TECHNICAL REPORT
ES-18

A METHOD FOR PREDICTING THE PROBABLE FREQUENCY OF DAILY MINIMUM TEMPERATURES FROM SUMMARIZED DATA

by

Earl E. Lackey, Ph.D.

Earth Sciences Division

December 1965

U. S. Army Materiel Command
U. S. ARMY NATICK LABORATORIES
Natick, Massachusetts

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A METHOD FOR PREDICTING THE PROBABLE FREQUENCY OF DAILY MINIMUM TEMPERATURES FROM SUMMARIZED DATA

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Earl E. Lackey, Ph. D.
General Environments Laboratory
Earth Sciences Division

Project Reference:
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FOREWORD

One of the responsibilities of the Earth Sciences Division is to develop methods for extracting the maximum amount of militarily useful information from available climatic summaries. This study describes a method whereby one may find, from a summary record, how often given low temperatures are likely to occur and is one in a series of studies by Dr. Lackey on estimating extreme values of climatic phenomena. It applies to minimum temperatures a method previously devised for maximum temperatures (ES-13 August 1964), which in turn was an improvement on the method presented in EP-88 (May 1958). Some of the data used in the study were tabulated material derived from an unpublished study by Mrs. Jane H. Westbrook.

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ABSTRACT

A method for assessing the relative frequency and level of daily minimum temperatures in various parts of North America in winter months is developed in this paper from the summary 10-year records of 10 widely separated weather stations representative of the numerous variables that are involved in the occurrence of low temperatures. The method is based on the following four items usually found in climatic summaries:

a. absolute minimum temperature

b. mean daily minimum temperature

c. mean daily maximum temperature

d. length of record.

The temperature and frequency estimates are achieved by use of a nomographic device with a grid of converted temperature values representing 36 varying degrees of asymmetry of temperature distribution, and a series of predictive curves designating the frequency of occurrence in days per month.

The efficiency of the method is demonstrated by tests of internal consistency, also by application to 12 widely separated test stations in the United States, and to 6 "Handbook" stations and by varying length of summary records (10 to 70 years) at a single station, Washington, D.C.

A METHOD FOR PREDICTING THE FREQUENCY OF DAILY MINIMUM TEMPERATURES FROM SUMMARIZED RECORDS

INTRODUCTION

The search for patterns of weather phenomena continues without ceasing. We must hope to find more and more useful meanings in the volumes of weather data that accumulate year after year. In this study, it is assumed that by reconstructing weather history from summarized records of accumulated data we will be able to discover patterns of recurrence that will help us estimate present and future probabilities and contingencies.

The present study is a refinement and extension of a previous one on this particular topic, and several on closely related topics. It presents newly-developed methods and techniques, which require less manual effort, encourage machine processing and amplify further the uses of summarized weather records. The previous study featured a graphical method, involving manual processing by use of a specially-designed probability scale and was not as flexible as the method herein described. The 10-year records on which the present work is based are sufficient for useful predictions. However, as shown later in this report, essential data for different decades at the same station may vary enough to affect predictions somewhat. Therefore, although the method worked out is sound, records of 20 or more years should be sought as a basis for prediction by data processing machines. This well-known principle is borne out by an examination of the varying records (10, 20, 30 to 70 years) of a single station, Washington, D. C., later in this report.

The summarized monthly data needed in order to make satisfactory predictions of daily minimum temperature probabilities in any given month are:

a. the absolute minimum temperature (AbMi)*

b. the mean daily minimum (MDMi)

c. the mean daily maximum (MDMx)

d. the length of the record

* All terms are defined in the Glossary.

From these condensed data, it is possible to predict the level at or below which the daily minimum temperature is likely to occur for a given number of days in a given month through any number of years from 1 year (1 day in 31, 1/31) to 100 years (1 day in 3100, 1/3100).

The key to the method is the varying (asymmetrical) position of the MDMi temperature between the MDMx and the AbMi, when the three measures are converted to a 100-unit scale. In brief, this means that the recorded AbMi is changed to a Converted Absolute Minimum (CAbMi), 100; the recorded MDMx is changed to a Converted Mean Daily Maximum (CMDMx); and the recorded MDMi becomes the Converted Mean Daily Minimum (CMDMi), varying in value from CMDMi 20 to CMDMi 55. Details of these conversions are explained later in connection with Table I.

The nomograph (Fig. 2) features 36 patterns of distribution, in which CMDMi values range from 20 to 55.* Each of the 36 patterns describes its own unique series of converted predictive temperature values (CPrT) ranging from Converted Predictive Temperature (CPrT) 0 (zero) to CPrT 100. The three summarized temperatures (AbMi, MDMi, MDMx) are the only values needed to identify the CMDMi pattern of converted predictive temperature values, which can then be changed back to Fahrenheit degrees.

* If predictions for summer months or for low latitudes are desired, the range of CMDMi patterns of asymmetry should be extended from CMDMi 20 to at least CMDMi 80.

PART I

BASIC DATA AND PROCESSING

1. Data Used: 10 Stations, 3 Winter Months, 10-Year Records

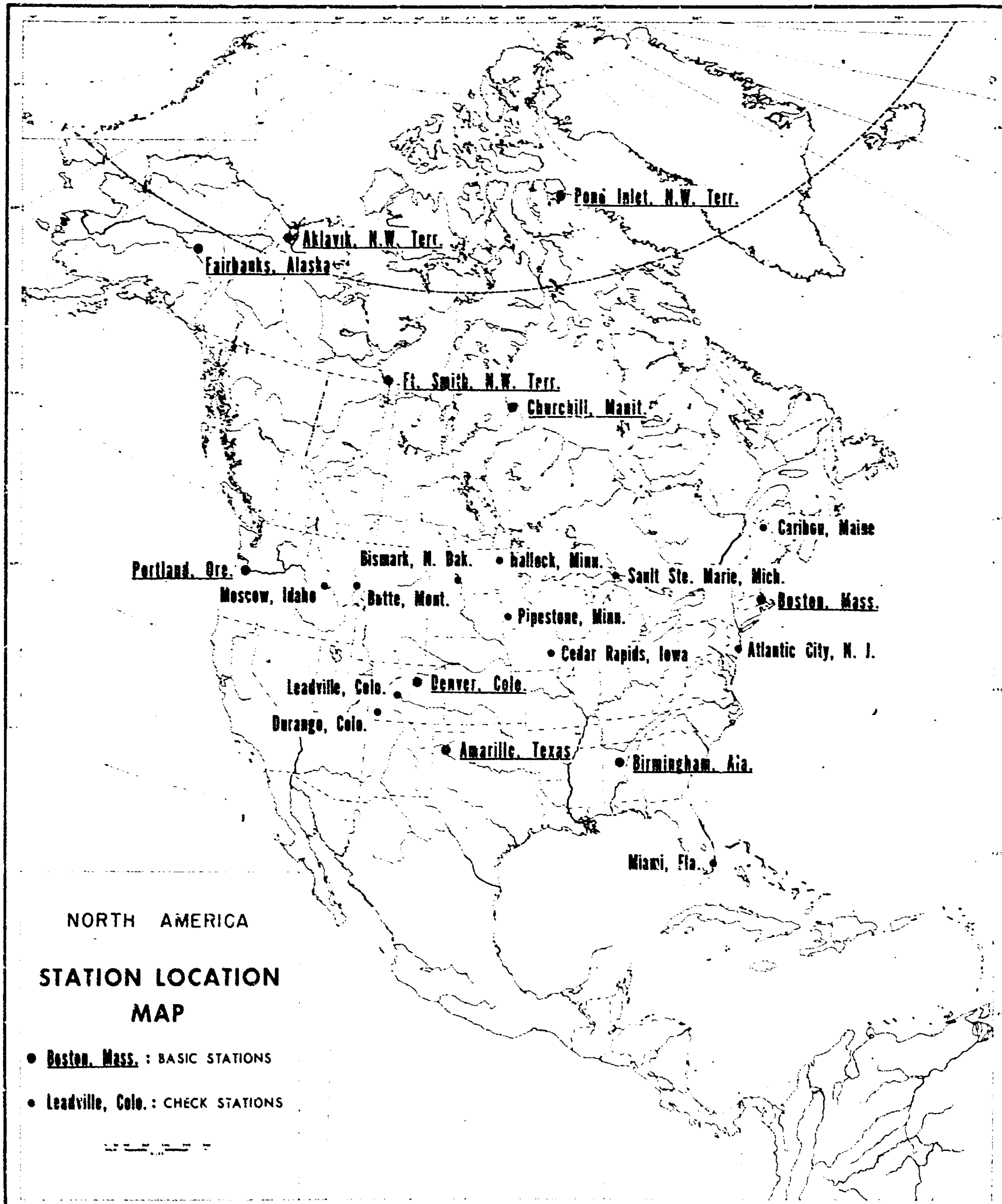
The frequency data for this study came from ten widely separated and diverse climatic areas throughout North America (stations underlined, Fig. 1). (The other 12 stations on the map are those whose records were used to test the reliability of the method, as described later in the report.) All the recorded daily minimum temperatures for November, January and March (representative winter months) through 10 years were assembled into time frequency tabulations for each of the ten stations. These tabulations serve as a model for the nomograph and for predictions based on it.

The method is illustrated in Table I for January at Aklavik, NWT, Canada, located near the mouth of the Mackenzie River. The necessary data appear in line 3, Table I, under two main column headings, the Essential Temperature Data and the Frequency Temperature Data. The Essential Temperature Data for this station and month (line 3, AbMi, -58°F; MDMi, -28°F; and MDMx, -10°F) are the only temperatures needed for prediction purposes, that is, to assess the probable minimum temperature that will be equalled or exceeded through the indicated number of years up to 100. They ordinarily appear in weather station summaries even where more complete data are not published.

In this study the Frequency Temperature Data for 30 months (10 stations, 3 winter months each) were tabulated manually from daily records for 10 years and the values obtained were used in constructing a predicting device - the Multiple Nomograph. An understanding of Table I can greatly help in understanding the discussion that follows.

2. Assembling and Tabulating the Essential and Frequency Data

The procedure in assembling and tabulating the temperature data is shown in part in Table I, that is, for one station (Aklavik) and one month (January). The so-called Essential Temperature Data are the a, b, and c items in line 3 (AbMi, -58°F; MDMi, -28°F; and MDMx, -10°F). Such values are given in nearly all of the world's summarized temperature records. The Frequency Temperature Data in line 3, columns d to n (310/310, -10°F; 250/310, -21°F; 200/310, -25°F; 150/310, -31°F; etc. to 1/310, -58°F), show in detail the number of days in 310 (10 Januarys) or the percentage of January days in 10 years that the minimum fell to at least the temperature levels indicated in line 3. For example, one day



Pond Inlet, N.W. Terr.
Aklavik, N.W. Terr.
Fairbanks, Alaska
Ft. Smith, N.W. Terr.
Churchill, Manit.
Caribou, Maine
Bismark, N. Dak.
Halleck, Minn.
Sault Ste. Marie, Mich.
Portland, Ore.
Moscow, Idaho
Butte, Mont.
Boston, Mass.
Pipestone, Minn.
Cedar Rapids, Iowa
Atlantic City, N. J.
Leadville, Colo.
Denver, Colo.
Durango, Colo.
Amarillo, Texas
Birmingham, Ala.
Miami, Fla.
NORTH AMERICA
STATION LOCATION MAP
Boston, Mass. : BASIC STATIONS
Leadville, Colo. : CHECK STATIONS

Figure 1

TABLE I. AKLAVIK, CANADA: JANUARY
(A sample of the 30 tabulations - 10 stations, 3 months each - used in constructing the Nomograph)

Lines	Nature of Data and Operation	Essential Temperature Data			Frequency Temperature Data by: (line 1) Percentage of the Time and (line 2) days per month (310 days = 10 Januaries)										
					100	80.6	64.5	48.4	32.2	16.1	9.68	3.2	1.08	.645	.32
					310/310	250/310	200/310	150/310	100/310	50/310	30/310	10/310	1/93	1/155	1/310
3	Observed Values, F°	-58 AbMi	-28 MDMi	-10 MDMx	-10	-21	-25	-31	-38	-45	-49	-52	-54	-56	-58
4	*Reduced Data	48 RAbMi	18 RMDMi	0 RMDMx	0	11	15	21	28	35	39	42	44	46	48
5	**Converted Observed Data 100-Unit Scale	100 CAbMi	<u>38</u> CMDMi	0 CMDMx	0	23	31	44	58	73	81	<u>88</u>	92	96	100
					------	------	------	------	------	CFT	------	------	------	------	------
6	***Converted Predictive Values 100-Unit Scale	100	38	0	0	15	26	40	48	63	74	81	90	95	100
					------	------	------	------	------	CPrT	------	------	------	------	------
7	****Predicted Values, F°	-58	-28	-10	-10	-17	-22	-27	-33	-40	-46	-49	-53	-56	-58
Columns		a	b	c	d	e	f	g	h	i	j	k	l	m	n

* Line 4 gives the value in line 3 subtracted from the MDMx (-10°F): all positive

** Line 5 gives the value in line 4 converted to 100-unit scale. These are the actual converted data (CFT) from one station (Aklavik).

