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### MODE THEORY APPROACH TO ARRAYS

FEBRUARY 1966

R. H. T. Bates

Prepared for

496L/474L SYSTEM PROGRAM OFFICE

ELECTRONIC SYSTEMS DIVISION  
AIR FORCE SYSTEMS COMMAND  
UNITED STATES AIR FORCE  
L. G. Hanscom Field, Bedford, Massachusetts



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Project 615.1

Prepared by

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## FOREWORD

The concept of interior grating lobes arose from discussions with J. L. Allen and W. P. Delaney, Lincoln Laboratory, who introduced the author to the phased-array, gain-directivity paradox and to Hannan's elegant reduction of it (see Reference [4]). The author's colleagues, J. H. Phillips and A. L. Murphy, The MITRE Corporation, are to be thanked for their encouragement and for the provision of experimental facilities. That the experiment demonstrated the anticipated result is largely due to the perserverance of P. Blasi.

## REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

## ABSTRACT

Attention is concentrated on the radiating and evanescent modes which must appear, both in free space and in the receiving transmission lines, when an infinite array of radiators is illuminated by a plane wave. If the radiators have a tendency to form a slow-wave structure (as all radiators must, even if to only a very slight degree), then for angles of incidence greater than a certain amount there can be propagating modes (surface waves capable of carrying power) supported by the radiators.

These propagating modes are dubbed "internal grating lobes" because they can appear even if the spacing of the radiators is close enough to prohibit the possibility of grating lobes in free space. The main effect of the internal grating lobes is to cause mismatching of the array, thereby deteriorating its scanning performance.

It was found possible to demonstrate the existence of internal grating lobes experimentally. The main conclusion is that the simplest types of radiators (slots in a flat sheet or dipoles above a ground plane) are probably best for phased arrays because, of all types considered both practical and imaginable, they have the least slow-wave character.



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## SECTION I

### INTRODUCTION

#### DEFINITION OF PROBLEM

It is known that the gain of a phased array of radiators is given by the sum of the gains of the individual radiators, provided that their gains are properly measured. <sup>[1]\*</sup> The gain of each radiator must be measured in the presence of the other radiators. The other radiators must be terminated in matched loads. When the radiators are identical and uniformly spaced, the study of the radiating performance of large arrays can reduce to the study of an individual radiator. This is a desirable simplification. Nevertheless, consideration of the array as a whole highlights certain aspects which are not immediately apparent when the array is considered as the sum of a number of independent, identical radiators. There is good precedent for this approach in Carter's classic paper <sup>[2]</sup> on dipole arrays around cylinders.

#### APPROACH OUTLINE

A plane wave is taken to be incident on an infinite planar array of identical, uniformly spaced radiators. There is a reradiated field in space and a transmitted field in each of the transmission lines which connect the radiators to their receivers. The reradiated field consists of a specularly reflected wave together with higher modes whose wave numbers are governed by the spacing between the radiators. This representation is complete because an infinite structure is considered. There is no need for the complex waves of Tamir and Oliner. <sup>[3]</sup> If the spacing between the radiators is less than half a wavelength, then the higher modes can never carry power.

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\* Numbers in brackets designate references listed at end of this report.

## SUMMARY OF CONCLUSIONS

A refinement of the model consists of breaking the system into three regions: I, free space; II, region containing the radiators; and III, transmission lines connecting the radiators to their receivers. The mode theory approach leads to the following conclusions:

- (1) Even in the simplest case when the radiators consist of holes in a conducting screen, Region II will have a higher refractive index than Region I. The more complicated the individual radiators, the more pronounced this effect. Dipoles above a ground plane will form a slightly slow-wave structure. Polyrods, Yagis or helices will form a significantly slow-wave structure, which will be, in general, anisotropic.
- (2) "Interior grating lobes" (higher order modes carrying power) can appear in Region II even if there are no grating lobes in free space. This is a consequence of the higher refractive index of Region II. When these interior grating lobes appear, the array will be very difficult to match. Thus, the major effect of the interior grating lobes will be to reduce the scan angle of the array. For this reason, it appears to be advantageous to use the simplest type of radiator. Also, it may be advantageous to match the array in Region III rather than in Region II, even if manufacturing problems and cost indicate that Region II matching would be more convenient.
- (3) Mutual impedance can be a dangerous concept to apply to phased arrays. The mutual coupling between radiators in an array can only be conveniently measured in Region III. If a measurement in Region III indicates that little power is coupled from a driven radiator into any of the other radiators of the array, it cannot

then be concluded that the radiators scarcely interfere with each other. It is most probable that the radiators behave as forward directional couplers with respect to the power they pick up from other radiators. This type of behavior is most likely if the radiators are end-fire antennas such as polyrods, Yagis, or helices.

- (4) An array cannot be matched for all scan angles unless the radiator matching networks can be varied with scan angle, or unless a condition similar to super-gain can be achieved. [4, 5]\*
- (5) For any given type of radiator, there appears to be an optimum size for maximizing the radiation efficiency of an array over a range of scan angles.

#### OUTLINE OF REPORT

The following sections contain the reasoning by which the above conclusions were reached. A simple two-dimensional problem is analyzed because it demonstrates all the pertinent idiosyncrasies of arrays which can be deduced from a mode theory. The two-dimensional problem is set up in Section II. Conclusion (4) is deduced in Section III. An approximate solution to the simple two-dimensional problem is given in Section IV, where conclusion (2) is deduced. Experimental evidence for the existence of internal grating lobes is given in Section V. A waveguide array covered with a dielectric sheet was used in the experiment. The results are compared with experiments by other workers. Section VI contains the conditions for the appearance of grating lobes in an array of two-dimensional polyrods. This substantiates conclusion (1). Conclusion (5) is illustrated in Section VII. The pertinence of the conclusions to array measurements is discussed in Section VIII.

