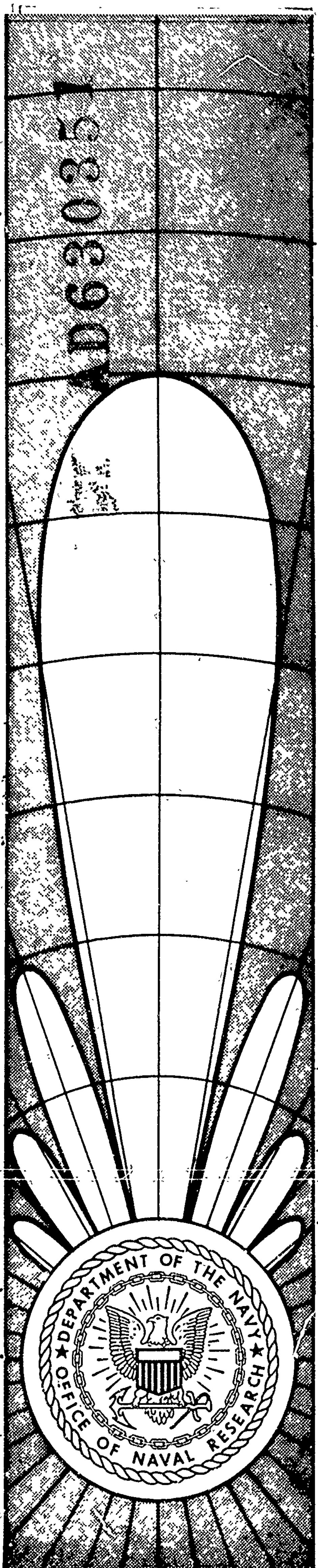


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# RECIPROCITY CALIBRATION IN A TUBE

# WITH ACTIVE-IMPEDANCE TERMINATION

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# Reciprocity Calibration in a Tube with Active-Impedance Termination

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A new application of the reciprocity principle has been developed for calibrating electroacoustic transducers in a closed vessel at static pressures to 8500 psi and frequencies from 100 to 1500 Hz. The necessary plane progressive wavefield is provided by sound propagation in the longitudinal mode within a sound channel terminated at both ends with active impedances. The technique is particularly well suited to the calibration of underwater-sound transducers because the high static pressure under which many of them must operate, as well as their size, mechanical construction, and operating frequency, often prevents the use of more conventional methods. The case of a rigid-walled, water-filled tube is analyzed theoretically. Results of measurements made by this method in a practical high-pressure calibration chamber are shown.

## INTRODUCTION

RECIPROCITY methods, because of their basic simplicity and inherent accuracy, offer a highly attractive technique for obtaining absolute calibrations of electroacoustic devices. On the other hand, calculating the appropriate reciprocity parameter usually requires the assumption that the acoustic field satisfy rather idealized boundary conditions that often cannot be approximated closely enough in practice. This situation exists in the calibration of underwater-sound transducers at the lower end of the audio-frequency range where it is often necessary to control the ambient temperature and static pressure and where the dimensions of the measurement chamber cannot be made small in comparison with the wavelength in the water, nor can reflections from the walls be eliminated with pulsing techniques or passive absorbers.

The coupler-reciprocity technique can be used at low audio frequencies, but it is applicable only to specially constructed probe hydrophones of high mechanical impedance. It cannot be used to calibrate other transducers of different size, shape, and type of construction.

The reciprocity procedure described here was developed to provide some relief from these problems. It is well-suited for use in a cylindrical vessel of high wall stiffness and moderate internal diameter. These characteristics are compatible with the requirement that the chamber accommodate a wide variety of transducer types and configurations and withstand high static pressure. Such a tube and its use for calibration purposes are described in a companion paper.[1] It is the purpose of this paper to present theoretical justification for the procedures.

The basic condition for valid calibration measurements in the tube is the existence of plane progressive waves in certain regions along the length of the tube. This condition is achieved by application of the principle of active impedance termination,[2] wherein the phase and amplitude of the electrical signal driving a transducer are varied relative to the phase and amplitude of the acoustic signal that it receives from a source transducer. The first transducer can be made to act as a controllable acoustic impedance with respect to the received signal. The reciprocity parameter applicable to the procedure is the plane-wave parameter first derived by Simmons and Urick,[3] where the area of the plane wave is the cross-sectional area of the channel.

In the analysis that follows, it is assumed that the walls of the calibration chamber are rigid and that the chamber is filled with airfree water and is excited by a sinusoidal signal in the frequency range for which only the longitudinal mode of sound propagation occurs in

[1] L. G. Beatty, R. J. Bobber, and David L. Phillips, "Sonar Transducer Calibration in a High-Pressure Tube," J. Acoust. Soc. Am. 39, 48-54 (1966).

[2] L. G. Beatty, "Acoustic Impedance in a Rigid-Walled Cylindrical Sound Channel Terminated at Both Ends with Active Transducers," J. Acoust. Soc. Am. 36, 1081-1089 (1964).

[3] B. D. Simmons and R. J. Urick, "The Plane Wave Reciprocity Parameter and Its Application to the Calibration of Electroacoustic Transducers at Close Distances," J. Acoust. Soc. Am. 21, 633-635 (1949).

