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TESTING SHIP MODELS IN TRANSIENT WAVES

by

Lt. Cdr. M. C. Davis, USN
and
Ernest E. Zarnick

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TESTING SHIP MODELS IN TRANSIENT WAVES

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## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>ADMINISTRATIVE INFORMATION</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>PRELIMINARY THEORY</td>
<td>3</td>
</tr>
<tr>
<td>MATHEMATICS OF TRANSIENT TESTING</td>
<td>3</td>
</tr>
<tr>
<td>VISUALIZATION OF SHIP-WAVE SYSTEM</td>
<td>5</td>
</tr>
<tr>
<td>WAVE TRANSIENTS</td>
<td>7</td>
</tr>
<tr>
<td>TESTING TECHNIQUES</td>
<td>11</td>
</tr>
<tr>
<td>TESTING TECHNIQUE USING TRANSIENT WAVES</td>
<td>11</td>
</tr>
<tr>
<td>PROGRAMMING FOR TRANSIENT WAVE GENERATION</td>
<td>12</td>
</tr>
<tr>
<td>FOURIER ANALYSIS OF RECORDED TRANSIENTS</td>
<td>13</td>
</tr>
<tr>
<td>TEST RESULTS</td>
<td>13</td>
</tr>
<tr>
<td>TEST SERIES 1</td>
<td>15</td>
</tr>
<tr>
<td>TEST SERIES 2</td>
<td>20</td>
</tr>
<tr>
<td>TEST SERIES 3</td>
<td>25</td>
</tr>
<tr>
<td>AREAS OF CONTINUING DEVELOPMENT EFFORT</td>
<td>34</td>
</tr>
<tr>
<td>GENERAL</td>
<td>34</td>
</tr>
<tr>
<td>SPECIAL USE OF THE TRANSIENT TESTING TECHNIQUE</td>
<td>36</td>
</tr>
<tr>
<td>GENERAL APPLICATIONS OF THE TRANSIENT TECHNIQUE</td>
<td>37</td>
</tr>
<tr>
<td>SUMMARY AND CONCLUSION</td>
<td>38</td>
</tr>
<tr>
<td>APPENDIX - WEIGHTING FUNCTION OR IMPULSE RESPONSE OF WATER</td>
<td>40</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>43</td>
</tr>
</tbody>
</table>

## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>General Linear System Representation</td>
<td>6</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Wave Height Defined as an Input Signal</td>
<td>6</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Representation of Wave Height and Ship Motion as &quot;Effects&quot; rather than &quot;Cause and Effect&quot;</td>
<td>6</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Weighting Function of Unidirectional Waves in Water</td>
<td>14</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Transient Analyzer Configuration on Analog Computer</td>
<td>14</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>6</td>
<td>Measured Transients for Model 4941 and Typical Analog Computer Frequency Response Measurement</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>Heave Response for Model 4941 at Zero Speed in Head Waves</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>Pitch Response for Model 4941 at Zero Speed in Head Waves</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td>Heave/Pitch Ratio for Model 4941 at Zero Speed in Head Waves</td>
<td>19</td>
</tr>
<tr>
<td>10</td>
<td>Pitch Response for Model 4941 in Head Waves at a Froude Number of 0.09</td>
<td>19</td>
</tr>
<tr>
<td>11</td>
<td>Transient Voltage Excitation to Wavemaking System during First and Second Series of Model Tests</td>
<td>21</td>
</tr>
<tr>
<td>12</td>
<td>Typical Transient Recording Taken during Second Series of Model Tests</td>
<td>21</td>
</tr>
<tr>
<td>13</td>
<td>Heave Response for Model 4606 at Zero Speed in Head Waves</td>
<td>22</td>
</tr>
<tr>
<td>14</td>
<td>Pitch Response for Model 4606 at Zero Speed in Head Waves</td>
<td>22</td>
</tr>
<tr>
<td>15</td>
<td>Comparison of Wave Height Measurements Made with Two Probes with and without Model in Water</td>
<td>24</td>
</tr>
<tr>
<td>16</td>
<td>Heave/Pitch Ratio for Model 4606 at Zero Speed in Head Waves</td>
<td>24</td>
</tr>
<tr>
<td>17</td>
<td>Excitation Voltage Program for Third Series of Transient Tests</td>
<td>26</td>
</tr>
<tr>
<td>18</td>
<td>Heave Response for Model 4889 at Zero Speed in Head Waves</td>
<td>26</td>
</tr>
<tr>
<td>19</td>
<td>Pitch Response for Model 4889 at Zero Speed in Head Waves</td>
<td>27</td>
</tr>
<tr>
<td>20</td>
<td>Heave/Pitch Ratio for Model 4889 at Zero Speed in Head Waves</td>
<td>27</td>
</tr>
<tr>
<td>21</td>
<td>Phase Angle between Pitch-Wave Height, Heave-Wave Height, and Pitch-Heave at Zero Speed</td>
<td>28</td>
</tr>
<tr>
<td>22</td>
<td>Heave Response for Model 4889 in Head Waves at a Froude Number of 0.14</td>
<td>28</td>
</tr>
<tr>
<td>23</td>
<td>Pitch Response for Model 4889 in Head Waves at a Froude Number of 0.14</td>
<td>29</td>
</tr>
<tr>
<td>24</td>
<td>Heave/Pitch Ratio for Model 4889 in Head Waves at a Froude Number of 0.14</td>
<td>29</td>
</tr>
<tr>
<td>Figure 25</td>
<td>Phase Angle between Pitch-Wave Height, Heave-Wave Height and Pitch-Heave at a Froude Number of 0.14</td>
<td>30</td>
</tr>
<tr>
<td>Figure 26</td>
<td>Record of Transient Test Conducted with Model 4889 at Zero Speed in Head Waves</td>
<td>30</td>
</tr>
<tr>
<td>Figure 27</td>
<td>Transient Wave Form Measured at Various Locations in Basin, Third Series of Test</td>
<td>32</td>
</tr>
<tr>
<td>Figure 28</td>
<td>Transient Wave Form Measured at Various Locations in Basin, Second Series of Test</td>
<td>32</td>
</tr>
<tr>
<td>Figure 29</td>
<td>Model Generated Wave Height/Forced Pitch Ratio; Effect of Motion Generated Waves on Forward Wave Height Measurement</td>
<td>33</td>
</tr>
<tr>
<td>Figure 30</td>
<td>Comparison of Transient Wave Height Spectra Measured during Zero and Forward Speed Runs with Model 4889</td>
<td>33</td>
</tr>
<tr>
<td>Figure 31</td>
<td>Comparison of Transient Wave Height Spectra Measured with and without Model in Water at Zero Speed</td>
<td>35</td>
</tr>
<tr>
<td>Figure 32</td>
<td>Comparison of Methods of Analysis of Heave Response Operators</td>
<td>35</td>
</tr>
</tbody>
</table>
ABSTRACT

