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# CALCULATED NATURAL FREQUENCIES AND NORMAL MODES OF THE GUIDED MISSILE CRUISER USS LONG BEACH (CG(N)-9)

by

Donald C. Robinson

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ACOUSTICS AND VIBRATION LABORATORY
RESEARCH AND DEVELOPMENT REPORT

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## ABSTRACT

Calculations were performed to determine the first six modes of vertical and horizontal flexural vibration, the first three modes of torsional vibration, and the first five modes of coupled torsion-horizontal bending vibration for the guided missile cruiser USS LONG BEACH (CG(N) 9) at a displacement of 16,103 tons. A calculation coupling propulsion system longitudinal vibration with vertical flexural hull vibration was performed on an electrical analog computer in an attempt to simulate the excitation of flexural hull modes by vibratory forces transmitted longitudinally through the propulsion-shafting system. Flexural calculations of hull horizontal vibrations were performed with the large forward superstructure deckhouse considered both as rigidly and flexibly attached to the hull.

Included in the results are the hull response to a sinusoidal force of 1000 lb acting at the stern in a vertical direction. The vertical flexural modes and natural frequencies were computed using both the standard two-node Lewis virtual mass coefficient and a variable virtual mass coefficient. The parameters used for the calculations are discussed in detail. Results of the calculations are compared with measured hull frequencies.

## ADMINISTRATIVE INFORMATION

This assignment was authorized by Bureau of Ships CONFIDENTIAL letter Serial 345-0280 of 8 January 1962 and was funded under Subproject S-F013 11 08, Task 01351.

## INTRODUCTION

The USS LONG BEACH (CG(N) 9) is the first of a class of nuclear-powered guided missile cruisers. The hull has no armor plating, and the superstructure has a long first deck and a tall aluminum forward deckhouse. An important consideration in the design and operation of a new ship class is the structural response to variable forces. Thus, the Bureau of Ships requested that the David Taylor Model Basin calculate the normal hull modes and natural frequencies for LONG BEACH.

The calculation of the natural frequencies and mode patterns of vibration of a ship are based on the idealization of the hull as a free-free nonuniform beam. The usual procedure in computing the resonant frequencies and modes of a ship is to consider the mass of the entire ship including the entrained mass of water, and to consider the stiffness characteristics of the primary hull girder only. Such a calculation was performed on LONG BEACH to determine the natural frequencies and modes of vertical, horizontal, torsional, and coupled torsion-horizontal vibration. The forced response of the hull was computed by applying a 1000-lb, single-amplitude sinusoidal exciting force at the stern and assuming several characteristic damping values. It should be noted that for the above procedures, it was assumed that the propulsion system and superstructure are rigidly attached to the hull and that the superstructure does not contribute to the overall hull strength.

A new additional evaluation was used to include the dynamic characteristics of the propulsion system by coupling longitudinal shaft vibration with the vertical flexural hull vibration. This calculation was performed on an electrical analog computer. The considerations involved in developing the electric model for this calculation and the interpretation of the results obtained are discussed in detail.

In order to obtain a more accurate estimation of the influence of the superstructure, a new calculation was also performed to determine the influence of the first superstructure deck on the fundamental mode of vertical vibration. In addition, a sprung mass calculation was made to determine the effect of the tall forward deckhouse on the horizontal modes of hull vibration.

Experimental data from underway vibration trials are compared with calculations.

## CALCULATION PROCEDURES

The vibration response of LONG BEACH was computed by assuming the hull to behave like a free-free nonuniform beam, including the effects of shear and rotary inertia. A detailed description of the formulation and solution of the governing differential equations of motion may be found in Reference 1.*

The inboard profile of LONG BEACH and the frame and station locations are shown in Figure 1. Table 1 gives the principal design characteristics of the ship. Figure 2 presents the longitudinal weight distribution and virtual mass of the ship, and Figure 3 shows graphically the moment of inertia distribution with respect to the horizontal and vertical axes through the centroids of the hull sections.

Since the parameters in the equations of motion cannot be expressed as continuous functions of the position along the ship, a numerical solution using finite difference equations was used. For the vertical, horizontal, and coupled torsion-horizontal hull modes, the hull was divided into 20 equally spaced sections, as shown in Figure 4. In these calculations, the propulsion shafts were first considered to be rigid and their mass was lumped with the hull mass. Then a coupled shafting-vertical flexural hull vibration calculation was performed in which the longitudinal flexibility of the propulsion shafts was considered and the shaft masses were lumped separately from the hull mass. For the latter calculation, the port and starboard shafts were divided into seven and ten equally spaced sections, respectively, as shown in Figure 4.

[1]References are listed on page 46.

The flexural response of the hull depends on the values used for ship mass, virtual mass, bending stiffness ($EI_y$ or $EI_z$), and the shear rigidity ($KAG$) at each station. For torsional hull response, the longitudinal mass inertia ($I_{mx}$) and torsional rigidity ($GJ_e$) become the pertinent torsional mass and stiffness parameters, respectively. Rotary inertia ($I_{mz}$) and terms involving the center of mass ($\bar{z}$) and the center of shear ($\bar{\bar{z}}$) were employed in the coupled torsion-horizontal hull vibration calculations. The longitudinal response of the propulsion system depends on the mass of the principal machinery components and on the longitudinal spring constant ($k$) of the shafts. The evaluation of the various mass and elastic parameters of the hull and propulsion system are discussed in Appendix A. These parameters are summarized in Tables 2, 3, 4, and 5.

Comparison of the calculations with experimental measurements made on LONG BEACH revealed that the computed fundamental vertical flexural mode was underestimated.[2] Since it is known that the superstructure of a ship can sustain bending stresses and thus contributes to the hull elastic characteristics, it was believed that neglecting the geometric properties of the superstructure in the original evaluation of the ship bending flexibility might have been the primary source of discrepancy. Hence a separate calculation was performed to estimate the effect of the first superstructure deck on the fundamental vertical hull mode; see Appendix B.

The calculation of the coupled longitudinal shafting-vertical flexural hull vibration on an electric analog computer was the first such calculation performed for a surface ship, although comparable calculations have been made on submarines in the analysis of coupled longitudinal shaft-hull vibrations. Since little experimental data were available with which to confirm the calculations, care was taken to check each step in the process of constructing the electric model used to represent the mechanical system. The details of this calculation are presented in Appendix C and include a description of the electric model.

An additional consideration in the vibration analysis of LONG BEACH was the effect of the large forward superstructure deckhouse on the beamlike modes of hull vibration. Comparison of the experimental measurements with the calculations revealed the occurrence of an additional resonant frequency between the computed second and third horizontal flexural hull modes. It was suspected that this frequency was due to the sprung mass effect of the superstructure forward deckhouse. The approximate mass of the superstructure house was known, but an analysis of the elastic flexibility of the forward deckhouse would have been quite complicated. Hence, for simplicity, the deckhouse was idealized as a single-degree-of-freedom mechanical system. Several values of elastic springs were assumed which, when taken in conjunction with the mass of the deckhouse, gave approximately the additional measured frequency. The primary purpose of this calculation was to estimate the effect of the forward deckhouse on computed hull resonant frequencies. Appendix D briefly reviews the theory underlying this calculation and presents results of the calculation of horizontal hull vibration with the superstructure considered both rigidly and flexibly attached to the hull.

## RESULTS OF THE CALCULATIONS

The normal mode profiles and natural frequencies of vertical hull vibration computed for a constant Lewis "J" virtual mass factor and for a varying "J" virtual mass factor are shown in Figures 5 and 6, respectively. The variation of the "J" factor versus frequency for the latter calculation is given in Figure 7. The moment distribution for vertical flexural vibration assuming a unit displacement (1 ft) at the stern for a constant "J" factor is presented in Figure 8. Since the reliability of the moment distribution profiles using a varying "J" virtual mass factor is questionable,[3] the profile computed by using the constant "J" factor was assumed to be the best analytical representation of the physical moment distribution. Table 6 compares computed vertical hull natural frequencies for constant and varying "J" factors.

Figure 9 gives the bending slope distribution for vertical flexural hull vibration assuming a unit displacement at the stern and a constant "J" virtual mass factor. The slope distribution is found useful in the interpretation of results obtained from the coupled longitudinal shaft-vertical flexural hull vibration calculation, which are reviewed in detail in Appendix C.

Figure 10 shows the normal mode profiles and natural frequencies of horizontal hull vibration. Table 7 summarizes the frequencies of the vertical and horizontal flexural hull modes computed by an electrical analog. In order to determine the influence of shearing on the hull modes, the vertical modes were computed considering (1) both bending and shearing flexibility of the hull and (2) bending flexibility only. The influence of the rotary inertia is considered to be negligible for the first six flexural modes of hull vibration. The rotary inertia was, however, considered in the coupled torsion-horizontal hull vibration calculations.

Figures 11 and 12 present, respectively, the horizontal and torsional amplitudes of coupled torsion-horizontal bending modes of hull vibration. Table 8 summarizes the frequencies of the coupled torsion-horizontal bending modes. Figure 13 shows the flexure-free torsional modes of vibration determined by an electric analog.

The forced response profiles at resonant frequencies of coupled longitudinal shafting-vertical flexural hull vibration for sinusoidal forces transmitted through the starboard shaft, through the port shaft, and through both shafts simultaneously are given in Figures 14, 15, and 16, respectively. The longitudinal shaft deflections and vertical hull deflections, designated as $L$ and $V$, respectively, are both shown in these figures. It should be noted that damping was not considered in this calculation and also that the shaft and hull deflections are not scaled with respect to one another. Table 9 summarizes the frequencies of the latter calculation with those obtained assuming a rigid propulsion shafting system whose mass is simply added to that of the hull.

Figures 17 and 18 show the vertical calculated response curves at several hull stations (1, 10, and 20) to a 1000-lb, single-amplitude sinusoidal forcing function at the stern (Station 0) for distributed damping values of 0.01 and 0.03 ton-sec/ft$^2$, respectively. A calculation which considers the influence of the first superstructure deck on the fundamental vertical hull mode is given in Appendix B.

The description of the electrical network used to represent the coupled shaft-hull system, shown in Figure 19, is presented in Appendix C. The hull parameters and the electrical network used for a horizontal hull vibration calculation in which the superstructure forward deckhouse was considered flexibly attached to the hull are presented in Table 10 and Figure 20, respectively. The results of this calculation, which is discussed in Appendix D, are shown in Figure 21 and Table 11.

Table 12 compares the calculated frequencies of the vertical and horizontal hull vibrations for LONG BEACH with the experimental frequencies obtained during underway vibration trials conducted by the Model Basin.[2]

## DISCUSSION

The natural frequencies and mode shapes of the hull of LONG BEACH computed by using the established beam theory of ship vibration compare reasonably well with experimental values. The computation of the coupled shafting-hull vibrations and the estimation of the dynamic effects of the superstructure first deck and forward deckhouse give results which seem compatible with measured hull frequencies. However, the latter calculations are based on certain assumptions and yield some results which could not be verified by measurements and hence require further discussion.

It is known that the longitudinal rigidity of ship hulls is large relative to their flexural rigidity. Hence, for the coupled shafting-hull vibration calculation, the simplifying assumption was made that the excitation of the hull by the propulsion system is created by periodically varying moments acting on the hull which excite the ship only into flexural motion. Since the beam theory of ship vibration yields best results for vertical vibration, only the vertical response of the ship was considered. Longitudinal exciting forces were considered to act on the hull at the stations where the shafting terminates, i.e., at the thrustbearings. These forces, in general, would be transmitted along the shafting, through the thrustbearing and machinery, and into the hull through the thrustbearing foundation. Because of the relatively large longitudinal rigidity of the hull, it was assumed that all of the longitudinal forces act through a moment arm or eccentricity (defined by the distance between the shaft terminals and the hull neutral bending axis) to produce only exciting moments on the hull in a vertical plane parallel to the hull vertical centerline plane. The calculated results indicate the extent to which the vertical beamlike hull modes are excited by the port and starboard propulsion systems acting separately. These results might be verified experimentally by conducting two mode surveys of the hull with each propulsion system operating separately and then comparing the measured hull response with the calculations.

A detailed analysis of the dynamic effects of superstructures on the ship vibration characteristics would involve, among other things, the determination of the load-carrying properties of the superstructure. Since no measurements were made on LONG BEACH which were useful in determining the effectiveness of the superstructure in sustaining either bending or shear loads, the evaluation of the effect of the first superstructure deck on the fundamental vertical mode should be considered as only a first approximation. Since the elastic characteristics of the superstructure forward deckhouse would have been quite difficult to determine analytically, the simplifying assumption was made that the deckhouse vibrated as a single-degree-of-freedom system. Several values of elastic constants were employed which, when used in conjunction with the known deckhouse mass, gave frequencies approximately equal to a measured frequency suspected to be a horizontal resonance of the superstructure.[2] The primary value of this calculation was to determine the extent to which the superstructure could influence the hull resonant modes. Since no mode survey was made on LONG BEACH, only the computed frequencies could be correlated with measurements. The comparison between the calculated and measured frequencies reveals a slight modification (spreading) of horizontal hull frequencies due to the dynamic effect of the deckhouse.

## CONCLUSIONS AND RECOMMENDATIONS

1. The beam theory of ship vibration is useful for predicting the beamlike modes of hull vibration. However, this theory should be supplemented by additional calculations to determine the dynamic effects of propulsion systems and/or superstructures on ship vibration or to investigate discrepancies between computations and measurements.

2. Whenever vibration tests are performed on multipropeller ships, it would be useful to determine the ship response when each of the propellers is operating separately for evaluation of the interaction between coupled propulsion-hull systems.

3. The influence of long superstructure decks should be considered in ship vibration calculations of the fundamental vertical hull mode.

4. Tall superstructure deckhouses can exhibit local vibrations which have frequencies lying within the range of beamlike hull modes and should be considered in ship vibration investigations.

## ACKNOWLEDGMENTS

The author gratefully acknowledges constructive criticism and suggestions from Mr. E. Noonan, Dr. E. Buchmann, and Mr. W. Hinterthan in the preparation of this report. The information and assistance provided by Messrs. R. Perkins, R. Farmer, and W. Fontaine are also gratefully acknowledged.

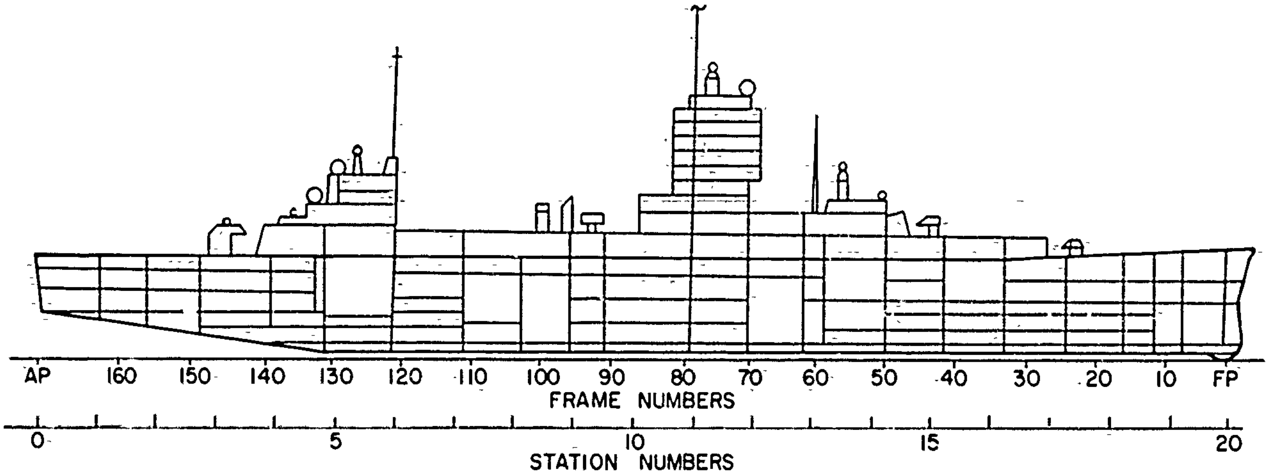




Figure 1 – Inboard Profile of LONG BEACH showing Frame and Station Locations

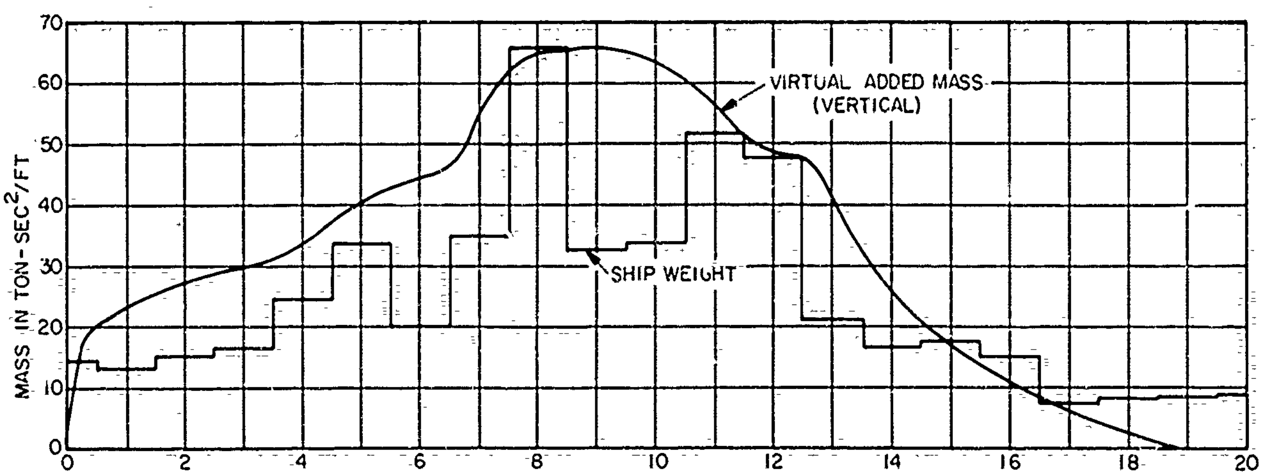



Figure 2 – Longitudinal Weight Distribution and Virtual Mass (Vertical) of LONG BEACH