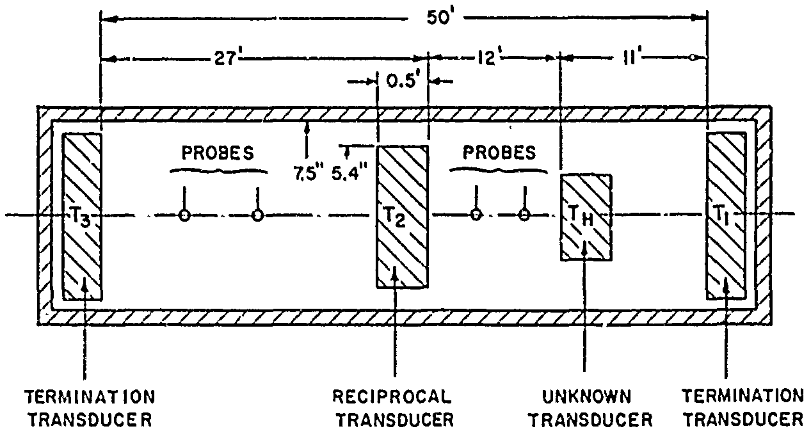


FIG. 1. Experimental arrangement.

the water without attenuation. These conditions can be approximated satisfactorily in a thick-walled steel tube of internal radius $b$ that satisfies the requirement

$$b \leq c/5f, \tag{1}$$

where $f$ is the highest operating frequency desired and $c$ is the longitudinal speed of sound in the water. The length of the chamber must be sufficient at all operating frequencies to ensure that there are two regions within the medium where only plane-wave components of the field exist. Further, each of the regions should extend at least the distance $\lambda/4$ at the frequency of measurement to ensure sufficient sensitivity in the detection of the plane progressive wave. This requirement is somewhat analogous to the criterion for spherical-wave reciprocity—that measurements are to be made at a sufficiently large distance from the reciprocal transducer to ensure that spherical-wave propagation exist there.

## I. MEASUREMENT PROCEDURE

Three transducers and four or more probe hydrophones are used in the measurement process. One of the transducers must be reciprocal and must satisfy certain impedance requirements, which are discussed later. The probe hydrophones serve to monitor the wave-propagation conditions in the tube. Preferably, the number and spacing of these probes should be such as to allow a good sampling of the conditions at all frequencies of interest. In measurements at the U. S. Navy Underwater Sound Reference Laboratory (USRL), nine probe hydrophones are spaced at various multiples of 1 ft along the axis of the chamber in the regions of interest.

The procedure follows that of the conventional reciprocity calibration, with the exception that conditions in the tube must be adjusted during each set of measurements to obtain plane progressive waves throughout certain regions of the chamber. Consider the physical arrangement depicted in Fig. 1, where $T_2$ is the reciprocal transducer and $T_H$ is the transducer that is used strictly as a receiver. First, transducer $T_1$ is driven while $T_2$ receives; next, $T_1$ serves as the termination transducer while $T_2$ is driven. Transducer $T_3$ serves solely as a termination device in both cases. The "double termination" procedure, so called because the terminal impedances at both ends of the tube are controlled when the reciprocal transducer is driving the tube, consists of the following steps.

1. Drive $T_1$ and $T_3$ at the same frequency, leaving $T_2$ open-circuited. Adjust the magnitude and phase relationships of the currents driving $T_1$ and $T_3$ so that plane progressive waves traveling toward $T_2$ are obtained in the region between $T_H$ and $T_2$, this condition being detected by probe hydrophones removed far enough from $T_H$ and $T_2$ to be out of the regions of field divergence associated with these transducers. Under these conditions, measure the open-circuit output voltages $E_{2r}$ of $T_2$ and $E_{Hr}$ of $T_H$. The method by which the required progressive-wave condition is detected with the probe hydrophones is discussed in detail in the companion paper[1]; in general, however, it involves the establishment of a particular phase-and-amplitude relationship between the outputs of a suitably chosen pair of probes.

2. Drive $T_2$, $T_1$, and $T_3$, adjusting the phase and amplitude relationships of the currents into $T_1$ and $T_3$ relative to the current into $T_2$ so that plane progressive waves again are obtained simultaneously in the regions between $T_2$ and $T_H$ traveling toward $T_H$ and between $T_2$ and $T_3$ traveling toward $T_3$. Thus, the reciprocal source transducer is radiating into a doubly terminated channel. Measure the current $I_{2S}$ driving $T_2$ and the open-circuit voltage $E_{HS}$ of $T_H$.

The values obtained for these parameters can be used now in the normal reciprocity equation to calculate the tube transmitting current response or voltage sensitivity of $T_2$, or the voltage sensitivity of $T_H$, which, within prescribed limitations, will be equivalent to the free-field values. The voltage sensitivity $M_H$ of $T_H$ is

$$M_H = \left[\frac{E_{Hr}E_{HS}}{E_{2r}I_{2S}} J_t\right]^{\frac{1}{2}}, \tag{2}$$

where $J_t$ is the reciprocity parameter in the tube. It is

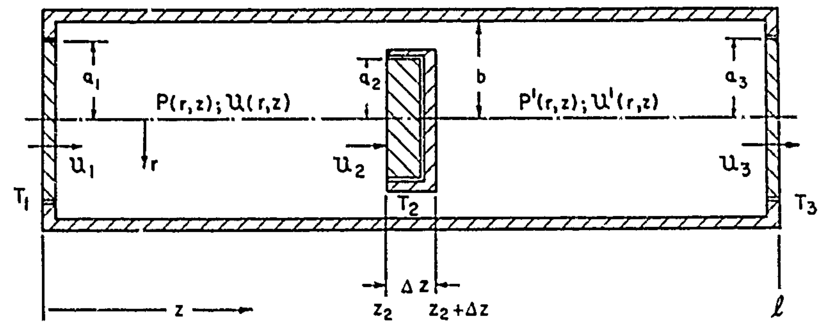


FIG. 2. Mathematical model.

shown in Sec. II-C1 that, for the prescribed measurement conditions and for certain limitations on the impedance of the reciprocal transducer,

$$|J_t| = 2A_t/\rho c, \tag{3}$$

where $A_t$ is the cross-sectional area of the tube and $\rho$ is the density of the water in the tube.