The seaworthiness characteristics of a ship design are often determined by a series of model tests in regular waves. This report describes a new model test procedure which makes use of a transient wave disturbance having energy distributed over all wave lengths of interest. With the use of this transient wave technique, the testing time required to characterize a model is reduced by an order of magnitude. In this report, the basic behavior of a unidirectional transient wave is discussed, and a simple Fourier transformation is developed to link wave height records measured at any two separated points along the path of such a wave. A particular form of transient wave, which is approximately sinusoidal with linearly varying frequency, has been used to test successfully a number of ship models. The results of these tests are presented and the practical problems in generating, measuring, and analyzing transient wave tests are discussed.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

The linear theory of ship motion in a random seaway has become generally accepted as a useful approximation to the actual nonlinear phenomena involved in ship-wave interaction. As outlined in the pioneering work of St. Denis and Pierson, the random sea surface can be visualized as a superposition of two-dimensional sinusoidal waves continuously distributed in amplitude, wave length, and direction. The total ship response in any degree of freedom is found from a summation of the responses to each individual wave component.

References are listed on page 23.
In providing information for quantitative full-scale motion prediction using this theory, the primary role of a ship model testing facility is to measure the response of a model to sinusoidal waves of unit amplitude over the entire range of ship speed, wave length, and direction of encounter. The Harold E. Saunders Maneuvering and Seakeeping Facility (MASK), located at the David Taylor Model Basin, is admirably suited to conduct such an investigation in head and oblique waves.

The scope of a complete model measurement program is without parallel in other engineering fields which perform frequency response testing of dynamic systems. A numerical example will illustrate the large number of tests which are required to characterize a model in regular waves. Assuming that the functions involved can be suitably approximated by tests at 10 wave lengths, 30-deg increments in direction from ahead to astern, and 5 speeds, 350 separate model tests are required, with measurement and analysis of a number of dynamic variables on each test. Such a program requires a major investment of time and money, and any techniques which can be developed to abbreviate the test time without technical compromise will reap high dividends.

It is clear that relative wave direction and model speed must remain fixed for any one test. However, under these constant conditions, a series of experiments in varying wave lengths is nothing more than a frequency response determination of a linear dynamic system, a common experiment in systems investigations for many engineering disciplines.

The frequency response characteristics of linear systems may be measured in three fundamental ways, that is, using sinusoidal, random, or transient excitations. The first two techniques are commonly employed in testing ship models. The third technique, using a transient water wave, is the subject of this report.

A transient wave will contain energy which, in general, is distributed over a range of wave lengths. Thus, a single motion test can yield information about the response of the ship at all wave lengths of interest. In the representative example just quoted, the number of tests required to characterize a ship design can be reduced from 350 to 35.
The following sections present the theoretical and practical aspects of transient wave testing. First, the basic mathematics of linear systems analysis are outlined. Then, the analytic peculiarities of the ship-wave system are discussed, stressing the fact that a wave is not properly an "input" as the term is usually understood. Next, the complex subject of unidirectional water-wave transients is treated in a simplified fashion by developing an expression which relates various wave-height time histories that might be recorded at various points along the path of the wave.

A particular wave which arises quite naturally from this analysis is the one which would, in theory, produce an impulse of infinite height and zero duration at some measurement point. An approximation to this theoretical waveform is quite easy to generate in a seaweeding facility; it has been extensively used at the Model Basin for model testing because of several attractive analytical properties.

Three sets of model tests are reported herein to support the theory and to illustrate some of the practical problems, especially those associated with the measurement of wave height, that can be anticipated in using transient waves to excite ship motion.

PRELIMINARY THEORY

MATHEMATICS OF TRANSIENT TESTING

Suppose that a linear dynamic system (such as shown in Figure 1) is under investigation, with an "input" $x(t)$ as the independent excitation and an "output" $y(t)$ as the dependent or forced variable. If $x(t)$ is a sinusoidal signal at a particular frequency, then, in general, $y(t)$ will asymptotically approach a steady-state sinusoidal response at the frequency. The ratio of the output amplitude to the input amplitude and the phase difference between output and input for all frequencies define the frequency response of this system, represented by the complex transfer function $G(j\omega)$, where $\omega$ is the frequency in radians per second.

When the system $G(j\omega)$ is at rest and a sudden transient $x(t)$ is applied at $t = 0$, then some response $y(t)$ will be measured, usually involving decaying transients. It is well known that a transient signal
can be decomposed into a continuous distribution of infinitesimal sinusoidal components with the aid of the Fourier transform. For example, the Fourier transform $X(j\omega)$ of a particular input signal $x(t)$ is given by the complex quantity

$$X(j\omega) = \int_{-\infty}^{\infty} dt \ x(t) \ e^{-j\omega t} \quad [1]$$

which represents the amplitude and phase of the incremental components at frequency $\omega$. Considering the output to be a summation of the response to each of the input frequencies, the well-known relation

$$Y(j\omega) = X(j\omega) \ G(j\omega) \quad [2]$$

gives a proper amplification and phase change to each of these components.

To summarize, the frequency response of a system $G(j\omega)$ can be found from a single transient experiment with input and output transforms $X(j\omega)$ and $Y(j\omega)$, respectively, with the relation

$$G(j\omega) = \frac{Y(j\omega)}{X(j\omega)} \quad [3]$$

In the transient testing of ship models, the input $x(t)$ is arbitrarily defined as the instantaneous amplitude of the undisturbed two-dimensional wave surface which would pass through the center of gravity of the model; see Figure 2. The output $y(t)$ is the time history of any one of the pertinent response variables, such as roll, pitch, or heave.

The use of a water wave input is the key distinction between transient tests of ship models and those conducted, for example, in control systems analysis where often a voltage is available for easy introduction of input transients.
The fact that the wave height referenced to the center of gravity of the model is defined as the input can lead to great mathematical difficulties. For example, in defining a linear system in the time domain, it is conventional to use a unit impulse as a standard input, which causes an "impulse response" whose Fourier transform is identical to the frequency response function. This is seen from Equation [2] where

\[ Y(j\omega) = G(j\omega) \]  

because the transform \( X(j\omega) \) of a unit impulse is unity.