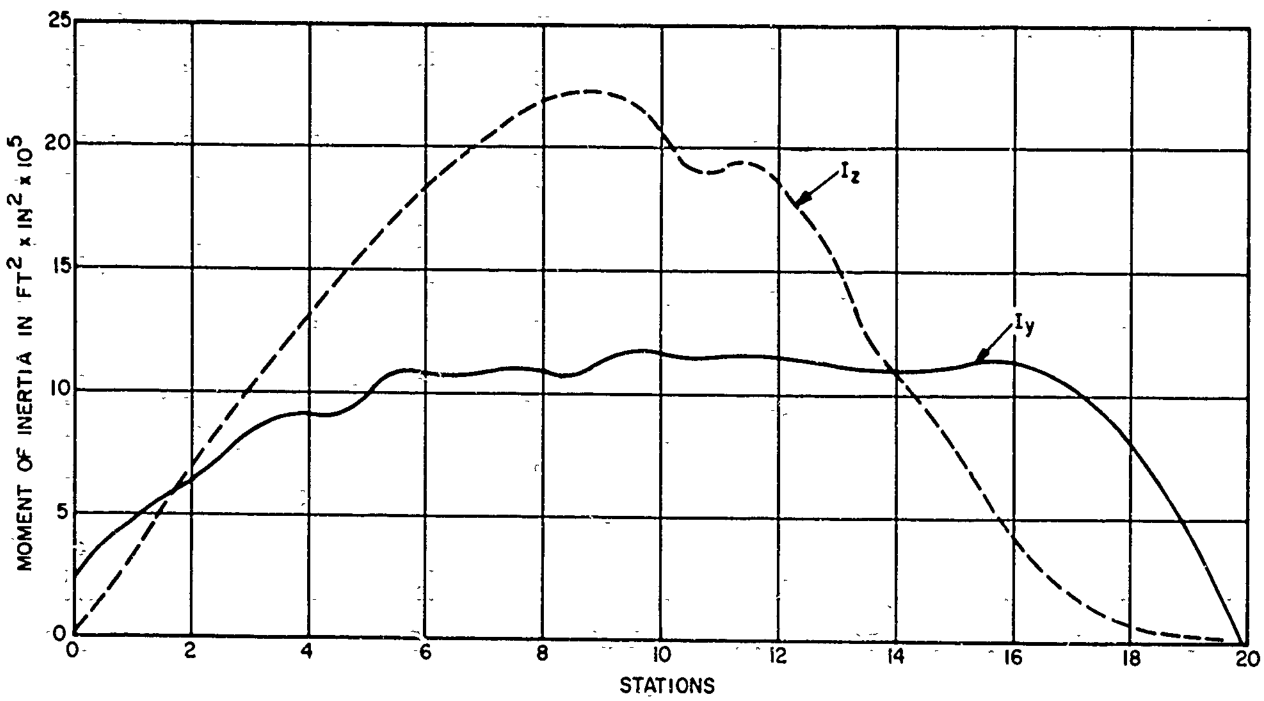
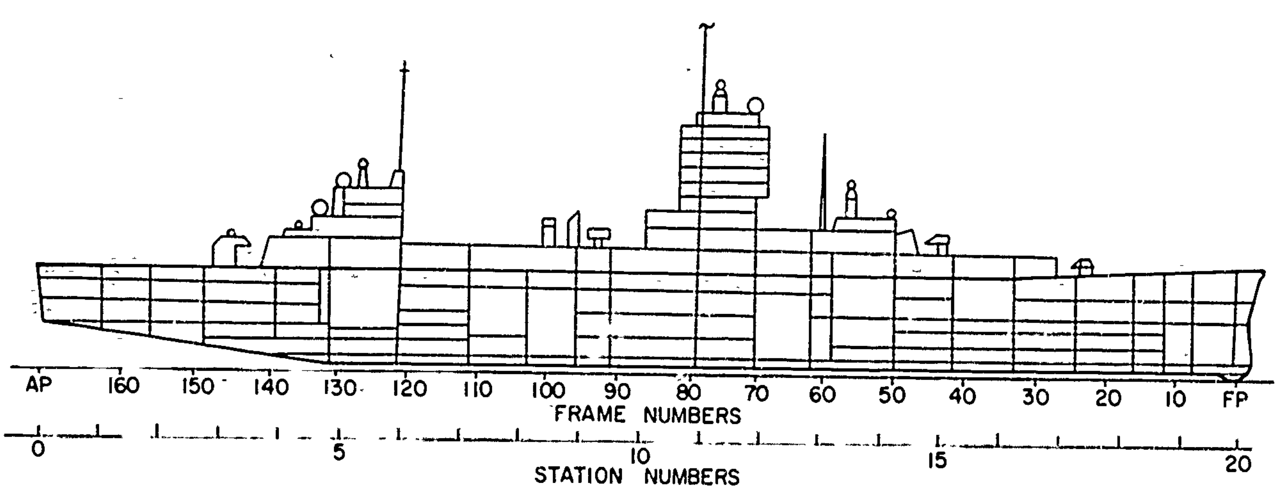



Figure 3 – Areal Moments of Inertia Distribution of LONG BEACH



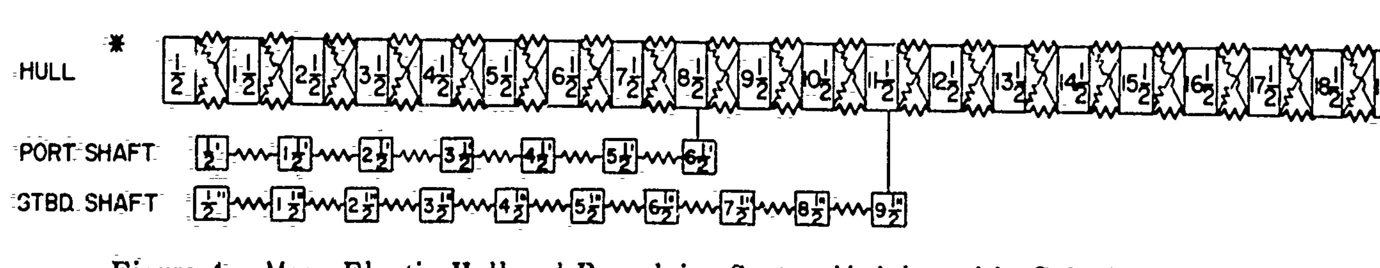




Figure 4 – Mass-Elastic Hull and Propulsion System Model used in Calculating Coupled Shafting-Hull Vibration

*The mechanical model for the hull represents bending and shearing flexibility of the hull, with translatory but no rotary inertia of the beam elements. The model is attributed to H. Schirmer (see Jacobsen and Ayre, "Engineering Vibrations," McGraw-Hill Book Co. (1958), p.498f).

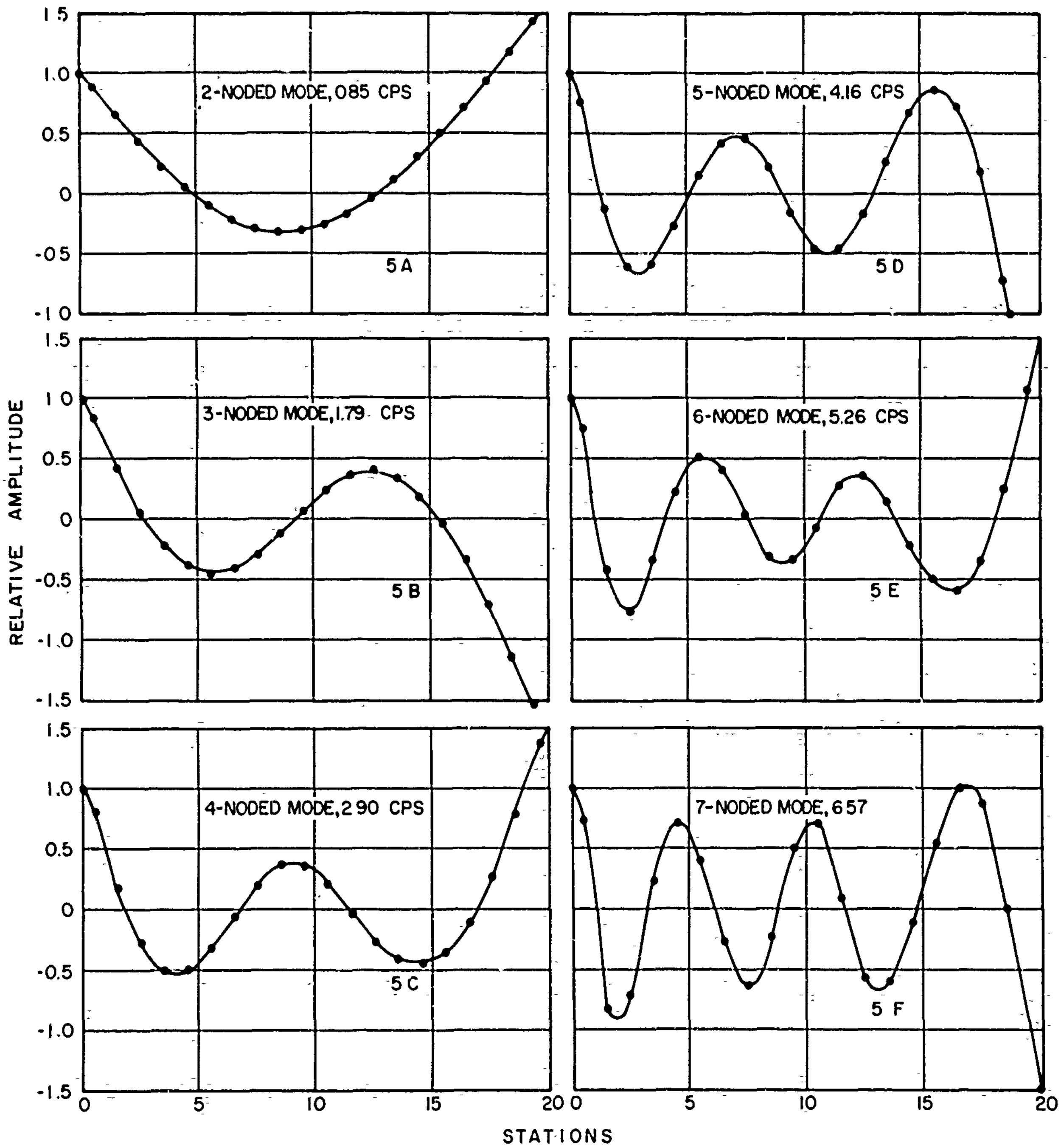



Figure 5 – Normal Mode Profiles and Natural Frequencies of Vertical Vibration with Constant "J" Factor

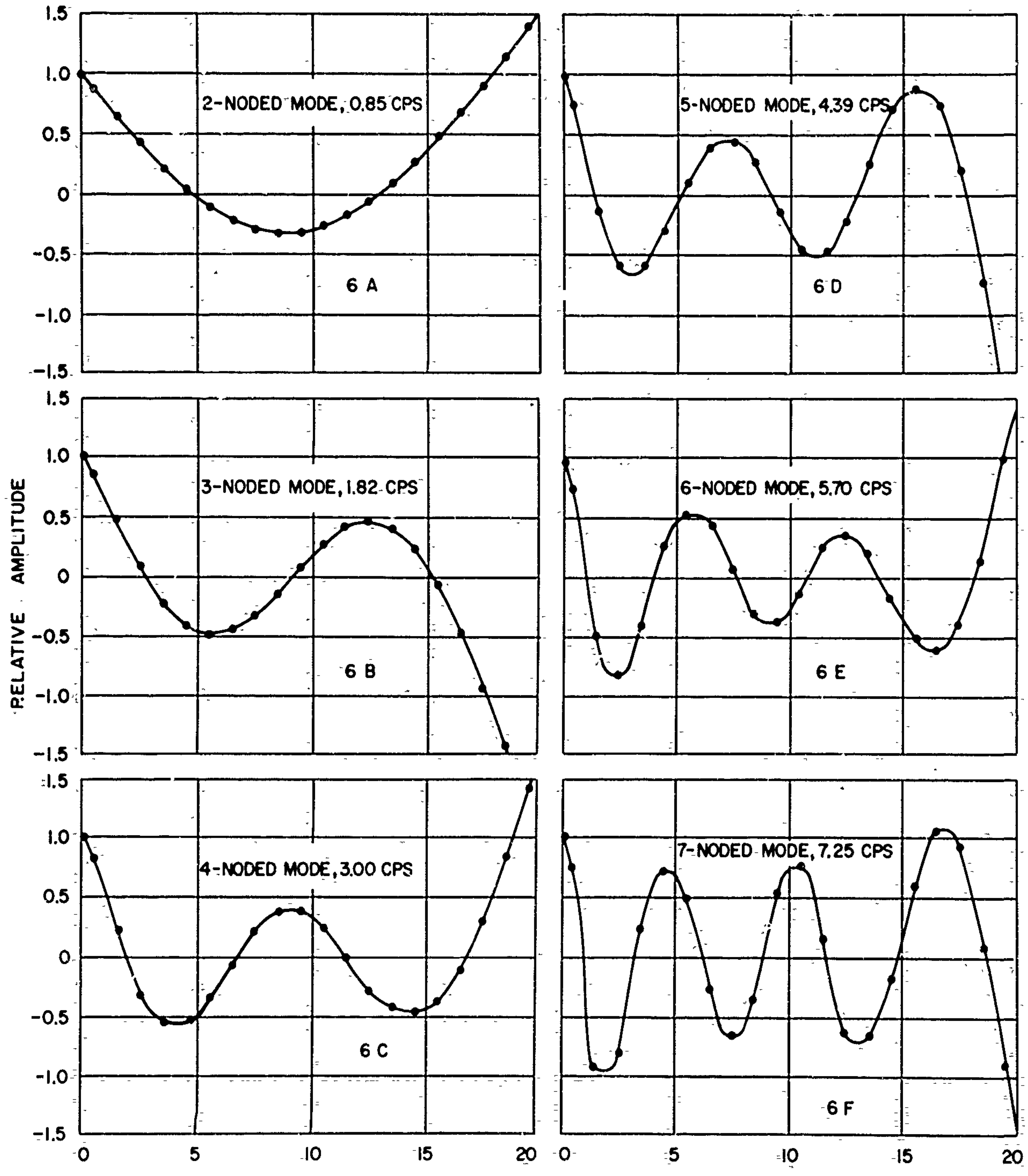



Figure 6 – Normal Mode Profiles and Natural Frequencies of Vertical Vibration with Varying "J" Factor

