A DIMENSIONLESS CONSOLIDATION OF WES DATA ON THE PERFORMANCE OF SAND UNDER TIRE LOADS

by

C. J. Nuttall, Jr.

December 1965

Sponsored by

U. S. Army Materiel Command

Prepared for

U. S. Army Engineer Waterways Experiment Station
CORPS OF ENGINEERS
Vicksburg, Mississippi

Under

Contract No. DA-22-079-eng-262

by

Wilson, Nuttall, Raimond Engineers, Inc.
Chestertown, Maryland

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Task 03

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FOREWORD

The study reported herein was conducted as a part of the re-
search program being pursued under Department of the Army Project
No. 1-V-0-21701-A-046, "Trafficability and Mobility Research," Task
1-V-0-21701-A-046-03, "Mobility Fundamentals and Model Studies." This
research is guided and sponsored by the Directorate of Research and De-
velopment, U. S. Army Materiel Command.

The analysis presented was performed by Wilson, Nuttall, Raimond,
Engineers, Inc. (WNRE), Chestertown, Maryland, under contract to the U. S.
Army Engineer Waterways Experiment Station (WES), Vicksburg, Mississippi,
the agency primarily responsible for the conduct of the research project.
This report was prepared by Mr. C. J. Nuttall, Jr., for presentation at a
Mobility Consultants Conference in September 1963. He was assisted in the
data analysis by Mr. C. W. Wilson and in performance of the many calcula-
tions by Mr. V. Kennedy, both of WNRE. Mr. Nuttall and Mr. R. P. McGowan
prepared Appendix A. Mr. C. J. Powell, former employee of WES, contributed
the discussion of strength variations in the laboratory soil bins that is
included as Appendix B.

The wheel tests upon which the analysis is based were accomplished at
WES by personnel of the Mobility Section, Army Mobility Research Branch,
Mobility and Environmental Division, under the supervision of Messrs. W. J.
Turnbull, W. G. Shockley, S. J. Knight, and D. R. Freitag. Col. Alex G.
Sutton, Jr., CE, and Col.John R. Oswalt, Jr., CE, were Directors of the
WES during the conduct of this study and preparation of this report.
Mr. J. B. Tiffany was Technical Director.

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CONTENTS

GLOSSARY

a   Tire lug depth, in.

A   Slope of surface of soil, dimensionless

A_c Contact area of a deflected tire on an unyielding surface, in.^{2}

b   Section width of undeflected tire, in.

B   Shear stress-strain parameter, in.

c   A generalized bearing-shear strength parameter for the in situ soil, lb/in.^{2}

c_s Structural cohesion of soil, lb/in.^{2}

c_t Effective, after-collapse cohesion of soil, lb/in.^{2}

C   Average slope of the curve of cone index versus depth (in.) of penetration, lb/in.^{3}

$\overline{CI}$ Before-traffic average cone index for the 0- to 6-in. layer of the test bed (CI_6), lb/in.^{2}*

$\overline{CI}_z$ Before-traffic average cone index for the 0- to 6-in. layer, lb/in.^{2}

d   Outside diameter of undeflected tire, in.

d_c Base diameter of the right circular cone of the cone penetrometer, in.

D   Drawbar pull of tire or vehicle, lb

E   Indicates a functional relation

f   Coefficient of friction between soil and tire

F   Indicates a functional relation

g   Acceleration due to gravity, in./sec^2

h   Depth of soil or soil layer, in.

j   Slip distance, or travel of an element in the tire contact patch relative to undisturbed soil during one contact cycle, in.

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* In previous trafficability reports cone index has been considered to be dimensionless but in this study it has the dimensions lb/in.^{2} to permit a dimensional analysis.

k A constant equal to π or 4 (see paragraph 37)

k' Dimensionless curve-fitting parameter in equation $p_z = k'z^n$, equation 18

k'' A constant in the numeric π_2, equation 23

K_2 $\dfrac{W_1}{\overline{CI}\ \delta\ b^{0.5}\ d}$, equation 14

ℓ Average length of the static tire contact surface at test load and inflation, determined on a hard, flat surface, in.

m Vehicle wheel base, in.

n Dimensionless curve-fitting parameter in the equation $p_z = k'z^n$, equation 18

N Indicates a function of n

p_c Average contact pressure of loaded tire on a hard surface (ignoring tread interruptions), $lb/in.^2$

p Nominal pressure based on total projected area of the wheel; equal to $\dfrac{W_1}{bd}$, $lb/in.^2$

p_z Unit penetration resistance experienced by a cone at depth of penetration z, $lb/in.^2$

Q Torque input (or output), in.-lb

R Rolling resistance of tire, lb

s Wheel spacing, in.

S Slip ratio $= V_s/V = j/\ell$

T Gross traction, determined by the shearing resistance of the soil in the contact area, lb

V Tire peripheral speed, in./sec

V_s Slip speed, or average speed of the elements in contact with the soil relative to undisturbed soil, in./sec

W Gross weight of vehicle, lb

W_1 Vertical load carried by each tire, lb

y Vehicle length, in.

z Tire or vehicle sinkage, or depth of penetration of cone penetrometer, in. It is the distance measured vertically from the soil surface downward

z_o A terminal value of z, in.

α Trim of vehicle, deg

β Plastic kinematic viscosity of soil, $in.^2/sec$

γ Bulk specific weight of soil, $lb/in.^3$

δ Tire deflection under test load, at test inflation, measured on hard, flat surface, in.

θ Indicates a functional relation

λ Linear scale ratio

π When used with a subscript, π denotes a numeric; when used without a subscript, π denotes the constant, 3.1416

π_1 $\dfrac{C\ell^3}{W_1}$, Freitag-Knight numeric

π_2 Final load numeric $= \left(\dfrac{1}{13}\right)^{\frac{n}{1+2n}} \left(\pi_3\right)^{\frac{1}{1+2n}}$, equation 31

π_3 $\dfrac{p_c}{\overline{CI}_{z_0} \left(\dfrac{d}{z_0}\right)^n}$, equation 30

σ Unit normal loading on a soil element, lb/in.^2

τ Static shearing resistance of soil, lb/in.^2

τ_0 Dynamic shearing resistance of soil, lb/in.^2

ϕ Effective, after-collapse angle of internal friction of soil, deg

SUMMARY

Data obtained in extensive field tests made by the U. S. Army Engineer Waterways Experiment Station (WES) to study the performance of pneumatic-tired vehicles in sands are correlated, by means of a semiempiric procedure, with WES Army Mobility Research Branch (AMRB) laboratory soil-bin data (ssentially on smaller tires) developed during the three years 1961-1963 in air-dry sand. The laboratory data are first shown to collapse reasonably well upon the basis of the empirically developed load coefficient, K_2.

$$K_2 = \frac{W_1}{\overline{CI} \, \delta \, b^{0.5} \, d} \tag{14}$$

where W_1 = vertical load carried by each tire, lb

$\overline{CI} = \overline{CI}_6$ = before-traffic average cone index for the 0- to 6-in. layer of the test bed, lb/in.^{2}*

δ = tire deflection under test load, at test inflation, measured on a hard, flat surface, in.

b = section width of undeflected tire, in.

d = outside diameter of undeflected tire, in.

This coefficient is not dimensionless. However, its success indicates that added insight and usefulness may be gained by seeking to make it so. It is reasoned that the shape of the cone index versus penetration curve may enter the results strongly, and a revised, dimensionless form of the load coefficient is developed accordingly. Use is made in this connection of the conventional gross logarithmic linearization of the pressure-penetration relation

$$p_z = k'z^n \tag{18}$$

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* In previous trafficability reports cone index has been considered to be dimensionless but in this study it has the dimensions lb/in.^{2} to permit a dimensional analysis.

where p_z = unit penetration resistance experienced by a cone at depth of penetration z , lb/in.^{2}

    k' = curve-fitting parameter

    z = tire or vehicle sinkage, in. (When used in relation to second and third passes in the same rut, refers to total or cumulative sinkage rather than increment occurring during the particular pass.)

    n = curve-fitting parameter

By forcing a correlation between the laboratory data (for which n = 0.7) and the field data (for which n = 1.0), an intermediate form of this numeric is developed which involves the tire sinkage or rut depth. Further manipulations and approximations yield a more useful form (which does not contain the dependent variable, z) as follows:

$$\pi_2 = \left(\frac{1}{13}\right)^{\frac{n}{1+2n}} \left[\frac{p_c}{\overline{CI}_{z_o}\left(\frac{d}{z_o}\right)^n}\right]^{\frac{1}{1+2n}} \tag{29}$$

where p_c = average contact pressure of loaded tire on a hard surface (ignoring tread interruptions), lb/in.^{2}

    $\overline{CI}_{z_o}$ = the average cone index of a bed of material to a depth z_o , lb/in.^{2}

    z_o = a terminal value of z , in.

This form is shown to correlate the laboratory and field data closely. Further internal checks are made and appear favorable. It is then shown that the final form for π_2 remarkably resembles that of the basic Bernstein equations for rigid wheels.

    The basic dimensional framework of the study is described in Appendix A, and in Appendix B a study of the variations of soil strength in the test cars is presented.

A DIMENSIONLESS CONSOLIDATION OF WES DATA

ON THE PERFORMANCE OF SAND UNDER TIRE LOADS

PART I: INTRODUCTION

Background

1. During the three years 1961-1963, the U. S. Army Engineer Water-
ways Experiment Station (WES) developed, in its laboratory soil bins,
extensive data on the performance of pneumatic tires in air-dry sands.
Over an even longer period, it has conducted a major field test program
on the performance of pneumatic-tired military vehicles in natural sand
conditions representative of those occurring on world deserts and beaches.
WES cone index values have been used throughout both programs as the
primary soil strength measurement parameter.

2. In a paper presented to the Society of Automotive Engineers in
January 1963, Messrs. Freitag and Knight[1]* demonstrated that the slope-
climbing ability of a wide range of wheeled vehicles in natural sand con-
ditions was primarily a function of the simple numeric

$$\pi_1 = \frac{C\ell^3}{W_1} \tag{1}$$

where C ≡ the average slope of the curve of cone penetration resistance
(cone index)** versus depth of penetration in the top 6 in. of
the sand† ($lb/in.^3$)

ℓ = the average length of the static tire contact surface at test
load and inflation, determined on a hard surface, in.

W_1 = the average vertical load carried by each tire, lb

Their concluding figure is reproduced herein as plate 1. Messrs. Freitag
and Knight noted in their paper that ". . . for the large majority of sands

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* Raised numbers refer to similarly numbered items in the Selected
Bibliography following the main text of this report.
** In previous trafficability reports cone index has been considered to be
dimensionless but in this study it has the dimensions $lb/in.^2$ to permit
a dimensional analysis.
† 0- to 6-in. average cone index divided by 3.

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tested, cone index increased generally in direct proportion to depth from the surface to depths well beyond 6 in."

Purpose

3. It was the purpose of this study to show that (a) an empiric load coefficient that correlates the results of the laboratory tests is actually dimensionless, (b) the numeric that correlates the field data and the numeric derived from the laboratory data are based on similar coefficients, and (c) a reasonable relation exists between the two sets of data.

Scope

4. The analyses presented in this report were based on existing laboratory and field data collected by WES personnel. The laboratory data are the results of tests conducted during the period 1961-1963 in conjunction with the test program "Performance of Soils Under Tire Loads."[2] The field data used in this analysis are found in WES reports of the "Trafficability of Soils" series.[3-5]

PART II: CONSOLIDATION OF THE LABORATORY SOIL-BIN DATA

Source of Data

5. The basic soil-bin facilities were used by WES in the laboratory program. Most of the tests were run in the high-speed, movable-bin facility known as the main mobility facility. Tests with larger tires, fewer in number, freely rolling only (towed), and generally more completely instrumented, were run in the auxiliary, fixed-bin mobility facility. The test facilities and techniques are described in WES Technical Report No. 3-666, Performance of Soils Under Tire Loads, Report 1, Test Facilities and Techniques.[2]

6. The data treated herein were derived from tests conducted prior to February 1963. In the complete program, diameters of tires tested varied from about 14 to about 40 in.; tire widths from 1-3/4 to 15 in.; width/diameter ratios from 0.06 to 0.91; tire loads from approximately 100 to 4500 lb; and tire deflections (on a hard surface) from 5 to 55 percent of the section height. The effects of ply ratings, construction methods (radial/circumferential versus conventional cord arrangement), and light treads were explored.

7. Powered and towed performances of each single tire were measured (at various loads and deflections) on the first through fifth passes in the same path. Data from the first three passes only have been considered in this report. The complete data record for each powered-wheel test consists of pull-slip, torque-slip, and sinkage-slip relations for slip values ranging from approximately 30 to 100 percent. Data considered representative of the wheel performance were taken from these curves at the zero torque (towed) point and the maximum pull point. The analysis which follows is relative to this portion of the laboratory data.

8. Tests were conducted primarily in Yuma desert sand in an air-dry condition at three general strength levels ranging from approximately 20 to over 60 in terms of WES cone index of the 0- to 6-in. layer. A smaller number of tests were conducted in an air-dry mortar sand.

Dimensional Considerations

9. A tire operating in soil constitutes a system composed of a highly

flexible structure of complex shape moving over and through a continually failing granular and/or plastic mass. The detailed relations that govern the behavior of the system undoubtedly are complex. One approach to practical quantitative solutions has been to conduct systematic laboratory tests on a number of tires covering a wide range of basic tire characteristics, loadings, and deflections in a wide range of soils of varied character and strength. The data from such systematic test series are most usefully presented and generalized by organization within the framework provided by a valid dimensional analysis of the system.[6]

Historical background

10. A dimensionless analysis for vehicle-soil systems was first proposed by Markwick in 1944.[7] This basic analysis has since been generally developed and validated by a number of investigators.[8,9] The last published form is reproduced in Appendix A.

The current analysis

11. In their current form, the equations for sinkage z, drawbar pull D, and torque input Q (or alternately for rolling resistance R, and slip ratio S) of a tire of fixed geometry (but not fixed size) operating in soil reduce to:

For powered elements:

$$z/d = \theta \ (W_1/cd^2, \ \emptyset, \ S, \ h/d) \tag{2}$$

$$D/W_1 = \theta' \ (W_1/cd^2, \ \emptyset, \ S, \ h/d) \tag{3}$$

$$2Q/dW_1 = \theta'' \ (W_1/cd^2, \ \emptyset, \ S, \ h/d) \tag{4}$$

For freely rolling towed elements:

$$z/d = E \ (W_1/cd^2, \ \emptyset, \ h/d) \tag{5}$$

$$R/W_1 = E' \ (W_1/cd^2, \ \emptyset, \ h/d) \tag{6}$$

$$S = E'' \ (W_1/cd^2, \ \emptyset, \ h/d) \tag{7}$$

where W_1 = vertical load on a tire, lb

d = outside diameter of undeflected tire, in., a characteristic linear dimension of the system

$\emptyset$ = dynamic internal friction of the soil (primarily in respect to its direct effects upon traction), deg

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h = depth of soil or controlling soil layer to any effective hard-
pan, in.

c = a generalized bearing-shear strength parameter for the in situ
bed of material, having the dimensions of a stress, $lb/in.^2$
A proper measure for this factor includes the contributions of
soil internal friction, cohesion, and density as they affect
bearing capacity, and may also reflect changes in these with
depth in the soil mass and/or with soil remolding

θ indicates a functional relation

E indicates a functional relation

Other factors, such as density (except as reflected in c), speed (per se),
and viscous properties, have proven, in systems and materials similar to
the tire-sand system here specifically under consideration, to have only a
minor influence on the behavior (z, D, Q, or z, R, S) of the tire. The
well-known importance of slip (S) in powered systems is considered to be
the result of essentially static stress-strain behavior of loose and/or
plastic materials rather than of speed, even though S is often expressed,
for convenience, in terms of speeds:

$$S = V_s/V$$

where V_s = slip speed, or average speed of tractive elements in contact
with the soil relative to undisturbed soil, in./sec

V = the average speed of the same elements in the same region rel-
ative to the supporting vehicular structure, in./sec

These findings and the other assumptions made in previous pertinent work
cited are accepted as the basis for further discussions. In the final
analysis, their justification lies primarily in the degree to which their
use leads to the desired semiempiric synthesis of the data.

Application to present data

12. Equations 2, 3, and 4 can be simplified still further. On
the basis of considerable testing with in situ devices, such as normally
loaded shear vanes, it appears that the dynamic or continuous after-
failure internal friction of most air-dry sands in nature is on the
order of

$$\tan \phi = 0.70 \quad (\text{from reference 10})$$

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For convenience, then, ϕ may (with some small loss in precision) be taken as a constant, and hence need not appear as a variable in the functional equations as they apply to the performance of tires in air-dry sands.

13. Slip ratio, S , which appears as an independent variable only when the wheel is powered, may be eliminated from the expressions relating to powered tires by considering data obtained at only one slip, i.e. by making S a constant. A useful and convenient point in treating the performance of powered tires in sands is the point, generally at about 20 percent slip, at which maximum pull occurs.

14. Finally, since no hardpan was involved in the present test series, terms involving h may be eliminated. (Note, however, that this is not always the case in field trials, particularly where vehicles are run in test lanes prepared to low surface densities by shallow tilling.)

15. These considerations reduce equations 2, 3, and 4 for powered wheels to the following simple expressions:

$$z/d = \theta \ (W_1/cd^2) \tag{8}$$
$$D/W_1 = \theta' \ (W_1/cd^2) \tag{9}$$
$$2Q/dW_1 = \theta'' \ (W_1/cd^2) \tag{10}$$

Equations 5, 6, and 7 for freely rolling, towed wheels become, correspondingly,

$$z/d = E \ (W_1/cd^2) \tag{11}$$
$$R/W_1 = E' \ (W_1/cd^2) \tag{12}$$
$$S = E'' \ (W_1/cd^2) \tag{13}$$

To make use of these equations, it is necessary to determine a value for the soil strength parameter c appropriate for each set of tire test results. The WES cone index has proved useful in correlating the field performance in sand of a variety of pneumatic-tired vehicles.[1,3-5] It is therefore logical to examine its usefulness in this extended context. There are other reasons for attempting to use the cone index for this purpose in preference to the indications of any of several other current instruments, as follows:

    <u>a</u>. Vast quantities of data are already available for sand areas of the world in terms of cone index.[3-5,11]

    <u>b</u>. The instrument for in situ field determination of cone

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indexes is light, compact, well developed, easy to use, and widely distributed.[11]

    <u>c</u>. Cone penetration data for the present test series appear to be the most consistent and reliable of all the types taken. Moreover, because of the simplicity of the test procedure, cone tests were regularly replicated in connection with each individual test treated herein.[10]

Results

16. After many regression attempts using different forms of the basic load numeric of equations 8-13, i.e. W_1/cd^2, it was found that a useful level of correlation was achieved by employing the coefficient K_2.

$$K_2 = \frac{W_1}{\overline{CI}\ \delta\ b^{0.5}\ d} \qquad (14)$$

where $\overline{CI} = \overline{CI}_6$* = average cone index for the 0- to 6-in. layer of the soil determined prior to any traffic, $lb/in.^2$

        d = outside diameter of the undeflected tire, in.

        b = maximum section width of the undeflected tire, in.

        δ = tire deflection under test load at test inflation measured on a hard surface, in.

The coefficient K_2 is not dimensionless. However, the degree of correlation resulting from its use is such as to warrant seeking valid means for making it so. This will be done following presentation of some data consolidated by its use.

Soil Strength and Tire Deflection

9.00-14 tire test data

17. Plates 2, 3, and 4 show the first-, second-, and third-pass maximum pull data for three smooth 9.00-14 tires (2-, 4-, and 8-ply rating)

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* The notation $\overline{CI}_{z_0}$ will henceforth be used to signify an average cone index taken before any traffic, measured for the 0 to z_0 layer, using the standard, WES, polished steel, 30-deg apex angle, right-circular cone, having a base area of 0.50 sq in. $\overline{CI}$ (without subscript) will denote $\overline{CI}_6$.

each at three deflections, and in soil strengths ($\overline{CI}$) ranging from 17 to 77. Dimensionless torque input $Q/0.5dW_1$, sinkage $z/d*$, and pull output D/W_1 from some 150 separate tests are presented as functions of

$$K_2 = \frac{W_1}{\overline{CI}\ \delta\ b^{0.5}d}$$

18. A high level of correlation is evident. (Plots of dimensionless towed force, R/W_1 , and corresponding sinkage, z/d , for over 120 towed tests on the same tires, as functions of the same coefficient, show a similar degree of correlation.) The scatter is, for the most part, within the error band introduced by variations in cone index along the length of a given test lane from the single average value assigned for the complete test. In general, this variation is on the order of ± 4 cone index which at lower sand strengths (around $\overline{CI} = 20$) amounts to approximately ± 20 percent (see Appendix B). Note that tests at low strengths tend to result in high values of the load coefficient K_2 , and that it is at the higher values of K_2 that the scatter is greatest. Some further expansion of the error band undoubtedly results from variations in the shape of the cone index versus penetration curve from which the average value is determined. The effects of curve shape will be discussed more fully later.

