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EFFECTIVENESS OF IMPERFECT DECOYS

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OPERATIONS EVALUATION GROUP
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ABSTRACT

Mathematical relations are developed to express the effectiveness of decoys in a situation which requires the attacker to assign his weapons on the basis of imperfect classification of the targets.

The analysis considers the effects of a number of pertinent parameters, such as the number of real and decoy targets and the enemy's classification ability and missile effectiveness, and calculates the probabilities of survival of real targets. A number of hypothetical examples are given to illustrate the application of the theory to practical problems.

Introduction

Alternative concepts for passive defenses of military objectives against enemy attack in the broadest sense include

- hardening
- mobility
- dispersion
- deception

Among these, deception presents perhaps the widest range of possibilities and the greatest challenge to the imagination of the military planner.

This paper deals with one facet of military deception, namely, the use of decoys to cause the attacker to assign his weapons incorrectly, and thereby increase the survivability of the real objectives. Mathematical relations are developed to express the effectiveness of decoys in a situation which requires the attacker to assign his weapons on the basis of imperfect classification of the targets. Typical applications include naval anti-air warfare, defense of missile sites against enemy bombers, and protection of striking warheads from defensive anti-missile missiles.

The analysis considers the effects of a number of pertinent parameters, such as the number of real and decoy targets and the enemy's classification ability and missile effectiveness, and calculates the probabilities of survival of real targets. A number of hypothetical examples are given to illustrate the application of the theory to practical problems.

Measures of Decoy Effectiveness

Since the purpose of employing decoys, in the final analysis, is to increase the survivability of "reals," a logical measure of decoy effectiveness is the drop in any appropriately sensitive measure of attack effectiveness due to the introduction of decoys. Three such measures of attack effectiveness to be discussed here are:

- probability of correct classification;
- expected number of "reals" attacked and of weapons assigned to each per "real" attacked;
- probability of survival of "reals."

Each of these measures is derived from the preceding one. The listing is thus in order of both increasing complexity and increasing applicability to an actual operational problem. It is possible that the assumptions used in the derivations do not hold for a specific problem at hand. In such case, a more appropriate measure of effectiveness may have to be derived from one of those listed above.

Symbology and Assumptions

The following symbols and general assumptions have been used in constructing the models:

Definition of Symbols

C	-	total cost of "reals" and decoys
c	-	fractional cost of decoy (cost of decoy divided by cost of "real")
D	-	number of decoys
p_d	-	probability of a decoy being classified as "real"
p_r	-	probability of a "real" being classified as "real"
p_h	-	one-shot hit probability against a "real" given one assignment
p_{ki}	-	cumulative conditional kill probability against a "real" given i hits
p_k	-	one-shot conditional kill probability against a "real" given one assignment
R	-	number of "reals"
S	-	number of "reals" surviving
W	-	total number of weapons assigned by enemy
w	-	number of weapons assigned to each "real" attacked

p_r, p_d, p_k (or p_h) are all assumed independent and invariant with time or outcome for any given problem. Both "reals" and decoys are assumed to be disposed at random.

Probability of Correct Classification

The method of calculating the probability of correct classification is a function of the attacker's investigation doctrine, among other factors. If the attacker investigates the targets either more or less simultaneously, or in sequence (but without knowing in advance the actual number of "reals,") he ends up looking at all targets before selecting targets for attack. If, on the other hand, the attacker investigates the targets in sequence, knowing in advance the actual number of "reals," he can stop looking as soon as he has classified that number of targets as "reals."

The probability of correct classification will be defined here as the probability that R' of the R reals are classified as "reals" and D' of the D decoys as decoys. R' and D' are arbitrary integers smaller than or equal to R and

D, respectively. Under the first or "simultaneous" investigating doctrine, this probability is the product of the probabilities of the classification outcomes for each target and the number of ways in which the particular combination of outcomes can be obtained.