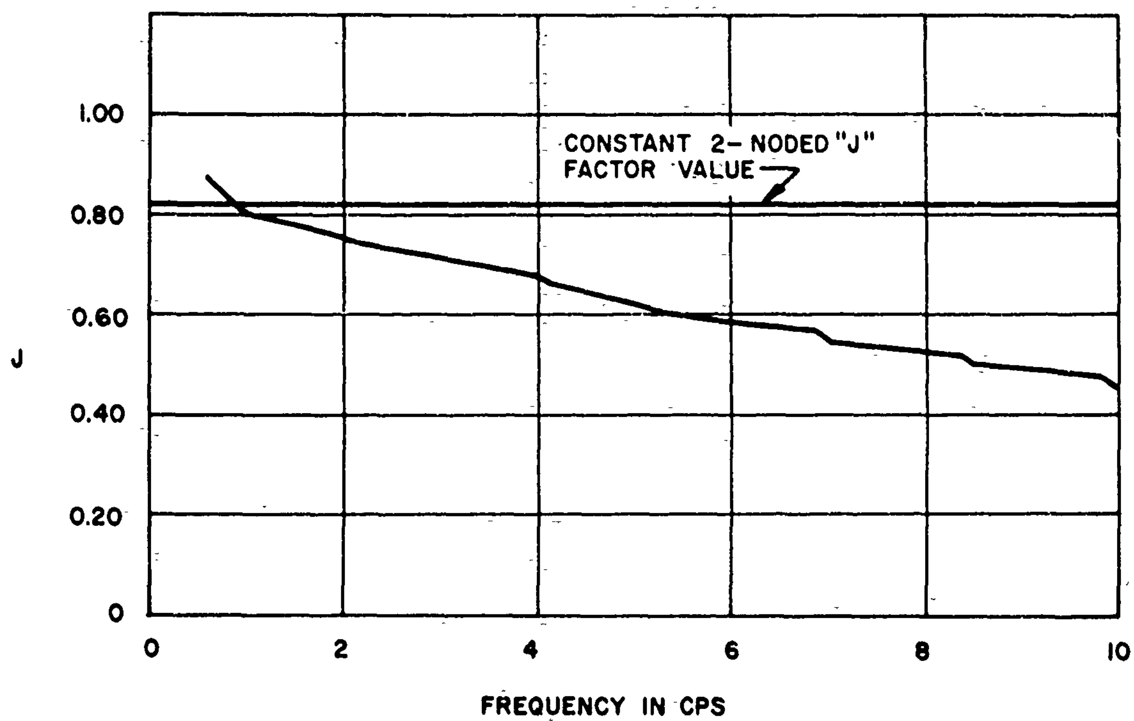



Figure 7 – Varied "J" Factor versus Frequency for Vertical Flexural Vibration

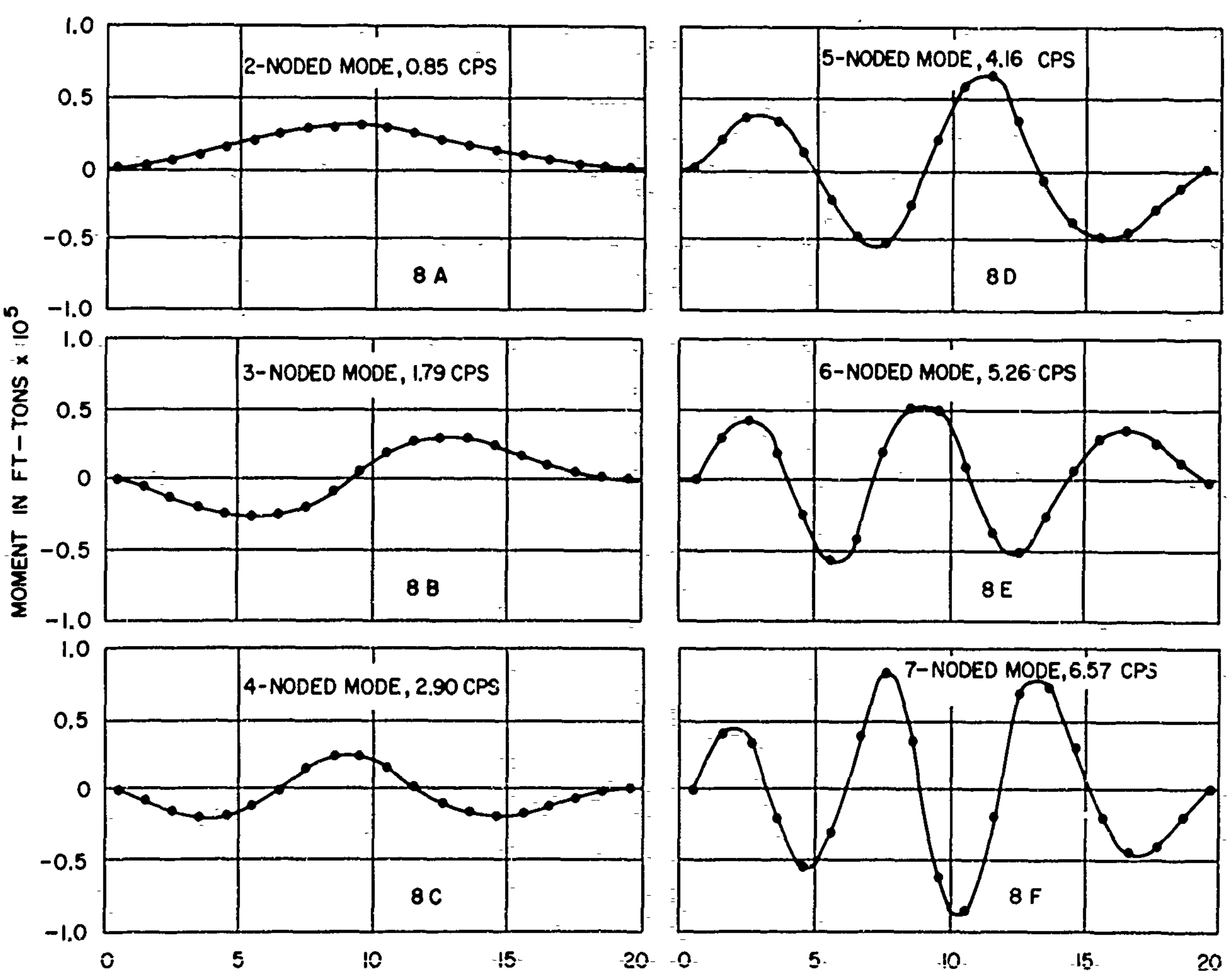



Figure 8 – Moment Distribution for Vertical Flexural Vibration Assuming Unit Deflection at Stern with Constant "J" Factor

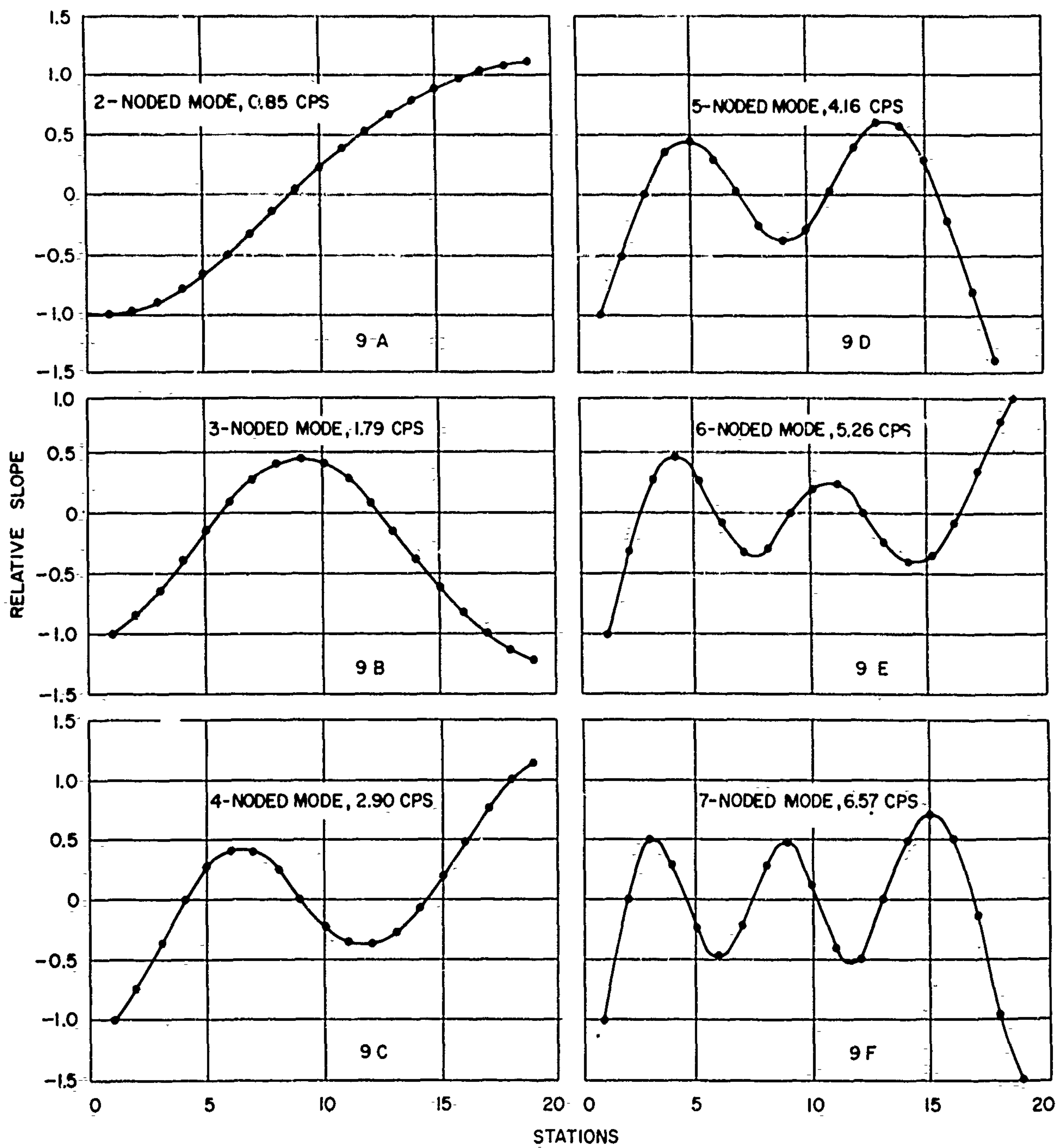



Figure 9 –Slope Distribution for Vertical Flexural Vibration, Assuming Unit Deflection at Stern with Constant "J" Factor

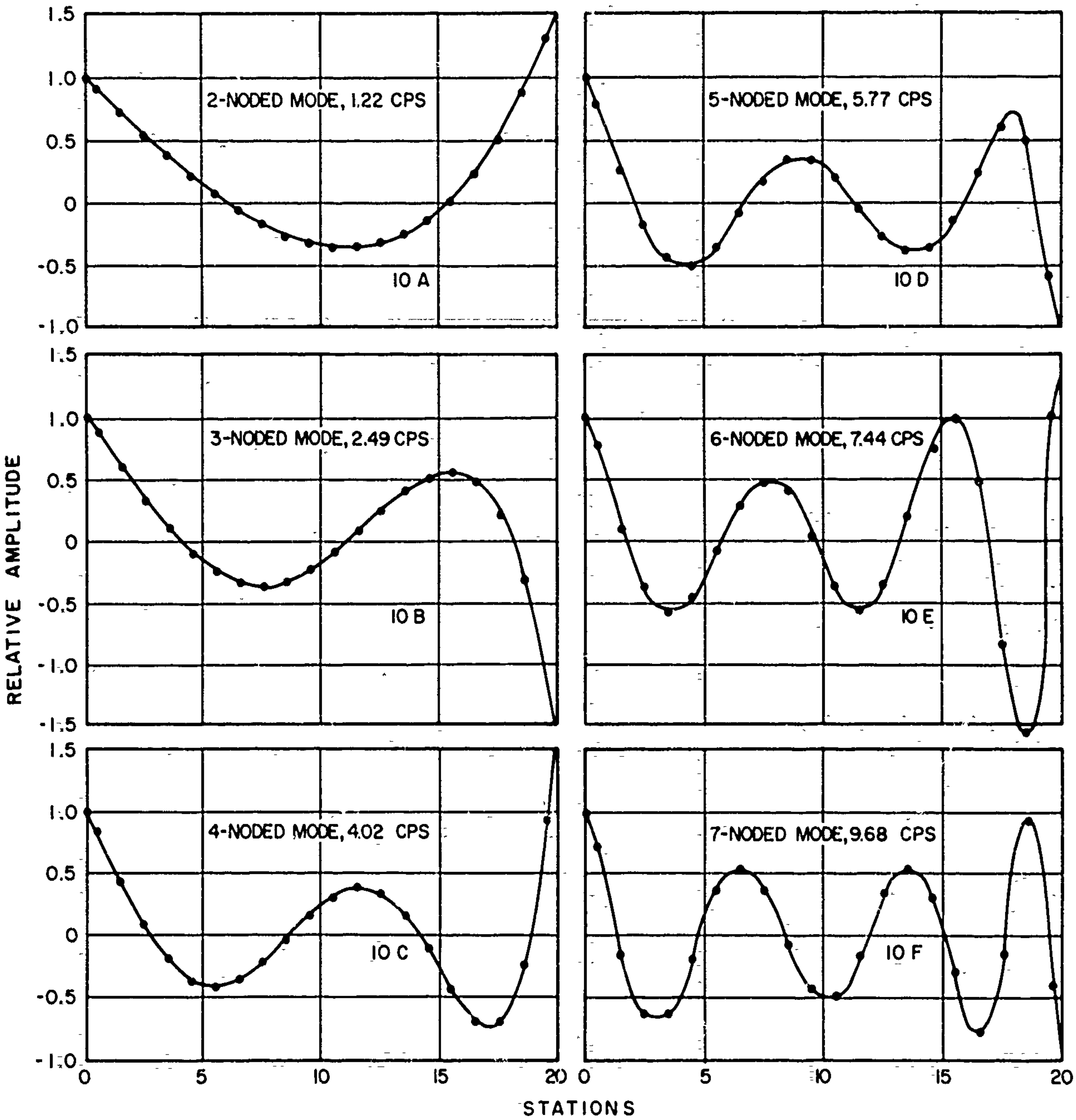



Figure 10 – Normal Model Profiles and Natural Frequencies of Horizontal Vibration

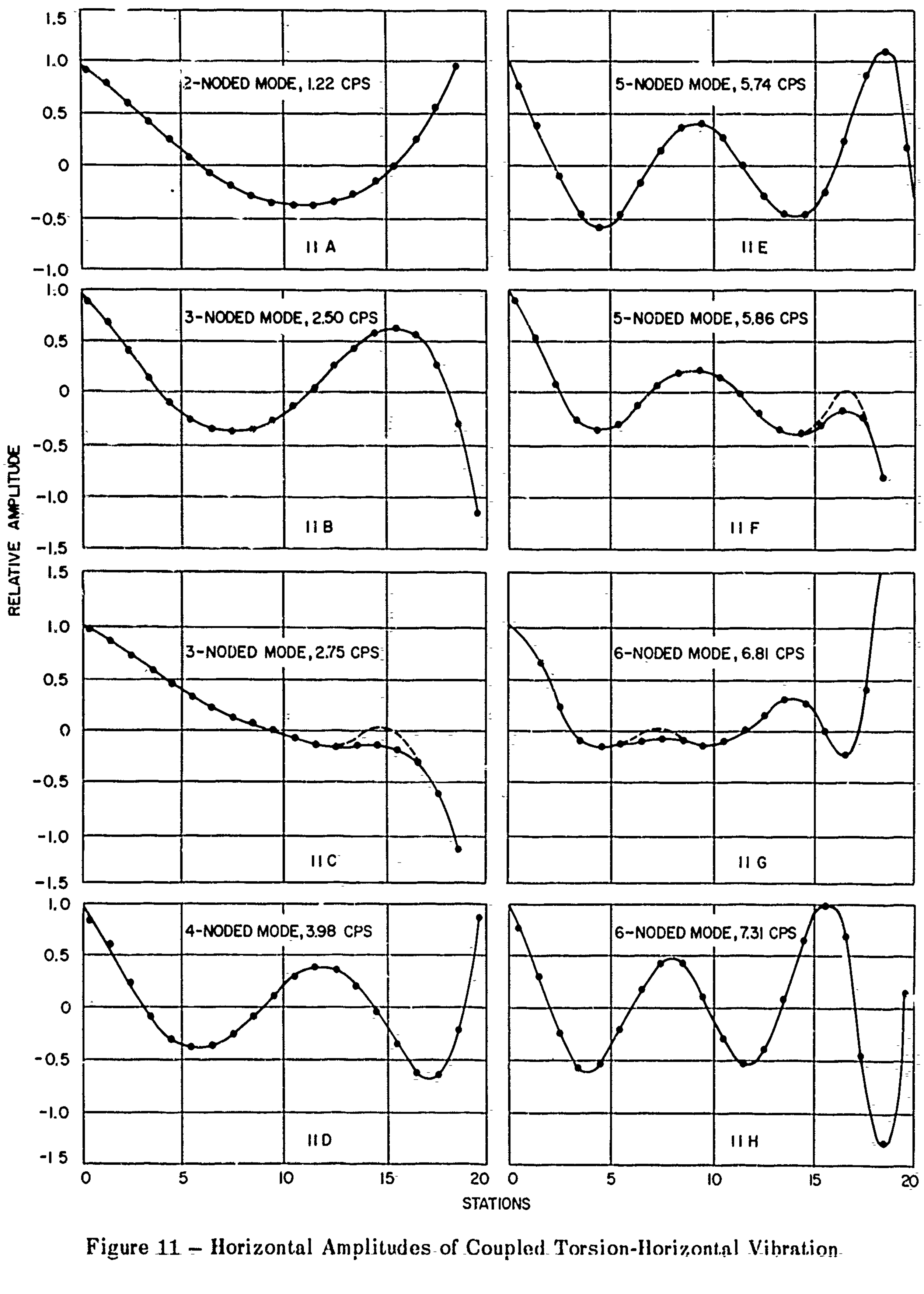



Figure 11 – Horizontal Amplitudes of Coupled Torsion-Horizontal Vibration

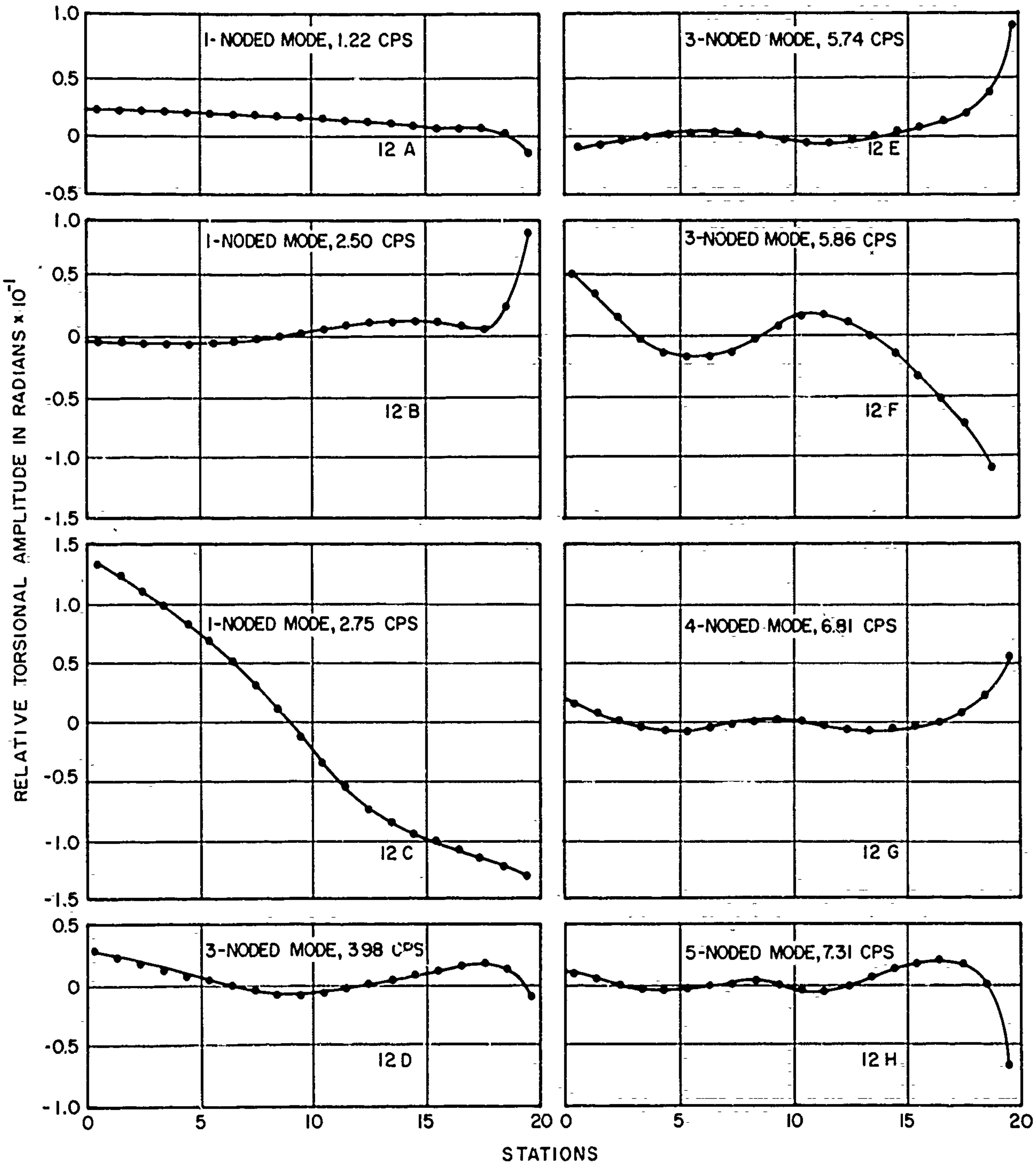



Figure 12 – Torsional Amplitudes of Coupled Torsion-Horizontal Vibration

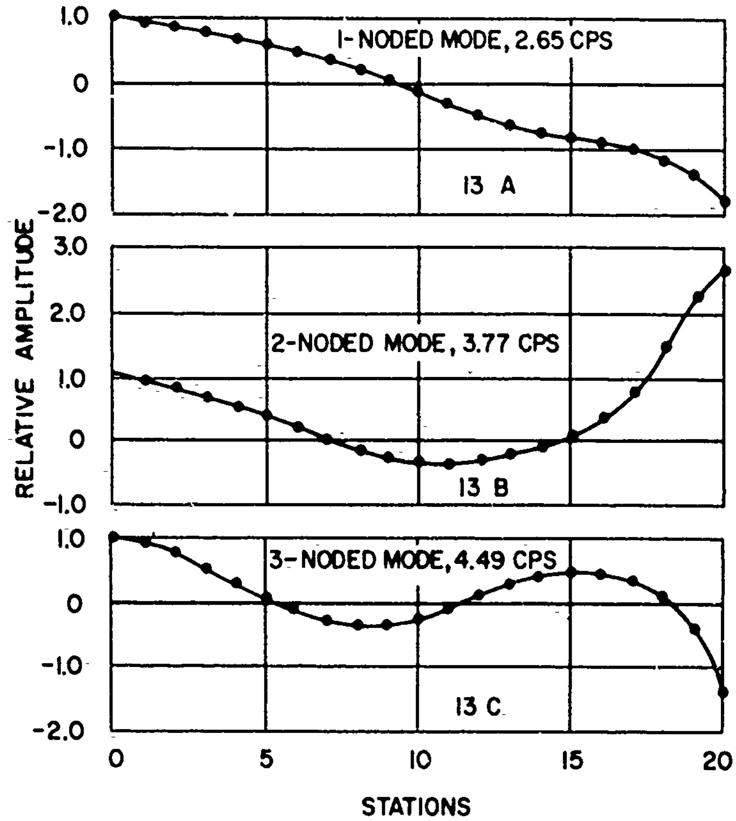



Figure 13 – Flexure-Free Torsional Modes of Vibration Determined by Electric Analog

