0.1 \cdot 10^{-14}$	$\pm$ 20 m

For convenience the problem is scaled by 10^{14} resulting in the P_X values of 10^4, 1, and 0.1, respectively. All results are related to a focal length of the photogrammetric camera, c = 0.316m.

Figures 10, 11, 12 are concerned with the 36 station net. The accuracies are shown which can be obtained for the determination of station coordinates. The assumed standard deviations on the baseline measurements, as outlined above, are assumed to be zero, ± 7 meters and ± 20 meters, respectively. The beneficial influence of one, two, three and four scalars on the accuracy of the determination of the station coordinates is demonstrated. The geometry of the triangulation is strengthened with each additional scalar and, consequently, the results show an improvement in the mean error of the station coordinates.

Additional scalars, even when measured with extreme accuracies, make little improvement on latitude and longitude determination. In fact, for all practical purposes one scalar is nearly as good as four. However, the height determination is decidedly improved with each additional scalar and with an increase in accuracy of scalar measurements. The ratio of the improvement in the mean error of station coordinates becomes smaller with the introduction of each additional scalar. The greatest relative improvement exists when two scalars are used to constrain the triangulation, and the addition of a third scalar will yield significant improvement. However, whether or not much is gained from four scalars over three is questionable. This fact is significant when considering the cost of obtaining baseline measurements over long distances. The influence of the measuring errors as they are propagated over the net can best be seen by examining Figure 13 which is a combined plot of Figures 10, 11, 12. Measuring errors of ± 7 meters using one baseline give about the same accuracy as four baselines with measuring errors of ± 20 meters. Similiarly, for one scalar with an assumed standard deviation of ± 20 meters the errors propagated over the net are twice as large as those obtained with the corresponding baseline measurements of ± 7 meters accuracy. The table that follows illustrates the situation. The fact should not be overlooked that only an insignificant improvement is obtained between assumed zero measurements and those with an assumed accuracy of ± 7 meters.

K_H - values for H-determination*			
No. of scalars	Assumed standard deviation of scalar ± 20	± 7	0
1	5.8	3.1	2.7
2	4.2	2.4	2.1
3	3.6	2.1	1.9
4	3.1	1.9	1.8

*Portions of these results were presented in [8].

The possibility of being unable to establish certain station locations led to the investigation of several other nets. The same identical runs that were computed for the 36 station net were computed for the following nets:

a) 35 station net without Russia (station No. 5 omitted) Figures 14-17.

b) 34 station net without Russia and the South Pacific (stations No. 5 and 30 omitted) Figures 18-21.

c) 28 station net without stations near the polar regions, Figures 22-25.

For each station not incorporated in the least squares adjustment the triangulation is weakened; this accounts for the more erratic appearance of each set of Figures for the 35, 34 and 28 station nets, respectively.

In the least squares adjustment for the 35 station net six triangles in the vicinity of Russia are omitted. The irregularity of Figures 14-17 for all three coordinates, latitude, longitude and height in this particular area of the net is apparent, particularly at stations which are close to the latitude and longitude of the omitted station No. 5. Station 6, which is about the same latitude as station 5, has the largest mean error in latitude over the net while stations 13 and 14, which have longitudes near that of station 5, have the largest mean error in longitude over the net. Furthermore, the height determinations have the largest mean errors occurring on stations 6 and 14 which are closest in both latitude and longitude to station 5. On the other hand the loss in accuracy is relatively small, on account of the basic favorable error propagation, which is unique for this type of three-dimensional satellite triangulation.

In Figures 26-49 the same information is presented in a different way, by comparing the accuracies obtainable for the four different net sizes (36, 35 34 and 28 stations) in terms of latitude, longitude and height.

EDNA L. LORTIE

36 STATION NET

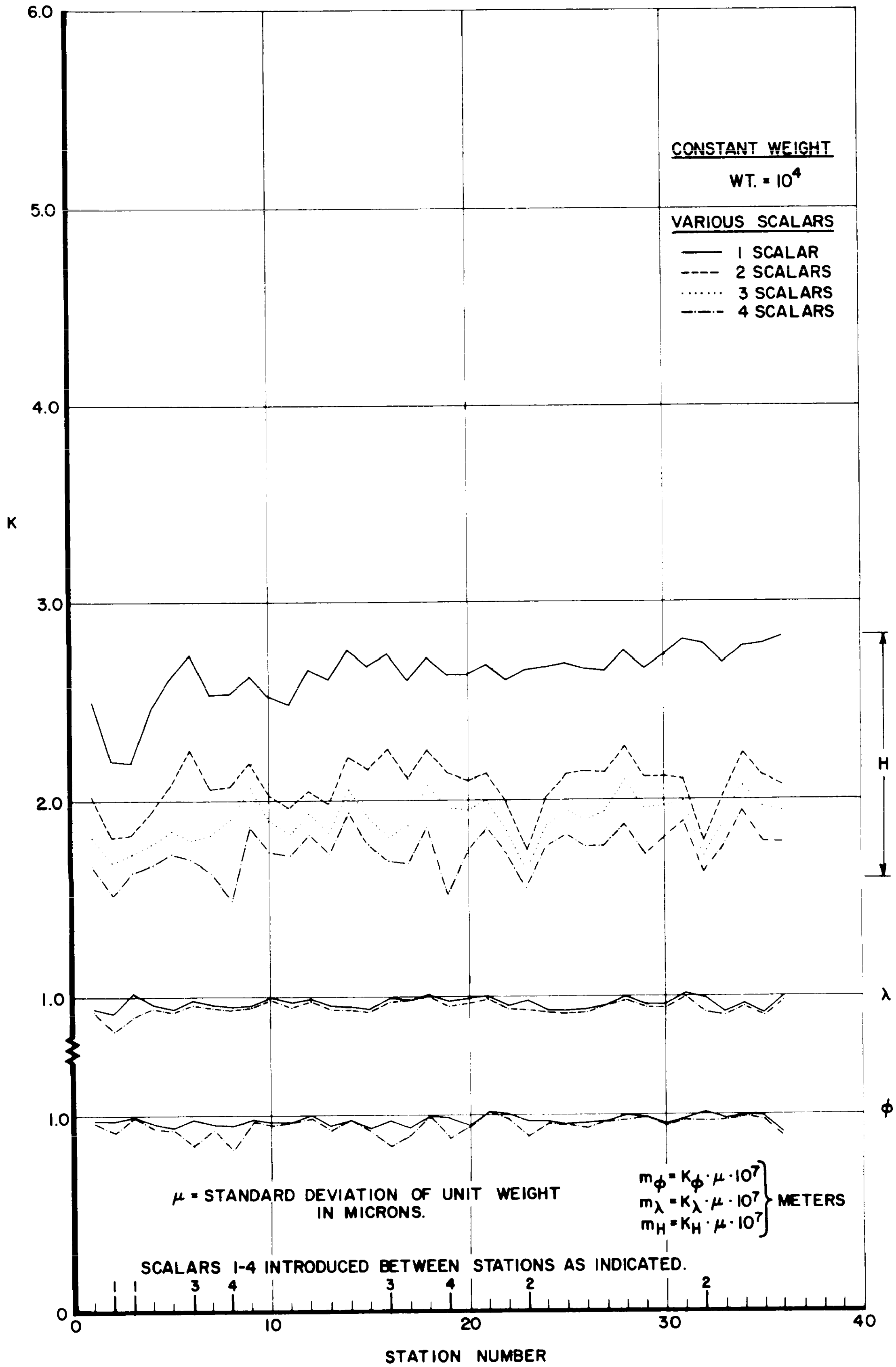


FIGURE 10

36 STATION NET

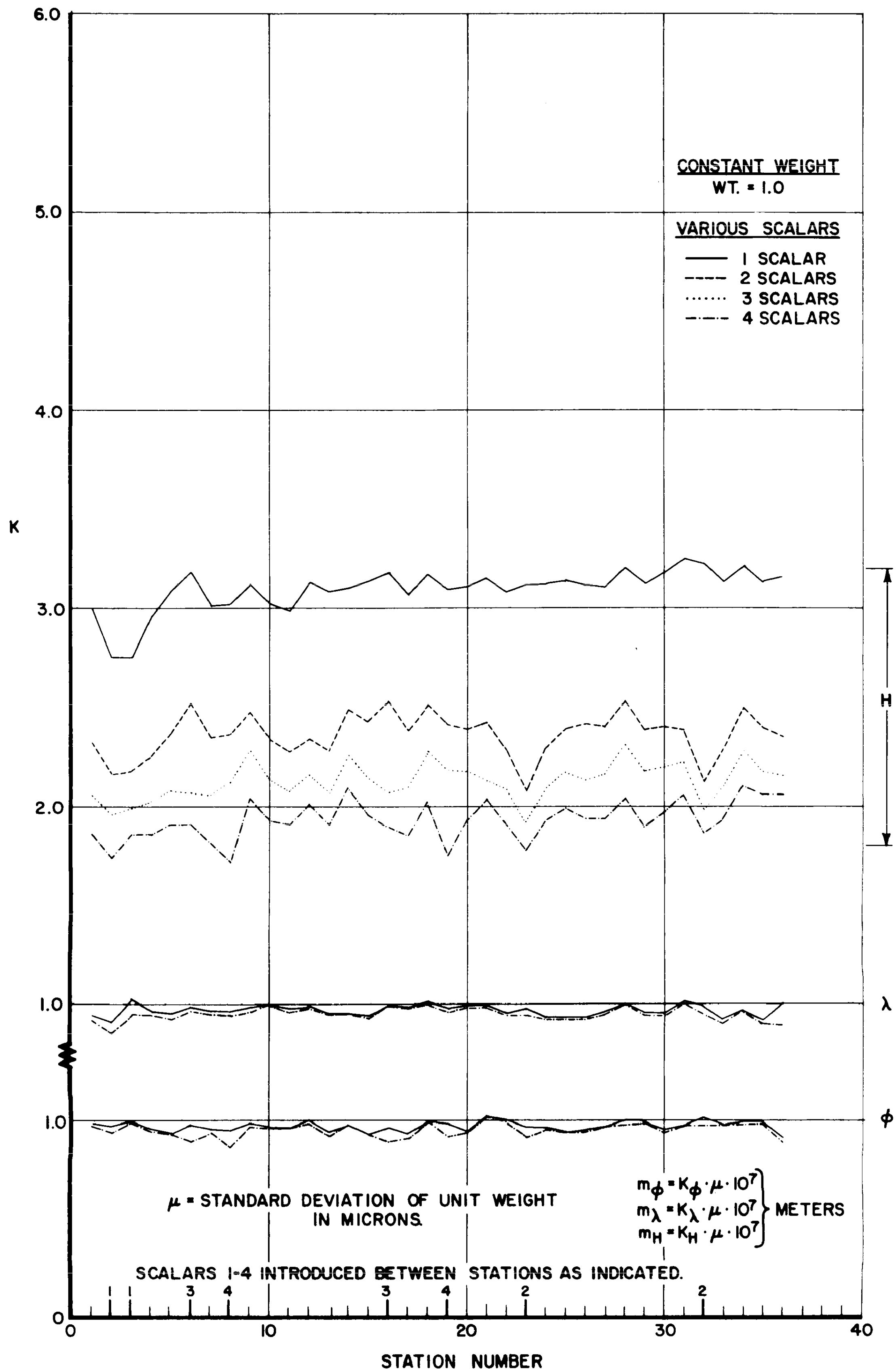


FIGURE 11

36 STATION NET

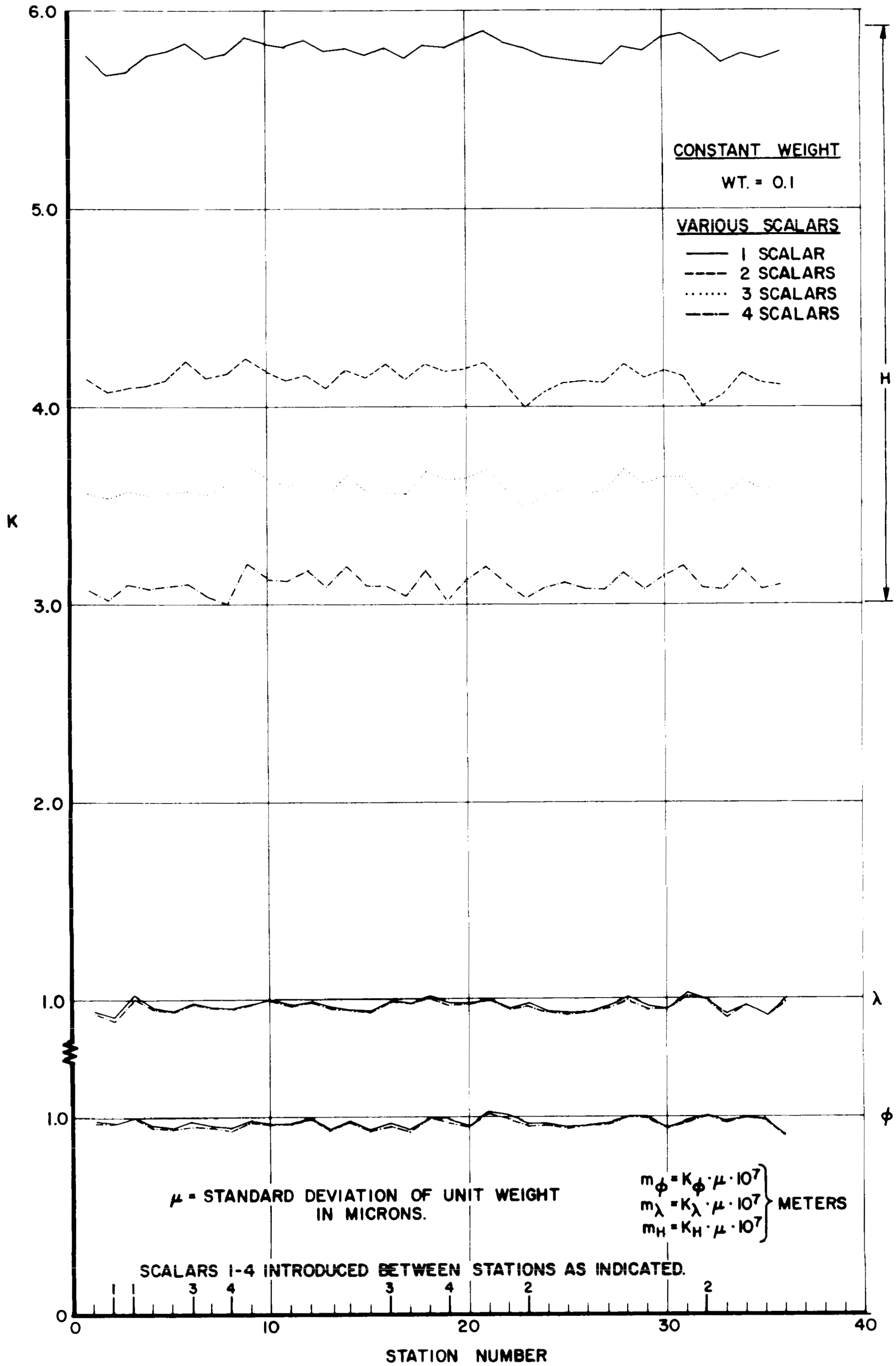


FIGURE 12

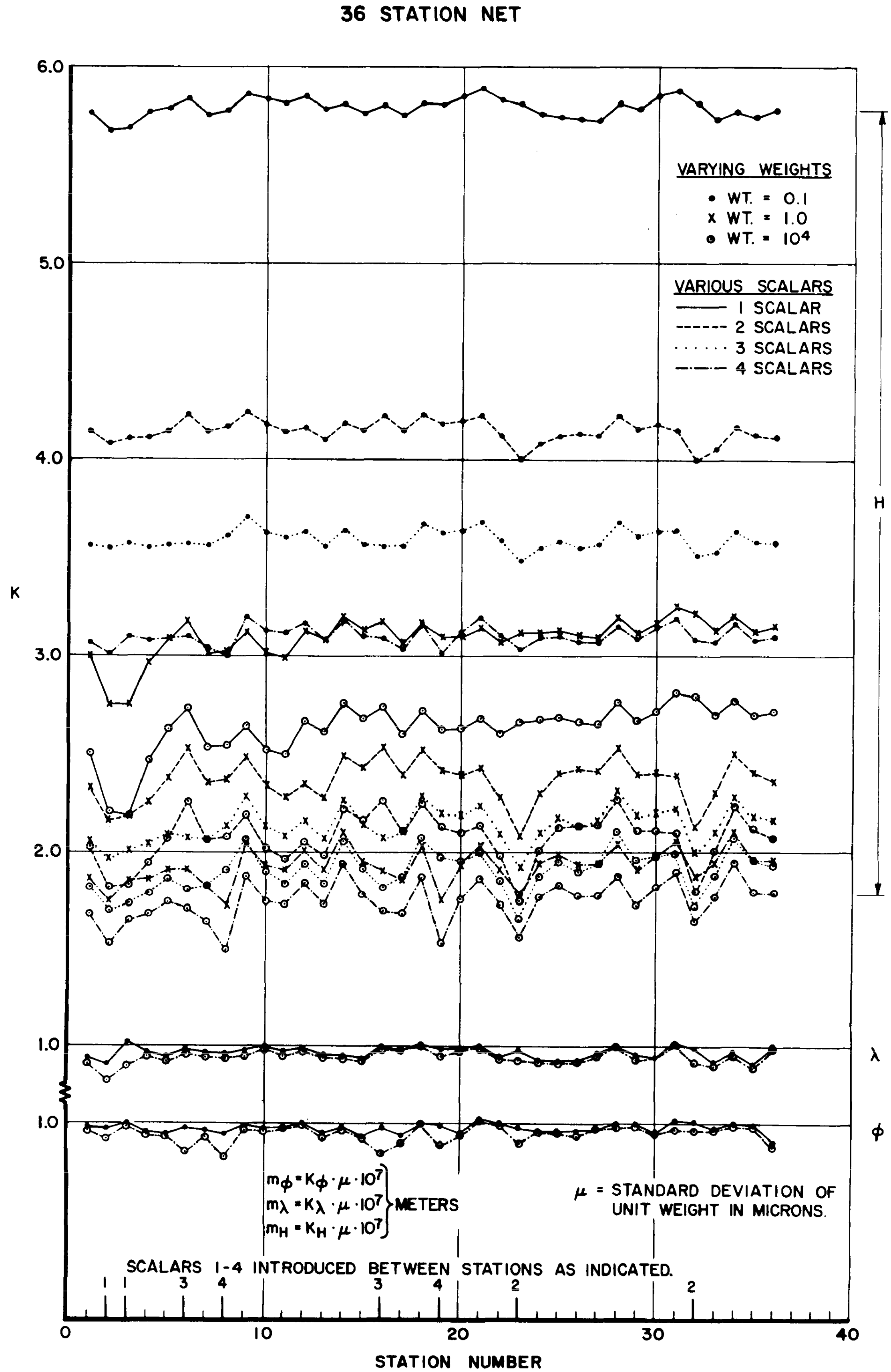

36 STATION NET
6.0
5.0
4.0
3.0
2.0
1.0
1.0
K
VARYING WEIGHTS
• WT. = 0.1
x WT. = 1.0
⊙ WT. = 10^4
VARIOUS SCALARS
1 SCALAR
2 SCALARS
3 SCALARS
4 SCALARS
H
λ
ϕ
$m_\phi = K_\phi \cdot \mu \cdot 10^7$
$m_\lambda = K_\lambda \cdot \mu \cdot 10^7$
$m_H = K_H \cdot \mu \cdot 10^7$
METERS
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED.
0
10
20
30
40
STATION NUMBER