The over-all expression for the probability that R' "reals" and D' decoys are classified correctly is thus:

$$P_{R'D'} = \binom{R}{R'} p_r^{R'} (1 - p_r)^{R-R'} \binom{D}{D'} (1 - p_d)^{D'} p_d^{D-D'} , \qquad (1)$$

where p_r and p_d are the probabilities that a "real" and a decoy, respectively, will be classified as a "real." In particular, the probability of completely correct classification is given by

$$P_{RD} = p_r^R (1 - p_d)^D , \qquad (1')$$

and the probability of completely incorrect classification by

$$P'_{RD} = (1 - p_r)^R p_d^D . \qquad (1'')$$

The "sequential" investigating doctrine requires the attacker to know the exact number of "reals." Here the probability of correct classification is a function of the sequence in which the targets are investigated. For any given sequence of "reals" and decoys, the probability of completely correct classification now contains the basic element $p_r^R (1 - p_d)^i$, where i is the number of decoys investigated before the last "real."

The number of different sequences in which i decoys can be investigated before the last "real" appears is the number of possible permutations of R - 1 + i objects taken R - 1 or i at a time and is given by

$$\binom{R-1+i}{i} = \frac{(R-1+i)!}{i!\,(R-1)!} .$$

The total number of different sequences of R "reals" and D decoys is the number of possible permutations of R+D objects taken R or D at a time and is given by

$$\binom{R+D}{R} = \frac{(R+D)!}{R!\,D!} .$$

Consequently, the expression for the probability of correct classification of all "reals" is

$$P_R = \frac{1}{\binom{R+D}{R}} p_r^R \sum_{i=0}^{D} \binom{R-1+i}{i} (1 - p_d)^i . \qquad (2)$$

Number of "Reals" Attacked

A more practical measure of decoy effectiveness, which will now be derived from the probability of correct classification, is the number of "reals" attacked and the number of weapons per "real" attacked. The development will be restricted to the "simultaneous" investigating doctrine. It will be assumed that the attacker knows that decoys may be present and that he assigns the same number of weapons to each target classified as "real."

Let us note at the outset that the presence of decoys has actually a dual effect. The decoys which are classified as "reals" attract a fraction of the attacker's weapons, but at the same time, some "reals" which are classified as decoys are assigned no weapons at all. The extents of these two effects vary with the combination of classification outcomes and will be considered separately.

The expected number of weapons assigned to each "real" attacked (or equivalently, to each target attacked), $\bar{w}$, is obtained from equation (1) by setting $R' = i$ and $D' = D - j$, where i "reals" and j decoys are classified as reals, and $i+j \neq 0$:

$$\bar{w} = W \sum_{i=0}^{R} \sum_{j=0}^{D} \frac{1}{i+j} \binom{R}{i} \binom{D}{j} p_r^i (1 - p_r)^{R-i} p_d^j (1 - p_d)^{D-j}, \tag{3}$$

while the expected number of "reals" not attacked is given by

$$\bar{R}_{na} = R - \sum_{i=0}^{R} i \binom{R}{i} p_r^i (1 - p_r)^{R-i} . \tag{4}$$

For values of R and D commonly encountered, equations (3) and (4) are cumbersome to evaluate. An approximate solution can be obtained in terms of $p_r R$, the average number of "reals" attacked, and $p_r R + p_d D$, the average number of targets attacked. In this formulation, the average number of weapons assigned to each "real" attacked and the average number of "reals" not attacked are expressed respectively by

$$\bar{w}' = W \frac{1}{p_r R + p_d D} \tag{3'}$$

and

$$\bar{R}'_{na} = (1 - p_r) R . \tag{4'}$$

Equations (3) and (3') assume that the number of weapons is sufficient to attack the required number of targets.

Survivability

Once the expected number of weapons assigned to each "real" has been determined, we can turn to the more meaningful concept of survivability of "reals."

Survivability of a "real" is a function of the number of weapons assigned to it and the conditional kill probability of a weapon against a "real." The latter quantity is defined here as the probability that a weapon assigned to a "real" will kill that "real."