19. Note that in plates 2-4 the complete form of the load coefficient K_2 is not under scrutiny. Despite some minor differences between the three different tires, tire diameter d and section width b are essentially constant for all of the points shown. The same degree of correlation could accordingly have been achieved using the incomplete function

$$\frac{W_1}{\overline{CI}\ \delta}$$

4.50-7 tire test data

20. Plate 5 shows the comparable first-pass maximum pull data from approximately 33 tests with the 4.50-7 tire. The 4.50-7 tire is nearly a one-half scale model (geometrically) of the basic 9.00-14 tire. As such, its behavior in relation to the 9.00-14 tires is theoretically of

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* Note that sinkage z for passes after the first is taken, in this report, to be total sinkage, rather than incremental sinkage due to the particular pass only.

importance from a dimensional viewpoint.

21. The complete data for the 4.50-7 tire show considerably more scatter than those for the larger tire, but still demonstrate a clear enough grouping to permit assignment of an independent "best-fit" line of the same character as that developed for the 9.00-14 tires. The increased scatter is thought to be the result, at least in part, of the three factors discussed below.

    a. The physical size of the tire (about one-half that of the 9.00-14 tire) and the loads (generally about one-fifth the size of the loads on the 9.00-14 tire) were intrinsically small for the test apparatus used, leading to larger possible errors in actual measurements due to mass effects, friction, basic load cell accuracy, etc. These errors are amplified by the ratio form of presentation.

    b. The small size of the tire also resulted in its "averaging" soil conditions less smoothly. (The validity of this observation is supported by a minor but noticeable tendency for the scatter to reduce on successive passes.)

    c. The 4.50-7 tire operated in the upper layers of the test bed, within which variations due to preparation differences probably are greatest, particularly when viewed on a ratio basis.

Scale Relations

22. The best-fit curves for the 9.00-14 and 4.50-7 tire data are plotted for direct comparison of maximum pull and towed force in plates 6-8 and 9-11, respectively. It will be observed that sinkage (z/d) correlates well with the independent numeric for all passes and for the maximum pull and the towed conditions. Thus, the two tires behave in geometrically similar fashion when subjected to the same type of test at the same load coefficient, i.e. they function properly as scale model and prototype. This indicates, but unfortunately does not prove, that the load coefficient K_2 is in fact dimensionless, but contains in its present form an important dimensioned constant parameter as yet unidentified.

23. Pull and torque also correlate well with K_2, as does first-pass towed force. Towed forces tend to diverge somewhat at higher loadings on the second pass, and still more so on the third. The final deviation is on the order of ± 15 percent from the curve halfway between them.

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(It is apparent, however, that no simple change in the load coefficient
can improve matters without disrupting the important sinkage relations.)

24. Note that the complete form of the load coefficient K_2 still
is not under scrutiny. Since the two sizes of tires are geometrically
similar, i.e. since $(b/d)_{9.00-14} = (b/d)_{4.50-7} = 0.30$, the plots could as
well have been presented on the basis of a load coefficient of the form

$$\frac{W_1}{CI \; \delta \; d^{1.5}}$$

Tire Shape

Results of tests
with various tire sizes

25. To demonstrate the full form of the coefficient, tests of two
tires of extreme proportions, plus one of more normal shape, were examined.
Data from about 10 tests with the 16x15 Terra-Tire ($b/d = 0.91$), from 12 or
more tests with the 1.75-26 bicycle tire ($b/d = 0.06$), and from tests with
three 6.00-16 tires ($b/d = 0.21$) were plotted, as described above, against
the coefficient K_2 .

26. For the Terra-Tire and the bicycle tire, the scatter was again
noticeably greater than in the tests of the 9.00-14 tires, probably for the
same general reasons (in somewhat lesser degree) just discussed (para-
graph 21) in relation to the scatter of the 4.50-7 tire test data.

27. Despite the decrease in precision, however, it appeared that the
load coefficient K_2 also correlated changes in measured behavior of each
of these three widely differing tires with load, deflection, and soil
strength as measured by the WES cone penetrometer. As in the case of the
other presentations of results with one tire size, only these three factors
actually entered the correlations at this point. Size and shape (d and b/d)
were constant in any one set of data.

28. Approximate, independent, best-fit curves of the same general
character as those derived for the data of the 9.00-14 and 4.50-7 tire
tests could readily be drawn. These were compared directly with the re-
sults from the 9.00-14 and 4.50-7 tire tests. For convenience and as a
matter of intrinsic interest, curves drawn midway between the faired results

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from the two latter tires were consolidated for maximum pull data (first, second, and third passes) in plate 12, and for towed force data (first, second, and third passes) in plate 13. These "average-average" curves are used in all further discussions to represent the combined results of these two series of tests.

29. In plates 14-19, the faired performance curves for the 16x15 Terra-Tire, the 1.75-26 bicycle tire, and the several 6.00-16 tires (which evidently can be pooled), for each of the first three passes, can be compared with the "average-average" curves for the tires of $b/d = 0.30$. All of the curves fall in the same general area, and they are clearly related. Just as clearly, they are not the same. Considerable trial and error has demonstrated that the agreement cannot be significantly improved by minor changes in the makeup of the single load coefficient.

30. The fact that complete data collapse has not been achieved, however, should not be suprising. In dealing with such a complex system as tires in sands, there is no reason, a priori, to expect even this level of consolidation and generalization on so simple a basis.

31. The series of curves in plates 14-19, interpreted as a family with the parameter (b/d), should have some practical usefulness. Illustrative "cross-curves" showing first- and third-pass maximum pull coefficients (D/W_1) as a function of (b/d) for various values of the loading coefficient K_2 are shown in plate 20.

Practical application

32. For practical applications, it is essential to evaluate all coefficients fully and to base design decisions on actual dimensioned values rather than upon maximum (or minimum) coefficients. For example, although the narrowest tire clearly gives the highest performance at a loading coefficient value of 0.60, it usually will be possible in a practical design problem to achieve higher actual performance by using a wider tire for which the loading coefficient is significantly lower. In this connection, note that permissible or practicable tire deflection (25 to 30 percent) is normally directly proportional to tire section height and that in conventional round-section tires this in turn bears a relatively constant relation to a maximum section width (about 90 percent). Thus, an increase in maximum section width in a round-section tire, at constant

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load, soil strength, and outside tire diameter, and at maximum permissible deflection, will reduce the loading coefficient approximately as $b^{1.5}$. For example, consider the two tires below:

Tire	d in.	b in.	b/d	δ_{max} in.	$\overline{CI}$ lb/in.^{2}	W_1 lb	$\dfrac{W_1}{\overline{CI}\,\delta\,b^{0.5}\,d}$	D/W_1*	D lb
A	56	3.5	0.0625	0.875	30	1650	0.60	0.06	100
B	56	14	0.250	3.50	30	1650	0.75	0.43**	700

* From plate 20.
** Extrapolated.

From this it is clear that narrow tires do not carry more load than wide ones in the test sand conditions, all other things being equal, despite the first impression that might erroneously be gathered from the coefficient curves.

Tests in a Second Type of Sand

9.00-14 tire test data

33. A series of 10 tests was run on the 9.00-14, 2-PR tire in an air-dry mortar sand. The results were compared with the "average-average" curve for this shape of tire in the Yuma desert sand. The correlations, while nowhere near exact, are good enough to suggest that the coefficient curves representing performance in the Yuma sand might be useful (albeit at reduced precision) for a broad range of other sands. This is consistent with extensive field demonstrations that a given cone index in any of a broad range of sand types has substantially the same meaning in terms of the performance of a given vehicle.[3-5]

11.00-20 tire test data

34. Towed tests were run by a different test group using completely different equipment. The tests were run in air-dry mortar sand with the 11.00-20, 12-PR, smooth (buffed) tire in the auxiliary facility. The 11.00-20 tire has a b/d ratio of 0.28, which is close to the b/d ratio of the 9.00-14 and 4.50-7 tires. The 11.00-20 tire may, in fact, be considered as a 1.4:1 scale model of the 9.00-14.

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35. The data from some 30 such tests are shown in plates 21-23 compared with the "average-average" curves for the smaller tires on Yuma sand. Despite considerable scatter, general agreement is apparent. Also evident is a tendency for the towed force coefficient of the large tire to be lower than that of the smaller tires at high values of the numeric, particularly as the number of passes increases. This may be due to differences in the character of cone index profiles in the two sands. The cone index in both sands increases with depth; but in the Yuma sand, there is generally very little increase in cone index below a depth of 12 in. In the mortar sand, however, the cone index normally continues to increase with depth at a nearly constant rate for the entire depth of measurement.

36. From the evidence and analysis presented in Part II, it is concluded that the load coefficient

$$K_2 = \frac{W_1}{\overline{CI}\,\delta\,b^{0.5}\,d}$$

is of substantially the optimum form to consolidate the complete test data to a useful degree. Accordingly, it is considered worthwhile to study its structure and to attempt, by further reasoning, to reduce it to a more acceptable dimensionless form.

37. First, consider that the area of contact A_c of an idealized, toroidal pneumatic tire deflected on a hard surface is given approximately by

$$A_c = k\,\delta\,b^{0.5}\,d^{0.5} \qquad (15)$$

where k = π for an elliptical contact patch

      = 4 for a rectangular contact patch[12]

That is, k is substantially constant (say 3.5) for the normal range of tires. Thus, the load coefficient may be rewritten

$$K_2 = \frac{W_1}{\overline{CI}\,\delta\,b^{0.5}\,d} = 3.5\,\frac{p_c}{\overline{CI}\,d^{0.5}} \qquad (16)$$

where $p_c = \dfrac{W_1}{A_c}$ = average hard-surface contact pressure (ignoring tread interruptions)

This form suggests that reexamination of the meaning of the cone index used in this coefficient may lead to an acceptable dimensionless form.

The Cone Index

38. The average cone index of a bed of soil material to a depth z_o is essentially the average unit resistance to vertical penetration experienced by a standard, polished steel, 30-deg apex angle, right-circular cone of base diameter d_c (= 0.80 in. for present tests) as it penetrates from the surface to a depth (measured at the cone

base) of z_o.* This may be written

$$\overline{CI}_{z_o} = \frac{1}{z_o}\int_0^{z_o} p_z \, dz \qquad (17)$$

where p_z = the unit penetration resistance (cone index) experienced by the cone at the depth of penetration z

In practice, the index assigned to a soil bed is, in turn, the average of the results from many cone probe tests.

39. In sands, the relation of p_z to z, even in the most carefully prepared, supposedly uniform test conditions in the laboratory, is one in which p_z steadily increases with z. It is convenient and, as will be seen, useful to resort for further discussions to a conventional, gross logarithmic linearization of the actual, complex curves determined experimentally, i.e.,

$$p_z = k' \, z^n \qquad (18)$$

where k' and n are curve-fitting parameters. Substituting equation 18 in 17 and integrating yields

$$\overline{CI}_{z_o} = \frac{k' \, z_o^{\,n}}{n + 1}$$

or

$$\overline{CI}_{z_o} = \frac{p_{z_o}}{n + 1} \qquad (19)$$

The relation between two average cone indexes in the same soil bed, taken to two different depths, z_1 and z_o, is accordingly given by

$$\overline{CI}_{z_1} = \overline{CI}_{z_o} \, (z_1/z_o)^n \qquad (20)$$

---

* The complete cone index concept has, over the years, been refined for some applications to reflect the average penetration resistance in a critical layer--6 to 12 in. for example--and to incorporate corrections for the changes from in situ strength resulting from remolding action.[11] The present discussion will, for simplicity, be limited to those past refinements that appear applicable to the present situation. Its extension to the more complete cone index analysis techniques is more cumbersome but entirely analogous.

A More Rational Form of the Strength Parameter

40. Equation 20, along with the basic observation already noted that cone index in sands increases with depth, suggests that the strengths (average cone index) assigned in relation to tire tests in sands should be related in some manner to the size and shape of the tire and/or to its sinkage. Assuming the validity of the cone index per se, it is reasonable to expect that performance of a normally proportioned tire, 2 ft in diameter, would be responsive to the cone index averaged over a range of the order of 0 to 12 in. (which is of the order of the depth to which its stress bulb would extend as its sinkage neared the immobilization point), while the performance of a similar 4-ft-diameter tire would correlate more closely with the cone index averaged over twice that range. As long as the penetration versus resistance relation is approximated by equation 18, the required values at such system-related depths can be estimated by means of equation 20, using cone index measured to some standard depth, such as 6 in. $(\overline{CI}_6)$, and an estimate of the exponent n which can be obtained from the usual cone index data by use of equation 19 in the form

$$n = \frac{p_{z_o}}{\overline{CI}_{z_o}} - 1$$

where p_{z_o} is the cone index at the depth z_o and $\overline{CI}_{z_o}$ is the average cone index to z_o. Alternatively, the estimate of n can be determined by use of equation 20 as follows:

$$n \log_{10}\left(\frac{z_1}{z_o}\right) = \log_{10}\left(\overline{CI}_{12}/\overline{CI}_6\right)$$

$$n = 3.32 \log_{10}\left(\overline{CI}_{12}/\overline{CI}_6\right) \tag{21}$$

where the data are for the 0- to 12-in. and 0- to 6-in. layers. Values of n calculated using equation 21 for a sample of some 50 small-bin tests in the Yuma sand are shown in table 1. Although there is considerable scatter, it is apparent that for most tests $n = 0.7$. In the light of the foregoing discussion, and in particular of the revised form of the load coefficient K_2 proposed in equation 16, it seems that the pr   dimensionless form for K_2 may be

16

$$\pi_2 = \frac{p_c}{\overline{CI}_{z'}} \qquad (22)$$

where $\overline{CI}_{z'}$ is the average cone index of the test soil bed taken over a depth appropriate for the scale of the system. If the tire diameter, d, is taken as a principal linear dimension of the wheel that characterizes the size of the entire system, then $z' = k''d$, where k'', at this point, is an unknown constant; and by equation 20

$$\overline{CI}_{z'} = \overline{CI}_{z_0} \left(\frac{k''d}{z_0} \right)^n$$

Substituting in equation 22 yields

$$\pi_2 = \frac{p_c}{\overline{CI}_{z_0} \left(\frac{k''d}{z_0} \right)^n} \qquad (23)$$

Then evaluating this expression at $z_0 = 6$ and using $n = 0.5$ as a first approximation,

$$\pi_2 = \frac{p_c}{\overline{CI} \left(\frac{k''d}{6} \right)^{0.5}}$$

or, using equation 16,

$$\pi_2 = \frac{0.7}{(k'')^{0.5}} K_2 \qquad (24)$$

41. Thus it appears that the load coefficient K_2 can be made dimensionless by rational means with reasonable physical significance. The unidentified "important dimensioned constant parameter" may accordingly be the depth over which the cone index is averaged (z_0). It remains to test the resulting hypothesis that the proper form for the load numeric is given essentially by equation 22 or 23.

A First-Order Check

42. The field data independently gathered, consolidated, and reported by Freitag and Knight[1], discussed at the beginning of this report, present an unusual opportunity to make an immediate first-order check. To do this, the relation between the proposed numeric π_2 and the Freitag-Knight numeric

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$$\pi_1 = \frac{C\ell^3}{W_1} \tag{1}$$

must first be developed.

43. It may readily be shown that, approximately,

$$\ell = 2\,\delta^{0.5}\,d^{0.5}$$

From the quoted observation that the cone indexes in the field generally increased directly with depth of penetration, it may be taken that, for the field tests, $n = 1$. From this and the definition of C (paragraph 2)

$$\overline{CI}_{z_o} = \frac{z_o\,C}{2}$$

Substituting these in equation 1 and inverting yields

$$\frac{1}{\pi_1} = \frac{1}{16}\frac{W_1\,z_o}{\overline{CI}_{z_o}\,\delta^{1.5}\,d^{1.5}}$$

or, by the approximation of equation 15,

$$\frac{1}{\pi_1} = \frac{1}{4.57} \cdot \frac{p_c}{\overline{CI}_{z_o}}\left(\frac{z_o}{d}\right)\left(\frac{b}{\delta}\right)^{0.5}$$

Finally, putting this in the form of equation 23,

$$\frac{1}{\pi_1} = \frac{k''}{4.57}\left(\frac{b}{\delta}\right)^{0.5} \cdot \frac{p_c}{\overline{CI}_{z_o}\left(\frac{k''d}{z_o}\right)^{1}}$$

or

$$\frac{1}{\pi_1} = \frac{k''}{4.57}\left(\frac{b}{\delta}\right)^{0.5}\pi_2$$

44. For the range of tires and deflections in the field tests $b/\delta = 3.7$ to 7.7, and accordingly

$$\frac{1}{4.57}\left(\frac{b}{\delta}\right)^{0.5} = 0.42 \text{ to } 0.61$$

In order to develop a simple approximate relation between π_2 and π_1, let this be taken as a constant, 0.5. Then

18

$$\pi_2 = \frac{2}{k''\pi_1} \tag{25}$$

45. If it is now assumed (a) that slope-climbing ability and drawbar coefficient are equivalent, (b) that the numeric π_2 is indeed valid, and (c) that the laboratory and field data should yield the same results when properly presented, an estimate of the constant k'' can be made. By assuming that when performance (slope-climbing ability or D/W) is equal, the actual numerical value of π_2 must be equal in the two systems, i.e. from equations 24 and 25 (each solved for π_2)

$$\frac{2}{k''\pi_1} = \frac{0.7}{(k'')^{0.5}} K_2$$

then

$$k'' = \frac{8.2}{(K_2\pi_1)^2} \tag{26}$$

To evaluate this, the data in table 2 were extracted from plate 1 for the field tests and from plates 12 and 13 for the laboratory tests. (Tire width/diameter ranged from 0.23 to 0.29 in the field tests. The laboratory data are for tires for which $b/d = 0.30$.)

46. Calculated values for k'', given in column 5 of table 2, turn out to be far from constant. Rather, it appears (see column 6 of table 2) that

$$k'' = 9 \ z/d$$

where z/d is second-pass sinkage from the laboratory tests. Substituting this in equation 23 yields

$$\pi_2 = \frac{p_c}{\overline{CI}_{z_o} \left(\frac{9z}{z_o}\right)^n} \tag{27}$$

In the light of preceding discussions, this indicates that the appropriate depth through which to average cone indexes for assigning a soil strength in a given test is approximately nine times the rut depth. This is not unreasonable for small rut depths.

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47. π_2 in this form is awkward, however, for z is a dependent variable, i.e.

$$z/d = \theta^{iv} (\pi_2)$$

Accordingly, an approximate relation may be sought within the data. Unfortunately, sinkage data have not been published for the field tests. From the laboratory results, however, as in plate 12 (powered tests)

$$z/d = 1.44 \ (\pi_2)^2 \tag{28}$$

Substituting this in equation 27 and solving yields

$$\pi_2 = \left(\frac{1}{13}\right)^{\frac{n}{1+2n}} \left[\frac{p_c}{\overline{CI}_{z_o} \left(\frac{d}{z_o}\right)^n}\right]^{\frac{1}{1+2n}} \tag{29}$$

or, by letting

$$\pi_3 = \frac{p_c}{\overline{CI}_{z_o} \left(\frac{d}{z_o}\right)^n} \tag{30}$$

equation 29 becomes

$$\pi_2 = \left(\frac{1}{13}\right)^{\left(\frac{n}{1+2n}\right)} \pi_3^{\left(\frac{1}{1+2n}\right)} \tag{31}$$

From equation 25, for the field data ($n = 1$)

$$\pi_3 = \frac{2}{\pi_1} \tag{32}$$

and, from equation 24, for the laboratory tests ($n = 0.5$)

$$\pi_3 = 0.70 \ K_2 \tag{33}$$

Values of π_2 calculated from the data in table 2 by means of equations 31-33 are given in table 3 and plate 24.

48. It is evident that a high degree of agreement and internal consistency has been reached. This first-order examination supports (but still does not prove) the hypothesis developed earlier, and suggests a still more refined form for the basic numeric. Note that when n is

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substantially constant, results can be correlated on the basis of the simple numeric π_3 , as was done (essentially) with both the laboratory and the field data, independently, prior to this analysis.

A Further Internal Check

49. A further check may be made with the laboratory data at hand by calculating π_2 (using equation 29) directly from the basic data, point by point, using an estimate of n from the cone indexes recorded as part of each individual test record. This has been done on a trial basis with the first-pass drawbar pull and sinkage from some 50 tests of the 9.00-14 tires. The results are shown in plate 25. In plate 26, the identical points are plotted to approximately the same scale against the original load coefficient K_2 . Comparison of these two plates shows that while the percentile range of scatter in the load coefficient, at constant D/W or z/d , is increased slightly by the use of π_2 , there appears to be a stronger central tendency for the bulk of the data when it is plotted against π_2 . Also, π_2 appears, by virtue of its approximate linearity, to be somewhat more discriminating at lower specific loads.