FIGURE 13

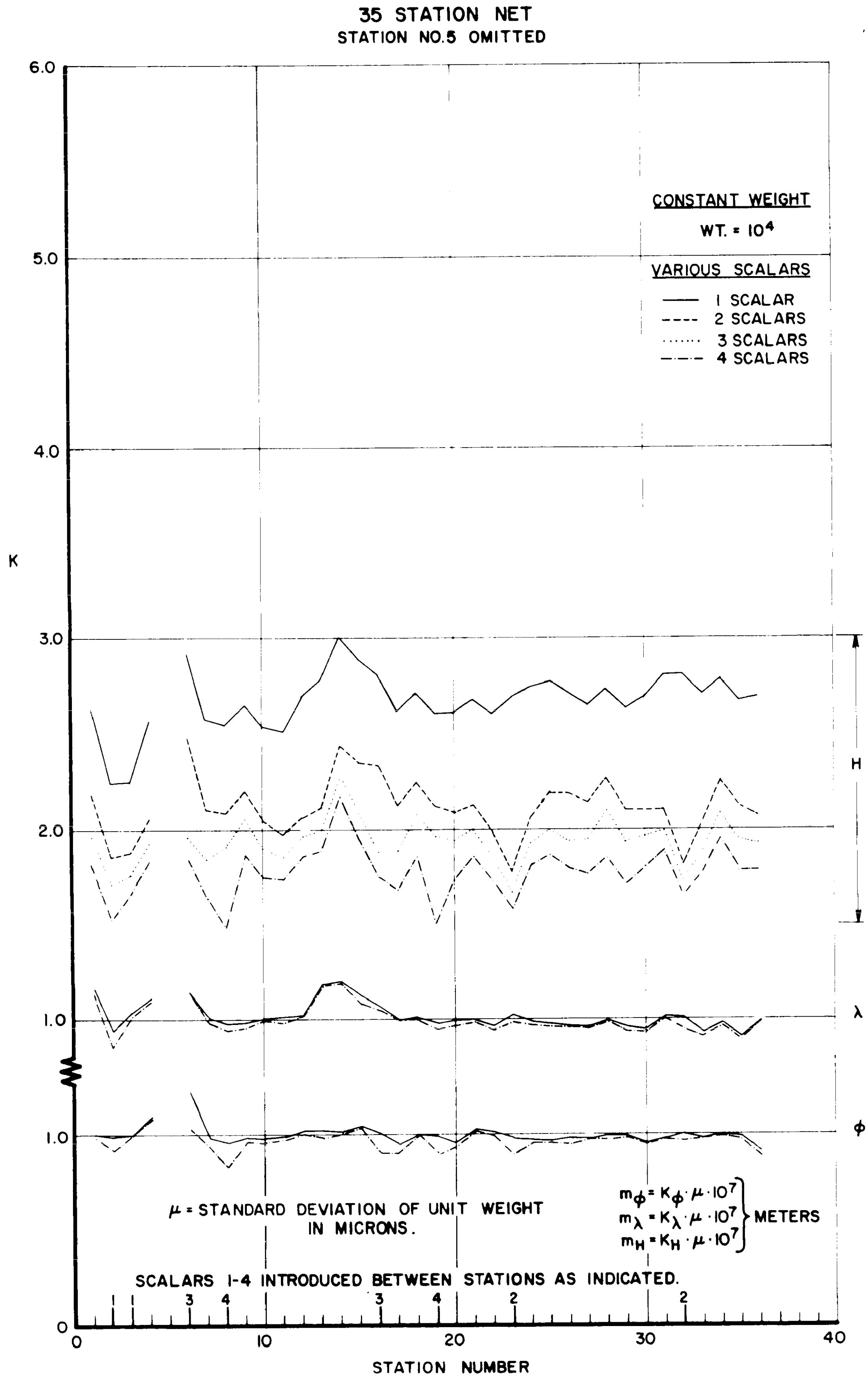

35 STATION NET
STATION NO.5 OMITTED
CONSTANT WEIGHT
WT. = 10⁴
VARIOUS SCALARS
1 SCALAR
2 SCALARS
3 SCALARS
4 SCALARS
6.0
5.0
4.0
K
3.0
2.0
1.0
1.0
0
H
λ
φ
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
$m_\phi = K_\phi \cdot \mu \cdot 10^7$
$m_\lambda = K_\lambda \cdot \mu \cdot 10^7$
$m_H = K_H \cdot \mu \cdot 10^7$
METERS
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED.
3 4 3 4 2 2
0 10 20 30 40
STATION NUMBER

FIGURE 14

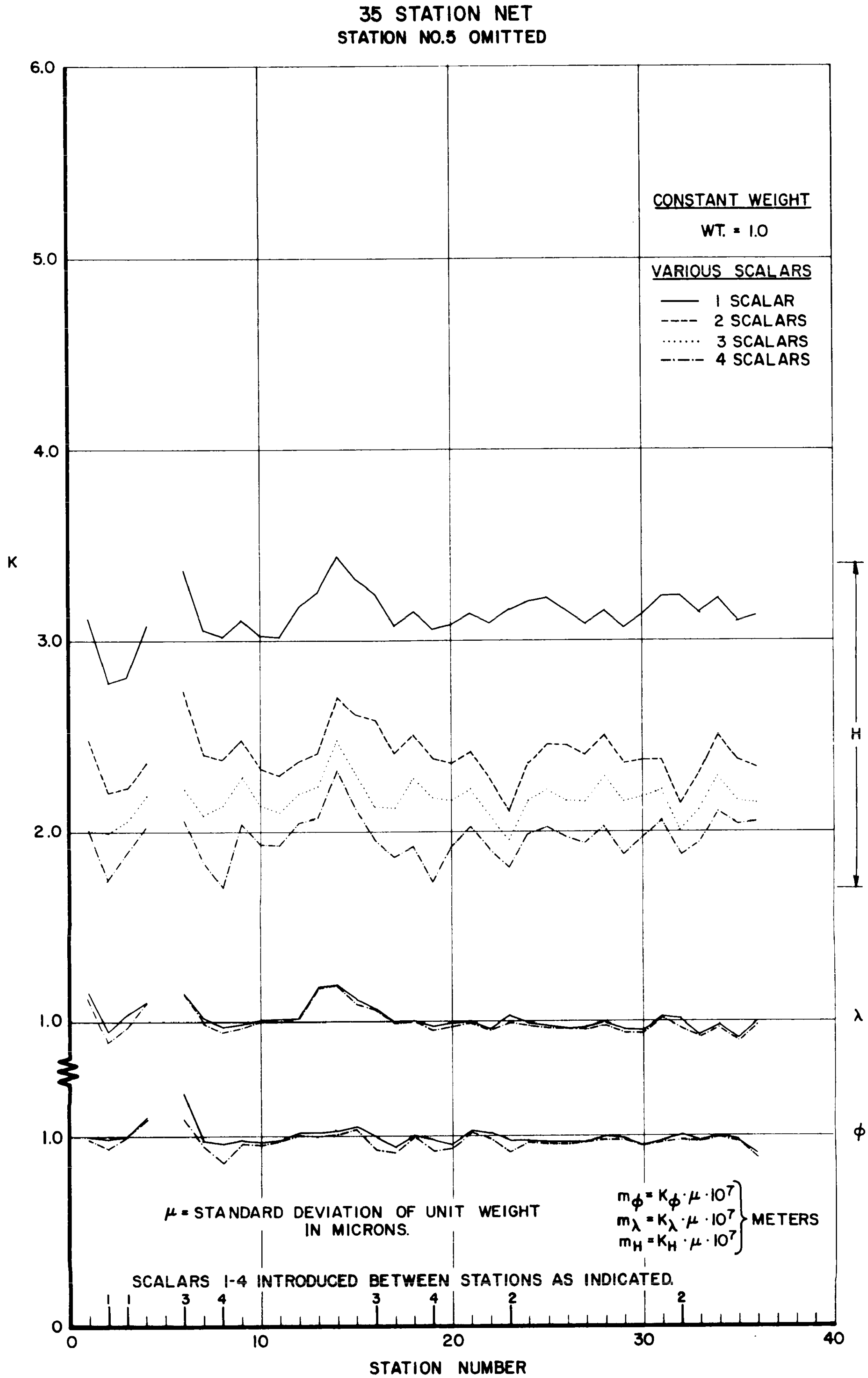

35 STATION NET
STATION NO.5 OMITTED
CONSTANT WEIGHT
WT. = 1.0
VARIOUS SCALARS
1 SCALAR
2 SCALARS
3 SCALARS
4 SCALARS
K
H
λ
φ
6.0
5.0
4.0
3.0
2.0
1.0
1.0
0
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
$m_\phi = K_\phi \cdot \mu \cdot 10^7$
$m_\lambda = K_\lambda \cdot \mu \cdot 10^7$
$m_H = K_H \cdot \mu \cdot 10^7$
METERS
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED.
1 1 3 4 3 4 2 2
0 10 20 30 40
STATION NUMBER

FIGURE 15

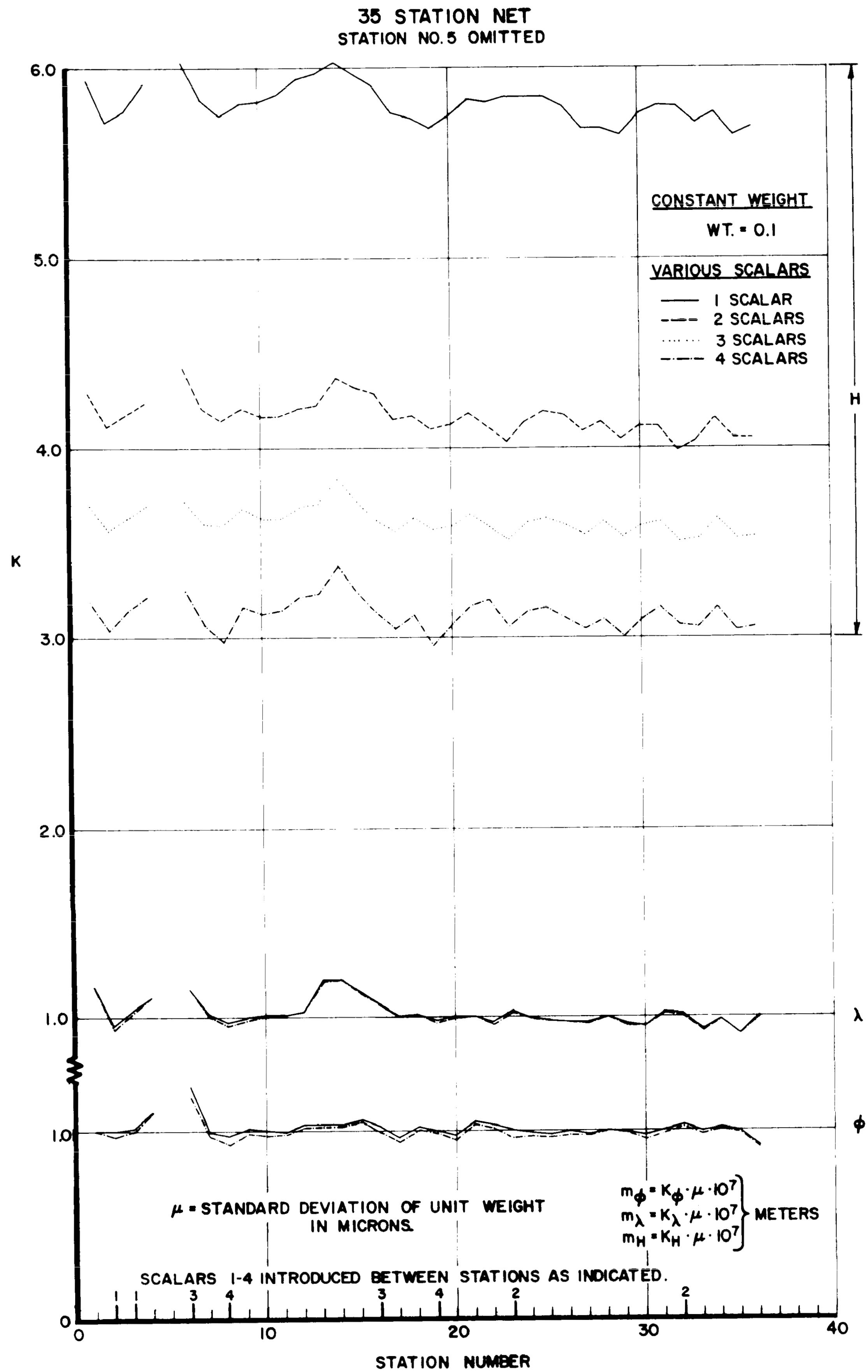

35 STATION NET
STATION NO. 5 OMITTED
CONSTANT WEIGHT
WT. = 0.1
VARIOUS SCALARS
1 SCALAR
2 SCALARS
3 SCALARS
4 SCALARS
H
λ
φ
K
6.0
5.0
4.0
3.0
2.0
1.0
1.0
0
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
$m_\phi = K_\phi \cdot \mu \cdot 10^7$
$m_\lambda = K_\lambda \cdot \mu \cdot 10^7$
$m_H = K_H \cdot \mu \cdot 10^7$
METERS
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED.
1 1 3 4 3 4 2 2
0 10 20 30 40
STATION NUMBER

FIGURE 16

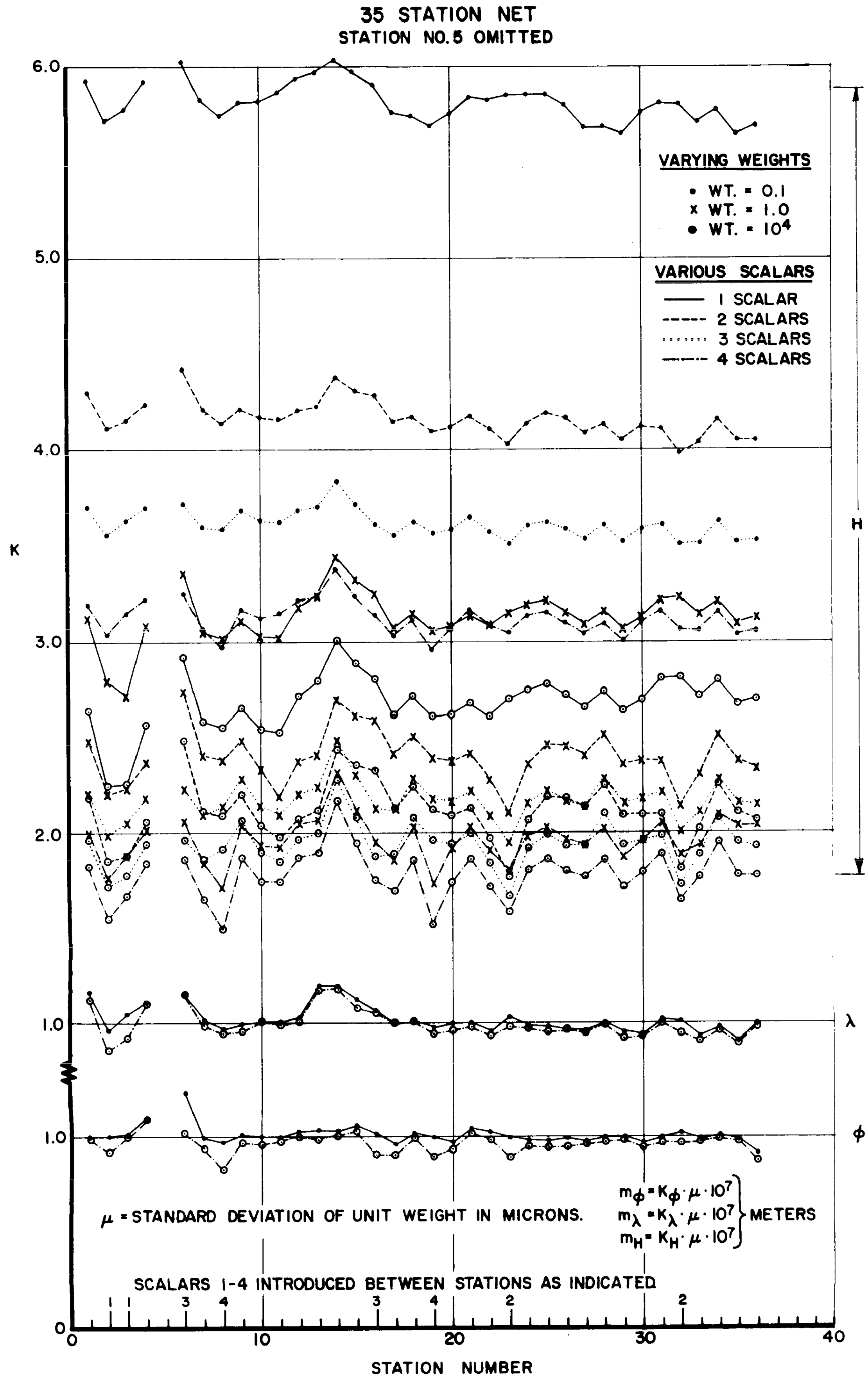

35 STATION NET
STATION NO. 5 OMITTED
VARYING WEIGHTS
WT. = 0.1
WT. = 1.0
WT. = 10⁴
VARIOUS SCALARS
1 SCALAR
2 SCALARS
3 SCALARS
4 SCALARS
K
H
λ
φ
6.0
5.0
4.0
3.0
2.0
1.0
1.0
0
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
mφ = Kφ · μ · 10⁷
mλ = Kλ · μ · 10⁷
mH = KH · μ · 10⁷
METERS
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED
1 1 3 4 3 4 2 2
0
10
20
30
40
STATION NUMBER

FIGURE 17

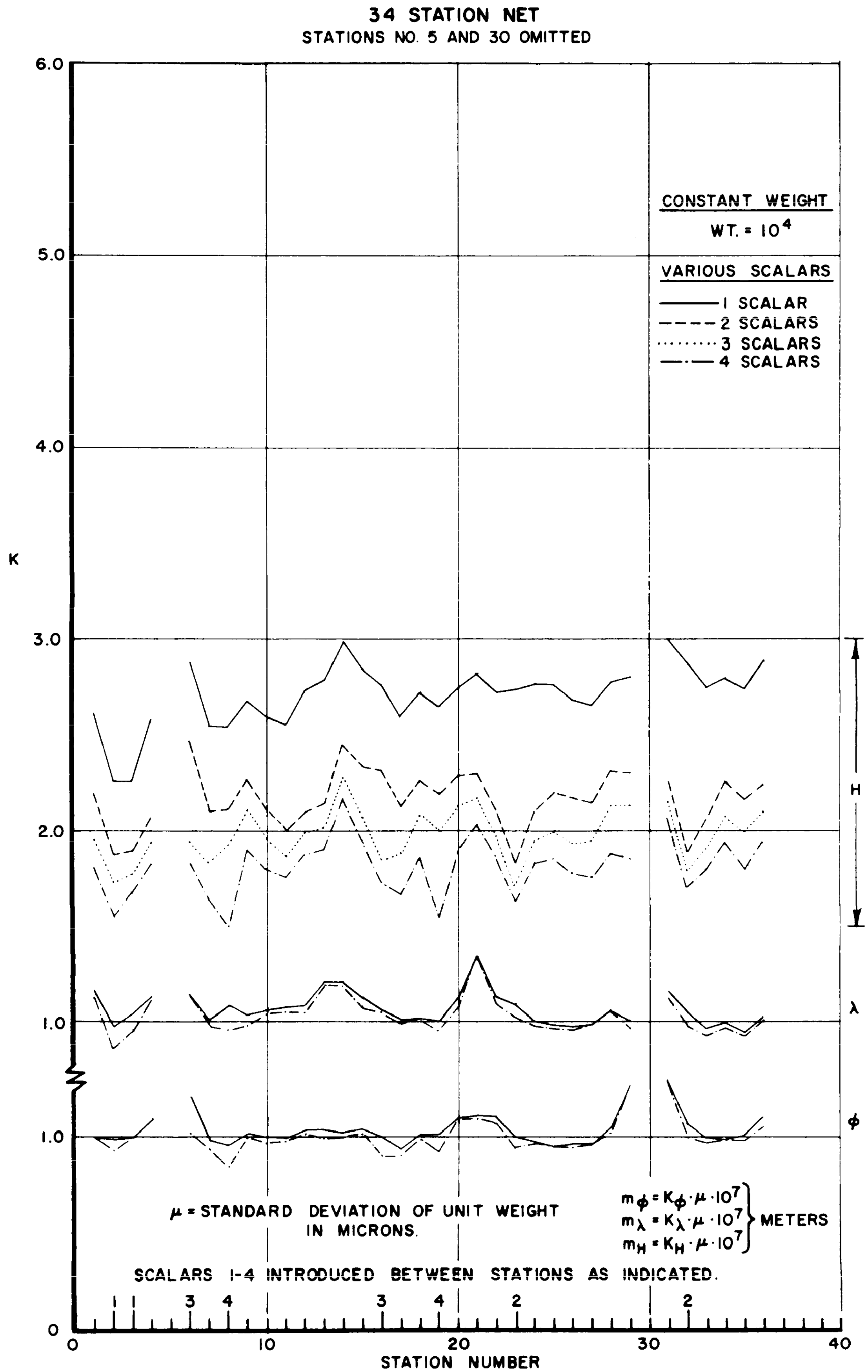

34 STATION NET
STATIONS NO. 5 AND 30 OMITTED
CONSTANT WEIGHT
WT. = 10^4
VARIOUS SCALARS
1 SCALAR
2 SCALARS
3 SCALARS
4 SCALARS
K
H
λ
φ
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
m_φ = K_φ · μ · 10^7
m_λ = K_λ · μ · 10^7
m_H = K_H · μ · 10^7
METERS
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED.
STATION NUMBER