Another, less-useful, arrangement is called "single termination." When this arrangement is used, Step 2 is changed to allow $T_3$ to be passive and open-circuited. Plane progressive waves are then present only in the region between $T_H$ and $T_2$, traveling toward $T_H$.

The magnitude of the reciprocity parameter for this case is

$$|J_t'| = A_t/\rho c|\cos\theta|, \tag{4}$$

where $\theta$ is an angle determined by the frequency and the distance between $T_2$ and $T_3$. This form of the reciprocity parameter, derived in Sec. II-C2, also imposes restrictions on the impedance of the reciprocal transducer and further imposes certain conditions on the effective acoustic impedance of $T_3$, open-circuited, to the plane-wave mode in the water in the tube.

## II. THEORY

### A. Solution of the Wave Equation

The derivations that follow are based on rather simple boundary conditions. In general, these simplifications do not restrict the practical application of the technique, provided that the basic assumptions of reciprocity are satisfied in practice and plane progressive waves are obtained.

Consider the mathematical model of Fig. 2 and assume the following conditions. The system consists of a rigid-walled, water-filled tube, of radius $b$ and length $l$, that is driven by coaxial transducers $T_1$, $T_2$, and $T_3$, whose diaphragms are pistons of radii $a_1$, $a_2$, and $a_3$, respectively. The face of transducer $T_1$ is at $z=0$; $T_2$, at $z=z_2$, facing $T_1$; $T_3$, at $z=l$, faces the back of $T_2$, which is considered rigid. Denote the piston velocities of the transducers by $U_1$, $U_2$, and $U_3$. The over-all thickness $\Delta z$ of $T_2$ in the $z$ direction is assumed to be small enough that its effect on the solution can be neglected.

The general solution of the wave equation for the system is considered to consist of the following parts:

$$\phi(r,z) = \sum_{n=0}^{\infty} [\alpha_n \exp(-\sigma_n z) + \beta_n \exp(\sigma_n z)] J_0(\kappa_n r) \tag{5}$$

for $0 \le z \le z_2$, and

$$\phi'(r,z) = \sum_{n=0}^{\infty} [\alpha_n' \exp(-\sigma_n z) + \beta_n' \exp(\sigma_n z)] J_0(\kappa_n r) \tag{6}$$

for $(z_2+\Delta z) \le z \le l$. In these expressions, $\phi(r,z)$ and $\phi'(r,z)$ represent the usual velocity potential functions having the common phase propagation constant $\sigma_n = (\kappa_n^2 + k^2)^{\frac{1}{2}}$, where $\kappa_n$ and $k$ are the radial and longitudinal components, respectively, of $\sigma_n$. The longitudinal component is related to the excitation frequency by $k=\omega/c$, where $\omega = 2\pi f$, $f$ being the excitation frequency. The constants $\alpha_n$, $\alpha_n'$, $\beta_n$, and $\beta_n'$ are amplitude coefficients of $\phi$ and $\phi'$. The radial dependency is determined by $J_0(\kappa_n r)$, the zero-order Bessel function of argument $(\kappa_n r)$, symmetry with respect to the $z$ axis having been assumed.

The constant $\kappa_n$ relates to the radial mode of wave propagation within the water. Its value is determined by the requirement that the radial velocity be zero at the walls of the tube; that is, the boundary condition

$$-\partial\phi/\partial r = -\partial\phi'/\partial r = 0 \quad \text{for} \quad r=b, \tag{7}$$

which is satisfied in both solutions by the same value of $\kappa_n$: namely,

$$\kappa_n = j_n/b; \quad n=0, 1, 2, 3, \cdots, \tag{8}$$

where $j_n$ is the $n$th zero of the first-order Bessel equation.

The remaining boundary conditions to be satisfied are

$$-\frac{\partial\phi}{\partial z} = \begin{cases} U_1 & \text{for } 0 \le r \le a_1, \\ 0 & \text{for } a_1 < r \le b, \end{cases} \quad z=0; \tag{9}$$

$$-\frac{\partial\phi}{\partial z} = \begin{cases} U_2 & \text{for } 0 \le r \le a_2, \\ U(r,z_2) & \text{for } a_2 < r \le b, \end{cases} \quad z=z_2; \tag{10}$$

$$-\frac{\partial\phi'}{\partial z} = \begin{cases} 0 & \text{for } 0 \le r \le a_2, \\ U(r, z_2+\Delta z) & \text{for } a_2 < r \le b, \end{cases} \quad z=z_2+\Delta z; \tag{11}$$

$$-\frac{\partial\phi'}{\partial z} = \begin{cases} U_3 & \text{for } 0 \le r \le a_3, \\ 0 & \text{for } a_3 < r \le b, \end{cases} \quad z=l; \tag{12}$$

where $U(r,z)$ is the longitudinal particle velocity at point $(r,z)$.