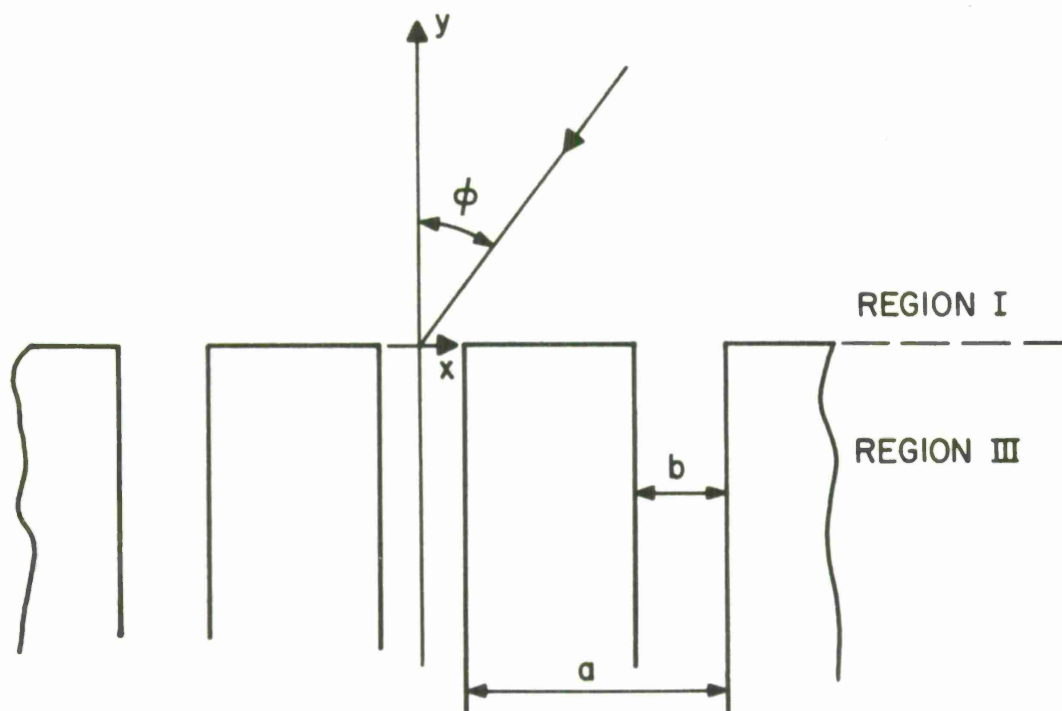
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\*The author of Reference [4] reached, and rigorously examined, the same conclusion from a point of view that is different from that taken in this paper. The paradox is well defined in Reference [5].

## SECTION II

### SIMPLE TWO-DIMENSIONAL PROBLEM

Figure 1 shows the geometry of the simple problem. A plane wave is incident at an angle  $\phi$  on a conducting sheet periodically punctured with parallel plate transmission lines. The structure is infinite in the plane perpendicular to the paper. In



NOTE:

THERE IS NO REGION II IN THIS SIMPLE MODEL

Figure 1. Simple Two-Dimensional Problem Geometry

order to have a match at normal incidence, it is supposed that the dielectric constant  $\epsilon_r$  of the medium between the parallel plates is not unity. The dielectric constant of Region I is unity. The incident wave is assumed to be magnetically polarized perpendicular to the paper. It follows that  $\epsilon_r$  must be less

than unity, which is impractical for very wide bandwidths but which can be achieved effectively over a useful bandwidth with the use of matching networks. The fields can be expressed in terms of a potential vector directed only perpendicular to the paper. Its component in this direction is  $V$ :

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \eta k^2 V = 0 ,$$

where

$$k = 2\pi/\lambda ,$$

$$\lambda = \text{free-space wavelength of incident wave} ,$$

$$\eta = 1 \text{ in Region I, and}$$

$$\eta = \epsilon_r \text{ in Region III.}$$

It follows that  $(\partial V/\partial y)$  and  $\eta V$  are continuous across an interface normal to the coordinate direction  $x$ . Also,  $(\partial V/\partial y)$  is equal to zero along a perfectly conducting interface. If

$$E(x, \phi) = \frac{\partial V}{\partial y} \bigg|_{y=0} \quad -\frac{b}{2} < x < \frac{b}{2} ,$$

then matching the fields at  $y=0$  gives

$$\begin{aligned} \frac{1+R}{1-R} = \frac{a \cos \phi}{E_{10}(\phi) E_{10}^*(\phi)} & \left[ \frac{\epsilon_r^{1/2} E_{20}^2(\phi)}{b} \right. \\ & \left. + \frac{j 2k \epsilon_r}{b} \sum_{n=1}^{\infty} \frac{E_{2n}^2(\phi)}{\Gamma_n} + \frac{jk}{a} \sum_{n=-\infty}^{\infty} \frac{E_{1n}^*(\phi) E_{1n}(\phi)}{\Gamma_n} \right] \end{aligned} \quad (1)$$

because of the periodicity of the geometry of Figure 1, where  $R$  is the current reflection coefficient of the specularly reflected wave in free space. Thus,

$$E_{10}(\phi) = \int_{(-b/2)}^{(b/2)} E(x, \phi) \exp(-jkx \sin \phi) dx ; \quad (2)$$

$$E_{20}(\phi) = \int_{(-b/2)}^{(b/2)} E(x, \phi) dx ; \quad (3)$$

$$E_{1n}(\phi) = \int_{(-b/2)}^{(b/2)} E(x, \phi) \exp \left( -jx \left[ k \sin \phi + \frac{2n\pi}{a} \right] \right) dx ; \quad (4)$$

$$E_{2n}(\phi) = \int_{(-b/2)}^{(b/2)} E(x, \phi) \cos \frac{n\pi}{b} \left[ \left( x + \frac{b}{2} \right) \right] dx ; \quad (5)$$

$$\gamma_n = k \left( \frac{n^2 \lambda^2}{4b^2} - \epsilon_r \right)^{1/2} \quad (6)$$

= imaginary wave number of  $n^{\text{th}}$  mode, of form

$$\cos \left[ \frac{n\pi}{b} \left( x + \frac{b}{2} \right) \right] ,$$

in a parallel plate line ;

$$\Gamma_n = k \left[ \left( \sin \phi + \frac{n\lambda}{a} \right)^2 - 1 \right]^{1/2} \quad (7)$$

= imaginary wave number of  $n^{\text{th}}$  mode, of form

$$\exp \left( j \frac{2n\pi x}{a} \right) , \text{ in free space ;}$$

$$\sum_n' C_n = \left( \sum_n C_n \right) - C_o .$$

For  $\gamma_n$  or  $\Gamma_n$  imaginary, the positive sign must be used. An asterisk denotes the complex conjugate. Equation (1) was derived in a standard manner. [6]