50. An approximate straight-line relation may readily be written (for the first pass)

$$D/W_1 = 0.78 - 2.84\ \pi_2$$

which suggests the conventional equation for pull

$$D/W = T/W - R/W$$

in which R = motion resistance of the tire, lb

T = the gross traction, determined by the shearing resistance of the soil in the contact area, lb

W = gross weight of vehicle, lb

On this basis, the motion resistance of the powered wheel at about 20 to 30 percent slip may, for further discussion, be taken as

$$R/W_1 = 2.84\ \pi_2 \tag{34}$$

51. The line representing the approximate second-pass sinkage

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relation (equation 28) used to develop the final form of π_2 (equation 29) is also drawn in plate 25. In relation to the first-pass data plotted, it appears as essentially an upper bound, which is entirely appropriate.

52. Plates 27 and 28 show the 4.50-7 tire test data replotted on the basis of π_2 and K_2, respectively, for direct visual comparison. The scatter is again great using the new numeric, but not noticeably more than before. (Some possible reasons for the greater scatter were suggested earlier.) Also, the degree of correlation between the data for the 9.00-14 and 4.50-7 tires, essential to the dimensional reasoning, remains substantially the same.

53. These calculations, by showing that the new numeric does not disturb the previous degrees of correlation even when applied on a point-by-point basis (rather than to the pooled data), provide a necessary demonstration of the possible validity of the new form. They do not, however, constitute a sufficient demonstration to prove its validity conclusively.

Relation to Bernstein Equations

54. The first use of the approximation

$$p = k' \, z^n$$

in treating of wheels in soils is generally credited to Bernstein.[13] Accordingly, it is instructive to compare the final form of the load numeric π_2, developed entirely empirically, with the generalized equations for the rolling resistance of a rigid, rectangular-section wheel derived following Bernstein's assumptions. The basic Bernstein approach and equations may be found in Bekker's Theory of Land Locomotion (pages 189-191),[14] or, with the equation in the final form used below, in The Rolling Resistance of Wheels in Soil by Nuttall.[15] The equations in their usual form are

$$R = k_o \, b \, \frac{z_o^{(n'+1)}}{n'+1}$$

and

$$W_1 = \frac{3-n'}{3} \, k_o \, b \, d^{0.5} \, z_o^{\frac{2n'+1}{2}}$$

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Eliminating z_o between them,

$$R/W_1 = \frac{\left(\frac{3}{3-n'}\right)^{\frac{2+2n'}{1+2n'}}}{1+n'} \left(\frac{W_1}{k_o \, b \, d^{1+n'}}\right)^{\frac{1}{1+2n'}} \tag{35}$$

If it is assumed that the probe used to determine k_o and n' for use in equation 35 may be a WES cone penetrometer, then

$$k_o = k'$$

and

$$n' = n$$

Also, from equations 18 and 19

$$k' = \frac{(1+n) \, \overline{CI}_{z_o}}{z_o^{\,n}}$$

Let $p_p = \frac{W_1}{bd}$, a nominal pressure based upon the entire projected area of the wheel. Then for the rigid wheel

$$R/W_1 = \left[\frac{3}{(3-n)\,(1+n)}\right]^{\frac{2+2n}{1+2n}} \left[\frac{p_p}{\overline{CI}_{z_o}\left(\frac{d}{z_o}\right)^n}\right]^{\frac{1}{1+2n}}$$

or

$$R/W_1 = N \left[\frac{p_p}{\overline{CI}_{z_o}\left(\frac{d}{z_o}\right)^n}\right]^{\frac{1}{1+2n}} \tag{36}$$

where N indicates a function of n . The correspondence between this independently derived expression for the towed force of a rigid, rectangular-section wheel (equation 36) and equation 34, developed semiempirically for powered, pneumatic tires

$$R/W_1 = N' \left[\frac{p_c}{\overline{CI}_{z_o}\left(\frac{d}{z_o}\right)^n}\right]^{\frac{1}{1+2n}} \tag{34}$$

23

or

$$R/W_1 = N' \left(\frac{p_c}{p_p}\right)^{\frac{1}{1+2n}} \left[\frac{p_p}{\overline{CI}_{z_o} \left(\frac{d}{z_o}\right)^n}\right]^{\frac{1}{1+2n}} \tag{37}$$

is striking, to say the least.

55. The Bernstein approach to motion resistance is based upon the assumption of a simple direct correspondence between the pressure-penetration relation of a small plate in vertical motion and that of a moving wheel. Both assumptions have been shown to be extremely crude,[10] and calculated values have not agreed closely with measured values. The present agreement in form indicates that Bernstein's assumptions lead to quantitative rather than qualitative differences, i.e. though crude, his approach and assumptions are not totally incorrect.

The Final Forms

56. Subject to further internal and external checking, it appears that the form of load numeric given by equation 29 (π_2) is both proper and potentially useful for the further consolidation and analysis of performance data on pneumatic tires in dry sands. With data for sands for which n is substantially constant, the simpler numeric π_3 (equation 30) can be used.

57. The complete data from laboratory and field tests in sand, analyzed on the basis of π_2, will provide information for the selection and design of tires for sand operations in a form readily accessible for the designer. While the numeric π_2 will not fully collapse the data for wide variations in tire proportions (b/d), or for sand having an angle of internal friction substantially different from that for the Yuma sand, it will bring them to about the same degree of agreement as achieved with the coefficient K_2 (plates 14-19). Should more precision be required, cross plots similar to those in plate 20 can be developed. Design data in this form, supplemented by information on probable values for the soil parameters $(\overline{CI}_6, \phi, n)$ and sand slopes, should prove extremely useful.

58. Examination of the numeric π_2 indicates that for accurate

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scale-model testing per se, values of the dimensionless parameter n in
the model tests should be made approximately equal to those anticipated for
the full-size vehicle. In this case, load-scaling can be based upon the
simpler numeric π_3 , which will accommodate minor variations in n , as
well as in $\overline{CI}_6$.

PART IV: SOME NOTES ON THE SEMIEMPIRIC APPROACH

59.   The foregoing exercise is, of course, largely an empiric development.   In such a development the exact steps, assumptions, and manipulations that precede the final result are normally of little relevance.   The usefulness of the expressions presented is determined primarily by the degree to which they consolidate the data.

60.   Such excuse as there may be for outlining the present arguments at such length may be found in four factors.   First, the analysis began with a theoretically sound question.   Why was the load coefficient K_2 , which appeared to work reasonably well for a wide range of tires and test conditions, not dimensionless?   The results suggest a reasonable answer.   They also, insofar as they may prove to have a proper theoretical foundation, offer answers to some important practical scale-modeling problems which were first posed some years ago,[16] and which have cropped up since, often unrecognized.[17]

61.   Second, some of the intermediate reasoning has been based, however crudely, upon theoretical considerations.   Its presentation permits formation of an independent estimate of the possible generality of the results, and suggests numerous points where some further research might greatly improve generality and/or precision.

62.   Third, detailing of the several steps in the development facilitates the making of further purely empirical refinements, should they appear desirable.   Several such possibilities exist.   For example, it has been shown in unpublished material by WES personnel working independently with the same laboratory data that scatter can be reduced, particularly for the narrower tires, by using an orderly variation in the powers of others of the dimensions appearing in the denominator of a load coefficient of the general form of K_2 .   The implications of any such refinements upon the important dimensional considerations can also be appreciated, and even evaluated.

63.   Finally, the possible importance of the unexpected correspondence between the semiempirically derived results and the 50-year-old Bernstein equations can only be assessed properly when the rationale of the former and its basic independence are understood.

PART V: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

64. The data and the analysis presented in this report are considered adequate basis for the following conclusions:

    a. The load coefficient K_2 can be made dimensionless by rational means using factors that have reasonable physical significance.

    b. It is evident that a high degree of agreement exists between the laboratory and field data and that the numerics which collapse these data are based on similar coefficients.

    c. The general form of the numeric has a certain relation to Bernstein's assumptions.

Recommendations

65. It is recommended that:

    a. Further checks, internal and independent, be made of a larger sample of the existing data.

    b. The laboratory and field data be reanalyzed point by point and presented in the form of load numerics for use by designers.

    c. Subsequent presentation, derived from further analyses of WES data (and similar information), include a thorough examination of pertinent soil parameters.

    d. Further studies include statistical information on slopes likely to be encountered in various types and locations of dry sand areas.

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Table 1

Calculation of n for a Random Sample of Small-Bin Tests, Yuma Sand

Test No.	CI_6	CI_{12}	n (Eq 21)		Test No.	CI_6	CI_{12}	n (Eq 21)
S234	49	76	0.63		S470	28	48	0.78
S240	23	35	0.50		S474	41	59	0.52
S246	23	39	0.76		S476	66	96	0.54
S253	48	76	0.66		S482	19	29	0.61
S259	69	118	0.77		S483	25	37	0.56
S261	63	108	0.78		S486	25	42	0.75
S268	73	124	0.77		S512	38	57	0.58
S279	26	43	0.73		S517	29	45	0.63
S283	20	35	0.81		S536	60	90	0.58
S292	49	76	0.63		S542	26	37	0.51
S293	60	97	0.69		S545	45	77	0.77
S306	72	116	0.69		S546	67	138	1.08
S337	23	36	0.65		S563	23	42	0.87
S342	39	61	0.64		S565	60	98	0.71
S350	40	64	0.68		S567	30	44	0.55
S399	37	60	0.70		S576	65	156	1.26
S407	26	43	0.73		S584	16	26	0.70
S411	23	35	0.61		S586	37	55	0.57
S420	15	24	0.68		S589	58	115	0.99
S432	25	44	0.53		S593	44	70	0.67
S433	49	71	0.53		S594	59	109	0.88
S438	59	104	0.82		S687	21	39	0.88
S442	55	100	0.86		S690	28	50	0.84
S463	33	54	0.71		S729	56	125	1.16
S468	54	95	0.81		S738	48	87	0.86
					S744	49	95	0.95
					S764	41	76	0.89

Table 2

Comparative Field and Laboratory Data

D/W or Slope	Field Tests (from Plate 1) $\frac{1}{\pi_1}$	Laboratory Tests (from Plates 12 & 13) Average 1st/2d Pass K_2	2d Pass z/d	Calculated Values Eq 26 k''	$\frac{k''}{z/d}$
0.0	0.182	0.47	0.14	1.23	8.8
0.1	0.083	0.32	0.09	0.55	6.1
0.2	0.046	0.20	0.05	0.43	8.6
0.3	0.025	0.12	0.03	0.35	11.7

Table 3

π_2, Calculated for Laboratory and Field Data

D/W or Slope	Field Tests $2/\pi_1 = \pi_3$	Eq 31 π_2	Laboratory Tests $0.7\,K_2 = \pi_3$	Eq 31 π_2	Eq 28 z/d	Test z/d
0.0	0.364	0.305	0.329	0.302	0.127	0.14
0.1	0.166	0.235	0.224	0.249	0.089	0.09
0.2	0.092	0.192	0.140	0.197	0.056	0.05
0.3	0.050	0.157	0.084	0.153	0.034	0.03
0.4	--	--	0.042	0.108	0.017	0.015

PLATE I

SLOPE IN PERCENT

$$\pi_i = \frac{Cg^3}{W_i}$$
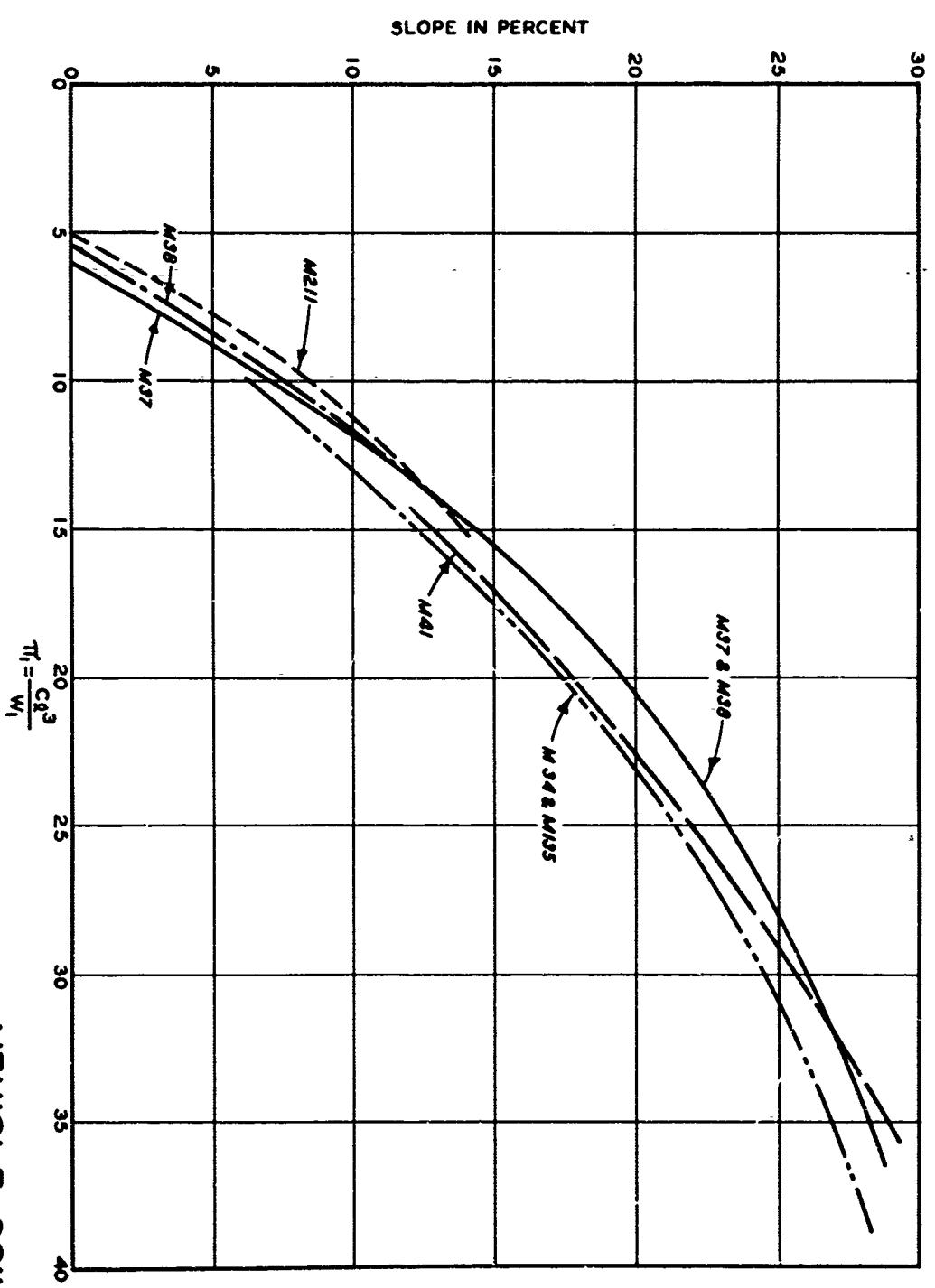
VEHICLE-SOIL
PERFORMANCE RELATIONS
M38, M211, M37, M34, M135,
AND M41 VEHICLES
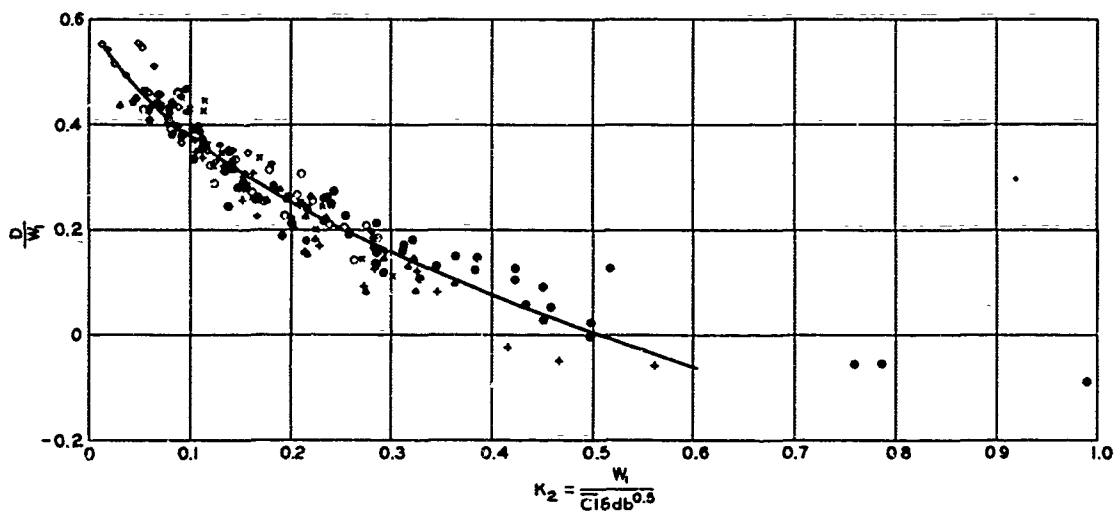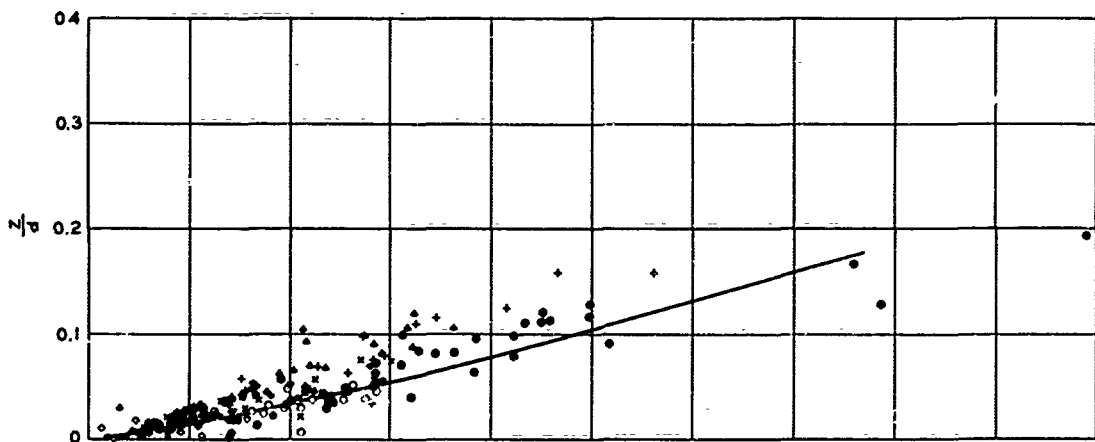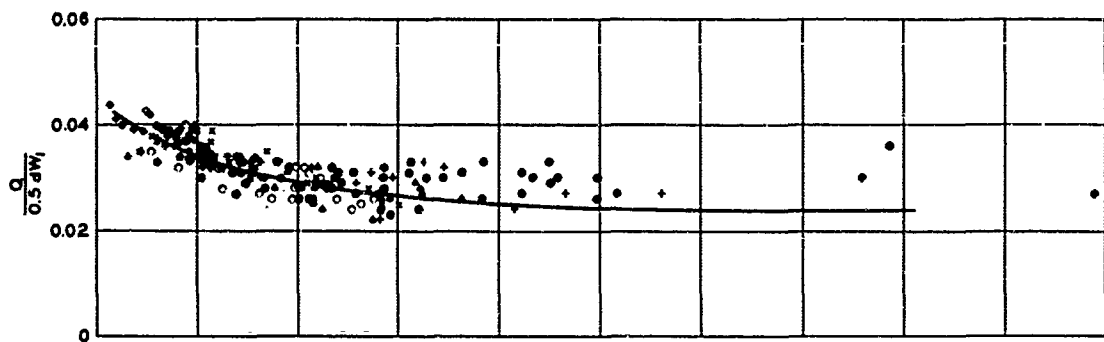
TORQUE, SINKAGE, AND
DRAWBAR PULL