Since it is impossible for a physical system to look into the future, to "laugh before it's tickled," the impulse response must be zero prior to \( t = 0 \) (when the impulse arrives). However, in the ship motion problem, there is no reason to believe that the inverse transform of an experimental frequency response will exist only in positive time. In fact, as will be shown later in this report, an "impulse" of wave height observed at \( t = 0 \) at the center of gravity of the model would be caused by the contraction of a wave train which had previously passed along the forward part of the ship, producing force on the hull and resultant motion in negative time. Thus, a more accurate description of the phenomena involved would be the general configuration pictured in Figure 3, where unidirectional wave height and ship motion are both viewed as responses to some undefined initial excitation, the mechanism which produces the waves. However, since wave height and ship motion are completely related in the sense that they respond to the single cause, it is proper to consider that ship motion can be related to wave height by the frequency relation

\[ Y(j\omega) = X(j\omega) \left\{ \frac{H_2(j\omega)}{H_1(j\omega)} = G(j\omega) \right\} \]  

where \( G(j\omega) \) is not properly the response of a physical system but the ratio of the frequency response of two physical systems.

With this philosophical restriction in mind, we will continue to call wave height an input and ship motion an output, but at no time will the intervening system be required to have the characteristics of a real physical system.
Figure 1 - General Linear System Representation

Figure 2 - Wave Height Defined as an Input Signal

Figure 3 - Representation of Wave Height and Ship Motion as "Effects" rather than "Cause and Effect"
WAVE TRANSIENTS

The study of transient waves on a free surface is an advanced topic in hydrodynamic theory, but it is amenable to the "systems" approach if linear wave behavior is assumed. Consider first that all waves are traveling in the same direction on a surface of infinite extent and in a fluid of infinite depth. Suppose further that a wave disturbance of finite energy per unit crest length has been traveling for all time and is observed at a single stationary point \( x_1 \). The wave height \( n(x_1, t) \) may be expressed in terms of its Fourier transform by the relation:

\[
  n(x_1, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \ N(x_1, \omega) \ e^{i\omega t} \tag{6}
\]

Following the technique of Stoker, the complex quantity \( N(x_1, \omega) \) is visualized as the infinitesimal wave component with frequency \( +\omega \). This wave at any instant of time extends over the entire plane and at any one point persists for all time. At another point \( x_2 \), which is a distance \( x \) along the direction of wave travel from \( x_1 \), this same infinitesimal wave is observed but with a phase lag of \( \omega^2 x / g \) radians, according to linearized theory. That is, the time history at \( x_2 \) is given by

\[
  n(x_2, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \ N(x_1, \omega) \ e^{-j\omega|x/g} \ e^{j\omega t} \tag{7}
\]

where the absolute value of \( \omega \) is used in the phase operator to ensure that the quantity

\[
  N(x_2, \omega) = N(x_1, \omega) \ e^{-j\omega|x/g} \tag{8}
\]

is conjugate with \( N(x_2, -\omega) \). This property is necessary in order that a Fourier transform represent a real function of time.
The operator $e^{-j\omega|x/g}$ can thus be viewed as the "transfer function" of water, or the frequency response function which relates wave heights measured at two points separated by a distance $x$ in the direction of travel.

To illustrate this result, suppose that

$$n(x_1, t) = \cos \omega_0 t$$  \[9\]

The Fourier transform of this wave height is given by

$$N(x_1, \omega) = \pi [\mu_0(\omega - \omega_0) + \mu_0(\omega + \omega_0)]$$  \[10\]

where $\mu_0(.)$ represents the unit impulse. The transform of $u(x_2, t)$ is given by

$$N(x_2, \omega) = \pi [\mu_0(\omega - \omega_0) + \mu_0(\omega + \omega_0)] e^{-j\omega|x/g}$$  \[11\]

We have for $n(x_2, t)$, then, the relation

$$n(x_2, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \pi [\mu_0(\omega - \omega_0) + \mu_0(\omega + \omega_0)] e^{-j\omega|x/g} e^{j\omega t}$$

$$= \frac{1}{2} (e^{-j\omega_0^2 x/g} e^{j\omega_0 t} + e^{j\omega_0^2 x/g} e^{-j\omega_0 t})$$  \[12\]

$$= \cos (\omega_0 t - \omega_0^2 x/g)$$

which is a well-known result from linear wave theory.

The remarks of the previous section apply to wave height pairs in the sense that the latter are both "effects" rather than "cause and effect." However, with knowledge of wave height at one point, the corresponding time history at another point can be determined by convolving the first wave height with the inverse transform of $e^{-j\omega|x/g}$ as is well known from linear systems analysis. This inverse transform is computed in the Appendix and yields the "weighting function" or "impulse response" of water.
\[ w(\tau) = a \cos \frac{\pi}{2} a^2 \tau^2 \left[ \frac{1}{2} + \frac{\tau}{|\tau|} C(\alpha \tau) \right] + a \sin \frac{\pi}{2} a^2 \tau^2 \left[ \frac{1}{2} + \frac{\tau}{|\tau|} S(\alpha \tau) \right] \]  \[13\]

where
\[ a = \left( \frac{g}{\pi x} \right)^{1/2} \]  \[14\]

and
\[ C(\alpha \tau) = \int_0^{\alpha \tau} dm \cos \frac{\pi}{2} m^2 \]
\[ S(\alpha \tau) = \int_0^{\alpha \tau} dm \sin \frac{\pi}{2} m^2 \]  \[15\]

This weighting function is shown in Figure 4 where
\[ w(\tau) \approx \sqrt{g/\pi x} \cos \left( \frac{2 \alpha \tau}{4x} - \frac{\pi}{4} \right) \]  \[16\]

as \( \alpha \tau \) becomes large. This function can be heuristically interpreted as a linear frequency sweep which looks back into the past of the wave height signal being processed and detects those frequency components that have gone by at a past time which would influence present wave height at a distance \( x \) in the direction of the wave. This is motivated by the convolution integral
\[ \eta(x_2, t) = \int_{-\infty}^{\infty} d\tau w(\tau) \eta(x_1, t-\tau) \]  \[17\]

which is the time domain equivalent of
\[ N(x_2, \omega) = N(x_1, \omega) e^{-j|\omega|x/g} \]  \[18\]
Suppose we ask the physically ridiculous but mathematically interesting question: "What signal would be observed at $x_1$ if a unit "impulse" of wave height were recorded at $x_2$?" A unit impulse is described as a signal which is zero except at an instant in time where it has infinite amplitude such that the integral over this point has a value of unity. The Fourier transform of a unit impulse is 1.0.