*** Line 6 values CPrT (d through n) are the predictive pattern CMDMi 38, taken from Table III, that is, the generalized converted data from 10 stations.

**** Line 7 is predicted from line 3, Essential Data. Compare predicted values in line 7 with observed values in line 3.

Note: CMDMi value (line 5, col. b) is a key value in predictive technique. CMDMi is found by the procedure outlined above, that is: a. subtract its line 3 value (-28) from the MDMx(-10), which gives 18, the Reduced MDMi; b. multiply the result (18) by 100 and divide by the range (MDMx-AbMi). This gives 38. From this a Formula has been constructed which is used in the problems:

$$CMDMi = \frac{100\ (MDMx - MDMi)}{MDMx - AbMi}$$

in 10 Januarys (1/310 or .32% of the time) the temperature fell to -58°F (AbMi); 50 days in 310 (50/310, an average of 5 days in January or 16.1% of the time) the temperature fell at least to -45°F.

3. Converting Data to the 100-Unit Scale

Conventional temperature values (Fahrenheit or Centigrade) cannot be used directly on the Nomograph. It is adapted only to the use of "converted" values, that is, values that have been changed from conventional temperature measures (F or C) to a 100-unit scale. The predicting is done in converted scale values, which are then "reconverted" back to the Fahrenheit (or Centigrade) measures.

For example, to convert the recorded temperature values and frequencies for Table I, line 3 (Aklavik, January) into 100-unit scale values, line 5, it was decided to proceed as follows:

a. Reduce the Fahrenheit Values

Reduce each of the values in line 3 by subtracting each from the mean daily maximum (MDMx = -10°F in line 3). The 3 items of "essential data" become the "reduced values" of line 4:

$$\text{RAbMi} = -10 - (-58) = 48$$
$$\text{RMDMi} = -10 - (-28) = 18$$
$$\text{RMDMx} = -10 - (-10) = 0$$

This gives the algebraic differences differences in line 4 which are all positive values but are reversed in numerical sequence to accord with the desired 100-unit scale. Thus, as seen in the tabulation above, the CMDMx becomes 0 and the AbMi becomes 48, with the frequency of other temperature values holding their relative positions between 0 and 48.

b. Convert Values to 100-Unit Scale

The reduced values in line 4 are then converted to the 100-unit scale by multiplying each by $\frac{100}{\text{Range}}$, that is, $\frac{100}{\text{MDMx} - \text{AbMi}}$. In this way the reduced data of line 4 become the converted values of line 5. For example, for column i, we have:

$$35 \times \frac{100}{-10 - (-58)} = 72.9 \text{ or } 73 \text{ (rounded)}.$$

This means that 73 in the 100-unit scale in line 5 holds the same relative position on the 100-unit scale that 35 does to 48 in line 4. In like manner, all the values in line 3 were converted to the 100-unit scale as shown in line 5. Note especially how MDMi (-28) was converted to CMDMi (38); the CMDMi determines the predictive pattern to be used. The steps in a. and b. above may be stated in the following Formula, which will be used in solving prediction problems:

$$\text{CMDMi} = \frac{100\ (\text{MDMx} - \text{MDMi})}{\text{MDMx} - \text{AbMi}}$$

The essential and the frequency temperatures for November and March at Aklavik were also processed in the same manner, and the same likewise for the other nine stations underlined in Figure 1; thirty processed records in all.

The unique pattern of frequency distribution and temperature levels of converted daily minima which may be expected in January at Aklavik can be identified by its CMDMi which is 38 (underlined in column b) and the associated converted observed temperature frequency values (CFT) in columns d through n, all in line 5, Table I. Note that the CMDMi 38 is the key to the frequency distribution (converted scale) of daily minimum temperatures in January at Aklavik. As mentioned above, this value must be found in solving any prediction problems.

The CMDMi values for the 30 monthly records analyzed in this study, as shown in Table II, ranged from CMDMi 25 (January in Portland) to CMDMi 54 (November in Amarillo) - which is the spread of asymmetry of the CMDMi's relative to their associated CAbMi 100 and CMDMx 0.*

4. Some Assumptions Basic to the Study

In working further with the problem of prediction which we have set for ourselves, the following generalizations are basic. They are supported by the nature of the climatic data, now on record, of the kind being used here.

a. The frequency and level of present and future daily minimum temperatures are reliably related to temperatures of the past.

b. The mean daily minimum and the mean daily maximum temperatures at a given station through a series of years are near constants.

c. The extreme daily minimum temperatures for a given month and place through a series of years, when arranged in descending sequence have a measurable downward trend which in general tends to be slightly less than it was at any given previous time (i.e., at a decelerated rate).

* On the Nomograph, this range (25 to 54) has been extrapolated to cover CMDMx's from 20 to 55. See footnote, Introduction.

TABLE II. CONVERTED MEAN DAILY MINIMUM TEMPERATURES (CMDMi) PAIRED WITH CONVERTED FREQUENCY MINIMA FOR 10 DAYS IN 310 (CFT 10/310) USED TO CONSTRUCT 1 DAY PER MONTH (1/31) CURVED PREDICTION LINE ON THE NOMOGRAPH

Station	Month	CMDMi	CFT 10/310	Station	Month	CMDMi	CFT 10/310
	Nov	38	74		Nov	35	86
Aklavik	Jan	38*	88*	Churchill	Jan	42	89
	Mar	37	87		Mar	45	86
	Nov	54	85		Nov	41	81
Amarillo	Jan	48	82	Denver	Jan	40	76
	Mar	46	76		Mar	43	86
	Nov	31	90		Nov	35	75
Fairbanks	Jan	29	82	Fort Smith	Jan	32	83
	Mar	39	85		Mar	33	76
	Nov	37	69		Nov	26	86
Birmingham	Jan	47	79	Pond Inlet	Jan	34	89
	Mar	46	81		Mar	37	81
	Nov	43	77		Nov	27	66
Boston	Jan	35	83	Portland	Jan	25	75
	Mar	30	74		Mar	47	75

* Underlined values CMDMi and CFT 10/310 are as developed in Table I, Aklavik, Canada, January, line 5.

d. The decelerated trend mentioned in c. above is dependably related to the asymmetrical position of the mean daily minimum between the mean daily maximum (0) and the absolute minimum temperature (100).

e. The decelerated trend mentioned in c. above may be discovered by use of Gumbel's Extreme Probability Scale and the Skew-Log Probability Scale. (See Fig. 3 and Ref 2.)

f. The mean daily minimum and mean daily maximum for a given station are near constant anchors from which to measure the oscillating daily minimum temperature probabilities.

g. The 3 items of essential data for predicting daily minimum temperatures are usually available in summary temperature tables.

PART II
THE NOMOGRAPH AND RELATED TABLE

5. Constructing the Basic Device (Basic Section, Nomograph)

As stated in paragraph 1, the Frequency Temperature Data (similar to those given in Table I) were used in constructing the Nomograph. This is because (as mentioned in par. 3), the unique pattern of frequency distribution and temperature levels of converted daily minima which may be expected can be identified by the CMDMi and the associated CFT values.

In constructing the Nomograph, the sloping 1/31 (10/310) line of the Nomograph was plotted in this way: In Table I, CFT 88 is the related (10/310) CFT value with CMDMi 38, for January, Aklavik. This pair of values appears in Table II (2d line, Aklavik, January).

These 30 pairs of converted values (CMDMi's and Converted Frequency Data, CFD) are plotted as stars on Figure 2. Through and among these stars, an appropriate curve was drawn* and labeled 10 days in 310 (10/310, or 1/31), an average of 1 day in 31.

In a similar manner, the other 9 prediction curves in Figure 2, Basic Section, were drawn, (e.g., 25/31 . . . 1/310) in each case appropriately corresponding to 30 converted frequency values (CFD) ranging between CMDMx (0) and AbMi (100). These separate CFD values, when thus integrated into smooth curves, become the curved prediction temperature frequency lines of the Nomograph, completed to the 10-year frequency (1/310). See Appendix B for a more complete diagram for the construction of the nomograph.

6. Using the Nomograph, Basic Section, for Predicting

Let us go back to Table I and see how near the minima predicted by the Nomograph for January at Aklavik come to the actual occurrences. Restating the problem:

* The trend of this curve is a visual "best fit" carefully modified to correlate with the observed trends of the associated curves, each of which was similarly derived and the whole group then mutually adjusted in the basic section of the nomograph. For the present, the best test of the fit achieved is the ability of the curves to predict minimum temperature frequencies. See Tables IV and VI.

