COLLAPSE, BUCKLING AND POST FAILURE BEHAVIOR OF CYLINDRICAL SHELLS UNDER ELEVATED TEMPERATURE AND DYNAMIC LOADS

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ABSTRACT

It has been shown experimentally that cylindrical shells subjected to side on air blast will go into two main types of failure. These are buckling and collapse. The buckling type of failure is described by a deformation pattern which consists of a number of lobes around the periphery of the shell and one half wave length along the length. The collapse is described by a straight failure hinge. The type of failure will depend upon the geometry of the shell and can be predicted from an elastic stress and buckling analysis of the shell as discussed in this report. The analytical details of representing the deformation patterns and the method for calculating the energy absorbed and the resulting deflection under normal and elevated temperature conditions due to a given loading is described completely in this report. In addition to energy and impulse methods of solving the problem a deformation type variational principle is employed to set up the governing nonlinear differential equation for the time dependent deflection in the plastic region. The biaxial stress strain law used for both the normal and elevated temperature cases is an elastic linear hardening law.

Of greatest importance in the report is the computation of the energy absorbed or work done by internal forces in the shell for very large plastic deformations. This work or energy can be used to compute the impulse to give a prescribed deformation; it can be used to compute the deformation for a given energy input to the shell (assuming all of it goes into plastic deformation); it can be used to compute static load for a given deformation; or it can be used as a design criterion itself.

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LIST OF SYMBOLS

$$\sigma_i = \frac{\sqrt{2}}{2}\sqrt{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)}$$

(Octahedral shear stress is $\sqrt{2}/_3\,\sigma_i$)

$$e_i = \frac{\sqrt{2}}{3}\sqrt{(\epsilon_x-\epsilon_y)^2+(\epsilon_y-\epsilon_z)^2+(\epsilon_z-\epsilon_x)^2+\frac{3}{2}(\gamma_{xy}^2+\gamma_{yz}^2+\gamma_{zx}^2)}$$

$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}$ six components of stress; σ's are direct stresses - τ's are shear stresses

$\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ six components of strain; ϵ's are direct strains - γ's are shear strains

$\theta = \epsilon_x + \epsilon_y + \epsilon_z$

K bulk modulus of material

Σ volume of the body

$\epsilon_1, \epsilon_2, \gamma$ strains in the midplane of the shell

z radial distance from the midplane of the shell to any element in the shell

x, ϕ cylindrical coordinates describing the position of the element along the length (x) and around the periphery (ϕ)

$y = a\phi$

a radius of the midplane of the shell (mean radius of the shell)

κ_1, κ_2, τ curvatures (κ) and twist (τ) associated with bending and twisting of the shell element

u, v, w displacements of the shell element; u longitudinal, v tangential, w radial

E modulus of elasticity of shell material

ν Poisson's ratio for shell material (assumed ½ for plastic region)

$\omega(e_i)$ function of e_i

e_s strain at which yielding in pure tension would occur

σ_s yield stress in pure tension

λ parameter involved in the plastic-linear hardening stress strain law; it is the slope of the σ_i-e_i curve in the plastic region

t	thickness of the shell
L	length of the shell
V	work done in deforming shell
dm	element of mass
$\dot{w}$	velocity of mass in radial direction
I	impulse per unit mass
ρ	mass density of shell material
T	initial kinetic energy put into shell by pressure pulse
I_t	total impulse
w_0	amplitude of w
$N_\phi, N_x, N_{x\phi}$	stress resultants
$f_1(\phi), f_2(\phi), f_3(\phi)$ $f_4(\phi)$	arbitrary functions of ϕ which are determined from the boundary conditions
$p(x,\phi)$	pressure distribution
β	parameter used to define the peripheral pressure distribution
n	number of circumferential waves in buckling pattern
$(p_0)_c$	pressure which initiates collapse
$(q_{cr})_b$	pressure which initiates buckling
$f(x,y)$	spatial distribution of the impulse
$F(x',\phi)$	spatial distribution of the deflection
x'	$= x/L$
d_0	width of plastic hinge
k	parameter defining periphery die out of spatial deflection distribution in buckling
p_0, p_1	pressure parameters defining distribution of pressure such that $p_0 + p_1$ is the pressure at $\phi = 0$
$\bar{p}$	$= p_1/(p_0+p_1)$
D	diameter of shell
p	$= p_0 + p_1$
$\bar{V}$	$= V\Big/\frac{\sigma_y t \pi L}{\sqrt{3}}$

$\bar{I}_1$ portion of the nondimensional energy function which arises when strain hardening is present (see eq. 19b and Figs.8, 9a-9c)

$\bar{V}_1$ portion of the nondimensional energy function which is present for both hardening and nonhardening cases; for a perfectly plastic material (λ =1) it is the only part of the function which remains (see eq. 19b and Figs.8, 9a-9c)

$(\bar{I}_1)_c$ the value of $\bar{I}_1$ for a collapse pattern

$(\bar{I}_1)_B$ the value of $\bar{I}_1$ for a buckling pattern

$(\bar{V}_1)_c$ the value of $\bar{V}_1$ for a collapse pattern

$(\bar{V}_1)_B$ the value of $\bar{V}_1$ for a buckling pattern

1. Introduction

A. General

Shell theory, especially the dynamic plasticity of shells, is in such a state at the present time that one can only hope to obtain approximate solutions to the problems that are of practical interest today. Very little work has been done on the dynamic plasticity of shells due in part to the mathematical complexities of the theory and probably due to the lack of experimental evidence with which to check the results of the theoretical developments. Fortunately the electronic compter will enable us to overcome some of the mathematical difficulties.

Extensive use is being made of cylindrical shells in missiles and submarines. Therefore more complete experiments are being conducted on these structures and more experimental evidence is becoming available to those working in the field of dynamic elasticity and plasticity. More theoretical develppment on this problem is needed and it is for this purpose that the present report has been written. This report contains an approximate method for predicting large elastic and plastic deformations of shells under static and dynamic loads.

Experiments have shown[1,2] that cylindrical shells subjected to side on blast can go into two main types of failure. These are buckling and collapse. The buckling type of failure is described by a deformation pattern which consists of a number of lobes around the periphery of the shell and one half wave length along the length as shown in Figure 1a. Both of these figures are taken from Schuman's experimental results.[1,2] The type of failure will depend upon the geometry of the shell and can be predicted from an elastic stress and buckling analysis of the shell as will be seen later in this report.

B. Objective of work, definition of problem and philosophy behind method of approach

The main objective of this work is to predict the final plastic deformation of a cylindrical shell of given geometry at a given temperature exposed to a side on blast of predetermined charge weight exploded at a known distance from the shell. There are two facits to the problem; the first is to obtain the blast pressure parameters and the second is to determine the plastic response of the shell under dynamic loading. The plastic problem is a large deflection problem involving deformations which are many times the thickness of the shell; deflections which may be of the order of the radius of the shell. Furthermore the blast almost always occurs as a side on load so that the shock wave progresses at some angle to the longitudinal axis of the shell. This will induce a nonaxisymmetrical loading response. The shell has end supports and the distance between these supports can be small. Thus a deformation pattern will result which depends upon both the longitudinal and peripheral coordinates.

A complete solution of this problem would involve solving a set of large deformation shell equations subject to a yield condition and an appropriate stress strain law of the incremental type.. A solution of the above type would involve a very great effort resulting in a very large computer program. Since the input parameters to the shell analysis such as material properties and details of the loading can only be obtained to a very limited accuracy and errors accrue in numerical solution of large computer problems one might well ask the question whether the large amount of numerical effort necessary to solve particular cases using numerical techniques will give any more accuracy than rough approximations. In view of the above evauluation of the problem, it only seems plausible to attempt an approximate solution based on the simplified deformation theory of plasticity.

Let it be clear what assumptions are to be made in this theory and what are the basic ideas that are being proposed. Firstly it is being assumed that the plastic stress strain law of the material can be approximated by a deformation type law with linear hardening in which the octehedral shear stress is a unique function of the octehedral shear strain. Secondly, it is assumed that once collapse or buckling is reached then the entire shell is in the plastic region. In this theory it is assumed that as the shell is loaded it can either buckle or go into a collapse failure. Once the type of failure is determined it is assumed that the shell will continue in this type of failure pattern as it deforms plastically. In order to determine whether collapse or buckling will occur several methods are proposed. The simplest method is to use elastic membrane shell theory to calculate the load at which yielding starts to occur at the center of the shell -- this can be called the yield load which starts collapse. The buckling load can then be computed by classical elastic buckling theory. If the yield load is less than the buckling load the shell should collapse and if it is greater, the shell should buckle.

C.

A more sophisticated approach would be to compute the static collapse load with a given collapse pattern and compare this with the plastic static buckling loads if the collapse load is less than the buckling load collapse should occur and vice versa. Once it is determined which type of failure will occur from the above simplified analysis then the work done by the internal forces in deforming the shell plastically is computed by assuming either a collapse or buckling pattern as previously determined. In the determination of this plastic work it is assumed that only the lateral deformation (i.e. deformation perpendicular to the cylinder axis) is of significance and that the longitudinal and tangential displacements can be neglected.

Several criteria are presented for computing the deformation under the blast. One such criterion is to equate the change in kinetic energy of the shell to the work done by the internal forces during this change.

The change in kinetic energy is computed in terms of the impulse given to the shell. The result is an expression for the impulse which will result in a given deformation. A second criterion is to estimate the energy flux (energy/unit area) delivered to the shell from the blast and then equate the energy flux times the shell surface area to the work done by the internal forces. The result will be an approximate expression for the deformation in terms of the charge weight and distance from the explosion.

A third criterion is to employ Hamilton's principle in the plastic region, assuming a given collapse or buckling pattern. The result will be a differential equation for the lateral deformation as a function of time. For this calculation the pressure distribution must be known explicitly. The use of the Hamilton's principle with the work computed on the basis of deformation theory is, of course, open to question since there are more accurate variational principles for plasticity. However, if only loading is considered and we assume no elastic recovery after the maximum deformation is reached it should give a reasonable approximation. The assumption of no elastic recovery has been checked for plates under static loading[17] and has been found to be a valid one.

C. Physical arguments backing up method of approach

The theory depends upon the assumption that the shell buckles or collapses and then continues deforming plastically in the same type pattern. The accuracy of the computation of buckling and collapse loads does not seem critical as long as we can be sure which pattern will occur. In the plastic calculations the detailed assumption of a collapse pattern is not critical since it is determined by one parameter -- the length of the plastic hinge. However the buckling pattern is quite a bit more complicated since it depends upon the number of buckling lobes and the assumption of how rapidly the deformation dies out around the periphery as one proceeds from the front where the blast load first hits. An exponentially decaying cosine wave is assumed in one particular calculation but other types of patterns are possible. One other such pattern is discussed in this report and consists of a series of hinges around the periphery.

The assumption on the degree of decay around the periphery is guided by the experimental results. The number of lobes is computed from classical buckling theory and has been found to agree with experimental results. The deformation patterns which have been employed in the plastic theory are likewise based upon experimental results.

The major item of contention concerning the validity of this theory is the fact that a deformation (total) type stress-strain law has been used instead of a flow (incremental) law and that a linear hardening law has been used as an approximation to the real material law; and that the elevated temperature is assumed to decrease the hardening and decrease the yield point. The latter assumptions concerning elevated temperature behavior have been demonstrated for one dimensional systems; it therefore seems plausible that the same type of behavior should hold true between the octohedral shear stress and strain for biaxial systems. If we are concerned with loading paths consisting of the first laoding and assume that there is no elastic recovery but that the shell remains permanently at its largest deformation (as was shown approximately for plates[17]), it would seem plausible that a total strain theory should give reasonable results.

II. Theory

A. Plastic energy absorption relations

Assume that the shell is exposed to an impulsive load of short duration which imparts an initial velocity to the structure. The problem is the equivalent to one in which the structure has an initial kinetic energy.[3,4] If no instabilities occur in the elastic region when the structure goes plastic there could be a plastic instability point. If instability does not occur at all and the structure is loaded further, then there will be a point at which the structure will collapse.

The deformation energy (or work done by the internal forces) per unit volume of an elastic-plastic body can be written:[5]

$$V = \int_0^{e_i} \sigma_i \, de_i + \frac{k\theta^2}{2} \qquad [1]$$

$$\sigma_i = \frac{\sqrt{2}}{2}\sqrt{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)}$$

$$e_i = \frac{\sqrt{2}}{3}\sqrt{(\epsilon_x-\epsilon_y)^2+(\epsilon_y-\epsilon_z)^2+(\epsilon_z-\epsilon_x)^2+\frac{3}{2}(\gamma_{xy}^2+\gamma_{yz}^2+\gamma_{zx}^2)}$$

$$\theta = \epsilon_x+\epsilon_y+\epsilon_z \quad , \qquad k = \text{Bulk Modulus}$$

where $\sigma_x, \sigma_y, \sigma_z$ are the direct stresses, $\tau_{xy}, \tau_{yz}, \tau_{zx}$ are the shear stresses, $\epsilon_x, \epsilon_y, \epsilon_z$ the direct strains and $\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ the shear strains.