### SECTION III

#### VARIATION OF MATCH WITH SCAN ANGLE

The form of Equation (1) directly demonstrates the impossibility of a match at all scan angles, unless the array contains matching networks which vary with scan angle. The condition for a match is  $R = 0$ , which cannot occur if the impedance of the space-array interface contains reactive terms such as in the infinite summations in Equation (1). Fixed matching networks in the parallel plate transmission lines could only cancel the reactance at a single scan angle because the reactance is seen to be a function of scan angle. It is, perhaps, possible to conceive of radiators which would exhibit negligible reactance change over a large range of scan angles. If this were so, then the condition for a match would be:

$$\frac{E_{10}(\phi)E_{10}^*(\phi)}{E_{20}^2(\phi)} = \frac{a\epsilon_r^{1/2}}{b} \cos \phi, \quad (8)$$

as can be seen from Equation (1). All types of radiators will be governed by equations similar in form to Equation (8), even though their reactance behavior will be widely different. The reason is that Equation (8) ensures conservation of power. Equations (2) and (3) indicate that Equation (8) implies that

$$\int_{(-b/2)}^{(b/2)} E(x, \phi) \exp(-jkx \sin \phi) dx \propto \cos^{1/2} \phi \int_{(-b/2)}^{(b/2)} E(x, \phi) dx, \quad (9)$$

in the interval  $(-\pi/2) < \phi < (\pi/2)$ . This relation could only be exactly true for a range of values of  $\phi$  under exceptional circumstances, probably similar to supergain, [4, 5] since the function  $E(x, \phi)$  cannot be controlled. The exact form of  $E(x, \phi)$  is governed by the initial boundary value problem.

It is instructive to consider the implications of the previous reasoning in terms of a transmitting array. In general, if a single radiator is excited, power will be

coupled into the transmission lines of the other radiators. When all the radiators are excited, the mutually coupled signals may interfere completely and destructively at a certain scan angle. However, when the scan angles changes, the relative phases of the mutually coupled signals will change, and there will be a net power flow into each of the radiator's generators. Since the consideration of an array as a receiver virtually precludes the possibility of matching over a wide scan angle, it appears that it is extremely unlikely that radiators with, effectively, zero mutual interaction can be found when they are spaced closely enough to avoid grating lobes. This reasoning is in agreement with Hannan's rigorous argument. <sup>[4]</sup>



# SECTION IV

## INTERNAL GRATING LOBES

Equation (1) expresses the normalized admittance of the space-array interface in a form which is known<sup>[6]</sup> to be stable under small variations in  $E(x, \phi)$ . Equation (1) is, therefore, suitable for an approximate evaluation of the normalized admittance, without fully solving the initial boundary value problem. Choose

$$E(x, \phi) = 1 . \quad (10)$$

This choice of  $E(x, \phi)$  becomes more valid with a decreasing ratio of  $b/a$ . It is worth noting that Equation (9) cannot hold for all values of  $\phi$  under Equation (10). When Equation (10) is applied to Equation (1), the result is:

$$\frac{1 + R}{1 - R} = \frac{\frac{a\epsilon_r^{1/2}}{b} - jk \sum_{n=-\infty}^{\infty} \frac{\text{sinc}^2 \left( \frac{kb \sin \phi}{2} + \frac{n\pi b}{a} \right)}{\Gamma_n}}{\frac{\text{sinc}^2 \left( \frac{kb \sin \phi}{2} \right)}{\cos \phi}} , \quad (11)$$

where

$$\text{sinc } x = \frac{\sin x}{x} .$$

Suppose that an attempt is made to match the array with a dielectric layer, as in Figure 2. The parallel plate lines are assumed to be in air, and  $\epsilon_r$  is the relative dielectric constant of the matching layer. Note that  $\epsilon_r$  in Equation (12) is equivalent to  $1/\epsilon_r$  in Equation (11). The normalized admittance\* of the dielectric-array interface is then given by

---

\* Note that  $R$  is the current (not voltage) reflection coefficient.

$$Y = \frac{1+R}{1-R} = \frac{a}{b\epsilon_r^{1/2}} - j \frac{\sum_{n=-\infty}^{\infty} \frac{\text{sinc}^2 \left( \frac{kb \sin \phi}{2} + \frac{n\pi b}{a} \right)}{\left[ \left( \sin \phi + \frac{n\lambda}{a} \right)^2 - \epsilon_r \right]^{1/2}}}{\frac{\text{sinc}^2 \left( \frac{kb \sin \phi}{2} \right)}{\cos \phi}} \quad (12)$$

from Equations (7) and (11), realizing that  $\Gamma_n$  is now the imaginary wave number of the  $n^{\text{th}}$  mode in a medium of fundamental velocity  $c/\epsilon_r^{1/2}$  rather than  $c$ . If  $a/\lambda = 0.5$ , there will be no grating lobes in free space. However, for

$$|\phi| > \arcsin \left( 2 - \epsilon_r^{1/2} \right), \quad (13)$$

there will be at least one propagating higher order mode in the dielectric. If  $\epsilon_r^{1/2} = 1.6$ , which is typical for cheap and robust dielectrics of low loss, then there