NOMOGRAPH FOR PREDICTING FREQUENCIES

Predicting from 10-Year Records or

BASIC SECTION: For 25 days in a Month (25/31) ; Up To 1 day in 10

Constructed from Actual Frequencies at 10 Stations in North Am

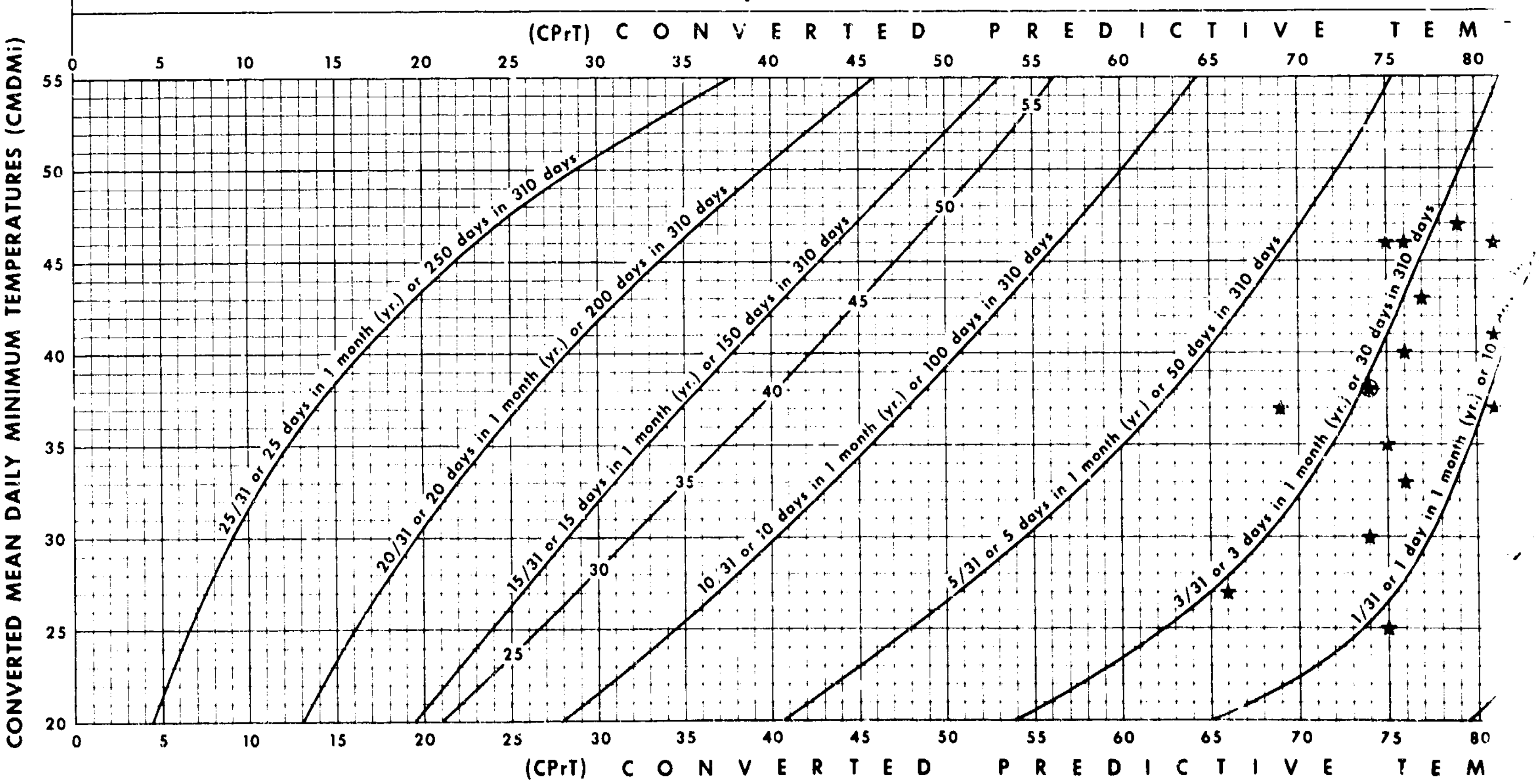


Figure 2

A

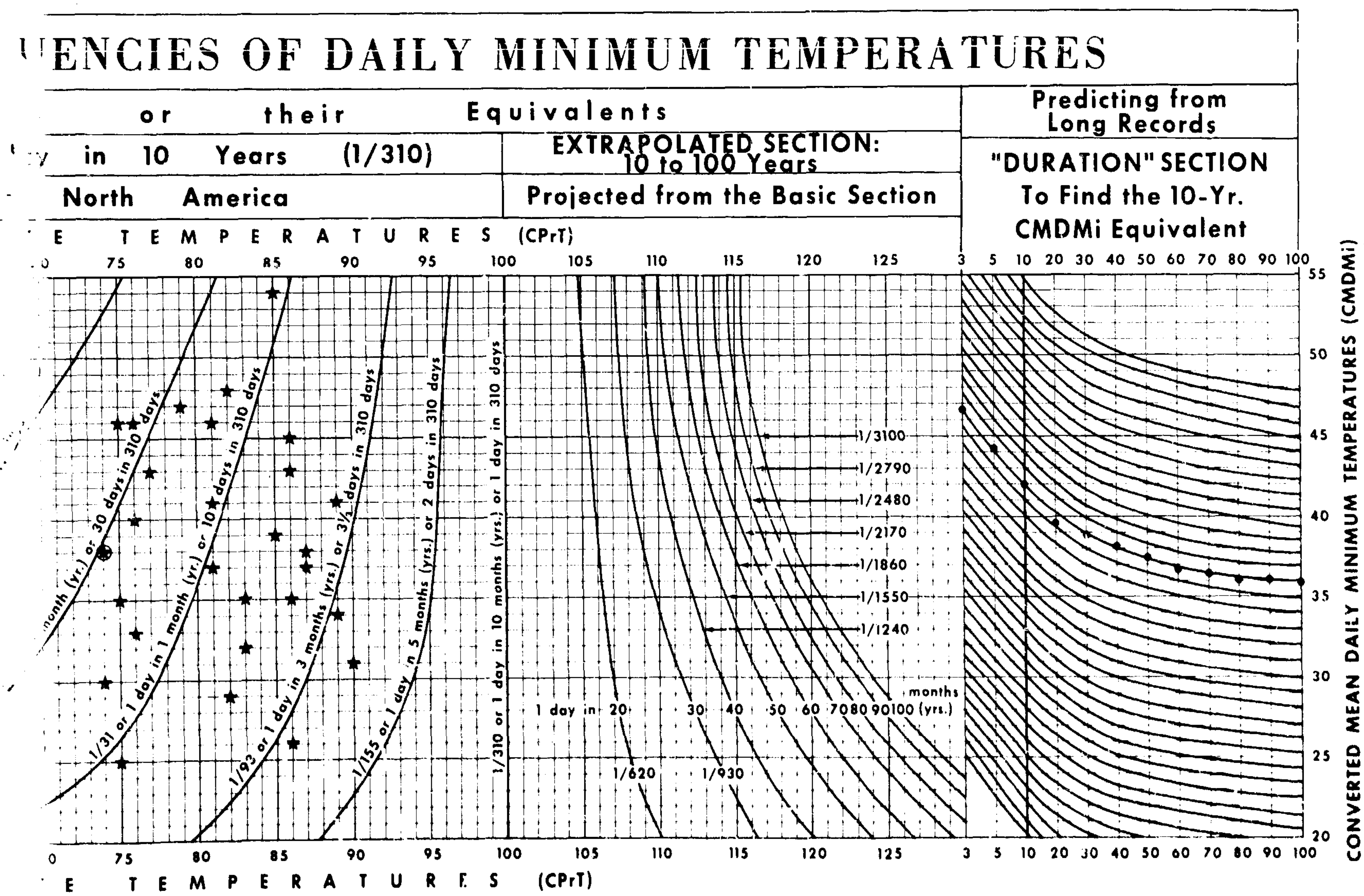

ENCIES OF DAILY MINIMUM TEMPERATURES
or their Equivalents
in 10 Years (1/310)
North America
E T E M P E R A T U R E S (CPrT)
EXTRAPOLATED SECTION: 10 to 100 Years
Projected from the Basic Section
Predicting from Long Records
"DURATION" SECTION
To Find the 10-Yr. CMDMi Equivalent
1/31 or 1 day in 1 month (yr.) or 10 days in 310 days
month (yr.) or 30 days in 310 days
1/93 or 1 day in 3 months (yrs.) or 3½ days in 310 days
1/155 or 1 day in 5 months (yrs.) or 2 days in 310 days
1/310 or 1 day in 10 months (yrs.) or 1 day in 310 days
1/3100
1/2790
1/2480
1/2170
1/1860
1/1550
1/1240
1 day in 20 30 40 50 60 70 80 90 100 (yrs.)
months
1/620
1/930
CONVERTED MEAN DAILY MINIMUM TEMPERATURES (CMDMi)

Figure 2

Given: For a 10-year January record at Aklavik:

AbMi = -58°F

MDMi = -28°F

MDMx = -10°F

Required: Using the Nomograph, what January minima may be expected:

1 day in 31 (1/31)	10 days in 31 (10/31)
1 day in 93 (1/93)	15 days in 31 (15/31)
1 day in 155 (1/155)	20 days in 31 (20/31)
1 day in 310 (1/310)	25 days in 31 (25/31)
5 days in 31 (5/31)	31 days in 31 (31/31)

Solution: (1) Find the CMDMi for January at Aklavik.

Formula:*

$$\text{CMDMi} = \frac{100\ (\text{MDMx} - \text{MDMi})}{\text{MDMx} - \text{AbMi}}$$

Substituting:

$$\text{CMDMi} = \frac{100\ [-10 - (-28)]}{-10 - (-58)} = 37.5 \text{ or } 38 \text{ (rounded)}$$

The CMDMi 38 pattern of CPrT's will be used for predicting.

(2) Find required CPrT values from the Nomograph.

Method: On Figure 2, Nomograph, follow horizontal line CMDMi 38 from the left margin to its intersection with designated prediction curves, then vertically upward to CPrT's

81 (1/31)	49 (10/31)
90 (1/93)	36 (15/31)
95 (1/155)	26 (20/31)
100 (1/310)	15 (25/31)
63 (5/31)	0 (31/31)

* Using the Formula developed in par. 3 and on Table I.

(3) Change above CPrT values to required F° values.

Formula:*

$$\text{Predicted F values} = \text{MDMx} - \frac{\text{CPrT (MDMx-AbMi)}}{100}$$

Substituting:

Probable DMi in °F

$$1/31 = -10 - \frac{81 \left[-10 - (-58)\right]}{100} = -49°F$$

This means that on 1 January day in 31, on the average, we may expect -49°F or lower.

The other desired predictions were similarly calculated and are given in line 7, (d to n), Table I. Comparing the predicted temperatures in line 7 with the recorded values in line 3, we find that the two do not differ at any of the 11 frequency levels by more than 5F°.

7. Nomographic Values, in Tabular Form (Prediction Table, Basic Section)

Each of the frequency prediction (curved) lines (25/31 . . . 1/310) on the Basic Section of the Nomograph, Figure 2, crosses 36 horizontal CMDMi lines; a total of nearly 400 nomographic values (CPrT) are thus fixed. These values are entered in Table III in an arrangement similar to that of the Nomograph. Note: the CPrT values are integrals on the table. These are the values for predicting from 10-year records. (The decimal values in the Table are CMDMi identification factors, to be used if the record is longer or shorter than 10 years and will be explained later in this report.) Thus pattern CMDMi 38 on the Nomograph and Table III (line CMDMi 38) furnish identical CPrT values. The same is true for each of the other 35 CMDMi's and their associated CPrT values.

8. Using Prediction Table, Basic Section, for Predictions

In the Aklavik problem in paragraph 6 the CPrT values associated with CMDMi 38 (81, 90, 100, 63 etc.) can also be found by using the Prediction Table. On the Table (Table III), follow the integral numbers on the horizontal CMDMi 38 line to the required time interval in the column

* Formula for reconversion of minimum temperature associated with given frequency into degrees Fahrenheit.

TABLE III. DATA TAKEN FR

CMDMi	CMDMx 31 in 31	BASIC SECTION — Constructed From 10-Years Actual Minimum Frequencies — Average Number of Days in One Month (CPrT): 25 in 31	20 in 31	15 in 31	10 in 31	5 in 31	3 in 31	1 yr 1 in 31	3 yrs 1 in 93	5 yrs 1 in 155	10 yrs 1 in 310
55	0	38	46	56	64	75	81	86	93 59.1	97 56.7	100 55
54	0	36	45	55	64	75	81	86	92 58.6	96 56.2	100 54
53	0	34	43	54	63	74	81	86	92 57.6	96 55.2	100 53
52	0	32	42	54	62	74	80	85	92 56.5	96 54.2	100 52
51	0	30	41	53	61	73	80	85	92 55.4	96 53.1	100 51
50	0	29	40	52	60	72	79	84 59.5	92 54.3	96 52.1	100 50
49	0	27	38	51	59	72	79	84 58.3	92 53.3	96 51.0	100 49
48	0	25	37	50	58	71	78	84 57.1	91 52.7	96 50.0	100 48
47	0	24	36	49	57	70	78	83 56.6	91 51.6	96 49.0	100 47
46	0	23	35	48	56	69	77	83 55.4	91 50.5	96 47.9	100 46
45	0	22	33	47	56	69	77	83 54.2	91 49.5	96 46.9	100 45
44	0	21	32	46	55	68	76 57.9	83 53.0	90 48.9	96 45.8	100 44
43	0	19	31	45	53	67	76 56.6	82 52.4	90 47.8	95 45.3	100 43
42	0	18	30	44	52	66	75 56.0	82 51.2	90 46.7	95 44.2	100 42
41	0	17	29	43	51	65 63.1	75 54.6	82 50.0	90 45.6	95 43.2	100 41
40	0	17	28	42	50	65 61.5	74 54.0	82 48.8	90 44.4	95 42.1	100 40
39	0	16	27	41	50	64 60.9	74 52.7	81 48.1	90 43.3	95 41.1	100 39
38	0	15	26	40	48	63 60.3	74 51.4	81 46.9	90 42.2	95 40.0	100 38
37	0	14	25	39	47	62 59.7	73 50.7	80 46.3	89 41.6	95 38.5	100 37
36	0	13	24	38	46	61 59.0	73 49.3	80 45.0	89 40.4	95 37.9	100 36
35	0	12	24	37	45	60 58.3	72 48.6	80 43.8	89 39.3	94 37.2	100 35
34	0	12	23	36	44	59 57.6	71 47.9	79 43.0	89 38.2	94 36.1	100 34
33	0	11	22	35	43	58 56.8	71 46.4	79 41.8	88 37.5	94 35.1	100 33
32	0	11	21	34	42	57 56.1	70 45.7	78 41.0	88 36.4	94 34.0	100 32
31	0	10	21	33	41	56 55.3	69 44.9	78 39.7	87 35.6	93 33.3	100 31
30	0	9	20	32	40	54 55.5	68 44.1	77 39.0	87 34.5	93 32.3	100 30
29	0	9	19	30	39	53 54.7	67 43.3	77 37.7	86 33.7	93 31.2	100 29
28	0	8	18	29	38	52 53.3	66 42.4	76 36.8	86 32.6	92 30.4	100 28
27	0	7	17	28	37 72.9	50 54.0	65 41.5	76 35.5	86 31.4	92 29.3	100 27
26	0	7	16	27	36 72.2	49 53.1	64 40.6	75 34.7	85 30.6	91 28.6	100 26
25	0	6	16	26	34 73.5	48 52.0	62 40.3	74 33.8	84 29.8	91 27.5	100 25
24	0	6	15	25	33 72.7	46 52.2	61 39.3	72 33.3	83 28.9	90 26.7	100 24
23	0	6	15	24	32 71.9	45 51.1	59 39.0	71 32.4	82 28.0	90 25.6	100 23
22	0	5	14	23	31 71.0	43 51.2	58 37.9	69 31.9	81 27.2	89 24.7	100 22
21	0	5	14	22	29 72.4	42 50.0	56 37.5	67 31.3	80 26.2	89 23.6	100 21
20	0	4	13	21	28 71.4	40 50.0	54 37.0	65 30.8	79 25.3	88 22.7	100 20