9.00-14, 2-, 4-, AND 8-PR TIRES
FIRST PASS, MAXIMUM PULL CONDITION
YUMA SAND
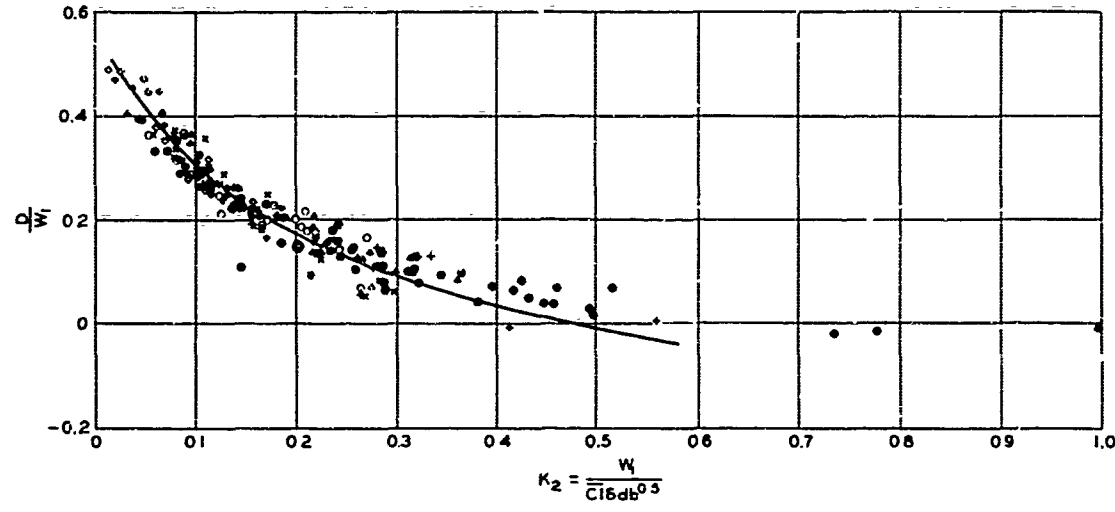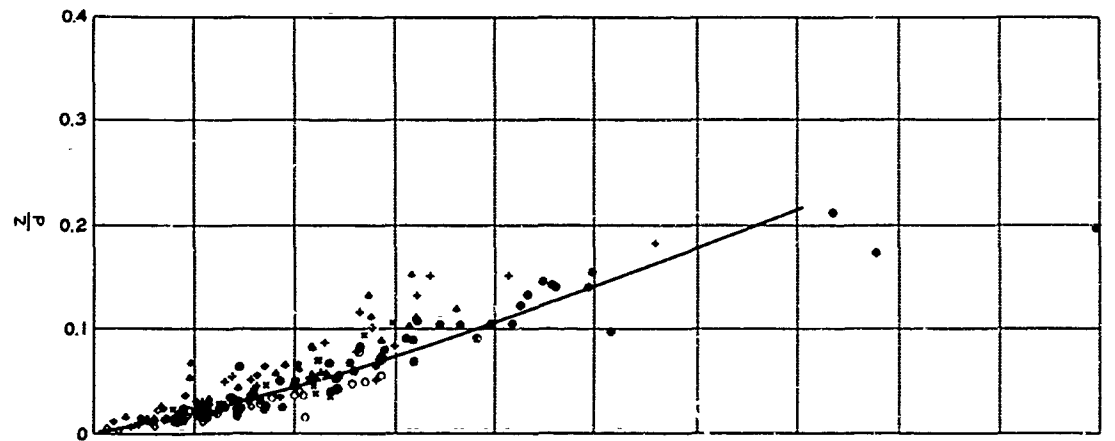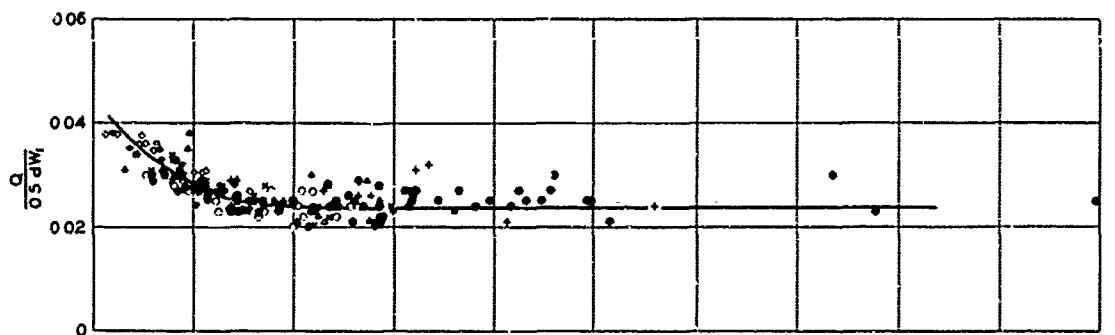
PLATE 2

LEGEND

TIRE DEFLECTION/SECTION HEIGHT IN PERCENT	CONE INDEX		
	20	40	60
15	●	●	●
25	+	×	●
35	●	●	●

NOTE: 152 TESTS.

TORQUE, SINKAGE, AND
DRAWBAR PULL

9.00-14, 2-, 4-, AND 8-PR TIRES
SECOND PASS, MAXIMUM PULL CONDITION
YUMA SAND
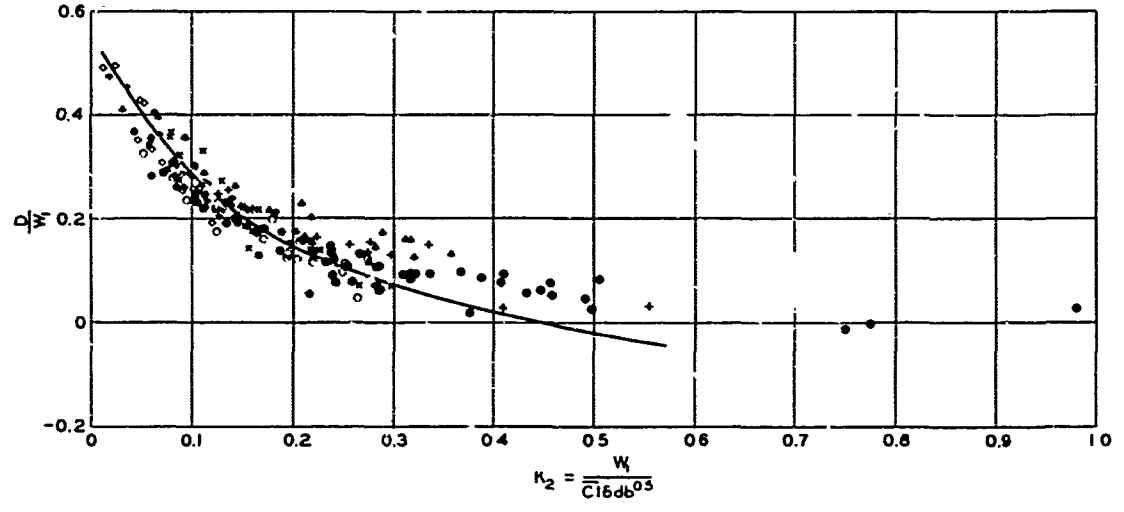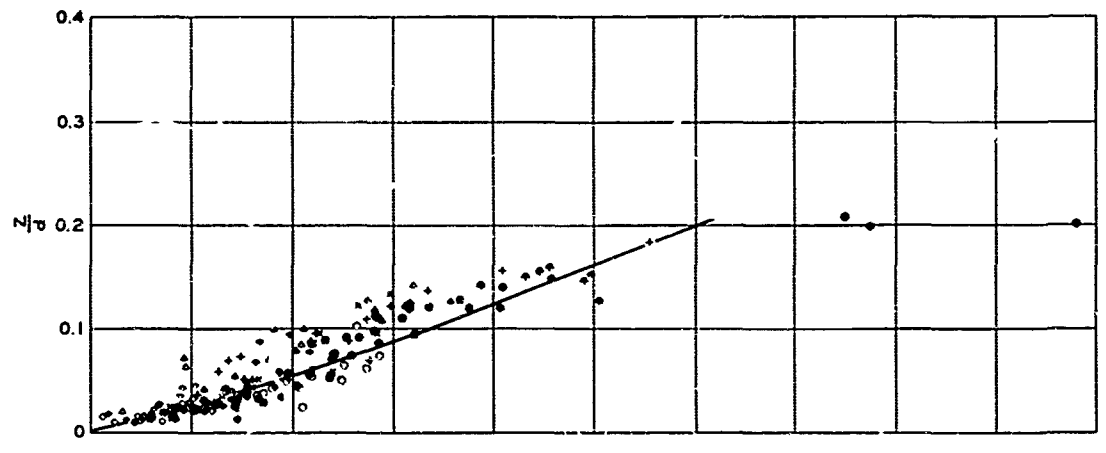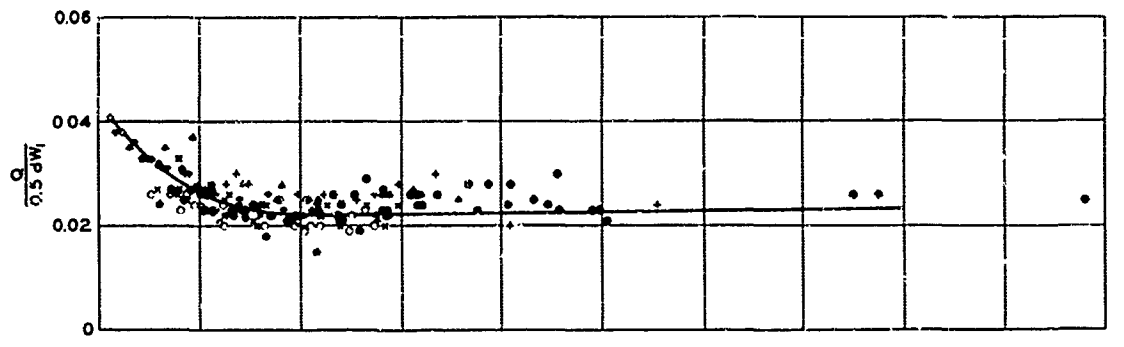
PLATE 3

TORQUE, SINKAGE, AND
DRAWBAR PULL

9.00-14, 2-, 4-, AND 8-PR TIRES
THIRD PASS, MAXIMUM PULL CONDITION
YUMA SAND
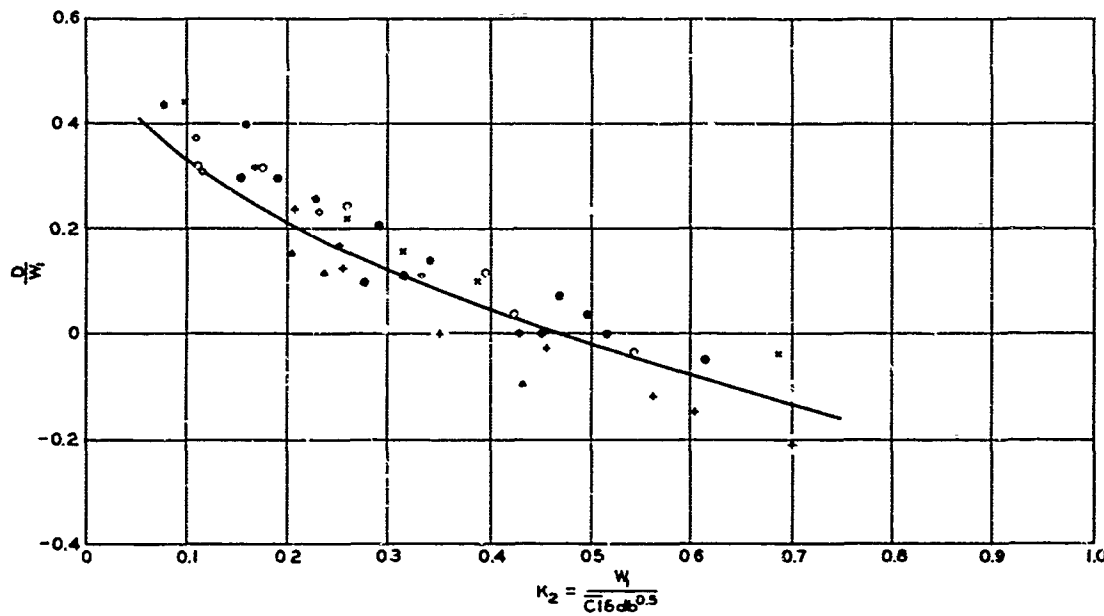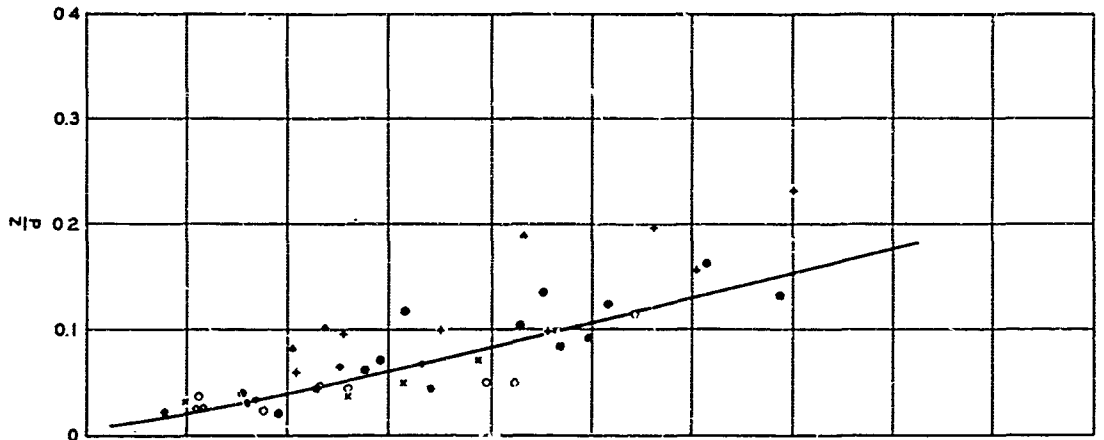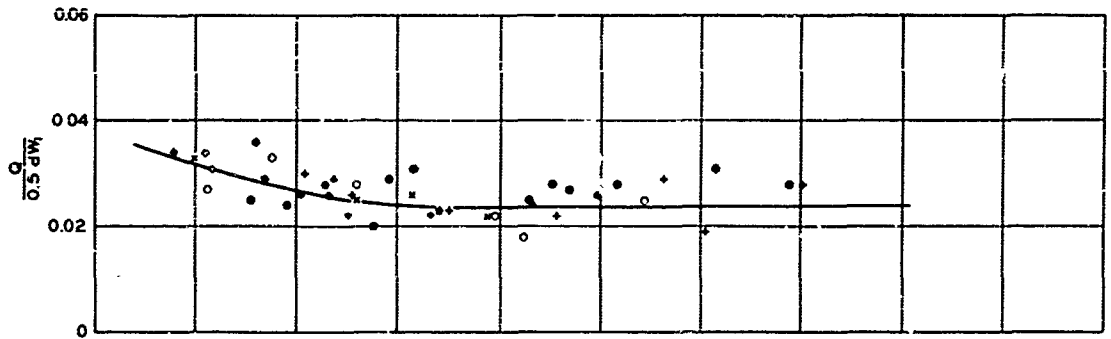
LEGEND

TIRE DEFLECTION/SECTION HEIGHT IN PERCENT	CONE INDEX		
	20	40	60
15	●	●	○
25	+	×	●
35	◆	◆	◇

NOTE: 145 TESTS.

$$K_2 = \frac{W_i}{C16db^{0.5}}$$

PLATE 4

TORQUE, SINKAGE, AND
DRAWBAR PULL

4.50-7, 2-PR TIRE

FIRST PASS, MAXIMUM PULL CONDITION
YUMA SAND
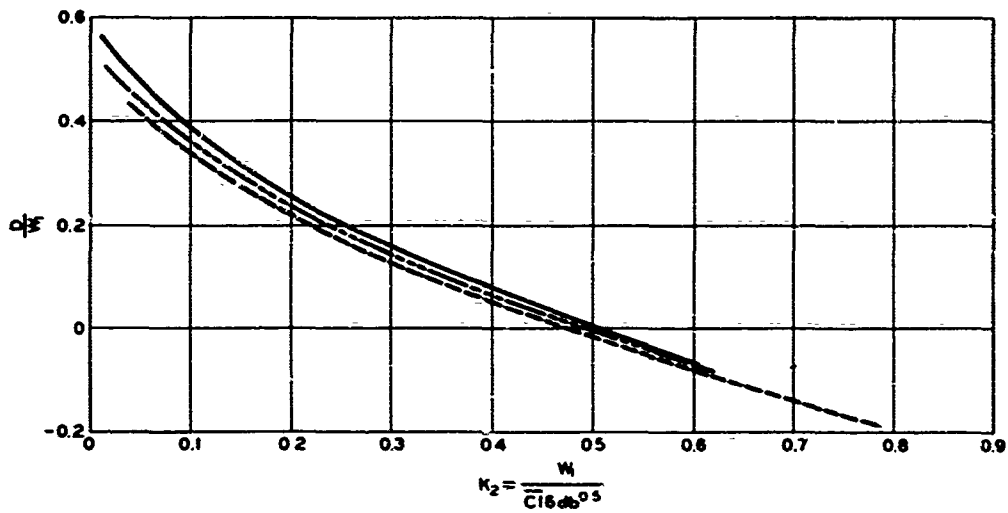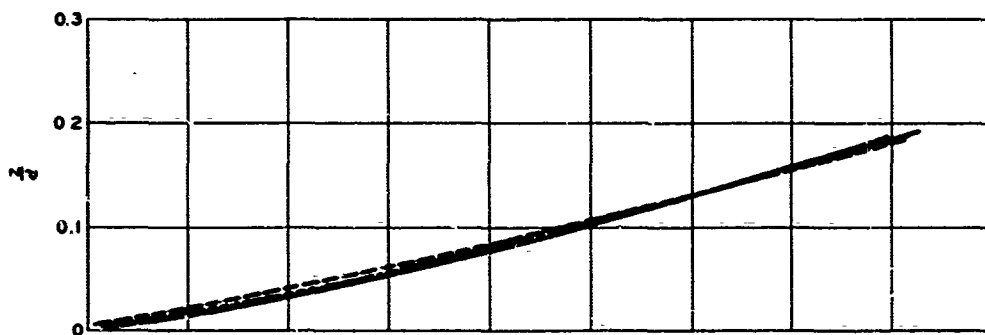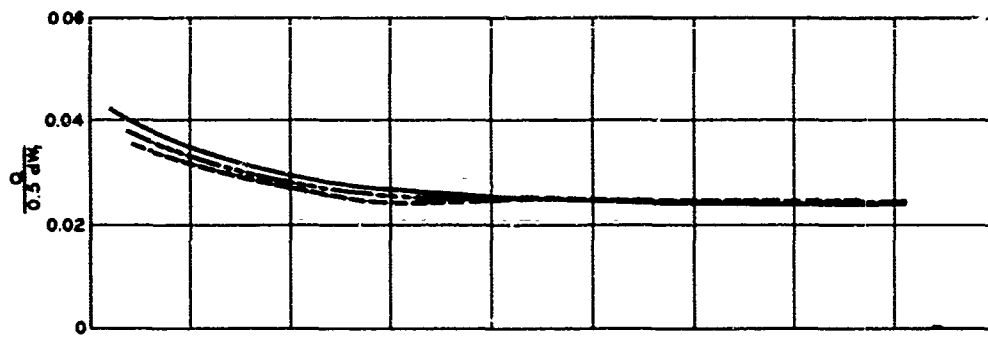
PLATE 5

LEGEND

——————— 9.00-14 TIRE
——·——·— 4.50-7 TIRE
——··——·· "AVERAGE-AVERAGE"
CURVE

AVERAGE AND "AVERAGE-AVERAGE"
TORQUE, SINKAGE, AND
DRAWBAR PULL
9.00-14 AND 4.50-7 TIRES
FIRST PASS, MAXIMUM PULL CONDITION
YUMA SAND
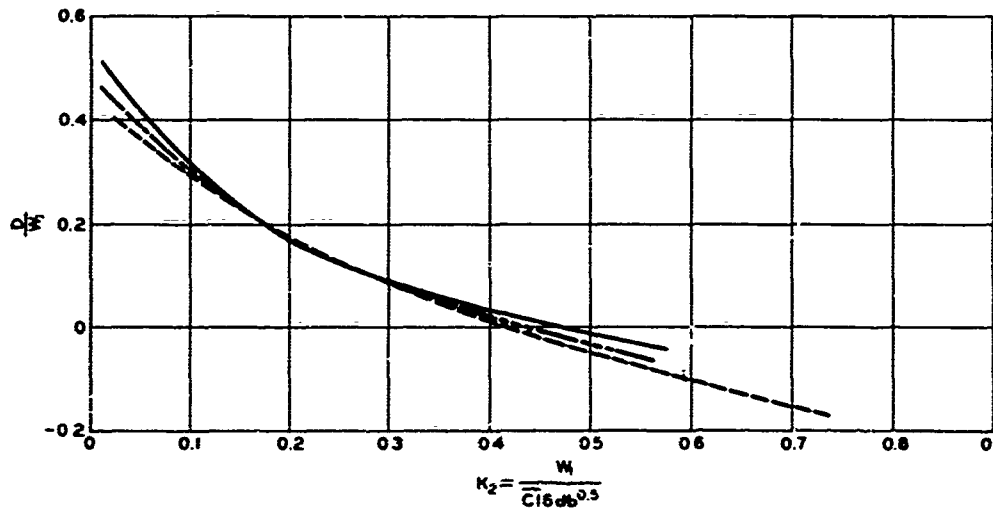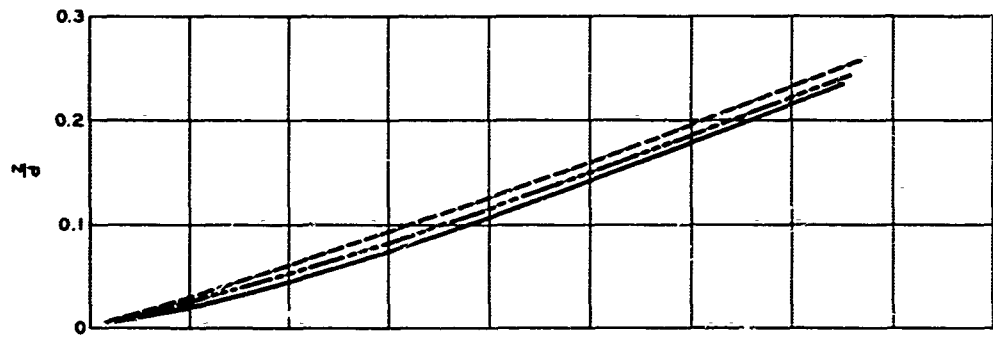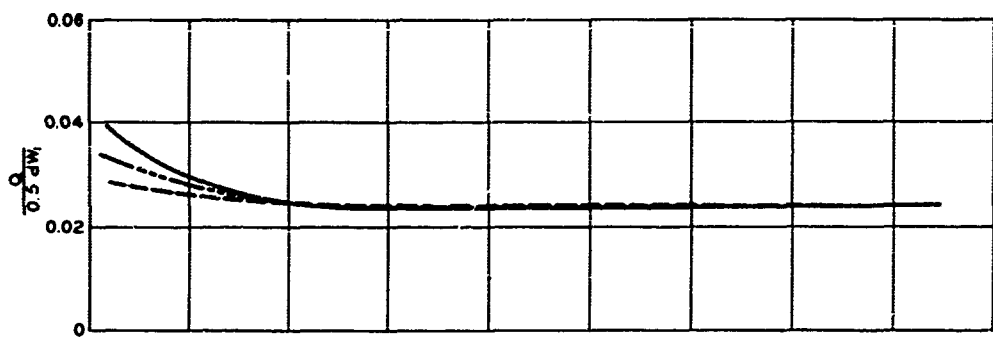
PLATE 6