The curve of σ_i vs e_i describes the stress-strain law of the material. Assuming an incompressible material ($\theta = 0$) and particularizing our analysis to a thin shell, we obtain

$$V = \int_\Sigma \left[\int_0^{e_i} \sigma_i \, de_i\right] d\Sigma$$

$$\sigma_i = \sqrt{\sigma_x^2-\sigma_x\sigma_y+\sigma_y^2+3\tau_{xy}^2} \quad , \quad e_i = \frac{2}{\sqrt{3}}\sqrt{\epsilon_x^2+\epsilon_x\epsilon_y+\epsilon_y^2+\frac{1}{4}\gamma_{xy}^2} \qquad [2]$$

The expressions for the strains are[6]

$$\epsilon_x = \epsilon_1 - z\kappa_1 \quad , \quad \epsilon_y = \epsilon_2 - z\kappa_2 \quad , \quad \gamma_{xy} = \gamma - 2z\tau \qquad [3]$$

where $\epsilon_1, \epsilon_2, \gamma$ are the midsurface strains, z is the radial distance from the midsurface to any element within the thickness of the shell and κ_1, κ_2, τ are the curvatures and twist. The values $\epsilon_1, \epsilon_2, \gamma$ are (allowing large deflections)[7]

$$\epsilon_1 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \quad , \quad \epsilon_2 = \frac{1}{a}\frac{\partial v}{\partial \varphi} + \frac{1}{2}\left(\frac{\partial w}{a\partial \varphi}\right)^2 - \frac{w}{a} \tag{4}$$

$$\gamma = \frac{\partial v}{\partial x} + \frac{1}{a}\frac{\partial u}{\partial \varphi} + \frac{\partial w}{\partial x}\frac{\partial w}{a\partial \varphi}$$

and the curvatures and twist are

$$\kappa_1 = \frac{\partial^2 w}{\partial x^2} \quad , \quad \kappa_2 = \frac{1}{a^2}\frac{\partial^2 w}{\partial \varphi^2} + \frac{1}{a^2}\frac{\partial v}{\partial \varphi} \tag{5}$$

$$\tau = \frac{1}{a}\frac{\partial^2 w}{\partial x \partial \varphi} + \frac{1}{a}\frac{\partial v}{\partial x}$$

A general deformation type elastic-plastic stress-strain law can be written[5] (assuming a Poissons ration of ½)

$$\begin{aligned} \sigma_x &= \frac{4}{3}\frac{\sigma_i}{e_i}\left(\epsilon_x + \frac{1}{2}\epsilon_y\right) \\ \sigma_y &= \frac{4}{3}\frac{\sigma_i}{e_i}\left(\epsilon_y + \frac{1}{2}\epsilon_x\right) \\ \tau_{xy} &= \frac{1}{3}\frac{\sigma_i}{e_i}\gamma_{xy} \end{aligned} \tag{6}$$

where

$$\frac{\sigma_i}{e_i} = E\left[1 - \omega(e_i)\right] \tag{7}$$

in which $\omega(e_i) = 0$ for the elastic region. Now neglect all terms in the strains which contain u and v, assuming that they are small compared to w and its derivatives. Then

$$\epsilon_x \approx \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2} \quad , \quad \epsilon_y \approx \frac{1}{2}\left(\frac{\partial w}{a\partial \varphi}\right)^2 - \frac{w}{a} - \frac{z}{a^2}\frac{\partial^2 w}{\partial \varphi^2} \tag{8}$$

$$\gamma_{xy} \approx \frac{\partial w}{\partial x}\frac{\partial w}{a\partial \varphi} - 2\frac{z}{a}\frac{\partial^2 w}{\partial x \partial \varphi}$$

Consider an <u>elastic-linear hardening incompressible</u> material which has a stress-strain law as shown in Figure 2. This stress-strain law can be written

$$\begin{aligned} &\text{Elastic Case} \qquad \omega(e_i) = 0 \quad \text{for} \quad e_i < e_s ,\ \sigma_i < \sigma_s \\ &\text{Plastic Case} \qquad \omega(e_i) = \lambda\left(1 - e_s/e_i\right) \quad \text{for} \quad e_i > e_s ,\ \sigma_i > \sigma_s \end{aligned} \tag{9}$$

$$\lambda = 1 - (1/E)(d\sigma_i/de_i)$$

(The special case of $\lambda = 1$ describes a perfectly plastic material)

Going back to the general expression for the energy, and substituting the stress-strain law of equ. [9]

$$V = \int_0^L \int_0^{2\pi} \int_{-t/2}^{t/2} \int_0^{e_i} E e_i\left[1 - \lambda\left(1 - e_s/e_i\right)\right] de_i \, a \, d\varphi \, dx \, dz \tag{10}$$

$$V = \int_0^L \int_0^{2\pi} \int_{-t/2}^{t/2} \left[\frac{E e_i^2}{2}(1-\lambda) + E\lambda e_s e_i \right] a\,d\phi\,dx\,dz - \frac{E\lambda e_s^2}{2} 2\pi a L t \qquad [11]$$

Substituting the expression for e_i from [2]

$$V = \int_0^L \int_0^{2\pi} \int_{-t/2}^{t/2} \left[\frac{E}{2}(1-\lambda)\frac{4}{3}\xi + \frac{E\lambda e_s}{\sqrt{3}} 2\sqrt{\xi} \right] a\,d\phi\,dx\,dz - E\lambda e_s^2 \pi a L t \,; \quad \xi = \epsilon_x^2 + \epsilon_x\epsilon_y + \epsilon_y^2 + \frac{\gamma_{xy}^2}{4} \qquad [12]$$

Using the strain expressions [8] and integrating with respect to z

$$V = \int_0^L \int_0^{2\pi} \Big\{ \frac{2}{3} E(1-\lambda)\left[t\alpha + \frac{t^3}{12}\beta\right] \qquad [13]$$

$$+ \frac{2\lambda E e_s}{\sqrt{3}} \left[\frac{(2\beta z+\gamma)\sqrt{\alpha + z\gamma + z^2\beta}}{4\beta} + \frac{4\alpha\beta - \gamma^2}{8\beta\sqrt{\beta}} \sinh^{-1}\left(\frac{2\beta z + \gamma}{\sqrt{4\alpha\beta - \gamma^2}}\right) \right]_{-t/2}^{t/2} \Big\} a\,d\phi\,dx$$

$$- E\lambda e_s^2 \pi a L t \qquad [14]$$

$$\alpha(x,\phi) = \frac{1}{4}\left(\frac{\partial w}{\partial x}\right)^4 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial w}{a\partial\phi}\right)^2 - \frac{1}{2}\frac{w}{a}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{4}\left(\frac{\partial w}{a\partial\phi}\right)^4 - \frac{w}{a}\left(\frac{\partial w}{a\partial\phi}\right)^2 + \frac{w^2}{a^2}$$

$$\beta(x,\phi) = \left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{1}{a^2}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial\phi^2} + \frac{1}{a^4}\left(\frac{\partial^2 w}{\partial\phi^2}\right)^2 + \frac{1}{a^2}\left(\frac{\partial^2 w}{\partial x\partial\phi}\right)^2$$

$$\gamma(x,\phi) = -\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial^2 w}{\partial x^2}\right) - \frac{1}{a^2}\frac{\partial^2 w}{\partial\phi^2}\left(\frac{\partial w}{a\partial\phi}\right)^2 + \frac{2}{a^2}\frac{\partial^2 w}{\partial\phi^2}\frac{w}{a} - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\frac{1}{a^2}\frac{\partial^2 w}{\partial\phi^2}$$
$$- \frac{1}{2}\left(\frac{\partial^2 w}{\partial x^2}\right)^2\left(\frac{\partial w}{a\partial\phi}\right)^2 + \frac{w}{a}\frac{\partial^2 w}{\partial x^2} - \frac{1}{a}\frac{\partial^2 w}{\partial x\partial\phi}\frac{\partial w}{\partial x}\frac{\partial w}{a\partial\phi}$$

The strain terms in equation [8] which involve z are associated with bending, the other terms are the membrane strains. For very thin shells sometimes the bending strains can be neglected and the resulting energy expression will be (neglecting the last term)

$$V \approx \int_0^L \int_0^{2\pi} \left\{ \frac{2}{3} E(1-\lambda)\, t\, \alpha(x,\phi) + \frac{2\lambda E e_s}{\sqrt{3}} t\sqrt{\alpha(x,\phi)} \right\} a\,d\phi\,dx \qquad [15]$$

Some materials can be considered perfectly plastic. For this case $\lambda = 1$ in the plastic region and

$$V_p \approx \int_0^L \int_0^{2\pi} \frac{2\lambda E e_s}{\sqrt{3}} t\sqrt{\alpha(x,\phi)}\, a\,d\phi\,dx \qquad [16]$$

The energy of deformation in the elastic region can be obtained from [13] by letting $\lambda = 0$. However a more accurate expression for the elastic region can be obtained by starting with the elastic stress-strain law for any Poisson ratio, i.e.

$$\sigma_x = \frac{E}{1-\nu^2}(\epsilon_x + \nu\epsilon_y) \quad , \quad \sigma_y = \frac{E}{1-\nu^2}(\epsilon_y + \nu\epsilon_x) \qquad [6a]$$
$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$$

The elastic energy is

$$V_e = \frac{1}{2}\int_z \frac{E}{1-\nu^2}\left(\epsilon_x^2 + 2\nu\epsilon_x\epsilon_y + \epsilon_y^2 + \frac{1-\nu}{2}\gamma_{xy}^2\right) dz$$

Thus

$$V_e = \int_0^L \int_0^{2\pi} \frac{E}{2(1-\nu^2)}\left[t\alpha_e(x,\phi) + \frac{t^3}{12}\beta_e(x,\phi)\right] a\,dx\,d\phi \qquad [13a]$$

where

$$\alpha_e(x,\varphi) = \frac{1}{4}\left(\frac{\partial w}{\partial x}\right)^4 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial w}{a\partial\varphi}\right)^2 - \nu\left(\frac{\partial w}{\partial x}\right)\frac{w}{a} + \frac{1}{4}\left(\frac{\partial w}{a\partial\varphi}\right)^4 - \frac{w}{a}\left(\frac{\partial w}{a\partial\varphi}\right)^2 + \frac{w^2}{a^2} + \frac{1-\nu}{2}\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial w}{a\partial\varphi}\right)^2$$

$$\beta_e(x,\varphi) = \left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{2\nu}{a^2}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial\varphi^2} + \frac{1}{a^4}\left(\frac{\partial^2 w}{\partial\varphi^2}\right)^2 + \frac{2(1-\nu)}{a^2}\left(\frac{\partial^2 w}{\partial x\partial\varphi}\right)^2$$

In some cases the deformation of the shell may be axisymmetric and the deflection w will then be independent of φ. For this case expression [13] becomes

$$V = 2\pi a\int_0^L\left\{\frac{2}{3}E(1-\lambda)\left[t\alpha + \frac{t^3}{12}\beta\right] + \frac{2\lambda E e_s}{\sqrt{3}}\left[\frac{(2\beta z+\gamma)\sqrt{\alpha+z\gamma+z^2\beta}}{4\beta} + \frac{4\alpha\beta-\gamma^2}{8\beta\sqrt{\beta}}\sinh^{-1}\left(\frac{2\beta z+\gamma}{\sqrt{4\alpha\beta-\gamma^2}}\right)\right]_{-t/2}^{t/2}\right\}dx - E\lambda e_s^2\pi aLt \qquad [17]$$

where

$$\alpha(x) = \frac{1}{4}\left(\frac{\partial w}{\partial x}\right)^4 - \frac{1}{2}\frac{w}{a}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{w^2}{a^2}\;;\quad \beta(x) = \left(\frac{\partial^2 w}{\partial x^2}\right)^2 \qquad [18]$$

$$\gamma(x) = -\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial^2 w}{\partial x^2}\right) + \frac{w}{a}\frac{\partial^2 w}{\partial x^2}$$

Now define a new variable $x' = x/L$ and let $w = w_0 f(x',\varphi)$ then the integrals can be written in terms of dimensionless quantities as follows:

We first combine the elastic and plastic cases in one set of integrals noting that:

a. For the elastic case $\lambda = 0$

b. For the plastic case $\nu = 1/2$

The combined integrals are then

$$V = \frac{E(1-\lambda)taL}{2(1-\nu^2)}\int_0^1\int_0^{2\pi}\bar{\alpha}\,dx'd\varphi + \frac{E(1-\lambda)taL}{6(1-\nu^2)}\int_0^1\int_0^{2\pi}\bar{\beta}\,dx'd\varphi$$
$$+\frac{\lambda E e_s taL}{\sqrt{3}}\int_0^1\int_0^{2\pi}\left\{\left[\frac{(2\bar{\beta}+\bar{\gamma})\sqrt{\bar{\alpha}+\bar{\gamma}+\bar{\beta}}}{4\bar{\beta}} + \frac{(4\bar{\alpha}\bar{\beta}-\bar{\gamma}^2)}{8\bar{\beta}}\frac{1}{\sqrt{\bar{\beta}}}\sinh^{-1}\left(\frac{2\bar{\beta}+\bar{\gamma}}{\sqrt{4\bar{\alpha}\bar{\beta}-\bar{\gamma}^2}}\right)\right]\right.$$
$$\left.-\left[\frac{(-2\bar{\beta}+\bar{\gamma})\sqrt{\bar{\alpha}-\bar{\gamma}+\bar{\beta}}}{4\bar{\beta}} + \frac{(4\bar{\alpha}\bar{\beta}-\bar{\gamma}^2)}{8\bar{\beta}}\frac{1}{\sqrt{\bar{\beta}}}\sinh^{-1}\left(\frac{-2\bar{\beta}+\bar{\gamma}}{\sqrt{4\bar{\alpha}\bar{\beta}-\bar{\gamma}^2}}\right)\right]\right\}dx'd\varphi$$
$$-E\lambda e_s^2\pi aLt \qquad [19]$$

where

$$\bar{\alpha}(x',\varphi) = \left(\frac{w_0}{a}\right)^4\left(\frac{a}{L}\right)^4\frac{1}{4}\left(\frac{\partial f}{\partial x'}\right)^4 + \left(\frac{w_0}{a}\right)^4\left(\frac{a}{L}\right)^2\frac{1}{2}\left(\frac{\partial f}{\partial x'}\right)^2\left(\frac{\partial f}{\partial\varphi}\right)^2 - \nu\left(\frac{w_0}{a}\right)^3\left(\frac{a}{L}\right)^2 f\left(\frac{\partial f}{\partial x'}\right)^2$$
$$+\frac{1}{4}\left(\frac{w_0}{a}\right)^4\left(\frac{\partial f}{\partial\varphi}\right)^4 - \left(\frac{w_0}{a}\right)^3 f\left(\frac{\partial f}{\partial\varphi}\right)^2 + \left(\frac{w_0}{a}\right)^2 f^2 \qquad [20]$$