Comment:

In addition to CPrT values (integral), this table gives CMDMi values to one decimal place. Use of these a

DATA TAKEN FROM NOMOGRAPH

	CAB Mi / CMD Mi	EXTRAPOLATED SECTION — Constructed by Extrapolation From 10-Year Frequency Records								
		One Day in a Given Number of Months or Days								
s 155	10 yrs 1 in 310	20 yrs 1 in 620	30 yrs 1 in 930	40 yrs 1 in 1240	50 yrs 1 in 1550	60 yrs 1 in 1860	70 yrs 1 in 2170	80 yrs 1 in 2480	90 yrs 1 in 2790	100 yrs 1 in 3100
.7	100 55	105 52.4	107 51.4	109 50.5	110 50.0	112 49.1	113 48.7	114 48.2	114 48.2	115 47.8
.2	100 54	105 51.4	107 50.5	109 49.5	110 49.1	112 48.2	113 47.8	114 47.4	114 47.4	116 46.6
.2	100 53	105 50.5	107 49.5	109 48.6	110 48.2	112 47.3	113 46.9	114 46.5	115 46.1	116 45.7
.2	100 52	105 49.5	107 48.6	109 47.7	110 47.3	112 46.4	113 46.0	114 45.6	115 45.2	116 44.8
.1	100 51	105 48.6	108 47.2	109 46.8	110 46.4	112 45.5	113 45.1	115 44.3	115 44.3	116 44.0
.1	100 50	105 47.2	108 46.3	110 45.5	111 45.0	112 44.6	113 44.2	115 43.5	115 43.5	116 43.1
.0	100 49	105 46.7	108 45.4	110 44.5	111 44.1	113 43.4	113 43.4	115 42.6	115 42.6	116 42.2
.0	100 48	105 45.7	108 44.4	110 43.6	111 43.2	113 42.5	114 42.1	115 41.7	116 41.4	116 41.4
.0	100 47	105 44.8	108 43.5	110 42.7	111 42.3	113 41.6	114 41.2	116 40.5	116 40.5	116 40.5
.9	100 46	105 43.8	108 42.6	110 41.9	111 41.4	113 40.7	114 40.4	116 39.7	116 39.7	117 39.3
.9	100 45	105 42.9	108 41.7	110 40.9	112 40.2	113 39.8	114 39.5	116 38.8	116 38.8	117 38.5
.8	100 44	106 41.5	108 40.7	110 40.0	112 39.3	114 38.6	115 38.3	116 37.9	116 37.9	117 37.6
.3	100 43	106 40.6	108 39.8	110 39.1	112 38.4	114 37.7	115 37.4	116 37.1	116 37.1	117 36.8
.2	100 42	106 39.6	108 38.9	110 38.2	112 37.5	114 36.8	115 36.5	116 36.2	116 36.2	117 35.9
.2	100 41	106 38.7	108 38.0	110 37.3	112 36.6	114 36.0	115 35.7	116 35.3	117 35.0	118 34.7
.1	100 40	106 37.7	108 37.0	110 36.4	112 35.7	114 35.1	116 34.5	116 34.5	117 34.2	118 33.9
.1	100 39	106 36.8	109 35.8	111 35.1	113 34.5	115 33.9	116 33.6	117 33.3	118 33.1	119 32.8
.0	100 38	106 35.8	109 34.9	111 34.2	113 33.6	115 33.0	117 32.5	117 32.5	118 32.2	119 31.9
.5	100 37	106 34.9	109 33.9	111 33.3	114 32.5	116 31.9	117 31.6	118 31.4	119 31.1	120 30.8
.9	100 36	106 34.0	110 32.7	112 32.1	114 31.6	116 31.0	117 30.8	118 30.5	119 30.3	121 29.9
.2	100 35	106 33.0	110 31.8	113 31.0	115 30.4	117 29.9	118 29.7	119 29.4	120 29.2	121 28.9
.1	100 34	106 32.1	110 30.9	113 30.1	115 29.6	117 29.1	118 28.8	119 28.6	120 28.3	121 28.1
.1	100 33	107 30.8	110 30.0	114 28.9	116 28.4	117 28.2	119 27.7	120 27.5	121 27.3	122 27.0
.0	100 32	107 29.9	111 28.8	114 28.1	116 27.6	118 27.1	119 26.9	120 26.7	121 26.4	122 26.2
.3	100 31	107 29.0	111 27.9	115 27.0	116 26.7	118 26.3	120 25.8	121 25.6	122 25.4	122 25.4
.3	100 30	107 28.0	111 27.0	115 26.1	117 25.6	118 25.4	120 25.0	122 24.6	123 24.4	123 24.4
.2	100 29	107 27.1	111 26.1	115 25.2	117 24.8	119 24.4	121 24.0	122 23.8	123 23.6	124 23.4
.4	100 28	107 26.2	111 25.2	115 24.3	117 23.9	119 23.5	121 23.1	123 22.8	123 22.8	125 22.4
.3	100 27	108 25.0	111 24.3	116 23.3	118 22.9	120 22.5	121 22.3	123 22.0	124 21.8	126 21.4
.6	100 26	108 24.1	111 23.4	116 22.4	118 22.0	120 21.7	122 21.3	124 21.0	125 20.8	127 20.5
.5	100 25	108 23.1	111 22.5	116 21.6	119 21.0	121 20.7	122 20.5	124 20.2	126 19.8	128 19.5
.7	100 24	108 22.2	112 21.4	117 20.5	120 20.0	122 19.7	123 19.6	125 19.2	127 18.9	130
.6	100 23	109 21.1	113 20.4	117 19.7	121 19.0	123	124	126	128	131
.7	100 22	109 20.2	114 19.3	118	122	124	125	128	130	132
.6	100 21	109 19.2	115	119	123	125	127	129	131	133
.7	100 20	110 18.1	116	120	124	127	129	131	133	135

Use of these additional values is discussed on p. 14 in the text.

B

heading at the top. This will give the same values as Step 2 of the problem in paragraph 6. Once you have the CPrT values, the procedure is the same. That is, in Step 3 you substitute the CPrT values in the Reconversion Formula.

Let us now consider an Example, using the CPr values from the Prediction Table.

<u>Given</u>: In December, at Grand Prairie, Alberta, during a 10-year period, the

AbMi was -48°F

MDMi " -2°F

MDMx " 17°F.

<u>Required</u>: What daily minimum temperatures in December may be expected within the following indicated intervals of time?

5 days in 31 (5/31)	1 day in 155 (1/155)
1 day in 31 (1/31)	1 day in 310 (1/310)
1 day in 93 (1/93)	310 days in 310 (310/310 or 31/31).

<u>Solution</u>: (1) Find the CMDMi pattern for December.

Formula:

$$\text{CMDMi} = \frac{100\ (\text{MDMx} - \text{MDMi})}{\text{MDMx} - \text{AbMi}}$$

Substituting:

$$\text{CMDMi} = \frac{100\ [17 - (-2)]}{17 - (-48)} = 29.23 \text{ or } 29 \text{ (rounded)}$$

Therefore, the CMDMi 29 pattern of CPrT's will be used for predicting daily minima.

(2) Find the CPrT values on CMDMi 29 in Table III at the required frequency intervals. These are the integral numbers in the Table:

5/31, CPrT 53	1/155, CPrT 93
1/31, CPrT 77	1/310, CPrT 100
1/93, CPrT 86	310/310, CPrT 0 or 31/31, CPrT 0

(3) Find required Fahrenheit predictions.

Formula:

$$PrDMi(F^\circ) = MDMx - \frac{CPrT\ (MDMx - AbMi)}{100}$$

Substituting:

$$PrDMi\ 5/31\ (F^\circ) = 17 - \frac{53\ [17 - (-48)]}{100} = -17^\circ F$$

5 days in December we may expect a minimum of -17°F or lower.

Other expected minima are:

PrDMi 1/31 = -33°F
PrDMi 1/93 = -39°F
PrDMi 1/155 = -43°F
*PrDMi 1/310 = -48°F
*PrDMi 310/310 = 17°F

---

* Note that we have here the recorded 10-year Absolute Minimum (-48°F) and the 10-year MDMx (17°F).

PART III
USING EXTRAPOLATED SECTION (NOMOGRAPH AND PREDICTION TABLE) TO PREDICT FOR DECADES BEYOND 10 YEARS

9. Trends in Daily Minimum Temperature

Thus far in this study, minimum temperature predictions by months from 10-year records have been confined to 10 years or less. If predictions for longer periods of time than 10 years are desired, say 20 to 50 years or more, the nomograph and Table III will require additional built-in features. These we will now proceed to construct.

The daily minimum temperatures for 10 years (310 January days) at a given station in a given month, when arranged in numerical sequence show a descending trend with the increase in the length of the record, but at a decelerated rate, corresponding to, but varying somewhat from, the Gumbel Theory of Extreme Values. Because the pattern of deceleration is not uniform, it is necessary in some cases to supplement the Gumbel Extreme Probability Scale by use of the Adjustable Skew-Log Probability Scale.*

10. Use of Probability Scales for Extrapolative Purposes

When CPrT 100, 95, 90, 81 and 74 for CMDMi 38 (10-year Basic Section Table III) are plotted in their respective places on Extreme Probability paper, Figure 3, the visual "best fit" straight line** through these points (encircled dots) when extended upward and to the right gives extrapolated values at decade levels (10 years to 100 years) of 100*** (10 years), 106 (20 years) 109, 111, 113, 115, 117, 117, 118 and 119 (100 years), respectively. In like manner, each of the other 35 CMDMi patterns were extrapolated from Table III, Basic Section, and then set up as a tentative Extrapolated Section to Table III.

11. Projecting Nomograph, Basic Section, into Extrapolated Section to Cover Predictions for Additional Decades

The 36 CMDMi series of CPrT values derived by extrapolation in Table III were plotted, smoothed and adjusted to make the prediction frequency

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* The Skew-Log Probability Scale is a Gumbel Extreme Probability Scale superimposed on a 2-cycle Log-Scale.

** If this is a curved line, it would be better to use the adjustable Skew-Log Probability Scale. See Technical Report ES-9, p. 15 for use of the Skew-Log Scale.