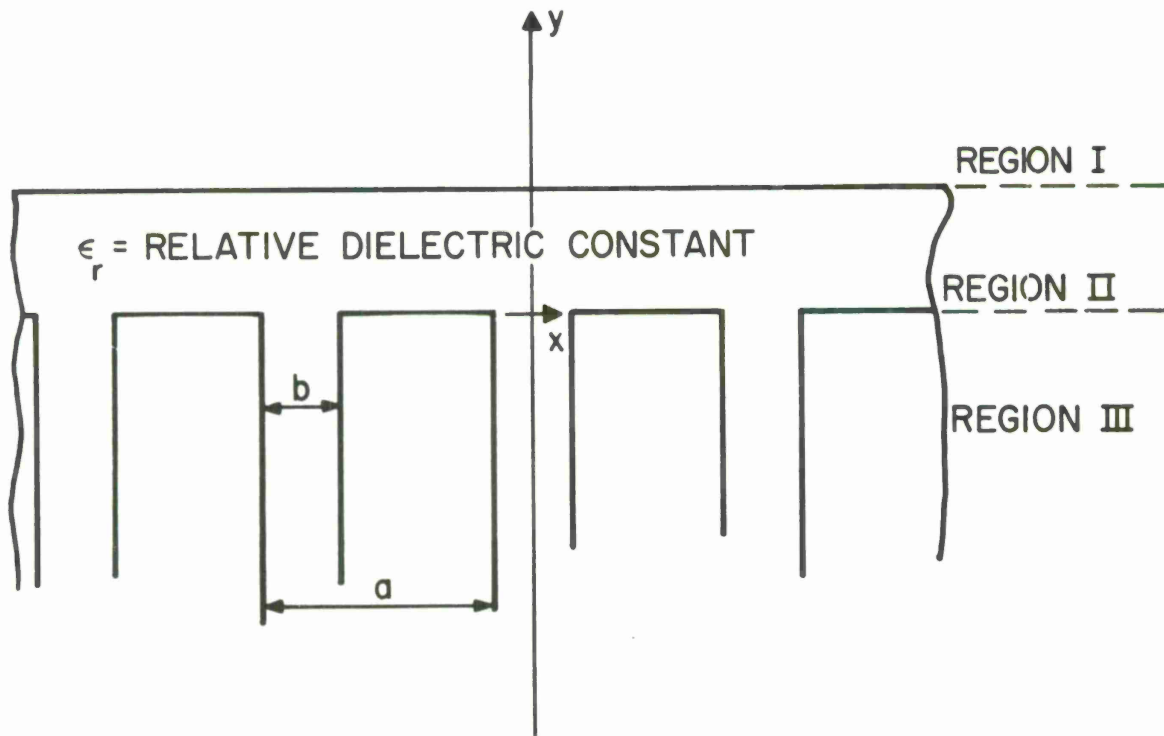


Figure 2. Dielectric Matching Layer in Region II

will be "internal grating lobes" for scan angles greater than 24 degrees from broadside. At the scan angle at which the internal grating lobe appears, the susceptible part of  $Y$  becomes infinite, as can be seen from the numerator of the infinite sum in Equation (12), unless

$$\frac{kb \sin \phi}{2} + \frac{n\pi b}{a} = m\pi, \quad m \neq 0, \quad (14)$$

at the same time. The coincidence of Equation (14) with the appearance of an internal grating lobe requires that

$$\frac{m\lambda}{b} = \epsilon_r^{1/2}. \quad (15)$$

Equation (15) is incompatible with  $a/\lambda = 0.5$ , unless  $\epsilon_r^{1/2} > 2$ , which itself means that there will be at least two internal grating lobes for  $0 < \phi < \pi/2$ . Equation (15) cannot be satisfied for more than one value of  $m$ . It is certainly true that the infinite susceptance at

$$\sin \phi + \frac{n\lambda}{a} = \pm \epsilon_r^{1/2}$$

is due to Equation (10). A more exact expression for  $E(x, \phi)$  would lead to a finite reactance. However, it is clear that it will be very difficult to maintain a good match between the array and free space if internal grating lobes are allowed to appear. This difficulty may be increased by another large susceptible admittance at the interface between Regions I and II. Any higher order modes which are propagating in the dielectric become evanescent in free space.

This reasoning indicates both that the radiators should be of the simplest types — slots or simple dipoles — because such radiators will form a medium with only slightly slow-wave properties, and also that the matching of the array should be effected in Region III rather than in Region II. If the array is covered with a dielectric layer for protection against the weather, the layer should be very thin compared with the wavelength so that the reactive impedances, due to internal grating lobes, at both surfaces of the layer effectively cancel each other.

## SECTION V

### EXPERIMENTAL DEMONSTRATION OF EXISTENCE OF INTERNAL GRATING LOBES

The mode theory indicates that high mismatches are to be expected when internal grating lobes appear. No attempt is made in this report to evaluate the mismatch quantitatively because of the analytical complexity of the problem. It was decided, however, to conduct a simple experiment at X-band to ascertain how bad the mismatch could be in practice. An array of seven waveguides in a 3-foot-square, flat, brass ground plane was used (see Figure 3). The six outer waveguides were terminated in matched loads. The center waveguide was terminated in a detector. The

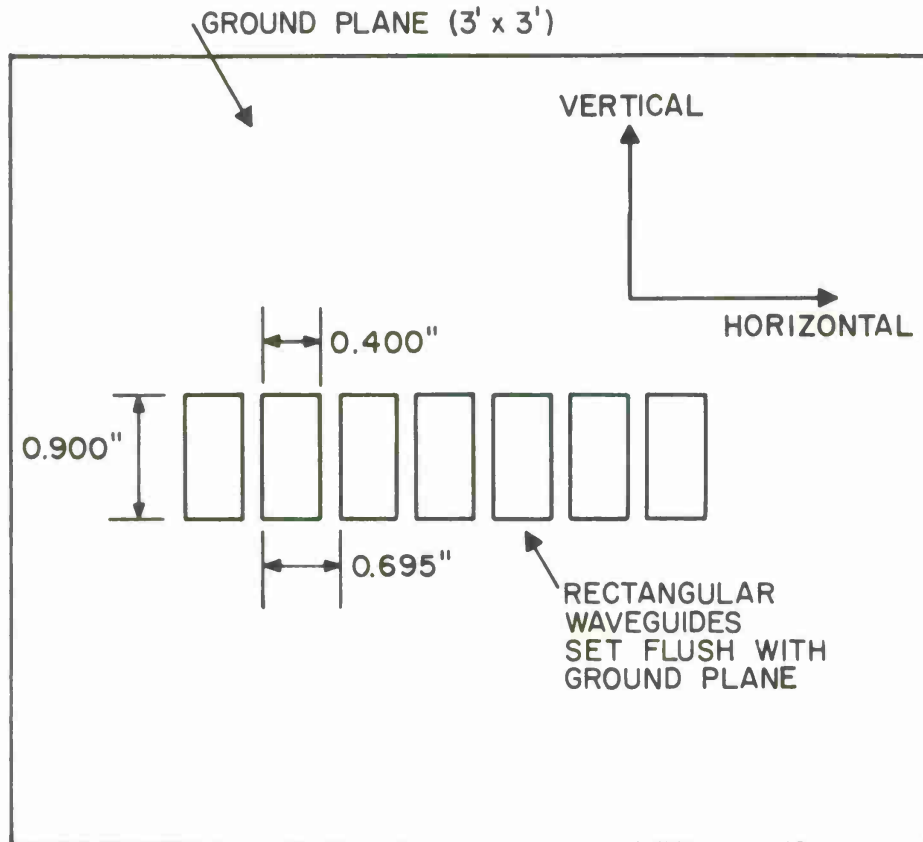


Figure 3. Experimental Array

horizontal plane polar diagram of the center waveguide was measured at a frequency of 8500 Mc, at which adjacent waveguides were separated by half a wavelength center-to-center (so that the array would not exhibit grating lobes in free space). Figure 4 shows the measured polar diagram. The ripples are due to the finite ground plane effect, as can be seen from diffraction lobes at  $\pm 82$ -degrees (note that the signal strength at  $\pm 90$  degrees, the edge of the ground plane, is almost exactly 3 db down on the peak of the diffraction lobe). The ripples have been smoothed out in Figure 5 (the dotted curve). The pattern of Figure 4 is identical with  $\cos \phi$ , within the limits set by experimental error.