$$\bar{\gamma}(x',\varphi) = \frac{t}{2}\gamma = -\left(\frac{w_0}{a}\right)^3\left(\frac{a}{L}\right)^4\frac{t}{2a}\left(\frac{\partial f}{\partial x'}\right)^2\left(\frac{\partial^2 f}{\partial x'^2}\right) - \left(\frac{w_0}{a}\right)^3\frac{t}{2a}\left(\frac{\partial^2 f}{\partial\varphi^2}\right)\left(\frac{\partial f}{\partial\varphi}\right)^2$$
$$+2\left(\frac{w_0}{a}\right)^2\frac{t}{2a}\frac{\partial^2 f}{\partial\varphi^2}f - \nu\left(\frac{w_0}{a}\right)^3\left(\frac{a}{L}\right)^2\frac{t}{2a}\left(\frac{\partial f}{\partial x'}\right)^2\left(\frac{\partial^2 f}{\partial\varphi^2}\right) - \nu\left(\frac{w_0}{a}\right)^3\left(\frac{a}{L}\right)^2\frac{t}{2a}\left(\frac{\partial^2 f}{\partial x'^2}\right)\left(\frac{\partial f}{\partial\varphi}\right)^2$$
$$+2\nu\left(\frac{w_0}{a}\right)^2\left(\frac{a}{L}\right)^2\frac{t}{2a}f\left(\frac{\partial^2 f}{\partial x'^2}\right) - 4\frac{(1-\nu)}{2}\left(\frac{w_0}{a}\right)^3\left(\frac{a}{L}\right)^2\frac{t}{2a}\left(\frac{\partial^2 f}{\partial x'\partial\varphi}\right)\left(\frac{\partial f}{\partial x'}\right)\left(\frac{\partial f}{\partial\varphi}\right)$$

$$\bar{\beta}(x',\varphi) = \left(\frac{t}{2}\right)^2\beta = \left(\frac{w_0}{a}\right)^2\left(\frac{a}{L}\right)^4\left(\frac{t}{2a}\right)^2\left(\frac{\partial^2 f}{\partial x'^2}\right)^2 + 2\nu\left(\frac{w_0}{a}\right)^2\left(\frac{t}{2a}\right)^2\left(\frac{a}{L}\right)^2\left(\frac{\partial^2 f}{\partial x'^2}\right)\left(\frac{\partial^2 f}{\partial\varphi^2}\right)$$
$$+\left(\frac{w_0}{a}\right)^2\left(\frac{t}{2a}\right)^2\left(\frac{\partial^2 f}{\partial\varphi^2}\right)^2 + 2(1-\nu)\left(\frac{w_0}{a}\right)^2\left(\frac{a}{L}\right)^2\left(\frac{t}{2a}\right)^2\left(\frac{\partial^2 f}{\partial x'\partial\varphi}\right)^2$$

Thus the integrals are dimensionless quantities which are functions of the dimensionless ratios w_0/a, a/L, $t/2a$. Equation [9] is the form that the electronic computer used to compute the integrals numerically for a given shape $f(x';\phi)$, given parameters a/L, $t/2a$ and a series of values of w_0/a.

B. Extension to thermoplastic deformation

1. Employment of a bilinear stress-strain law

One dimensional thermoplastic tests have been conducted by Alder and Phillips[18] and Bell,[19,20] some of the results of which are discussed in a book by Goldsmith.[21] The effect of temperature on the stress-strain law is also discussed qualitatively by Rhinehart and Pearson.[22] To this author's knowledge there have been no biaxial thermoplastic stress-strain tests to date. In fact the basic elements of thermoplasticity are just starting to be discussed in the literature.[23] Therefore any biaxial thermoplastic stress-strain law which is assumed will have to be based mostly upon intuition and partly upon extrapolation of the one dimensional tests. Based upon the above mentioned references it seems that the effect of temperature on the stress-strain law is to lower the yield stress, lower the elastic modulus and decrease the degree of hardening of the material. If a bilinear (elastic-linear hardening) stress-strain law is assumed for the room temperature case, then it seems logical to use the same type law for the thermal case with the degree of hardening decreased and the yield stress lowered in the plastic region. The assumption will be that the relation between σ_i and ϵ_i for the biaxial case is the same as that between σ (stress) and ϵ (strain) in a one dimensional test.

In terms of the previous parameters this would mean as the temperature is increased the λ is increased (with an upper limit of $\lambda = 1$) and the yield stress, σ_s, is decreased. In the elastic region the modulus of elasticity would decrease. The effect of temperature on σ_s and E is contained in the literature,[24] but the effect on hardening (λ) must be determined at the present time by extrapolation from one dimentional tests.[18,19,20] In materials which are almost perfectly plastic ($\lambda = 1$) at room temperature we can only assume that the total effect of elevated temperature will be to decrease the yield stress.

Keeping in mind the above concepts let us rewrite equation [19] as follows:

$$\frac{V}{\frac{\lambda E e_s t a L}{\sqrt{3}}} = \frac{\sqrt{3}(1-\lambda)}{2\lambda e_s(1-\nu^2)}\left\{\int_0^1\!\!\int_0^{2\pi}\bar{\alpha}\,dx'\,d\phi + \frac{1}{3}\int_0^1\!\!\int_0^{2\pi}\bar{\beta}\,dx'\,d\phi\right\} + \left\{\int_0^1\!\!\int_0^{2\pi}\Delta\,dx'\,d\phi\right\} - \frac{E\lambda e_s^2 \pi a L t}{\frac{\lambda E e_s t a L}{\sqrt{3}}} \qquad [19a]$$

where Δ is the term in the bracket of the last integral in eq. [19]. This expression can be further rewritten as

$$\bar{V} = \frac{V}{\frac{\sigma_s t a L}{\sqrt{3}}} = \frac{\sqrt{3}(1-\lambda)}{2e_s(1-\nu^2)}\{\bar{I}_1\} + \lambda\{\bar{V}_1\} - \lambda\sqrt{3}\pi e_s \qquad [19b]$$

where I denotes the sum of the $\bar{\alpha}$ and $\bar{\beta}$ integrals and V_1 corresponds to the Δ integral.
We need only compute the values of I and V_1 (which are both independent of λ) and the above expression gives us V for any λ. Thus in the plastic region the elevated temperature case can be obtained directly from the results of the room temperature case by assuming the appropriate values for ϵ_s, σ_s and λ. All three of these parameters can be extrapolated by fitting a bilinear curve to the one dimensional thermoplastic tests.[18,19,20]

2. Extrapolation of the Bell Parabolic Law[19,20]

In lieu of assuming a bilinear law one might extend the one dimensional Bell Law[19,20] which is

$$\sigma_x = \beta_0 \left(1 - \frac{T}{T_m}\right) \sqrt{\epsilon_x}$$

where β_0 is a universal constant ($58.0\ kg/mm^2$), σ_x is the normal stress and ϵ_x the strain; T is the temperature in degrees Kelvin and T_m is the melting point of the metal. It could be assumed that the same functional relationship holds between the octehedral shear stress and strain, i.e.

$$\sigma_i = \beta_0 \left(1 - \frac{T}{T_m}\right) \sqrt{\epsilon_i}$$

However in the absence of any biaxial tests we will consider here only the approximation given by the bilinear curve. At some late date either an extension to the Bell Law for the biaxial case or a modification of it will be studied further.

C. Impulse-energy relation

Let I be the impulse per unit mass applied to the shell. The impulse momentum relation for an elemental mass dm can be written: (only the lateral velocity $\dot{w}$ is being considered, $\dot{u}$ and $\dot{v}$ are being neglected)

$$\dot{w}\, dm = I\, dm \qquad [21]$$

where $\dot{w}$ is the lateral velocity imparted to the mass by the impulse. Thus

$$\dot{w} = I \qquad [22]$$

The Kinetic energy imparted to the shell is

$$T = \int_A \tfrac{1}{2} \rho t \dot{w}^2 dA = \tfrac{1}{2} \int_A \rho t I^2 dA \qquad [23]$$

where ρt is the mass per unit area and dA is an elemental area. The impulse can vary over the surface. Therefore write the impulse as

$$I(x,y) = I_0 f(x,y) \qquad [24]$$

Thus from [21]

$$T = \tfrac{1}{2} \rho t\, I_0^2 \int_A f^2(x,y)\, dA \qquad [25]$$

Equating the initial kinetic energy to the energy of deformation absorbed by the shell the expression for the impulse per unit mass becomes:

$$I_0 = \sqrt{V \frac{2}{\rho t \int_A f^2(x,y)\, dA}} \qquad [26]$$

The total impulse on the shell will then be

$$I_t = \int_A I_0 \rho t f(x,y)\, dA = \int_A \sqrt{V \frac{2\rho t}{\int_A f^2(x,y)\, dA}}\, f(x,y)\, dA \qquad [27]$$

The impulse per unit area will be

$$I_0 \rho t = \sqrt{V \frac{2\rho t}{\int_A f^2(x,y)\, dA}} \qquad [28]$$

If the impulse is uniform

$$f(x,y) = 1 \qquad [29]$$

and

$$I_0 \rho t = \sqrt{V \frac{\rho t}{\pi a L}} \qquad [30]$$

It should be mentioned that the expressions for the impulse hold when the shell has reached its final state of deformation and has come to rest and all the initial kinetic energy is used up in energy or work of deformation.

D. Approximation to energy flux delivered from explosion

The energy flux density (energy per unit area) that is directed toward the cylinder from the blast is a function of the charge weight and distance of the charge from the cylinder. It has been found[3,25] that the energy flux for an underwater explosion can be written approximately as

$$E_f \approx C\, W/R^2$$

where E_f is the energy per unit area, C is a constant, W is the charge weight and R is the distance from the explosion to the target. Assuming a constant energy distribution over the cylinder the total energy available to do damage to the cylinder would be

$$E_t = C \frac{W}{R^2} 2\pi a L$$

where a is the cylinder radius and L is the length.

Using an energy balance this total energy can be equated to the work done by the internal forces during deformation of the cylinder (this work was denoted by V in the previous sections of the report)
Thus

$$V = E_t$$

or

$$C \frac{W}{R^2} 2\pi a L = V = \bar{V} \frac{\sigma_s t a L}{\sqrt{3}}$$

where $\bar{V}$ is the nondimensional energy function as given by [19b]. This function is computed later in the report for a variety of cases. Using this $\bar{V}$ function together with experimental results on buckling and collapse of steel and aluminum cylinders as given by Schuman[1,2] we could derive a value for C and thereby obtain a seim-empirical formula for obtaining the plastic deformation of cylinders for any charge weight and distance. Such a semi-empirical relation will be discussed later in the report,

E. Approximate pressure dustribution and impulse delivered to shell from blast

1. Pressure distribution

If one proposes to use a more exact approach such as the governing differential equations or variational principles in the plastic region for computing the response of the shell to the blast, the pressure distribution in space and time must be known over the entire cylinder. Approximate pressure results for overpressures less than 25 psi are given in the literature.[26] Tests are currently being conducted at the Aberdeen Proving Ground[27] to verify these results. One such test result is illustrated in Fig. (10). Qualitatively the pressute time plots look similar to the plots that are obtained by employing the curves in the Nuclear Effects Handbook.[26] The Aberdeen Experiments seem to show a finite duration of the positive phase whereas the results form the Handbook show an exponential decay of the pressure with time. A calculation of the pressure vs. time for the test case at three locations around the cylinder using the curves in the Nuclear Effects Handbook together with the plots of Baker and Schuman[28] and the work of Goodman[29] is shown in Figure (11). It is plainly seen that there is no good agreement between the Handbook theory[26] and the experiment. This indeed points to the fact that ~~many~~ more pressure measurements and comparisons are in order.

2. Impulse distribution

Along with the pressures at various locations around the periphery the impulse was also measured by integrating the pressure time relation at the various points. For the test cylinder described above the impulse distribution was as shown below:

Angle	0°	90°	180°	240°	300°	330°
Impulse (psi-ms)	8	4	8	6	6	7

This indicates that the total impulse does not vary drastically from point to point around the periphery as was originally anticipated in earlier work[15] when an exponentially decaying impulse was assumed.

F. Simplifications in the energy expression

The complete energy expression can be written

$$V = \iint_{\Sigma}\left[\int_0^{e_s}\sigma_i\,de_i + \int_{e_s}^{e_i}\sigma_i\,de_i\right]d\Sigma \qquad [31]$$

$$= \int_0^L\int_0^{2\pi}\int_{-\frac{t}{2}}^{\frac{t}{2}}\left[\frac{Ee_i^2}{2}(1-\lambda) + E\lambda e_s e_i\right]a\,d\phi\,dx\,dz - \frac{E\lambda e_s^2}{2}2\pi a L t$$

For very thin shells undergoing very large plastic deformations $e_i >> e_s$ if $\lambda = 1$ (perfectly plastic material), then

$$V \approx \int_0^L\int_0^{2\pi}\frac{2Ee_s}{\sqrt{3}}t\sqrt{\alpha(x,\phi)}\,a\,d\phi\,dx \qquad [32]$$

For some physical cases $w_0/a << 1$, $a/L << 1$ so that

$$\alpha \approx \left(\frac{w_0}{a}\right)^2 f^2$$

Thus

$$V \approx \frac{2\sigma_s t w_0 L}{\sqrt{3}}\int_0^1\int_0^{2\pi} f(x',\phi)\,d\phi\,dx' \qquad [33]$$

III. Elastic analysis

A. Buckling

A rather extensive study of elastic buckling of a cylindrical shell under nonuniform lateral pressure is given by Almroth.[8] He represents the pressure on the shell by the relation

$$p(\phi) = p_0 + p_1\cos\phi \qquad [34]$$

The pressure at $\phi = 0$ is $(p_0 + p_1)$. Almroth obtains the critical value of $(p_0 + p_1)$ for buckling of the shell. The nonuniformity of the pressure is described by a parameter $\bar{p}$ defined as

$$\bar{p} = p_1/(p_0 + p_1) \qquad [35]$$

He obtains a variety of critical load curves for $\bar{p} = 0.5$; this is the case which will be considered here. These buckling curves are shown as solid lines in Figure 3. The yield load corresponding to a nonuniform pressure as given above with $\bar{p} = 0.5$ will be considered in the next section. In figure 3 it was assumed that $E = 1000\sigma_s$, σ_s being the yield stress in pure tension.