FIGURE 18

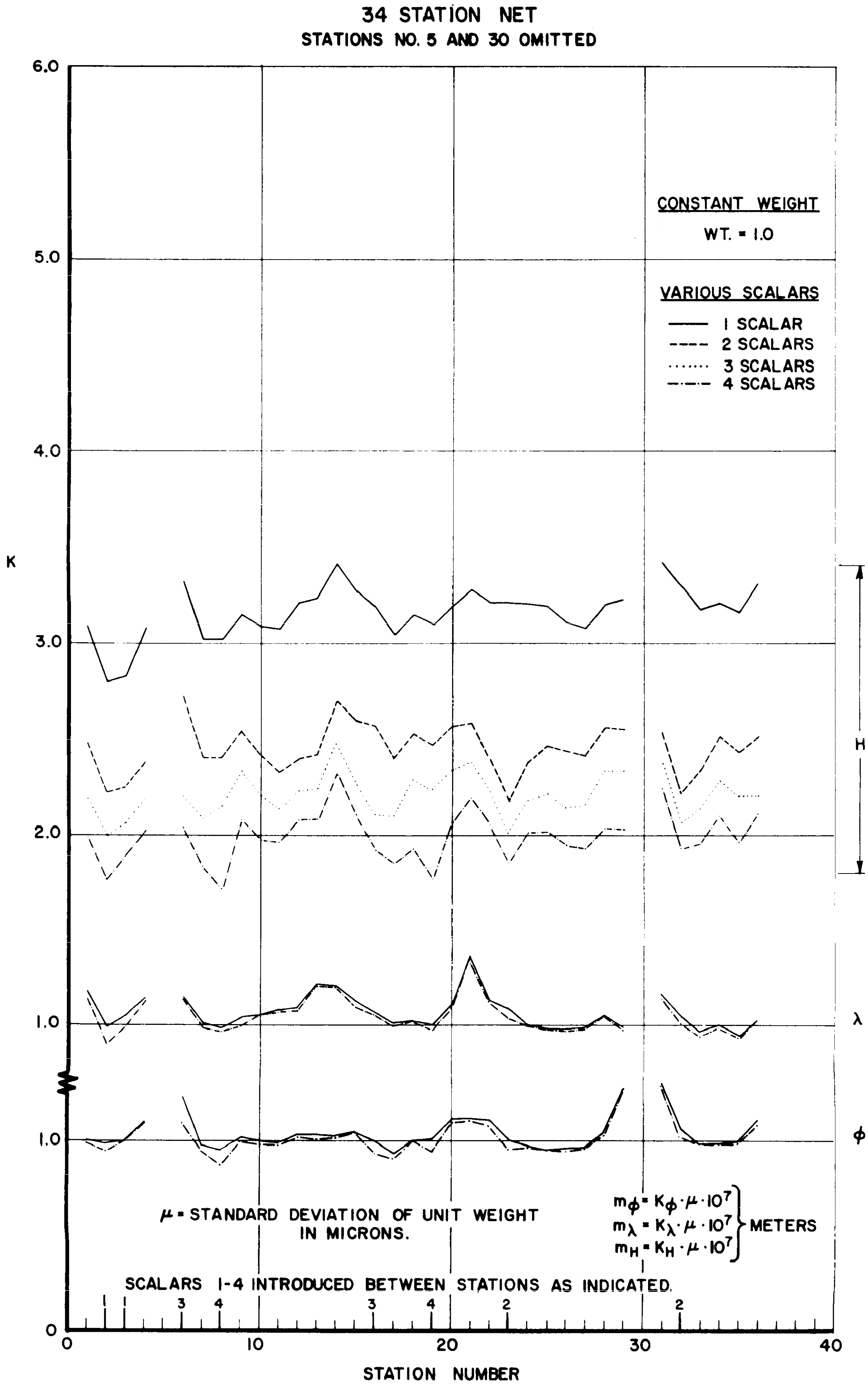

34 STATION NET
STATIONS NO. 5 AND 30 OMITTED
CONSTANT WEIGHT
WT. = 1.0
VARIOUS SCALARS
1 SCALAR
2 SCALARS
3 SCALARS
4 SCALARS
K
H
λ
φ
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
$m_\phi = K_\phi \cdot \mu \cdot 10^7$
$m_\lambda = K_\lambda \cdot \mu \cdot 10^7$
$m_H = K_H \cdot \mu \cdot 10^7$
METERS
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED.
STATION NUMBER

FIGURE 19

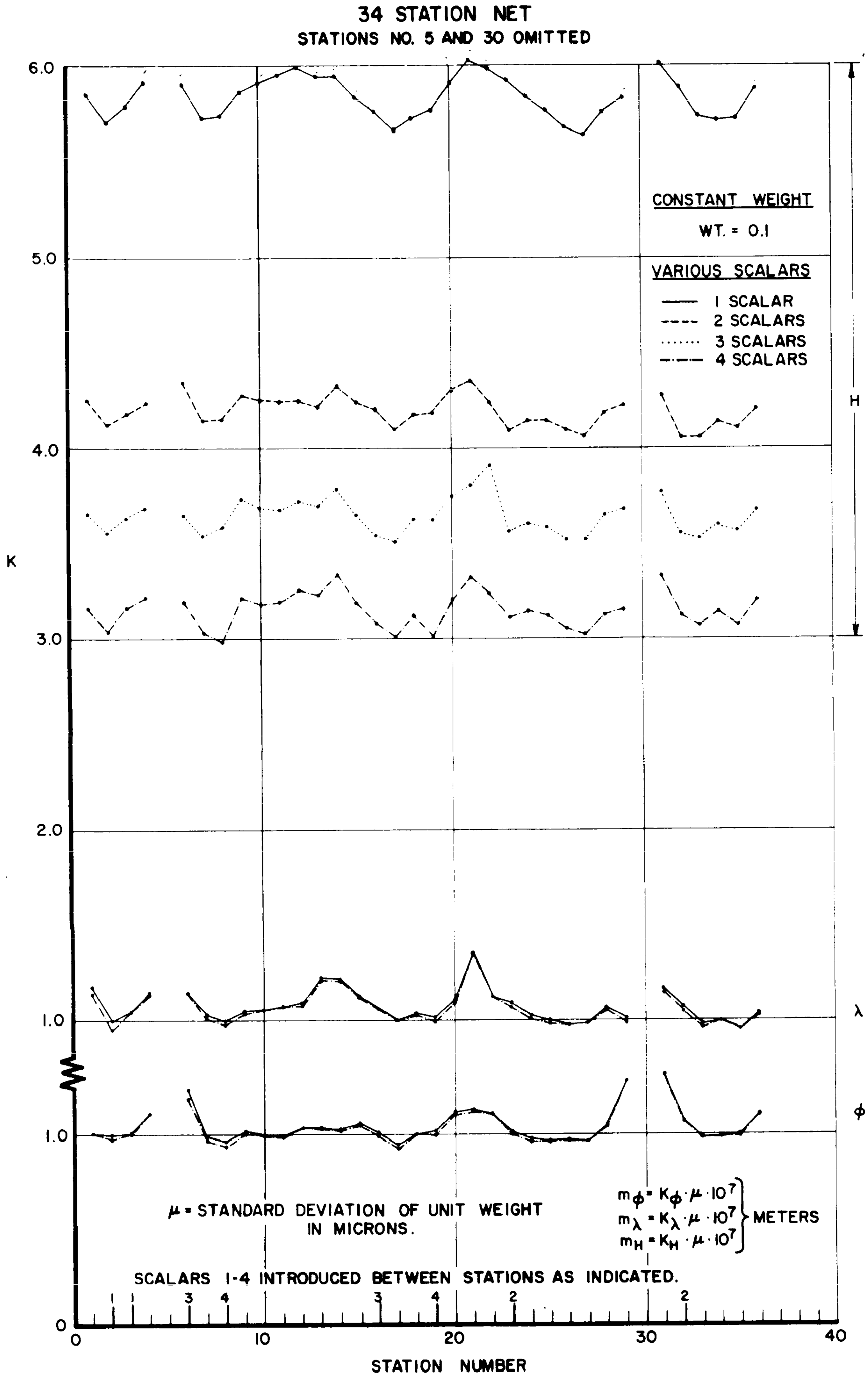

34 STATION NET
STATIONS NO. 5 AND 30 OMITTED
CONSTANT WEIGHT
WT. = 0.1
VARIOUS SCALARS
1 SCALAR
2 SCALARS
3 SCALARS
4 SCALARS
K
H
λ
φ
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
mφ = Kφ · μ · 10^7
mλ = Kλ · μ · 10^7
mH = KH · μ · 10^7
METERS
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED.
STATION NUMBER

FIGURE 20

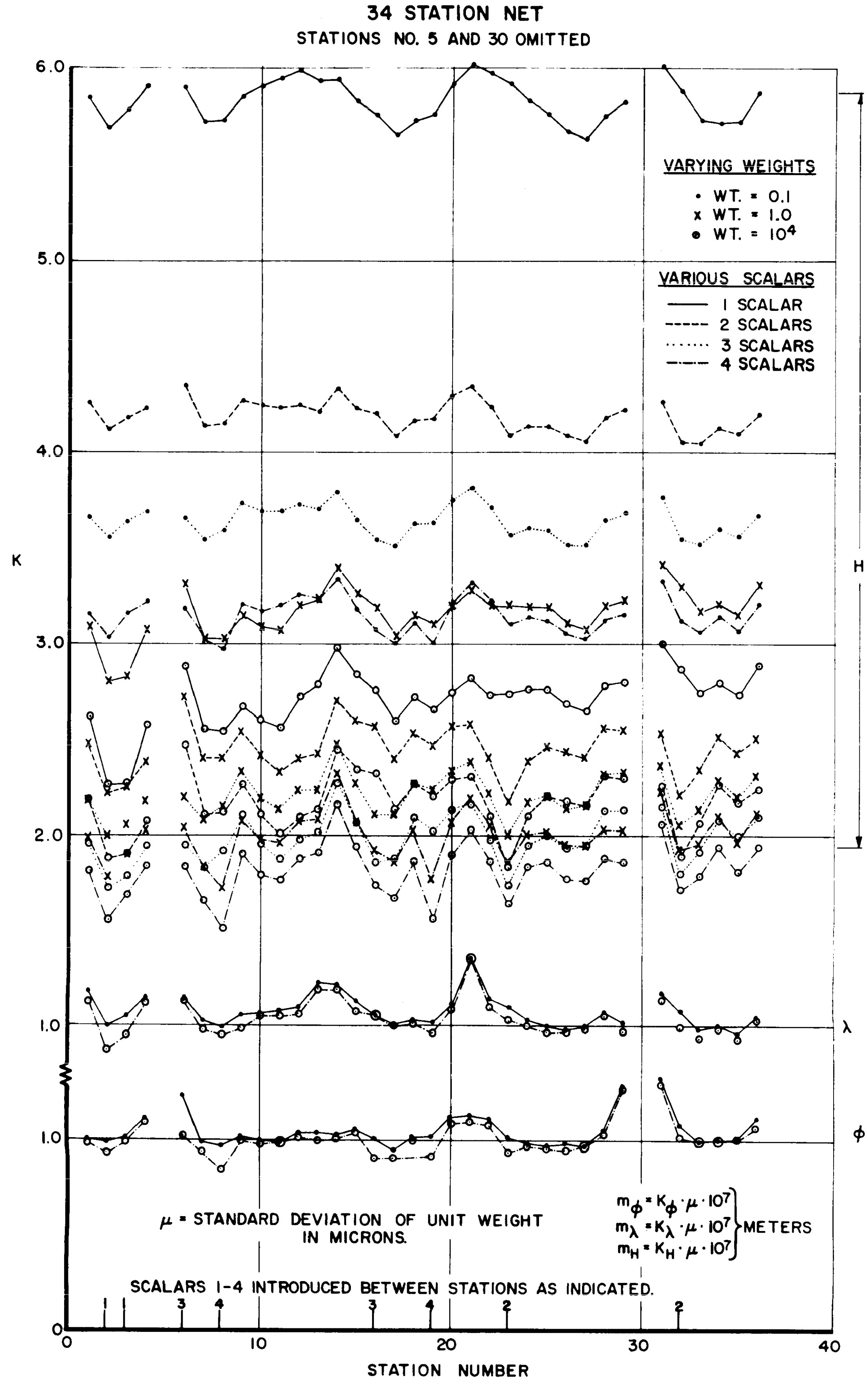

34 STATION NET
STATIONS NO. 5 AND 30 OMITTED
VARYING WEIGHTS
• WT. = 0.1
x WT. = 1.0
⊙ WT. = 10⁴
VARIOUS SCALARS
1 SCALAR
2 SCALARS
3 SCALARS
4 SCALARS
K
H
λ
φ
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
m_φ = K_φ · μ · 10⁷
m_λ = K_λ · μ · 10⁷
m_H = K_H · μ · 10⁷
METERS
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED.
STATION NUMBER

FIGURE 21

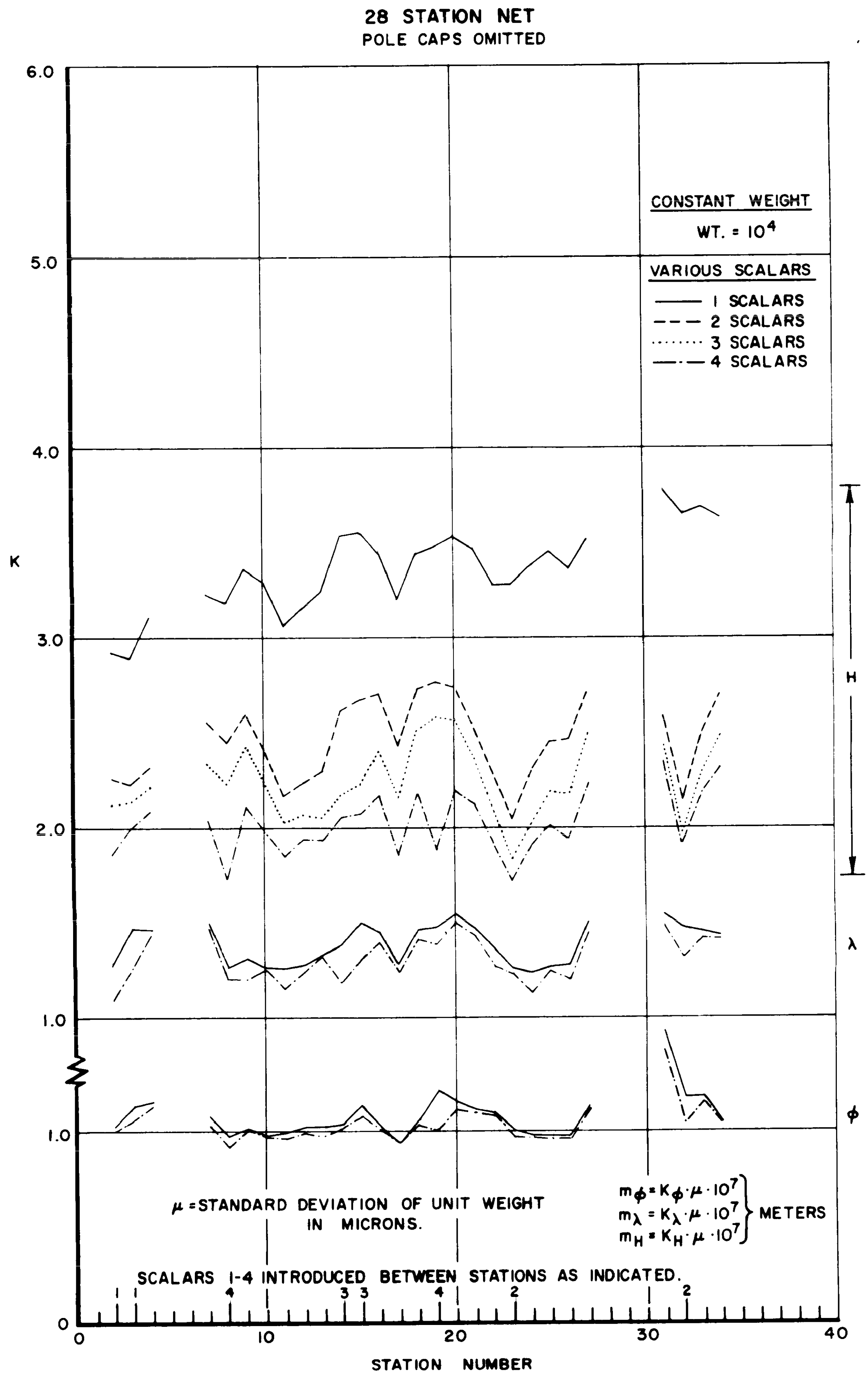

28 STATION NET
POLE CAPS OMITTED
CONSTANT WEIGHT
WT. = 10^4
VARIOUS SCALARS
1 SCALARS
2 SCALARS
3 SCALARS
4 SCALARS
K
H
λ
φ
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
$m_\phi = K_\phi \cdot \mu \cdot 10^7$
$m_\lambda = K_\lambda \cdot \mu \cdot 10^7$
$m_H = K_H \cdot \mu \cdot 10^7$
METERS
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED.
STATION NUMBER

FIGURE 22

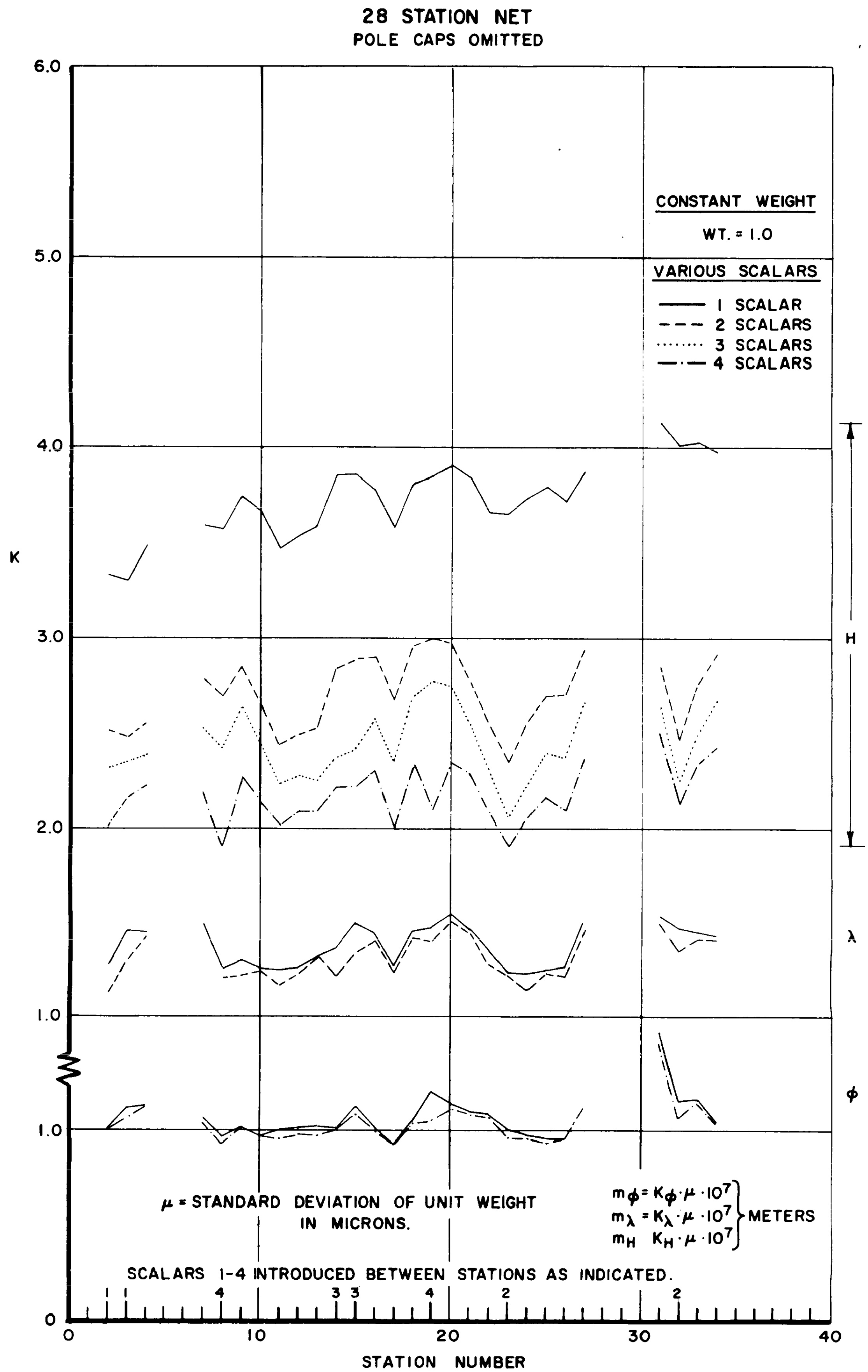

28 STATION NET
POLE CAPS OMITTED
CONSTANT WEIGHT
WT. = 1.0
VARIOUS SCALARS
1 SCALAR
2 SCALARS
3 SCALARS
4 SCALARS
K
H
λ
φ
6.0
5.0
4.0
3.0
2.0
1.0
1.0
0
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
$m_\phi = K_\phi \cdot \mu \cdot 10^7$
$m_\lambda = K_\lambda \cdot \mu \cdot 10^7$
$m_H \; K_H \cdot \mu \cdot 10^7$
METERS
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED.
1 1 4 3 3 4 2 2
0 10 20 30 40
STATION NUMBER