It was decided that to demonstrate the existence of internal grating lobes, the simplest method was to cover the ground plane with a dielectric sheet. The dielectric used was plexiglass, which, by the theory of Section IV, should exhibit internal grating lobes for  $\phi > 24$  degrees. Figure 5 shows the smoothed, measured polar diagrams of the center waveguide for two thicknesses of plexiglass, together with the polar diagram for no plexiglass. The effect of the internal grating lobes is apparent. The waveguides were matched to better than 1.5 for each thickness of plexiglass.

The experiments of King and Peters<sup>[7]</sup> on polyrod arrays show an effect similar to that displayed in Figure 5. King and Peters explained their results on the basis of element interaction, a broader concept than mutual impedance. Their Figure 2 is strikingly similar to the curve for the 0.225-inch plexiglass sheet in Figure 5 of this report. Their Figure 3 shows a smooth main beam, but the element spacing is large enough for the existence of several grating lobes in free space.

Effects similar to those reported by King and Peters have also been noted by J. L. Allen's group at Lincoln Laboratory<sup>[8]</sup> with arrays of horns loaded with dielectric.

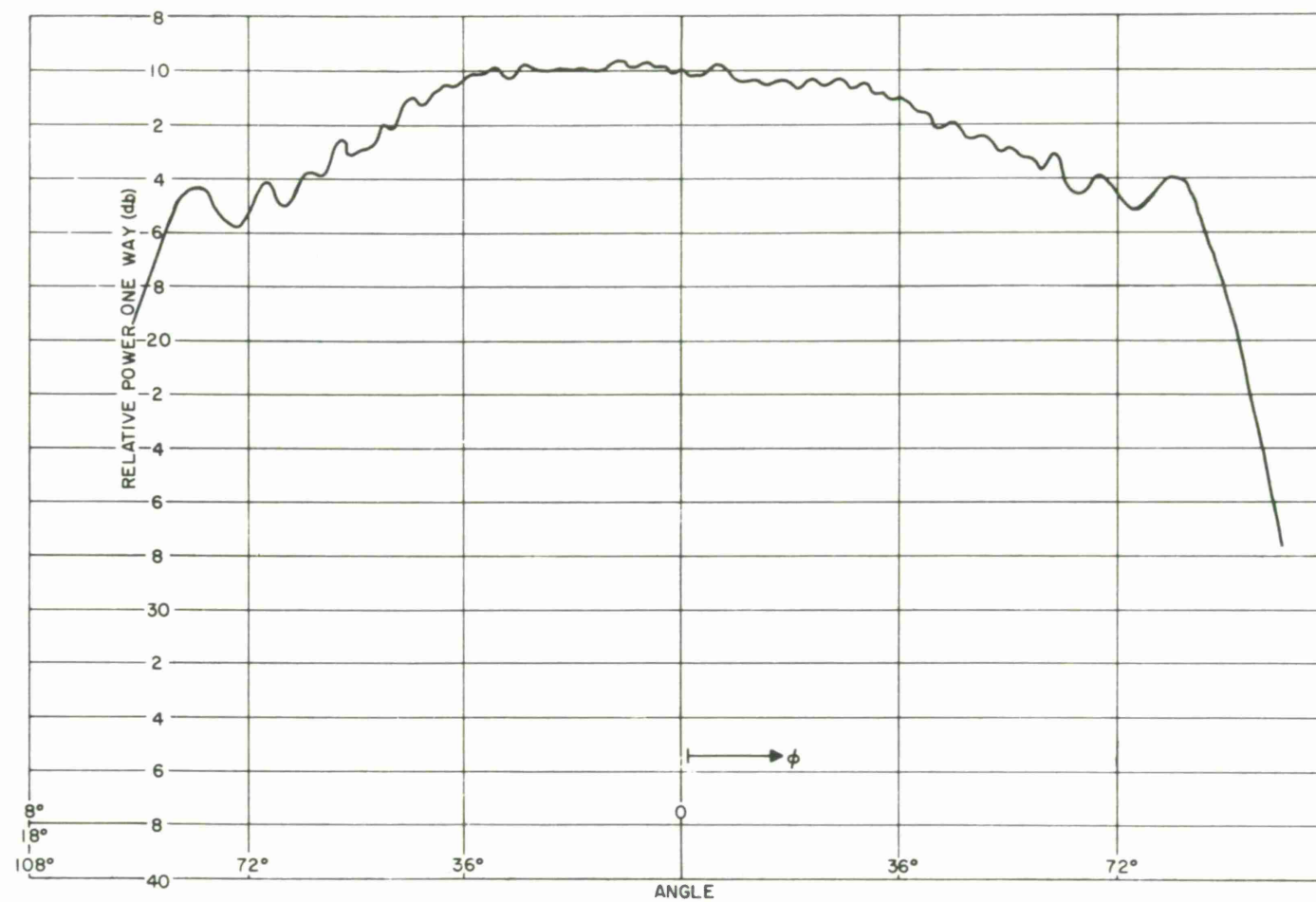


Figure 4. Measured Polar Diagram Center Waveguide

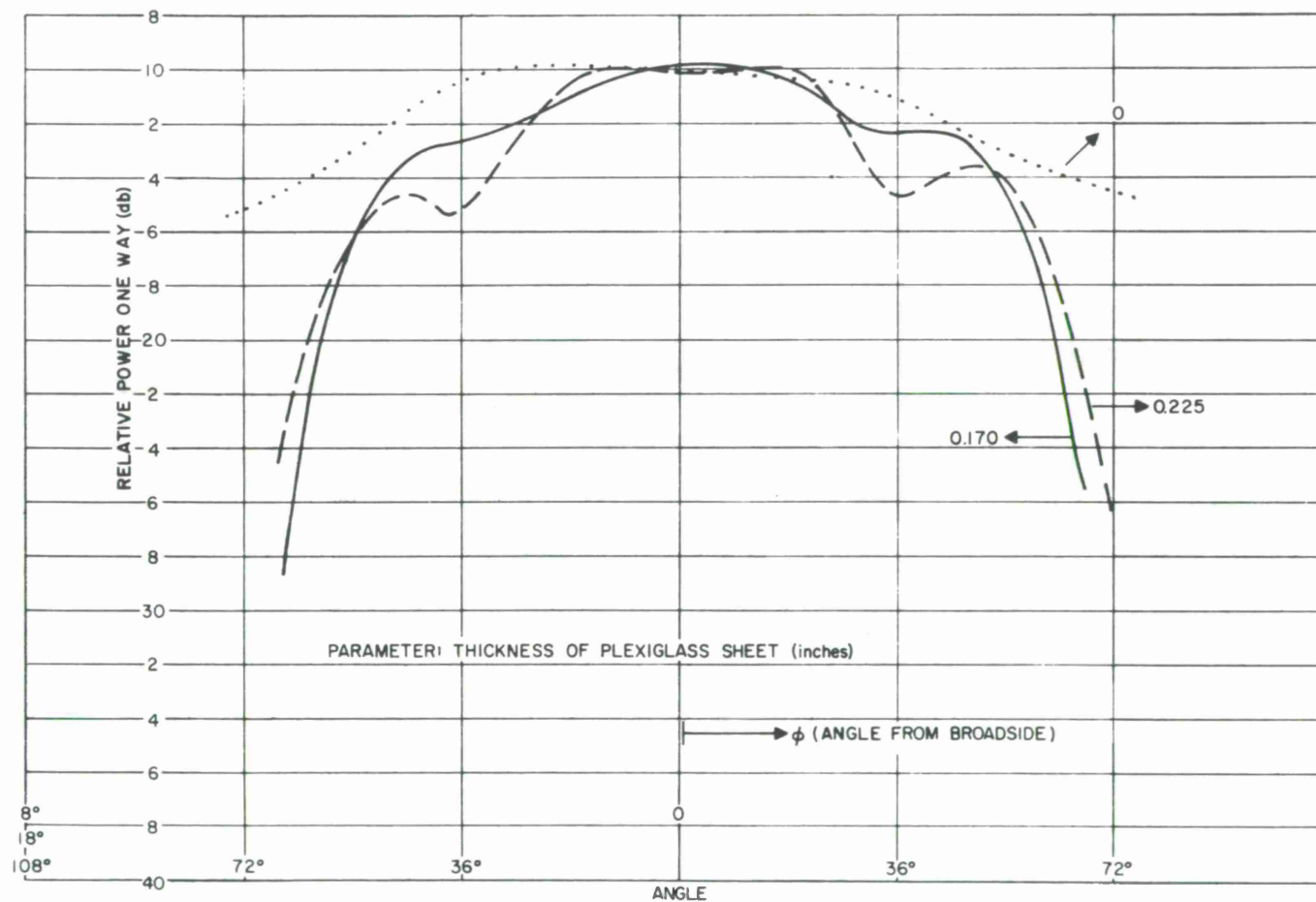


Figure 5. Smoothed Measured Polar Diagrams of Center Waveguide



## SECTION VI

### INTERNAL GRATING LOBES IN POLYROD ARRAY

The purpose of this section is to show that end-fire radiators probably exhibit internal grating lobes at modest scan angles. Only the simplest theoretical problem is treated, but it is sufficient to demonstrate the importance of internal grating lobes. Figure 6 shows the geometry. For a wave incident

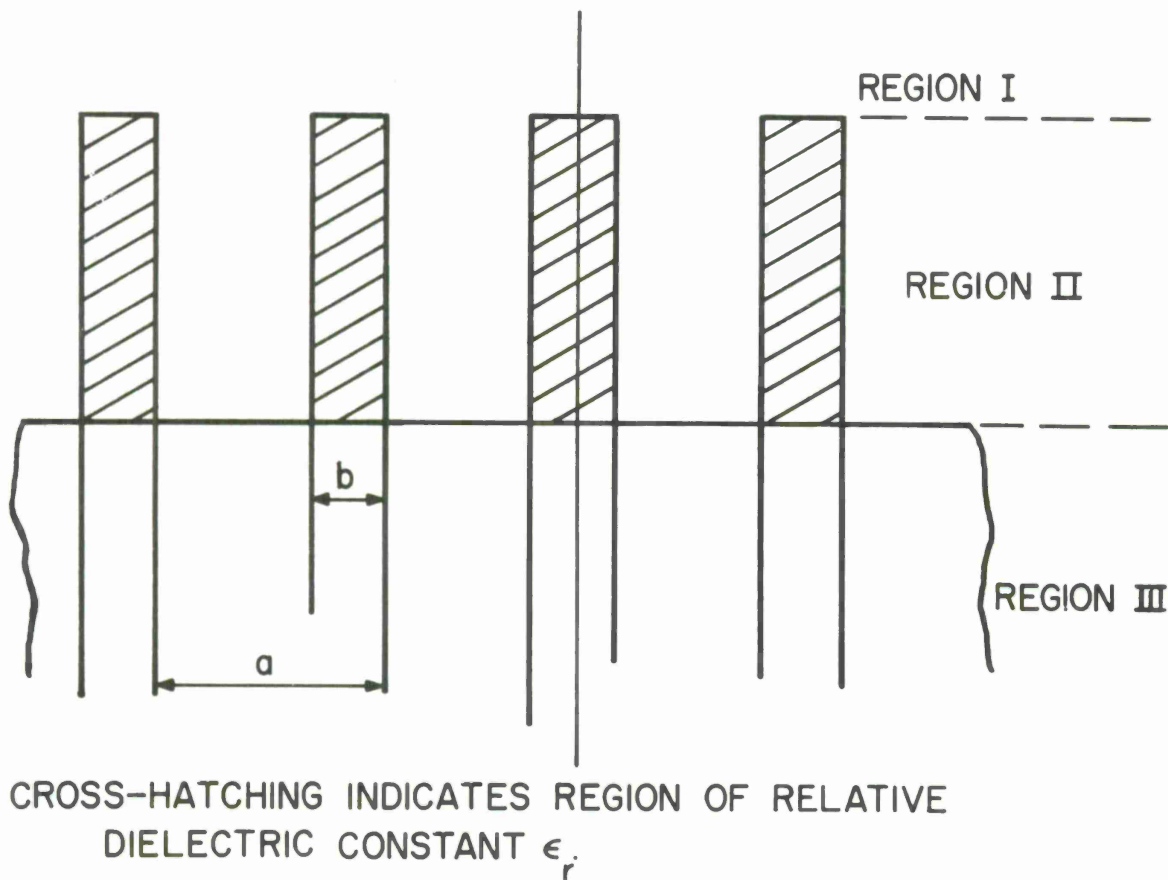


Figure 6. Array of Two-Dimensional Polyrods

at angle  $\phi$ , the transverse resonance condition for the initial appearance of an internal grating lobe is