For uniform loading the classical theory of buckling of cylindrical shells is presented by Timoshenko.[9] It is found that the lateral pressure at which buckling under uniform loading will occur is given by the following relation:[10]

$$(q_{cr})_b = \frac{Et}{a(1-\nu^2)}\left[\frac{1-\nu^2}{(n^2-1)\left(1+\frac{n^2L^2}{\pi^2a^2}\right)^2} + \frac{t^2}{12a^2}\left(n^2-1+\frac{2n^2-1-\nu}{1+\frac{n^2L^2}{\pi^2a^2}}\right)\right] \qquad [36]$$

where n is the number of full waves around the periphery of the shell. It has been found by Reynolds[11] that in lobar buckling such as this, the circumferential parameter n approximately satisfies the relation

$$\frac{1}{1+ n^2L^2/\pi^2a^2} \approx 1.23 \frac{\sqrt{at}}{L} \qquad [37]$$

and that buckling will always occur with only one half wave along the length. In other words, if m denotes the number of axial half waves along the length, then m = 1 and n is determined from equation [37]. The factor 1.23 has to be adjusted so that n turns out as a whole number. Once n is determined then equation [36] will give the value of the buckling load for uniform loading.

B. Yield or collapse

Assume that the shell is thin and that membrane theory is adequate to describe the stress patterns in the shell. Assume also that the shell is of length l and is supported at each end. Take the origin or coordinates at the center of the shell as shown in Figure 4. The membrane forces are shown on the differential element in Figure 5.

If $p(x,\varphi)$ is the static load per unit area applied laterally to the shell then it follows from the basic membrane equations[12,13] that

$$\begin{aligned} N_\varphi &= a\,p(x,\varphi) \\ N_{x\varphi} &= -\int \frac{1}{a}\frac{\partial N_\varphi}{\partial \varphi}dx + f_1(\varphi) \\ N_x &= -\int \frac{1}{a}\frac{\partial N_{x\varphi}}{\partial \varphi} + f_2(\varphi) \end{aligned} \qquad [38]$$

where $f_1(\varphi)$ and $f_2(\varphi)$ are functions of φ which are to be determined from the boundary conditions on N_x, $N_{x\varphi}$, N_φ. If some of the boundary conditions are given in terms of displacements then the following membrane equations in terms of displacements must be utilized:[12]

$$\begin{aligned} Etu &= \int (N_x - \nu N_\varphi)\,dx + f_3(\varphi) \\ Etv &= 2(1+\nu)\int N_{x\varphi}\,dx - \frac{Et}{a}\int \frac{\partial u}{\partial \varphi}dx + f_4(\varphi) \\ Etw &= a(N_\varphi - \nu N_x) + Et\frac{\partial v}{\partial \varphi} \end{aligned} \qquad [39]$$

where t is the shell thickness, E is the modulus of elasticity, u,v,w are the displacements (see Fig. 5) and $f_3(\varphi)$, $f_4(\varphi)$ are arbitrary functions to be determined from the boundary conditions.

Now assume that the pressure $p(x,\varphi)$ can be represented as

$$p = p_0 f(\varphi) \quad , \quad f(\varphi) = (1 + {}^{p_1}/_{p_0} \cos\varphi) \qquad [40]$$

The solution will be obtained for the boundary conditions

$$u = 0 \quad \text{at} \quad x = \pm L/2$$
$$N_{x\phi} = 0 \quad \text{at} \quad x = 0 \text{ by symmetry} \qquad [41]$$

By straight forward integration of equations [38] and [39] subject to boundary conditions [41] it is found that

$$N_\phi = a p_0 f(\phi) \qquad [42]$$
$$N_{x\phi} = -p_0 x f'(\phi)$$
$$N_x = \frac{p_0}{a}\frac{x^2}{2} f''(\phi) + \nu a p_0 f(\phi) - \frac{p_0}{a}\frac{L^2}{24} f''(\phi)$$

in which

$$f'(\phi) = -p_1/p_0 \sin\phi \quad , \quad f''(\phi) = -p_1/p_0 \cos\phi$$

So at the center of the shell $x = 0$, $\phi = 0$

$$N_\phi = a(p_0 + p_1) \quad , \quad N_{x\phi} = 0 \qquad [43]$$
$$N_x = \nu a(p_0+p_1) + \frac{L^2}{24a} p_1 = \nu a(p_0+p_1) + \frac{L^2}{24}(p_0+p_1)\bar{\beta}$$

Now let $p = (p_0 + p_1)$ (the critical yield pressure at $\phi = 0$; corresponds to Almroth's calculations for buckling) [44]

Then

$$N_\phi = a p \qquad [45]$$
$$N_x = \nu a p + \frac{L^2}{24} p \bar{\beta}$$

Using the Von Mises yield condition

$$N_x^2 - N_x N_\phi + N_\phi^2 + 3 N_{x\phi}^2 = \sigma_0^2 t^2 \qquad [46]$$

The critical pressure for yield at $x = \phi = 0$ is

$$(p)_c = \sigma_s \frac{t}{a} \frac{1}{\sqrt{(1-\nu+\nu^2) + \bar{\beta}(L^2/24a^2)(2\nu-1) + \bar{\beta}^2 (L^2/24a^2)^2}} \qquad [47]$$

which for $\bar{\beta} = 0.5$, $\nu = .3$ reduces to

$$(p)_c = \sigma_s \frac{2}{D/t} \frac{1}{\sqrt{.79 - .03328 (L/D)^2 + .00696 (L/D)^4}} \qquad [48]$$

These yield curves are shown as dotted lines in Figure 3.

C. The criterion for buckling or yield

It is clear from Schuman's experiments[12] that both buckling and yield collapse can occur. The main problem is to be able to predict which type will take place. Once buckling or collapse has commenced the plastic deformation will take place in that particular pattern into which the failure has started. It should be made clear at this point that the collapse and buckling criteria are not to be used to approximate any of the dynamical parameters of the shell; they are to be used only to determine which type of failure will start. Once this is known then the plastic analysis as given in the previous sections will be employed to determine the plastic deflections and impulse values. Buckling or yield could be predicted by the above relations in the previous section by employing the following criterion:

> If the buckling load is less than the yield load the shell should buckle; if the yield load is less than the buckling then the shell should collapse.

One might argue that we need a more accurate criterion such as the collapse load which has been defined as[30] the load for which the structure remains in equilibrium, but the displacements can increase indefinitely, geometry changes being ignored. As will be seen later, due to the nonlinearity of the displacements and due to possible hardening effects, we cannot determine a "collapse load" as such. A more positive criterion (as to whether the shell will collapse or buckle) such as the yield point load and elastic buckling load used above may seem over simplified, but it is, very subtlely, a more realistic outlook.

IV. Initial Plastic Analysis

A. General

If we attempt to define a static collapse load by equating the work done by the external forces to the work done by internal forces, the following relation would result

$$\int_0^{\ell}\int_0^{2\pi} p(x,\varphi)\, w(x,\varphi)\, a\, dx\, d\varphi = V \text{ or } W = V$$

where W is the work done by the external load, $p(x,\varphi)$, during the deformation, $w(x,\varphi)$, and V is the work done by the internal forces during this deflection as defined previously in the report. Using this criterion let us attempt to define a collapse load for the cases of interest.

B. Axisymmetrical Collapse

Consider the axially symmetric lateral loading of a perfectly plastic shell subjected to both static and dynamic loading which is assumed uniform over the shell. The problem is to estimate the static collapse load and to determine the impulse-deflection relationship in the plastic region.

Shells under axially symmetric external load usually collapse with a deformation pattern resembling two frustrums of a cone with a hinge circle at the center of the cylinder as shown in Figure 6. According to the sign convention used in this analysis w is positive inward. The work done by the internal forces in this deformation is

$$V = \frac{2}{\sqrt{3}}\sigma_s t\, 2\pi a \left\{ \int_{-L/2}^{0} \sqrt{\frac{1}{4}\left(\frac{w_0}{L/2}\right)^4 - \frac{1}{2}\left(\frac{w_0}{L/2}\right)^3 \frac{x}{a} + \left(\frac{w_0}{L/2}\right)^2 \frac{x^2}{a^2}}\; dx + \int_0^{L/2} \sqrt{\frac{1}{4}\left(\frac{w_0}{L/2}\right)^4 + \frac{1}{2}\left(\frac{w_0}{L/2}\right)^3 \frac{x}{a} + \left(\frac{w_0}{L/2}\right)^2 \frac{x^2}{a^2}}\; dx \right. \qquad [49]$$

Integrating, we obtain

$$V = \frac{4\pi a \sigma_s t}{\sqrt{3}} \left\{ \frac{L}{8}\left(\frac{w_0}{L/2}\right)^3 a + \frac{3}{16}\left(\frac{w_0}{L/2}\right)^3 a \sinh^{-1}\frac{1}{\sqrt{3}} + \frac{L + \frac{1}{2}\left(\frac{w_0}{L/2}\right)a}{2}\sqrt{\frac{1}{4}\left(\frac{w_0}{L/2}\right)^4 + \frac{1}{2}\left(\frac{w_0}{L/2}\right)^3\frac{L}{2a} + \left(\frac{w_0}{L/2}\right)^2\frac{L^2}{4a^2}} + \frac{3}{16}\left(\frac{w_0}{L/2}\right)^3 a \sinh^{-1}\left(\frac{\frac{L}{a} + \left(\frac{w_0}{L/2}\right)}{\sqrt{3}\left(w_0/L_2\right)}\right)\right\} \qquad [50]$$

Assume that the shell and deformation pattern are such that

$$w_0/L_2 \ll 1 \quad , \quad L/a \gg 1 \qquad [51]$$

the energy expression reduces to

$$V = \frac{2\pi\sigma_s t}{\sqrt{3}} w_0 L \qquad [52]$$

For static loading the work done by the external uniform lateral load is

$$W = 2\pi a\, p_0\, w_0 L/2 \qquad [53]$$

Equating the internal energy absorbed by the shell to the work done by the external load it is found that

$$p_0 = \frac{2}{\sqrt{3}}\frac{\sigma_s t}{a} \quad ; \quad \sigma_s = E\epsilon_s \qquad [54]$$

Expression [54] implies that the load will not be dependent on the deflection throughout the plastic region. Therefore this load will correspond to any deflection in the plastic region and can therefore be termed the static collapse load.

It is interesting to note that the load corresponding to yield in an elastic shell with ordinary hoop tension is

$$(p_0)_e = \frac{\sigma_s t}{a} \qquad [55]$$

Thus [54] predicts that the static collapse load is about 15% higher than the load at which yielding will start. For shells in which L/a is not much greater than unity, higher order terms in the energy must be included. If second order terms in w_0/L_2 are retained, the energy becomes

$$V = \frac{4\pi a\sigma_s t}{\sqrt{3}}\left(\frac{w_0 L}{2a} + \frac{1}{4}\frac{w_0^2}{L_2}\right) \qquad [56]$$

Therefore the static load-deflection relation in the plastic region will be

$$p_0 = \frac{2}{\sqrt{3}}\frac{\sigma_s t}{a}\left(1 + \frac{1}{4}\frac{w_0}{L_2}\frac{a}{L_2}\right) \qquad [57]$$

The effect of the nonlinear terms is to stiffen the shell so that the plastic deflection for a given load will be definite value. As the deflection increases these nonlinear terms become more predominant and it is no longer possible to define a load at which the deflection increases indefinitely. Using relations [52] or [56] with [28] the impulse-deflection curve in the plastic region can be obtained. With equation [52] we obtain:

$$(I_0 \rho t)^2 = \frac{2\pi\sigma_s t}{\sqrt{3}} w_0 L \frac{\rho t}{\pi a L} \qquad [58]$$

With [56] we obtain

$$(I_o \rho t)^2 = \frac{4\pi a \sigma_y t}{\sqrt{3}}\left(\frac{w_o L}{2a} + \frac{1}{4}\frac{w_o^2}{L^2}\right)\frac{\rho t}{\pi a L} \qquad [59]$$

The static collapse load does not define collapse in a dynamic problem. In fact if the load is applied dynamically the actual magnitude of the load can be much greater than the static collapse load and still not result in plastic deformation. It is the load-time relation that is important and the impulse is a lumped parameter which essentially is a measure of the load time history.

C. Nonaxisymmetric buckling and collapse

If we compute the work done by the pressure during the deformation a linear function of the deflection will be obtained, i.e.

if $\quad w = w_o f(x,\phi)$

then $\quad W = w_o \int_0^l \int_0^{2\pi} p(x,\phi) f(x,\phi)\, a\, dx\, d\phi$

The work done (V) by internal forces is in general a nonlinear function of w_o as we see from equation [19], [20] and the curves plotted in Fig. (8), (9). By equating the work done by the external pressure to the work done by internal forces an expression will result between the load p_o and the deflection w_o . It is therefore certainly not plausible to talk about a collapse load for large deflections. If a power series in terms of w_o is fitted to V = $f(w_o)$ then one of the terms will be linear in w_o ; if all terms except the linear one are neglected then a collapse load could be defined in the same sense as in the axisymmetric case.