Because the conditions expressed by Eqs. 10 and 11 are not independent, an additional set of relationships is required. This set is obtained from the force equations over the regions $a_2<r\leq b$ at $z=z_2$ and $z_2+\Delta z$:

$$F(z_2)=i2\pi\omega\rho\int_{a_2}^{b}\phi(r,z_2)rdr, \tag{13}$$

$$F(z_2+\Delta z)=i2\pi\omega\rho\int_{a_2}^{b}\phi'(r,z_2+\Delta z)rdr, \tag{14}$$

where, as usual, $i=\sqrt{-1}$.

From these boundary relationships and the assumption that $z_2\approx z_2+\Delta z$, the following expressions are obtained for the constants $\alpha_n$, $\alpha_n'$, $\beta_n$, and $\beta_n'$:

$$\alpha_n=(S_nj_n/i4\pi\rho\omega b)[U_3a_3J_3+U_2a_2J_2\cosh\sigma_n(l-z_2)-U_1a_1J_1\exp(\sigma_nl)], \tag{15}$$

$$\alpha_n'=(S_nj_n/i4\pi\rho\omega b)[U_3a_3J_3+(U_2a_2J_2\cosh\sigma_nz_2-U_1a_1J_1)\exp(\sigma_nl)], \tag{16}$$

$$\beta_n=(S_nj_n/i4\pi\rho\omega b)[U_3a_3J_3+U_2a_2J_2\cosh\sigma_n(l-z_2)-U_1a_1J_1\exp(-\sigma_nl)], \tag{17}$$

$$\beta_n'=(S_nj_n/i4\pi\rho\omega b)[U_3a_3J_3+(U_2a_2J_2\cosh\sigma_nz_2-U_1a_1J_1)\exp(-\sigma_nl)], \tag{18}$$

where

$$S_n\equiv-i4\pi\rho\omega/\sigma_nj_n^2J_0^2(j_n)\sinh\sigma_nl,$$
$$J_1\equiv J_1(a_1j_n/b),\ J_2\equiv J_1(a_2j_n/b),\ J_3\equiv J_1(a_3j_n/b), \tag{19}$$

and $J_1$ on the right-hand side of these expressions for $J_1$, $J_2$, and $J_3$ indicates the Bessel function of order one and argument indicated in the parentheses.

### B. Related Force Equations

The relation of the field equations to the driving forces $F_1$, $F_2$, and $F_3$, applied to transducers $T_1$, $T_2$, and $T_3$ by their electrical currents is obtained by considering the transducers to be driven by Thévenin generators of internal mechanical impedances $Z_{1m}$, $Z_{2m}$, and $Z_{3m}$. The following equations can be written for this condition:

$$F_1=U_1[K_{10}+(U_2/U_1)K_{12}+(U_3/U_1)K_{13}], \tag{20}$$

$$F_2=U_2[K_{20}+(U_1/U_2)K_{21}-(U_3/U_2)K_{23}], \tag{21}$$

$$F_3=U_3[K_{30}+(U_1/U_3)K_{31}-(U_2/U_3)K_{32}], \tag{22}$$

where $K_{10}\equiv Z_{1m}-K_{11}$, $K_{20}\equiv Z_{2m}-K_{22}$, $K_{30}\equiv Z_{3m}-K_{33}$, and the terms $K_{pq}$ $(p,q=1,2,3)$ represent the interaction impedance functions (in mechanical units) of the acoustical system associated with transducers $T_1$, $T_2$, and $T_3$, as follows:

$$\begin{aligned}
K_{11}&\equiv a_1^2\sum_{n=0}^{\infty}S_nJ_1^2\cosh\sigma_nl,\\
K_{22}&\equiv a_2^2\sum_{n=0}^{\infty}S_nJ_2^2\cosh\sigma_n(l-z_2)\cosh\sigma_nz_2,\\
K_{33}&\equiv a_3^2\sum_{n=0}^{\infty}S_nJ_3^2\cosh\sigma_nl,\\
K_{12}&=K_{21}\equiv a_1a_2\sum_{n=0}^{\infty}S_nJ_1J_2\cosh\sigma_n(l-z_2),\\
K_{13}&=K_{31}\equiv a_1a_3\sum_{n=0}^{\infty}S_nJ_1J_3,\\
K_{23}&=K_{32}\equiv a_2a_3\sum_{n=0}^{\infty}S_nJ_2J_3\cosh\sigma_nz_2.
\end{aligned} \tag{23}$$

### C. Derivation of the Reciprocity Parameter

The preceding equations, in association with the measurement conditions previously outlined, now can be used to derive the reciprocity parameter for the tube. In the derivations that follow, for convenience, we use the superscript notation 0 to indicate that the series terms involved in the expressions contain only the terms for which $n=0$, and the superscript $n+1$ to indicate that the series terms are formed from $n+1$ to $\infty$. Also, we use the symbols $A_1$, $A_2$, $A_3$, and $A_t$ to indicate the areas of the disphragms of the projectors and the cross-sectional area of the tube bore.

#### *1. Double Termination*

For Step 1 of the double-termination measurement procedure, the reciprocal transducer $T_2$ acts as an open-circuited receiver so that $F_2=0$; hence, from Eq. 21,

$$U_{2r}=[-U_{1r}K_{21}+U_{3r}K_{23}]/K_{20}, \tag{24}$$

where the subscript r indicates the receiving condition. Also, for this condition, the $\beta_0$ term, Eq. 17, equals zero, which requires that

$$A_3U_{3r}/A_2U_{2r}-(A_1U_{1r}/A_2U_{2r})e^{-ikl}=-\cos k(l-z_2). \tag{25}$$

The force equation in the region $0\leq z\leq z_2$ over the area $A_2$ can be expressed as the sum

$$F_r(z,A_2)=F_r^0(z,A_2)+F_r^{n+1}(z,A_2). \tag{26}$$

In terms of the plane progressive-wave pressure $P_r^0(z)$, it can be shown that

$$P_r^0(z)=(1/A_2)F_r^0(z,A_2)=A_1Z_0U_{1r}e^{ikz}, \tag{27}$$

where $Z_0=\rho c/A_t$.