$$\begin{aligned}
& \cos 2\pi \left( n + \frac{a}{\lambda} \sin \phi \right) \\
& \cos \frac{2\pi(a-b)}{\lambda} \cos \frac{2\pi \epsilon_r^{1/2} b}{\lambda} \\
& - \frac{1}{2} \left( \epsilon_r^{1/2} + \epsilon_r^{-1/2} \right) \sin \frac{2\pi(a-b)}{\lambda} \sin \frac{2\pi \epsilon_r^{1/2} b}{\lambda} ,
\end{aligned}$$

as can be seen by matching the fields at the interfaces between free space and the dielectric in Region II of Figure 6. The value of  $\phi$  satisfied by the above expression is shown in Figure 7 as a function of  $b/a$ . It is clear that internal grating lobes need to be considered in the design of polyrod arrays.

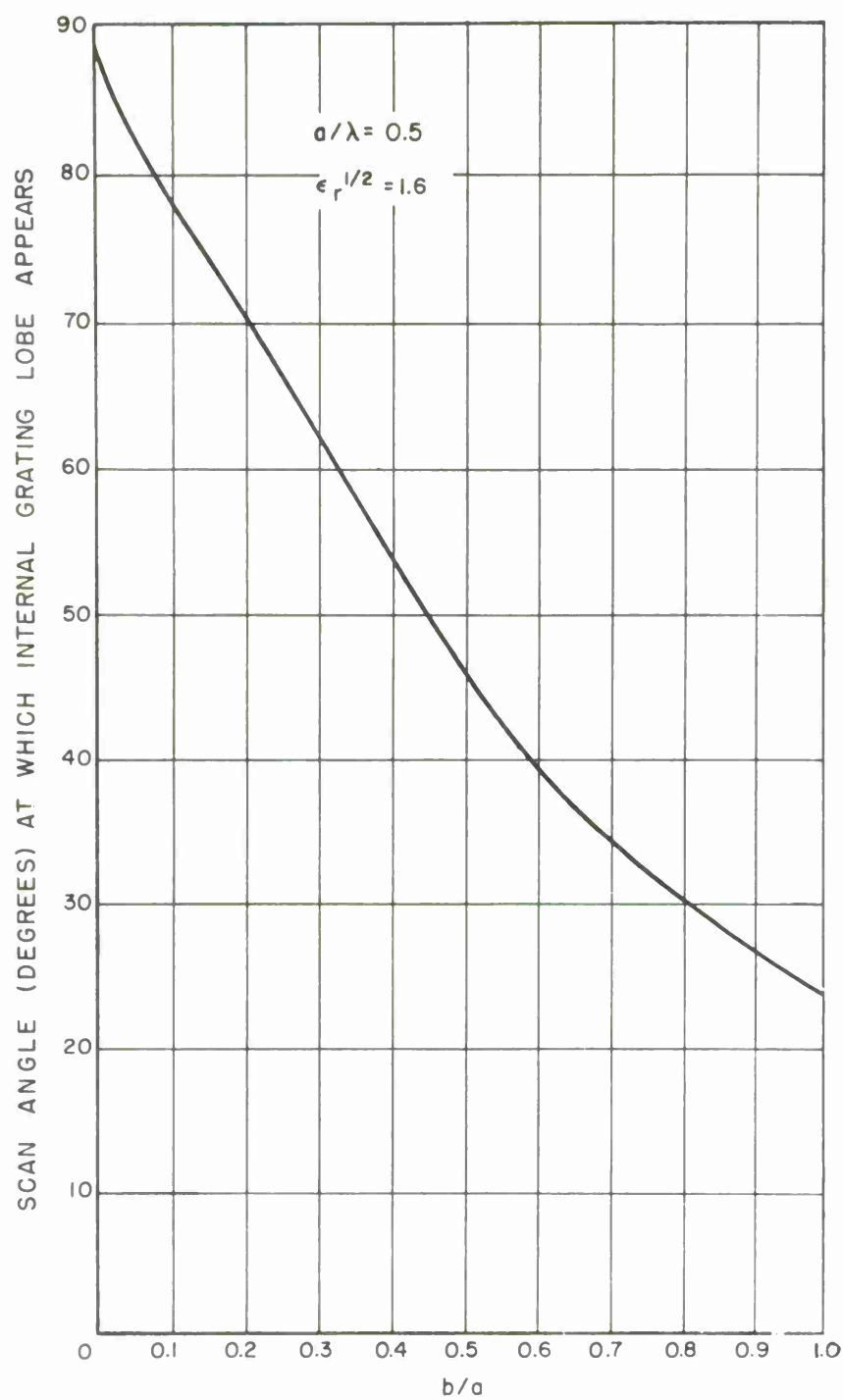


Figure 7. Appearance of Internal Grating Lobes in Polyrod Array

## SECTION VII

### OPTIMUM SIZE OF RADIATOR

Consider the simple array shown in Figure 1. The matching of the array will be considered to be entirely in Region III so that Equation (11) will apply. It will be assumed that  $a/\lambda$  is sufficiently small so that there will be no grating lobes for the scan angles of interest. Only the resistive part of the impedance of the space-array interface will be considered, because it is possible to imagine that a radiator could be found which exhibited a negligible reactance change over a useful range of scan angles. The conductive part  $g$  of the impedance, for the geometry of Figure 1, is given by

$$g = \frac{\cos \phi}{\text{sinc}^2 \left( \frac{kb \sin \phi}{2} \right)}, \quad (16)$$

with  $\epsilon_r = b^2/a^2$ . Remember that the effect of the matching networks in Region III has been approximated by filling the parallel plate lines with a fictitious dielectric which is less dense than free space. Figure 8 shows  $g$  versus  $\phi$ , with  $b/\lambda$  as the parameter. It is seen that, for large scan angles, the optimum value of  $b/\lambda$  is 0.5. Greater values of  $b/\lambda$  are not allowed because  $b$  cannot be greater than  $a$ , and if  $a/\lambda$  is appreciably greater than 0.5, there will be grating lobes in free space for wide scan angles. An interesting point is Equation (11) shows that the change of reactance with scan angle is less for small values of  $b/\lambda$ . It is thus probable that a pronounced optimum size exists for a given type of radiator. In practice, this optimum size will only be found from the type of measurement outlined in the following section.