FIGURE 23

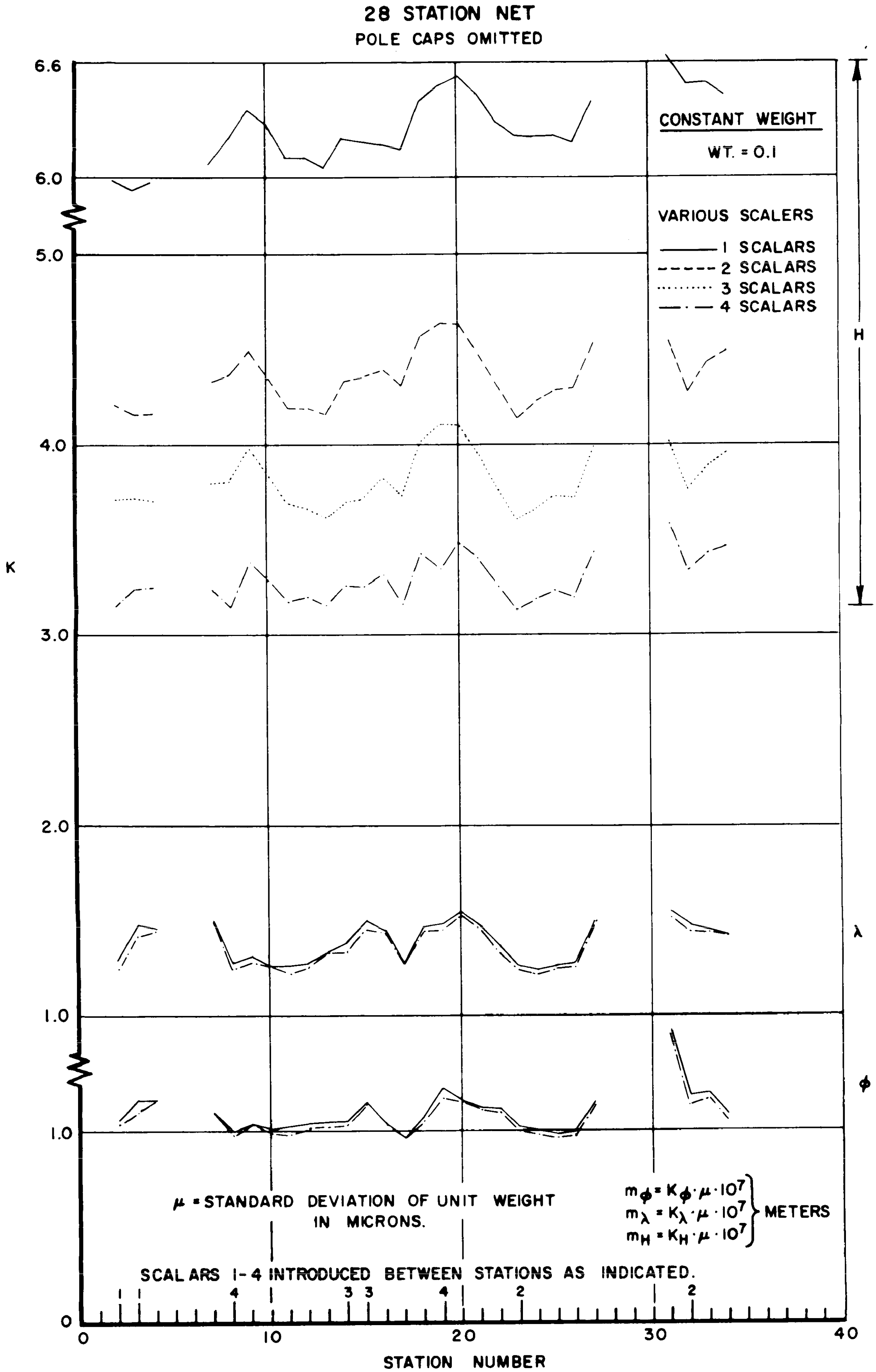

28 STATION NET
POLE CAPS OMITTED
CONSTANT WEIGHT
WT. = 0.1
VARIOUS SCALERS
1 SCALARS
2 SCALARS
3 SCALARS
4 SCALARS
H
λ
φ
K
6.6
6.0
5.0
4.0
3.0
2.0
1.0
1.0
0
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
$m_\phi = K_\phi \cdot \mu \cdot 10^7$
$m_\lambda = K_\lambda \cdot \mu \cdot 10^7$
$m_H = K_H \cdot \mu \cdot 10^7$
METERS
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED.
0
10
20
30
40
STATION NUMBER

FIGURE 24

28 STATION NET

POLE CAPS OMITTED

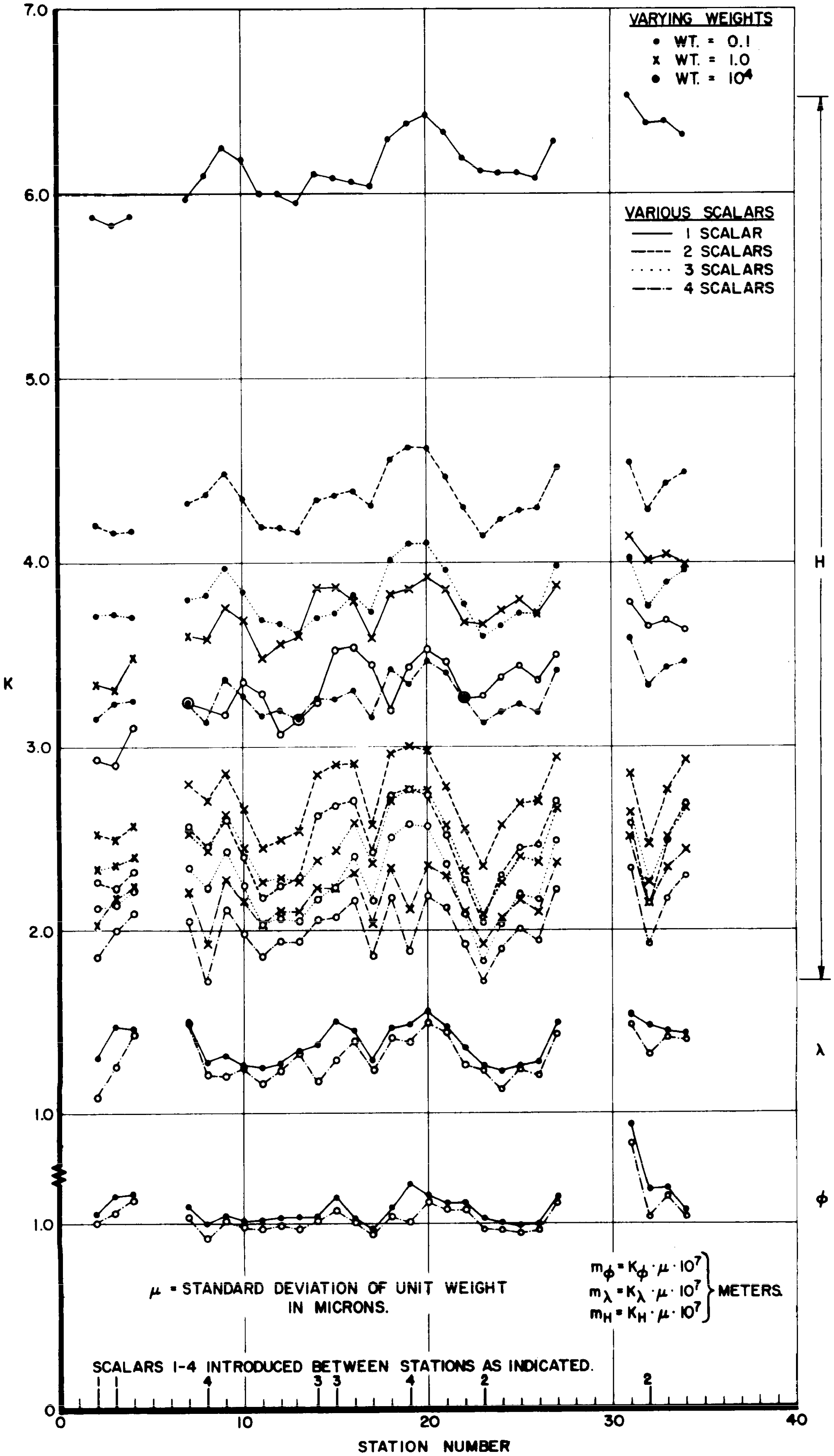


FIGURE 25

VARIOUS SIZE NETS

ONE SCALARS CONSTANT WT. = 10^4

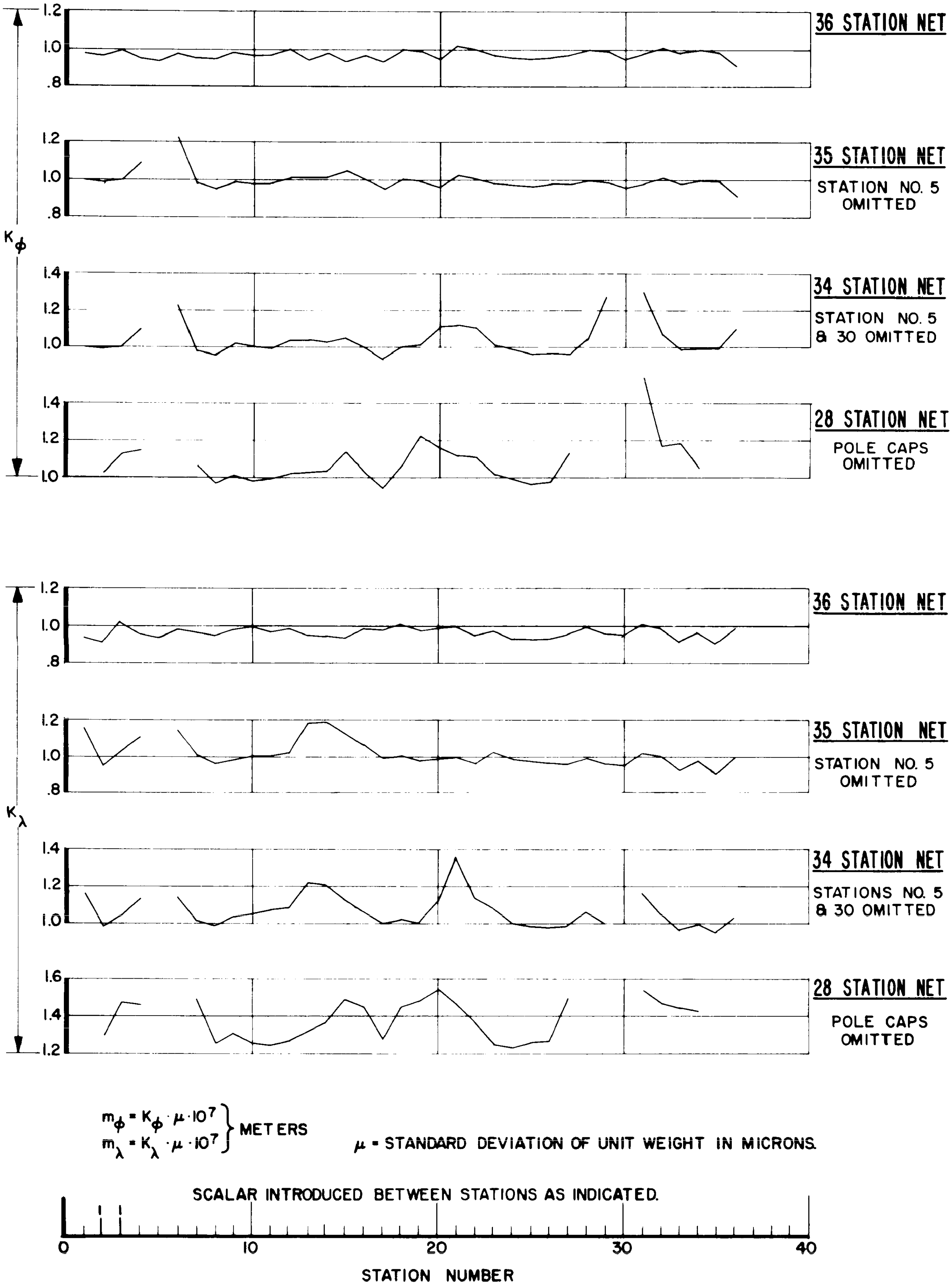
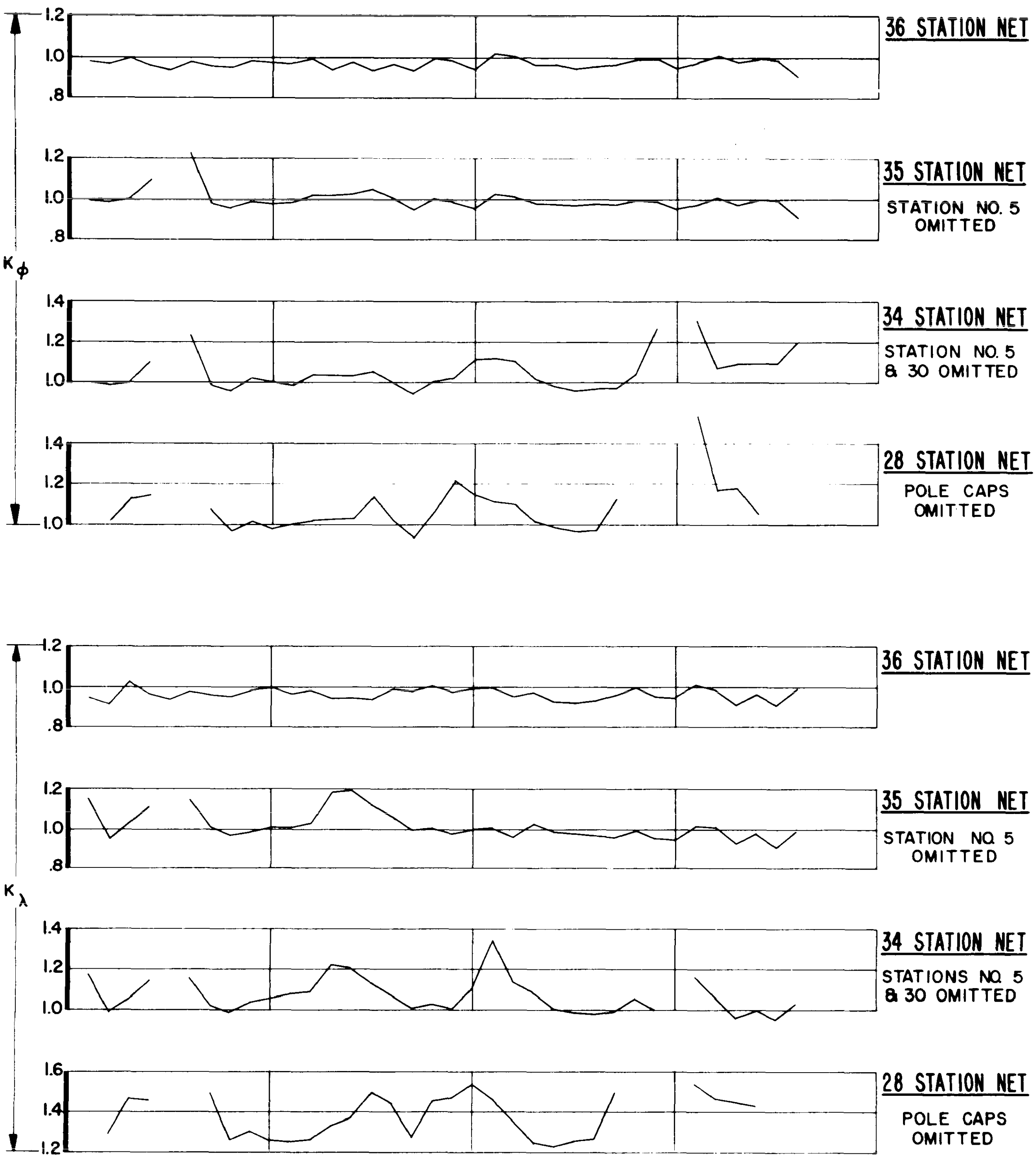


FIGURE 26

VARIOUS SIZE NETS

ONE SCALARS CONSTANT WT. = 1.0



$$\left.\begin{array}{l} m_\phi = K_\phi \cdot \mu \cdot 10^7 \\ m_\lambda = K_\lambda \cdot \mu \cdot 10^7 \end{array}\right\} \text{METERS}$$

μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.

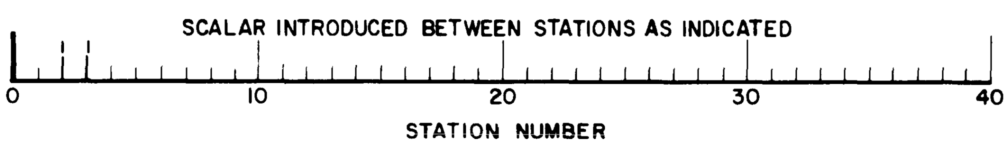


FIGURE 27

VARIOUS SIZE NETS

ONE SCALAR CONSTANT WT. = 0.1

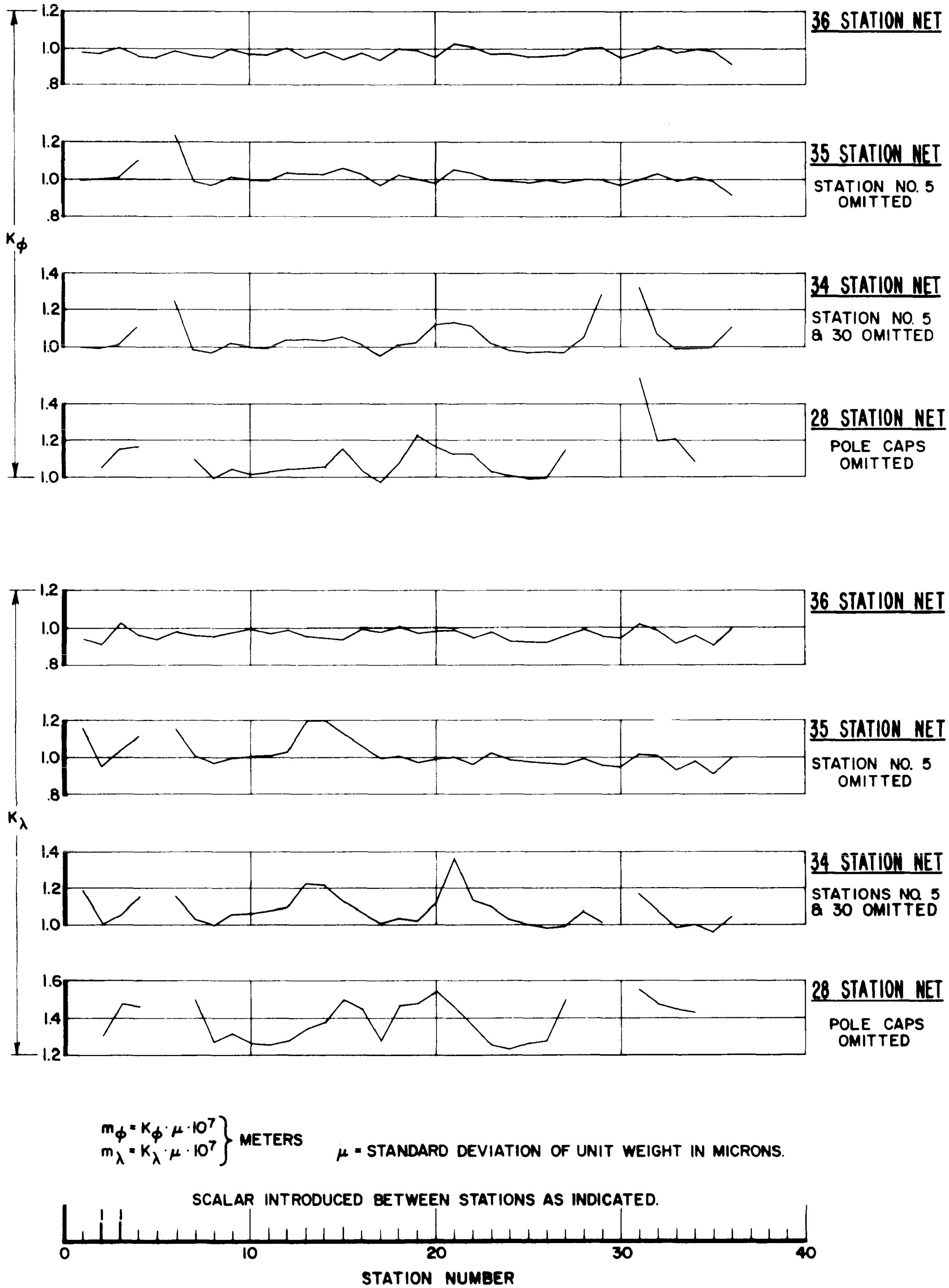
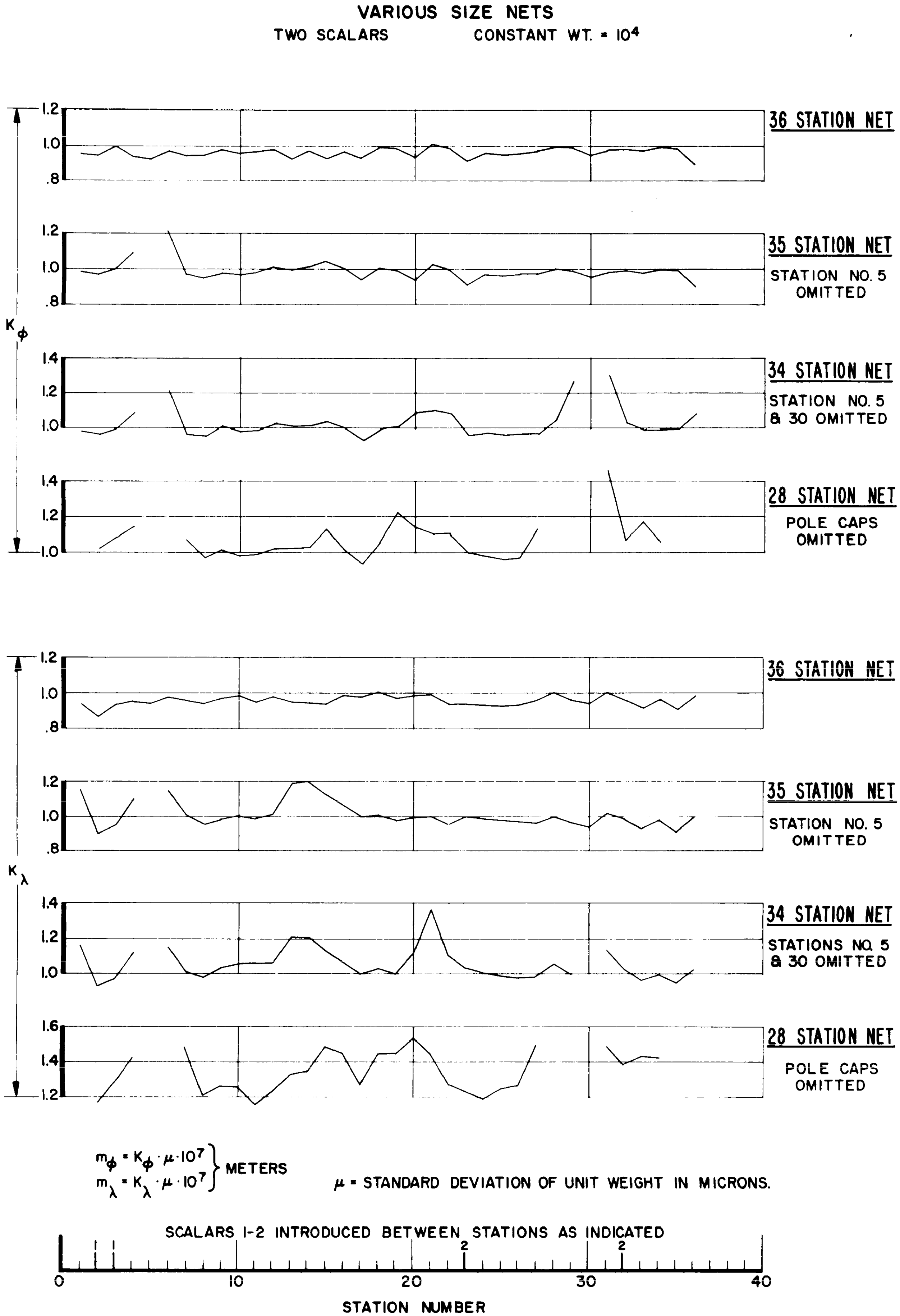