Finally, for the imposed conditions, the effect of mutual interaction of the higher-order modes among the transducers can be neglected in the interaction

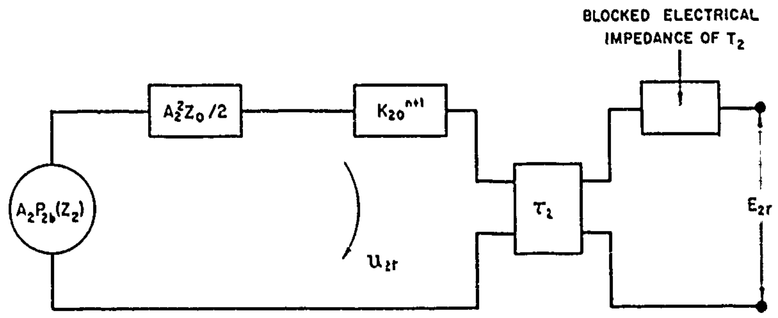


FIG. 3. Electrical analog circuit of transducer $T_2$ as receiver.

impedance functions; that is, we can use the approximations

$$K_{pq}^{n+1}(p\neq q)\approx 0. \tag{28}$$

These results can be used to put Eq. 24 in the form

$$U_{2r}=A_2P_r^0(z_2)/K_{20}^{n+1}. \tag{29}$$

Now, $T_2$ can be represented as a receiver by the electrical analog circuit shown in Fig. 3, where the Thévenin generator pressure or blocked pressure over the area $A_2$ has been labeled $P_{2b}(z_2)$. This blocked pressure is proportional to the plane progressive-wave pressure $P_r^0(z_2)$; that is,

$$P_{2b}(z_2)=D_{2t}P_r^0(z_2), \tag{30}$$

where $D_{2t}$ is a constant. In a free field, this constant is called the diffraction constant of the transducer and normally is considered to be a function of the shape, dimensions relative to the wavelength of sound in the acoustic medium, and the orientation of the transducer in the sound field. In the tube, the constant $D_{2t}$, which will be called the tube diffraction constant of transducer $T_2$, not only is a function of these parameters, but it depends upon the mechanical impedance of the transducer as well.

To obtain the expression for $D_{2t}$, we note from Fig. 3 and Eq. 30 that

$$U_{2r}=A_2D_{2t}P_r^0(z_2)/(\tfrac{1}{2}A_2^2Z_0+K_{20}^{n+1}). \tag{31}$$

Equating the right-hand members of Eqs. 29 and 31 and solving for $D_{2t}$ gives

$$D_{2t}=1+\tfrac{1}{2}A_2^2Z_0/K_{20}^{n+1}. \tag{32}$$

By definition, the receiving sensitivity $M_2$ of $T_2$ is

$$M_2=E_{2r}/P_r^0(z_2)=U_{2r}\tau_2/P_r^0(z_2), \tag{33}$$

where $\tau_2$ is the electromechanical transfer constant of transducer $T_2$.

Thus, through the use of Eq. 31,

$$M_2=A_2D_{2t}\tau_2/(\tfrac{1}{2}A_2^2Z_0+K_{20}^{n+1}). \tag{34}$$

For the transmission conditions of Step 2, it is required that $\alpha_0=\beta_0=0$. For this condition to be satisfied, it is necessary that

$$U_{1S}/U_{2S}=\tfrac{1}{2}(A_2/A_1)\exp(-ikz_2), \tag{35}$$

and

$$U_{3S}/U_{2S}=-\tfrac{1}{2}(A_2/A_3)\exp[-ik(l-z_2)], \tag{36}$$

so that

$$U_{1S}/U_{3S}=-(A_3/A_1)\exp[ik(l-2z_2)], \tag{37}$$

where the subscript S denotes the transmitting condition.

It should be noted here that solutions of the form $\alpha_0=\alpha_0'=0$ or $\beta_0=\beta_0'=0$ give rise to mathematically indeterminate forms, implying that they are physically unattainable. This, of course, merely confirms that plane progressive waves proceeding in identical directions in both sections of the tube cannot be achieved unless the physical effect of transducer $T_2$ can be neglected—that is, unless $T_2$ appears essentially as the characteristic impedance $Z_0$ without being driven.