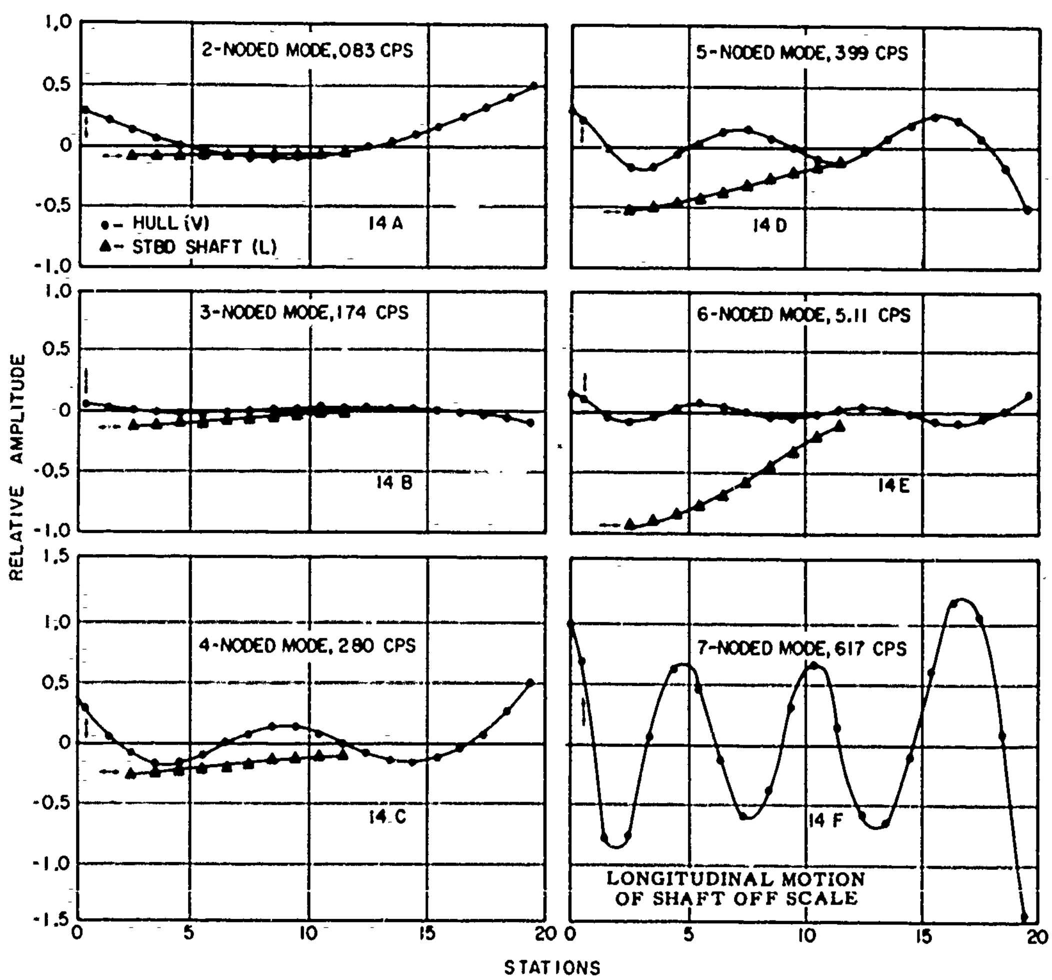



Figure 14 – Forced Response Profiles at Resonant Frequencies of Coupled Longitudinal Shafting-Vertical Flexural Hull Vibration (Excitation through Starboard Shaft)

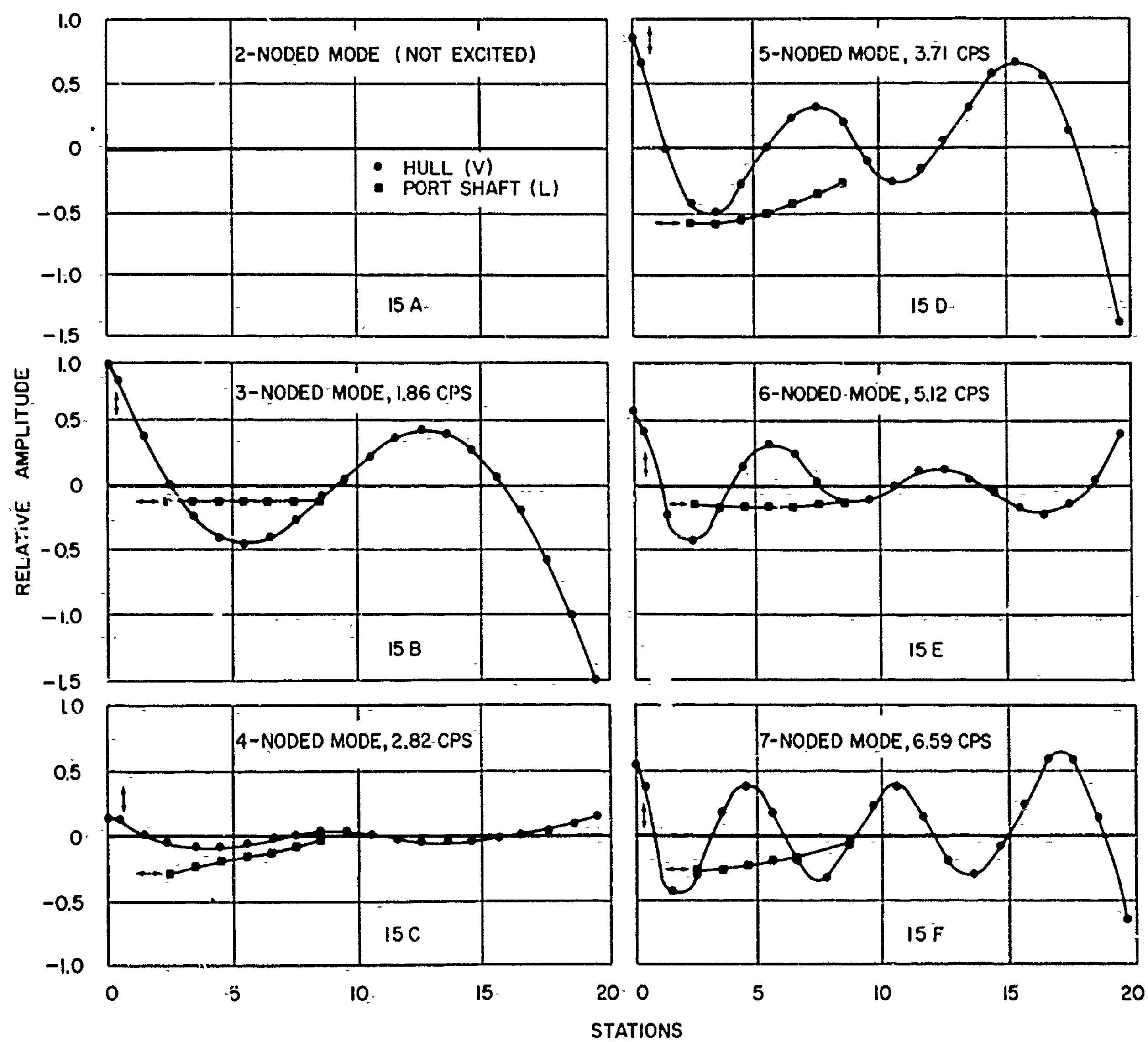



Figure 15 – Forced Response Profiles at Resonant Frequencies of Coupled Longitudinal Shafting-Vertical Flexural Hull Vibration (Excitation through Port Shaft)

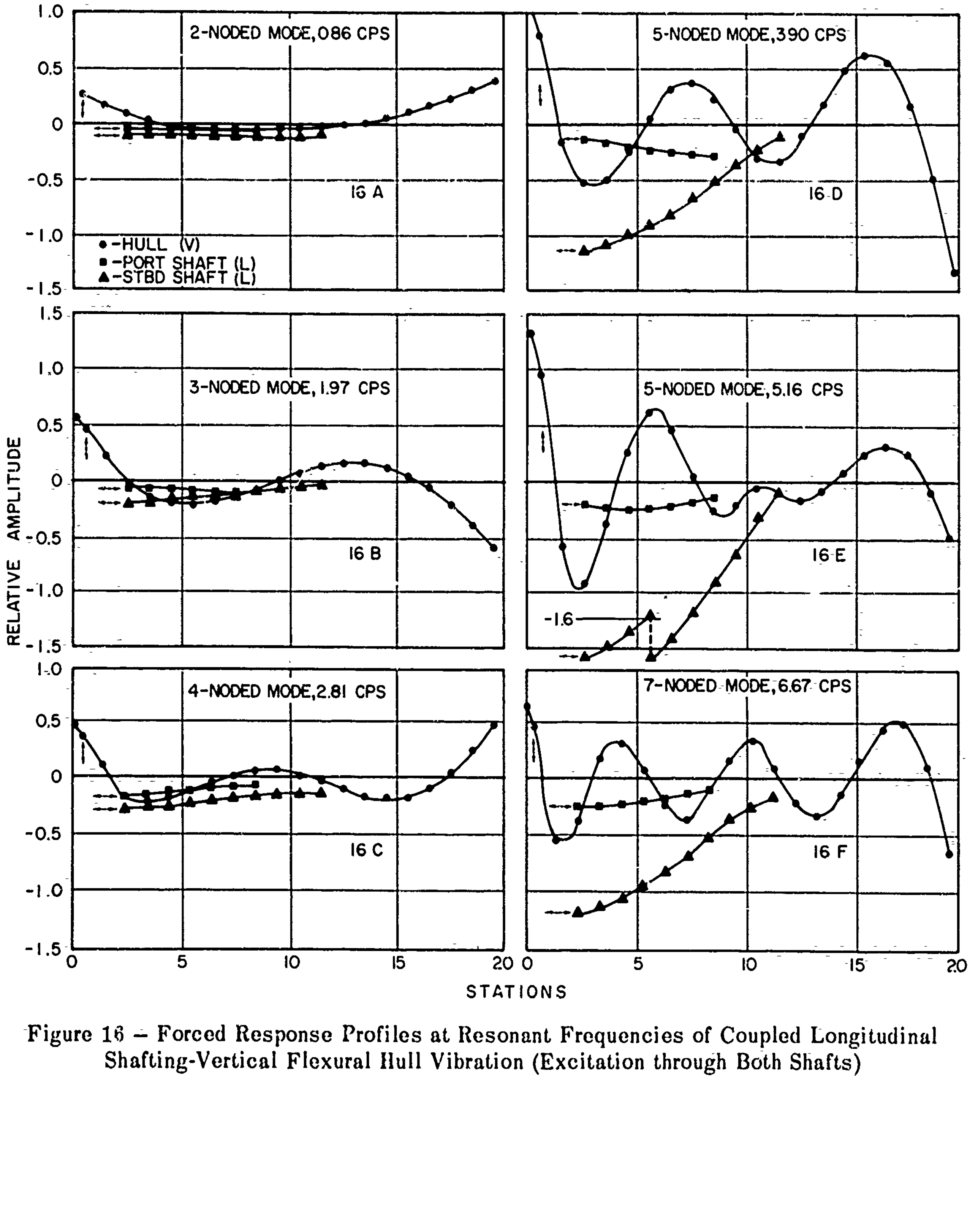



Figure 16 – Forced Response Profiles at Resonant Frequencies of Coupled Longitudinal Shafting-Vertical Flexural Hull Vibration (Excitation through Both Shafts)

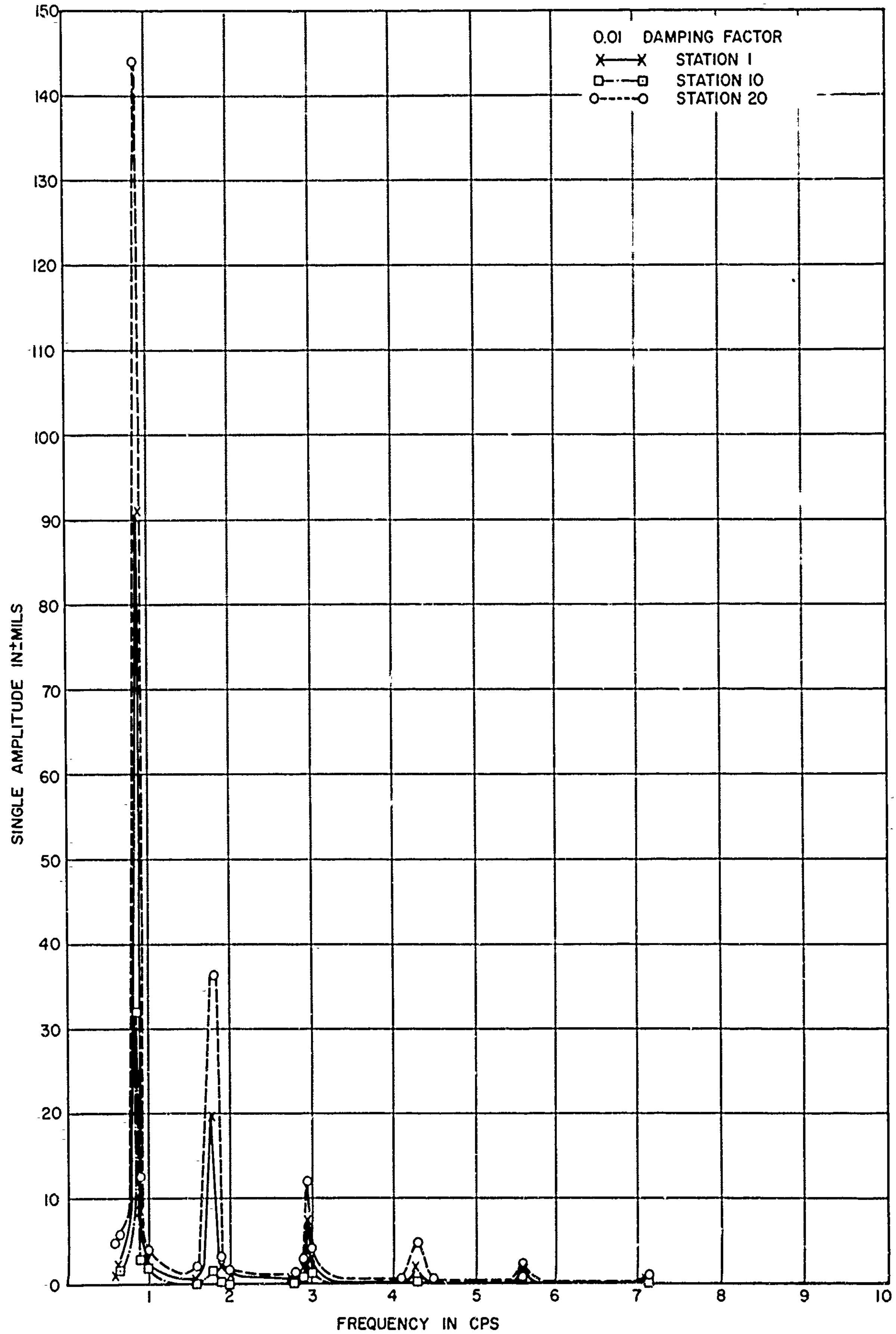



Figure 17 – Vertical Response at Selected Hull Stations to a 1000-Pound, Single-Amplitude Sinusoidal Forcing Function at the Stern (0.01 Damping)