*** This (100) is the converted minimum for a 10-year record for all predictive patterns from CMDMi 20 to CMDMi 55.

PROJECTION OF 10-YEAR PREDICTIVE PATTERNS
BY USE OF EXTREME PROBABILITY AND SKEW-LOG SCALES

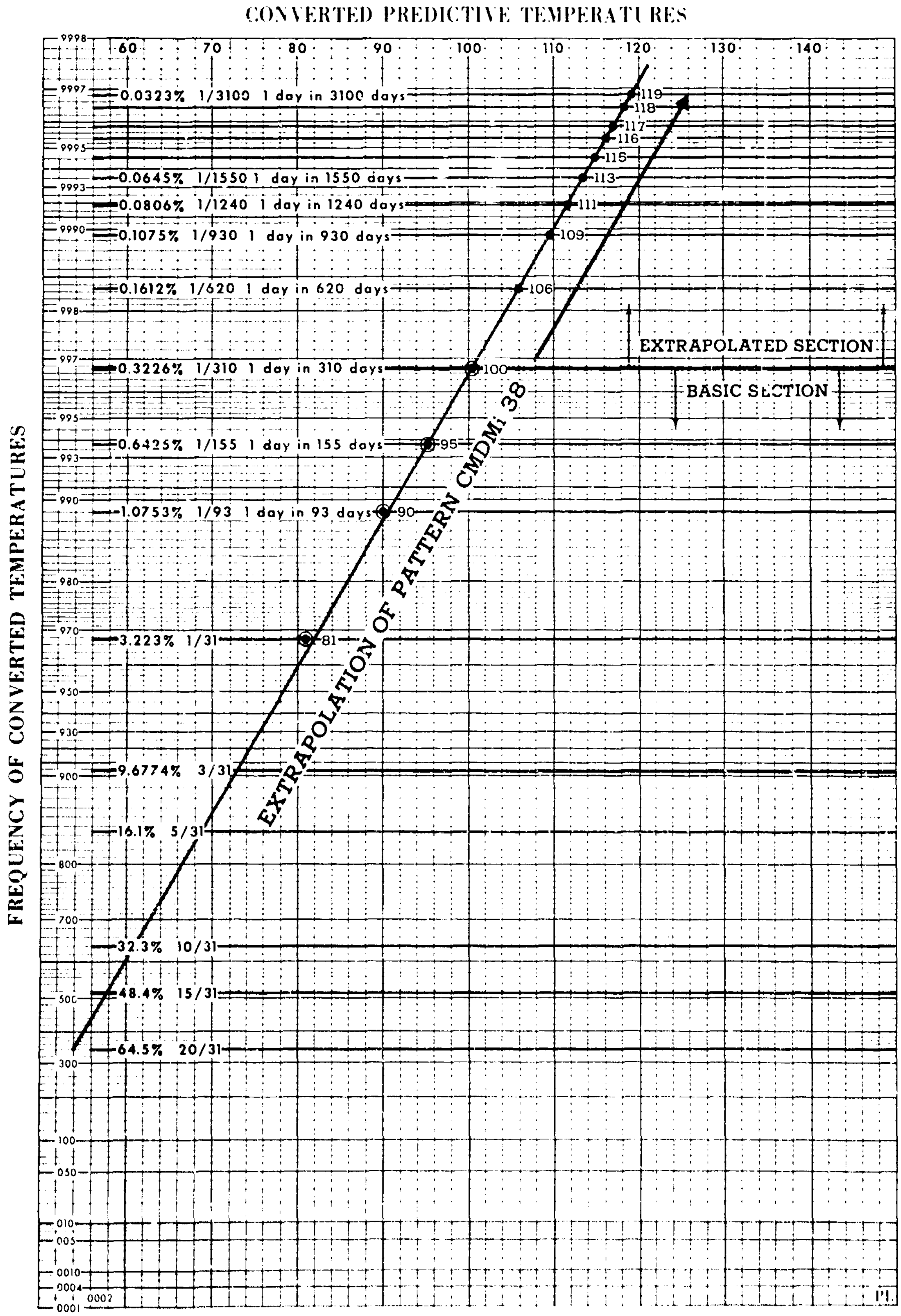

Figure 3

curved CPrT lines (1/620, 1/930, 1/1240, etc. to 1/3100) in the Extrapolated Section of the Nomograph. This in turn called for a few correlative adjustments in the tentative Table III, so that the Nomograph and Table III values would synchronize in a satisfactory manner.

12. Using the Expanded Nomograph (Including the Extrapolated Section) for Predicting for Decades Beyond 10 Years

In Figure 2, each of the 36 CMDMi lines (horizontal) in the extrapolated section is crossed by 10 CPrT* predictive curved frequency lines, making in all 360 intersections. Thus, the converted predictive temperature values (CPrT) for January at decade intervals 10 years and above at Aklavik with its CMDMi 38 are, respectively (using method described in Step 2, par. 6):

1/310, 100	1/1860, 115
1/620, 106	1/2170, 117
1/960, 109	1/2480, 117
1/1240, 111	1/2790, 118
1/1550, 113	1/3100, 119

When these CPrT values are converted to F°, using reconversion formula as in Step 3, paragraphs 6 and 8, the most probable daily minimum January temperatures by decades at Aklavik are:

-61°F or lower in 20 years
-62°F or lower in 30 years
-63°F or lower in 40 years
-64°F or lower in 50 years
-65°F or lower in 60 years
-66°F or lower in 70 years
-66°F or lower in 80 years
-67°F or lower in 90 years
-67°F or lower in 100 years

13. Use of Extrapolated Section of Prediction Table to Predict for Decades Beyond 10 Years

Prediction of daily minimum temperatures by use of Table III, (Extrapolated Section) can be done more rapidly than by use of the nomograph. This part of Table III is simply an extension of the Basic Section and is designed to use Table III for prediction of probable minima by decades up to 100 years. The steps are the same as for predicting from Basic Section or for predicting from the Nomograph (par. 12):

a. Find the CMDMi

b. Find on Table III, on the CMDMi pattern in Extrapolated Section, the CPrT for the required frequency.

c. Use the Reconversion Formula to reconvert to °F.

* Both the Basic and Extrapolated sections include the key CPr value 100 (the converted 10-year minimum).

PART IV
USING THE 10-YEAR TABLE OR NOMOGRAPH TO PREDICT FROM RECORDS LONGER THAN 10 YEARS

14. Identification of 10-Year CMDMi (Converted Mean Daily Minimum) Patterns from Multiple-Decade Records

It must be remembered that the CPrT values of both the Basic and Extrapolated Sections of Table III are keyed exclusively to 10-year records. Therefore Table III may be used for predicting a DMi temperature and frequencies when the essential data are for approximately 10 years. When the essential data for processing come from records longer (or shorter) than 10 years, we must first identify the 10-year equivalent pattern (CMDMi) to substitute for the CMDMi for the longer period of record.

Solution of problems involving records of more than 10 years can be done more easily by use of Table III than by the Nomograph.

The decimal values in the decade columns Table III, are the converted mean daily minima (CMDMi) associated with the family of CPrT values in the same line - all keyed to the 10-year basic records used in constructing the nomograph. For example, in the first line, 55 is the CMDMi of a temperature frequency distribution lying asymmetrically between CPrT 0 (CMDMx) and CPrT 100 (CAbMi). Its value was determined by study of 10-year records as outlined earlier. In the same line (extrapolated) the 10-year CPrT value in 70-year column is 113. That is, in 70 years the CMDMx is 0 and the CAbMi is 113 on the equivalent 10-year 100-unit scale.

The decimal value 48.7, associated with the 113 in CMDMi 55 pattern is found thus:

$$\frac{(\text{10-yr CAbMx})\ (\text{10-yr CMDMi})}{\text{10-yr CPrT value}} = \frac{100 \times 55}{113} = 48.7 \text{ or } 49 \text{ (rounded)}$$

and is the CMDMi between 0 and 113 for the 70-year record. Thus, the CMDMi 55 pattern of CPrT values for a 10-year record becomes the exact equivalent for a CMDMi 49 pattern for a 70-year record. This obviates the need for constructing more than a dozen 100-unit conversion tables similar to Table III - one for each frequency interval of time. All the fractional values in the decade columns were computed in the same manner.

This device enables one to use the 10-year basic Table III for predicting the frequency and temperature level of daily minima from any decile record for any winter month from 3 years to 100 years.

15. Constructing the "Duration" Section of the Nomograph

The decimal CMDMi values in the several columns of Table III enable us to construct the "Duration" Section of the Nomograph. For example, the data for the black dots in the "Duration" Section were taken from the decimal values in line 42, Table III (10-year CMDMi 42). The curved line drawn through and among these dots defines the decelerated downward temperature trend corresponding to the 10-year CMDMi 42 as the length of the record increases from 3 years to 100 years. Each of the other curved lines in Section III were drawn in like manner, and defines similar identification values for predicting daily minimum temperatures from records of various lengths.

16. Predicting from Multiple-Decade Records by Use of the 10-Year Nomograph

The "Duration" Section of the nomograph enables us to predict probable daily minimum temperatures from summary records of any length up to 100 years. The Duration Section is used to identify the 10-year equivalent pattern to substitute for the CMDMi for the longer period. Once we have the 10-year CMDMi equivalent and the 10-year AbMi, we can proceed much as we did with the previous problems.

Given: In January at Omaha a 50-year summary record was:

$$PMi = -32°F$$

$$PMDMi = 13°F$$

$$PMDMx = 30°F$$

Required: What probable January daily minimum may be expected:

1 day in 10 years (1/310)?
1 day in 30 years (1/930)?
1 day in 50 years (1/1550)?
1 day in 80 years (1/2480)?

Solution: (1) Find the 50-year CMDMi.

Formula:*

$$CMDMi = \frac{100\ (PMDMx - PMDMi)}{(PMDMx - PMi)}$$

* As developed in par. 3.

Substituting:

$$CMDMi = \frac{100\,[(30) - (13)]}{30 - (-32)} = 27.4$$

(2) Find the 10-year CMDMi equivalent to the 50-year CMDMi 27.4.

Method: On the nomograph ("Duration" Section) on the 50-year vertical line, find curved (identification) line nearest to 27.4. This curved line crosses the 10-year vertical line (accentuated) at CMDMi 32 (horizontal line). Therefore, the CMDMi 32 pattern of CPrT values is to be used for prediction.

(3) <u>Find the required CPrT values</u>. On the Nomograph, follow the CMDMi 32 pattern to the left to each required time interval - 1/310, 1/930, 1/1550, 1/2480, thence vertically upward to CPrT values 100, 111, 116, 120, respectively.

(4) Find the 10-year PMi.

Formula:

$$\text{10-yr PMi} = PMDMx - \frac{100\,(PMDMx - PMi)}{\text{10 yr CPrT at 50-yr level}}$$

Substituting:

$$\text{10-yr PMi} = 30 - \frac{100\,[30 - (-32)]}{116} = -23°F$$

The expected 10-year minimum is -23°F.

(5) Find expected January minimum temperatures °F at specified decade intervals.

Formula: (Reconversion to °F)

$$\text{Required PMi} = PMDMx - \frac{\text{10-yr CPrT* } (MDMx - \text{10-yr AbMi})^{**}}{100}$$

* As found in Step 3
** As found in Step 4

Substituting:

$$\text{Required 10-yr PMi} = 30 - \frac{100\,[30 - (-23)]}{100} = -23°F$$

$$\text{Required 30-yr PMi} = 30 - \frac{111\,[30 - (-23)]}{100} = -29°F$$

$$\text{Required 50-yr PMi} = 30 - \frac{116\,[30 - (-23)]}{100} = -32°F$$

$$\text{Required 80-yr PMi} = 30 - \frac{120\,[30 - (-23)]}{100} = -34°F$$

PART V
TESTING THE RELIABILITY OF THE METHOD

17. Internal Consistency

When Table III is used for predictions using the Essential Temperature Data (Table I, line 3) for January at Aklavik, the predicted F° values in line 7 are secured. These predicted values do not differ from the recorded values (line 3) at any frequency level by more than 5F°. This is a measure of internal consistency. The other stations in Table II furnished satisfactory additional tests for internal consistency.