For the imposed conditions, Eq. 21 can be written in the form

$$U_{2S}=F_2/(\tfrac{1}{2}A_2^2Z_0+K_{20}^{n+1}). \tag{38}$$

The pressure in the plane progressive wave can be expressed as

$$P_S^0(z)=\tfrac{1}{2}Z_0A_2U_{2S}\exp i[\pi-k(z_2-z)]. \tag{39}$$

By using Eqs. 38 and 39, the transmitting response of $T_2$ in the tube for the case of double termination can be expressed as

$$S_2=\tau_2(\tfrac{1}{2}Z_0)A_2\exp i[\pi-k(z_2-z)]/(\tfrac{1}{2}A_2^2Z_0+K_{20}^{n+1}) =P_S^0(z)/I_{2S}. \tag{40}$$

Now, we define the reciprocity parameter for the case of double termination as the ratio of Eq. 34 to Eq. 40:

$$J_t=M_2/S_2=(2D_{2t}/Z_0)\exp\{-i[\pi-k(z_2-z)]\}. \tag{41}$$

For the imposed frequency limitations, the term $-K_{22}^{n+1}$ in the expression for $D_{2t}$ has the form of a mass reactance $iX_{22}$. Hence, Eq. 32 can be written in the form

$$D_{2t}=1+\tfrac{1}{2}A_2^2Z_0/(Z_{2m}+iX_{22}). \tag{42}$$

The sum $\frac{1}{2}A_2^2Z_0+iX_{22}$ is the mechanical equivalent of the radiation load on transducer $T_2$ in the tube for the case of double termination, the reactive portion or the $n+1$ terms of $K_{22}$ being attributed to the effects of divergence in the vicinity of the transducer.

Now, if we assume that the reciprocal transducer is so constructed that its mechanical impedance satisfies the condition

$$(A_2^2\rho c/A_t)(1/|Z_{2m}+iX_{22}|)\ll 2, \tag{43}$$

then $|D_{2t}|\approx 1$ for the tube, and the magnitude of the reciprocity parameter becomes

$$|J_t| = 2A_t/\rho c. \tag{44}$$

This restriction, Eq. 43, implies, in practice, that the reciprocal transducer appear as a stiffness-controlled device over the frequency range of interest.

### 2. *Single Termination*

For the case of single termination, Step 2 of the measurement procedure is modified. Transducer $T_3$ is open-circuited instead of being driven. This condition gives rise to standing waves in the region between $T_2$ and $T_3$, and both the reciprocity parameter and the diffraction constant are affected thereby.

The tube diffraction constant in this case takes the form

$$D_{2t}' = 1 + A_2^2 Z_0 |\cos k(l-z_2)| \exp(ikz_2)/K_{20}^{n+1}. \tag{45}$$

The receiving sensitivity, Eq. 34, becomes

$$M_2 = \tau_2 A_2 D_{2t}'/[A_2^2 Z_0 |\cos k(l-z_2)| \times \exp(ikz_2) + K_{20}^{n+1}]. \tag{46}$$

For the transmitting response, we obtain

$$S_2 = \frac{Z_0 A_2 \tau_2 \exp[ik(z-\pi)] |\cos k(l-z_2)|}{A_2^2 Z_0 |\cos k(l-z_2)| \exp(ikz_2) + K_{20}^{n+1}}, \tag{47}$$

in the derivation of which we have used the relationship obtained from Eqs. 22 and 25:

$$\frac{U_{2S}}{U_{1S}} = e^{ikl}\frac{A_1}{A_2}\frac{1+Z_0A_3^2/K_{30}^{n+1}}{\cos k(l-z_2)+i(Z_0A_3^2/K_{30}^{n+1})\sin k(l-z_2)}. \tag{48}$$

When it is assumed that $|Z_0A_3^2/K_{30}^{n+1}|\ll 1$, we obtain as an approximation for the magnitude of Eq. 48.

$$|U_{2S}/U_{1S}| \approx (A_1/A_2)/|\cos k(l-z_2)|. \tag{49}$$

It should be noted that, when $\cos k(l-z_2)=0$, the expression in Eq. 48 remains finite unless $Z_0A_3^2/K_{30}^{n+1}$ also is zero—that is, unless the termination transducer appears as an infinite impedance. If $K_{30}^{n+1}$ is purely reactive, then it is also possible for the denominator to become zero for nonzero values of $\cos k(l-z_2)$. In practice, the ratio $U_{2S}/U_{1S}$ will remain finite because of the losses always present in the real system.

We now require that

$$|A_2^2 Z_0 \cos k(l-z_2) \exp -ik(l-z_2)/K_{20}^{n+1}| \tag{50}$$

be negligibly small in comparison with 1 so that the tube diffraction constant can be taken as approximately equal to 1, $D_{2t}'\approx 1$. Then, for the single termination,

$$|J_t'| \approx \frac{A_t}{Z_0}\left|\frac{1}{\cos[(\omega/c)(l-z_2)]}\right|$$

or

$$|J_t'| = A_t/\rho c|\cos\theta|, \tag{51}$$

where $\theta = (\omega/c)(l-z_2)$.

## III. COMPARISON OF TUBE RECIPROCITY AND FREE-FIELD RECIPROCITY RESULTS

Let $S_t$ and $S_f$ be the transmitting current responses in the tube and in the free field, respectively, and note that, if the transducer is reciprocal, the ratio of these responses is

$$S_f/S_t = (J_t/J_f)(M_f/M_t), \tag{52}$$

where $M_f$ and $M_t$ are the receiving responses in the free field and in the tube, and $J_f$ and $J_t$ are the reciprocity parameters for the free field and the tube. Equation 52 can also be written in the form

$$\frac{S_f}{S_t} = \left|\frac{Z_m+Z_{rt}}{D_t}\right|\left|\frac{D_f}{Z_m+Z_{rf}}\right|\frac{A_tf}{dc}, \tag{53}$$

where $Z_{rt}$ and $Z_{rf}$ are, respectively, the radiation loads in mechanical units in the tube and in the free field, $d$ is the distance to which the free-field measurements are referred, and $D_t$ and $D_f$ are the diffraction constants.