Solving Equation [18], we find that

$$N(x_1, \omega) = e^{j\omega |x|/g}$$

[19]

This can be readily shown to have an inverse transform which is equivalent to that shown in Figure 4 except for a reversal of the time variable.

Thus, if we observe a transient wave in the water which initially has a very high frequency (associated with slow velocity) and if this frequency linearly decreases toward zero with constant amplitude and tapers off as in Figure 4 with time reversed, then at some point in space and time, a very large wave would be created for a brief instant, assuming that linear wave theory holds. This phenomenon can be viewed as the simultaneous meeting of a large number of wave components whose individual speeds and starting times were properly adjusted so that the faster traveling waves were behind but catching up with the slower ones.

For the purposes of wave generation in a model-testing facility, it is manifestly impossible to provide a sinusoidal wave at infinite frequency. However, it is certainly possible to generate a wave train which has a frequency varying linearly from the highest desired value toward zero. Such waves have been the backbone of the Model Basin transient studies and will be described more fully along with experimental results in the following sections. Briefly, the linear theory appears to hold quite well, and in the early exploratory studies, very large peaked waves—approximating the impulse—were formed, although they were limited by cresting and other near mechanisms.

An important property associated with a transient water wave is that the magnitude of the Fourier transform remain constant regardless of.
where it is observed and when the origin in time is fixed; i.e., the water transfer function is solely a phase operator. For a pair of moving probes separated by a fixed distance \( x \), the same frequency response relation is applicable. However, transforms of wave height measured at nonzero speed are computed using the frequency-of-encounter time scale, where each wave length component corresponding to a stationary frequency \( \omega \) is measured at the frequency

\[
\omega_e = \omega + \frac{2v}{g}
\]

where \( v \) is the speed of the wave probes against the direction of wave travel. If a Fourier transform of a transient is computed for one wave height measurement, the companion wave measurement in the direction of wave travel will have the same magnitude at each frequency but the phase will be shifted by \( e^{-j\omega |x|/g} \) where \( \omega \) is the stationary frequency of the wave component concerned. For waves traveling in the same direction as the wave probes, ambiguities exist, and special techniques, which are beyond the scope of this report, must be employed.

The preceding treatment of transient water waves does not follow a conventional path in that spatial effects are suppressed and initial conditions or exciting forces on the water are not considered. If an instrument measures unidirectional wave height at some point for all time, then the time history at any other point is readily estimated through transform techniques. Even though a wavemaker may be generating the transient wave in the testing basin, so far as the height-measuring probe and the ship model are concerned, the wave is considered to be one that has been traveling forever on an infinite surface.

TESTING TECHNIQUES

TESTING TECHNIQUE USING TRANSIENT WAVES

Transient testing with a model in ahead waves is accomplished quite easily. As currently conducted, the first waves to be generated are slowly progressing high-frequency waves. When these first waves have traversed a portion of the test basin, the model is brought up to speed in calm water.
and measurement of all dynamic variables is commenced. The wave train passes, induces motion, and then the water and model return to the quiescent condition where recording is stopped.

Each time record is used to compute a Fourier transform from a common time base. The wave height transform must be corrected to the location of the model center of gravity by the transfer function $e^{-j\omega |\omega| x/g}$, where $x$ is the distance to the ahead wave probe and $\omega$ is the wave frequency. The ratio of motion transform to corrected wave height transform defines the transfer function, amplitude and phase for that motion.

PROGRAMMING FOR TRANSIENT WAVE GENERATION

The wavemaking system MASK, described recently by Brownell, is quite well suited for the generation of transient waves. Eight electro-hydraulic servosystems can be used to control the flow of air to and from domes along the shorter side of the basin, thus imparting energy to the water which travels away in the form of waves. These servosystems can be driven in unison by an electrical signal from either a low-frequency sine wave generator or a tape recorder.

The actuator servosystem has proved to be a considerable improvement over the previous electromechanical arrangement for wave generation, which provided a constant-amplitude, variable-frequency, sinusoidal excitation to the water. The actuator system can allow independent control of both amplitude and frequency, or it can introduce transient or random disturbances of more general form. Random wave generation has been described recently in Reference 4.

The transient waveform which has been used to date is characterized by a linearly decreasing frequency, starting at the highest frequency of interest for model testing-nominally 1.0 cps. The electrical signal that produces these waves is recorded on magnetic tape by the crude but effective procedure of linearly decreasing the frequency of a low-frequency sinusoidal source.

The frequency response characteristics of the basin, relating wave height to actuator motion, are such that frequency components near 0.4 to 0.5 cps are greatly amplified. Two modes of transient waves have been
used. One has the program amplitudes weighted so as to counteract this frequency behavior; the other has a constant amplitude with varying frequency.

FOURIER ANALYSIS OF RECORDED TRANSIENTS

The Fourier transform of a transient record is defined by the mathematical relation

$$F(j\omega) = \int_{-\infty}^{\infty} dt \, f(t) \cos \omega t - j \int_{-\infty}^{\infty} dt \, f(t) \sin \omega t \quad [21]$$

and is readily accomplished by digital computation or by special-purpose devices designed for this application.

For this exploratory investigation, it was decided to use a particular analog computer configuration which has interesting properties. A single channel is shown in Figure 5 where conventional analog computer symbols are used. As described in Reference 5, this undamped resonant circuit is driven by a transient input and oscillates as $t \to \infty$ at an amplitude corresponding to the magnitude of the transform of the input transient at the particular frequency $\omega$ and with a phase corresponding to the phase of the transform. A number of similar computer configurations, all adjusted to the same frequency, were driven simultaneously by the tape-recorded transients resulting from a particular experiment in order to maintain a common time base for phase measurements.

The use of this scheme for transient analysis was motivated by considerations of accessibility and operator control of the computations. However, other techniques will be employed for mass handling of transient records on an assembly-line basis.