18. Inter-areal Reliability

a. Twelve Representative Stations in North America

For each of the 12 stations in Table IV, line a gives the actual manually tabulated January frequencies of daily minimum temperatures - most of them for 20-January records. Line b for each station gives the Essential Predictive Data and the predicted January minimum temperature probabilities for 19 levels of frequency ranging from 25 days in one January (25/31) to one day in 100 Januarys (1/3100). In general, the divergence between the tabulated minima and the predicted minima up to the length of the tabulated record (1/620), is probably not greater than might be expected when the variability and caprices of January minimum temperatures through the years are taken into account. It may be seen that at several stations the summary minimum did not include the lowest minimum of the tabulated record, e.g., at Moscow, Durango, Butte and Sault St. Marie. This is understandable when we realize that the very low minimum may occur any year, but on the average at increasingly long intervals. The departure of the recorded temperatures from the predicted in Table IV are shown graphically in Figure 4.

b. Six "Handbook" Stations

Each of the six handbooks listed as related Earth Sciences Division Publications (see back of Title Page) contains monthly summarized temperature data and line diagrams of frequency and level of daily extreme temperature for each of 12 months. The frequencies for the diagrams in the handbooks were manually tabulated.

In Table V is given the temperature frequencies and levels as predicted from the summaries in the handbooks (MDMx, MDMi and AbMi), and just below the predicted values are given the corresponding values taken from the line drawings in the handbooks. The summarized data used for predictive probabilities are given in columns 1, 2, 3 and 4, and the frequencies and temperature levels from the line drawings in columns 5 to 15 inclusive.

DEPARTURE OF ACTUAL TEMPERATURES FROM PREDICTED TEMPERATURES AT GIVEN FREQUENCIES

(DATA FROM TABLE IV)

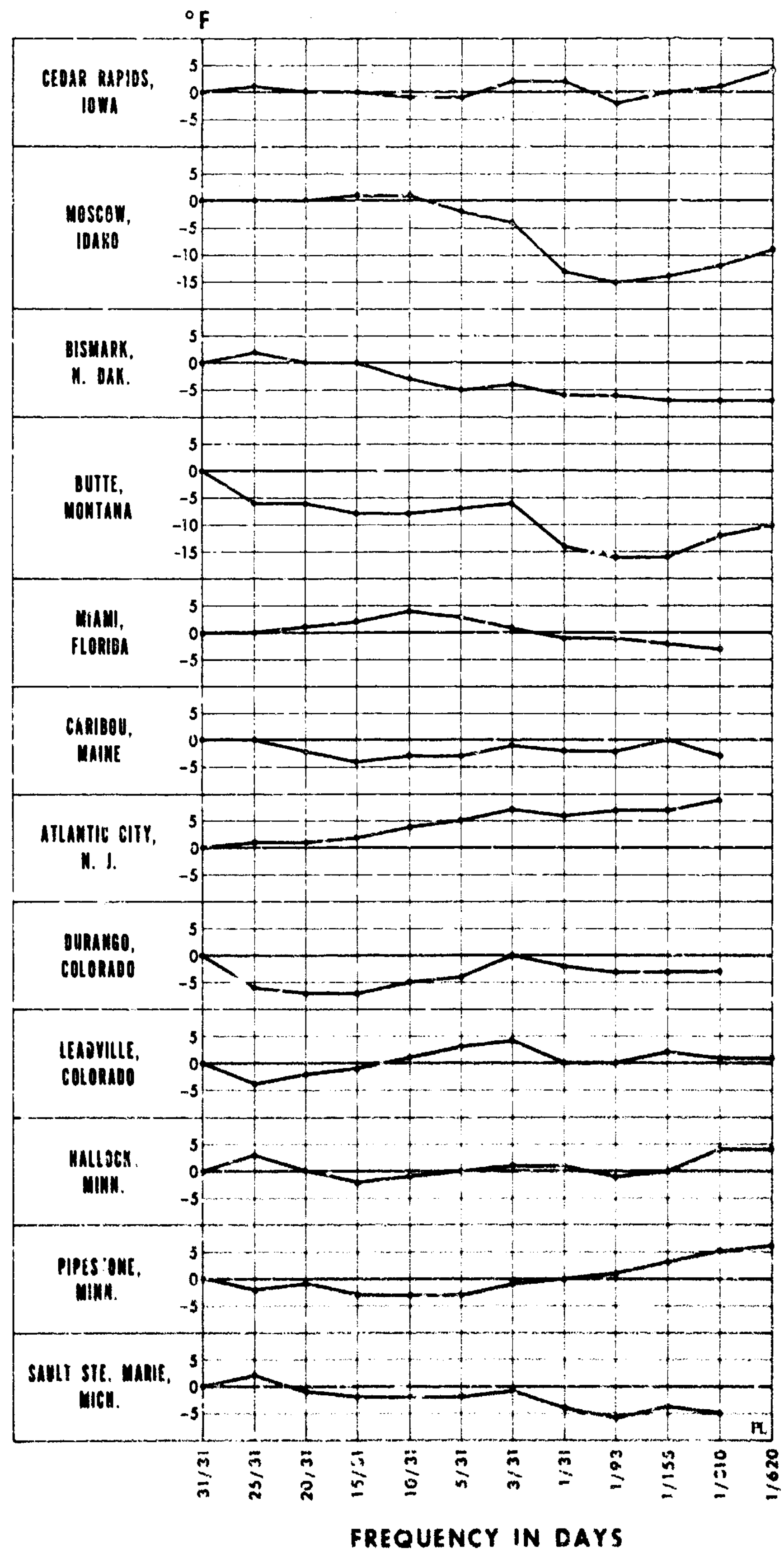

Figure 4

TABLE IV. TABULATED DAILY MINIMUM TEMPERATURE FREQUENCIES FROM RANDOM FREQUENCY RECORDS COMPARED WITH PREDICTED FREQUENCIES FROM SUMMARIZED LONGER OR SHORTER RECORDS FROM THE SAME STATIONS RESPECTIVELY.

(Actual Frequencies from Mrs. Jane H. Westbrook's Tabulations)

Stations		Record in yrs	Essential Temp. Data AbMi	MDMi	MDMx	CMD Mi	Tabulated and Predicted Temperature Data: 80.6% 25/31	64.5% 20/31	48.4% 15/31	32.3% 10/31	16.1% 5/31	9.7% 3/31	3.23% 1/31	1.08% 1/93	.645% 1/155	.323% 1/310	.161% 1/620	.017% 1/930	.081% 1/1240	.065% 1/1550	.054% 1/1860	.046% 1/2170	.040% 1/2480	.036% 1/2790	.032% 1/3100
Cedar Rapids, Iowa	r	20					25	18	13	5	-3	-7	-15	-20	-22	-24	-25								
	p	68	-36	12	30*	27.3	24	18	13	6	-2	-9	-13	-18	-22	-25	-29	-31	-33	-34	-35	-35	-36	-37	-37
Moscow, Idaho	r	20					31	28	24	18	9	0	-14	-21	-23	-25	-26								
	p	37	-20	22	34	22.2	31	26	23	17	11	4	-1	-6	-9	-13	-17	-18	-21	-21	-22	-23	-24	-25	-26
Bismark, N. Dakota	r	20					12	3	-4	-11	-20	-24	-30	-36	-38	-41	-44								
	p	76	-43	-2	19	38.9	10	3	-4	-8	-15	-20	-24	-29	-31	-34	-37	-38	-39	-40	-41	-42	-42	-43	-44
Durango, Colorado	r	20					18	12	8	4	-1	-5	-11	-16	-19	-23	-27								
	p	3[illegible]	-26	12	28	25.9	24	19	15	9	3	-5	-9	-13	-16	-19	-22	-24	-26	-27	-27	-28	-28	-30	-30
Leadville, Colorado	r	20					13	9	5	0	-5	-8	-16	-20	-22	-24	-27								
	p	33	-29	5	30	42.4	17	11	6	-1	-8	-12	-16	-20	-23	-25	-28	-29	-32	-31	-32	-33	-34	-34	-34
Hallock, Minnesota	r	20					5	-4	-12	-18	-25	-29	-35	-40	-41	-41	-43								
	p	32	-48	-11	11	37.3	2	-4	-10	-17	-25	-30	-34	-39	-41	-45	-47	-48	-50	-51	-52	-53	-53	-53	-54
Pipestone, Minnesota	r	20					14	8	1	-7	-14	-17	-23	-27	-28	-29	-29	-29							
	p	29	-40	3	23	31.8	16	9	4	-3	-11	-18	-23	-28	-31	-34	-37	-40	-41	-43	-44	-44	-45	-45	-46
Butte, Montana	r	20					16	9	1	-6	-15	-21	-34	-43	-46	-47	-48								
	p	5	-30	8	26	32.1	21	14	9	-2	-8	-15	-20	-26	-30	-34	-38	-41	-43	-44	-45	-46	-47	-48	-48
Sault Ste. Marie, Michigan	r	19					18	10	5	0	-7	-10	-16	-22	-23	-25									
	p	21	-24	6	22	34.8	16	11	7	2	-5	-9	-12	-15	-19	-21	-24	-25	-26	-27	-28	-28	-29	-29	-30
Atlantic City, New Jersey	r	10					39	33	30	26	20	16	11	8	5	4									
	p	18	-8	27	43	31.4	38	32	28	22	15	9	5	1	-2	-5	-8	-10	-12	-13	-13	-14	-15	-15	-16
Miami, Florida	r	10					69	65	62	58	52	47	42	39	35	33									
	p	19	34	59	78	43.2	69	64	60	54	49	46	43	40	38	36	34	33	32	31	31	30	29	29	29
Caribou, Maine	r	14					12	5	-2	-7	-14	-17	-22	-26	-27	-32									
	p	22	-32	1	18	34.0	12	7	2	-4	-11	-16	-20	-24	-27	-29	-32	-34	-35	-36	-37	-37	-37	-38	-39

Comments:
1. Line r for each station gives the recorded minimum temperatures for the indicated frequency intervals. For example, at Cedar Rapids, -20°F occurred on an average, once in 93 January days (1/93); -3°F, an average of 5 times in January (5/31).
2. Line p for each station gives the corresponding predicted temperature - using the AbMi, MDMi, and MDMx for the given number of years as the basis of prediction.
3. Cedar Rapids, Iowa: The r and p values for 20 and 68 years, respectively, parallel each other rather closely.
4. Bismark, N. Dakota: The AbMi in the 70-year record was only 1F° lower than that for the 20-year record.

* Note: the four items needed for prediction.

TABLE V. PREDICTED MINIMUM TEMPERATURES COMPARED WITH MANUALLY TABULATED DATA FOR SIX "HANDBOOK" STATIONS*, VARIOUS MONTHS

Columns				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Station	Month	Years Record	Operation	High Mi	MD Mx	MD Mi	Ab Mi	MD Mx	25/31	20/31	15/31	10/31	5/31	3/31	1/31	1/93	1/155	1/310
Big Delta	Jan	9	Pred.					3	-1	-9	-15	-23	-32	-44	-49	-55	-59	-65
			Tabul.	+29	+3	-13	-65*	3	3	-3	-10	-20	-37	-46	-56			
	Jul		Pred.					69	55	52	48	46	42	40	39	37	36	35
			Tabul.	60	69	49	35	69	54	51	49	47	45	44	41			
	Oct		Pred.					30	30	25	21	15	8	-1	-4	-8	-12	-15
			Tabul.	44	30	20	-15	30	30	25	21	16	0	-4	-9			
	Dec		Pred.					4	1	-7	-12	-20	-28	-38	-46	-52	-56	-62
			Tabul.	33	4	-13	-62	4	5	-4	-11	-19	-34	-39	-47			
Devils Lake, N.D.	Jan	25	Pred.					13	7	2	-5	-9	-17	-24	-28	-33	-36	-43
			Tabul.	30	13	-6	-44	13	9	0	-7	-16	-23	-27	-32			
	Jul		Pred.					81	63	60	55	53	48	46	44	42	40	39
			Tabul.	79	81	56	37	81	64	60	57	54	50	47	46			
	Dec		Pred.					20	14	8	1	-3	-11	-17	-22	-26	-29	-32
			Tabul.	44	20	2	-37	20	15	8	1	-6	-14	-17	-24			
Whittier, Alaska	Jan	7	Pred.					30	26	22	19	14	8	3	-2	-3	-8	-11
			Tabul.	44	30	18	-4	30	30	26	23	16	9	3	-6			
	Jun		Pred.					60	50	47	45	41	38	36	34	32	31	30
			Tabul.	58	60	44	30	60	48	45	42	40	38	36	34			
Fort Lee, Va.	Jan	25	Pred.					52	46	40	35	29	21	15	10	6	3	0
			Tabul.	69	52	32	-5	52	42	34	31	26	21	16	9			
Lake Cochituate, Mass.	Jan	25	Pred.					38	31	24	16	11	2	-5	-10	-15	-18	-22
			Tabul.	45	38	15	-28	38	27	21	15	10	1	-5	-9			
	Jul		Pred.					84	79	74	68	65	58	53	50	46	44	41
			Tabul.	75	84	58	39	84	64	61	57	54	50	47	44			
Fort Churchill, Manitoba	Jan	19	Pred.					-11	-15	-21	-25	-30	-37	-42	-45	-49	-52	-54
			Tabul.	23	-11	-27	-57	-11	-14	-21	-27	-31	-36	-39	-45			
	May		Pred.					38	33	27	21	17	10	5	-1	-5	-3	-11
			Tabul.	44	38	22	-14	38	31	27	23	19	15	11	1			

* Essential data were gleaned from the six Handbooks listed on back of title page.
Note: The high minimum is sometimes above the MDMx.