For the axisymmetric case we were able to define such a collapse load by neglecting the higher order terms in V. It was found that the collapse load was 15% higher than the load at which yield would start in the shell. If the buckling load is as close as 15% to the yield point load there is a possibility that we will predict the wrong pattern of deformation. In cases where the two loads are far apart (say 100% apart) we should be able to use the buckling-yield criterion to predict the correct pattern.

V. Post failure, nonaxisymmetric collapse and buckling

A. Collapse

The deflection pattern for collapse is shown in Figure 7 and can be written analytically as follows:

$$w(x,\phi) = a\cos\phi - \sqrt{a^2 - \tfrac{1}{4}\left[d_o - \tfrac{d_o}{L/2}x\right]^2} \qquad [60]$$

Letting $x' = x/L$

$$\begin{aligned} w(x,\phi) &= a\cos\phi - \sqrt{a^2 - \tfrac{d_o^2}{4}[1-2x']^2} \qquad \text{for } 0<x'<\tfrac{1}{2} \\ &= a\cos\phi - \sqrt{a^2 - \tfrac{d_o^2}{4}[1+2x']^2} \qquad \text{for } -\tfrac{1}{2}<x'<0 \end{aligned} \qquad [61]$$

where d_o is the width of the hinge line as shown in Figure 7. Using these deformation expressions and equation [33] the work done on the shell in deforming it plastically in collapse can be written

$$V = \frac{2\sigma_s t L}{\sqrt{3}} \left\{ \int_{-\frac{1}{2}}^{0} \int_{-\phi_1(x')}^{\phi_1(x')} \left[a\cos\phi - \sqrt{a^2 - \frac{d_o^2}{4}(1+2x')^2} \right] d\phi\, dx' + \int_{0}^{\frac{1}{2}} \int_{-\phi_1(x')}^{\phi_1(x')} \left[a\cos\phi - \sqrt{a^2 - \frac{d_o^2}{4}(1-2x')^2} \right] d\phi\, dx' \right\} \qquad [62]$$

where

$$\phi_1 = \sin^{-1} \frac{d_o(1-2x')}{2a} \text{ for } 0<x'<\tfrac{1}{2}\,; \quad \phi_1 = \sin^{-1} \frac{d_o(1+2x')}{2a} \text{ for } -\tfrac{1}{2}<x'<0 \qquad [63]$$

After some mathematical manipulation and substitution of

$$y = \frac{d_o}{2a}[1+2x'] \;, \quad y' = \frac{d_o}{2a}[1-2x'] \qquad [64]$$

it is found that V can be written

$$V = \frac{2\sigma_s t L}{\sqrt{3}} \frac{4a}{2\,d_o/2a} \int_0^{d_o/2a} \left(y - \sqrt{1-y^2}\,\sin^{-1} y \right) dy \qquad [65]$$

or finally

$$V = \frac{2\sigma_s t L D}{\sqrt{3}} \frac{1}{d_o/D} \left[\frac{3}{4}\left(\frac{d_o}{D}\right)^2 - \frac{\Phi^2}{4} - \frac{\Phi}{2}\frac{d_o}{D}\sqrt{1-\left(\frac{d_o}{D}\right)^2} \right]; \quad \Phi = \sin^{-1} d_o/D, \quad D = 2a \qquad [66]$$

Now using the impulse equation [27] and assuming that

$$f(x,y) \equiv f(x,\phi) = e^{-\beta\phi} \qquad [67]$$

The relation between the impulse and the deformation can be written as (assuming $\beta = 2$)

$$I_t \approx 2tLD\sqrt{K}\sqrt{\frac{\sigma_s \rho}{\sqrt{3}}} \qquad [68]$$

$$K = \frac{1}{d_o/D}\left[\frac{3}{4}\left(\frac{d_o}{D}\right)^2 - \frac{\Phi^2}{4} - \frac{\Phi}{2}\frac{d_o}{D}\sqrt{1-\left(\frac{d_o}{D}\right)^2} \right]$$

The deflection is actually described by d_o/D

B. Buckling

1. First approximation - continuous curve

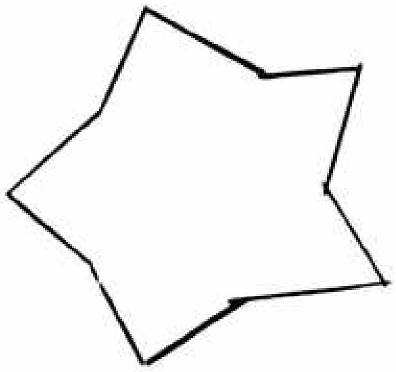
   For the post failure buckling region the circumferential parameter takes on great importance and the simplification given by equation [33] cannot be employed. Instead, the plastic work has to be computed from the more general integral expressions. We use equation [19] and assume a post failure deflection pattern of the form

$$\bar{f}(x';\phi) = \sin \pi x'\, e^{-\lambda\phi} \cos n\phi \quad \text{for } 0<\phi<\pi$$
$$= \sin \pi x'\, e^{-\lambda(2\pi-\phi)} \cos n(2\pi-\phi) \quad \text{for } \pi<\phi<2\pi \qquad [69]$$

2. Second approximation - series of hinges around periphery

   Upon closer examination of the buckling failures it is seen that the ideal buckling pattern of a continuous cosine wave as defined above does not exactly describe the post failure deflection in all cases.

Possibly a more realistic pattern for some cases might be a series of hinges, the depth of which becomes smaller around the periphery as shown below:



The number of such hinges could be determined by the number of elastic buckling lobes as described before. It is expected to do more work with such deformation patterns in the future.

C. Energy expression from numerical integration of the general integrals using first buckling approximation

The curves of $\bar{V}$ are plotted in Figures 8 and 9 for buckling and collapse failure as a function of the deflection and the geometric parameters of the shell. The buckling curves were computed for the parameters $\nu = \frac{1}{2}$, $\ell = \frac{1}{4}$. The values of n satisfying relation [37] were used in these calculations. It is seen that the collapse results using the simplified formula (equation [66]) compare well with the more exact curves shown in Figure 8 for $.2 < d_0/D < .6$.

D. Variational principle and governing differential equation in plastic region

The principle of extremum potential energy can be applied if we are dealing with a deformation theory as is the case in this work. Greenberg[31] writes this principle as follows:

$$\delta [U] = 0 \qquad [69]$$

where $$U = \int_{V'} \left(\int_0^{e_i} \sigma_i \, de_i \right) dv' - \int_{\Sigma} T_i u_i \, d\Sigma = V - W$$

The integral over V' is the potential energy or work done by internal forces during deformation and the integral over Σ is the work done by surface tractions.

If we extend the principle to the dynamic case then its extension can be written as Hamilton's Principle[32]

$$\delta \int_{t_1}^{t_2} [T - U] \, dt = 0 \qquad [70]$$

where $T = \frac{1}{2}\int_A \mu \dot{w}^2 dA$ the kinetic energy

μ = mass per unit area of shell
dA = element of surface area
$\dot{w}$ = velocity normal to shell surface (the longitudinal and tangential inertia are being neglected)

The variation is taken just as in the elastic problem by Love.[32]

This problem is equivalent to the following problem in the calculus of variations:

> Find the function $y(x)$ which takes on a given value for x = a and x = b and which minimizes the definite integral

$$I = \int_a^b F(x, y, y')\,dx \qquad [71]$$

The result is that F must satisfy the Euler Equation

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0 \qquad [72]$$

In our case

$$F = T - U$$
$$y' \to \dot{w}_0 \quad , \quad x \to t \quad , \quad y \to w_0 \qquad [73]$$

Thus

$$T = \frac{1}{2}\int_A \mu \dot{w}^2 dA = \frac{1}{2}\int_A \mu \dot{w}_0^2 f^2(x,\phi)\, dA \qquad [74]$$

We fit a power series curve to V from results like the ones in Figs. (8) (9). Thus

$$V = [A + Bw_0 + Cw_0^2 + Dw_0^3 + ---] \qquad [75]$$

and

$$W = \int_A p(x,\phi,t)\, w_0 f(x,\phi)\, dA$$

where $p(x, \phi, t)$ is the pressure applied to the shell

This results in the following nonlinear differential equation in time for the determination of w_0

$$\ddot{w}_0 \int_A \mu f^2(x,\phi)\, dA + [B + 2Cw_0 + 3Dw_0^2 ---] = \int_A p(x,\phi,t)\, f(x,\phi)\, dA \qquad [76]$$

This will be the governing differential equation for the loading regime. The initial conditions are

$$w_0(0) = \dot{w}_0(0) = 0 \qquad [77]$$

This equation will be valid as long as the deflection increases and unloading has not yet started to occur. If it is assumed that the shell deformation increases to a maximum and has no elastic recovery then the maximum deflection determined from this equation will give the permanent

Based on careful study of the existing plasticity flow theories of today it is difficult for this author to see how the important complications such as nonlinearity in the strians and stress-strain law in addition to nonuniform dynamic loading can be included and result in a solvable problem. In such plastic dynamic biaxial stress problems this author sees very little hope for any but the deformation theories and the energy type procedure for actually getting reasonable answers in a reasonable time.

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set. If we assume that elastic unloading occurs then the initial conditions for the elastic unloading regime can be determined from the solution of this equation. An elastic equation using the elastic potential energy can then be solved for the elastic recovery regime. A closer look at this approach to the problem will be given during the next several months. However, for the present time the energy approach as given in the next section will be used to approximate the plastic deformations.

VI. Practical use of the energy criterion

A. The energy criterion

Using the energy relations described in Section D earlier in this report we arrived at the basic relation

$$C \frac{W}{R^2} 2\pi a L = \bar{V} \frac{\sigma_s t a L}{\sqrt{3}}$$

where C is to be determined from experimental results. Using results for <u>collapse behavior</u> of steel shells we find that $C \approx 16000$ (where all units are in pounds and inches). A very rough expression governing the plastic behavior of steel shells in the collapse region is

$$\bar{V}_c \approx 174{,}000 \frac{W}{R^2} \frac{L}{\sigma_s t} \qquad [77]$$

B. Future work with the energy criterion

Since it has not as yet been determined how much hardening there is in the material and since there are some doubts about the true buckling post failure pattern, these items shall have to be determined more definitively in the near future before an adequate semi-empirical buckling formula can be obtained similar to the collapse formula above. Also more accurate collapse measurements will have to be taken in order to obtain a better collapse relation than above. More work is also in order to obtain a more accurate expression for the energy flux density.

VII. Discussion

The great advantage of the energy absorption method as described here and elsewhere in the literature[3,4,14,15,16] is that many complications can be considered in the analysis and still result in a tractable problem. For example it is seen that we have considered here nonlinearity in the strains, strain hardening in the stress-strain law and nonuniform dynamic loading. This type of theory has one main disadvantage - we have to assume a deflection pattern. However it must be realized that even in some of the more spohisticated plasticity theories assumptions on the pattern must also be made.

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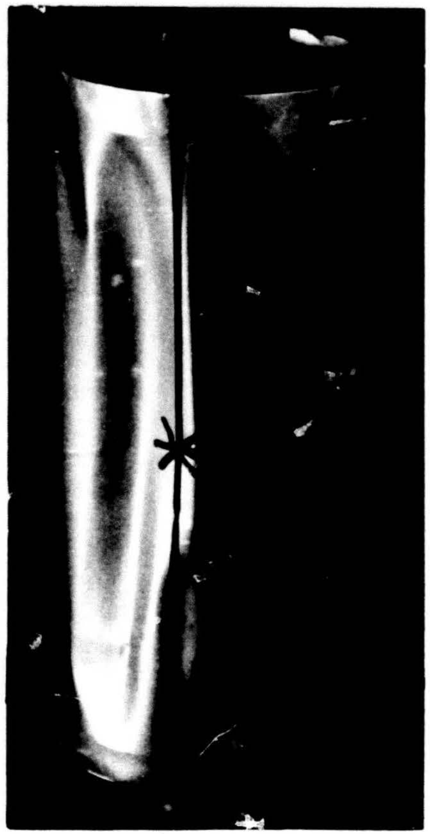


Fig. 1a. Buckling

Fig. 1b. Collapse

Figure 1 Types of Failure (from Schuman's experiments[1,2])