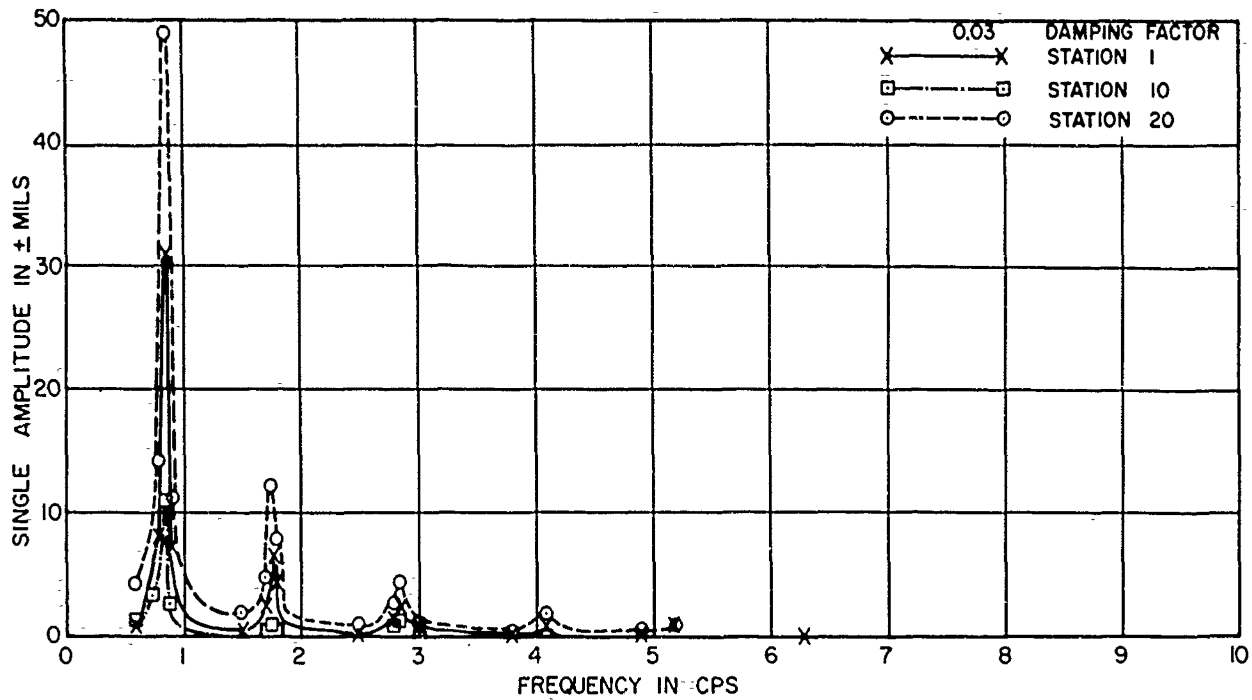



Figure 18 – Vertical Response at Selected Hull Stations to a 1000-Pound, Single-Amplitude Sinusoidal Forcing Function at the Stern (0.03 Damping)

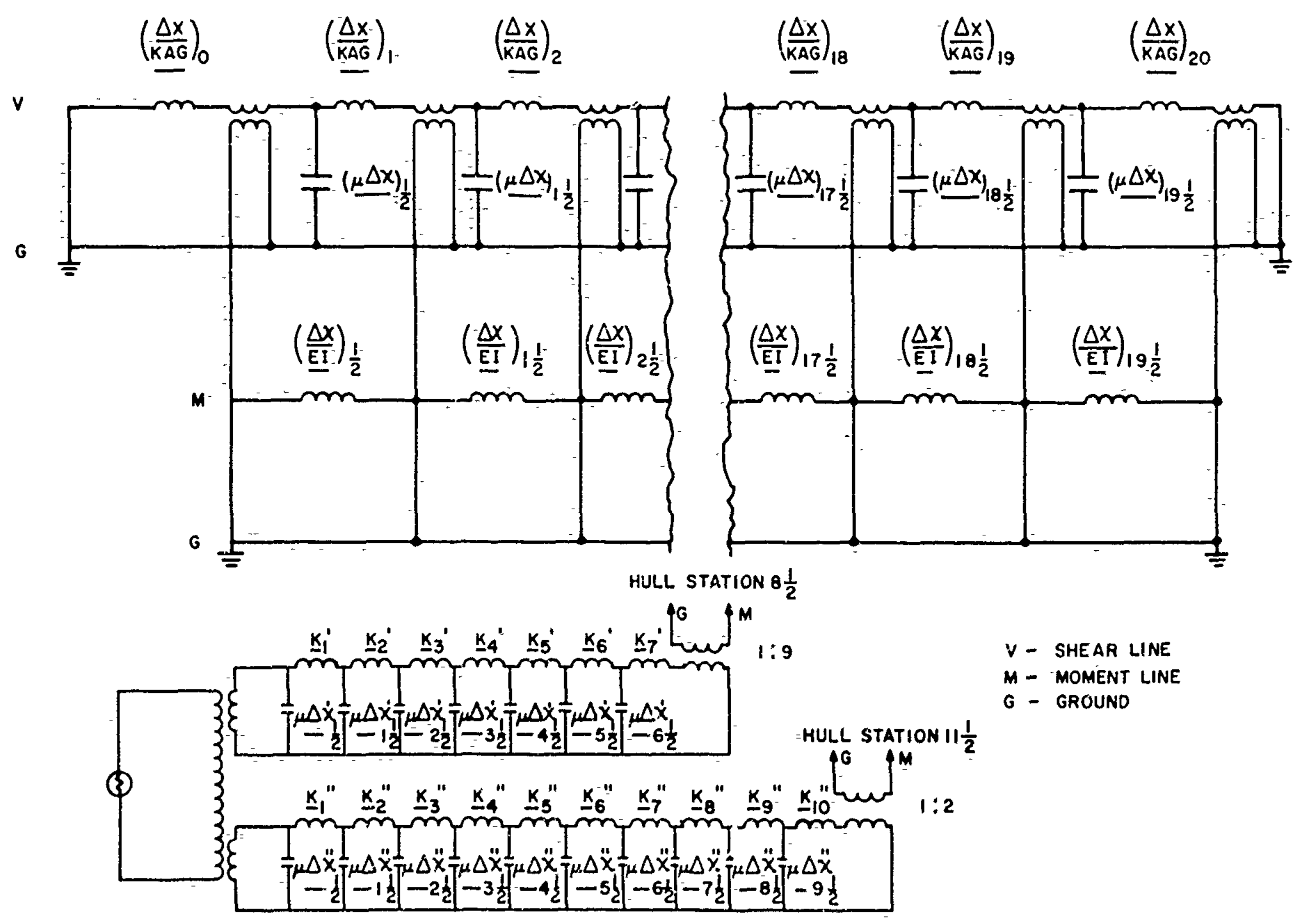



Figure 19 – Scheme of Mobility Analog Circuit used to Represent Coupled Longitudinal Shafting-Vertical Flexural Hull Vibration on TMB Network Analyzer

Rotary inertia is neglected; ‾ represents values of network elements used to simulate mechanical quantities in parentheses.

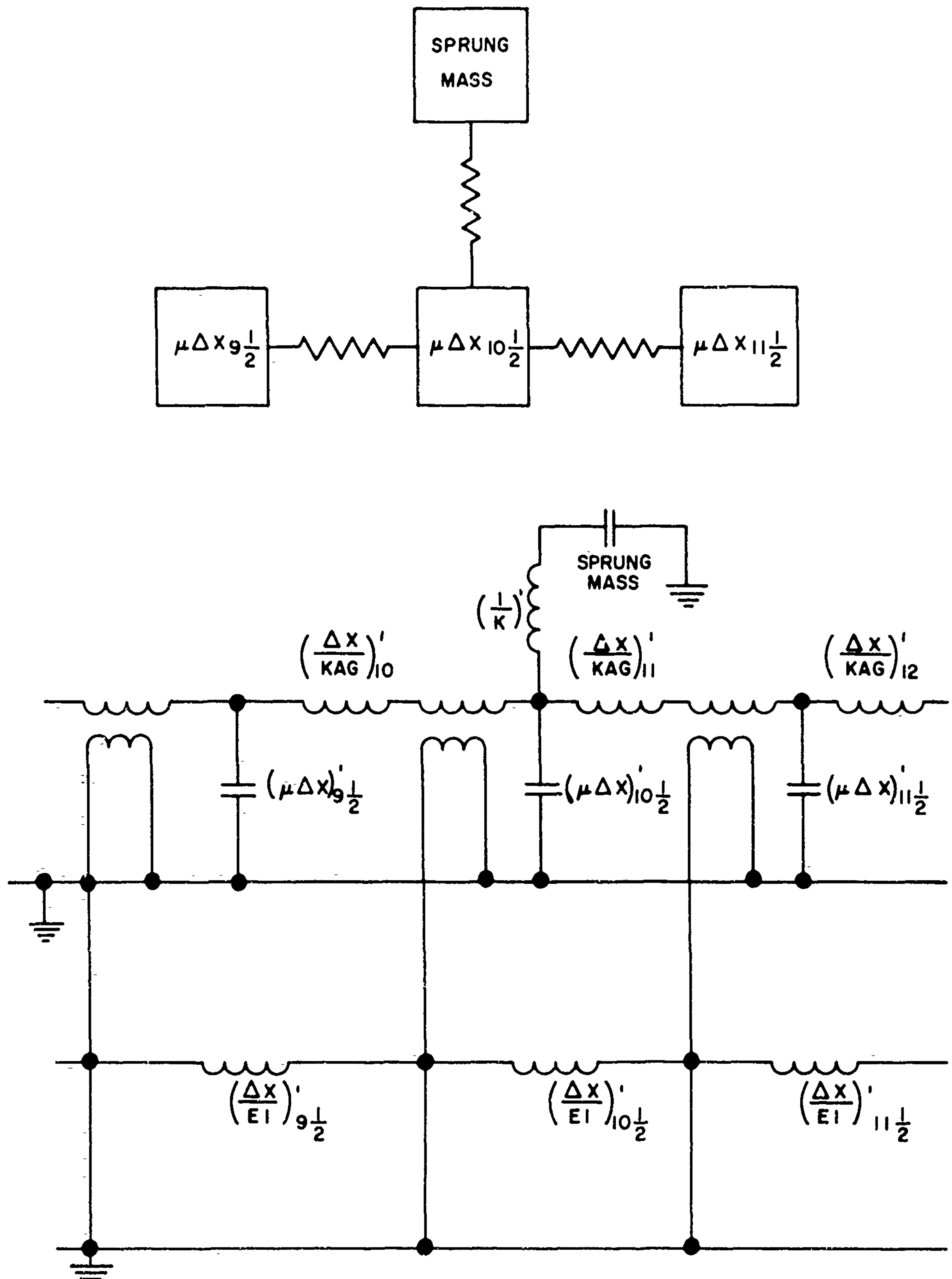



Figure 20 – Mechanical and Electric Analog Diagrams for Forward Deckhouse Attached as a Sprung Mass to Station 10 1/2

Rotary inertia is neglected; ′represent values of network elements used to simulate mechanical quantities.

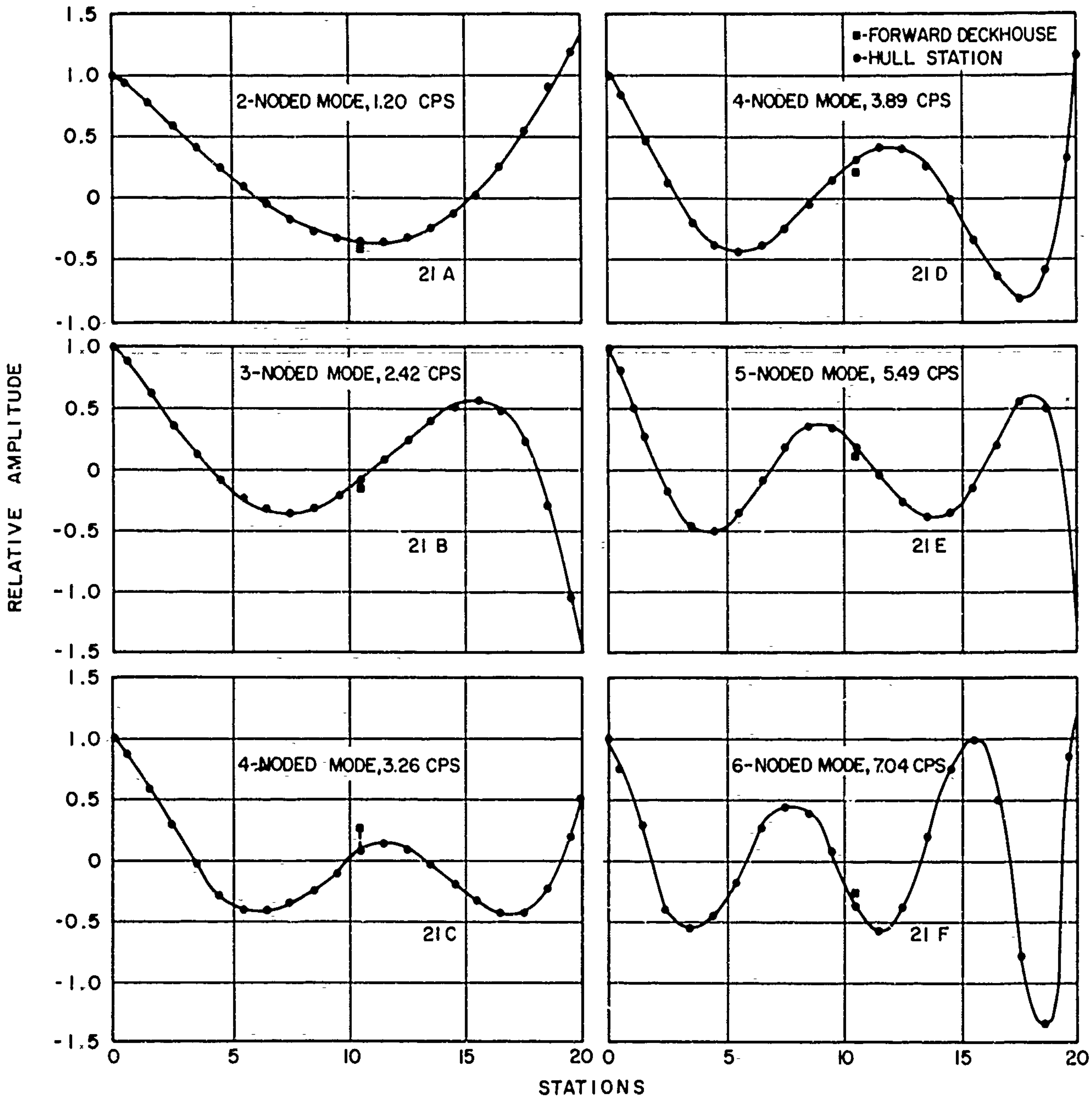



Figure 21 – Horizontal Normal Mode Profiles for Forward Deckhouse Attached to Hull as a Sprung Mass at Station 10 1/2

## TABLE 1

### Principal Characteristics of LONG BEACH

| | |
|---|---|
| Length (overall) | 721 ft, 3 in. |
| Length (waterline) | 691 ft, 10 in. |
| Length (BP) | 690 ft, 0 in. |
| Beam (extreme) | 73 ft, 4 in. |
| Depth (to main deck, molded) | 45 ft, 0 in. |
| Draft (mean) | 22 ft, 3 in. |
| Trim by stern | 1 ft, 4 in. |
| Displacement (design) | 14,981 long tons |
| Length to Beam Ratio, L/B | 9.41 |
| Beam to Depth Ratio, B/D | 1.63 |
| Length to Depth Ratio, L/D | 15.33 |

## TABLE 2

### Mass and Stiffness Data for Computing Vertical Flexural Modes of Vibration

| Station | Total Mass $\mu \Delta x$ ton-sec$^2$/ft | | $\frac{\Delta x}{EI} \times 10^8$ 1/ton-ft | $\frac{\Delta x}{KAG} \times 10^6$ ft/ton |
|---|---|---|---|---|
| | Ship | Virtual | | |
| 0 | | | | 62.26 |
| 1/2 | 14.41 | 20.00 | 2.176 | |
| 1 | | | | 24.35 |
| 1 1/2 | 13.31 | 23.10 | 0.9827 | |
| 2 | | | | 15.17 |
| 2 1/2 | 15.50 | 26.00 | 0.6273 | |
| 3 | | | | 11.59 |
| 3 1/2 | 16.70 | 29.20 | 0.4495 | |
| 4 | | | | 8.31 |
| 4 1/2 | 24.51 | 33.20 | 0.3508 | |
| 5 | | | | 6.24 |
| 5 1/2 | 33.68 | 37.20 | 0.2798 | |
| 6 | | | | 5.61 |
| 6 1/2 | 20.15 | 43.90 | 0.2458 | |
| 7 | | | | 5.26 |
| 7 1/2 | 34.94 | 58.80 | 0.2289 | |
| 8 | | | | 4.64 |
| 8 1/2 | 65.94 | 63.40 | 0.2288 | |
| 9 | | | | 4.57 |
| 9 1/2 | 32.79 | 60.50 | 0.2189 | |
| 10 | | | | 4.78 |
| 10 1/2 | 33.84 | 57.50 | 0.2319 | |
| 11 | | | | 5.15 |
| 11 1/2 | 51.68 | 49.50 | 0.2313 | |
| 12 | | | | 4.20 |
| 12 1/2 | 47.93 | 44.90 | 0.2569 | |
| 13 | | | | 4.27 |
| 13 1/2 | 21.08 | 27.80 | 0.2823 | |
| 14 | | | | 5.19 |
| 14 1/2 | 16.90 | 20.90 | 0.3216 | |
| 15 | | | | 5.38 |
| 15 1/2 | 17.83 | 10.60 | 0.41[illegible]4 | |
| 16 | | | | 7.28 |
| 16 1/2 | 14.98 | 5.50 | 0.5497 | |
| 17 | | | | 10.86 |
| 17 1/2 | 7.74 | 2.90 | 0.7764 | |
| 18 | | | | 17.57 |
| 18 1/2 | 8.31 | 0.80 | 1.148 | |
| 19 | | | | 30.67 |
| 19 1/2 | 8.63 | 1.20 | 2.301 | |
| 20 | | | | 101.61 |

