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Thermal Cracking  
of Waveguide Windows

9 February 1966

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## Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
LINCOLN LABORATORY

THERMAL CRACKING OF WAVEGUIDE WINDOWS

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*Group 46*

TECHNICAL NOTE 1966-19

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A B S T R A C T

A theory is derived to explain the cracking of waveguide windows due to stresses generated by heat dissipated in the window when subject to the RF fields in a waveguide. Calculations of the temperature rise and power-handling ability of half-wavelength block windows at X-band are presented.

Accepted for the Air Force  
Franklin C. Hudson  
Chief, Lincoln Laboratory Office

# THERMAL CRACKING OF WAVEGUIDE WINDOWS

by C. E. Muehe

Many microwave tube failures have been attributed to mechanical stresses induced in the output window by heat sources which cause a nonuniform temperature distribution throughout the window. Any quantitative discussion of this phenomena involves three parts:

1. The calculation of the stresses induced in a window by assumed temperature distributions,
2. The calculation of temperature distributions due to assumed heat source distributions and,
3. The calculation of the transmitted power as it relates to the heat sources.

## 1. Induced Stresses

When a flat window is heated more or less uniformly the center gets hotter than the edges. The center tries to expand by an amount depending on the temperature coefficient of expansion  $\alpha$ . Since it is restrained from expanding by the cool edges, the center is in compression. The edges, on the other hand, must be in tension because they are restraining the expansion of the center. We will consider only the case where the window is mounted in a relatively soft material like copper which yields at stresses lower than the tensile stress of the ceramics being considered. Windows of this sort will break in tension by a crack starting out from the edge toward the center of the window. For a rectangular window with a parabolic temperature distribution from top to bottom the induced tensile stress at the center of the top or bottom edge<sup>(1)</sup> is

$$\sigma = \frac{2}{3} \alpha E \psi_m \quad (1)$$

where  $E$  is the modulus of elasticity in tension (Young's modulus) and  $\psi_m$  is the temperature rise of the center of the window over the edge. For a round

<sup>(1)</sup>F. Paschke, "Heating of Waveguide Windows as a Limit to the Output Power of Microwave Tubes," RCA Review (September, 1962) p. 311.

window with a parabolic temperature distribution along a diameter the tensile stress at the edge is<sup>(2)</sup>

$$\sigma = \frac{1}{2} \alpha E \tau_m. \quad (2)$$

Table I shows the allowable temperature rise for rectangular windows of various materials where  $\sigma = \sigma_T$ , the tensile strength. In each case the coefficients,  $\alpha$ ,  $\sigma_T$  and  $E$  used were those quoted for the maximum temperature rise plus the assumed ambient temperature of 40°C. In the case of quartz the temperature rise is so high that it will glow red and melt before it cracks. For the remainder of the calculations we use a value of 400°C for quartz because the loss tangent starts to increase rapidly for this temperature rise.

TABLE I

	Alumina	Lucalox and Synthetic Sapphire	Beryllia	Fused Quartz
$\sigma_T$	25,000	55,000	21,000	15,500 PSI
$\alpha$	$7 \times 10^{-6}$	$6 \times 10^{-6}$	$6 \times 10^{-6}$	$0.55 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$
$E$	$5 \times 10^7$	$5 \times 10^7$	$5 \times 10^7$	$1.07 \times 10^7$ PSI
$\tau_m$	107	275	105	3950 (400) °C
$k$	0.27	0.42	2.0	0.017 watt/cm°C
$\epsilon'$	9	9.4	6.0	3.9
$\epsilon''$	0.002	0.0006	0.0027	0.0004
$P_T$	146	1950	750	151 kw
$c$	0.66	0.65	0.81	0.98 cm
$P_d$	206	810	1840	71 watts
$P_d/P_T$	1.4	0.41	2.5	0.47 watts/kw

Maximum temperature rise  $\tau_m$ , power-handling ability  $P_T$ , and total dissipation  $P_d$  in half-wavelength block windows at 7750 MHz in WR 137 waveguide. Constants used in this table are average values quoted by various manufacturers.

(2) C. T. Wang, Applied Elasticity, (McGraw-Hill, 1953) p. 71.

## 2. Temperature Distribution

Calculation of the temperature distribution in a rectangular window carrying the  $TE_{01}$  mode shows that  $h_o$ , the heat dissipated per unit volume at the center of the window, is given to within 20 percent by<sup>(3)</sup>

$$h_o = \frac{1}{m} k \left( \frac{11.4}{a^2} + \frac{8}{b^2} \right) , \quad (3)$$

where  $k$  is the thermal conductivity,  $a$  is the window width and  $b$  is the window height (parallel to the electric field). This equation assumes no heat loss from the face of the window. All the heat is conducted to metal waveguide in which the window is mounted.