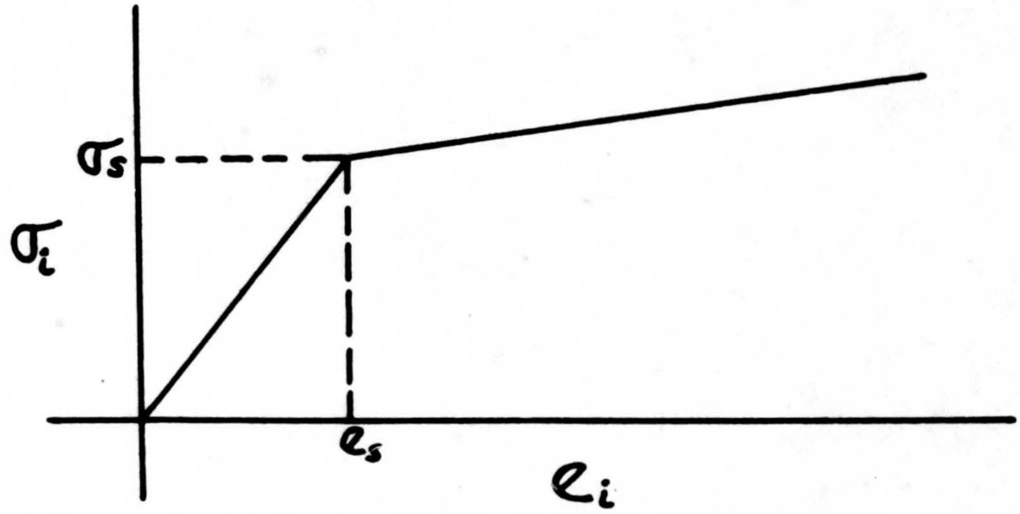


Figure 2 Elastic-Linear Hardening Law

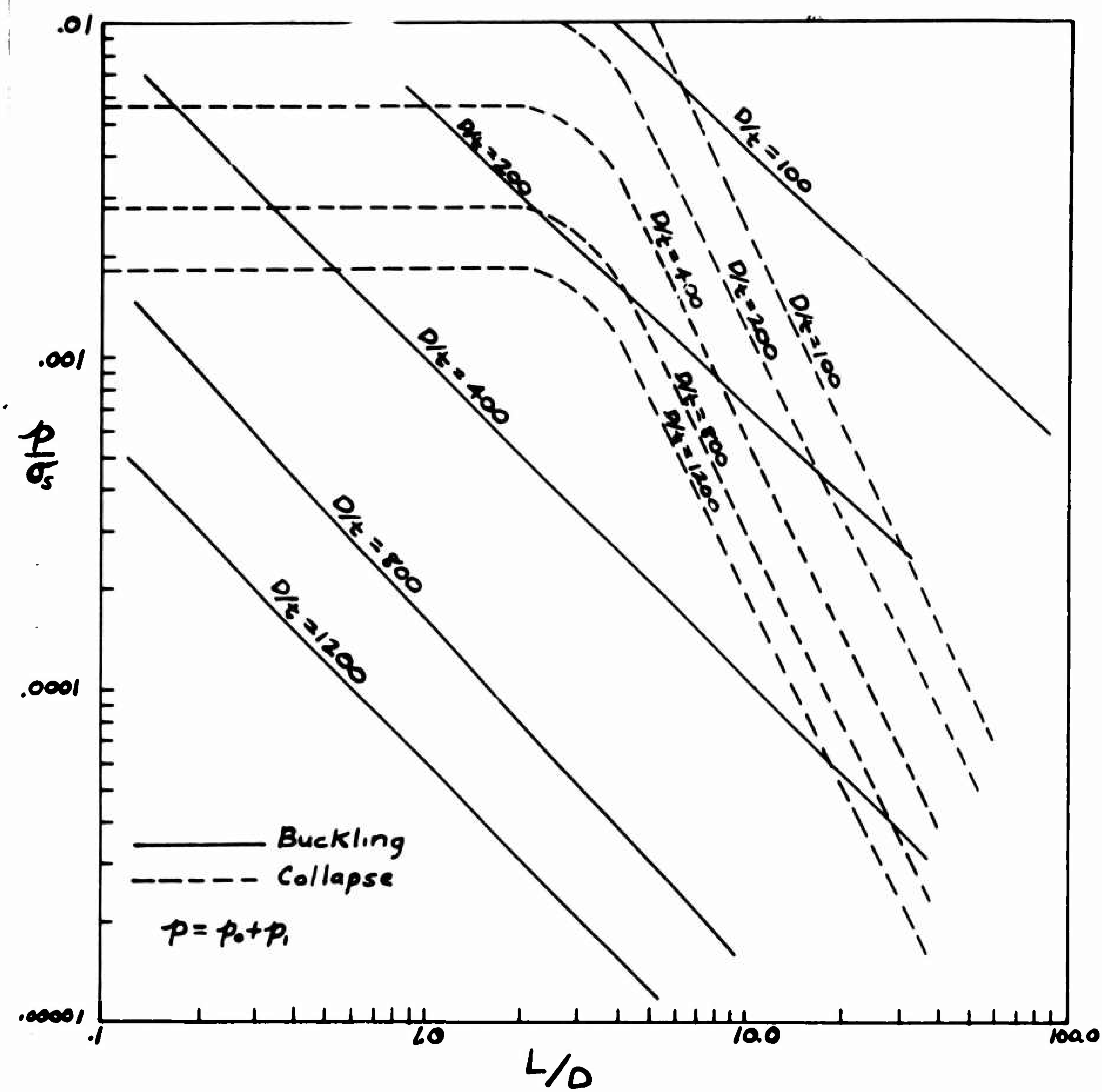


Figure 3 Buckling and Collapse Loads

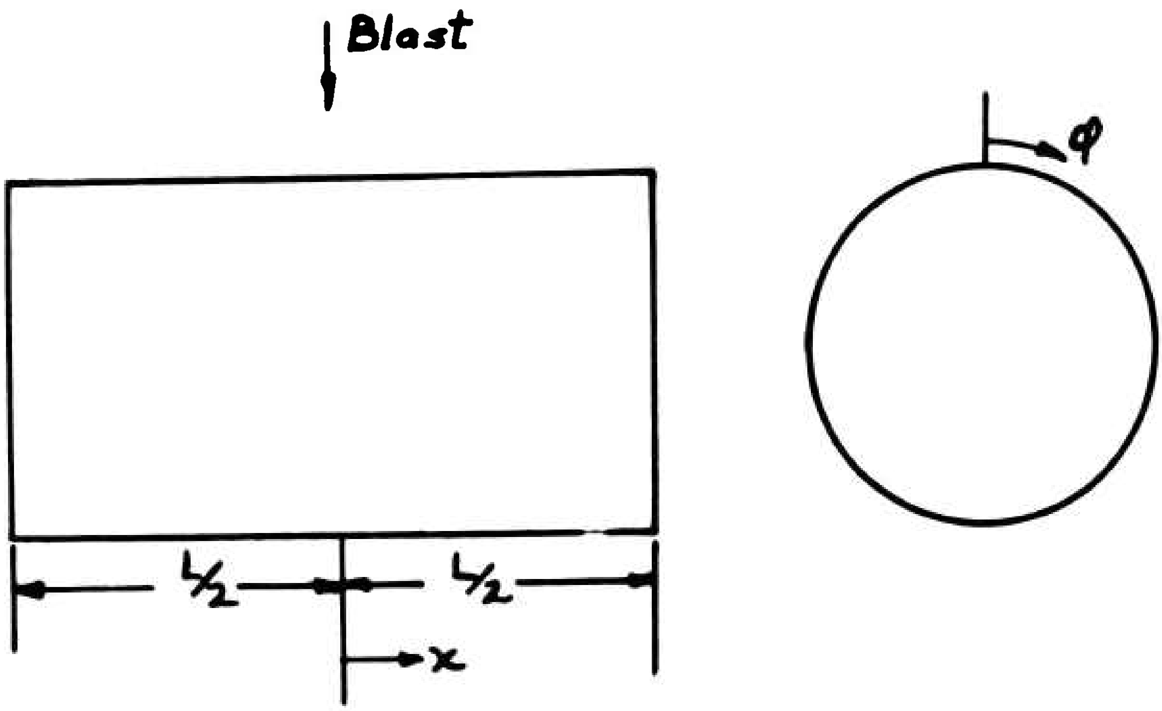


Figure 4 Origin of Coordinates

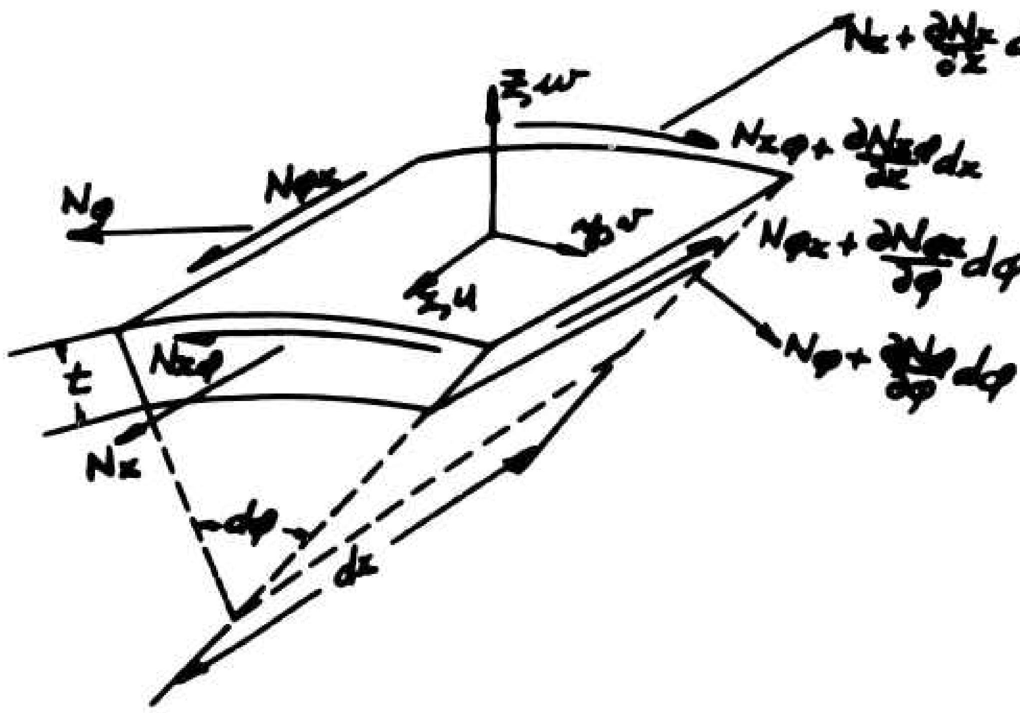


Figure 5 Shell Element Membrane Forces

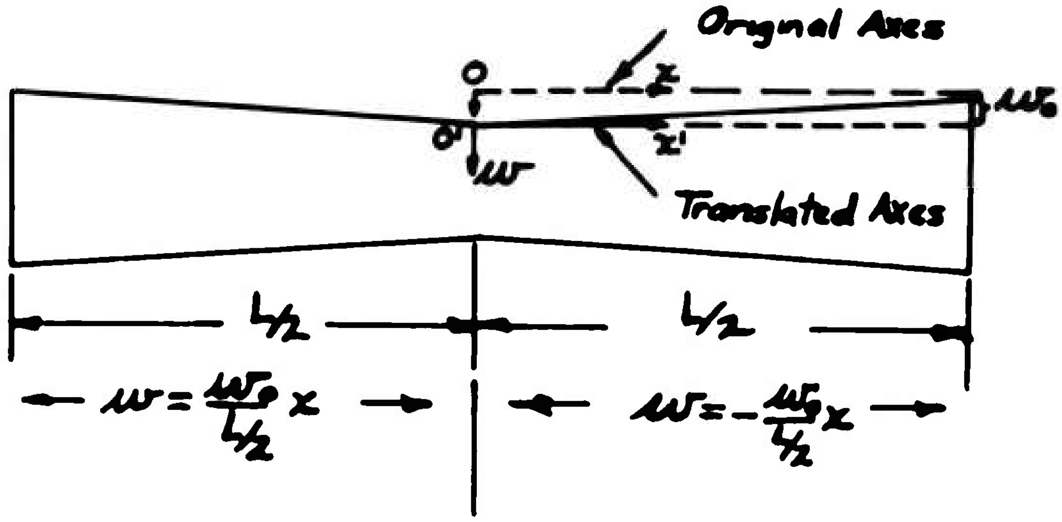


Figure 6 Deformation Pattern of Shell for Axisymmetric Collapse

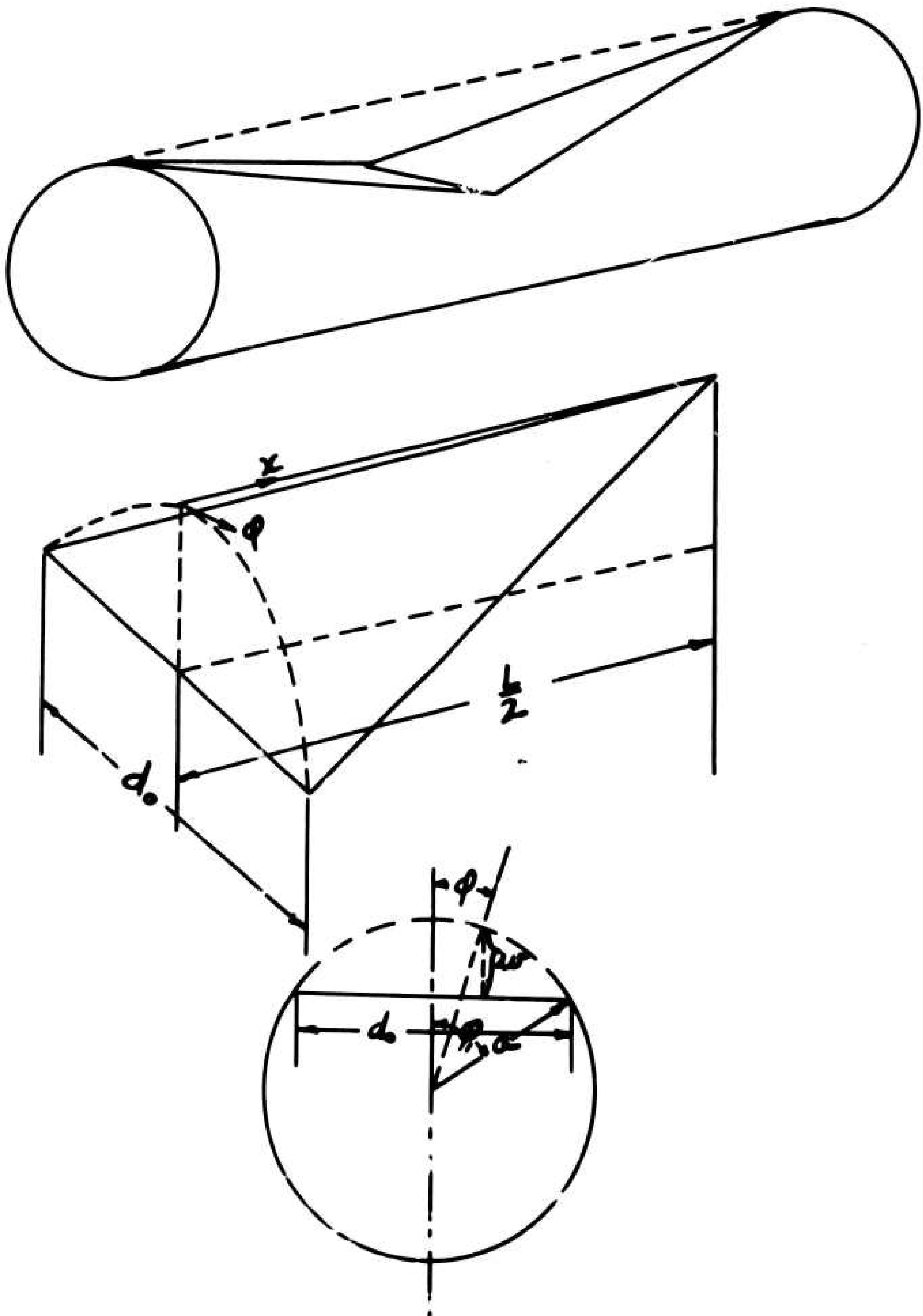


Figure 7 Deformation Pattern of Shell for Nonaxisymmetric Collapse

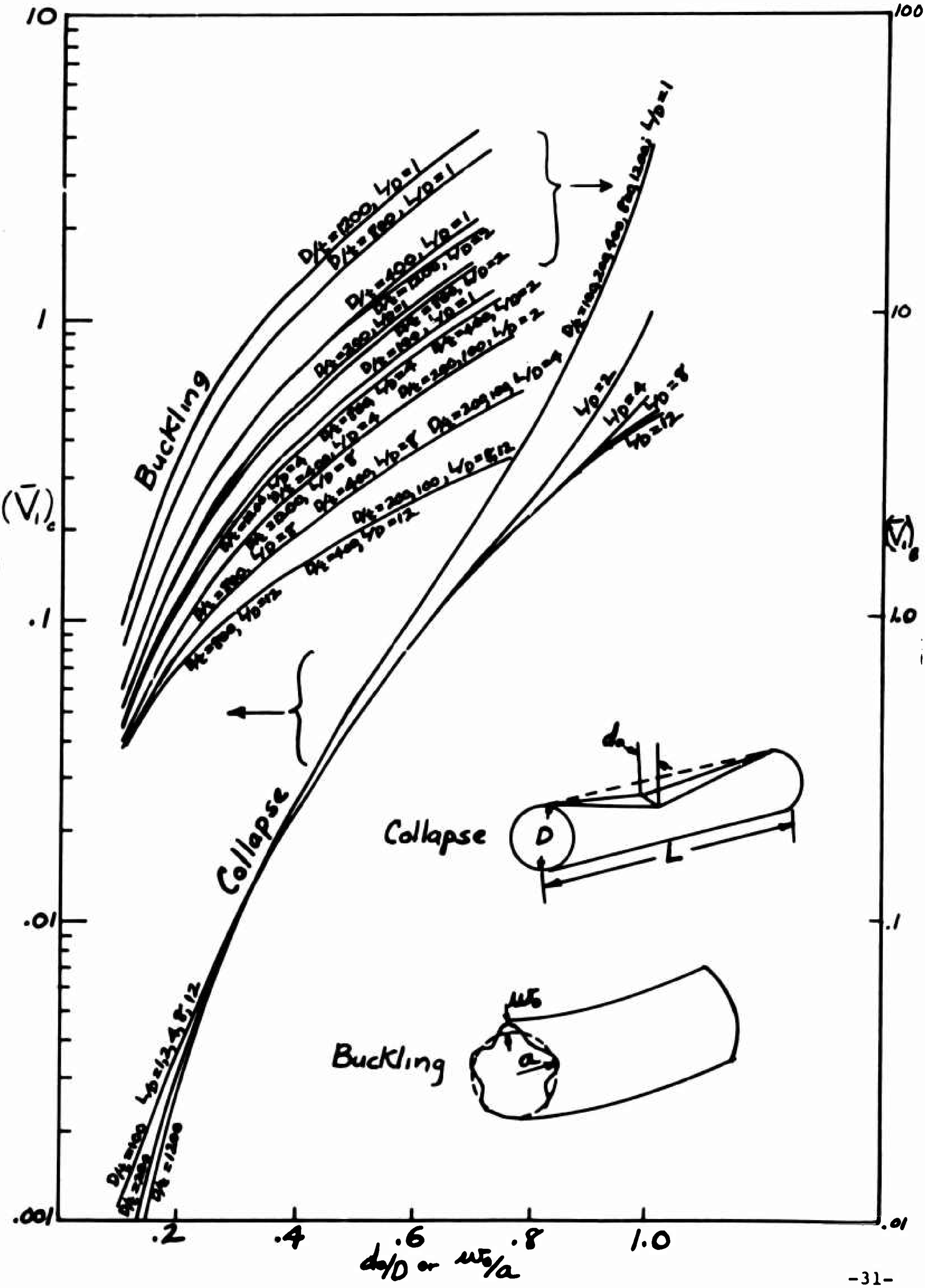


Figure 8 Post Failure Collapse and Buckling Curves

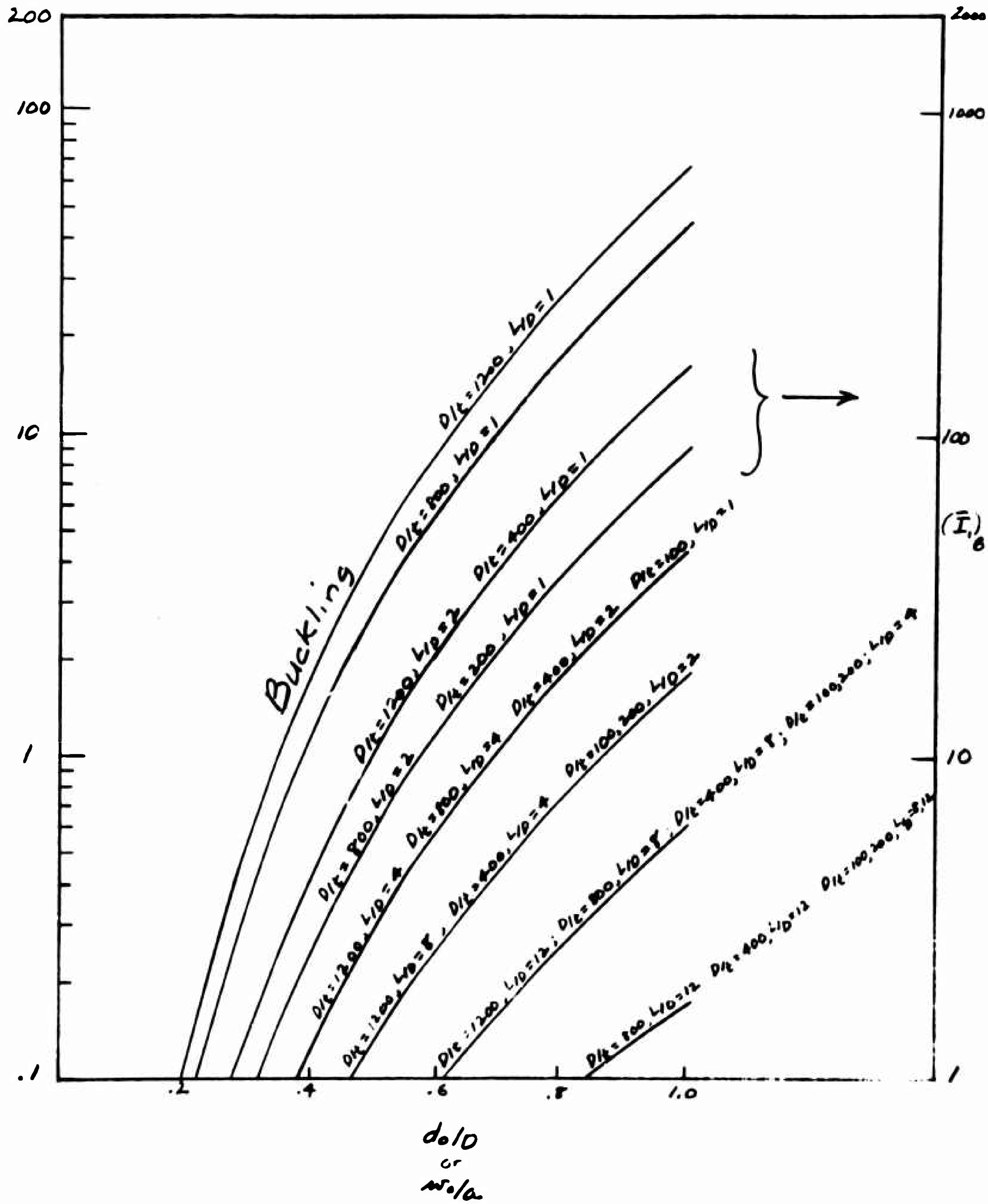


Fig. 9c. Post Failure Collapse and Buckling Curves

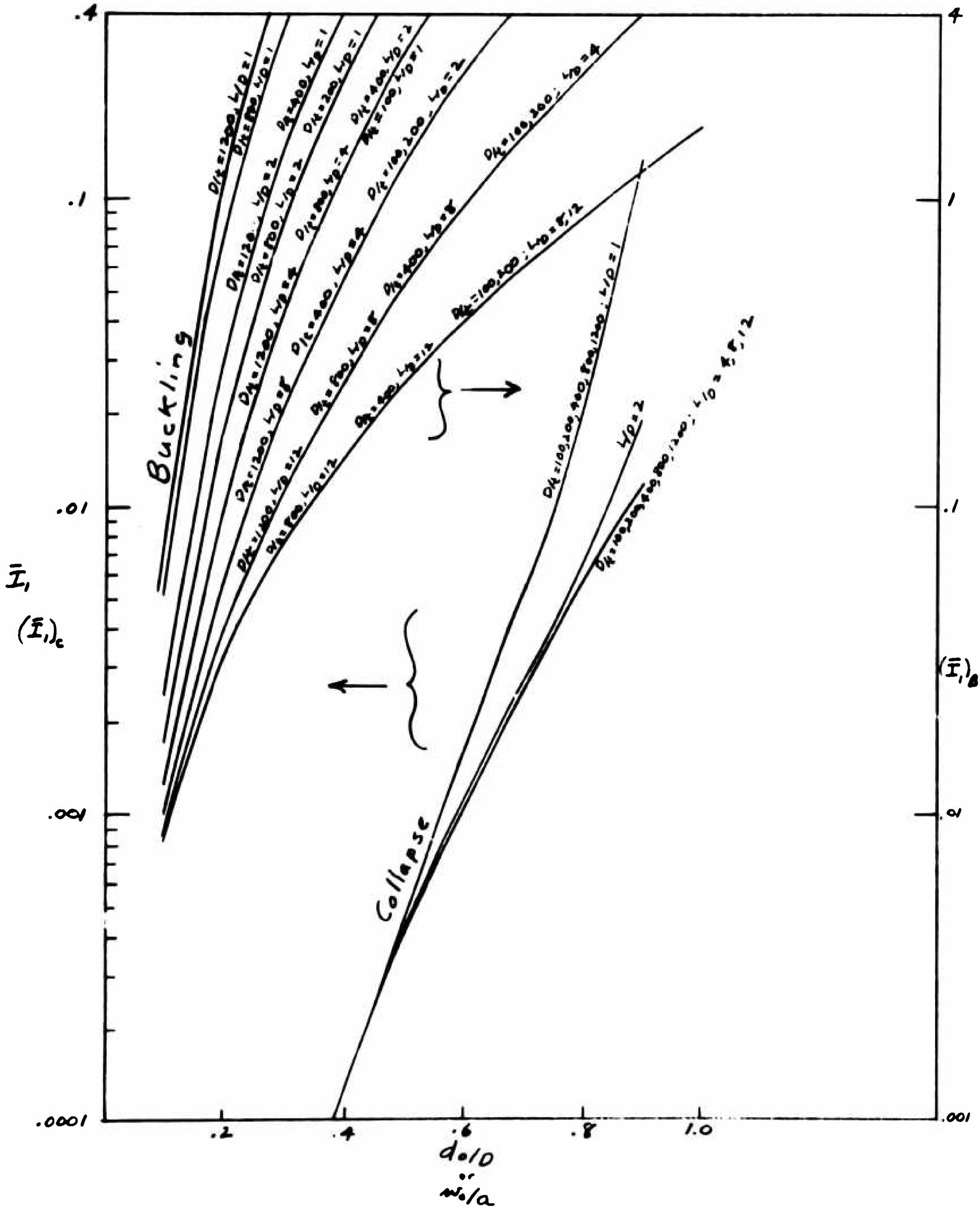


Fig. 9b Post Failure Collapse and Buckling Curves

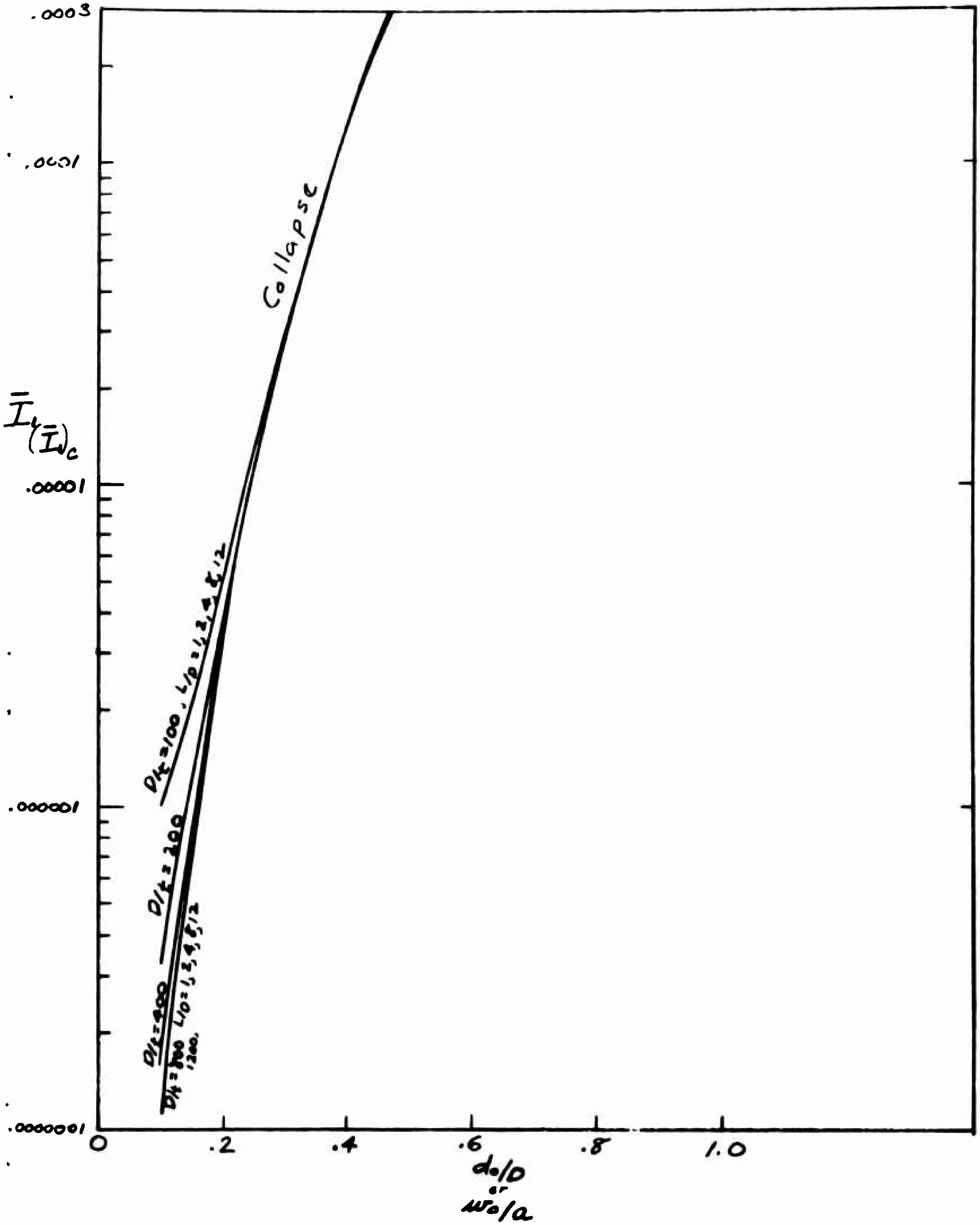
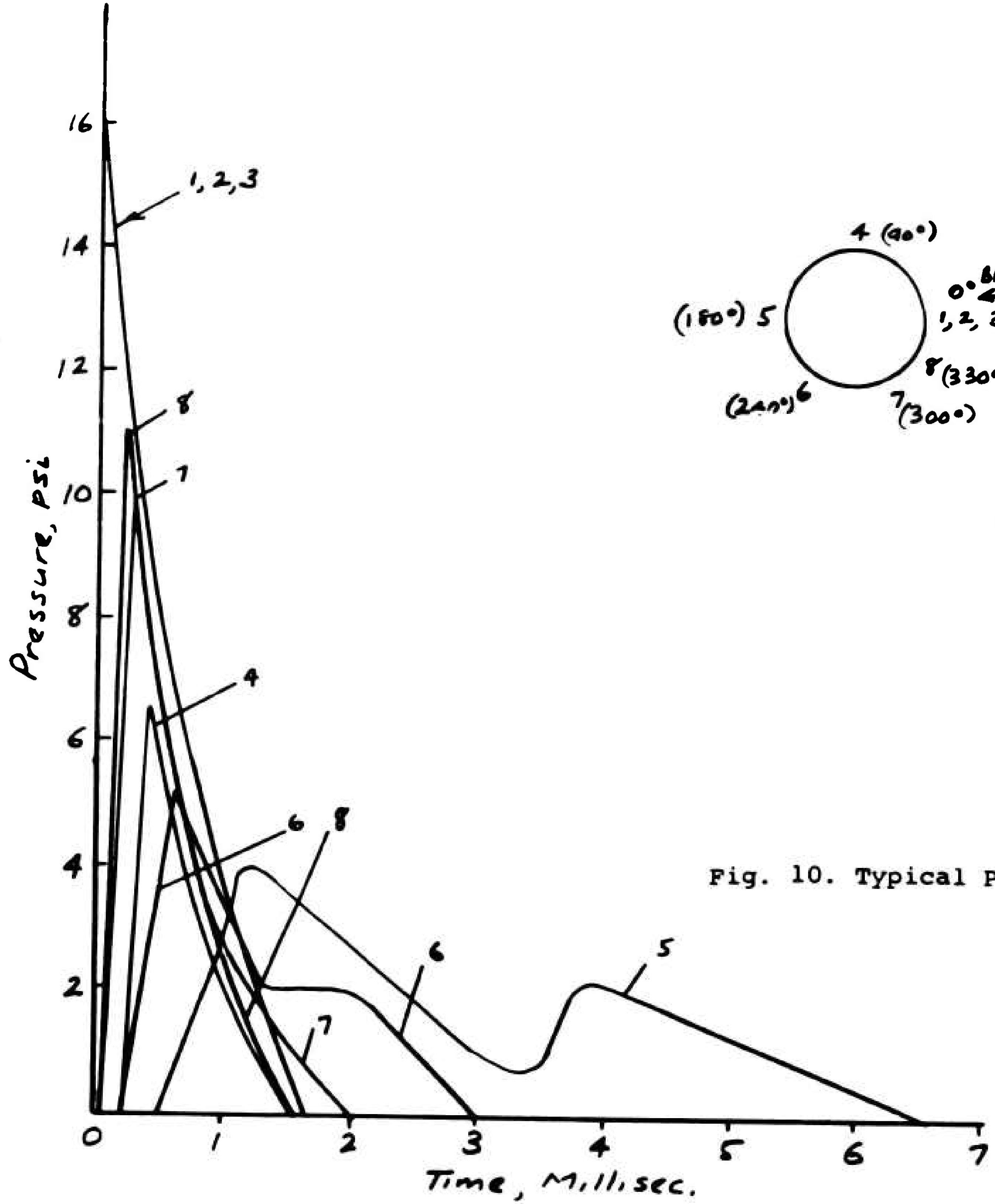
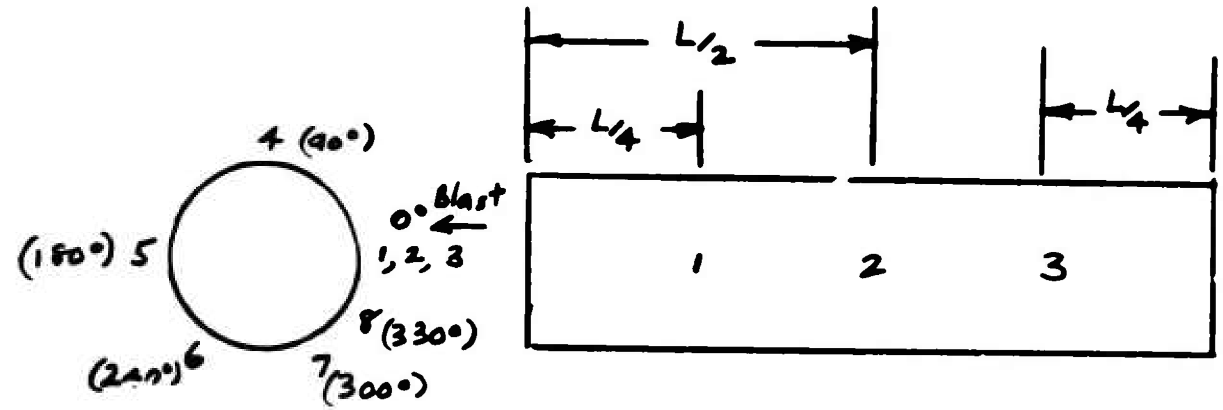


Fig 9a. Post Failure Collapse and Buckling Curves