LEGEND
——————— 9.00-14 TIRE
——————— 4.50-7 TIRE
——·——·—— "AVERAGE-AVERAGE"
            CURVE

AVERAGE AND "AVERAGE-AVERAGE"
TORQUE, SINKAGE, AND
DRAWBAR PULL
9.00-14 AND 4.50-7 TIRES
SECOND PASS, MAXIMUM PULL CONDITION
YUMA SAND

PLATE 7

LEGEND

—————— 9.00-14 TIRE
— — — — 4.50-7 TIRE
— · — · — "AVERAGE-AVERAGE" CURVE

AVERAGE AND "AVERAGE-AVERAGE"
TORQUE, SINKAGE, AND
DRAWBAR PULL
9.00-14 AND 4.50-7 TIRES
THIRD PASS, MAXIMUM PULL CONDITION
YUMA SAND

PLATE 8
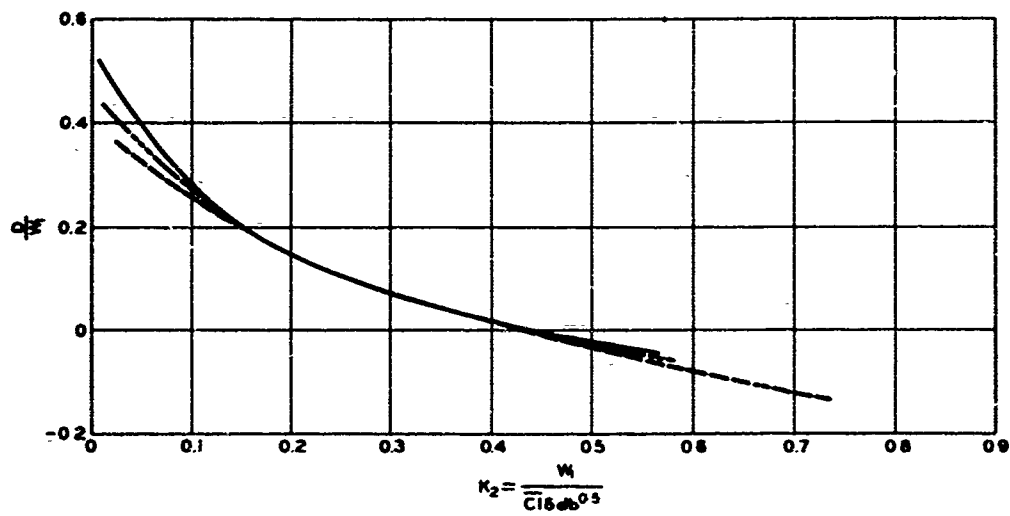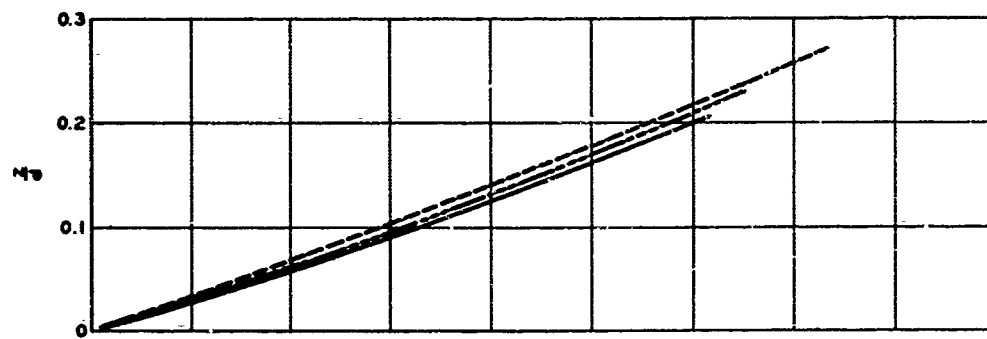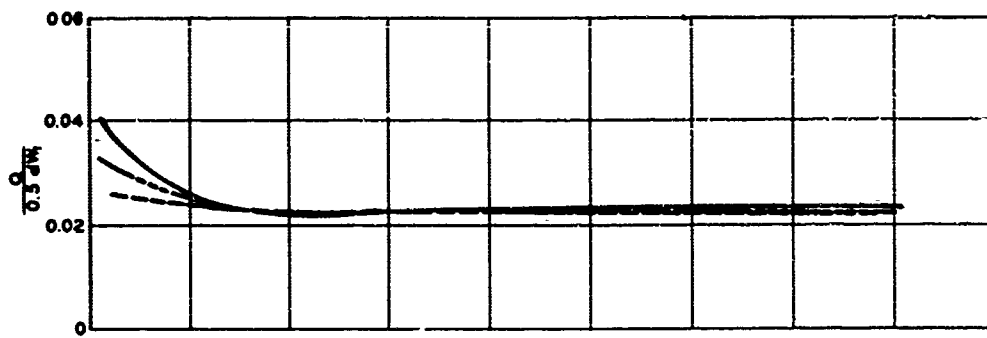
AVERAGE AND "AVERAGE–AVERAGE"
SINKAGE AND TOWED FORCE
9.00–14 AND 4.50–7 TIRES
FIRST PASS, TOWED CONDITION, YUMA SAND

LEGEND

———— 9.00–14 TIRE
– – – – 4.50–7 TIRE
—·—·— "AVERAGE–AVERAGE" CURVE

PLATE 9

AVERAGE AND "AVERAGE-AVERAGE"
SINKAGE AND TOWED FORCE
9.00-14 AND 4.50-7 TIRES
THIRD PASS, TOWED CONDITION, YUMA SAND

PLATE 11

PLATE 10

LEGEND

9.00-14 TIRE
4.50-7 TIRE
"AVERAGE-AVERAGE" CURVE

$\frac{R}{W_i}$

$\frac{z}{d}$

$K_2 = \dfrac{W_i}{C16db^{0.5}}$
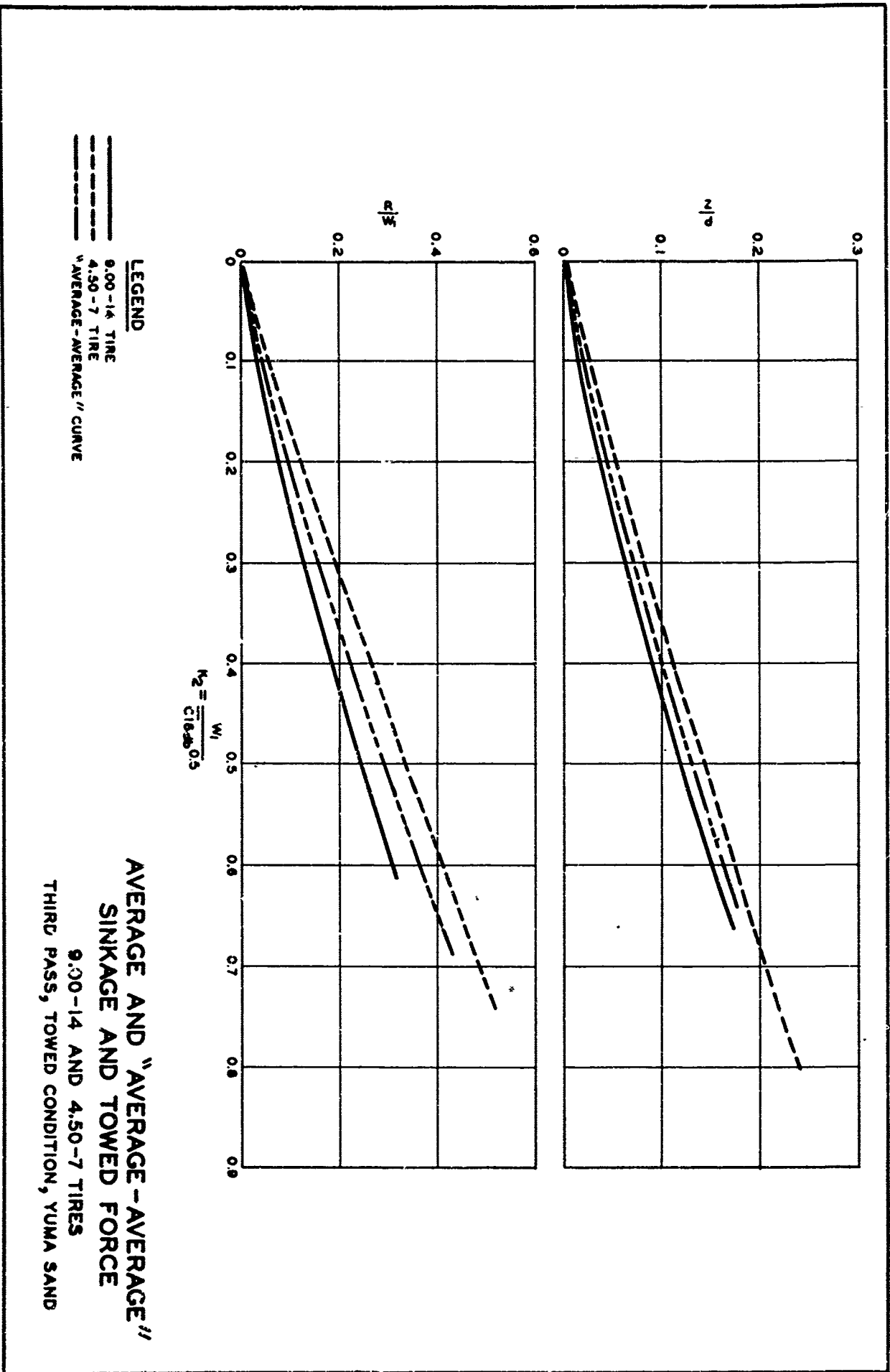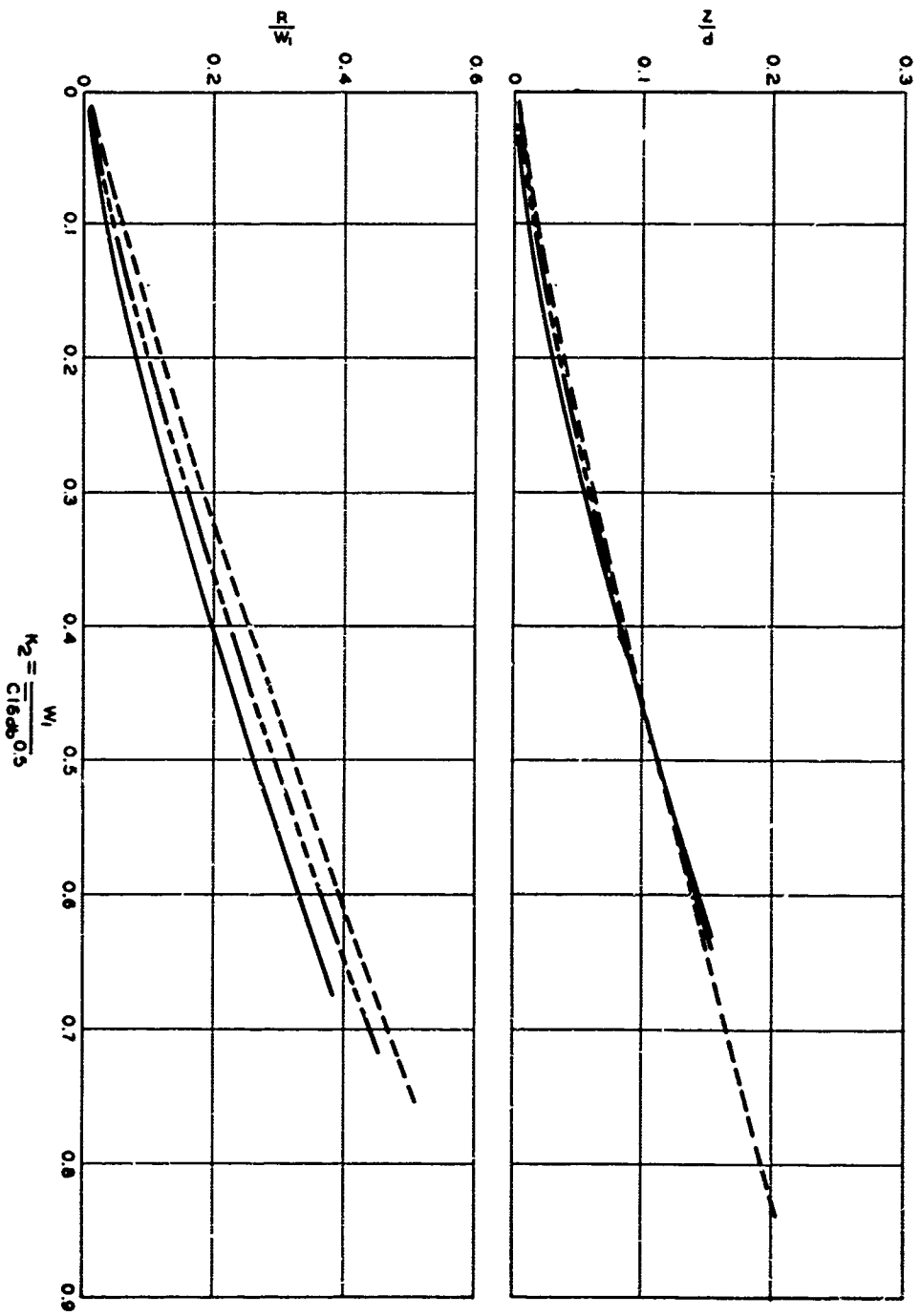
AVERAGE AND "AVERAGE-AVERAGE"
SINKAGE AND TOWED FORCE
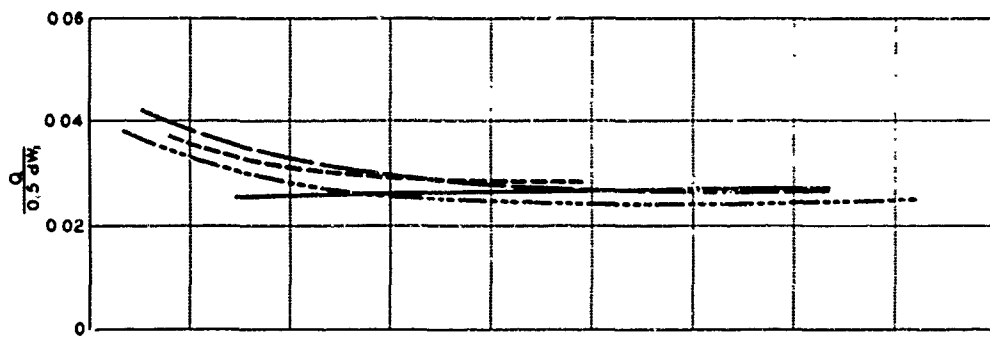9.00-14 AND 4.50-7 TIRES
SECOND PASS, TOWED CONDITION, YUMA SAND
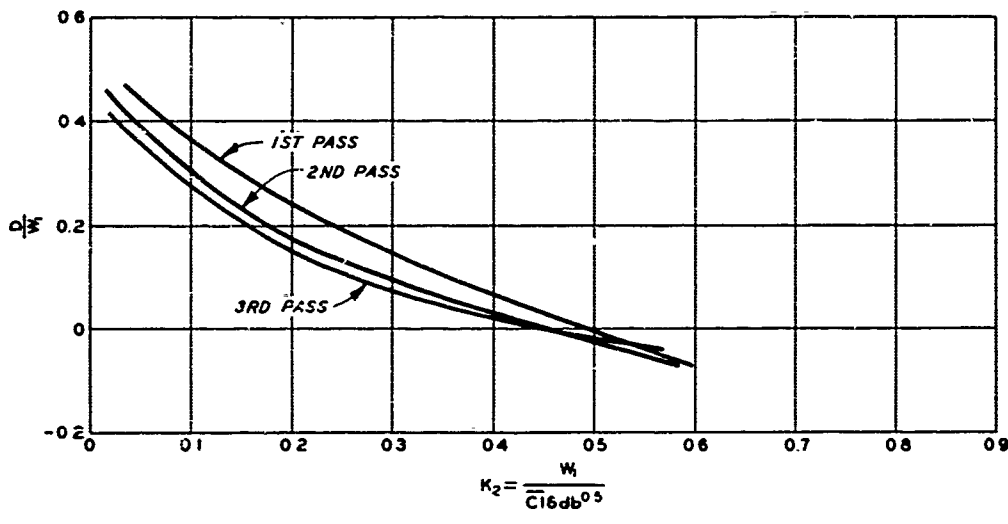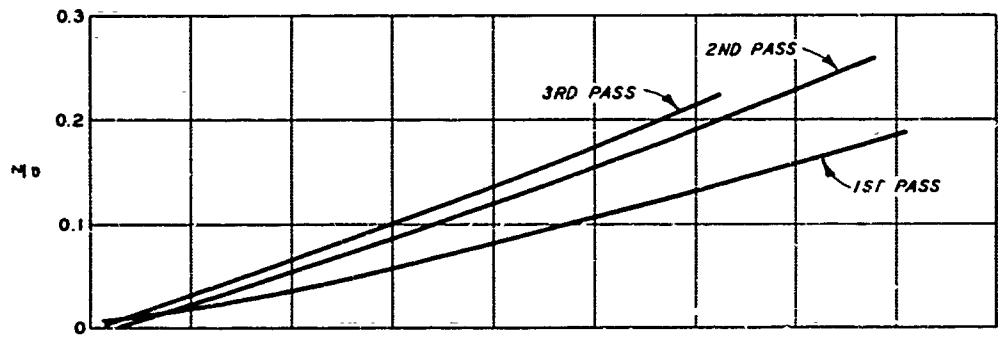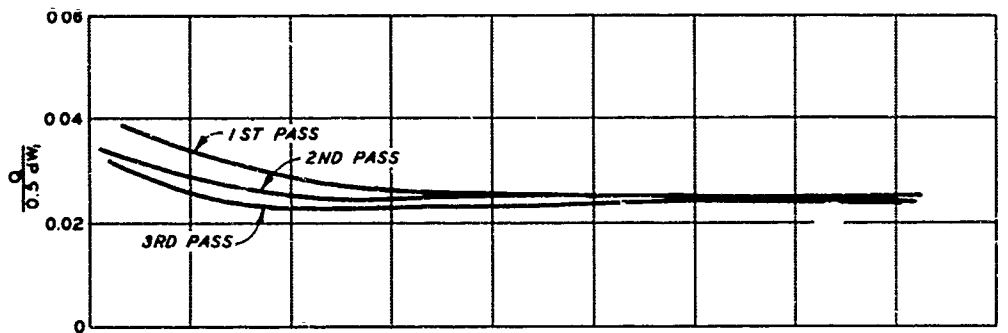
NOTE: CURVES REPRESENT THE AVERAGES
OF THE 9.00-14, 2-, 4-, AND 8-PR
AND THE 4.50-7, 2-PR TIRES.