FIGURE 28



VARIOUS SIZE NETS
TWO SCALARS
CONSTANT WT. = 10⁴
36 STATION NET
35 STATION NET
STATION NO. 5 OMITTED
34 STATION NET
STATION NO. 5 & 30 OMITTED
28 STATION NET
POLE CAPS OMITTED
K φ
36 STATION NET
35 STATION NET
STATION NO. 5 OMITTED
34 STATION NET
STATIONS NO. 5 & 30 OMITTED
28 STATION NET
POLE CAPS OMITTED
K λ
mφ = Kφ · μ · 10⁷
mλ = Kλ · μ · 10⁷
METERS
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
SCALARS 1-2 INTRODUCED BETWEEN STATIONS AS INDICATED
STATION NUMBER

FIGURE 29

VARIOUS SIZE NETS

TWO SCALARS CONSTANT WT. = 1.0

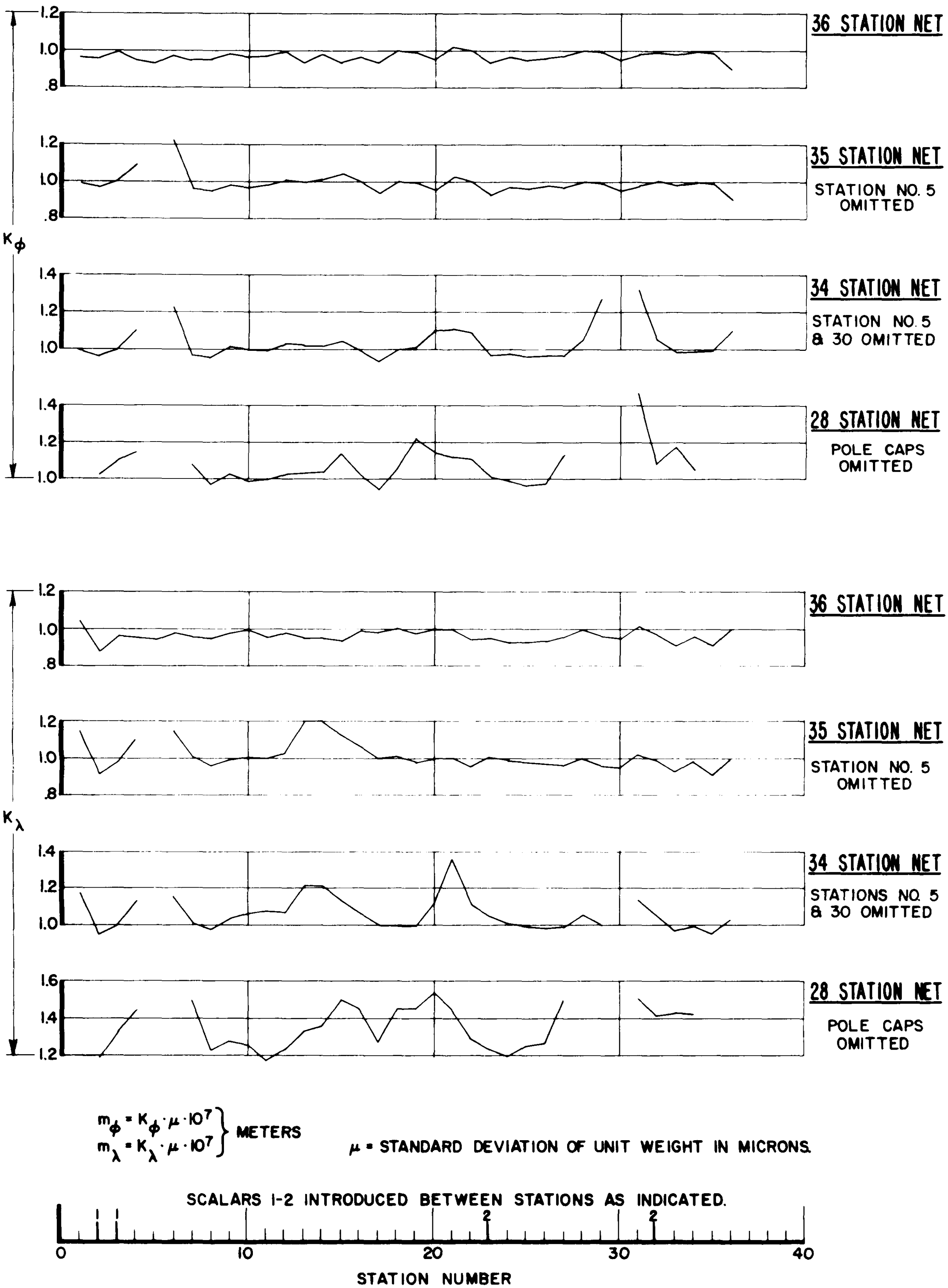


FIGURE 30

VARIOUS SIZE NETS

TWO SCALARS CONSTANT WT. = 0.1

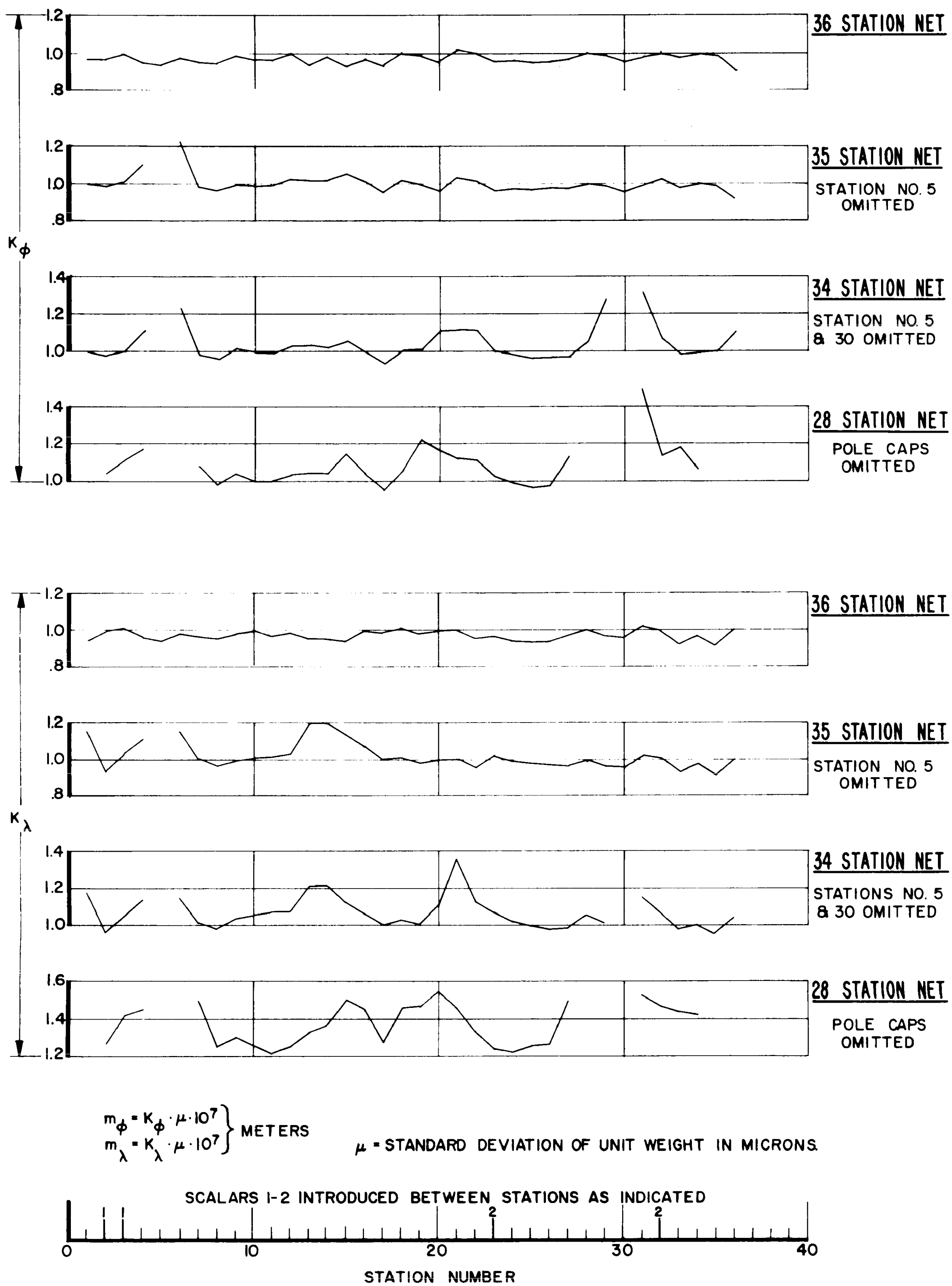


FIGURE 31

VARIOUS SIZE NETS

THREE SCALARS CONSTANT WT. = 10^4

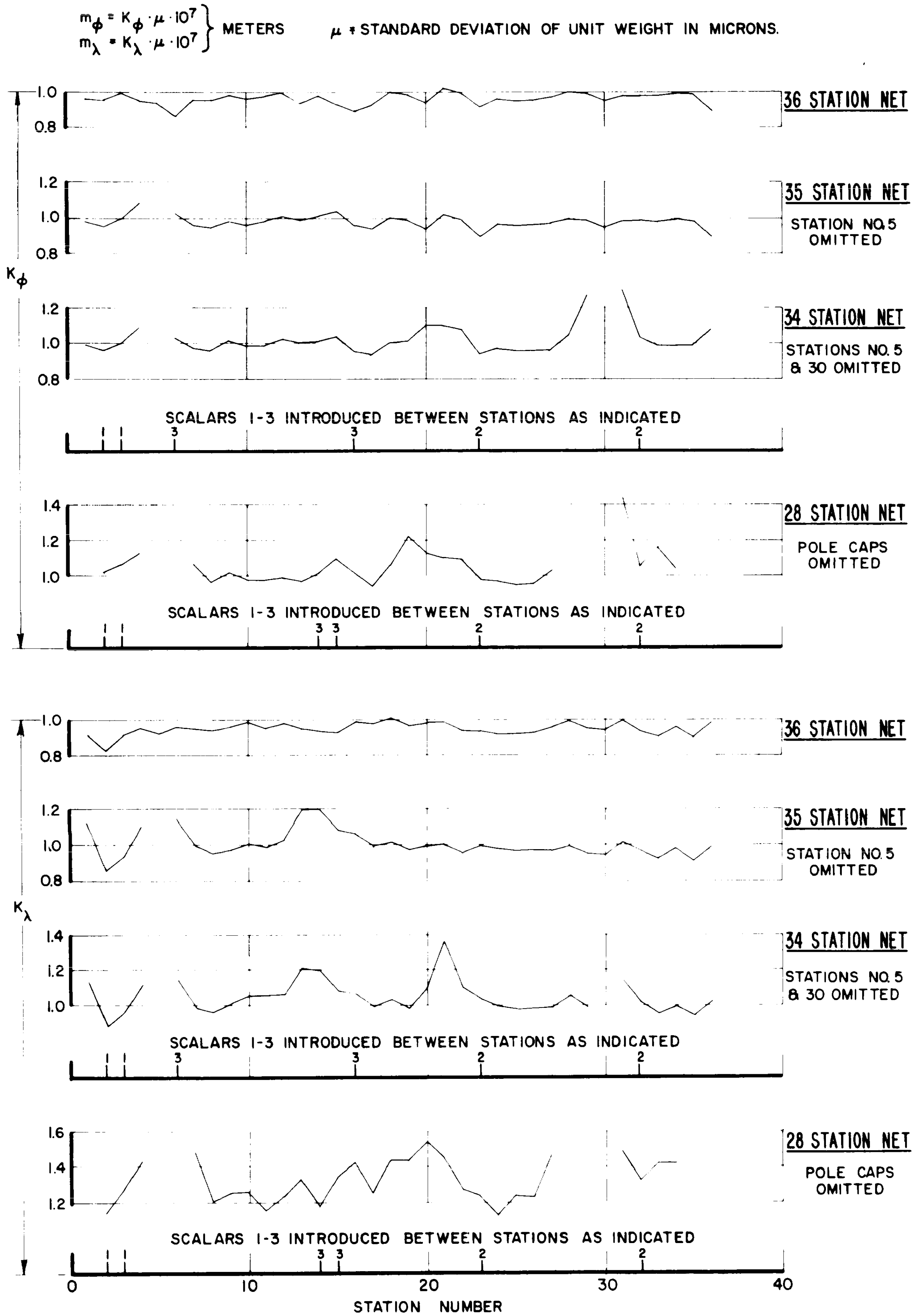


FIGURE 32

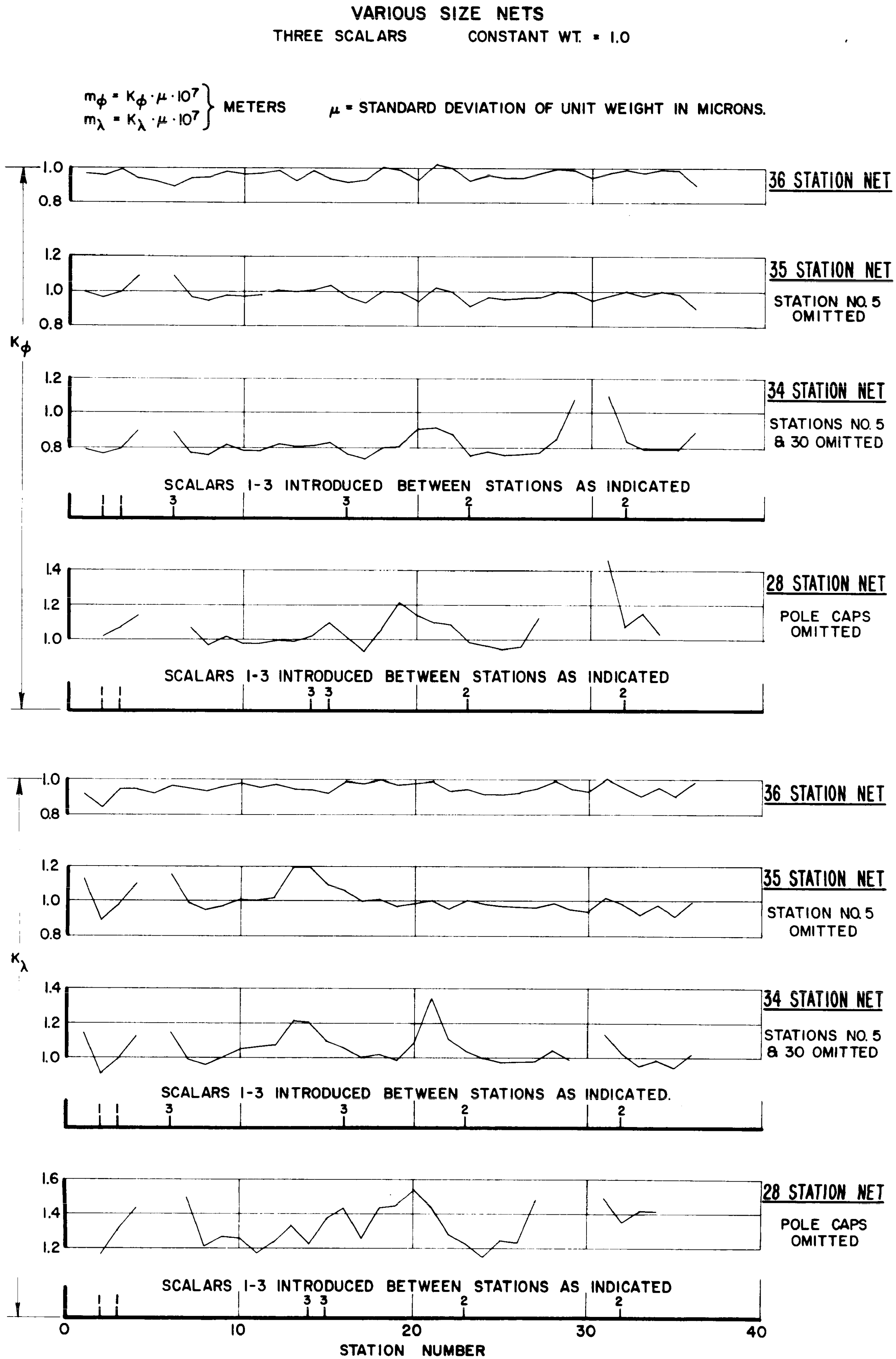

VARIOUS SIZE NETS
THREE SCALARS CONSTANT WT. = 1.0
$m_\phi = K_\phi \cdot \mu \cdot 10^7$
$m_\lambda = K_\lambda \cdot \mu \cdot 10^7$
METERS
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
K_ϕ
36 STATION NET
35 STATION NET
STATION NO. 5 OMITTED
34 STATION NET
STATIONS NO. 5 & 30 OMITTED
SCALARS 1-3 INTRODUCED BETWEEN STATIONS AS INDICATED
28 STATION NET
POLE CAPS OMITTED
SCALARS 1-3 INTRODUCED BETWEEN STATIONS AS INDICATED
K_λ
36 STATION NET
35 STATION NET
STATION NO. 5 OMITTED
34 STATION NET
STATIONS NO. 5 & 30 OMITTED
SCALARS 1-3 INTRODUCED BETWEEN STATIONS AS INDICATED.
28 STATION NET
POLE CAPS OMITTED
SCALARS 1-3 INTRODUCED BETWEEN STATIONS AS INDICATED
STATION NUMBER

FIGURE 33

VARIOUS SIZE NETS

THREE SCALARS CONSTANT WT. = 0.1

$\left.\begin{array}{l} m_\phi = K_\phi \cdot \mu \cdot 10^7 \\ m_\lambda = K_\lambda \cdot \mu \cdot 10^7 \end{array}\right\}$ METERS μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.