$E = 13.4 \times 10^3$ ton/in$^2$

$G = 5.36 \times 10^3$ ton/in$^2$

## TABLE 3

**Mass and Stiffness Data for Computing Horizontal Flexural Modes of Vibration when Considered as Torsion-Free**

| Station | Ship Mass $\mu\Delta x$ ton-sec$^2$/ft | $\frac{\Delta x}{EI} \times 10^8$ 1/ton-ft | $\frac{\Delta x}{KAG} \times 10^6$ ft/ton |
|---|---|---|---|
| 0 | | | 24.67 |
| 1/2 | 14.81 | 1.055 | |
| 1 | | | 9.12 |
| 1 1/2 | 13.31 | 0.4858 | |
| 2 | | | 5.84 |
| 2 1/2 | 16.40 | 0.3043 | |
| 3 | | | 5.07 |
| 3 1/2 | 24.10 | 0.2206 | |
| 4 | | | 4.63 |
| 4 1/2 | 30.11 | 0.1757 | |
| 5 | | | 4.69 |
| 5 1/2 | 37.18 | 0.1514 | |
| 6 | | | 3.86 |
| 6 1/2 | 26.35 | 0.1325 | |
| 7 | | | 3.08 |
| 7 1/2 | 41.54 | 0.1213 | |
| 8 | | | 2.95 |
| 8 1/2 | 73.14 | 0.1153 | |
| 9 | | | 2.96 |
| 9 1/2 | 28.29 | 0.1176 | |
| 10 | | | 3.09 |
| 10 1/2 | 39.94 | 0.1343 | |
| 11 | | | 3.14 |
| 11 1/2 | 59.08 | 0.1323 | |
| 12 | | | 3.26 |
| 12 1/2 | 53.93 | 0.1511 | |
| 13 | | | 4.11 |
| 13 1/2 | 25.18 | 0.2060 | |
| 14 | | | 4.56 |
| 14 1/2 | 24.80 | 0.2693 | |
| 15 | | | 6.07 |
| 15 1/2 | 23.53 | 0.4256 | |
| 16 | | | 8.39 |
| 16 1/2 | 22.18 | 0.9396 | |
| 17 | | | 11.31 |
| 17 1/2 | 17.04 | 1.800 | |
| 18 | | | 12.70 |
| 18 1/2 | 17.91 | 4.527 | |
| 19 | | | 27.21 |
| 19 1/2 | 16.43 | 9.536 | |
| 20 | | | 83.99 |

# TABLE 4

## Mass and Stiffness Data for Computing Coupled Torsion-Horizontal Flexural Modes of Vibration

| Station | Ship Mass $\mu\Delta x$ ton-sec$^2$/ft | $m\bar{z}$ ton-sec$^2$ | $I_{mz} \times 10^{-2}$ ton-ft-sec$^2$ | $\frac{\Delta x}{EI} \times 10^8$ 1/ton-ft | $I_{mx} \times 10^{-2}$ ton-ft-sec$^2$ | $\bar{\bar{z}}$ ft | $\frac{\Delta x}{KAG} \times 10^6$ ft/ton | $\frac{\Delta x}{GJ_e} \times 10^8$ 1/ton-ft |
|---|---|---|---|---|---|---|---|---|
| 0 | | | 25.02 | | | 9.16 | 24.67 | 4.60 |
| 1/2 | 14.81 | 132.3 | | 1.055 | 84.71 | | | |
| 1 | | | 40.41 | | | 8.93 | 9.12 | 3.16 |
| 1 1/2 | 13.31 | 107.5 | | 0.4858 | 104.43 | | | |
| 2 | | | 45.84 | | | 8.70 | 5.84 | 2.02 |
| 2 1/2 | 16.40 | 116.2 | | 0.3043 | 137.07 | | | |
| 3 | | | 62.19 | | | 8.48 | 5.07 | 1.31 |
| 3 1/2 | 24.10 | 176.0 | | 0.2206 | 191.60 | | | |
| 4 | | | 77.49 | | | 5.65 | 4.63 | 0.840 |
| 4 1/2 | 30.11 | 146.3 | | 0.1757 | 271.46 | | | |
| 5 | | | 140.33 | | | 5.43 | 4.69 | 0.645 |
| 5 1/2 | 37.18 | 78.8 | | 0.1514 | 367.97 | | | |
| 6 | | | 113.06 | | | 2.72 | 3.86 | 0.570 |
| 6 1/2 | 26.35 | 124.8 | | 0.1325 | 318.01 | | | |
| 7 | | | 132.58 | | | 2.41 | 3.08 | 0.548 |
| 7 1/2 | 41.54 | 20.8 | | 0.1213 | 658.85 | | | |
| 8 | | | 295.50 | | | 0.69 | 2.95 | 0.528 |
| 8 1/2 | 73.14 | −308.5 | | 0.1153 | 939.79 | | | |
| 9 | | | 235.87 | | | 0.45 | 2.96 | 0.514 |
| 9 1/2 | 37.49 | 136.5 | | 0.1176 | 619.04 | | | |
| 10 | | | 133.14 | | | 0.25 | 3.09 | 0.536 |
| 10 1/2 | 39.94 | 73.5 | | 0.1343 | 599.19 | | | |
| 11 | | | 183.81 | | | 0.75 | 3.14 | 0.565 |
| 11 1/2 | 59.08 | −132.0 | | 0.1323 | 469.14 | | | |
| 12 | | | 292.53 | | | 0.73 | 3.26 | 0.652 |
| 12 1/2 | 53.93 | −234.0 | | 0.1511 | 587.04 | | | |
| 13 | | | 93.78 | | | 0.30 | 4.11 | 0.765 |
| 13 1/2 | 25.18 | 133.0 | | 0.2060 | 198.75 | | | |
| 14 | | | 61.27 | | | 1.08 | 4.56 | 0.890 |
| 14 1/2 | 24.80 | 137.3 | | 0.2693 | 149.08 | | | |
| 15 | | | 50.41 | | | 1.14 | 6.07 | 1.20 |
| 15 1/2 | 23.53 | 106.3 | | 0.4256 | 104.73 | | | |
| 16 | | | 39.47 | | | 2.75 | 8.39 | 1.96 |
| 16 1/2 | 22.18 | 175.3 | | 0.9396 | 65.52 | | | |
| 17 | | | 21.79 | | | 6.40 | 11.31 | 3.96 |
| 17 1/2 | 17.04 | 157.6 | | 1.800 | 25.17 | | | |
| 18 | | | 11.38 | | | 6.64 | 12.70 | 9.34 |
| 18 1/2 | 17.91 | 139.2 | | 4.527 | 24.36 | | | |
| 19 | | | 13.78 | | | 6.85 | 27.21 | 38.57 |
| 19 1/2 | 16.43 | 160.2 | | 9.536 | 23.89 | | | |
| 20 | | | 5.54 | | | 7.10 | 83.99 | 118.50 |

# TABLE 5

## Mass and Stiffness Data for Computing Coupled Longitudinal Shafting-Vertical Flexural Hull Modes of Vibration

| Hull | | | |
|---|---|---|---|
| Station | $(\mu\Delta x)$ ton-sec$^2$/ft | $\frac{\Delta x}{EI} \times 10^8$ 1/ton-ft | $\frac{\Delta x}{KAG} \times 10^6$ ft/ton |
| 0 | | | 62.26 |
| 1/2* | 33.10 | 2.176 | |
| 1 | | | 24.35 |
| 1 1/2* | 34.64 | 0.9827 | |
| 2 | | | 15.17 |
| 2 1/2* | 41.03 | 0.6273 | |
| 3 | | | 11.59 |
| 3 1/2* | 44.96 | 0.4495 | |
| 4 | | | 8.31 |
| 4 1/2* | 57.15 | 0.3508 | |
| 5 | | | 6.24 |
| 5 1/2* | 70.23 | 0.2798 | |
| 6 | | | 5.61 |
| 6 1/2* | 63.64 | 0.2458 | |
| 7 | | | 5.26 |
| 7 1/2* | 93.38 | 0.2289 | |
| 8 | | | 4.64 |
| 8 1/2* | 124.78 | 0.2288 | |
| 9 | | | 4.57 |
| 9 1/2* | 93.13 | 0.2189 | |
| 10 | | | 4.78 |
| 10 1/2* | 91.03 | 0.2319 | |
| 11 | | | 5.15 |
| 11 1/2* | 96.93 | 0.2313 | |
| 12 | | | 4.20 |
| 12 1/2 | 92.83 | 0.2569 | |
| 13 | | | 4.27 |
| 13 1/2 | 48.88 | 0.2823 | |
| 14 | | | 5.19 |
| 14 1/2 | 37.80 | 0.3216 | |
| 15 | | | 5.38 |
| 15 1/2 | 28.43 | 0.4124 | |
| 16 | | | 7.28 |
| 16 1/2 | 20.48 | 0.5497 | |
| 17 | | | 10.86 |
| 17 1/2 | 10.64 | 0.7764 | |
| 18 | | | 17.57 |
| 18 1/2 | 9.11 | 1.148 | |
| 19 | | | 30.67 |
| 19 1/2 | 8.83 | 2.301 | |
| 20 | | | 101.61 |

*Mass of propulsion system subtracted from these stations.

| Port Shaft and Propulsion Machinery | | |
|---|---|---|
| Station | $(\mu\Delta x)$ ton-sec$^2$/ft | $k \times 10^{-5}$ ton/ft |
| 1/2′ | 1.30 | |
| 1 ′ | | 1.142 |
| 1 1/2′ | 0.47 | |
| 2 ′ | | 1.142 |
| 2 1/2′ | 0.47 | |
| 3 ′ | | 1.142 |
| 3 1/2′ | 0.47 | |
| 4 ′ | | 1.142 |
| 4 1/2′ | 0.20 | |
| 5 ′ | | 0.812 |
| 5 1/2′ | 0.20 | |
| 6 ′ | | 1.085 |
| 6 1/2′ | 4.25 | |
| 7 ′ | | 1.435 |

| Starboard Shaft and Propulsion Machinery | | |
|---|---|---|
| Station | $(\mu\Delta x)$ ton-sec$^2$/ft | $k \times 10^{-5}$ ton/ft |
| 1/2″ | 1.30 | |
| 1 ″ | | 1.142 |
| 1 1/2″ | 0.47 | |
| 2 ″ | | 1.142 |
| 2 1/2″ | 0.47 | |
| 3 ″ | | 1.142 |
| 3 1/2″ | 0.47 | |
| 4 ″ | | 1.142 |
| 4 1/2″ | 0.31 | |
| 5 ″ | | 0.812 |
| 5 1/2″ | 0.31 | |
| 6 ″ | | 0.812 |
| 6 1/2″ | 0.31 | |
| 7 ″ | | 0.812 |
| 7 1/2″ | 0.31 | |
| 8 ″ | | 1.043 |
| 8 1/2″ | 0.31 | |
| 9 ″ | | 1.338 |
| 9 1/2″ | 4.25 | |
| 10 ″ | | 1.435 |

## TABLE 6

### Comparison of Natural Frequencies of Vertical Vibration for Constant and Varying "J" Factors

(Calculated values were obtained by digital computer)

| Modes | Calculated Frequency, CPS | |
|---|---|---|
| | Constant "J" Factor | Varying "J" Factor |
| First | 0.85 | 0.85 |
| Second | 1.79 | 1.8 |
| Third | 2.90 | 3.0 |
| Fourth | 4.16 | 4.4 |
| Fifth | 5.26 | 5.7 |
| Sixth | 6.57 | 7.2 |

## TABLE 7

### Frequencies of Vertical and Horizontal Modes (Considered as Torsion Free) Computed by Electrical Analog

| Modes | Number of Nodes | Frequencies in CPS | | |
|---|---|---|---|---|
| | | Vertical | | Horizontal |
| | | Bending and Shear | Bending Only | Bending and Shear |
| First | 2 | 0.83 | 0.86 | 1.20 |
| Second | 3 | 1.73 | 1.93 | 2.43 |
| Third | 4 | 2.80 | 3.39 | 3.84 |
| Fourth | 5 | 3.98 | 5.16 | 5.50 |
| Fifth | 6 | 5.06 | 7.30 | 7.04 |
| Sixth | 7 | 6.31 | 9.66 | 9.15 |

## TABLE 8

### Frequencies of Coupled Torsion-Horizontal Modes Calculated by Digital Computer

| Modes | Number of Nodes | | Frequencies in CPS |
|---|---|---|---|
| | Horizontal | Torsional | |
| First | 2 | 1 | 1.24 |
| Second | 3 | 1 | 2.54 |
| | 3 | 1 | 2.79 |
| Third | 4 | 3 | 4.04 |
| Fourth | 5 | 3 | 5.82 |
| | 5 | 3 | 5.95 |
| Fifth | 6 | 4 | 6.91 |
| | 6 | 5 | 7.42 |

## TABLE 9

### Comparison of Natural Frequencies of Vertical Hull Vibration Assuming Rigid and Flexible Propulsion Shafting

(Calculated values given here were computed by the analog computer)

| Modes | Calculated Frequency, CPS | |
|---|---|---|
| | Rigid Propulsion Shafting* | Flexible Propulsion Shafting** |
| First | 0.83 | 0.86 |
| Second | 1.73 | 1.97 |
| Third | 2.80 | 2.81 |
| Fourth | 3.98 | 3.90 |
| Fifth | 5.06 | 5.16 |
| Sixth | 6.31 | 6.67 |

*Frequencies determined from vertical flexural vibration assuming constant "J" virtual mass factor.

**Frequencies determined from coupled longitudinal shaft-vertical flexural hull vibration.

## TABLE 10

Mass and Stiffness Data for Computing Horizontal Flexural Modes of Vibration when Considered as Torsion Free (Forward Deckhouse Attached to the Hull as a Sprung Mass)

| Station | Ship Mass $\mu \Delta x$ ton-sec$^2$/ft | $\frac{\Delta x}{EI} \times 10^8$ 1/ton-ft | $\frac{\Delta x}{KAG} \times 10^6$ ft/ton |
|---|---|---|---|
| 0 | | | 24.67 |
| 1/2 | 14.81 | 1.055 | |
| 1 | | | 9.12 |
| 1 1/2 | 13.31 | 0.4858 | |
| 2 | | | 5.84 |
| 2 1/2 | 16.40 | 0.3043 | |
| 3 | | | 5.07 |
| 3 1/2 | 24.10 | 0.2206 | |
| 4 | | | 4.63 |
| 4 1/2 | 30.11 | 0.1757 | |
| 5 | | | 4.69 |
| 5 1/2 | 37.18 | 0.1514 | |
| 6 | | | 3.86 |
| 6 1/2 | 26.35 | 0.1325 | |
| 7 | | | 3.08 |
| 7 1/2 | 41.54 | 0.1213 | |
| 8 | | | 2.95 |
| 8 1/2 | 73.14 | 0.1153 | |
| 9 | | | 2.96 |
| 9 1/2 | 37.49 | 0.1176 | |
| 10 | | | 3.09 |
| 10 1/2 | 39.94 | 0.1343 | |
| 11 | | | 3.14 |
| 11 1/2 | 59.08 | 0.1323 | |
| 12 | | | 3.26 |
| 12 1/2 | 53.93 | 0.1511 | |
| 13 | | | 4.11 |
| 13 1/2 | 25.18 | 0.2060 | |
| 14 | | | 4.56 |
| 14 1/2 | 24.80 | 0.2693 | |
| 15 | | | 6.07 |
| 15 1/2 | 23.53 | 0.4256 | |
| 16 | | | 8.39 |
| 16 1/2 | 22.18 | 0.9396 | |
| 17 | | | 11.31 |
| 17 1/2 | 17.04 | 1.800 | |
| 18 | | | 12.70 |
| 18 1/2 | 17.91 | 4.527 | |
| 19 | | | 27.21 |
| 19 1/2 | 16.43 | 9.536 | |
| 20 | | | 83.99 |

TABLE 11

Effect of Superstructure Forward Deckhouse on Horizontal Modes of Vibration

| Hull Mode | Computed Natural Frequency, CPS: Deckhouse Rigidly Attached to Hull | Computed Natural Frequency, CPS: Deckhouse Flexibly Attached to Hull | Measured Frequency CPS |
|---|---|---|---|
| First | 1.20 | 1.20 | 1.3 |
| Second | 2.43 | 2.42 | 2.5 |
| | --- | 3.26* | 3.3 |
| Third | 3.83 | 3.89 | 4.1 |
| Fourth | 5.48 | 5.49 | 5.6 |
| Fifth | 7.04 | 7.04 | 6.9 |

*Natural frequency of the hull due to the superstructure

TABLE 12

Comparison of Calculated and Experimental Frequencies for LONG BEACH

(Calculated frequencies corrected for loading of the ship during underway trials)

| Modes | Theoretical Frequencies, CPS | | | | Experimental Frequencies, CPS (Values Obtained from Reference 2) |
|---|---|---|---|---|---|
| | Digital Calculations | | Analog Calculations | | |
| | Rigid Propulsion Shafting | | Constant "J" Factor | | |
| Vertical | Constant "J" Factor | Varying "J" Factor | Rigid Propulsion Shafting | Flexible Propulsion Shafting | |
| First | 0.85* | 0.85 | 0.83 | 0.86 | 1.0 |
| Second | 1.8 | 1.8 | 1.7 | 2.0 | 2.0 |
| Third | 2.9 | 3.0 | 2.8 | 2.8 | 3.0 |
| Fourth | 4.2 | 4.4 | 4.0 | 3.9 | 4.5 |
| Fifth | 5.3 | 5.7 | 5.1 | 5.2 | 5.3 |
| Sixth | 6.6 | 7.2 | 6.3 | 6.7 | 6.7 |
| | Rigid Forward Deckhouse | | Torsion-Free | | |
| Horizontal | Torsion Free | Torsion Included | Rigid Forward Deckhouse | Flexible Forward Deckhouse | |
| First | 1.2 | 1.2 | 1.2 | 1.2 | 1.3 |
| Second | 2.5 | 2.5 | 2.4 | 2.4 | 2.5 |
| | --- | 2.8 | --- | --- | |
| Third | --- | --- | --- | 3.3 ** | 3.3** |
| | 4.0 | 4.0 | 3.8 | 3.9 | 4.1 |
| Fourth | 5.8 | 5.8 | 5.5 | 5.5 | 5.6† |
| | --- | 6.0 | --- | --- | --- |
| Fifth | 7.4 | 6.9 | 7.0 | 7.0 | 6.9† |
| | --- | 7.4 | --- | --- | --- |

*The first vertical hull mode is estimated as 0.92-cps if the effect of the first superstructure deck is considered (see Appendix B).