**AVERAGE TORQUE, SINKAGE
AND DRAWBAR PULL**
9.00-14 AND 4.50-7 TIRES
FIRST, SECOND, AND THIRD PASSES
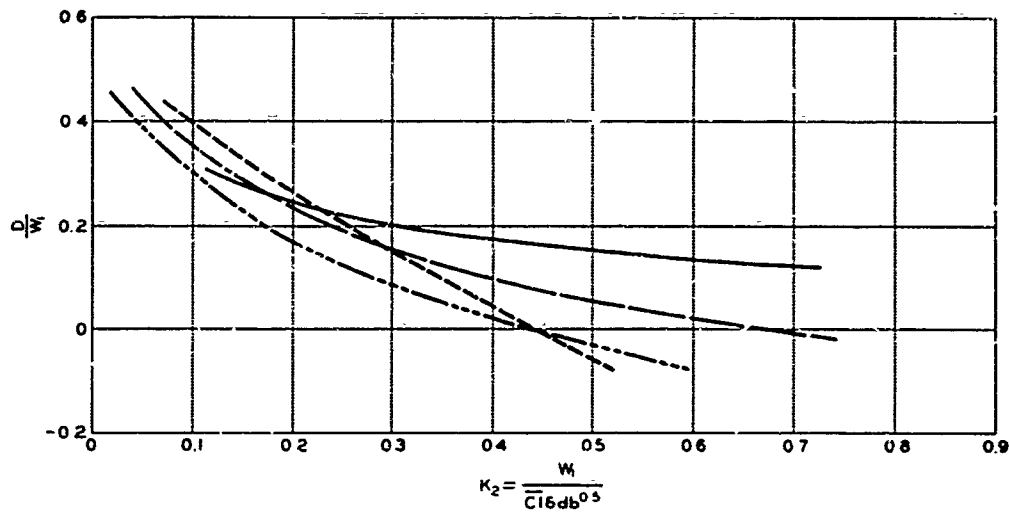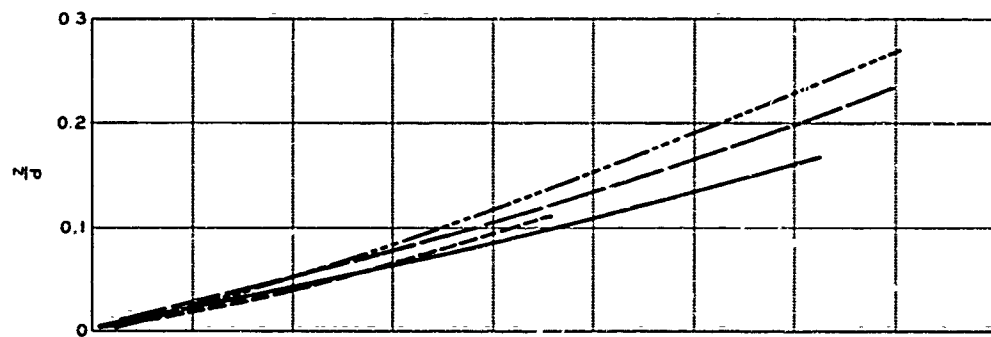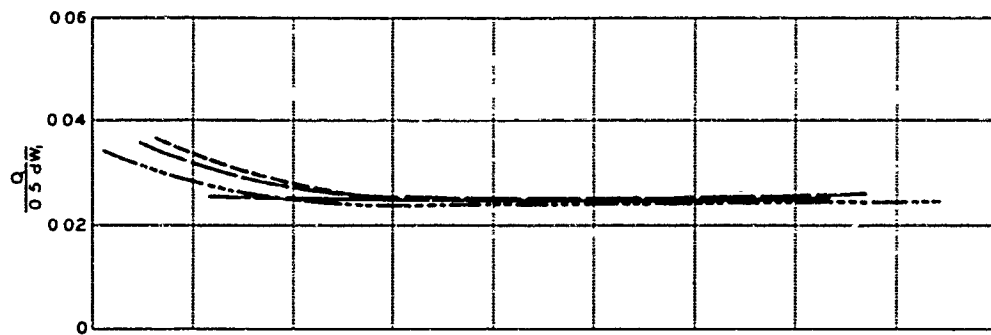MAXIMUM PULL CONDITION
YUMA SAND

PLATE 12

PLATE 13
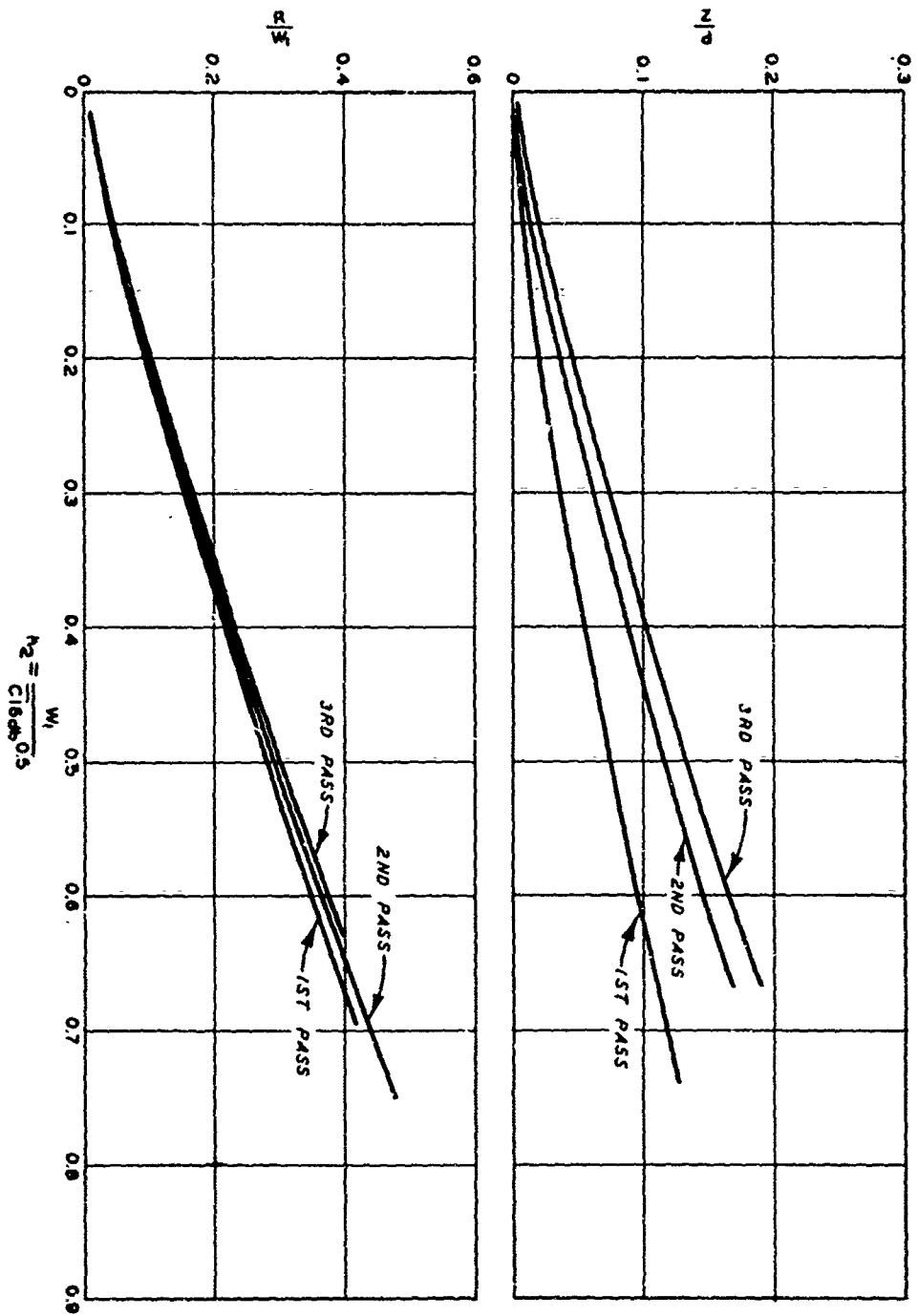
NOTE: CURVES REPRESENT THE AVERAGES
OF THE 9.00-14, 2-, 4-, AND 8-PR
AND THE 4.50-7, 2-PR TIRES.

AVERAGE SINKAGE AND TOWED FORCE
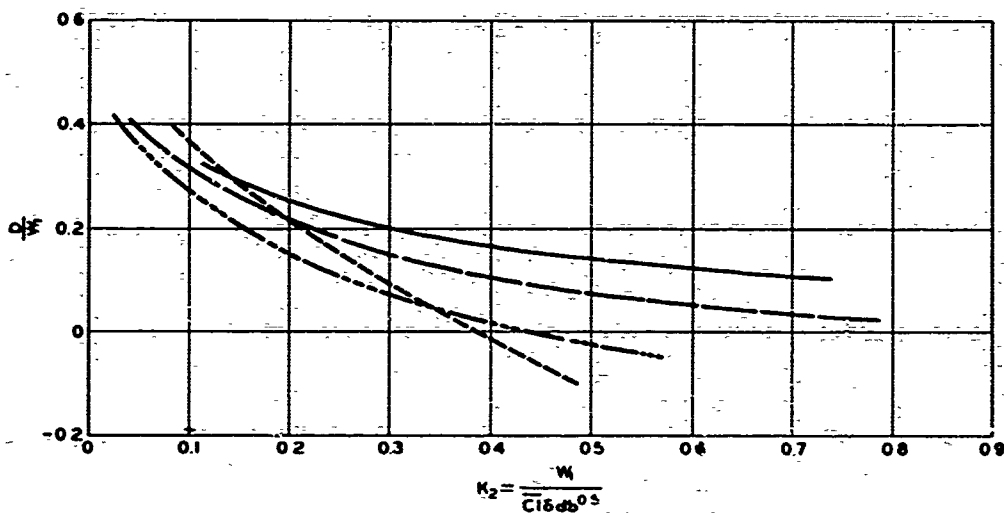FIRST, SECOND, AND THIRD PASSES
TOWED CONDITION
YUMA SAND
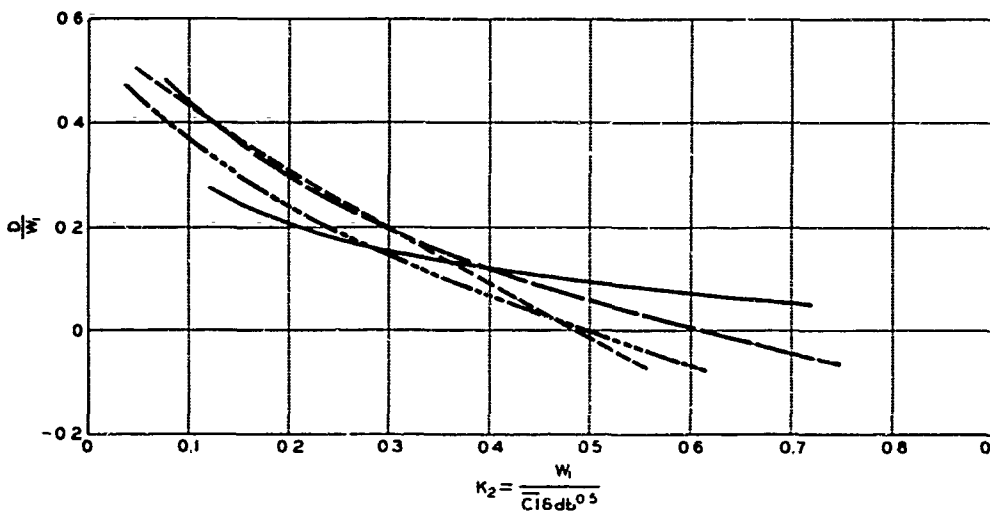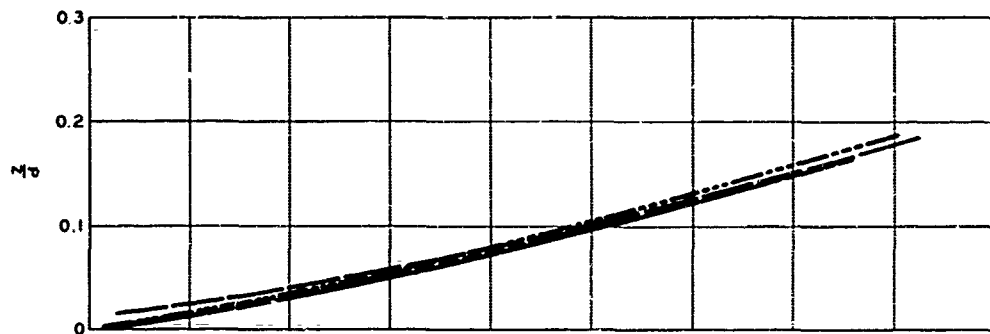
PLATE 13

LEGEND

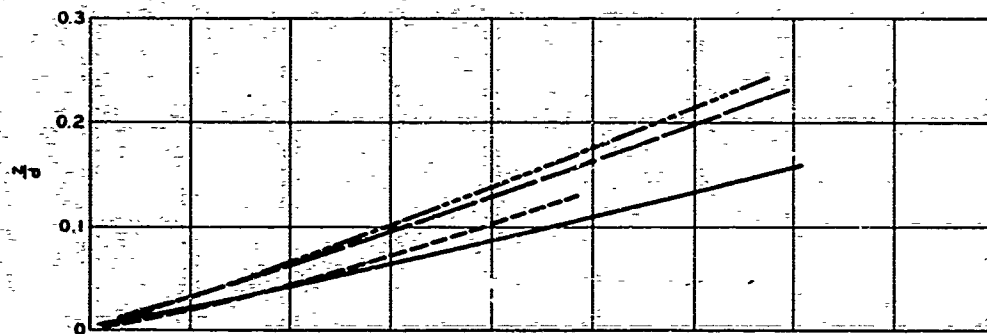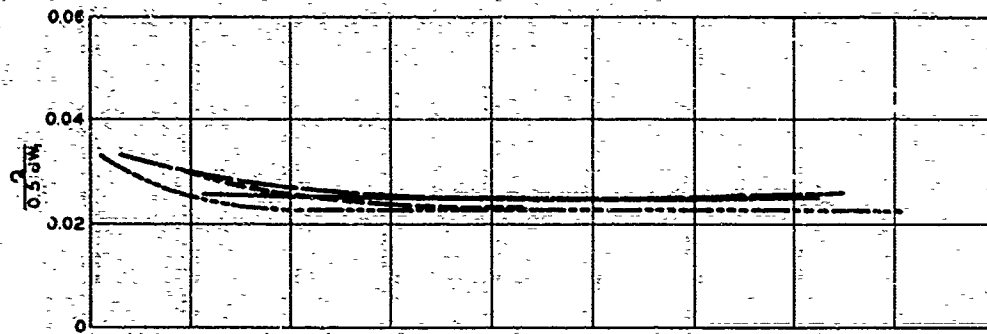——————— 1.75-26 BICYCLE TIRE
- - - - - - 16×15 TERRA-TIRE
—·—·—·— AVERAGE 9.00-14 AND 4.50-7 CURVE
——··——··— 6.00-16, 2- AND 4-PR SMOOTH TIRES

TORQUE, SINKAGE, AND
DRAWBAR PULL
VARIOUS TIRE SIZES
FIRST PASS, MAXIMUM PULL CONDITION
YUMA SAND

PLATE 14

LEGEND
———————— 1.75-26 BICYCLE TIRE
-------- 16×15 TERRA-TIRE
—·—·— AVERAGE 9.00-14 AND 4.50-7 CURVE
———— 6.00-16, 2- AND 4-PR SMOOTH TIRES

**TORQUE, SINKAGE, AND
DRAWBAR PULL**

VARIOUS TIRE SIZES

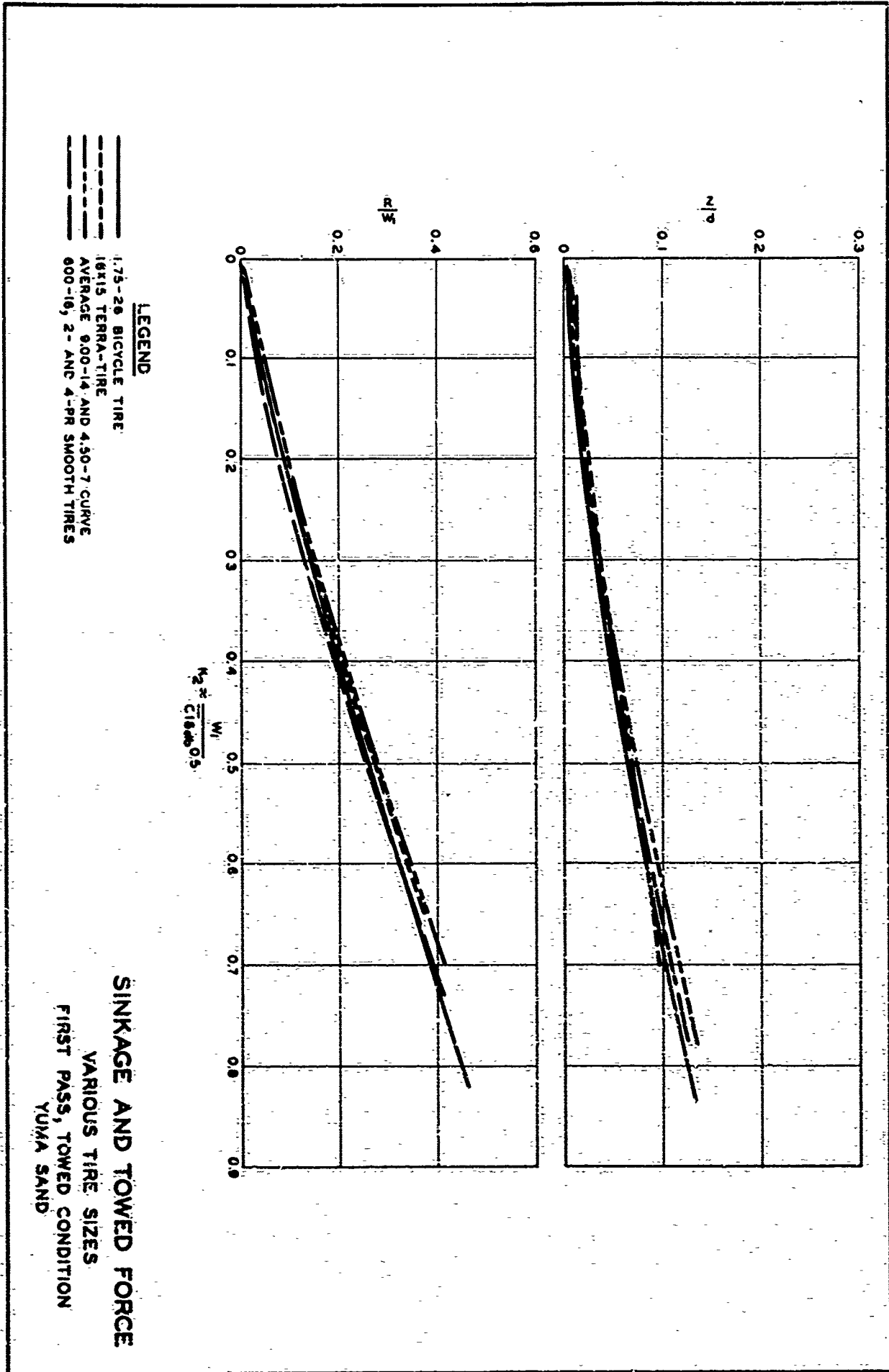SECOND PASS, MAXIMUM PULL CONDITION
YUMA SAND

PLATE 15

TORQUE, SINKAGE, AND
DRAWBAR PULL

VARIOUS TIRE SIZES

THIRD PASS, MAXIMUM PULL CONDITION
YUMA SAND

PLATE 16

SINKAGE AND TOWED FORCE
VARIOUS TIRE SIZES
FIRST PASS, TOWED CONDITION
YUMA SAND

LEGEND

——— 1.75-26 BICYCLE TIRE
—·—· 16x15 TERRA-TIRE
———— AVERAGE 9.00-14 AND 4.50-7 CURVE
——— 600-16, 2- AND 4-PR SMOOTH TIRES