If the kill probability of a weapon is completely independent of the effects of previous weapons, then the probability that an attacked "real" survives w weapon assignments is given by

$$P_{as} = (1 - p_k)^w; \tag{5}$$

the probability that any given "real" survives is given by

$$P_{1s} = p_r (1 - p_k)^w + (1 - p_r) ; \tag{6}$$

the probability that all "reals" survive is given by

$$P_{Rs} = \left[p_r (1 - p_k)^w + (1 - p_r)\right]^R ; \tag{7}$$

the probability that exactly N "reals" survive is given by

$$P_{Ns} = \binom{R}{N} \left[p_r (1 - p_k)^w + (1 - p_r)\right]^N \left[p_r - p_r (1 - p_k)^w\right]^{R-N} ; \tag{8}$$

the expected number of surviving "reals" is given by

$$\overline{S} = \sum_{N=0}^{R} N P_{Ns} = RP_{1s} = p_r R (1 - p_k)^w + (1 - p_r) R, \tag{9}$$

and the corresponding standard deviation by

$$\sigma(S) = \left[R P_{1s} (1 - P_{1s})\right]^{1/2} = \left\{R\left[p_r (1 - p_k)^w + (1 - p_r)\right]\left[p_r - p_r (1 - p_k)^w\right]\right\}^{1/2}. \tag{10}$$

An approximate expression for the expected number of surviving "reals" can be obtained by a simpler approximation technique, whenever the product $p_k w$ is small enough. If this is the case, the average number of all kills (both "reals" and decoys) is approximately $p_k W$ and the approximate average number of "reals" killed is

$$\bar{K} \cong p_k W \frac{p_r R}{p_r R + p_d D} . \tag{11}$$

The approximate average number of surviving "reals" includes both the survivors among the $p_r R$ "reals" which were attacked and the $(1 - p_r)R$ "reals" which were not:

$$\bar{S} \cong p_r R - p_k W \frac{p_r R}{p_r R + p_d D} + (1 - p_r) R = R - p_k W \frac{p_r R}{p_r R + p_d D} . \tag{9'}$$

This expression can be obtained from equation (9) by neglecting quadratic and higher terms in the binomial series expansion. The approximation is fairly accurate when $p_k w \leq 0.1$.

When the damage inflicted by successive hits is cumulative, it is more meaningful to speak in terms of p_{ki}, the cumulative conditional kill probability of weapons, than in terms of the one-shot kill probability. The over-all kill probability for i hits is the product of p_{ki}, the conditional probability of killing a real, given i hits, and the probability of i hits. The probability that an attacked "real" survives is expressed by

$$P_{asc} = 1 - \sum_{i=0}^{w} \binom{w}{i} p_h^i (1 - p_h)^{w-i} p_{ki} , \tag{12}$$

where p_h is the one-shot hit probability; the probability that any given "real" survives is expressed by

$$P_{1sc} = p_r \left[1 - \sum_{i=0}^{w} \binom{w}{i} p_h^i (1 - p_h)^{w-i} p_{ki} \right] + (1 - p_r) ; \tag{13}$$

the probability that all "reals" survive is expressed by

$$P_{Rsc} = P_{1sc}^R = \left[1 - p_r \sum_{i=0}^{w} \binom{w}{i} p_h^i (1 - p_h)^{w-i} p_{ki} \right]^R ; \tag{14}$$

the probability that exactly S "reals" survive is expressed by

$$P_{Ssc} = \binom{R}{S} P_{1sc}^{S} (1 - P_{1sc})^{R-S} ; \qquad (15)$$

the expected number of surviving "reals" is expressed by

$$\bar{S}_c = R\, P_{1sc} = R - p_r R \sum_{i=0}^{w} \binom{w}{i} p_h^{i} (1 - p_h)^{w-i}\, p_{ki} , \qquad (16)$$

and the corresponding standard deviation by

$$\sigma (S_c) = \left[R\, P_{1sc} (1 - P_{1sc}) \right]^{1/2} . \qquad (17)$$

Applications

Having established some of the fundamental concepts of decoy theory, we can now turn to applying them to operational problems. Three representative problems are discussed below.