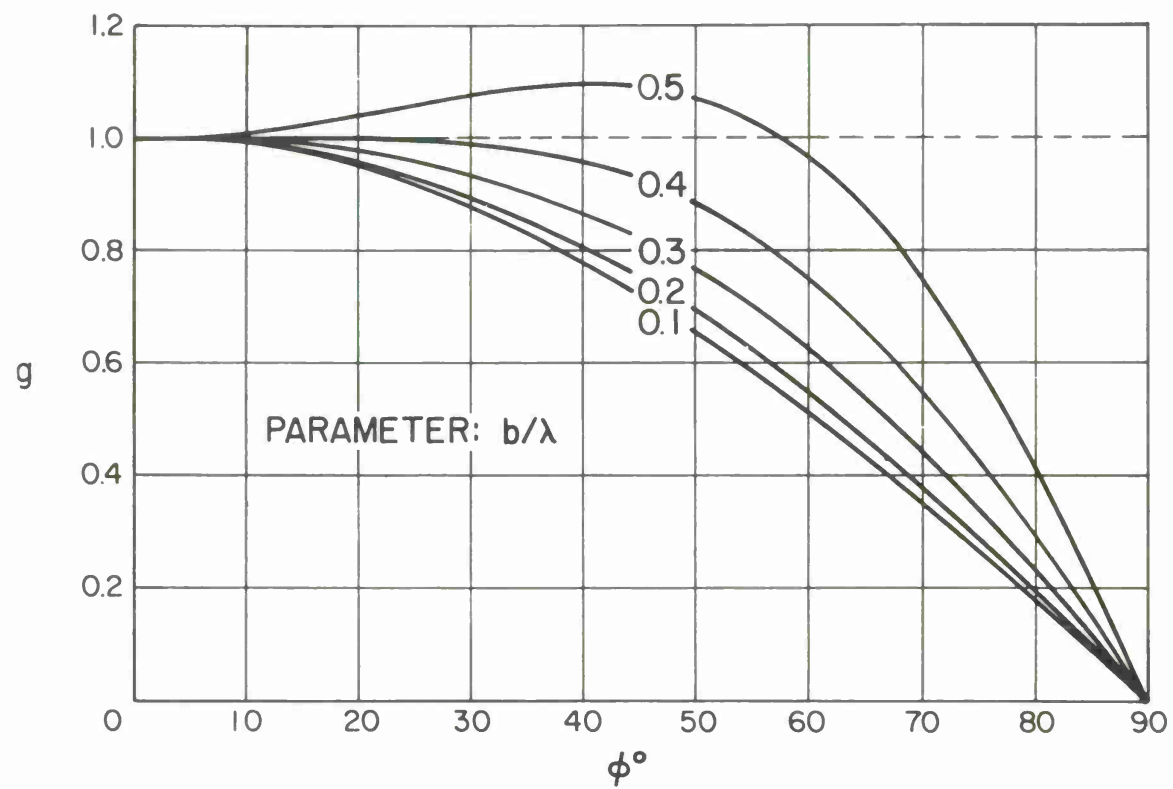


Figure 8. Normalized Conductance Space-Array Interface

## SECTION VIII

### RELATION OF MODE THEORY TO ARRAY MEASUREMENTS

The mode theory has a particular impact on array measurements which could be significant. The scanning performance of an array can be predicted from measurements on a single radiator, in the presence of the other radiators, when they are properly terminated. The gain pattern of the single radiator determines the gain of the whole array. If the scanning performance of a particular array is found to be unacceptably inferior, then measurements on a single radiator will not indicate how the radiators should be modified in order to improve the scanning performance of the array. In general, the gain pattern of a single radiator will not be smooth.

The question is: What should be attempted in practice to force the gain pattern closer to the ideal  $\cos \phi$  ? The mode theory approach indicates that any attempt to reduce mutual impedance by using end-fire radiators may compound the problem because of the increased possibility of the appearance of internal grating lobes. This is not to suggest that there is anything wrong with mutual impedance as a concept, only that mutual impedance does not completely govern the problem. The mode theory indicates that radiators should be sought that have a minimum reactance change over a maximum range of scan angles. Also, there is likely to be an optimum size for each type of radiator. It follows that, besides the simple measurement on a single radiator, one of the two following types of array measurement is required:

- (a) array of matched radiators illuminated with, essentially, a plane wave; amplitude and phase of specularly reflected wave measured as a function of scan angle; and
- (b) array of radiators all driven; ability for phasing radiators required, so beam can be scanned; slotted line inserted in feed to one of central radiators; impedance of radiator measured as a function of scan angle.

Both types of measurement are more delicate and less convenient than the simple measurement on a single radiator. Nevertheless, the diagnostic power of either measurement is superior to that of the simple measurement.

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13. ABSTRACT <p>Attention is concentrated on the radiating and evanescent modes which must appear, both in free space and in the receiving transmission lines, when an infinite array of radiators is illuminated by a plane wave. If the radiators have a tendency to form a slow-wave structure (as all radiators must, even if to only a very slight degree), then for angles of incidence greater than a certain amount there can be propagating modes (surface waves capable of carrying power) supported by the radiators.</p> <p>These propagating modes are dubbed "internal grating lobes" because they can appear even if the spacing of the radiators is close enough to prohibit the possibility of grating lobes in free space. The main effect of the internal grating lobes is to cause mismatching of the array, thereby deteriorating its scanning performance.</p> <p>It was found possible to demonstrate the existence of internal grating lobes experimentally. The main conclusion is that the simplest types of radiators (slots in a flat sheet or dipoles above a ground plane) are probably best for phased arrays because, of all types considered both practical and imaginable, they have the least slow-wave character.</p>			

14. KEY WORDS	LINK A		LINK B		LINK C	
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