TEST RESULTS

Tests of three different models are described in this report. Emphasis is placed on a correlation of transient wave results with regular wave results rather than on a complete description of the characteristics.
Figure 4 - Weighting Function of Unidirectional Waves in Water

Figure 5 - Transient Analyzer Configuration on Analog Computer
of any particular hull form. A chronological description is used to indicate the problems encountered and the progress obtained to date.

TEST SERIES 1

The first transient tests were conducted in December 1962 using Model 4941, a 13-ft model of a C4-S-A3 Mariner hull which had been previously tested in regular waves. Pitch and heave in ahead waves were measured as well as wave height with a sonic wave probe mounted approximately 12 ft abeam of the center of gravity of the model.

The philosophy of wave generation during this series of tests was to create a wave which would contract and "peak," as described in an earlier section, at a point somewhere near the model. Figure 6 shows the records taken during a typical test where the wave transient reached a peak height at the location of the stationary model. Along with these signals, the outputs of the analog computer circuitry (described in the previous section) are displayed for a particular frequency of analysis.

These sinusoidal outputs have magnitudes which are essentially equal to the magnitudes of the Fourier transforms of the respective signals up to that point. Although the variation of these amplitudes at the end of the transient has been a vexing problem with this method of transient analysis, sources of test error have been uncovered such as waves reflected from the sloping beach.

Another practical difficulty is the transient wave height record where the energy in the water does not decay rapidly after passage of the main signal. Fortunately, much of this disturbance is above the frequency range of interest.

A zero-speed transient test was analyzed at many different frequencies using the relative lull at the end of the passage of the main wave as the defined end of the transient. Figures 7 and 8 display the resulting heave and pitch frequency responses; they show good agreement with those of the regular wave tests. The only severe deviation is a sharp peak near 0.35 cps in both responses; it was caused by an unexplained sharp null in the measured wave height transform.

The heave/pitch ratio is plotted in Figure 9 for the zero-speed case. Although heave/pitch ratio plays no part in prediction of motion, it
Figure 6 - Measured Transients for Model 4941 and Typical Analog Computer Frequency Response Measurement
Figure 7 - Heave Response for Model 4941 at Zero Speed in Head Waves

Figure 8 - Pitch Response for Model 4941 at Zero Speed in Head Waves
is a ratio which has as much significance as any other in the isolated transient experiment since all three dynamic quantities are "effects," as previously discussed. This ratio has the virtue of being independent of errors in wave height measurement and has provided a useful index of the accuracy of the motion measurements throughout the series of transient tests.

A forward speed transient test was conducted at a speed corresponding to a Froude number of 0.09. Unfortunately, heave calibration was lost and only pitch results are valid, as given in Figure 10. These results show a moderate agreement with the regular wave pitch response.

In analyzing the results of these initial tests, it was felt that the transient technique had demonstrated considerable promise but that there were many possibilities for improvement.

First of all, it was recognized that there is no particular virtue in having a model at the point of contraction of the wave transient since the amplitude of an ideal wave transform is invariant with position. In fact, there is a strong possibility that nonlinear water or model behavior would be accentuated near the point of highest wave amplitude and that surge modulation effects would be significant. In addition, passing through a longer program before it coalesced would mean that more controlled energy could be imparted to the wave excitation, which, other things being equal, would lessen the effects of extraneous noise. And finally, the practical virtue of not having to conduct a precisely timed meeting of model and wave is still another motivation for altering and extending the duration of the wave transient.

A second major source of error was believed to be in the wave height measurement. Besides the previously mentioned test error attributed to wave reflections from the beach, there was a considerable possibility that waves generated by the model were being picked up by the side wave probe. Although there are some advantages in measuring a wave signal at its geometric reference point, they seem to be outweighed by the readily demonstrated fact that waves are generated much more efficiently by the model in the abeam direction rather than in the ahead direction.
Figure 9 - Heave/Pitch Ratio for Model 4941 at Zero Speed in Head Waves

Figure 10 - Pitch Response for Model 4941 in Head Waves at a Froude Number of 0.09
TEST SERIES 2

A second series of transient tests was conducted during January 1963; in these tests, an attempt was made to profit from the lessons learned in the initial tests. The model selected was a Series 60, Block 0.60 hull form, Model 4606. Heave and pitch were measured in ahead waves. Attention was focused on the zero-speed case in order to minimize the number of factors affecting the experiment.

During these tests, wave height was measured with two sonic probes. One was placed in the same location as in the previous tests, approximately 12 ft abeam of the model center of gravity, and the other was located 19 ft 2 in. ahead of the center of gravity.

The wave program employed was considerably longer than that used in the previous test series. A comparison of the excitation signals from both Series 1 and 2 (Figure 11) shows that the duration of the control voltage for the second test was doubled, that is, raised from 40 to 80 sec.

A typical transient test recording of this series is shown in Figure 12. Note that the forms of the wave height and motions differ considerably from those observed during the first tests; they resemble the varying frequency and amplitude characteristics that would be predicted by theory. One immediately obvious result is that the side wave measurement contains a peculiar null in its envelope which is not present in the ahead wave measurement, an anomaly which is most likely due to model-created waves generated to the side.

In Figures 13 and 14, respectively, frequency response operators obtained from four transient tests in heave and pitch are compared with regular wave measurements made during the same series of tests. Agreement seems to be quite good over most of the frequency range. The solid curves shown on these plots were computed on the basis of slender-body hydrodynamic theory by Newman for a roughly similar hull, Series 60, Block 0.70.

The Newman computation neglects everything but buoyancy. For many models tested at the Taylor Model Basin, the resonant frequencies of heave and pitch are sufficiently high so that in a zero speed test, the wave length components that produce significant pitch and heave motions act at frequencies which are considerably below resonance. The resulting motions
Figure 11 - Transient Voltage Excitation to Wavemaking System during First and Second Series of Model Tests

Figure 12 - Typical Transient Recording Taken during Second Series of Model Tests
Legend

- Regular Waves
- Transient Run 4
- Transient Run 6
- Transient Run 3
- Transient Run 5

Figure 13 - Heave Response for Model 4606 at Zero Speed in Head Waves

Figure 14 - Pitch Response for Model 4606 at Zero Speed in Head Waves
are essentially a wave force measurement or the response of the ship without ship dynamics. The comparison between computed and measured zero speed response is very impressive.