Cylinder 12" Dia., 36" long, 1/2" thick
(filled with sand and two
intermediate bulkheads)
Charge 1# at 8'9"

Fig. 10. Typical Pressure-time Curve

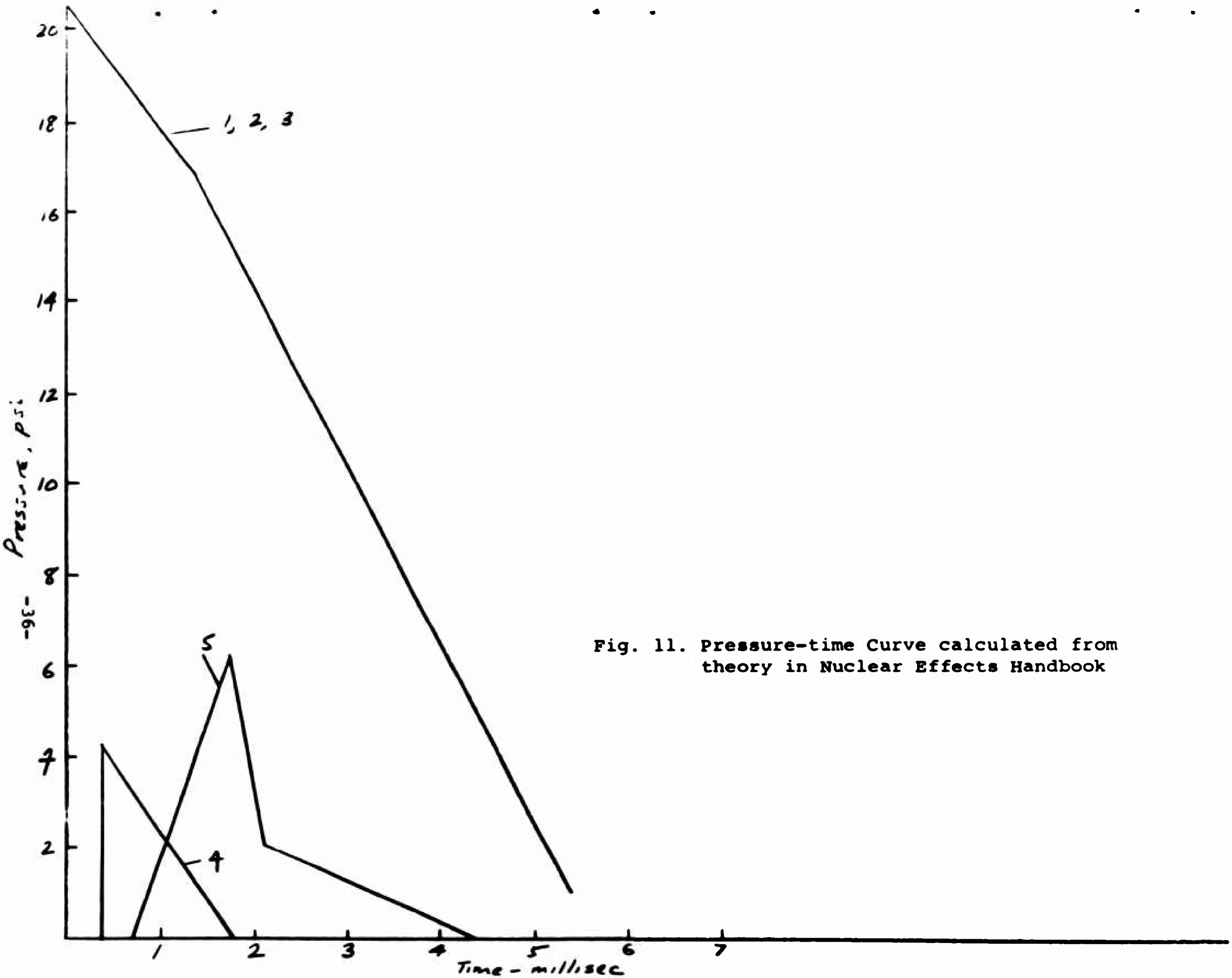


Fig. 11. Pressure-time Curve calculated from theory in Nuclear Effects Handbook

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13. ABSTRACT

It has been shown experimentally that cylindrical shells subjected to side on air blast will go into two main types of failure. These are buckling and collapse. The buckling type of failure is described by a deformation pattern which consists of a number of lobes around the periphery of the shell and one half wave length along the length. The collapse is described by a straight failure hinge. The type of failure will depend upon the geometry of the shell and can be predicted from an elastic stress and buckling analysis of the shell as discussed in this report. The analytical details of representing the deformation patterns and the method for calculating the energy absorbed and the resulting deflection under normal and elevated temperature conditions due to a given loading is described completely in this report. In addition to energy and impulse methods of solving the problem a deformation type variational principle is employed to set up the governing nonlinear differential equation for the time dependent deflection in the plastic region. The biaxial stress strain law used for both the normal and elevated temperature cases is an elastic linear hardening law.

Of greatest importance in the report is the computation of the energy absorbed or work done by internal forces in the shell for very large plastic deformations. This work or energy can be used to compute the impulse to give a prescribed deformation; it can be used to compute the deformation for a given energy input to the shell (assuming all of it goes into plastic deformation); it can be used to compute static load for a given deformation; or it can be used as a design criterion itself.

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