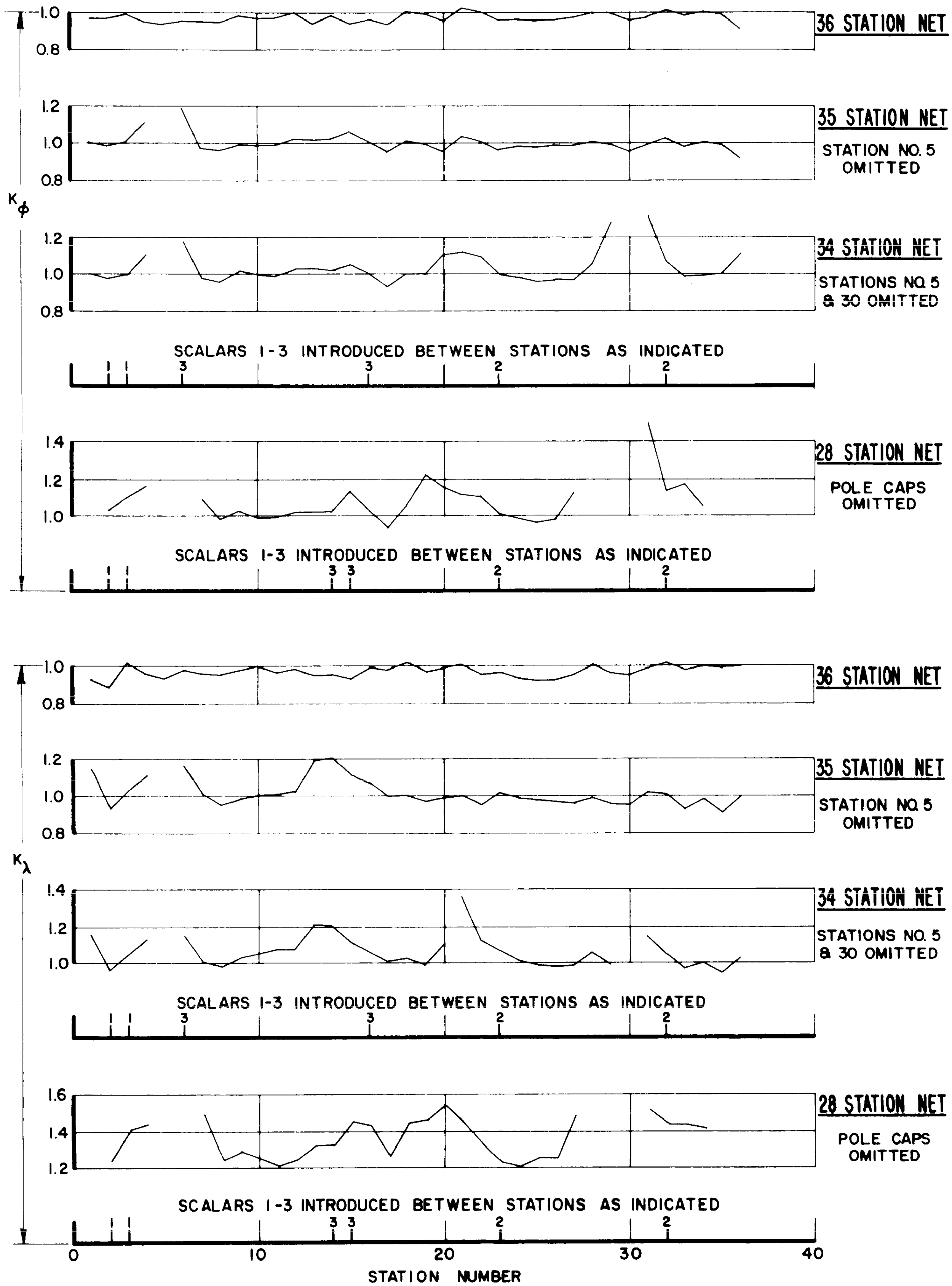


FIGURE 34

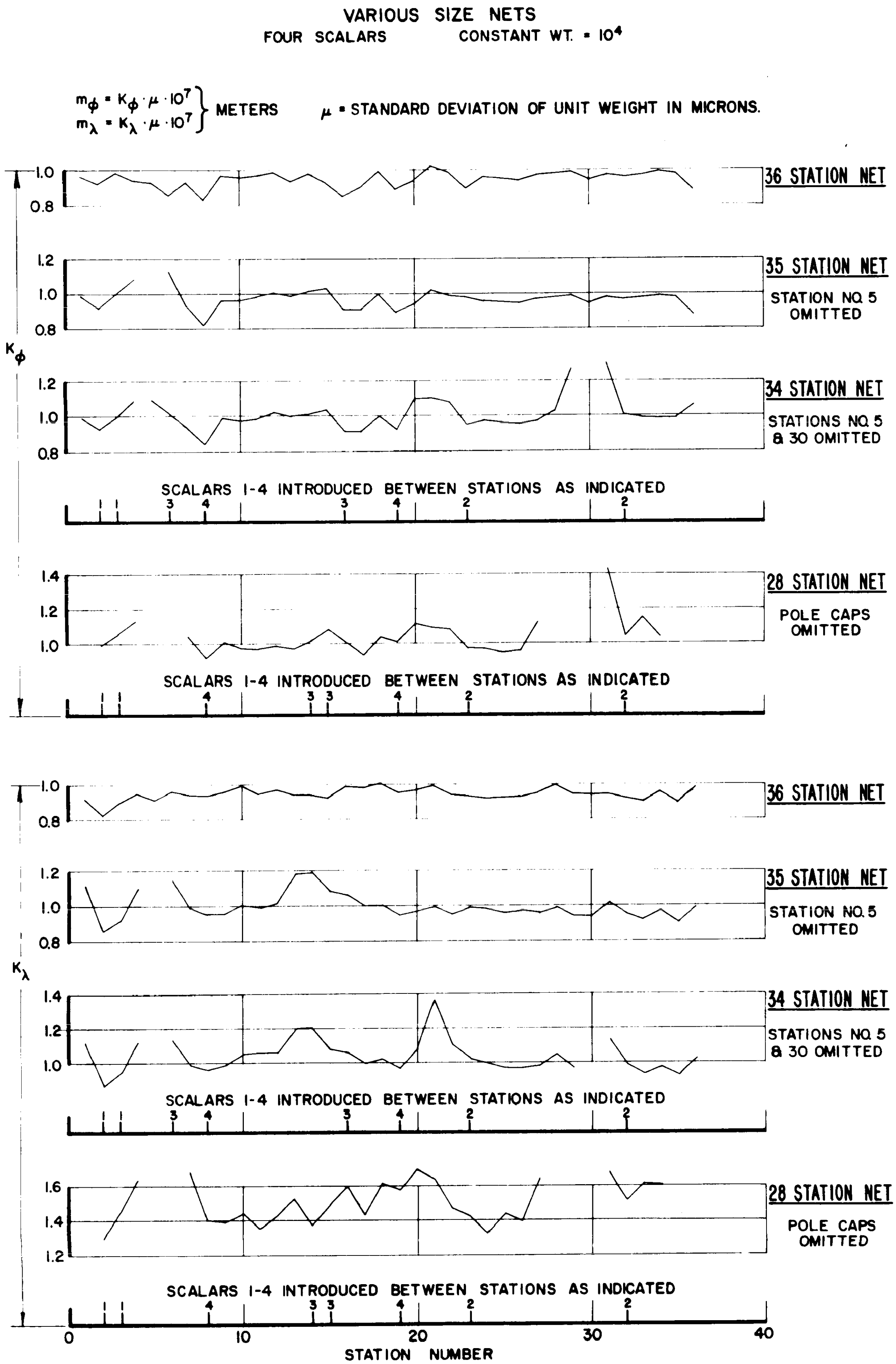

VARIOUS SIZE NETS
FOUR SCALARS
CONSTANT WT. = 10^4
$m_\phi = K_\phi \cdot \mu \cdot 10^7$
$m_\lambda = K_\lambda \cdot \mu \cdot 10^7$
METERS
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.
K_ϕ
36 STATION NET
35 STATION NET
STATION NO. 5 OMITTED
34 STATION NET
STATIONS NO. 5 & 30 OMITTED
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED
28 STATION NET
POLE CAPS OMITTED
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED
K_λ
36 STATION NET
35 STATION NET
STATION NO. 5 OMITTED
34 STATION NET
STATIONS NO. 5 & 30 OMITTED
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED
28 STATION NET
POLE CAPS OMITTED
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED
STATION NUMBER

FIGURE 35

VARIOUS SIZE NETS

FOUR SCALARS CONSTANT WT. = 1.0

$$\left.\begin{array}{l} m_\phi = K_\phi \cdot \mu \cdot 10^7 \\ m_\lambda = K_\lambda \cdot \mu \cdot 10^7 \end{array}\right\} \text{METERS} \qquad \mu = \text{STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS.}$$

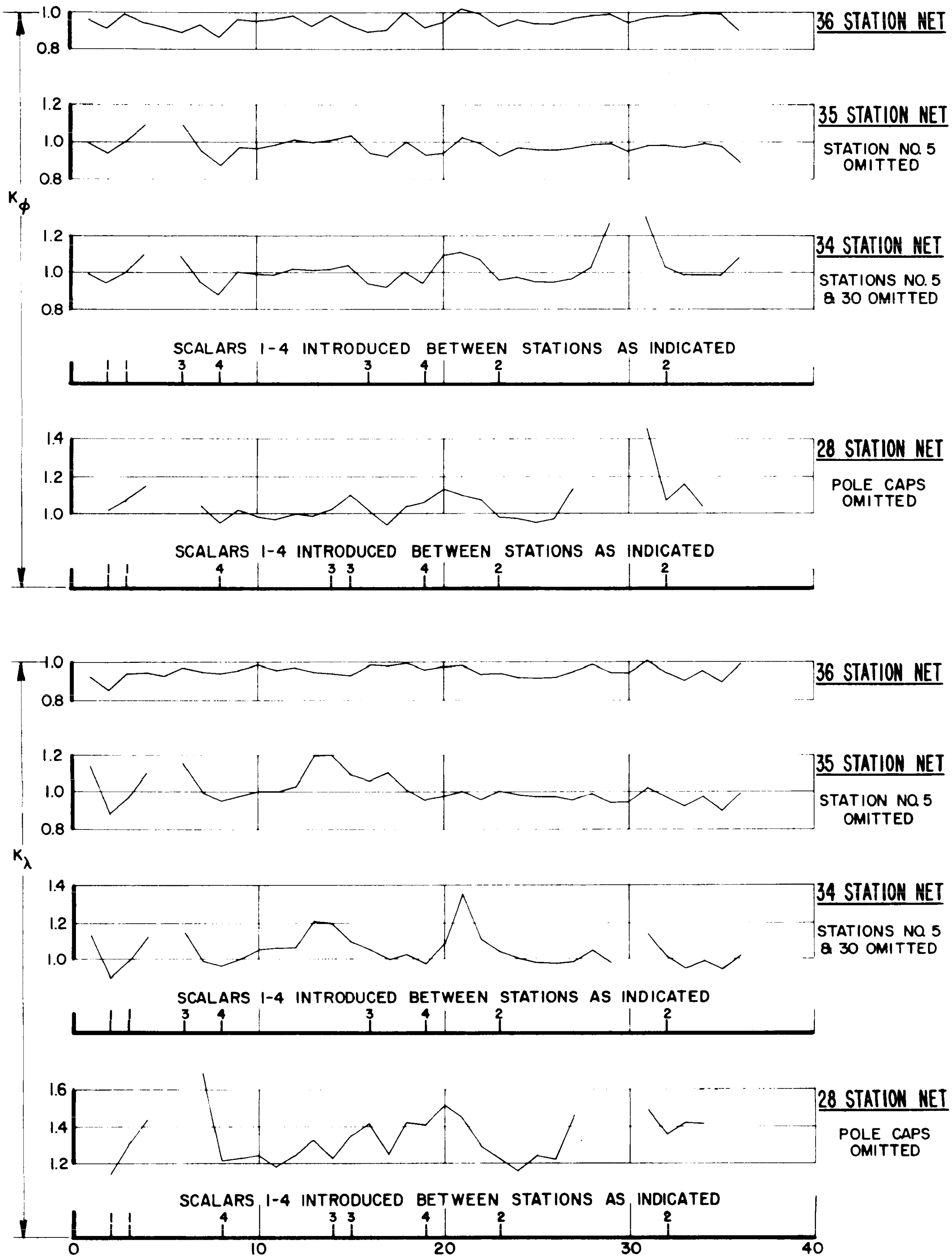


FIGURE 36

VARIOUS SIZE NETS

FOUR SCALARS CONSTANT WT. = 0.1

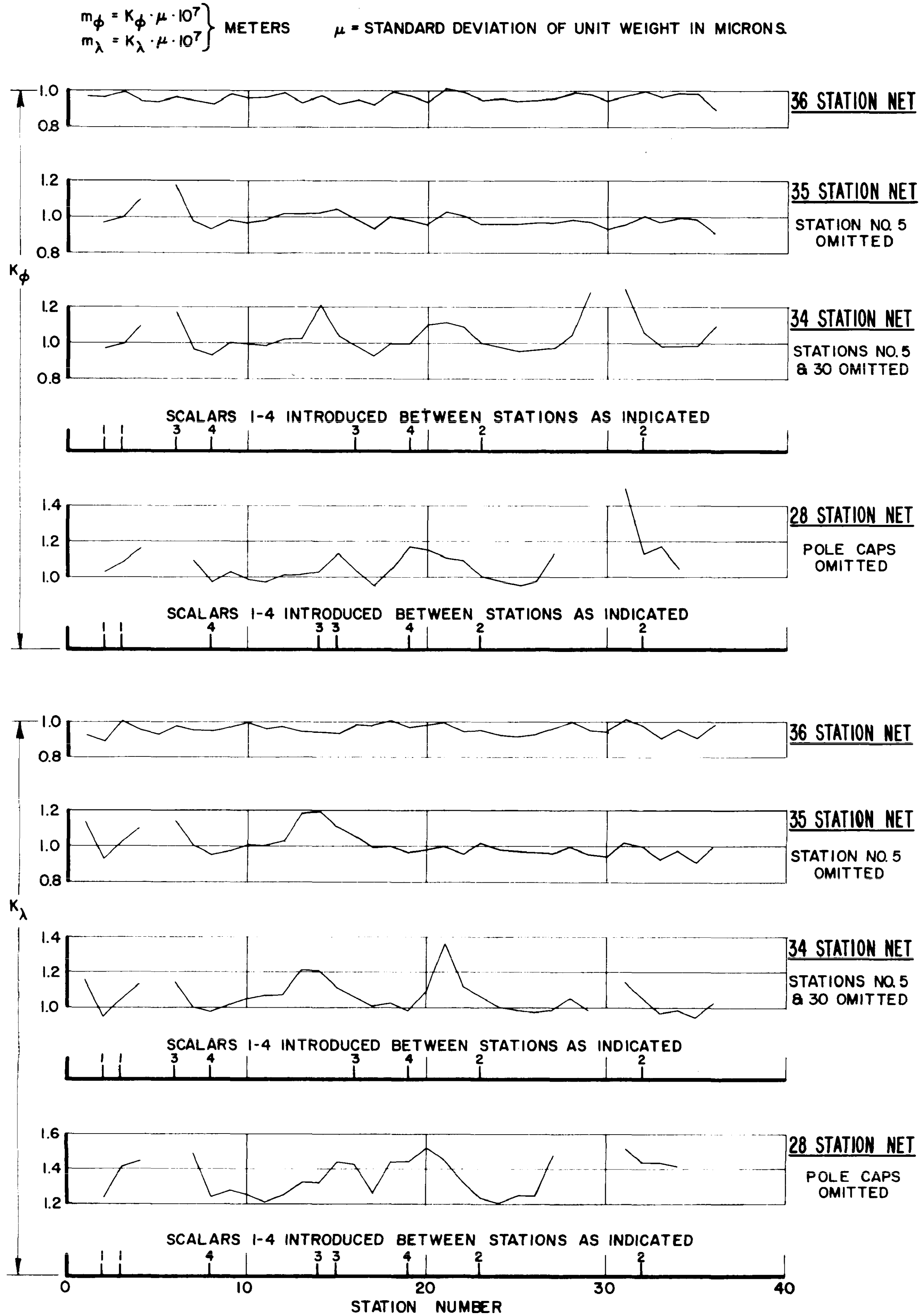
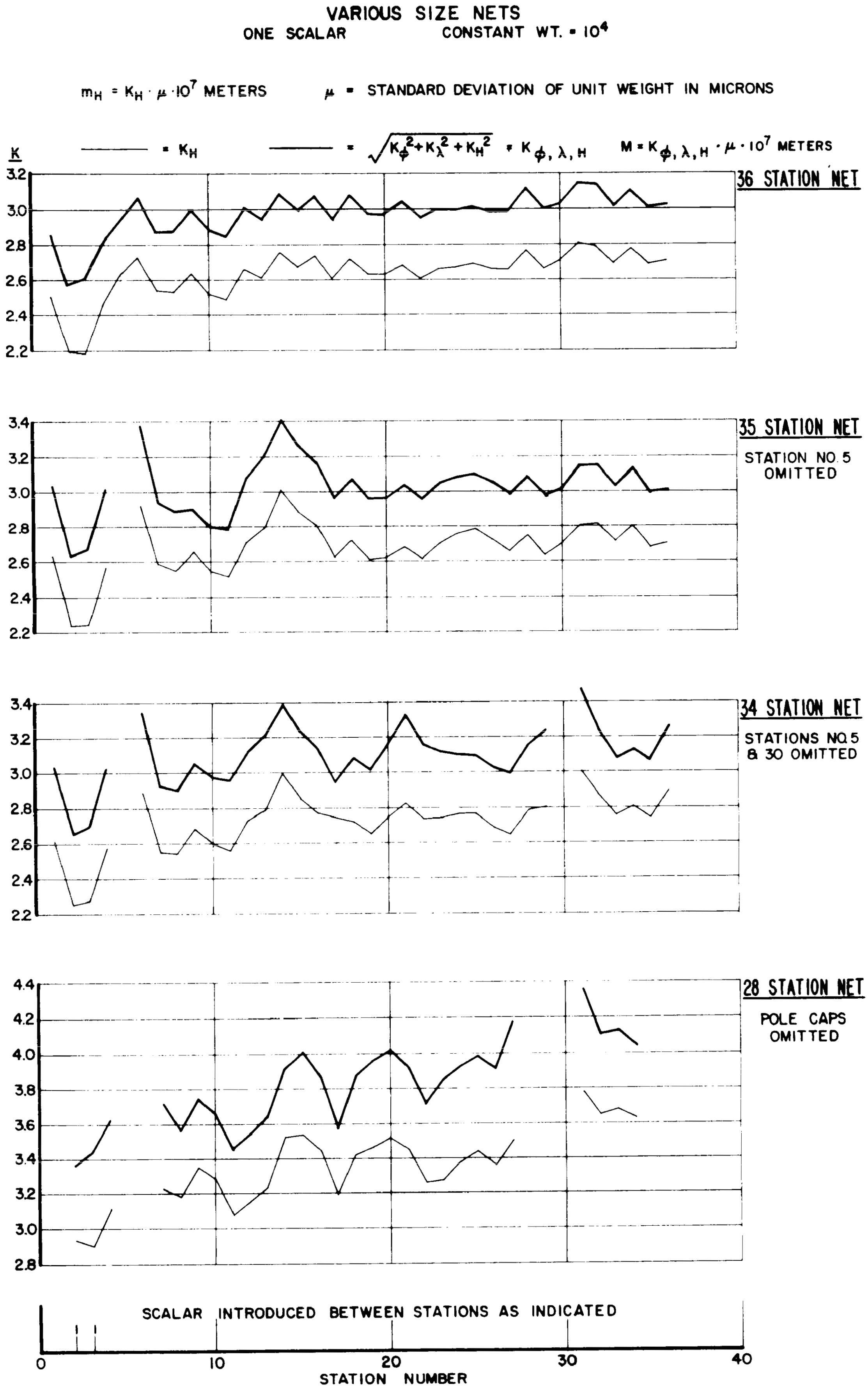


FIGURE 37



VARIOUS SIZE NETS
ONE SCALAR
CONSTANT WT. = 10⁴
mH = KH · μ · 10⁷ METERS
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS
= KH
= √(Kφ² + Kλ² + KH²) = Kφ, λ, H
M = Kφ, λ, H · μ · 10⁷ METERS
K
36 STATION NET
35 STATION NET
STATION NO. 5 OMITTED
34 STATION NET
STATIONS NO. 5 & 30 OMITTED
28 STATION NET
POLE CAPS OMITTED
SCALAR INTRODUCED BETWEEN STATIONS AS INDICATED
STATION NUMBER