A detailed frequency analysis was conducted for one test shown in Figures 13 and 14, and wave height transform amplitudes were computed for the same wave program measured without the model in the water. This was done in order to remove the hypothesized effect of model-generated waves. A comparison of analyzed wave heights measured under these various conditions is presented in Figure 15 which shows a large percentage variation in the high-frequency range.

The heave/pitch ratio for these tests (Figure 16) is reasonably consistent except for some of the regular wave values. These deviations, together with Figure 13, lead to a strong suspicion that the heave measurements during the regular wave tests at 0.4 and 0.55 cps were low.

The results of this second series of transient tests added further experimental support for the utility of transient waves in ship model testing. In most cases, frequency response functions could be estimated with about 10 percent accuracy. By far the major source of error concerned the measurement of wave height, either through the corruption of the incoming wave with model-generated waves or through some nonlinear mechanism associated with water dynamics or wave measurement.

Several further improvements in test technique appeared feasible as a result of this second series. First, the use of an excitation voltage with varying frequency and constant amplitude resulted in a water wave having essentially the same frequency but an amplitude which reflected the sensitivity of the wavemaking system to certain frequencies. This type of excitation influences the magnitude of the wave height transform; see Figure 15 which shows these large variations over the frequency range. It was felt that preliminary amplitude weighting of the control voltage would counteract the known frequency characteristics of the basin and produce a wave with more uniform distribution of energy over the frequency interval used in testing. The result would be that (1) extraneous "noise" in wave measurement would be overridden by significant amplitude of the exciting wave and (2) transfer function computation would not involve the ratio of two rapidly varying transform amplitudes.
Figure 15 - Comparison of Wave Height Measurements Made with Two Probes with and without Model in Water

Figure 16 - Heave/Pitch Ratio for Model 4606 at Zero Speed in Head Waves
Another desirable feature of an adequate comparison between transient and regular wave testing was highlighted by the variations in heave transfer function measurement using regular waves (as observed in Figure 13). To minimize questions as to the accuracy of tests in regular waves, a large number of tests should be conducted throughout the frequency range—many more than are usually called for in routine testing.

TEST SERIES 3

The third and final series of transient tests to be presented in this report was conducted in April 1963, using Model 4889. Heave and pitch were again measured in ahead waves, and wave height was provided by two sonic probes mounted 12 and 20 ft ahead of the model center of gravity.

The program used in this series is shown in Figure 17. Based on the observed frequency behavior of previous transient tests, both amplitude and frequency sweep rate were varied so as to yield the proper cancellation of basin frequency response.

Regular and transient tests were conducted at zero speed and at a model speed corresponding to a Froude number of 0.14. The transfer function plots—heave, pitch, and heave/pitch ratio—for zero speed—are presented in Figures 18, 19, and 20, respectively. The corresponding phase data are presented in Figure 21. The agreement between the regular wave tests and the transient tests is impressive. Of special interest is the heave/pitch ratio, which, of course, is independent of wave-height measurement error. The agreement between regular and transient wave tests shown in these figures presents the strongest indication of the potential accuracy of the transient technique and, incidentally, of the linearity of motion response of a ship in waves. Note also that the pitch transfer function in Figure 19 demonstrates convincingly, through a close-spaced series of regular wave tests, the lack of smoothness of pitch response when examined in detail.

Good correlation was also obtained in the ahead speed results; see Figures 22, 23, 24, and 25.

The appearance of transient wave records taken during this series is somewhat different from that of earlier test records. Figure 26 shows the zero-speed test with a wave height record which appears to be distorted
Figure 17 - Excitation Voltage Program for Third Series of Transient Tests

Figure 18 - Heave Response for Model 4889 at Zero Speed in Head Waves
Figure 19 - Pitch Response for Model 4889 at Zero Speed in Head Waves

Figure 20 - Heave/Pitch Ratio for Model 4889 at Zero Speed in Head Waves
Figure 21 - Phase Angle between Pitch-Wave Height, Heave-Wave Height, and Pitch-Heave at Zero Speed

Figure 22 - Heave Response for Model 4889 in Head Waves at a Froude Number of 0.14
Figure 23 - Pitch Response for Model 4889 in Head Waves at a Froude Number of 0.14

Figure 24 - Heave/Pitch Ratio for Model 4889 in Head Waves at a Froude Number of 0.14
Figure 25 - Phase Angle between Pitch-Wave Height, Heave-Wave Height and Pitch-Heave at a Froude Number of 0.14

Figure 26 - Record of Transient Test Conducted with Model 4889 at Zero Speed in Head Waves
in contrast with the smooth quasi-sinusoidal wave behavior presented earlier in Figure 12. This is a result of one of the techniques which was used to vary the transform amplitude of the input voltage to the wave-makers, namely a varying frequency sweep rate which caused a nonuniform deformation of the wave shape.

The progressive transformation of this wave shape is shown in Figure 27, which presents measurements taken on the program at three locations. For contrast, similar measurements for the transient of the second series of tests are given in Figure 28, where the contraction of the shape is much more orderly. In both figures, waves were measured without a model in the water.

Another interesting test performed during this series employed a human transient generator. To investigate the degree of corruption of wave height recording by model-generated waves, the model was made to oscillate in pitch by manually forcing the bow over a range of frequencies. Generated waves were measured by the two forward wave probes and analyzed for their Fourier content, along with the motion record. The transfer functions, wave height/pitch motion, are given in Figure 29 where for the nearer probe, there was an average wave height of 1 in. for a pitch motion of 5 deg over the high-frequency band. The effect of heave is neglected.

The results of this final series of tests show clearly that the transient technique is a usable tool for the investigation of ship response to waves; however, further improvement is possible. When considering (1) the very close agreement between transient and regular heave/pitch ratios observed in Figures 20 and 24 and (2) the variation in the other frequency response estimates that is due solely to choice of forward or after wave height probe, an unwavering finger of suspicion points to the measurement of the dynamic wave disturbance.

Figure 30 compares the wave height transform of the zero-speed transient with that of the forward speed run properly transferred to the frequency scale of the stationary measurement. The general agreement is quite good, but the difference resulting from measurements made only 8 ft apart on the same wave-with forward and after probes-is puzzling.
Figure 27 - Transient Wave Form Measured at Various Locations in Basin, Third Series of Test

Figure 28 - Transient Wave Form Measured at Various Locations in Basin, Second Series of Test
Figure 29 - Model Generated Wave Height/Forced Pitch Ratio; Effect of Motion Generated Waves on Forward Wave Height Measurement

Figure 30 - Comparison of Transient Wave Height Spectra Measured during Zero and Forward Speed Runs with Model 4889
To weigh the possibility that this difference is associated with model-generated waves, Figure 31 displays the same wave height transform at zero speed compared with that of a similar measurement made under identical conditions except that the model was not in the water. The agreement is not impressive, and the wave transforms of the latter measurement do not correlate well, forward probe with after probe. The remaining sources of error in the wave height transform estimate have obviously not been isolated by these tests.