## 3. Transmitted and Dissipated Power

We will assume that fields of one mode only are present within the window. Any matching structures are far enough removed that fringing fields do not exist at the window. For the  $TE_{01}$  mode in a rectangular window completely filling the guide the power transmitted is given by

$$P_T = (1 - |\Gamma|^2) \frac{E_1^2 ab}{2Z_d} \quad (4)$$

where we have assumed a standing wave with reflection coefficient  $\Gamma$  within the dielectric,  $Z_d$  is the wave impedance within the dielectric and  $E_1$  is the rms value of the electric field of the incident wave at the center of the guide.

The power dissipated within the window will vary with distance through the window. We will interpret the dissipation per unit volume  $h_o$  as the average value along the center line so that

$$h_o = E_1^2 \omega \epsilon_o \epsilon'' \frac{1}{c} \int_0^c |1 + |\Gamma| e^{j2\theta}|^2 dc$$
$$h_o = E_1^2 \omega \epsilon_o \epsilon'' \left( 1 + |\Gamma|^2 + \frac{1}{c} \int_0^c 2|\Gamma| \cos 2\theta dc \right) \quad (5)$$

<sup>(3)</sup>C. E. Muehe, "Some Aspects of High-Power Window Design," M.I.T. Lincoln Laboratory Report 46G-0003, October, 1960.

where  $\epsilon''$  is the loss factor of the window and  $c$  is its thickness. Substituting  $E_1$  from (5) into (4) and using (3), we find:

$$\begin{aligned}
 P_T &= K \frac{abh_o}{2Z_d \omega \epsilon_o \epsilon''} \\
 &= K \frac{\Psi_m^k}{2Z_d \omega \epsilon_o \epsilon''} \left( \frac{11.4b}{a} + \frac{8a}{b} \right) \quad (6)
 \end{aligned}$$

where  $K$  is the matching parameter given by

$$K = \frac{1 - |\Gamma|^2}{1 + |\Gamma|^2 + \frac{1}{c} \int_0^c 2|\Gamma| \cos 2\theta \, dc} \quad (7)$$

Finally, we will use (1) in (6) to obtain

$$P_T = K \frac{k \sigma_T^3}{\alpha E \epsilon'' 4\omega \epsilon_o Z_d} \left( \frac{11.4b}{a} + \frac{8a}{b} \right) \quad (8)$$

#### 4. X-Band Half-Wavelength Block Window

For any multiple half-wavelength-thick window the integral term in the denominator of (7) is zero so that

$$K = \frac{1 - |\Gamma|^2}{1 + |\Gamma|^2} \quad (9)$$

Consider a rectangular half-wavelength block window mounted in a piece of waveguide of the same size without any matching structures, then

$$|\Gamma| = \frac{Z_o - Z_d}{Z_o + Z_d} \quad (10)$$

where  $Z_o$  is the wave impedance of the empty waveguide. Then

$$K = \frac{2Z_o Z_d}{Z_o^2 + Z_d^2} \quad (11)$$

and the quantity

$$K \frac{Z_0}{Z_d} = \frac{4a^2 \epsilon' - \lambda^2}{2a^2 \epsilon' - \lambda^2 + 2a^2} \quad , \quad (12)$$

so that the power-handling ability is

$$P_T = \frac{4a^2 \epsilon' - \lambda^2}{2a^2 \epsilon' - \lambda^2 + 2a^2} \frac{k \sigma_T}{\alpha E \epsilon''} \frac{3}{8\pi} \frac{\lambda^2}{\lambda_g} \left( 11.4 \frac{b}{a} + \frac{8a}{b} \right) \quad (13)$$

where we have used the fact that

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{\lambda_g}{\lambda} \quad , \quad (14)$$

and a similar expression applied inside the window.

To compare various window materials let us calculate the power-handling ability  $P_T$  and the total dissipation  $P_d$  of half-wavelength block windows of various materials at 7750 MHz. Let us assume the use of WR 137 waveguide. Putting in the correct values, we find from (13)

$$P_T = 5.6 \frac{k \psi_m}{\epsilon''} \frac{4a^2 \epsilon' - \lambda^2}{2a^2 \epsilon' - \lambda^2 + 2a^2} \quad . \quad (15)$$

Using (3) and integrating over the volume of the window

$$\begin{aligned} P_d &= \frac{1}{2} abc h_0 = \frac{k \psi_m c}{2} \left( \frac{11.4b}{a} + \frac{8a}{b} \right) \\ &= 10.8 c \psi_m k \quad . \end{aligned} \quad (16)$$

Using the values of  $\psi_m$  calculated previously, we have completed Table I.