PLATE 17

SINKAGE AND TOWED FORCE
VARIOUS TIRE SIZES
THIRD PASS, TOWED CONDITION
YUMA SAND

LEGEND

1.75-26 BICYCLE TIRE
16X15 TERRA-TIRE
AVERAGE 9.00-14 AND 4.50-7 CURVE
600-16, 2- AND 4-PR SMOOTH TIRES

$$N_2 = \frac{W_1}{C16d_b^{0.5}}$$

$\frac{R}{W}$

$\frac{z}{d}$

PLATE 19

PLATE 18

LEGEND

—————— 1.75-26 BICYCLE TIRE
— — — — 16X15 TERRA-TIRE
— - — - — AVERAGE 9.00-14 AND 4.50-7 CURVE
— -- — -- 600-16, 2- AND 4-PR SMOOTH TIRES

$$N_2 = \frac{W_i}{C i b d b^{0.5}}$$

SINKAGE AND TOWED FORCE

VARIOUS TIRE SIZES

SECOND PASS, TOWED CONDITION

YUMA SAND

DRAWBAR PULL VS
WIDTH/DIAMETER

FIRST AND THIRD PASSES
YUMA SAND

LEGEND
O   1.75-26   BICYCLE TIRE
△   6.00-16   TIRE
□   9.00-14   AND 4.50-7 TIRES
▽   16×15   TERRA-TIRE

PLATE 20

SINKAGE AND TOWED FORCE
11.00-20 TIRE
FIRST PASS, TOWED CONDITION
MORTAR SAND
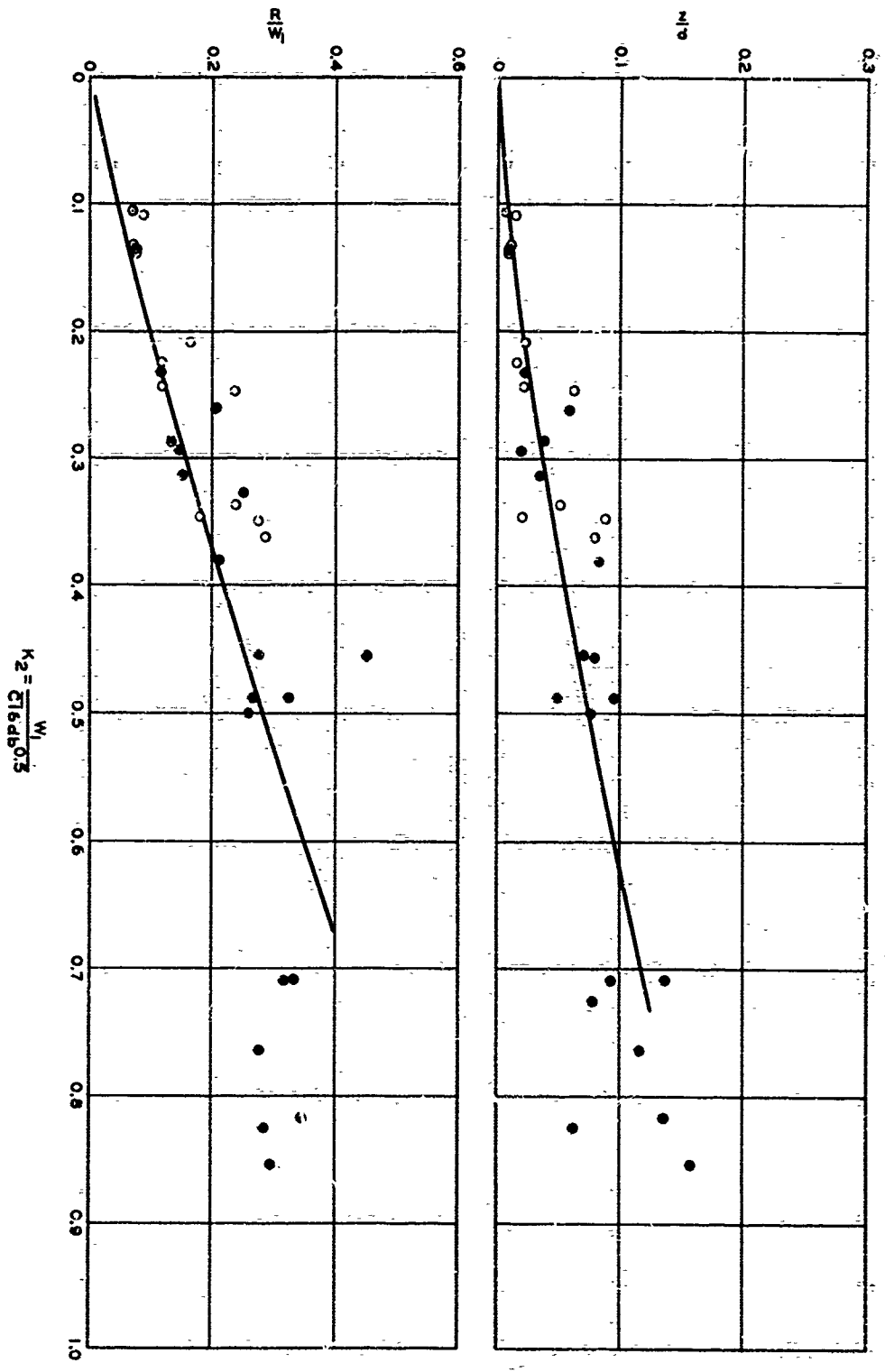
LEGEND
CONE INDEX
20   40   60
●   ●   ○
NOTE: 31 TESTS.

NOTE: CURVE IS "AVERAGE-AVERAGE"
CURVE FOR SMALLER TIRES ON
YUMA SAND.

$$K_2 = \frac{W_1}{c16db^{0.3}}$$

PLATE 21

SINKAGE AND TOWED FORCE
11:00-20 TIRE
THIRD PASS, TOWED CONDITION
MORTAR SAND

PLATE 23

PLATE 22

SINKAGE AND TOWED FORCE
11.00 - 20 TIRE
SECOND PASS, TOWED CONDITION
MORTAR SAND
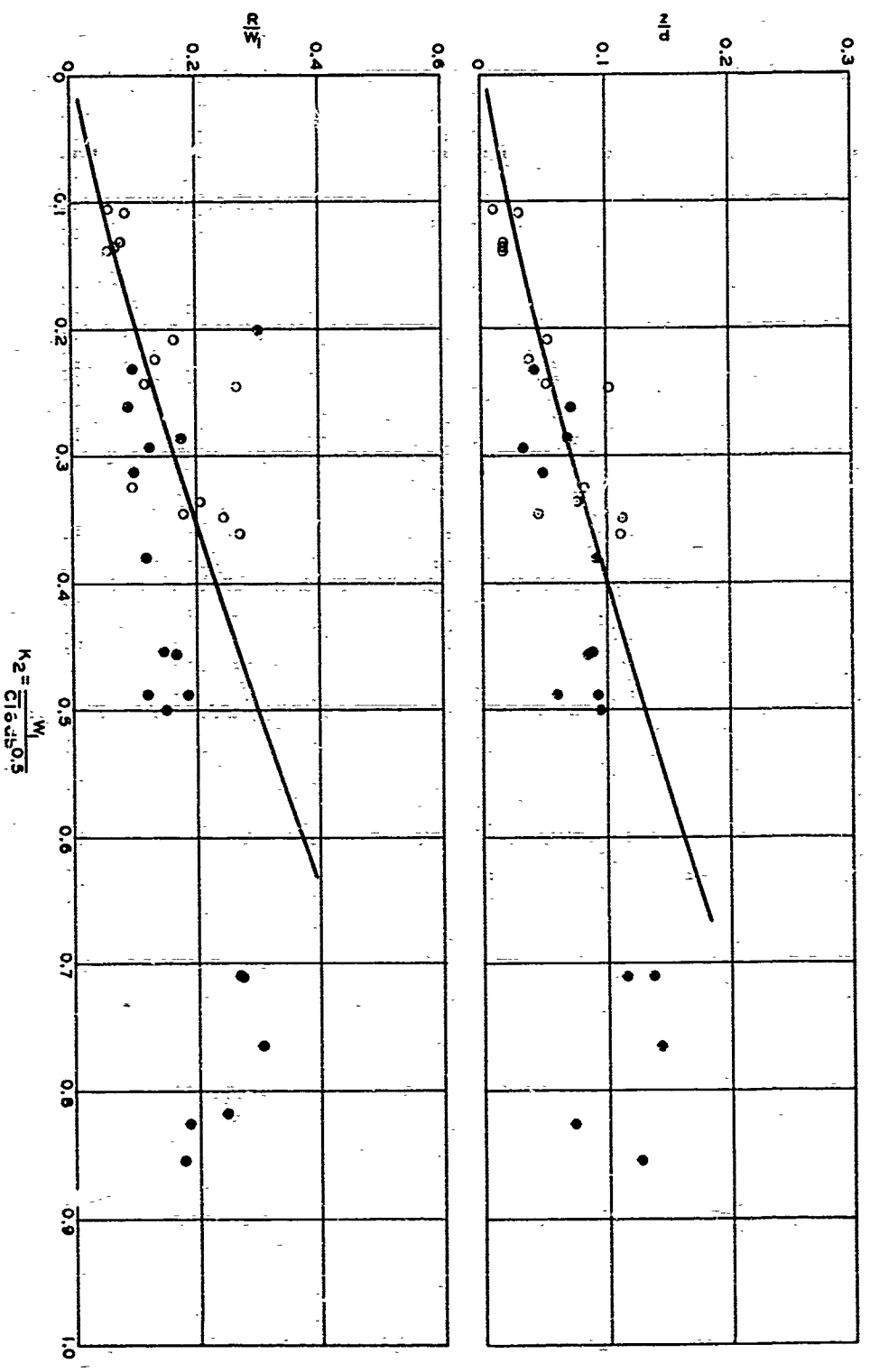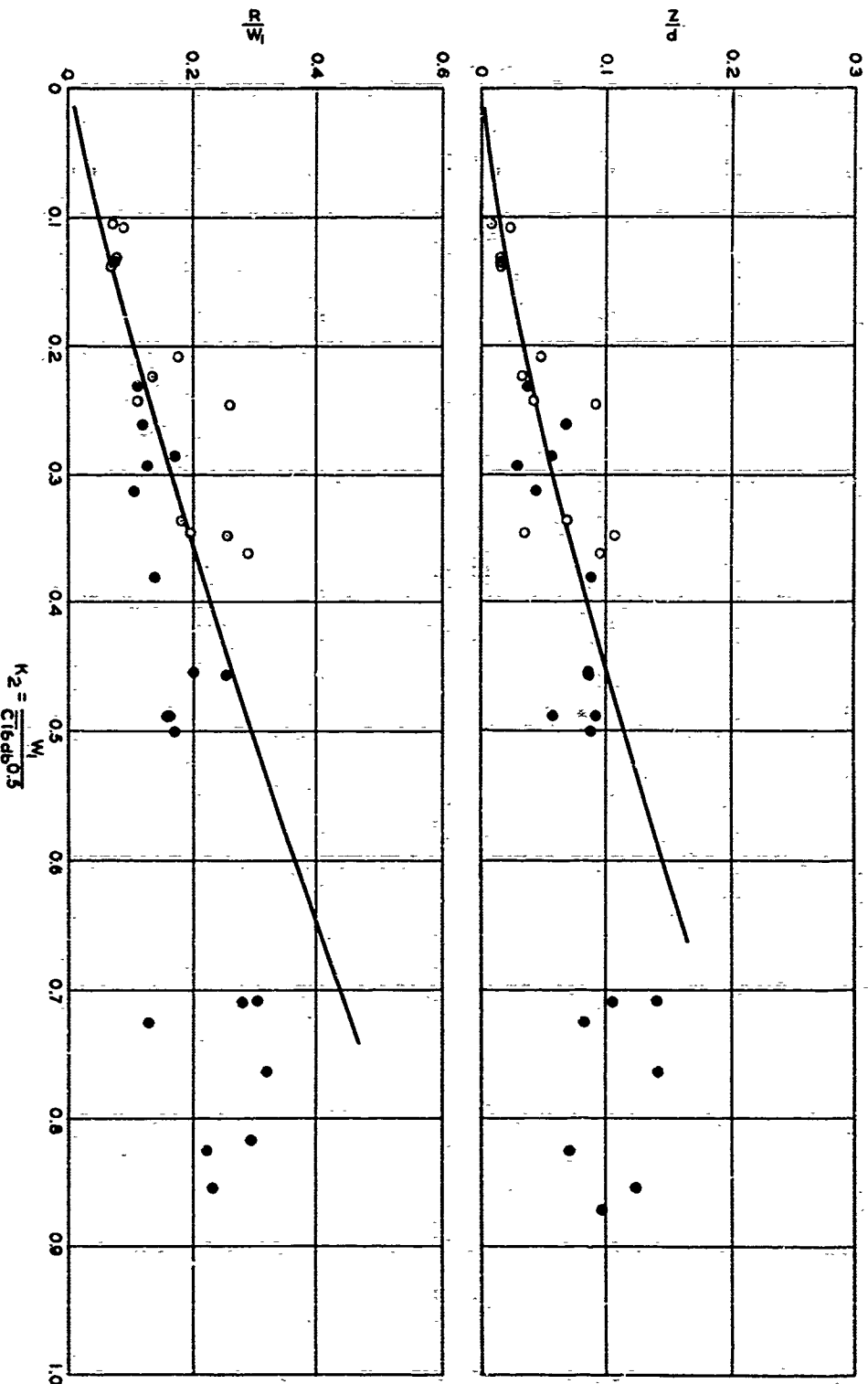
LEGEND
CONE INDEX
20   40   60
NOTE: 32 TESTS

$K_2 = \dfrac{W_l}{\overline{C}i6cb^{0.5}}$

$\dfrac{R}{W_l}$

$\dfrac{z}{d}$

$$\frac{z}{d} = 1.44 \left(\pi_2\right)^2$$
(EQUATION 28)

LEGEND
O   LABORATORY DATA $(n \doteq 0.5)$
□   FIELD DATA $(n \doteq 1.0)$
(REFERENCE 1)

$\frac{D}{W}$ OR SLOPE

π_2 (EQUATION 29)
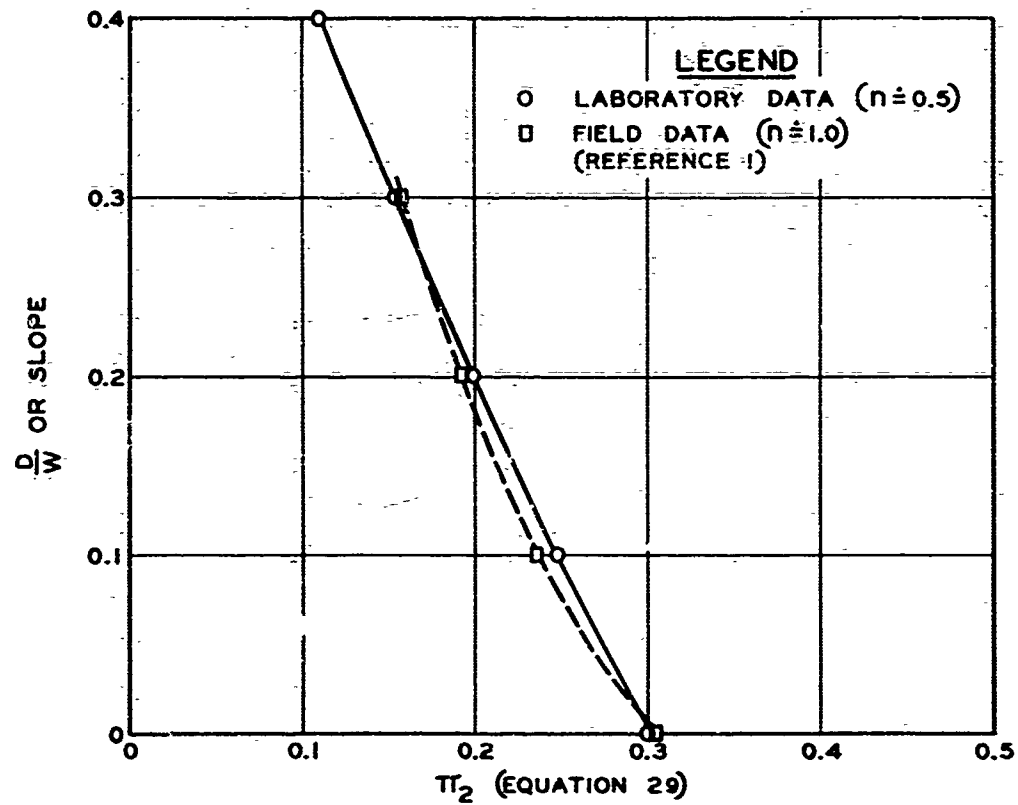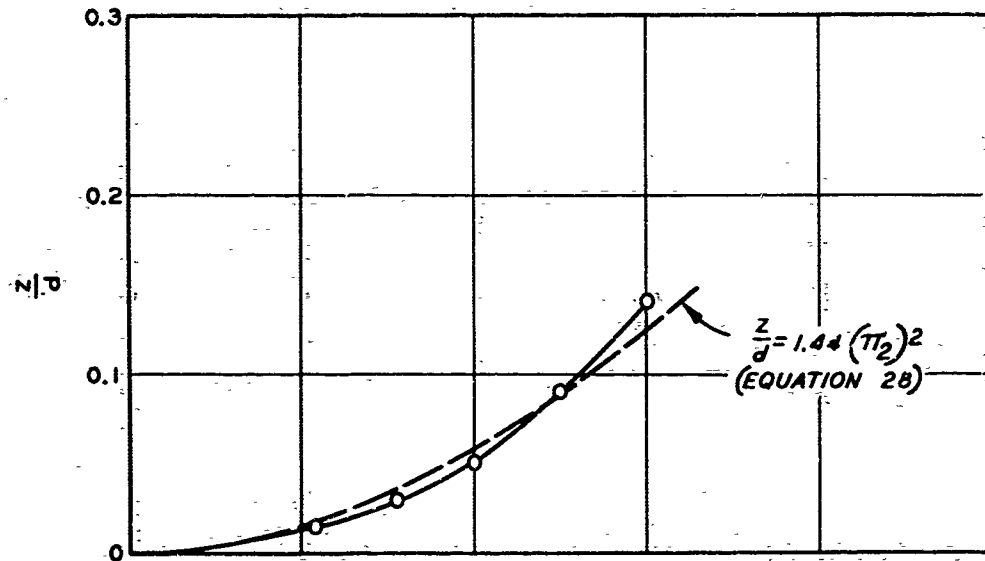
CORRELATION OF
LABORATORY AND FIELD DATA

PLATE 24

$$\frac{z}{d} = 1.44\,(\pi_2)^2 \quad (EQ.\ 28)$$
$$(SECOND\ PASS)$$

$$\frac{D}{W_1} = 0.78 - 2.84\,\pi_2$$

π_2 (EQUATION 29)