**Assumed natural frequency of superstructure.

†Assumed natural frequency of coupled torsion-horizontal hull mode.

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# APPENDIX A

## EVALUATION OF PARAMETERS

The methods of determining the parameters required for the calculation of natural frequencies, normal modes, bending moments, and bending slope for the various types of hull vibration and the resonant frequencies and modes of coupled shaft-hull vibration are described. These parameters are summarized in Tables 2, 3, 4, and 5.

### SHIP MASS $(\mu \Delta x)_s$

The distribution of ship mass was obtained from weight curves prepared by the Bethlehem Steel Company, Quincy, Massachusetts. The mass of the propulsion-shafting system was included in the ship mass. The longitudinal weight curve is shown in Figure 2 together with the computed virtual mass for vertical vibration. The total ship mass is summarized in Tables 2 and 3 for vertical and horizontal flexural vibration, respectively.

### VIRTUAL MASS $(\mu \Delta x)_v$

The virtual mass was computed using procedures given in References 4 and 5 and included the influence of the bilge keels. The effect of bilge keels on a vibrating ship is to increase the virtual mass coefficient $C$ of the hull relative to that obtained for a ship having no bilge keels.[4] The percentage increase in the virtual mass coefficient due to the bilge keels was determined from Figure 27 of Reference 4. This figure is applicable to a body having a square cross section vibrating in an infinite fluid or to a body having a submerged rectangular cross section vibrating in a semi-infinite fluid. Although the hull cross section of LONG BEACH is not rectangular, the hull is full-formed in the region over the length of the bilge keels, and hence does not depart significantly from a rectangular section.

### BENDING FLEXIBILITY $\Delta x/EI$

Area moments of inertia were obtained from inertia sections of the hull supplied by the Bureau of Ships (Plan SK 1669-11-612, Inertia Sections Fore and Aft). The influence of the superstructure on the moments of inertia at the various ship stations was neglected. However, an estimate of the influence of the first superstructure deck on the fundamental vertical flexural mode is presented in Appendix B. The area moments of inertia, $I_y$ and $I_z$, used to compute the vertical and athwartship hull bending flexibility, respectively, are shown for the various hull stations in Figure 3. Computations of the hull bending flexibility or compliance $\Delta x/EI$ (reciprocal of stiffness) of each $\Delta x$ hull section for vertical and horizontal flexural vibrations are summarized in Tables 2 and 3, respectively.

## SHEAR FLEXIBILITY $\Delta x/KAG$

The area $A$ used in the determination of the shear rigidity at various hull stations is the total cross-sectional area of all the continuous members crossing the section up to and including the main deck. The factor $KA$ was approximated by $A'G$(i.e., $K = A'/A$).[1] For vertical vibrations, $A'$ is the total cross-sectional area of all the vertical hull plating that runs continuously and consists of side-shell plating and continuous longitudinal bulkhead plating. For horizontal vibrations, $A'$ is the total cross-sectional area of all deck and bottom plating. The areas $A'$ were plotted against the station equivalence of the length of the ship, and a curve was faired through them to give an approximate area for each section for use in evaluating the shear rigidity $KAG$. Computations of hull shear flexibility $\Delta x/KAG$ of each hull $\Delta x$ section for vertical and horizontal flexural vibrations are summarized in Tables 2 and 3, respectively.

## TORSIONAL FLEXIBILITY $\Delta x/GJ_e$

The effective polar moment of inertia $J_e$ used in the determination of the torsional rigidity at various hull stations was found from[1]

$$J_e = \frac{4F_0^2}{\sum \frac{\Delta s}{\delta}} \qquad \text{[A-1]}$$

where $F_0$ is the enclosed area of the hull cross section at the station,

$\delta$ is the hull plating thickness, and

$\Delta s$ is the element of length of the plating enclosing the cross section.

This formula is based on the consideration of only a single outer shell consisting of shell plating and the main deck. The inner decks and inner bottom were neglected since the shear strains of the hull which sustain torsional forces are primarily carried by the outer shell. The values of the torsional compliance $\Delta x/GJ_e$ for the various hull stations are summarized in Table 4. The effect of the superstructure on the hull torsional flexibility was neglected in computing these values.

## MASS INERTIA $I_{mx}$ and $I_{mz}$

The mass inertias $I_{mx}$ and $I_{mz}$ were computed for the hull including the effect of the superstructure. These values were determined by using the procedures outlined in Reference 1. The estimation of $I_{mx}$ included consideration of inertia due to the ship structure, virtual mass, and cargo. The rotary inertia $I_{mz}$ was considered to be negligible for the first six flexural modes of hull vibration. This inertia was, however, considered in the coupled torsion-horizontal calculations. Values for $I_{mx}$ and $I_{mz}$ are summarized in Table 4.

## CENTER OF MASS ($\bar{z}$) AND CENTER OF SHEAR ($\bar{\bar{z}}$)

The center of mass $\bar{z}$ and center of shear $\bar{\bar{z}}$ were used in the computation of the coupled torsion-horizontal hull vibrations; the procedures are given in Reference 1. To calculate $\bar{z}$, the masses of the hull, superstructure, machinery, cargo, ballast fuel, and virtual mass were all considered. A precise determination of the shear center of a hull transverse section is quite difficult and hence the shear center is commonly assumed to coincide with the centroid or point of intersection of the natural axes of the cross section. Values of $m\bar{z}$ (where $m$ is the total mass of a hull section) and $\bar{\bar{z}}$ are summarized in Table 4.

## PROPULSION SYSTEM PARAMETERS

The mass and elastic parameters for the propulsion system consisting of propellers, shafting, and machinery were obtained from Reference 6. The port and starboard shafting was subdivided into equally spaced sections consisting of lumped masses connected by weightless springs having only longitudinal flexibility. The masses of the propellers and the machinery components were added at the ends of the shafting for the coupled shafting-hull vibration calculation. The schematic diagram for this calculation is shown in Figure 4. The port propulsion system was subdivided into seven approximately equal spaced sections, designated from Station 1/2′ to 6 1/2′ and connected to the hull at Station 8 1/2′. Likewise, the mass of the starboard propulsion system was subdivided into ten approximately equal spaced sections, designated from Station 1/2″ to 9 1/2″ and connected to the hull at Station 11 1/2. For the coupled shafting-hull vibration calculation, the propulsion system mass was subtracted from the hull mass. The mass and elastic characteristics of the propulsion-hull system are summarized in Table 5. Considerations involved in the representation of the coupled shafting-hull system on an electric analog are discussed in Appendix C.

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## APPENDIX B

## ESTIMATED EFFECT OF THE SUPERSTRUCTURE FIRST DECK ON THE FUNDAMENTAL VERTICAL HULL MODE

The superstructure of LONG BEACH, shown in Figure 1, is characterized by a long steel first deck extending for a distance of approximately 65 percent of ship length and a large aluminum forward deckhouse extending 86 ft above the main deck. A superstructure contributes to both the mass and elastic ship vibration parameters. The mass of superstructures is considered in ship vibration computations, but the elastic or geometric characteristics of the superstructure are not always considered. Because of a discrepancy between the computed and measured values of the fundamental vertical hull frequency, it was believed that the geometric characteristics of the superstructure first deck should be considered in the computation of the fundamental vertical hull mode. The forward deckhouse affects the computation of higher hull modes and is considered separately in Appendix D.

The method used to estimate the influence of the superstructure on the fundamental vertical hull mode follows the procedure presented in Chapter 8 of Reference 7. The frequency for the vertical 2-noded mode for ships without superstructures may be found from Todd's formula.

$$N = \beta \sqrt{\frac{BD^3}{\Delta_1 L^3}} \qquad \text{[B-1]}$$

where $\beta$ is a coefficient which depends on the ship type,

$B$ is the beam of the ship in feet,

$D$ is the depth of the ship to strength deck in feet,

$L$ is the length between perpendiculars in feet,

$\Delta_1$ is equal to $\Delta\left(1.2 + \frac{1}{3}\,\frac{B}{d}\right)$,

$\Delta$ is the ship displacement in tons,

$d$ is the draft in feet, and

$N$ is the frequency of vertical 2-noded mode in cycles per minute.

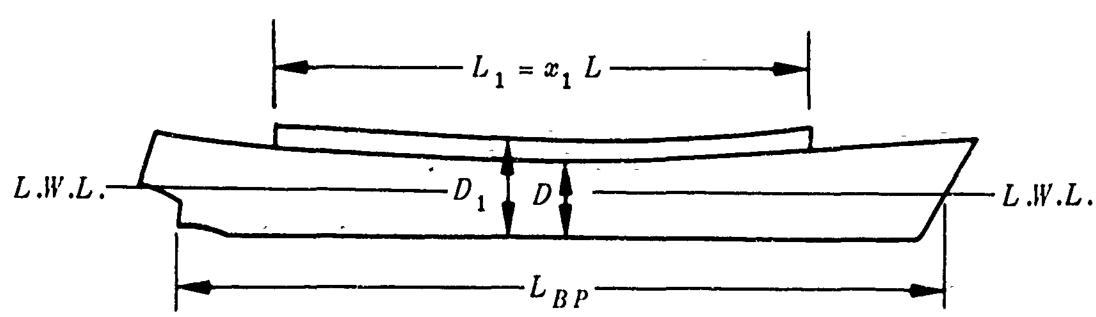
For a ship with one long superstructure of length $L_1$ and depth from the superstructure deck $D_1$, as shown in Figure B-1, an equivalent depth $D_E$ may be found from

$$D_E = \sqrt[3]{D^3(1-x_1) + D_1^3 x_1} \qquad \text{[B-2]}$$

where $x_1 = L_1/L$.

In order to obtain a value of the coefficient $\beta$ in Todd's formula, $N$ was set equal to 51 cpm, which was the frequency of the first vertical hull mode obtained from digital computations in which only the mass characteristics of the superstructure were considered. The equivalent depth $D_E$ of LONG BEACH was computed from Equation [B-2], with $D_1$ being the depth from the first superstructure deck. Using these values for $\beta$ and $D_E$, Equation [B-1] was solved to obtain a frequency of 55 cpm (0.92 cps) for the vertical 2-noded mode.

This frequency is approximately 8 percent larger than the frequency computed in which only the mass of the superstructure was considered, and the result compares more favorably with the measured fundamental vertical frequency of 60 cpm (1.0 cps).[2]




$$D_E = \sqrt[3]{D^3(1-x_1) + D_1^3 x_1}\ , \quad x_1 = L_1/L$$

Figure B-1 - Definition of Equivalent Depth $D_E$ for a Ship with one Superstructure Deck

## APPENDIX C

## DESCRIPTION OF COUPLED LONGITUDINAL SHAFTING-VERTICAL FLEXURAL HULL VIBRATION CALCULATION ON THE TMB ELECTRIC ANALOG (NETWORK ANALYZER)

The main propulsion machinery of LONG BEACH consists of two water-cooled nuclear reactors which supply steam to geared low and high pressure turbines. The turbines are connected to the port and starboard shafts and propellers by a reduction gear, and the thrust-bearings are located at the aft end of the gear cases. Propulsion machinery in Engineroom 1 powers the starboard shaft and the machinery in Engineroom 2 powers the port shaft, which is shorter in length. Mass and elastic characteristics of the propulsion system are given in Reference 6.

Propeller-excited vibratory forces on a ship are transmitted from the propeller through the shafting to the thrustbearing and its foundation and into the hull. The analysis of the longitudinal vibration of the propulsion system of LONG BEACH reported by Bethlehem Steel Company[6] had not considered the vibratory characteristics of the hull. Accordingly, in order to analyze the response of LONG BEACH to propeller-excited forces in a more complete manner, a calculation was performed on the TMB Network Analyzer in which the propulsion system was considered coupled to the hull. Similar analyses have been made in the study of submarine longitudinal vibrations.[8] Since the only deformations considered in the present calculation were longitudinal shafting deformation and hull bending and shear deformation in a vertical plane, the coupled propulsion-hull vibration analysis is referred to as a coupled longitudinal shafting-vertical flexural hull vibration calculation.

In developing the electrical model of the shafting-hull system of LONG BEACH, some assumptions were required in order to connect the network representing the hull vertical flexural characteristics with the networks representing the shafting longitudinal characteristics. The simplifying assumption was made that the hull is excited into vertical vibration by periodically varying moments acting on the hull at the stations where the shafting forces are transmitted to the hull, i.e., at the thrustbearing foundations. The exciting moments originate from longitudinal forces which are transmitted along the shafting and through a moment arm or "eccentricity" determined by the distance from the thrustbearing foundation to the neutral bending axis of the hull for vertical vibration of the ship. Because the port and starboard thrustbearing foundations are located at different hull stations and the "eccentricities" are correspondingly different, the response of the ship to identical forces acting through the port and starboard shafts would generally be dissimilar.

In order to construct the electric model for the calculation, the networks for the port and starboard propulsion systems were first established. As an initial step, the mass and elastic parameters reported in Reference 6 were converted to the parameters used in Model Basin shafting vibration calculations. In the vibration analysis given in Reference 6, each propulsion system was subdivided into three masses and two springs, and the first two resonant frequencies were predicted. For purposes of the coupled shafting-hull calculation, the propulsion system parameters were subdivided into smaller mass and stiffness elements to allow for a more complete analysis including prediction of mode shapes. These converted parameters are given in Table 5. Measurements reported in Reference 2 indicate good agreement with the shafting fundamental longitudinal modes predicted in Reference 6. Hence a basis existed for checking the frequency response of the port and starboard shafting networks.

The hull network was checked by comparing the frequency response of the network excluding the mass of the propulsion system with measured hull frequencies and with digital calculations in which the propulsion system mass was included. These frequencies compared very closely, which was expected, since the mass of the propulsion system is only a small percentage of the total ship mass.

In the construction of the complete electric model, each of the shafting networks was connected to the hull network before both shafting networks were simultaneously connected to the hull network. This procedure permitted a separate investigation of the effect of each propulsion system in exciting the various hull vertical modes.

The procedure for coupling the hull and shafting networks involved the use of transformers to couple the networks. Turns ratios of 2:1 and 9:1, corresponding to the previously defined eccentricity values, were used to couple the starboard and port shafting networks, respectively, to the hull. The electrical basis for the use of transformers to couple the networks is that in the design of the TMB Network Analyzer, described in Reference 1, mechanical forces and moments are both represented by electric currents. Hence, the conversion of a force into a moment can be modeled by modifying the current representing the force by a transformer.

The complete mobility analog circuit used to represent the coupled shafting-hull vibration of LONG BEACH is given in Figure 19. For this calculation, a dual transformer arrangement was used to simulate sinusoidal exciting forces applied to both shafts simultaneously. Since the magnitude of the propeller-excited forces was not known, no attempt was made to scale the input current to the coupled networks to represent the physical propeller-excited forces. However, the same magnitude of current was used to excite the various modes of coupled vibration.

The results of the coupled shafting-hull vibrations, given in Figures 14, 15, and 16, require some interpretation since the results are not self-explanatory. For purposes of interpreting the mode shapes obtained from the coupled calculation, it is useful to employ the normal mode influence relation.[9]

The normal mode influence relation states that the normal mode pattern determines the influence of the point of application of a simple harmonic driving force on the magnitude of the amplitude excited in that mode. For the coupled shafting-hull vibration analysis, it was assumed that a harmonic driving moment excites the hull into vertical vibration, and the corresponding amplitude governed by the influence relation is the angular displacement or slope in the vertical plane of motion. Thus, the influence relation for this analysis infers that a given exciting moment will not excite a hull vertical mode which has a nodal point in the normal mode profile of the hull slope at its point of application. Conversely, the influence relation indicates that an exciting moment will excite the maximum hull amplitude in a mode when applied at the points for which the slope is a maximum.

In order to apply the influence relation to the results of the coupled shafting-hull calculation, a qualitative tabulation of the hull vertical flexural response together with the nodal points of the hull normal mode slope profiles is shown in Table C-1.

Note that, in general, the hull is poorly excited into vertical vibration whenever either of the shafting terminals are located within one station of a node in the hull normal mode slope profile for vertical vibration, given in Figure 9. Comparison of the last three columns in Table C-1 also indicates generally that forces transmitted through the port shafting system govern the vibration response of the hull due to forces transmitted through both shafting systems. In other words, the hull response to forces acting through both shafting systems is never better than the response due to forces acting through the port shafting only. On the other hand, for the first and third hull vertical modes, the poor response of the hull due to excitation through the port shafting appears to reduce the relatively greater hull response because of excitation through the starboard shafting when both shafts are coupled to the hull. It appears from this qualitative argument that the vertical hull vibration response of LONG BEACH to propeller-excited forces is primarily governed by longitudinal forces transmitted through the port shafting.

## TABLE C-1

### Qualitative Results of Coupled Shafting-Hull Vibration Calculation

| Hull Vertical Flexural Mode | Location of Nodes in Normal Mode Slope Profile (Figure 9) | Location of Port Shafting Terminal | Location of Starboard Shafting Terminal | Hull Vertical Response Excited Through: | | |
|---|---|---|---|---|---|---|
| | | | | Starboard Shaft (Figure 14) | Port Shaft (Figure 15) | Port and Starboard Shafts (Figure 16) |
| 1 | Station 8 1 2– 9 | Station 8 1 2 | Station 11 1 2 | G | P | A |
| 2 | Station 5 1 2– 6<br>12 –12 1 2 | | | P | G | G |
| 3 | Station 4<br>9<br>14 –14 1 2 | | | G | P | A |
| 4 | Station 3<br>7<br>11<br>15 1 2 | | | A | G | G |
| 5 | Station 2 1 2<br>5 1 2– 6<br>9<br>12<br>16 –16 1 2 | | | P | G | G |
| 6 | Station 2<br>4 1 2<br>7 1 2<br>10 –10 1 2<br>13<br>15 | | | G | G | G |

A – Moderately excited
G – Well excited
P – Poorly excited

## APPENDIX D

## SPRUNG MASS CALCULATION OF THE SUPERSTRUCTURE FORWARD DECKHOUSE

When the amplitudes of vertical vibration at points lying in the same transverse plane are not the same, a local flexibility may be present. In dealing analytically with local flexibility, it has proven useful to consider the vibratory characteristics of an ideal beam with one or more masses elastically attached to it. The attached mass is called a "sprung mass," and the effect of a local flexibility is often termed the "sprung mass effect."