AREAS OF CONTINUING DEVELOPMENT EFFORT

GENERAL

The understanding and counteracting of the various factors leading to an inaccurate wave height measurement will be undertaken as the major effort in developing further the capability of the transient wave test. A series of tests in waves, with and without a model, is planned in order to investigate the error contributions from (1) nonlinear water dynamics, (2) nonlinear measurement by the wave height probe, (3) spatial variations in basin waves, (4) side or end reflections, (5) residual waves after passage of the main wave, (6) faulty Fourier analysis, (7) electronic interaction between adjacent wave probes, and (8) model-generated waves.

A second major goal will be the development of a program for a transient wave with linear sweep rate, constant amplitude over the frequency range of interest, and smooth starting and ending to minimize initial and terminal transients and the extraneous "noise" at the end of the wave train.

To assist in these investigations and to speed up data analysis, digital computer programs are being written for Fourier transform computations. An important part of these programs will be the "smoothing" of the transient records prior to transformation, or the multiplication of all time histories by a quantity which is unity over the duration of the test and eases to zero at the beginning and end of the test. This smoothing, standard in spectral analysis, will considerably reduce the effect of residual noise in the water near the end of a run.
Figure 31 - Comparison of Transient Wave Height Spectra Measured with and without Model in Water at Zero Speed

Figure 32 - Comparison of Methods of Analysis of Heave Response Operators
After more experience has been gained in forward speed runs, a critical analysis must be made to determine the effect of surge variations in transient analysis. This very knotty theoretical problem might require conducting wave transient tests at reduced wave height levels in order to avoid the time distortion of motion records.

The particular problems associated with tests in astern waves must be resolved. Unfortunately, a given frequency measured in the water with respect to the moving model can originate from astern waves at three different wave lengths. This ambiguity may force the use of transient waves with energy in more limited frequency bands, the use of multiple wave probes to make use of phase information, or both.

SPECIAL USE OF THE TRANSIENT TESTING TECHNIQUE

In one interesting use of a transient test, the variance of all motions in a given unidirectional random seaway can be found directly without need for frequency analysis. To show how this can be done, consider the equation which relates mean square motion \( \overline{m^2} \) to the wave power spectral density at the frequency of encounter \( \Phi(\omega) \) and to the applicable transfer function \( G(\omega) \):

\[
\overline{m^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, \Phi(\omega) \, |G(\omega)|^2 \tag{22}
\]

In a transient test, the integral square motion is given by

\[
\int_{-\infty}^{\infty} dt \, m^2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, |N(\omega)|^2 \, |G(\omega)|^2 \tag{23}
\]

where \( N(\omega) \) is the Fourier transform of the measured wave height, using a well-known relation from linear systems theory. Thus, if \( N(\omega)^2 \) is programmed to be equal to the wave height spectrum \( \Phi(\omega) \), we have
Simple analog data processing, conducted during the model test, would square and integrate the motions of interest and, at the end of the run, would yield voltages proportional to motion variances in the defined random seaway.

Although such a simplified scheme of data processing would not extract much of the significant information available from a transient test, it is conceivable that there might be occasions when a very fast answer would be required regarding the seaworthiness characteristics of a ship form in a given seaway. One example would be a search for the worst combination of speed and heading.

GENERAL APPLICATIONS OF THE TRANSIENT TECHNIQUE

The method for producing a transient water wave described in this report was developed in order to take account of the behavior of waves on a free surface. The form of this wave, however, would appear to have considerable promise for applications in many linear systems investigations.

Conventionally, a pulse-like transient is used for system excitation. The frequency range of the excitation is determined primarily by the concentration of the transient about a single point in time, and the amount of signal needed to produce measurable response is a function of the amplitude levels of the pulse. As a consequence, the faithful reproduction of a rapidly changing signal imposes several measurement requirements. Moreover, the system under test will probably exhibit nonlinear behavior because of the large inputs often required to override the effects of measurement noise in the recorded output signal.

The use of a signal with linearly varying frequency and constant amplitude as an input signal removes these strong drawbacks to the pulse technique, however. Since it has constant amplitude, the input can be constrained to lie within the linear range. With proper choice of sweep
rate and starting frequency, the controlled signals can have any desired energy level in each frequency band and thus defeat to a great extent the effects of random measurement noise.

Another strong advantage in the use of transients with a linear sweep rate is that in many cases, the entire transfer function of a system can be obtained by cursory analysis of the transient records alone. If the rate of change of frequency and amplitude is slow enough, the signals involved behave very much like sinusoidal waves. From the integration method of stationary phase, it can be shown that the amplitude of the transform at frequency $\omega$ of such a signal is equal to one-half of the single amplitude of the signal at the apparent local frequency $\omega$ divided by the square root of the rate of frequency change (in cycles per second). Such a computation was performed for the heave measurement of the transient test shown in Figure 13. The results of this computation for the heave transfer function are shown in Figure 32; there is good agreement between results obtained by the measurement and the calculation techniques.

**SUMMARY AND CONCLUSION**

The complete determination of the response of a ship in regular waves involves a large and expensive testing program. When the transient wave technique is used properly, the total testing time can be reduced by an order of magnitude.

In a theoretical discussion of ship dynamics, it has been stated that the usual systems representation of ship motion as an "output" and wave height as an "input" is a misconception; both dynamic quantities are "output." Water wave transients traveling in a single direction can be readily analyzed with the use of Fourier transforms; the transforms of two measurements on a single wave transient are related by the so-called "transfer function" of water $e^{-j\omega x/g}$, where $x$ is the distance separating the measurement points in the direction of wave travel.

The wave used at the Model Basin for transient testing has a continuously increasing wave length which results in an intense concentration of wave energy for a short period of its travel. Model transient tests are commenced in calm water, then passed through a wave having energy in
all frequencies of interest, and eventually returned to the smooth-water condition. The transfer function for a particular motion variable is found for all frequencies by dividing the Fourier transform of the motion transient by that of the wave height record, referenced to the model center of gravity. With a suitably tailored wave transient, mean square motion levels in a particular random seaway can be found immediately by squaring and integrating the motions during a transient test.