In the case of quartz, we have used 400°C for  $\psi_m$  since it melts at a lower temperature than its predicted thermal cracking temperature. The values for  $P_T$  clearly point out the advantages of beryllia and synthetic sapphire (Linde's single crystals or G. E.'s Lucalox) over alumina and fused quartz.

## 5. General Discussion

Examination of Eq. (13) shows that there are five material constants to which the power-handling ability is directly related and a sixth  $\epsilon'$  which is of minor importance. Of the five the one which probably varies the most from

batch to batch of a material is the loss factor  $\epsilon''$ . The others are probably known to within about 20 percent. ( $\sigma_T$  for Lucalox is not known but is assumed equal to that for Linde synthetic sapphire.) Because it may vary over wide ranges, it is especially important to know  $\epsilon''$  for the particular batch of material from which a window is being made.

If one makes the window larger by putting it in a larger size waveguide, Eq. (13) shows that the principle effect is on  $\lambda_g$ . For instance, the ratio of the power-handling ability in WR 137 is only 1.13 greater than a window of the same material in WR 112 at 7750 MHz.

In actual use there may be excess loading on a window, both thermal and mechanical. Three sources are: (1) a multipactor discharge on the vacuum side or an arc on the pressure side, (2) excess electric fields caused by a high VSWR on the load side of the window, (3) mechanical stress transmitted to the window from the waveguide due to improper mounting. Thin windows are hurt more by these excess loadings than thick windows. The first two can be guarded against by providing a very sensitive fast-acting arc detector and reflected power monitor. Because of the possibility of excess loading, it seems imperative to have excess power-handling ability considerably over and above the actual power output of the tube. A safety factor of two seems reasonable. Probably the safest way to assure a long-lived window would be to check each one at about twice the power level of its intended use. Another would be to measure the temperature rise at the center of the window as a function of power level and extrapolate to the window failure point. This could only be done after testing several windows of the same design to failure so as to find out what the failure temperature is. Certainly at the very least all incoming window material ought to be measured to determine whether the five parameters in Eq. (8) are within tolerance.

A study of Eq. (7) seems to show that the matching parameter  $K$  and thus the power-handling ability can be varied over wide ranges. To a certain extent this is true. For instance if a very thin window ( $c \ll \lambda_{gd}$ ) is put at a voltage minimum in a standing-wave pattern purposely introduced in a waveguide,  $\cos 2\theta$  will be -1 and Eq. (7) will be

$$K = \frac{1 - |\Gamma|^2}{(1 - |\Gamma|)^2} \quad (17)$$

This quantity approaches infinity as  $|\Gamma|$  approaches unity. One should be cautious in using the thin-window approach, however, because the standing wave needed outside the window may cause voltage breakdown. Also any added loads such as those mentioned above cause a greater stress in proportion to those caused by dielectric losses since the added stresses are generally distributed over the volume of the window.

For half-wavelength windows there is some advantage to be gained by matching. With  $|\Gamma| = 0$ ,  $K = 1$ . For a typical alumina window operating near the high end of the waveguide band, Eq. (11) gives  $K = 0.5$ . The proper impedance to reduce the standing waves inside the window to zero could be provided by either (a) mounting the window in a smaller waveguide which is extended a quarter-wavelength on either side of the window before connecting onto the main waveguide, or (b) mounting the window in a larger waveguide which is extended a half-wavelength on either side of the window before connecting onto the main waveguide. The first matching method may limit peak-power breakdown and the second may run into trapped or ghost-mode problems. Still another method to reduce the standing waves inside the window to zero is to put susceptances on either side of the window of such a value that a voltage minimum will exist at the faces of the window and the ratio of E to H at the minimum will just equal the wave impedance of the window. These susceptances will lower the breakdown power for the waveguide.

Caryotakis<sup>(4)</sup> has found an exact solution for the temperature distribution in a window mounted in a round waveguide carrying the  $TE_{11}$  mode. His result for the equation corresponding to (6) is

$$P_T = 16.0 \frac{K \rho k}{2Z_d \omega \epsilon_o \epsilon''} \quad (18)$$

When we put  $b/a = 1$  in (6), we find the coefficient to be 19.4 instead of 16.0 and the exact solution<sup>(3)</sup> for a square window gives a coefficient of 18.5 so we see that there is practically no difference in power handling between a square and a round window.

(4)

G. Caryotakis, "R.F. Heating in Disc Windows," EIMAC Report No. TR65-5, February 3, 1965.

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