* * *

A. Given p_r, p_d, p_k, W, and R, how many decoys are required, in order that an expected number $\bar{S}$ of "reals" survive?

By suitable transformation of equation (3'), D may be expressed in terms of the known inputs and w $(=\bar{w}')$:

$$D = \frac{1}{p_d} \left(\frac{W}{w} - p_r R\right) . \qquad (3'')$$

According to equation (9), the expected number of surviving "reals" is

$$\bar{S} = p_r R (1 - p_k)^{w} + (1 - p_r) R .$$

The value of w can be determined from this expression and substituted in equation (3") to obtain D.

* * *

B. Given p_d, p_{ki}, p_h, D, R, and W, what is the probability P_{os} of killing all "reals?"

A necessary conditions for killing all "reals" is that $p_r = 1$. The probability that no "reals" survive is obtained from equation (15) by setting $S = 0$ and $p_r = 1$:

$$P_{0sc} = \left[\sum_{i=0}^{w} \binom{w}{i} p_h^i (1 - p_h)^{w-i} p_{ki} \right]^R . \tag{18}$$

The value of w is obtained from equations (3) or (3') and substituted in equation (18), to obtain P_{0sc}. Whenever w is a fractional number, P_{0sc} can be evaluated by interpolation between probability values corresponding to the two nearest integral values of w.

C. Given $p_r = 1$, p_d, p_k, W, and c (cost of a decoy divided by cost of a "real"), what is the cheapest ratio of decoys to "reals" which will result in an expected number $\overline{S}$ of surviving "reals?"

The total cost of "reals" and decoys, C, in terms of cost of a "real," is

$$C = R + cD = R\,(1 + c\,\frac{D}{R}) . \tag{19}$$

To illustrate the mathematical procedure, assume $p_k w \sim 0.1$ and use the simplified equation (9') to express R as a function of $\overline{S}$ and D/R:

$$R = \overline{S} + p_k W \frac{R}{R + p_d D} = \overline{S} + p_k W \frac{1}{1 + p_d\,(D/R)} . \tag{20}$$

Then the total cost in terms of S and D/R is

$$C = \overline{S} + \frac{p_k W}{1 + p_d\,(D/R)} + \overline{S}c\,\frac{D}{R} + \frac{p_k W}{1 + p_d\,(D/R)}\,c\,\frac{D}{R} . \tag{21}$$

In order to minimize total cost with respect to D/R, equation (21) is differentiated and the derivative set equal to 0:

$$\frac{d\,C}{d\,(D/R)} = - \frac{p_d p_k W}{(1 + p_d\,(D/R))^2} + Sc + \frac{(1+p_d\,(D/R))p_k Wc - p_k Wc\,(D/R)p_d}{(1 + p_d\,(D/R))^2}$$

$$= Sc - \frac{p_k W\,(p_d - c)}{(1 + p_d\,(D/R))^2} = 0 , \tag{22}$$

or

$$p_d^2 (D/R)^2 + 2\, p_d (D/R) + 1 - (p_k W/Sc)(p_d - c) = 0. \tag{23}$$

Solution of equation (23) for D/R yields the desired ratio:

$$\frac{D}{R} = \frac{1}{p_d} \left(\sqrt{(p_k W/Sc)(p_d - c)} - 1 \right) . \tag{24}$$

* * *

A number of other decoy problems can be solved by applying either the basic relations developed here or their logical extensions.

The increase in survival time of "reals" due to the presence of a given number of decoys can be calculated if the attacker's delay in supplying the additional weapons is known. A similar calculation can give the attacker's materiel losses due to the increased number of weapons required, if the effectiveness of active defenses is known.

Improved target classification gained by exchange of information between different observers, or decreased kill probabilities due to conflicting weapon assignments can be treated by appropriate adjustments of p_r, p_d, and/or p_k.

The attacker can use these relations to develop an optimum firing doctrine. Should he fire w weapons at every target in sight, until he runs out of weapons? Or should he wait until the entire formation is in sight before deciding on weapon assignment? Is it more advantageous to fire more weapons at more real-like targets, or fewer weapons at every target? And so on.

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