Reference 9 applies the term "sprung mass" to (1) local elastic structures, (2) relatively rigid assemblies that are supported in the hull by means of resilient mountings, and (3) heavy items of equipment that are installed on foundations nominally rigid but which in practice exhibit flexibility as a consequence of the large mass attached to them. It was considered quite possible that the forward deckhouse of LONG BEACH could be termed a sprung mass in the sense of the first application of this term.

The forward deckhouse, shown in Figure D-1, nas a weight of approximately 300 tons, which is about 2 percent of the design displacement of the ship. The deckhouse, which is principally of aluminum construction, is not heavy relative to the ship displacement. However, because of its large size and relatively flexible support, it was felt that a sprung mass analysis was in order. Furthermore, vibration measurements revealed the presence of a low order horizontal resonant frequency not predicted from the hull vibration calculation which considered the deckhouse as rigidly attached to the hull.

According to Reference 9, certain modifications of the beamlike vibratory response characteristics of a hull are caused by the sprung mass effect. Hull frequencies below the local natural frequency are lowered, and those above this frequency are raised. The effect of the local natural frequency on the hull frequencies decreases the farther the hull frequencies are from the local natural frequency. In addition, the sprung mass may introduce an extra mode into the vibratory characteristics of the ship. That is, there may be found in the ship's overall characteristics two modes with the same number of nodes in the displacement profile of the hull. In such a case, the phase relation between the displacement of the sprung mass and that of the hull in its vicinity will be reversed in these two modes.

The effect of local elasticities on the natural frequencies are explained in References 10 and 11 as follows. Let a mass $m$ be attached by a spring to point $P$ on a beam; see Figure D-2. The beam is free from external influences other than a force exerted on it by the spring. An oscillatory force $(-F_p)$ will act on the beam at $P$ and a force $(-F_p)$ will act on $m$ during any vibration of the combined system at a fixed frequency, whether this vibration is free or produced by a force action on $m$. The vibration of the beam can be regarded as a forced vibration maintained by the force $F_p$.

The unlabeled curves in Figure D-3 show the response ratio $Y_p/F_p$ of the beam plotted against $\omega$, with resonances indicated at $\omega_{n-2}$, $\omega_{n-1}$, $\omega_n$, $\omega_{n+1}$, and $\omega_{n+2}$ and antiresonances at $\omega_{a,\,n-1}$; $\omega_{a,\,n}$; $\omega_{a,\,n+1}$; and $\omega_{a,\,n+2}$.

During free vibration of the combined system, no other forces act on $m$, and its equation of motion is

$$m\,\frac{d^2Y_m}{dt^2} = -F_p = -m\omega_0^2\,(Y_m - Y_p) \qquad \text{[D-1]}$$

where $m\omega_0^2$ represents the spring constant of the spring connecting the sprung mass to the beam expressed in terms of $\omega_0$, which is the circular frequency for free vibration of $m$ when $P$ is held fixed. From this it follows that in any steady vibration at frequency $\omega$, during which $d^2y_m/dt^2 = -\omega^2 y_m$, then

$$\frac{Y_m}{Y_p} = \frac{\omega_0^2}{\omega_0^2 - \omega^2} \qquad \text{[D-2]}$$

and

$$\frac{Y_p}{F_p} = \frac{\omega_0^2 - \omega^2}{m\omega_0^2\,\omega^2} \qquad \text{[D-3]}$$

Four members, labeled $A$, $B$, $C$, and $D$, of the family of curves indicated by Equation [D-3] are shown in Figure D-3.

The qualitative effect of $m$ on the natural frequencies of the system can be seen from the relationship between the two sets of curves. All resonant frequencies of the beam alone are repelled by $\omega_0$; those beam frequencies $\omega_n < \omega_0$ are replaced by lower frequencies (in the case of Curve $A$, $\omega'_{n-1,\,A}$ replaces $\omega_{n-1}$); those frequencies $\omega_n < \omega_0$ are replaced by higher frequencies ($\omega'_{n,\,A}$ for $\omega_n$, $\omega'_{n+1,\,A}$ for $\omega_{n+1}$).

In addition, a new resonant frequency is added for the combined system, lying between $\omega_0$ and the adjacent antiresonant frequency of the beam alone (at $\omega'_{0A}$ between $\omega_{0A}$ and $\omega_{an}$ for Curve $A$, or at $\omega'_{0c}$ between $\omega_{0c}$ and $\omega_{a,\,n+1}$ for Curve $C$). If $\omega_0$ happens to equal an antiresonant frequency $\omega_a$, this new added frequency equals $\omega_0$, and $Y_p$ stands still during vibration in this mode, so that $m$ might be said to act as a local dynamic vibration absorber at $P$. If $\omega_0$ equals a resonant frequency, then $\omega_n$ for the system is replaced by two frequencies, either of which can be regarded as the new one ($\omega'_{n1}$ and $\omega'_{n2}$ in place of $\omega_n$ for Curve $B$).

If $m$ is small, the added mode is essentially one of local vibration of $m$. The $Y_p/F_p$ curve, as given by Equation [D-3], is quite steep for small $m$; see Curve $D$ in Figure D-3. It can be seen that because of the steepness of the curve, the new value $\omega'_{0D}$ is close to $\omega_{0D}$ and that $Y_m/Y_p$ (in Equation [D-2]) is quite large. Thus in this added mode, $m$ is vibrating almost as though the beam were stationary. Also, it can be seen that the effect of a small mass on any resonant frequency is slight.

For a relatively large $m$, the angular frequency $\omega'$ of the added mode lies close to an antiresonant $\omega_a$ of the beam. The conclusion may be made that in this mode, a large mass forces the beam to have a node in the vicinity of the mass. The beam frequencies near $\omega_0$ are strongly repelled when the mass is large.

Vibration measurements on LONG BEACH[2] indicated the presence of deckhouse horizontal and longitudinal resonant frequencies at about 3.3 cps and a vertical resonance at about 10 cps. The 3.3-cps horizontal frequency occurred between the second and third horizontal modes of the hull, and the 10-cps resonance occurred at a frequency higher than the sixth vertical hull mode. It was assumed that the longitudinal resonance of 3.3 cps lies below the fundamental longitudinal hull mode, which is generally too high in frequency to be of interest for vibration analyses of surface ships and therefore was not computed for LONG BEACH. Thus the horizontal resonance measured on the deckhouse was considered to be the most significant frequency with regard to the influence of the superstructure on the beam-like hull modes of LONG BEACH.

The 3.3-cps horizontal resonance was observed at several stations on the main deck of the hull in addition to the deckhouse. The horizontal hull frequencies predicted from calculations which considered the deckhouse as rigidly attached to the hull agreed well with the measured hull frequencies for the first five horizontal modes. However, sprung mass theory predicts the spreading of the hull frequencies adjacent to a sprung mass frequency (in this case, the second and third hull modes) which was not detected by comparison of the measurements with the calculations.

To determine the sprung mass effect of the deckhouse, a calculation was performed on the TMB Network Analyzer considering the deckhouse as flexibly attached to the hull at Station 10 1/2. The mobility analog used for this calculation is shown in Figure 20. Table 10 presents the mass and stiffness parameters of the hull for computing horizontal flexural vibration. The weight of the deckhouse was subtracted from that of the ship. A mass of 9.2 ton-sec$^2$/ft was subtracted from the mass parameter at Station 10 1/2. This mass was attached to the hull with assumed values for the spring constant $K$ of $0.280 \times 10^4$, $0.373 \times 10^4$, and $0.466 \times 10^4$ tons/ft.

The normal mode profiles for this calculation, shown in Figure 21, are presented for the value of $0.373 \times 10^4$ tons/ft for $K$, since the computed frequencies for this spring constant agreed best with the measured frequencies. The computations, which are summarized in Table 11, indicated that the hull modes adjacent to the sprung mass frequency of 3.26 cps are only slightly shifted as a result of the sprung mass effect. This result is principally due to the relatively small mass of the deckhouse compared with the hull mass.

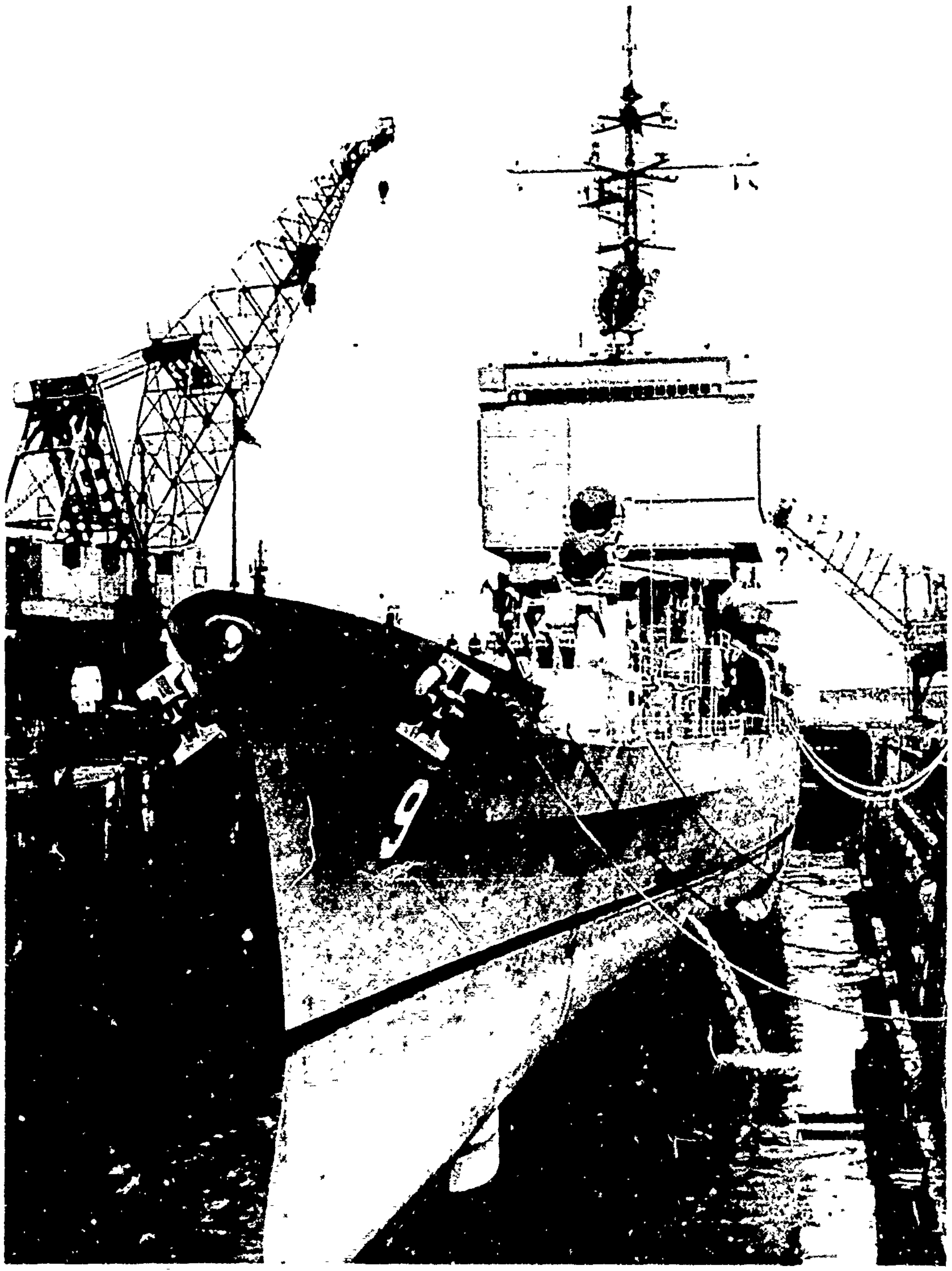


Figure D-1 – USS LONG BEACH Forward View, showing Superstructure Forward Deckhouse

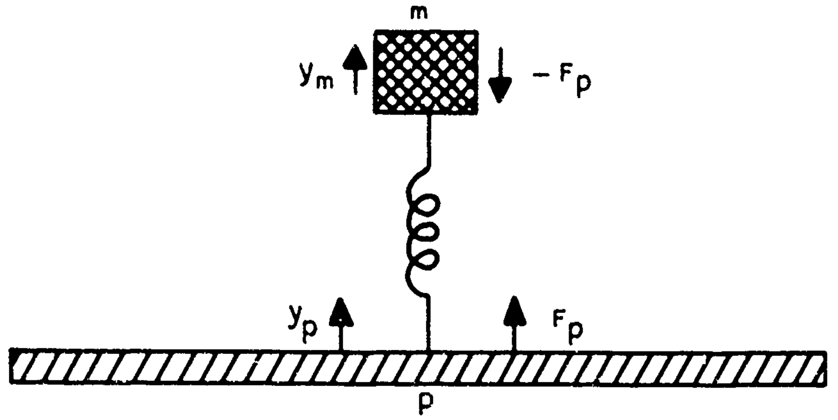



Figure D-2 – Beam with Attached Spring Mass System

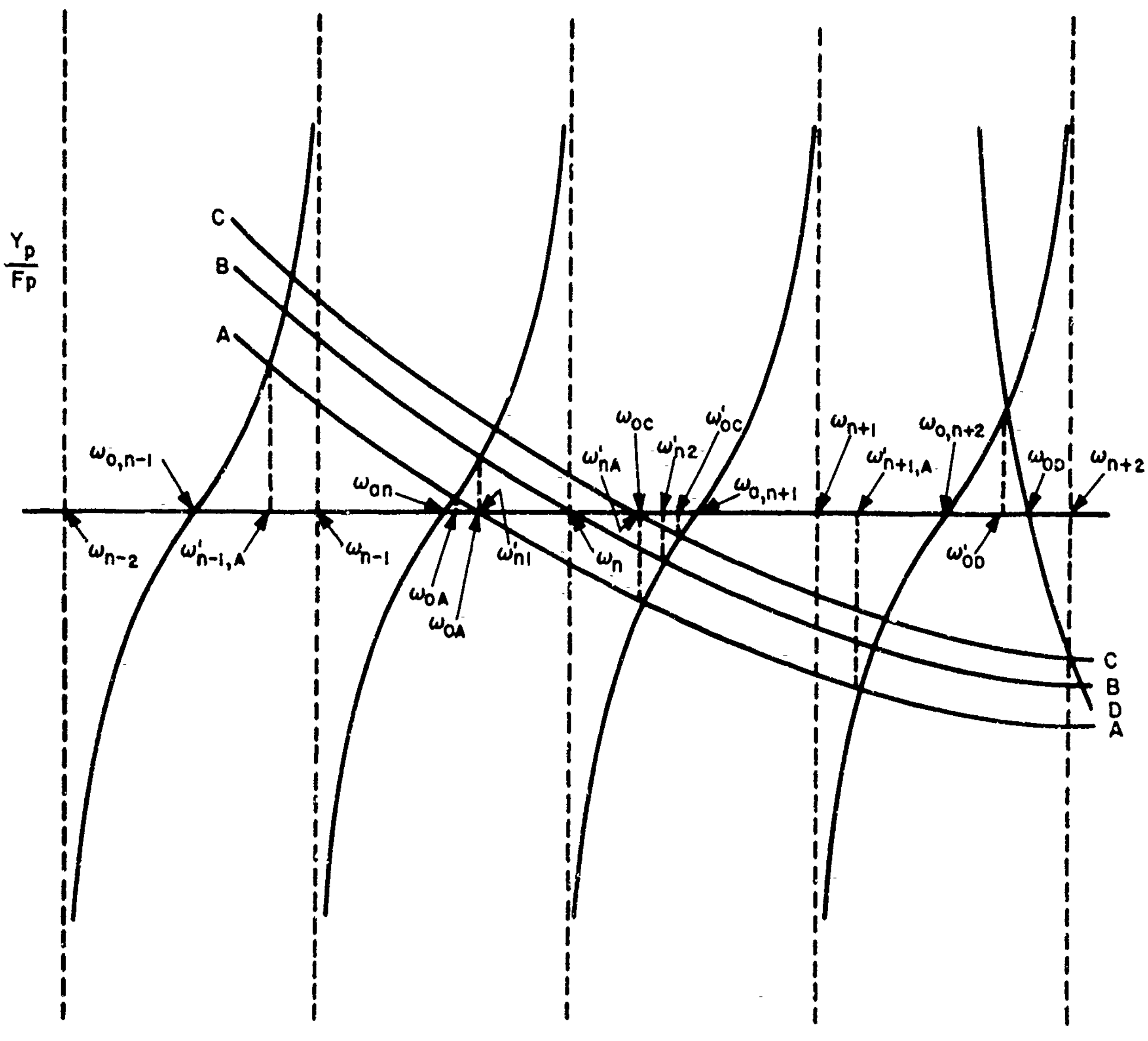



Figure D-3 – Diagram Illustrating the Effect of an Added Sprung Mass or Beam Frequencies

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13. ABSTRACT

Calculations were performed to determine the first six modes of vertical and horizontal flexural vibration, the first three modes of torsional vibration, and the first five modes of coupled torsion-horizontal bending vibration for the guided missile cruiser USS LONG BEACH (CG(N)9) at a displacement of 16,103 tons. A calculation coupling propulsion system longitudinal vibration with vertical flexural hull vibration was performed on an electrical analog computer in an attempt to simulate the excitation of flexural hull modes by vibratory forces transmitted longitudinally through the propulsion-shafting system. Flexural calculations of hull horizontal vibrations were performed with the large forward superstructure deckhouse considered both as rigidly and flexibly attached to the hull.

Included in the results are the hull response to a sinusoidal force of 1000 lb acting at the stern in a vertical direction. The vertical flexural modes and natural frequencies were computed using both the standard two-node Lewis virtual mass coefficient and a variable virtual mass coefficient. The parameters used for the calculations are discussed in detail. Results of the calculations are compared with measured hull frequencies.

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