For the imposed restrictions on the ratio of wavelength to transducer dimensions, $D_f\approx 1$. Thus, when $D_{2t}$ is substituted for $D_t$, Eq. 53 may be expressed in the form

$$\left|\frac{S_f}{S_t}\right| = \left|\frac{Z_m+iX_{rt}}{Z_m+R_{rf}+iX_{rf}}\right|\frac{A_tf}{dc}, \tag{54}$$

where, in terms of mechanical units, $X_{rt}$ and $X_{rf}$ are the mass reactive portions of the radiation load in the tube and in the free field, respectively, and $R_{rf}$ is the free-field radiation resistance.

Thus, equality between the two responses depends on the magnitude of the differences that occur in the acoustic radiation loads and on the factor $A_tf/dc$. Equality between the open-circuit voltage receiving responses depends only on the former.

For transducers operating in the stiffness-controlled portion of their frequency range, the voltage sensitivities are equal. Above resonance, the mass reactive terms in the sensitivities control. Hence, if the net change in radiation mass in the tube is small in comparison with the total free-field mass, then the two sensitivities are approximately equal in this portion of the frequency range also.

For a resonant transducer, two effects are predominant: (1) a shift occurs in the frequency of resonance in the sensitivity-versus-frequency curves, reso-

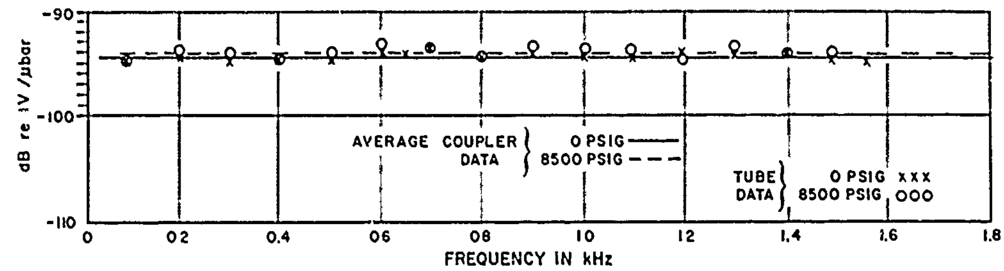


FIG. 4. Free-field voltage sensitivity, G20 hydrophone.

nance in the tube occurring at the higher frequency, and (2) a change occurs in the broadness of the peaks in the vicinity of resonance on the sensitivity curves.

Both of these changes are attributed to the differences in the radiation mass under the respective conditions. In addition, however, the change in the $Q$'s that is due to mass difference is opposed by the effects of $R_{rf}$ in the free-field case.

The expressions for the $Q$'s are,

$$Q_f \approx f_f/(f_{2f}-f_{1f}) = 2\pi f_f(m_m+m_{rf})/(R_m+R_{rf}), \quad (55)$$

$$Q_t \approx f_t/(f_{2t}-f_{1t}) = 2\pi f_t(m_m+m_{rt})/R_m, \quad (56)$$

where $R_m$ is a real portion of the mechanical impedance, $m_m$ is the mass of the mechanical impedance, and $m_{rt}$ and $m_{rf}$ are the radiation mass in the tube and the free field, respectively. Also $f_f$, $f_{1f}$, $f_{2f}$ and $f_t$, $f_{1t}$, $f_{2t}$ are the frequencies of resonance and of the lower and upper 3-dB down points on the sensitivity curves in the free field and in the tube, respectively.

Thus, the ratio of the frequency difference between the 3-dB down points is

$$\frac{f_{2f}-f_{1f}}{f_{2t}-f_{1t}} = \left(1+\frac{R_{rf}}{R_m}\right)\left(\frac{f_f}{f_t}\right)^2. \quad (57)$$

The factor $(1+R_{rf}/R_m)$ represents the ratio of the tube sensitivity to the free-field sensitivity at the respective resonant frequencies. Thus, if $R_m \gg R_{rf}$, the sensitivities at these frequencies are equal.

In Eqs. 56 and 57, it has been assumed that, near resonance, the impedances involved are derivable from frequency-independent constants. For the same assumption, an approximate correlation can be obtained between the voltage sensitivities by plotting the tube results against the normalized frequency scale $f'=f(f_f/f_t)$.

On the $f'$ scale in terms of voltage ratio in decibels, the error in the level of the tube response sensitivity will not be greater than $20\log(f_t/f_f)$ on the slopes of the curve. At the peak, it will be $20\log(Q_t/Q_f)+20\log(f_t/f_f)$.

We have discussed the case of resonant transducers without considering the fact that, to perform a reciprocity calibration, the tube diffraction constant $D_{2t}$ must equal 1. As has been mentioned previously, this condition requires that the reciprocal transducer $T_2$ be nonresonant. On the other hand, this restriction does not apply to the receiving response of the unknown transducer $T_H$; hence, the fact that this transducer can be resonant is immaterial to the calibration process. The equivalence of the resonse to that in a free-field still depends, however, on the criteria previously discussed.

For the frequency limitations imposed, the magnitude of the radiation mass load of the medium on the transducer is essentially a function of the frequency, the density of the medium, and the effective piston area. The frequency is independent of variation in static pressure, and the density (when water is the medium) is virtually independent of it. Thus, differences between the voltage response measured in the tube and that measured in a free field for a particular transducer will vary little with static pressure unless the pressure affects the mechanical impedance of the transducer. It is justifiable to assume, therefore, that the changes encountered are, to a good approximation, a measure of the effect of the same pressure on the free-field response. Differences between the transmitting responses as a function of change of static pressure will likewise reflect actual effects of static pressure change on the free-field behavior of the transducer when allowance is made for the effect of pressure on sound speed in the water in the tube.