Transient tests conducted on three models in ahead waves have verified the theory presented. Close agreement between transient and regular wave tests has been obtained for heave and pitch motions at zero and forward speed. The major difficulty encountered has been in the generation and measurement of waves, and further refinements and research are proposed. Digital programs are being written for the bulk processing of transient records.

Finally, the technique of using a transient excitation which is a linear frequency sweep is an original contribution to general linear systems analysis; it has virtues of linearity, noise-immunity, and the capability of determining frequency response of a system by visual inspection of the transient records.
APPENDIX
WEIGHTING FUNCTION OR IMPULSE RESPONSE OF WATER

It has been shown in this report that the operator $e^{-j\omega |x|/g}$ is the frequency response function that relates wave height measured at two points separated by a distance $x$ in the direction of travel. The weighting function $w(\tau)$ can be determined by taking the inverse Fourier transform of the frequency response function:

$$w(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-j\omega |x|/g} e^{j\omega \tau} d\omega$$  \[25\]

Expanding into trigonometric terms and noting the symmetrical properties of the function, we have after simplification

$$w(\tau) = \frac{1}{\pi} \int_{0}^{\infty} \cos (-\omega^2 x/g + \omega \tau) d\omega = \frac{1}{\pi} \int_{0}^{\infty} \cos (\omega^2 x/g - \omega \tau) d\omega$$  \[26\]

Employing the technique used by Lamb\(^7\) we let

$$\zeta = \frac{x^{1/2}}{g^{1/2}} \left( \omega - \frac{g \tau}{2x} \right)$$

and

$$\sigma = \frac{g^{1/2} \tau}{2x^{1/2}}$$

These terms are substituted into Equation [26], yielding

$$w = \frac{1}{\pi} \frac{g^{1/2}}{x^{1/2}} \int_{-\sigma}^{\infty} \cos (\zeta^2 - \sigma^2) d\zeta$$
\[= \frac{1}{\pi} \frac{1}{x^{1/2}} \left[ \int_{-\sigma}^{\sigma} \cos (\zeta^2 - \sigma^2) \, d\zeta + \int_{0}^{\infty} \cos (\zeta^2 - \sigma^2) \, d\zeta \right]\]

\[= \frac{1}{\pi} \frac{1}{x^{1/2}} \left\{ \cos \sigma^2 \left[ \int_{-\sigma}^{0} \cos \zeta^2 \, d\zeta + \int_{0}^{\infty} \cos \zeta^2 \, d\zeta \right] \right\} + \sin \sigma^2 \left[ \int_{-\sigma}^{0} \sin \zeta^2 \, d\zeta + \int_{0}^{\infty} \sin \zeta^2 \, d\zeta \right] \quad [27] \]

We can make the following identification:

\[\int_{-\sigma}^{0} \cos \zeta^2 \, d\zeta = \int_{0}^{0} \cos \zeta^2 \, d\zeta = \frac{\sigma}{|\sigma|} \sqrt{\pi/2} \, C(\mu);\]

\[\int_{-\sigma}^{0} \sin \zeta^2 \, d\zeta = \int_{0}^{0} \sin \zeta^2 \, d\zeta = \frac{\sigma}{|\sigma|} \sqrt{\pi/2} \, C(\mu);\]

\[\int_{0}^{\infty} \sin \zeta^2 \, d\zeta = \int_{0}^{\infty} \cos \zeta^2 \, d\zeta = 1/2 \, \sqrt{\pi/2}\]

where

\[C(\mu) \triangleq \int_{0}^{\mu} \cos \left( \frac{1}{2} \pi \mu^2 \right) \, d\mu; \quad S(\mu) \triangleq \int_{0}^{\mu} \sin \left( \frac{1}{2} \pi \mu^2 \right) \, d\mu\]
Substituting the above into Equation [27] yields

\[ w = \left( \frac{g}{2\pi} \cos \omega^2 \right)^{1/2} \left\{ \cos^2 \left[ \frac{\omega^2}{2} + \frac{\alpha}{|\alpha|} C \left( \sqrt{\frac{2}{\pi}} \cdot \sigma \right) \right] + \sin^2 \left[ \frac{\omega^2}{2} + \frac{\alpha}{|\alpha|} S \left( \sqrt{\frac{2}{\pi}} \cdot \sigma \right) \right] \right\} \]

For simplification, let

\[ a = \left( \frac{g}{2\pi} \right)^{1/2} \]

Then

\[ \sigma = \sqrt{\frac{2}{\pi}} \cdot a \tau \]

and

\[ w(\alpha \tau) = a \left\{ \cos \frac{\pi}{2} a^2 \tau^2 \left[ \frac{1}{2} + \frac{\tau}{|\tau|} C(\alpha \tau) \right] + \sin \frac{\pi}{2} a^2 \tau^2 \left[ \frac{1}{2} + \frac{\tau}{|\tau|} S(\alpha \tau) \right] \right\} \]

[28]

For large values of \( a \) or \( \sigma \), going back to Equation [27], we find that

\[ w(\alpha \tau) \approx a \left\{ \cos \frac{\pi}{2} a^2 \tau^2 + \sin \frac{\pi}{2} a^2 \tau^2 \right\} \]

\[ = \sqrt{2} a \cos \left( \frac{\pi}{2} a^2 \tau^2 - \frac{\pi}{4} \right) \]

\[ = \sqrt{g/\pi} \cos \left( \frac{\pi}{4} \cdot \frac{\tau^2}{4} - \frac{\pi}{4} \right) \]

[29]

\( \tau \gg a \)
REFERENCES


**Abstract**

The seaworthiness characteristics of a ship design are often determined by a series of model tests in regular waves. This report describes a new model test procedure which makes use of a transient wave disturbance having energy distributed over all wave lengths of interest. With the use of this transient wave technique, the testing time required to characterize a model is reduced by an order of magnitude. In this report, the basic behavior of a unidirectional transient wave is discussed, and a simple Fourier transformation is developed to link wave height records measured at any two separated points along the path of such a wave. A particular form of transient wave, which is approximately sinusoidal with linearly varying frequency, has been used to test successfully a number of ship models. The results of these tests are presented and the practical problems in generating, measuring, and analyzing transient wave tests are discussed.
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