It should be noted that comparisons were made for several different months as well as from six different stations. For example, Big Delta in January according to a 9-year record had a MDMx of 3°F; MDMi, -13°F; and AbMi, -65°F (columns 2, 3 and 4). The predicted temperatures (columns 5 to 15) were adjusted for a 10-year record (the Big Delta record was for 9 years). The differences between the tabulated and predicted values are not greater than might be expected.

The diagrams in the handbooks did not extend to cover more than a 1-month span. This accounts for the three vacant spaces in the tabulated lines. The nomograph enables us to extend the predictions to any decade period up to 100 years. Of the 14 samples, it seems the first one presents about the poorest correlation. It may be noted in columns 1 and 2 that the MDMx is frequently below the highest minimum.

19. Variation in 10-Year Summary Records at the Same Station

Let us examine how predictions from several 10-year summary records (i.e. in different decades) at a given station may differ.

The Essential Summary Data for 7 consecutive 10-year (January) intervals at Washington, D.C. are given in Table VI. In the decade 1921-1930, for example, the January AbMi was 3°F; the MDMi, 27°F; and the MDMx, 43°F. The predicted minima to be expected for 1, 3, 5, 10, 20, 30, 40 Januarys, etc. to 100 are given in the same line. Corresponding predictions up to 100 years are made for each from the other six 10-January records.

The several 10-January actual minima (first column, Table VI) differ by 17F° (+3°F to -14°F). The several 100-January predicted minima (last column, Table VI) differ by 24F° (-4°F to -28°F). Some apparent inconsistencies in the Washington data are discussed in the next section.

This shows that the essential data for different decades at the same station may vary enough to affect predictions.

20. Optimal Length of Record

As stated in the Introduction this is considered a pilot study, based largely on manually tabulated data. The method seems to suggest far-reaching possibilities. By machine processing of more and longer records, certain refinements of the method seem possible. For example, Table VII indicates a fair degree of success of the method, but at the same time offers some suggestions for refinement.

In that table, seven separate 10-January daily minimum temperature records of Washington, D. C. were assembled by decade-cumulative accretion from 10 years to 70 years, beginning with 1871 and continuing to 1940.

TABLE VI. WASHINGTON, D.C.: SEVEN SEPARATE DECADES OF ESSENTIAL DATA WITH FREQUENCY OF MINIMUM TEMPERATURE PREDICTIONS

Decades for which essential January data are used	Essential Data (F°)			CM DMi	Frequency in Given Month of Predicted Minimum Temperatures (F°)												
	Ab Mi	MD Mi	MD Mx		Number of Januaries for One Occurrence of Given Temperature Levels												
					1	3	5	10	20	30	40	50	60	70	80	90	100
1921-1930	3	27	43	40	10	9	5	3	1	0	-1	-2	-3	-3	-3	-4	-4
1931-1940	-2	30	44	30	8	3	-1	-2	-6	-8	-10	-11	-11	-12	-13	-14	-14
1901-1910	-2	26	41	35	7	3	1	-2	-5	-6	-8	-8	-9	-10	-10	-11	-11
1891-1900	-6	26	41	32	4	0	-3	-6	-9	-11	-13	-14	-14	-15	-15	-16	-16
1871-1880	-6	24	42	38	3	-1	-4	-6	-9	-10	-11	-12	-13	-14	-14	-15	-15
1911-1920	-13	27	43	29	0	-5	-9	-13	-17	-19	-21	-23	-24	-25	-25	-26	-26
1881-1890	-14	25	43	32	-1	-7	-11	-14	-18	-20	-22	-23	-24	-25	-25	-26	-27

Comments:

1. Predictions are based on 10-year records according to Table III.
2. The recorded minima (AbMi) for January through the seven separate decades range in [illegible] from +3°F (1921-1930) to -14°F (1881-1890).
3. The 70-year minimum predictions according to the 1921-1930 decade is -3°F, but for the decades 1881-1890 and 1911-1920 it is -25°F. Since essential data for different decades may vary enough to affect predictions, it would appear that data for 20 years or more should be sought.

TABLE VII. PREDICTIONS FROM 1 TO 7 DECADES (CUMULATIVE) OF TEMPERATURE RECORDS: WASHINGTON, D.C. 1871-1940

Periods for which essential data are used	Essential Data (F°)			Converted Values		Frequency in Given Month of Predicted Minimum Temperatures (F°)												
	Ab Mi	MD Mi	MD Mx	(A) CM DMi	(B) CM DMi	Number of Januaries for One Occurrence of Given Temperature Levels												
						1	3	5	10	20	30	40	50	60	70	80	90	100
10 Years 1871-1880	-6	24	42	37.5	38	3	-1	-4	-6	-9	-10	-11	-12	-13	-14	-14	-15	-15
20 Years 1871-1890	-14	25	41	29.1	31	2	-2	-5	-9	-14	-15	-17	-17	-18	-19	-20	-20	-20
30 Years 1871-1900	-14	25	41	29.1	32	2	-3	-7	-9	-13	-14	-16	-17	-18	-19	-19	-20	-20
40 Years 1871-1910	-14	25	41	29.1	33	3	-1	-4	-7	-10	-12	-14	-15	-15	-16	-17	-17	-18
50 Years 1871-1920	-14	25	42	30.4	35	3	-2	-4	-7	-10	-12	-13	-14	-15	-16	-16	-17	-17
60 Years 1871-1930	-14	26	42	28.6	33	4	-1	-3	-6	-9	-11	-12	-13	-14	-15	-15	-16	-16
70 Years 1871-1940	-14	26	42	28.6	34	4	-1	-3	-6	-9	-11	-12	-13	-14	-14	-15	-16	-16

Comment:

Column (A) CMDMi for the number of years in the period of record.
Column (B) CMDMi for the 10-year equivalent of (A). To be used for predictive purposes.

The lowest minimum occurred in the second decade of the sequence. From the second decade on through the whole series of decade predictions, the results seem quite satisfactory. That is, it seems that predictions from records of 20 years or more may give increasingly stable results. In general, at each decade level beyond 20 years the predicted minimum rises 1 or 2 degrees. Note the Fahrenheit change in predicted minima at the 60-year level: -18°, -18°, -15°, -15°, -14°, -14°. This might mean that the decelerated trend of daily minimum temperature with increased length of record should be greater than the nomograph indicates, and that the extrapolated section of the Nomograph may need some adjustments - a task which probably calls for some large-scale machine processing.

On the other hand, it might signify that the assumption of the near constancy of the MDMx and MDMi needs further consideration, and that confirmation or rejection of this assumption awaits the processing of larger masses of raw data.

Conclusions and Implications

1. A more reliable Nomograph would result from a use of 20-year records instead of 10-year record.

2. Converting every record to a 100-unit scale makes records of any length numerically comparable to any other similar record.

3. The method requires that:

   a. data to be compared must be converted from the conventional scale to the 100-unit scale

   b. the converted predictions are then reconverted to the conventional (e.g., °F) scale.

4. The frequency distribution of predictive measures takes on a variety of patterns, due to skewness, varying widely within the 0-100 distribution.

5. By extrapolating, the asymmetric patterns or trends of the tails of the distributions may be discovered by the use of various kinds of probability devices, as described in this study.

6. The position on the 100-unit scale of the CMDMi between the relatively stable CMDMx (0) and the oscillating AbMi or PMi (100) is the measure of asymmetry or skewness that gives uniqueness to this method, and thus furnishes the numerous (36) CMDMi patterns of frequency that are featured in the study.

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APPENDIX A

ABBREVIATIONS AND GLOSSARY OF TERMS

AbMi = Absolute Minimum: The lowest temperature ever recorded in a given month at a given station.

PMi = Period Minimum: Lowest temperature during the given period of record.

MDMi = Mean Daily Minimum: The average of the daily minima in any given month.

PMDMi = Period Mean Daily Minimum: The average of daily minima for a given month during the given period of record.

MDMx = Mean Daily Maximum: The average of the daily maxima in any given month.

PMDMx = Period Mean Daily Maximum: The average of the daily maxima for a given month during a given period of record.

RED = Recorded Essential Data

CFT = Converted Frequency Data

FD = Frequency Data

RT = Reduced Temperature: Values derived by subtracting RD and FD from the MDMx.

CAbMi = Converted Absolute Minimum: The reduced minimum converted to 100 in the 100-unit scale as in Table I.

CMDMx = Converted Mean Daily Maxima: Average of the reduced daily maxima converted to (0) in the 100-unit scale as in Table I.

CMDMi = Converted Mean Daily Minima: Average of the reduced daily minima converted to 100 on the 100-unit scale as in Table I.

CPrT = Converted Predictive Temperature: Nineteen numerical values on the 100-unit scale associated with each CMDMi (20 to 55) in Table III and the Nomograph.

APPENDIX B

CONSTRUCTION OF THE NOMOGRAPH FOR PREDICTING THE FREQUENCIES OF DAILY MINIMUM TEMPERATURES

The location of the circles shown in blue at the right of Figure 5 corresponds exactly to the data in Table II for 10 days in 310 (1/31), as also shown on the Nomograph, Figure 2. (See par 5 in the text.)

A table like Table II was set up for each of the frequency curves in Figure 5. The corresponding circles and triangles determined the trend of the nine associated frequency curves of the Nomograph.