## IV. EXPERIMENTAL RESULTS

The theory has been applied to practical measurements in the tube for both the single- and the double-termination conditions. The essentials of the physical arrangement are shown in Fig. 1. The termination and reciprocity transducers are oil-filled crystal units that act as stiffness-controlled piston drivers over the frequency range of interest, thus ensuring the fulfillment of the impedance requirements imposed by the theory. The value of $A_2{}^2Z_0/[A_t|Z_{2m}+iX_{22}|]$ is estimated to be about $2\times10^{-2}$ at 1.5 kHz.

The results of measurements made on a special hydrophone by double-termination reciprocity at 0 and 8500 psig are shown in Fig. 4. This hydrophone has been designed so that it can be calibrated also in a coupler chamber in the desired frequency range as a function of high static pressure. The results of coupler measurements as a function of frequency at static pressure also are shown in Fig. 4.

Figure 5 shows results obtained from single-termination measurements on a pressure-sensitive hydro-

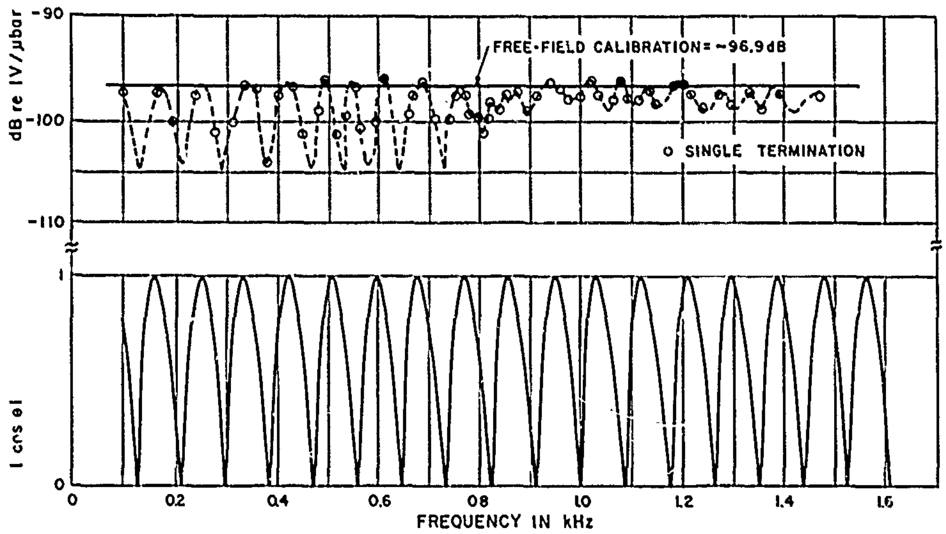


FIG. 5. Free-field voltage sensitivity by single-termination reciprocity, H35 hydrophone.

phone whose free-field response is constant within the frequency range of the measurements.

In computing the sensitivity from the single-termination measurement data, the reciprocity parameter was taken as the magnitude of the actual parameter when $\cos\theta = \pm 1$. Thus, the measured response reflects the oscillations of the reciprocity parameter.

A magnitude-versus-frequency plot of the cosine function for $(l-z_2)=27$ ft, the distance used in the experiment, also is shown in Fig. 5. As can be seen, the period of these oscillations is in close agreement with the experimental results.

Of course, infinite terminal impedance at $z=l$ is assumed for the ideal oscillations, whereas the effective impedance of the transducer and tube closure only approximate the ideal conditions. In addition, the frequency dependence of these effective impedances results in an apparent compression and expansion of the ideal oscillation period. The cause of the rather abrupt transition in the magnitude of the oscillation starting at about 0.8 kHz has not been determined.

Additional calibration results are given in the companion paper.[1]

## V. CONCLUSION

A method has been described for making reciprocity calibration measurements on electroacoustic transducers under boundary conditions that, in the past, have prevented the use of the reciprocity technique. Adaptability of this technique to the case of a rigid-walled tube makes the method well suited for calibrating underwater sound transducers at various temperatures and high static pressures and at the lower audio frequencies.

Within the imposed theoretical restrictions, the technique can be applied to resonant as well as nonresonant transducers. Theoretical justification has been given and the theory has been verified in a practical calibration facility. Results obtained are in acceptable agreement with free-field and coupler measurements.

The need for achieving plane progressive waves adds some complexity to the measurement procedure as well as to the associated electroacoustic equipment, but the measurement process is not unduly tedious. The maximum error due to failure to attain a standing-wave ratio of 1 is approximately proportional to the square root of the product of the standing-wave ratios achieved under sending and receiving conditions.

It is emphasized here that the procedures used to make absolute measurements are directly applicable to comparison calibrations also. Thus, the discussion relating measurements in the tube to those in a free field can be applied to comparison measurements as well.


## ACKNOWLEDGMENTS

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13. ABSTRACT

A new application of the reciprocity principle has been developed for calibrating electroacoustic transducers in a closed vessel at static pressures to 8500 psi and frequencies from 100 to 1500 Hz. The necessary plane progressive wavefield is provided by sound propagation in the longitudinal mode within a sound channel terminated at both ends with active impedances. The technique is particularly well suited to the calibration of underwater-sound transducers because the high static pressure under which many of them must operate, as well as their size, mechanical construction, and operating frequency, often prevents the use of more conventional methods. The case of a rigid-walled, water-filled tube is analyzed theoretically. Results of measurements made by this method in a practical high-pressure calibration chamber are shown.

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