FIGURE 38

VARIOUS SIZE NETS
ONE SCALAR CONSTANT WT. = 1.0

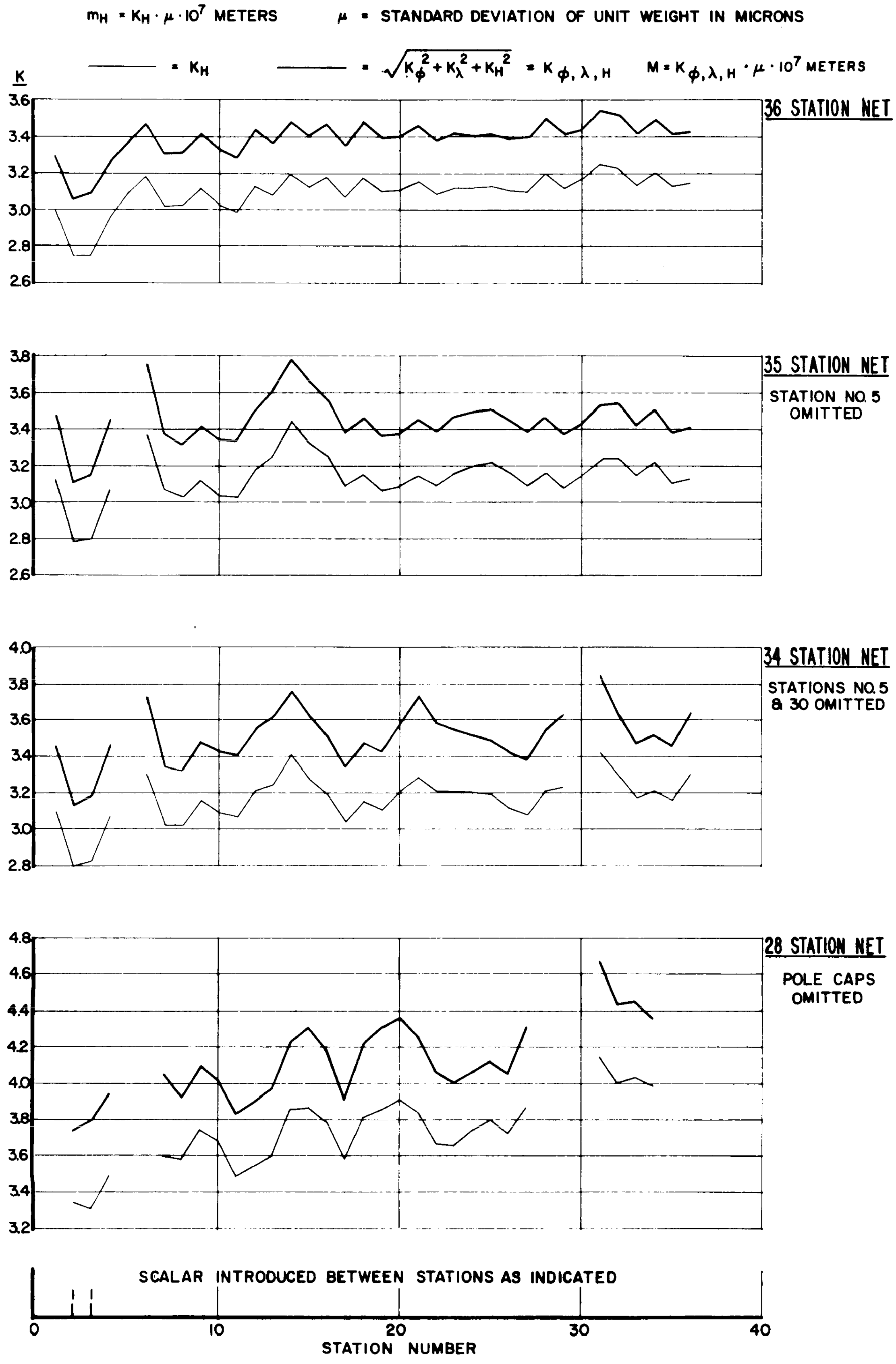


FIGURE 39

VARIOUS SIZE NETS

ONE SCALAR CONSTANT WT. = 0.1

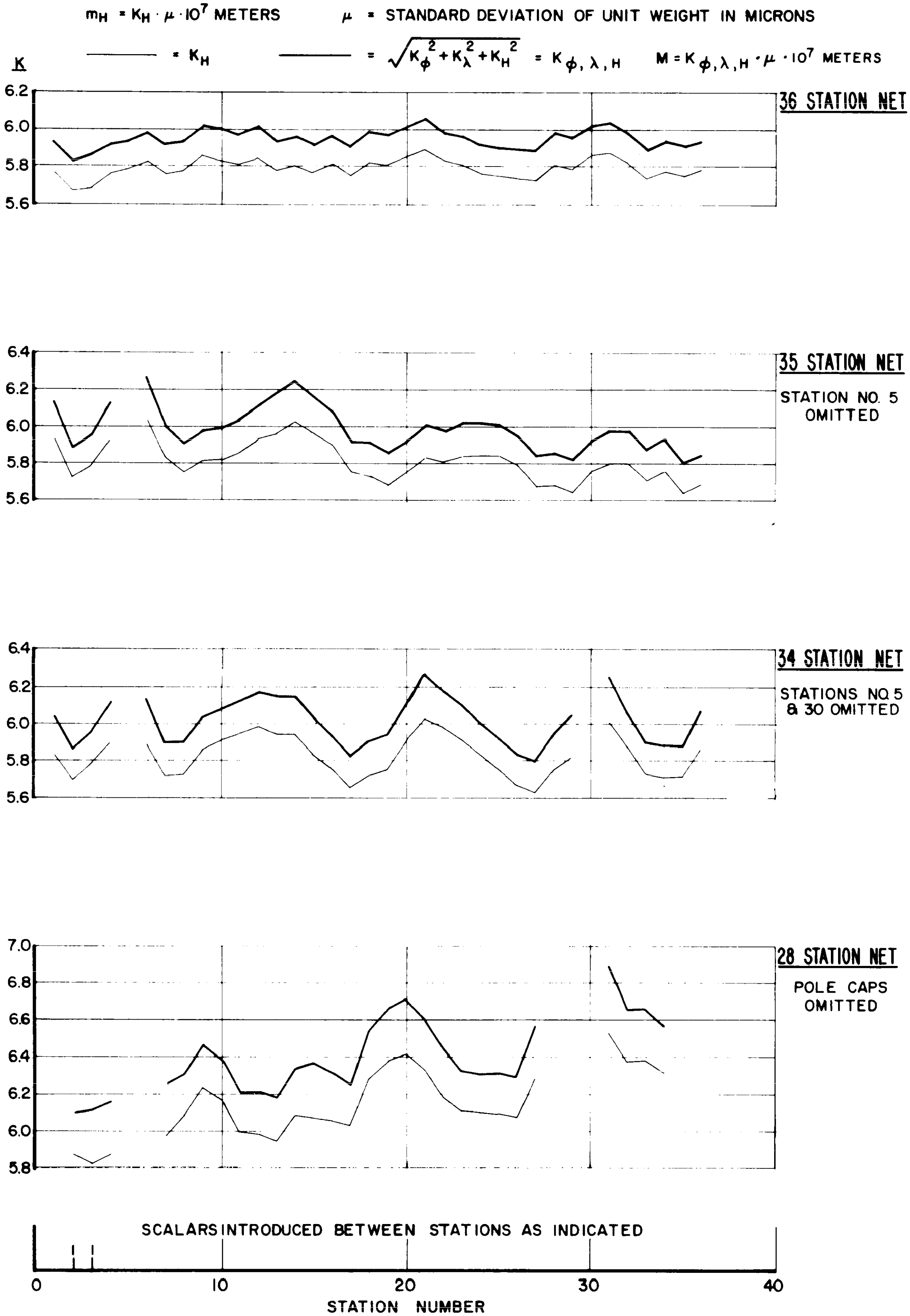


FIGURE 40

VARIOUS SIZE NETS
TWO SCALARS CONSTANT WT. = 10^4

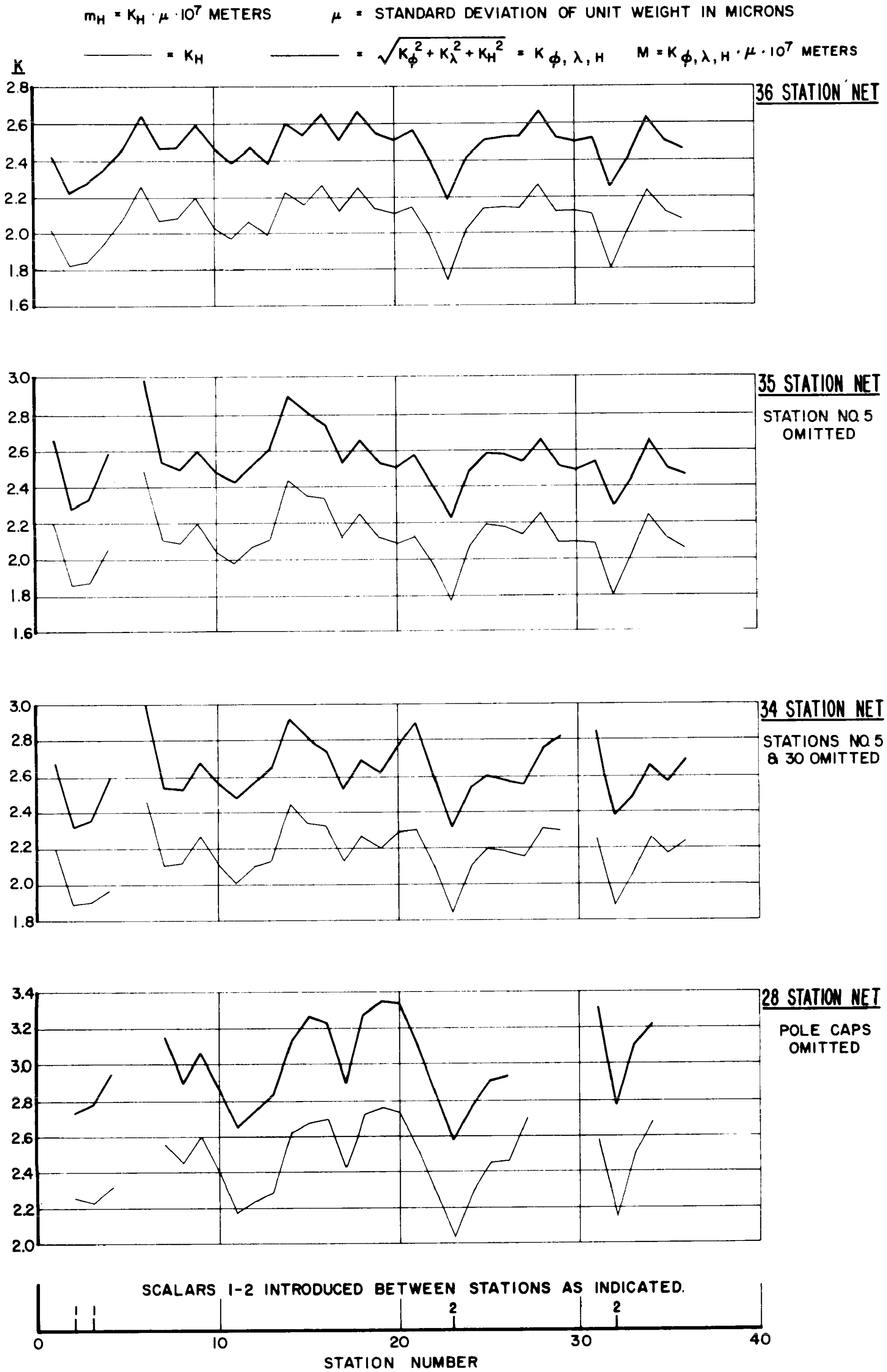


FIGURE 41

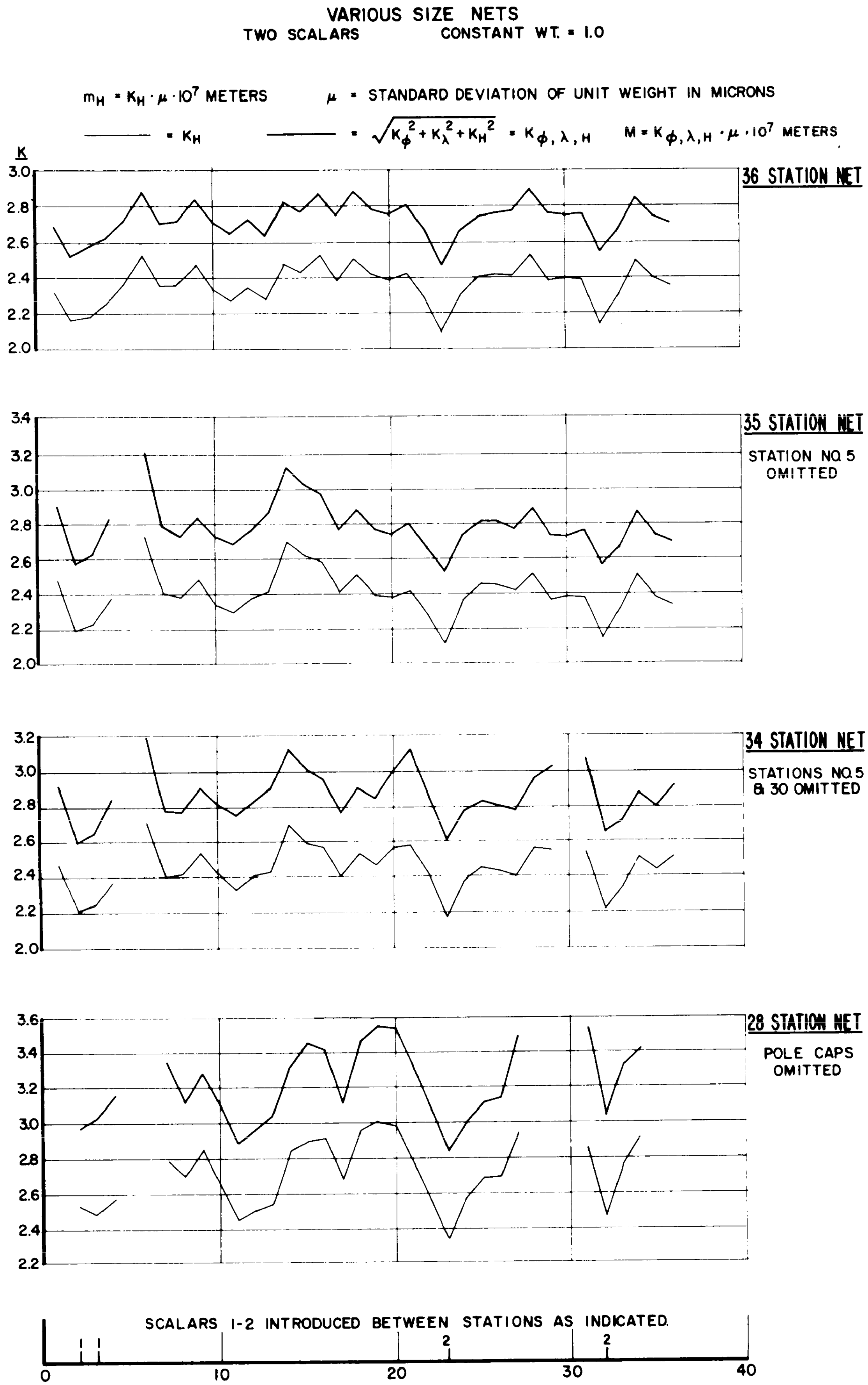
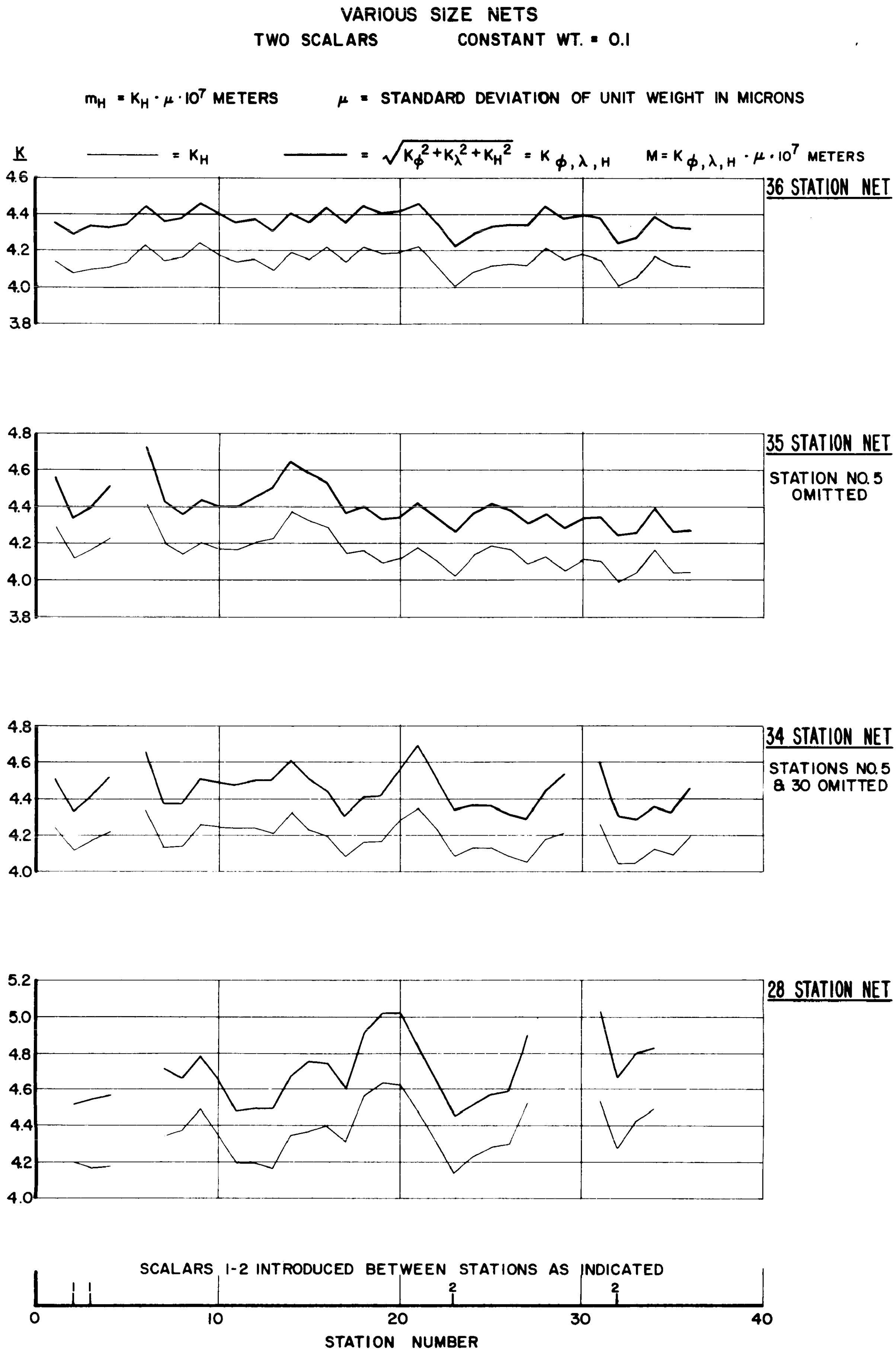

VARIOUS SIZE NETS
TWO SCALARS CONSTANT WT. = 1.0
$m_H = K_H \cdot \mu \cdot 10^7$ METERS
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS
$= K_H$
$= \sqrt{K_\phi^2 + K_\lambda^2 + K_H^2} = K_{\phi,\lambda,H}$
$M = K_{\phi,\lambda,H} \cdot \mu \cdot 10^7$ METERS
K
36 STATION NET
35 STATION NET
STATION NO. 5 OMITTED
34 STATION NET
STATIONS NO. 5 & 30 OMITTED
28 STATION NET
POLE CAPS OMITTED
SCALARS 1-2 INTRODUCED BETWEEN STATIONS AS INDICATED.
STATION NUMBER

FIGURE 42



VARIOUS SIZE NETS
TWO SCALARS CONSTANT WT. = 0.1
$m_H = K_H \cdot \mu \cdot 10^7$ METERS
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS
K
$= K_H$
$= \sqrt{K_\phi^2 + K_\lambda^2 + K_H^2} = K_{\phi,\lambda,H}$
$M = K_{\phi,\lambda,H} \cdot \mu \cdot 10^7$ METERS
36 STATION NET
35 STATION NET
STATION NO. 5 OMITTED
34 STATION NET
STATIONS NO. 5 & 30 OMITTED
28 STATION NET
SCALARS 1-2 INTRODUCED BETWEEN STATIONS AS INDICATED
STATION NUMBER

FIGURE 43

VARIOUS SIZE NETS

THREE SCALARS CONSTANT WT. = 10^4

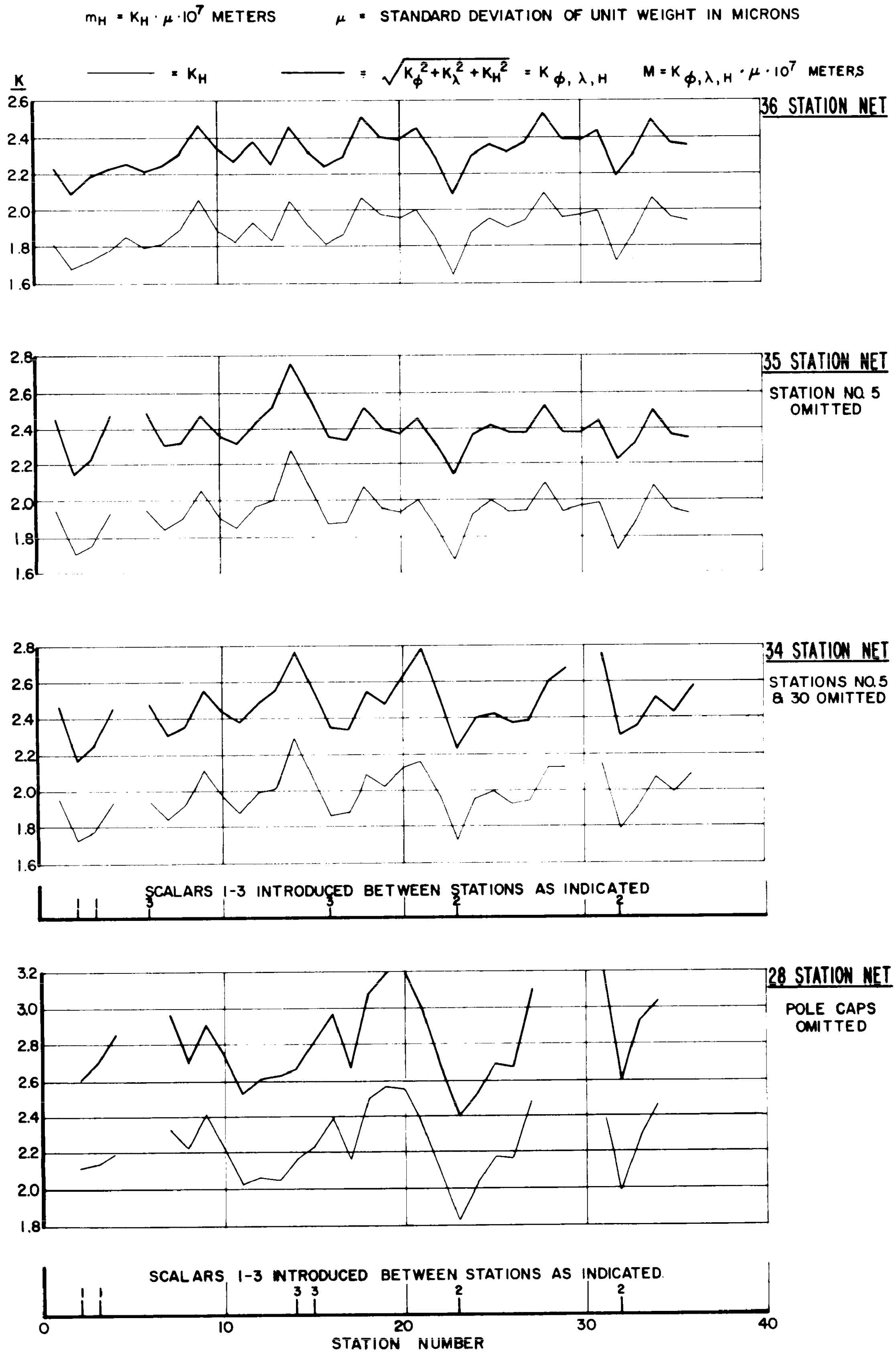


FIGURE 44

VARIOUS SIZE NETS

THREE SCALARS CONSTANT WT. = 1.0

$m_H = K_H \cdot \mu \cdot 10^7$ METERS μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS

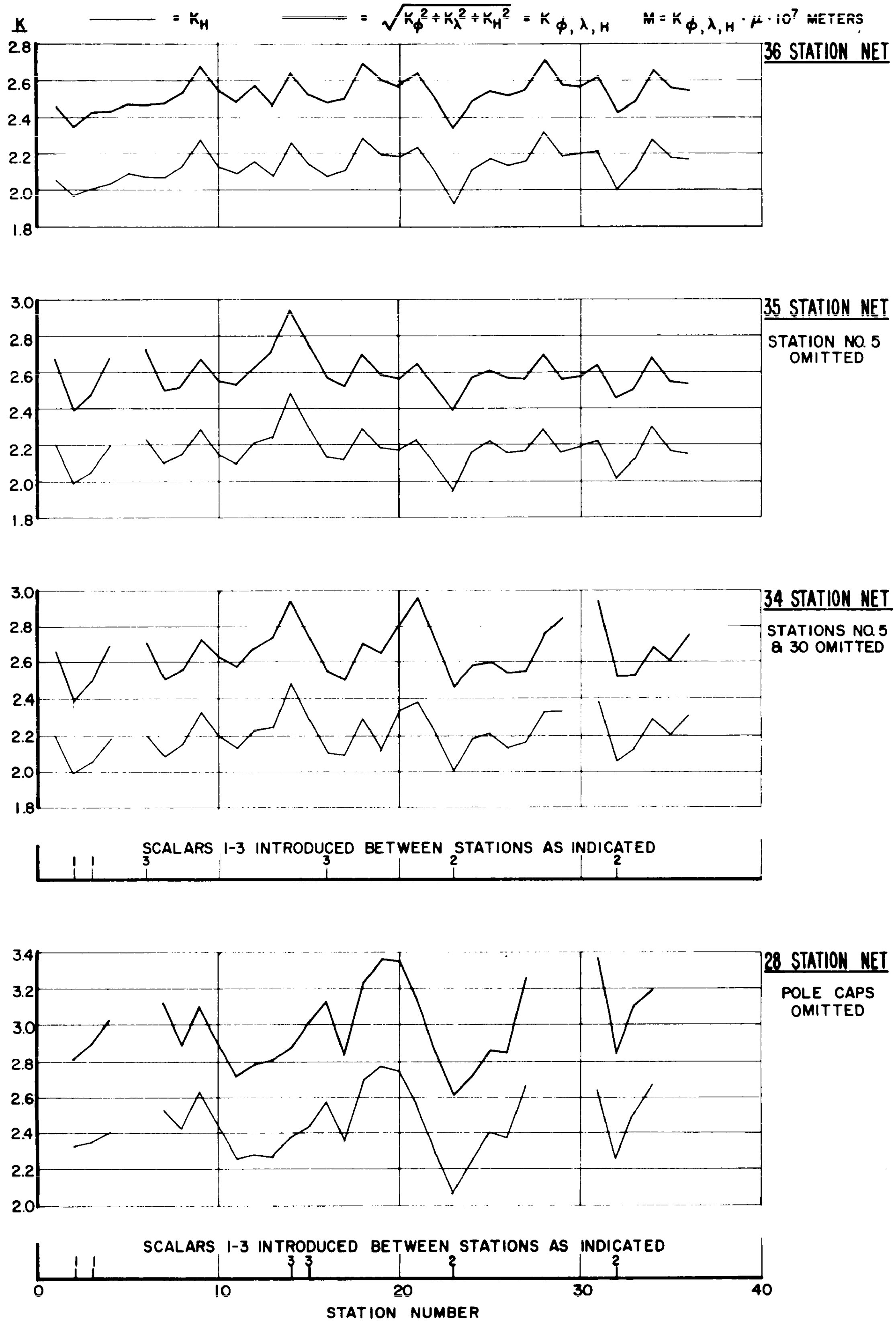


FIGURE 45

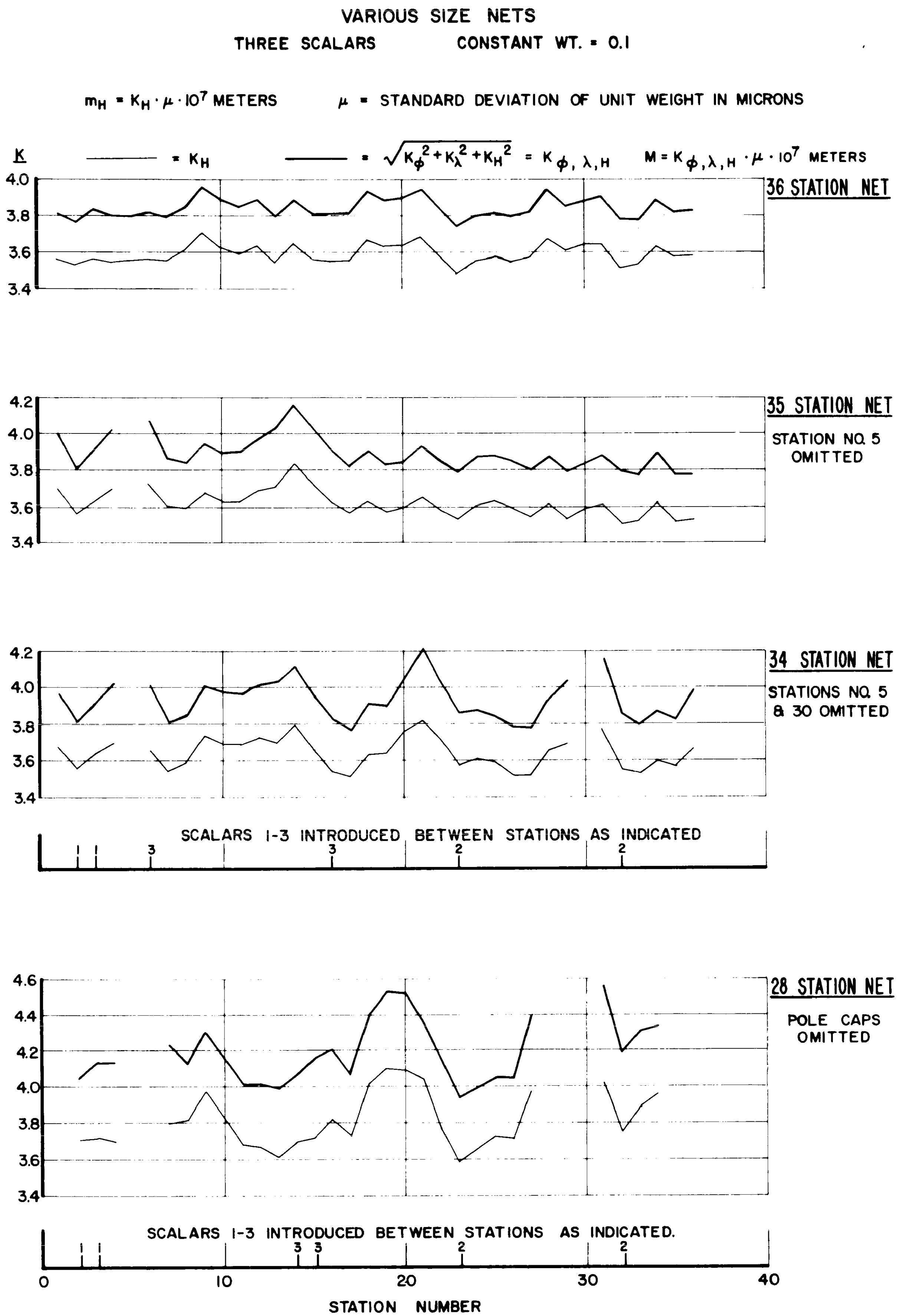

VARIOUS SIZE NETS
THREE SCALARS CONSTANT WT. = 0.1
$m_H = K_H \cdot \mu \cdot 10^7$ METERS
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS
K
= K_H
= $\sqrt{K_\phi^2 + K_\lambda^2 + K_H^2} = K_{\phi,\lambda,H}$
$M = K_{\phi,\lambda,H} \cdot \mu \cdot 10^7$ METERS
36 STATION NET
35 STATION NET
STATION NO. 5 OMITTED
34 STATION NET
STATIONS NO. 5 & 30 OMITTED
SCALARS 1-3 INTRODUCED BETWEEN STATIONS AS INDICATED
28 STATION NET
POLE CAPS OMITTED
SCALARS 1-3 INTRODUCED BETWEEN STATIONS AS INDICATED.
STATION NUMBER

FIGURE 46

VARIOUS SIZE NETS

FOUR SCALARS CONSTANT WT. = 10^4

$m_H = K_H \cdot \mu \cdot 10^7$ METERS μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS

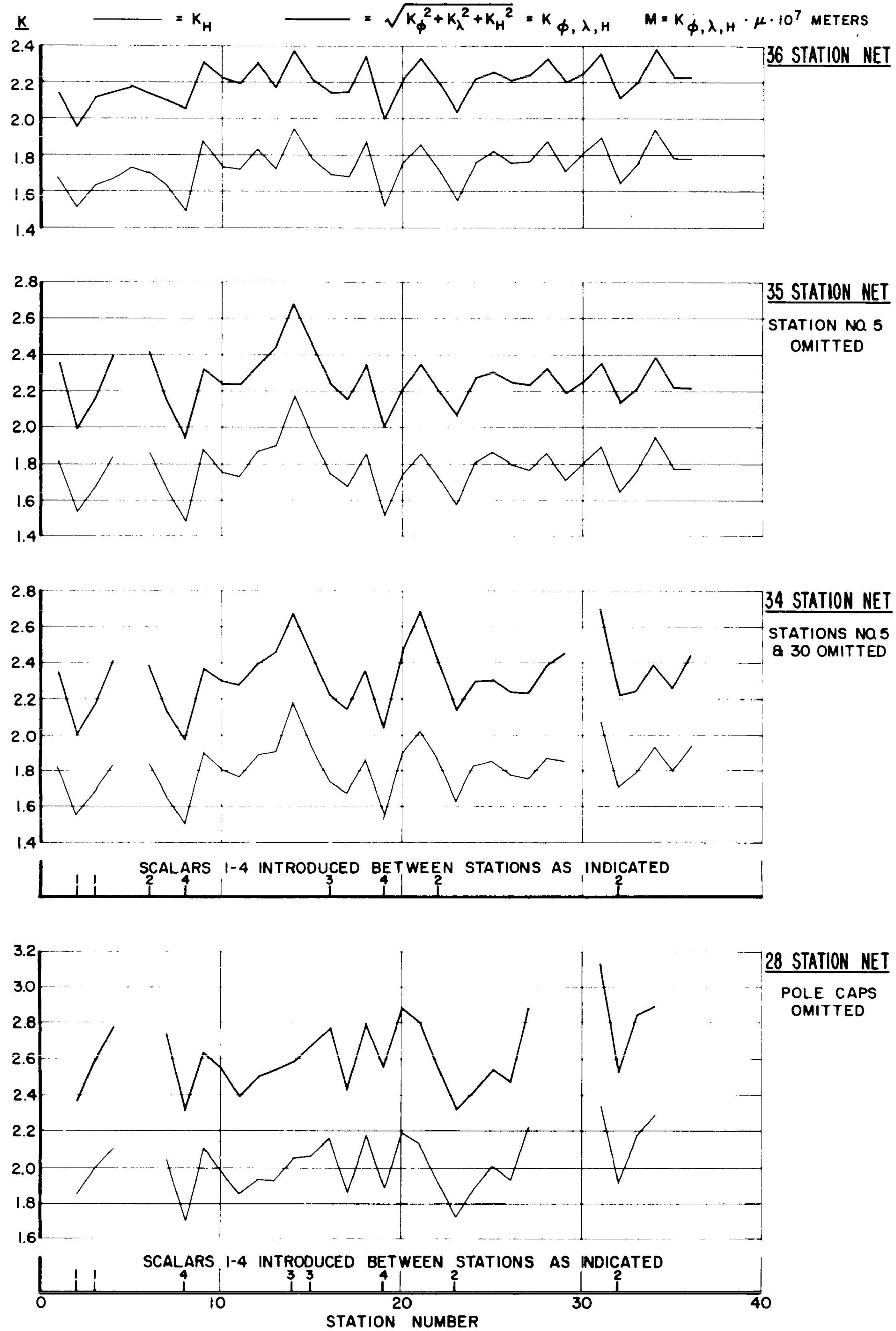


FIGURE 47

VARIOUS SIZE NETS

FOUR SCALARS CONSTANT WT. = 1.0

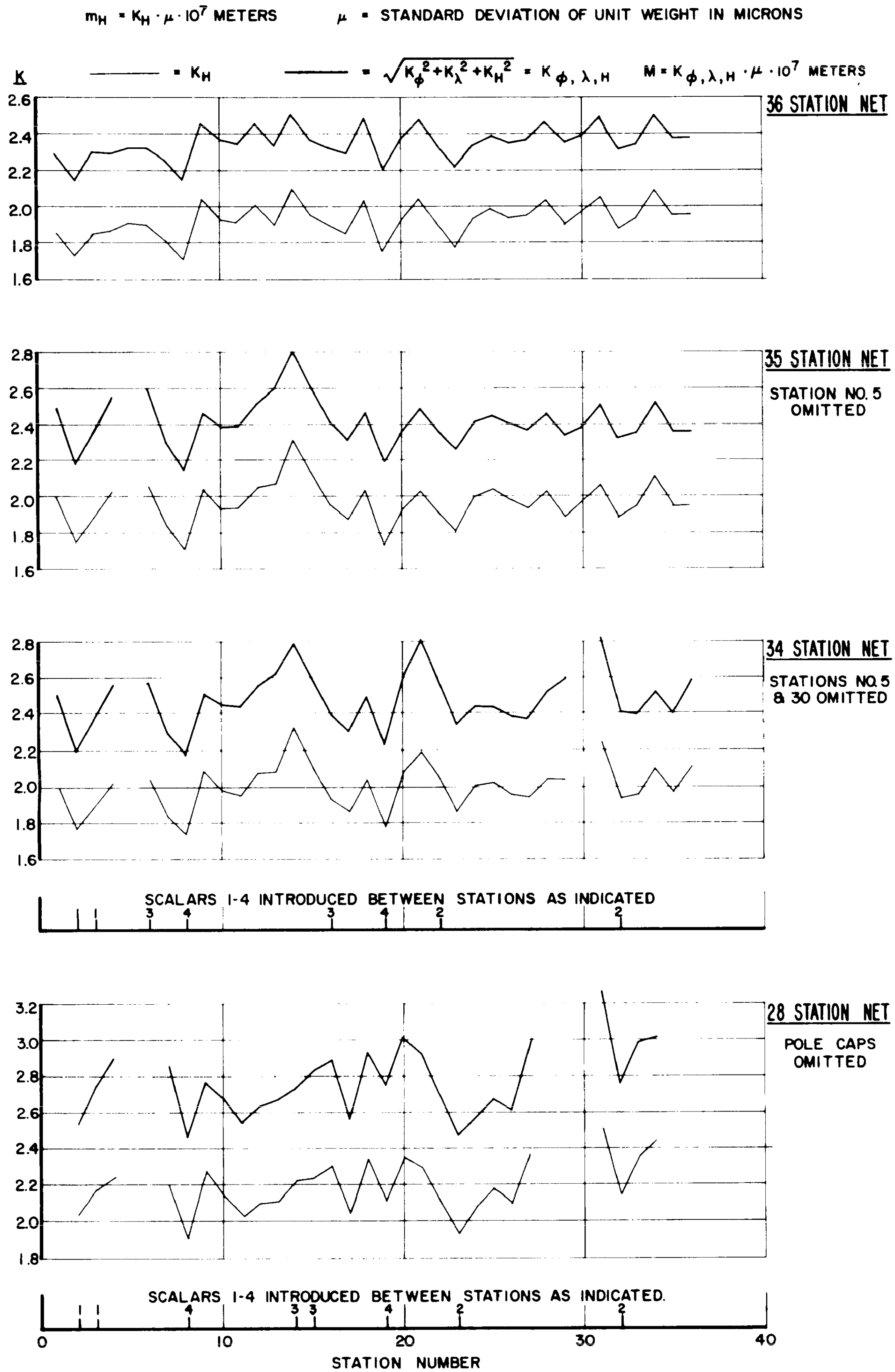


FIGURE 48

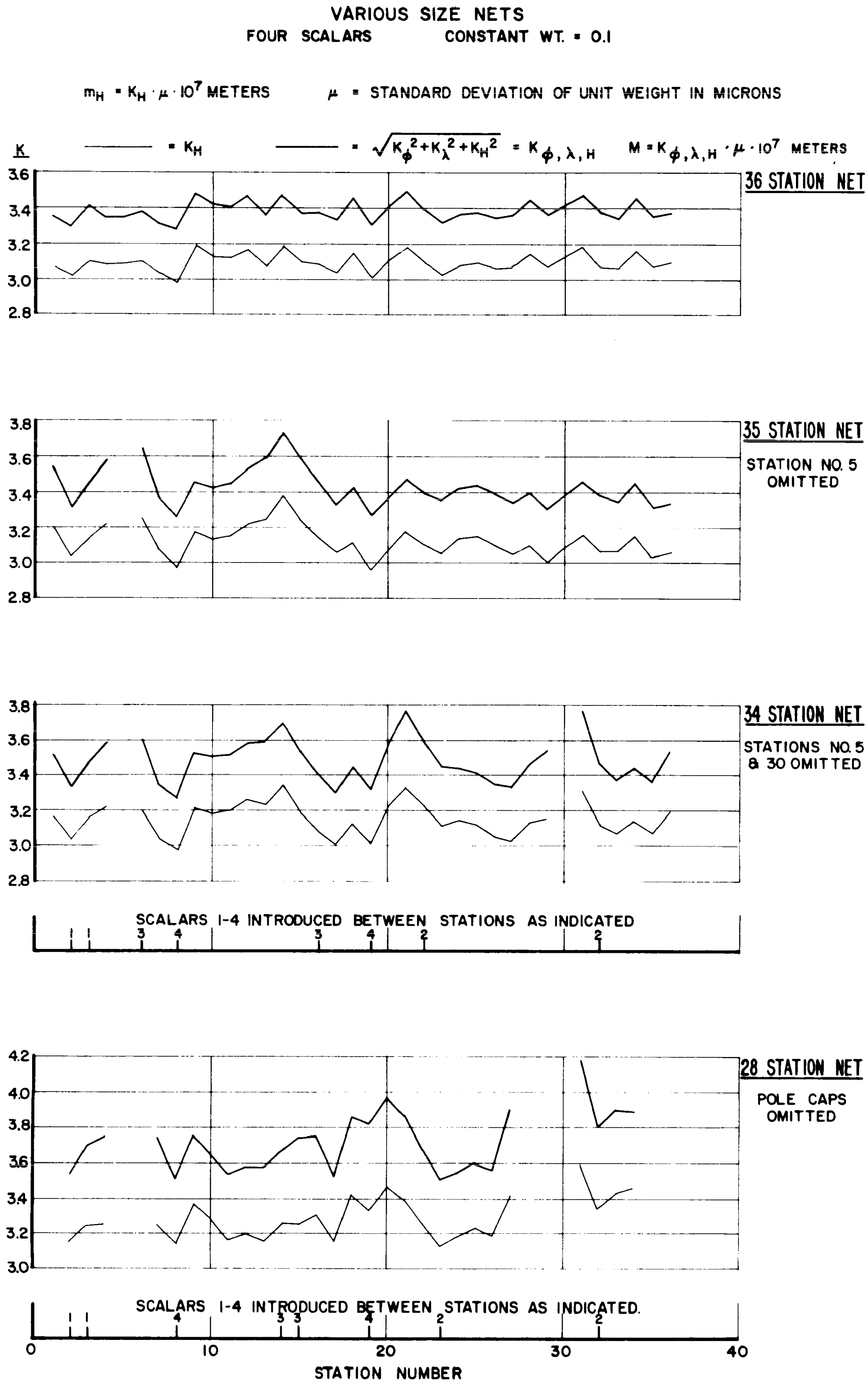

VARIOUS SIZE NETS
FOUR SCALARS CONSTANT WT. = 0.1
$m_H = K_H \cdot \mu \cdot 10^7$ METERS
μ = STANDARD DEVIATION OF UNIT WEIGHT IN MICRONS
K
$= K_H$
$= \sqrt{K_\phi^2 + K_\lambda^2 + K_H^2} = K_{\phi,\lambda,H}$
$M = K_{\phi,\lambda,H} \cdot \mu \cdot 10^7$ METERS
36 STATION NET
35 STATION NET
STATION NO. 5 OMITTED
34 STATION NET
STATIONS NO. 5 & 30 OMITTED
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED
28 STATION NET
POLE CAPS OMITTED
SCALARS 1-4 INTRODUCED BETWEEN STATIONS AS INDICATED.
STATION NUMBER

FIGURE 49

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5. AUTHOR(S) *(Last name, first name, initial)*

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A proposed three-dimensional geodetic reference system, consisting of 36 stations so situated on the surface of the earth as to form a closed system of approximate equilateral triangles surrounding the globe, is studied in an attempt to find out to what degree of accuracy the station positions can be determined when triangulating the corresponding station coordinates by photogrammetric techniques. The polyhedron of the spatially oriented triangles is formed by photographing a high altitude satellite against the star background. The triangulation method depends on applying analytical photogrammetric techniques to satellite observations where the satellite serves as a high altitude auxiliary triangulation target point. Numerical analyses on the whole net and portions of it are presented from which the problem of error propagation and the influence of the number of baselines can be studied.

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
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