PREDICTIVE MINIMUM TEMPERATURE FREQUENCY CURVES

(DAYS IN A GIVEN 31-DAY MONTH IN 10 YEARS)

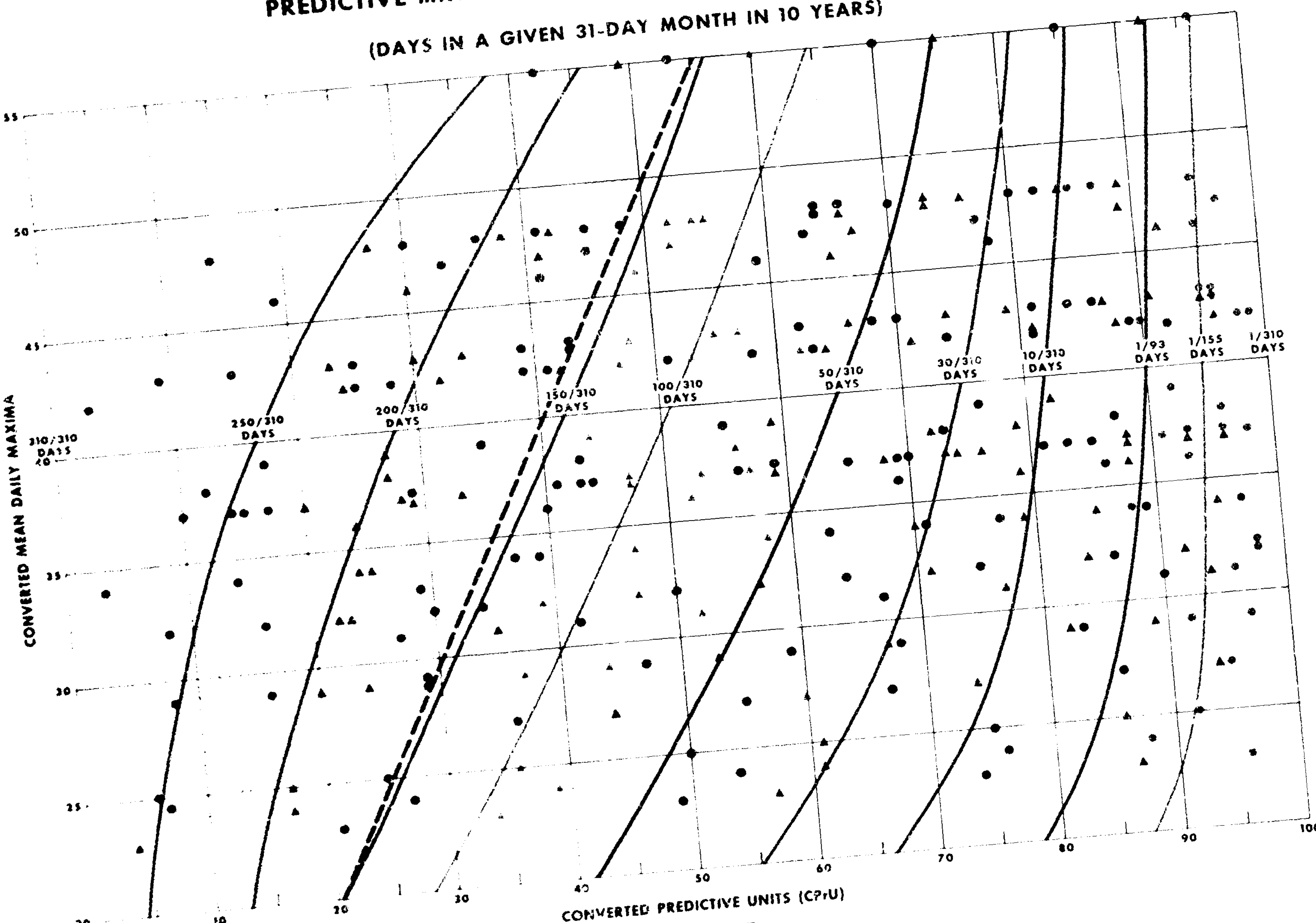


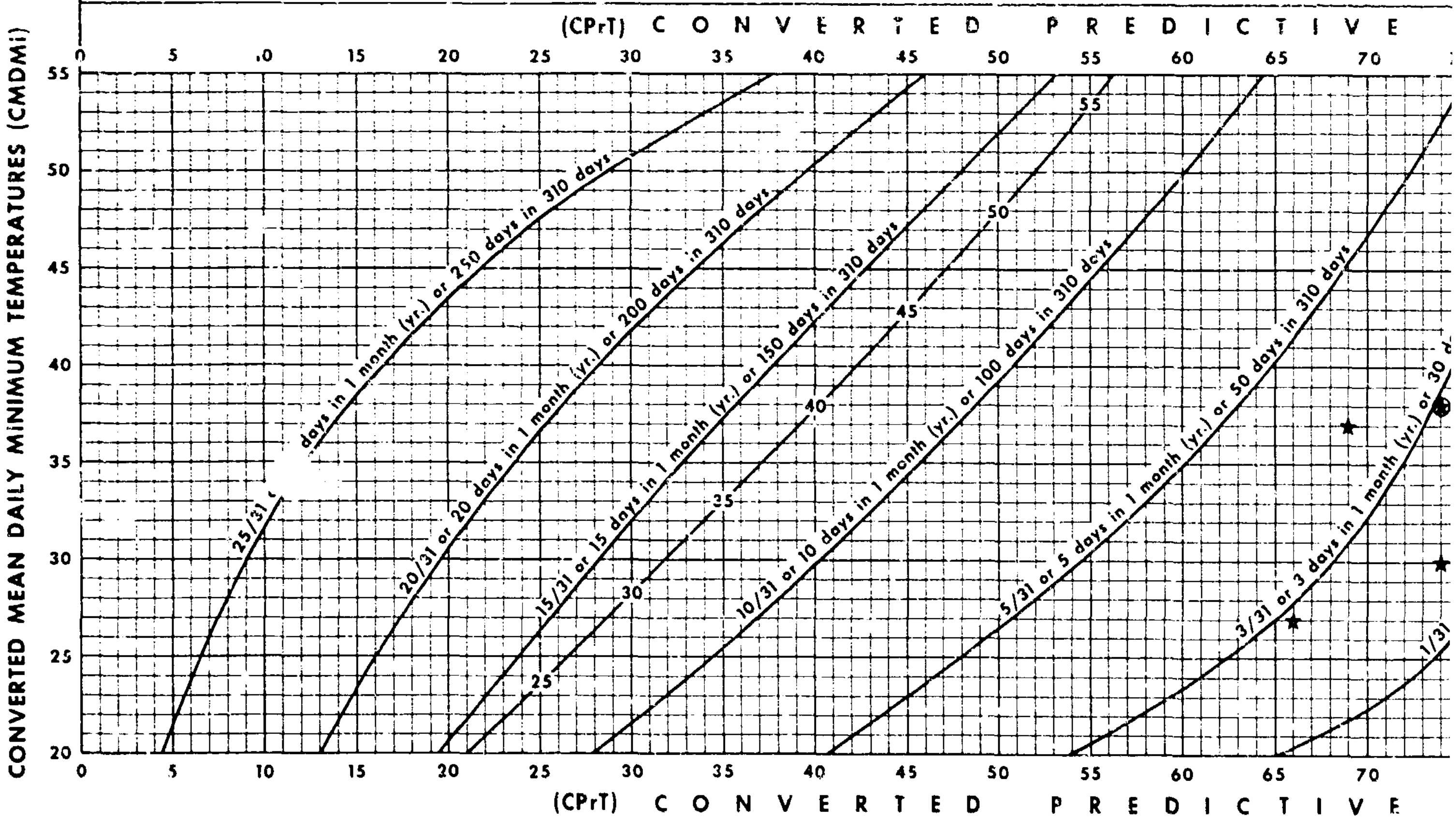
Figure 5

NOMOGRAPH FOR PREDICTING FREQUEN

Predicting from 10-Year Records

BASIC SECTION: For 25 days in a Month (25/31) ; Up To 1 day in

Constructed from Actual Frequencies at 10 Stations in Nor



Fig

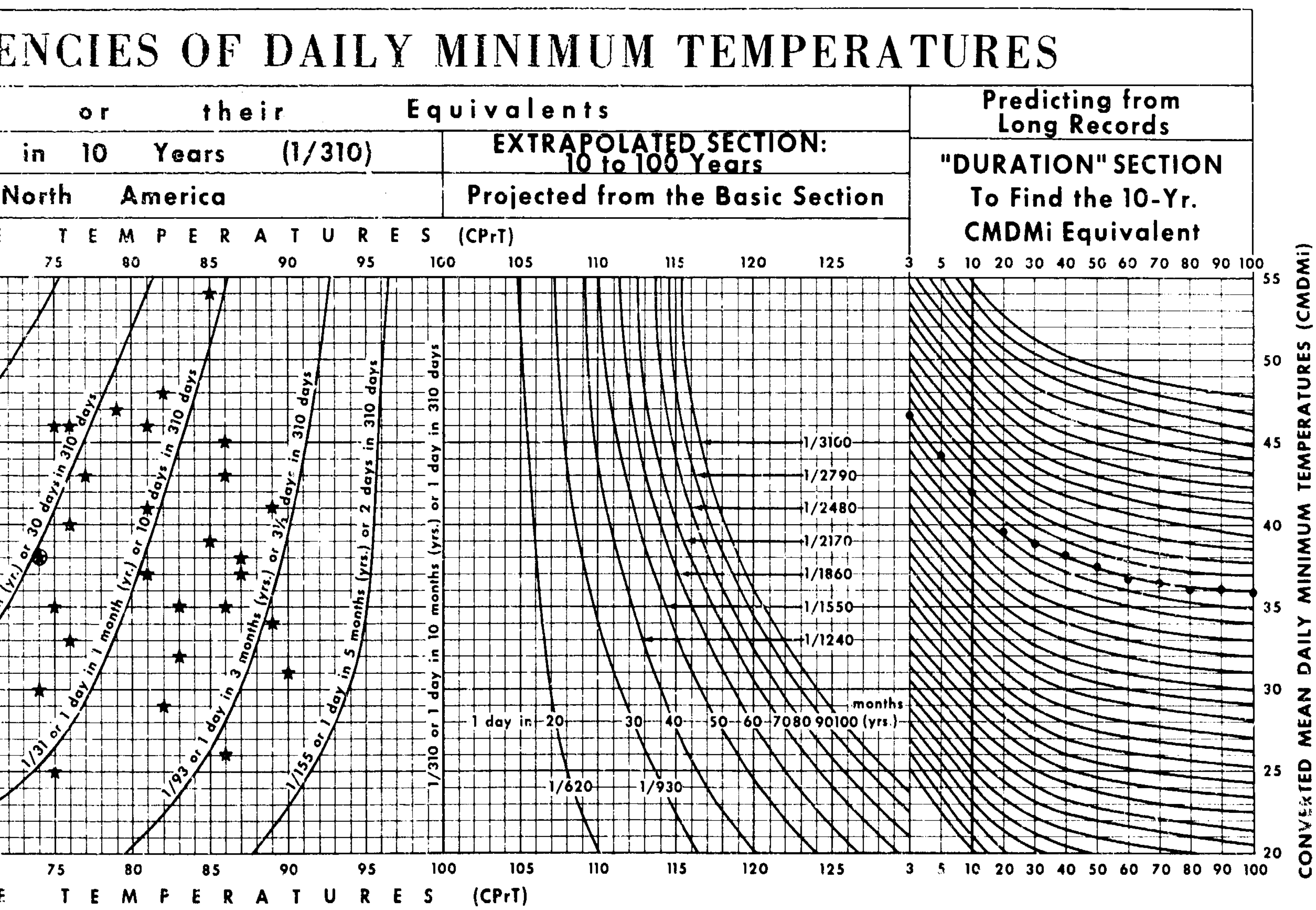

ENCIES OF DAILY MINIMUM TEMPERATURES
or their Equivalents
in 10 Years (1/310)
North America
EXTRAPOLATED SECTION: 10 to 100 Years
Projected from the Basic Section
Predicting from Long Records
"DURATION" SECTION To Find the 10-Yr. CMDMi Equivalent
TEMPERATURES (CPrT)
CONVERTED MEAN DAILY MINIMUM TEMPERATURES (CMDMi)
1/310 or 1 day in 10 months (yrs.) or 1 day in 310 days
1 day in 20 30 40 50 60 70 80 90 100 (yrs.) months
1/620
1/930
1/1240
1/1550
1/1860
1/2170
1/2480
1/2790
1/3100

Figure - 2

Unclassified
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4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

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13. ABSTRACT

A method for assessing the relative frequency and level of daily minimum temperatures in various parts of North America in winter months is developed in this paper from the summary 10-year records of 10 widely separated weather stations representative of the numerous variables that are involved in the occurrence of low temperatures. The method is based on the following four items usually found in climatic summaries:

a. absolute minimum temperature
b. mean daily minimum temperature
c. mean daily maximum temperature
d. length of record.

The temperature and frequency estimates are achieved by use of a nomographic device with a grid of converted temperature values representing 36 varying degrees of asymmetry of temperature distribution, and a series of predictive curves designating the frequency of occurrence in days per month.

The efficiency of the method is demonstrated by tests of internal consistency, also by application to 12 widely separated test stations in the United States, and to 6 "Handbook" stations and by varying length of summary records (10 to 70 years) at a single station, Washington, D.C.

DD FORM 1 JAN 64 1473

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Prediction	8					
Temperature	1,2					
Daily	0					
Minimum	0					
Nomographs	10					
Grids (Coordinates)	10					

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