SINKAGE AND
DRAWBAR PULL VS π_2
9.00-14 TIRE
FIRST PASS, YUMA SAND

PLATE 25
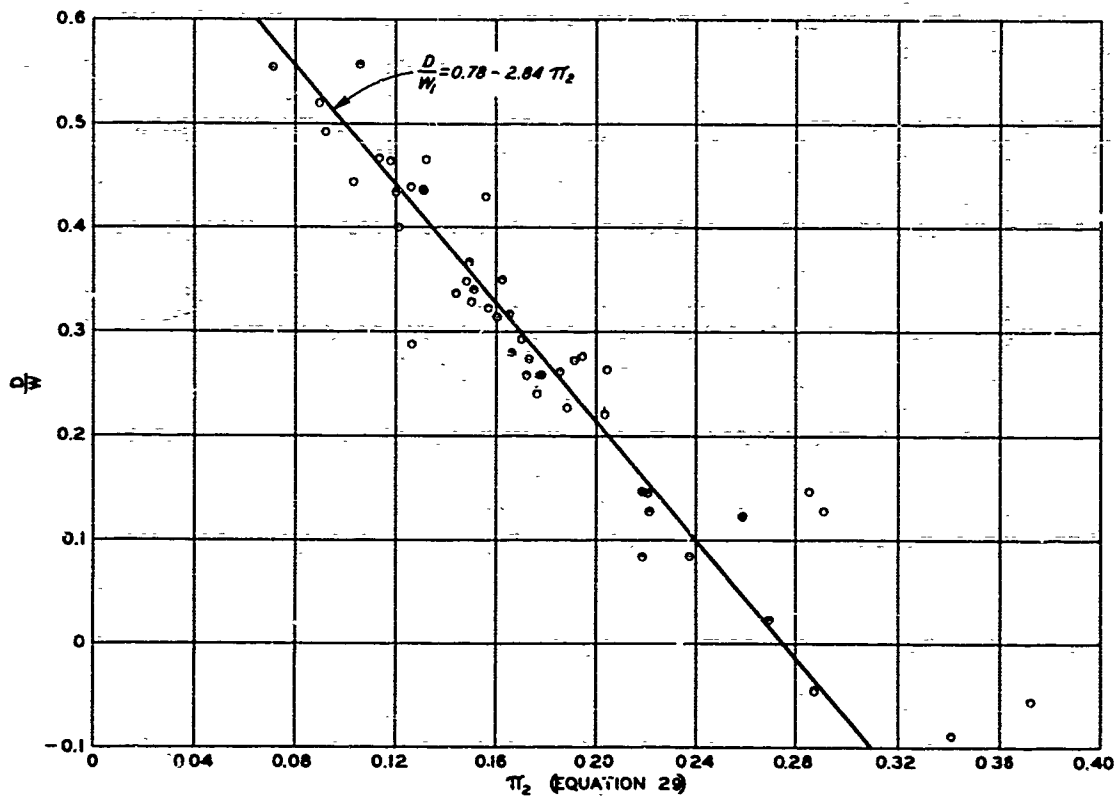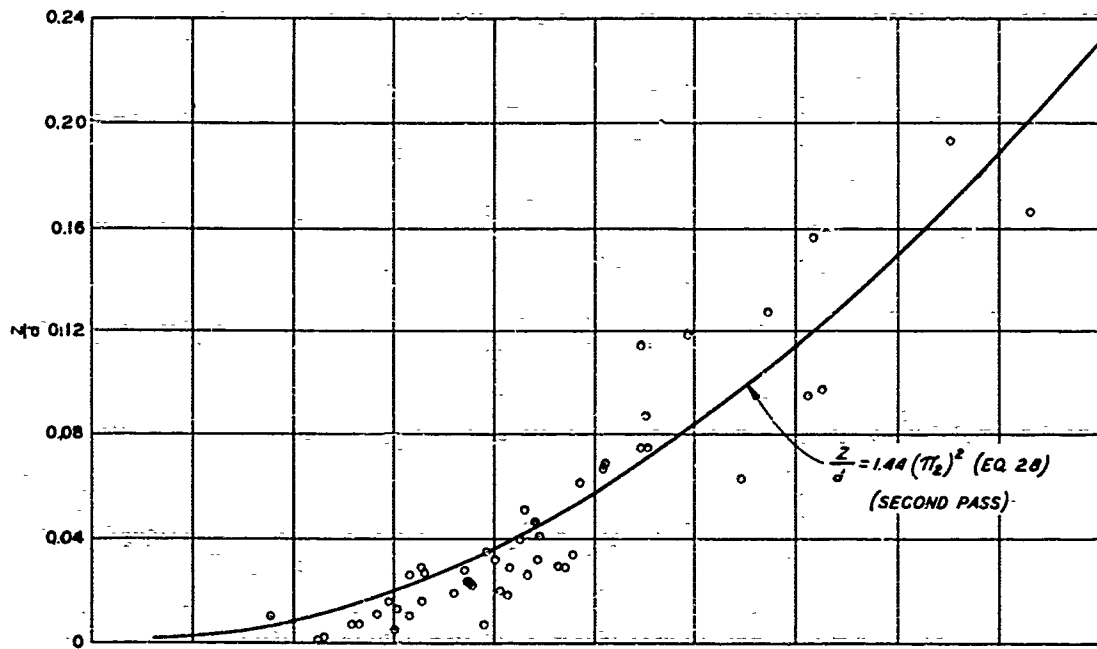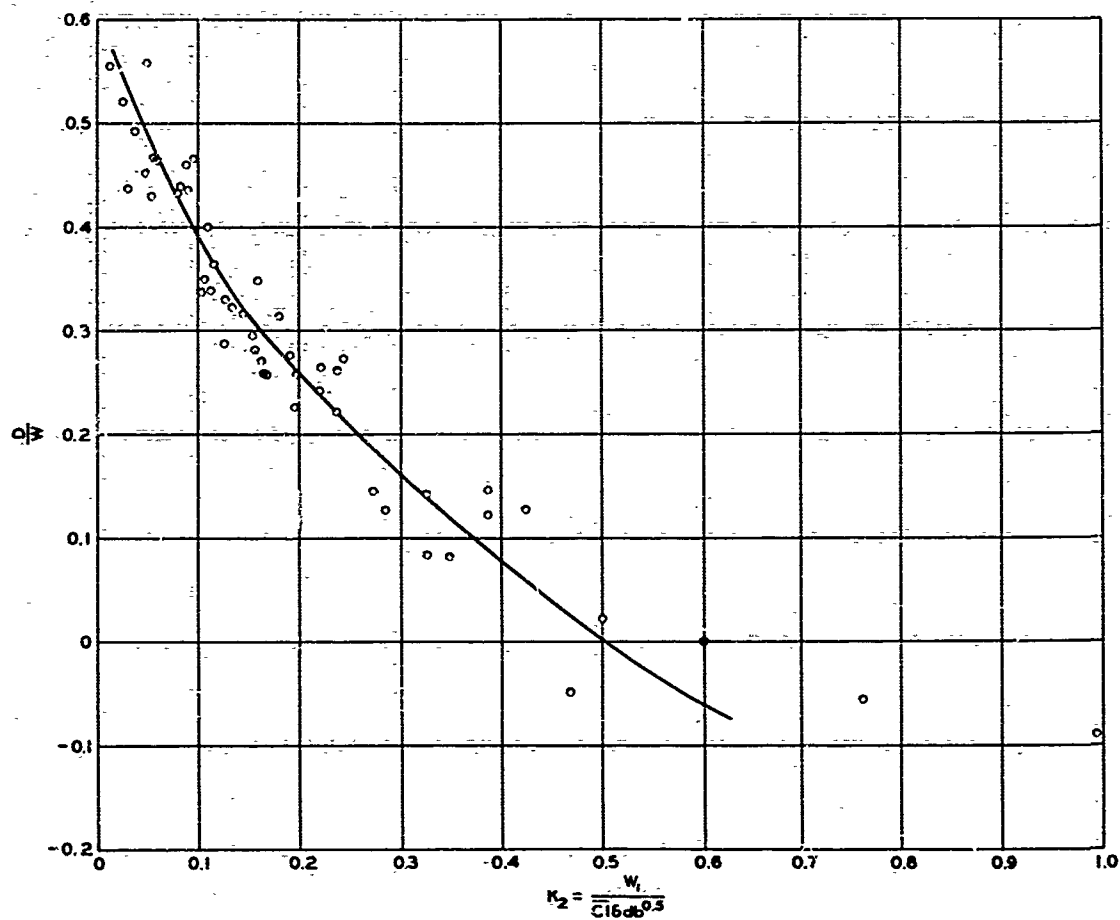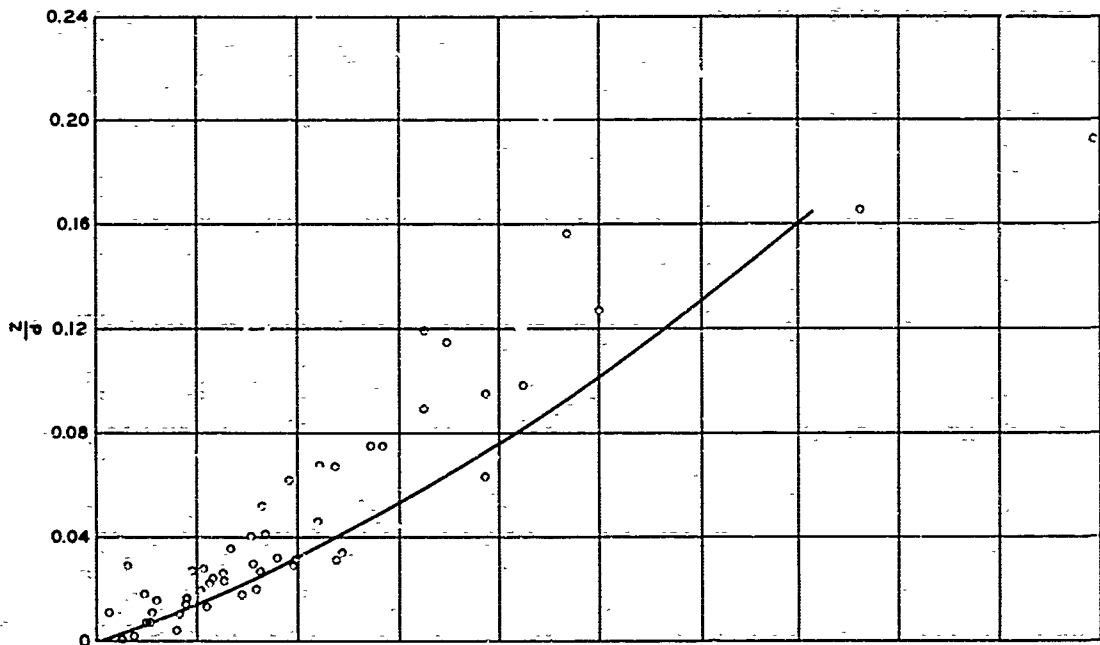
SINKAGE AND
DRAWBAR PULL VS K_2
9.00-14 TIRE
FIRST PASS, YUMA SAND

$$K_2 = \frac{W_1}{C16d_t b^{0.5}}$$

PLATE 26

SINKAGE AND
DRAWBAR PULL VS π_2
4.50-7 TIRE
FIRST PASS, YUMA SAND

Equations shown on the figure:

$$\frac{z}{d} = 1.44\,(\pi_2)^2 \quad (EQ.\ 28)$$
(SECOND PASS)

$$\frac{D}{W_1} = 0.78 - 2.84\,\pi_2$$

Axis labels: $\frac{z}{d}$ (vertical, top plot), $\frac{D}{W}$ (vertical, bottom plot), π_2 (EQUATION 29) (horizontal)

PLATE 27
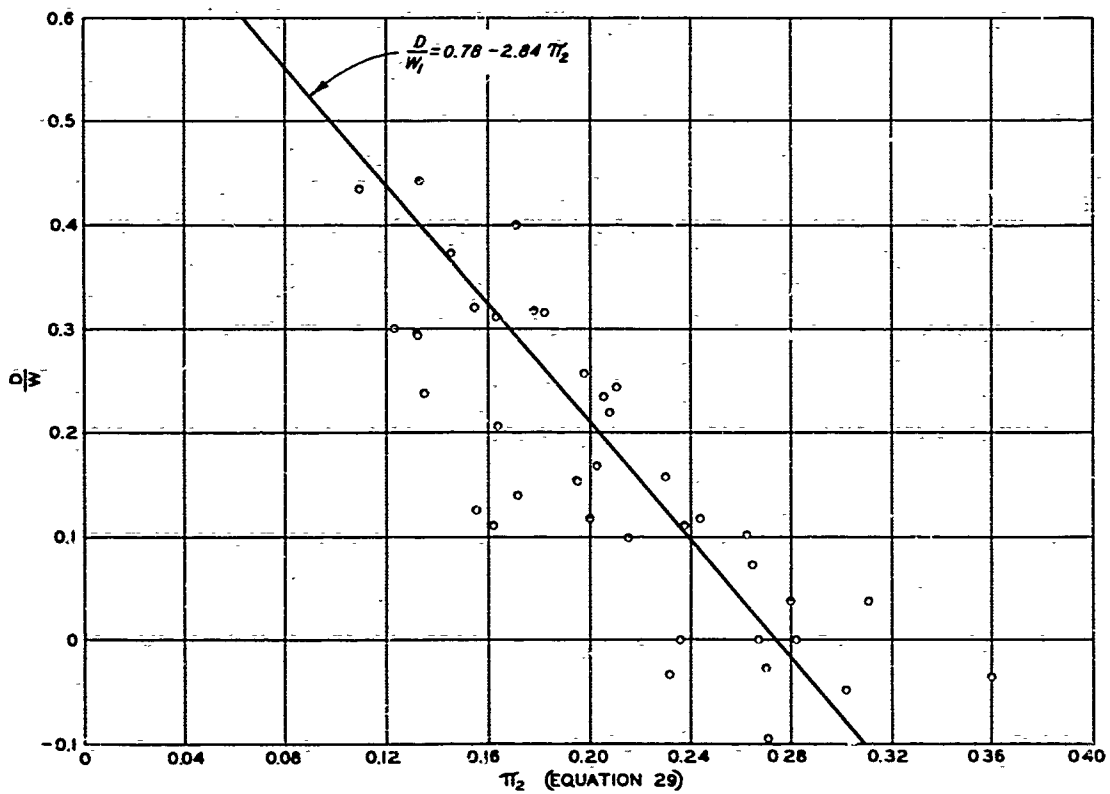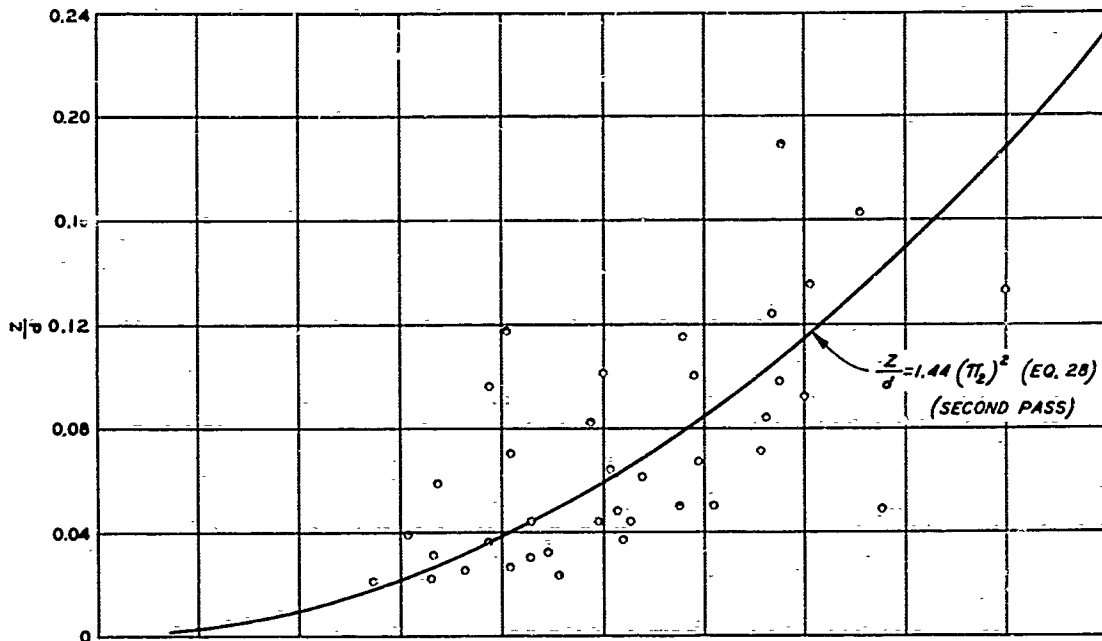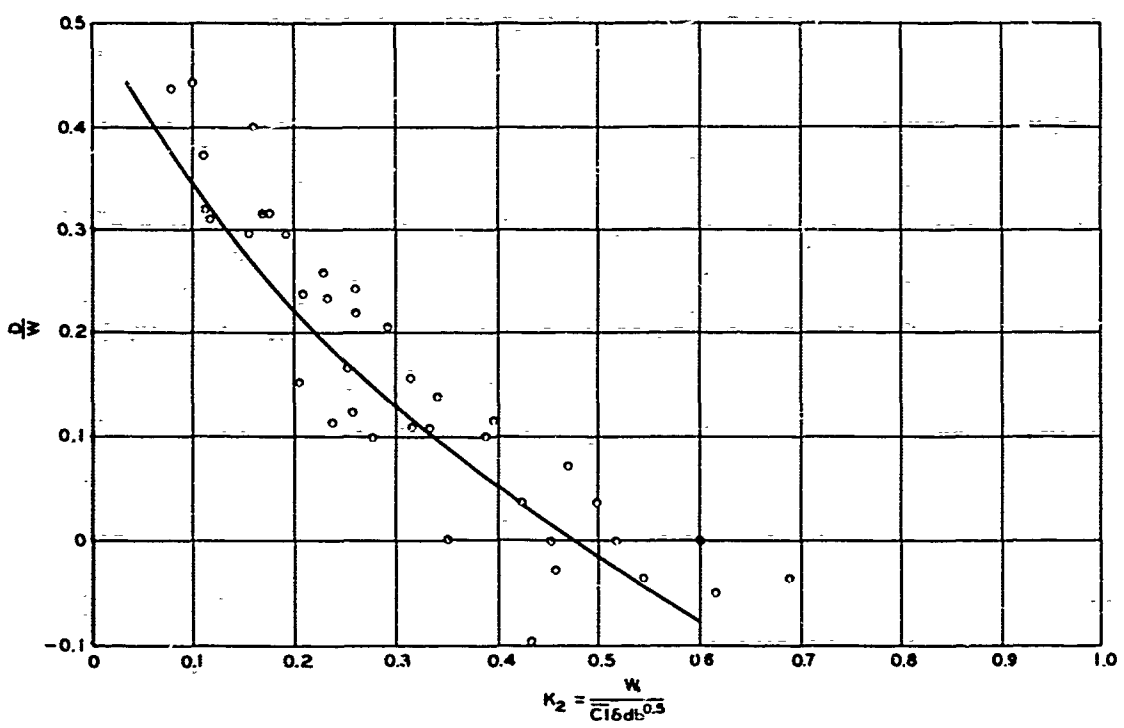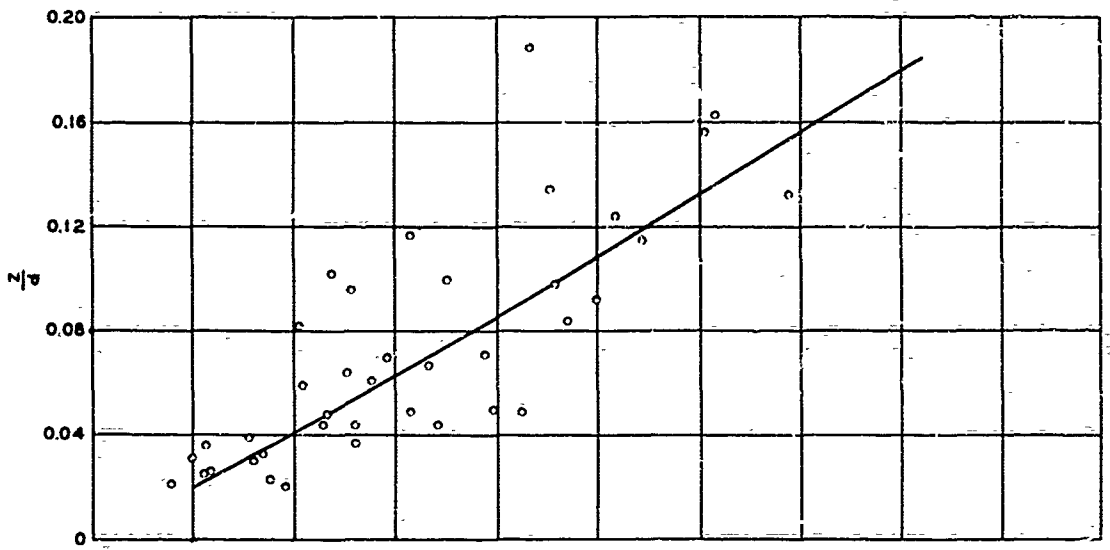
SINKAGE AND
DRAWBAR PULL VS K_2
4.50-7 TIRE
FIRST PASS, YUMA SAND

Chart axes:

Top chart: y-axis $\frac{z}{d}$ with values 0, 0.04, 0.08, 0.12, 0.16, 0.20

Bottom chart: y-axis $\frac{D}{W}$ with values -0.1, 0, 0.1, 0.2, 0.3, 0.4, 0.5

x-axis: $K_2 = \frac{W}{C16db^{0.5}}$ with values 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0

PLATE 28

APPENDIX A: THE DIMENSIONAL FRAMEWORK FOR TREATING SCALE-MODEL VEHICLES IN SOFT TERRAINS[18, 19]*

1. The principal requisite for a valid, useful dimensional analysis of a given type of mechanical system is a complete, minimum list of the truly important independent parameters influencing its behavior. An analysis will be incorrect if any major variable is overlooked, and will be misleading and less useful than it might be if the list is not pared of all factors whose influence is negligible.

Independent Parameters

Factors relating to the vehicle

2. The geometry or configuration of the vehicle, which may be specified by the ratio of each important linear dimension of the machine, such as length (y), wheel base (m), lug depth (a), tire section width (b), and wheel spacing (s), tire deflection (δ), etc., to a single, characteristic linear dimension such as the wheel diameter (d). Thus,

$$y/d, \ m/d, \ a/d, \ b/d, \ s/d, \ \delta/d, \ \text{etc.}$$

3. The statement of geometric similarity between a model and a full-size prototype means that each such ratio for the model equals the corresponding ratio for the full-scale vehicle. Their influence on the complete system is thus constant for a given configuration, and may be pooled with other constants relating to the system. They need not appear in the functional equation that is the final expression of a dimensional analysis performed in relation to the use of geometrically similar scale models.

4. d = a characteristic linear dimension of the vehicle, such as its wheel diameter, which expresses its size. The ratio d_m/d_f , in which the subscripts m and f refer to scale model and full-size prototype, respectively, is the linear scale ratio λ .

5. W = the gross weight of the vehicle. Where important, the ratios

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* Raised numbers refer to similarly numbered items in the Selected Bibliography following the main text of this report.

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of component weights to the gross weight must also be specified. However, the requirement for complete dynamic similarity between two sizes of vehicles includes the condition that corresponding ratios on each be equal, so that, as in the case of the geometry, these ratios need not appear in the final equations.

Factors relating to the
operation of the vehicle

6. $V =$ a characteristic speed related to the vehicle, such as peripheral wheel speed relative to the vehicle. At a given moment, all other velocities and components in the system may be expressed in terms of ratios to this speed.

7. One ratio of particular importance in dynamic systems is the slip ratio S : $S = V_s/V$, where V_s = the slip speed, or average speed of elements in the contact area relative to the undisturbed soil or snow.

8. It is to be noted that while the slip ratio is usually conceived in terms of speeds, it also has a simple geometric interpretation:

$$S = j/\ell$$

where j is the slip distance or travel of an element in the contact area relative to the undisturbed material during one cycle in contact with the material and ℓ is the distance from front to rear of running gear contact area, along the same path over which j is measured (usually parallel to the average surface of the material).

9. In terms of speeds, the slip ratio implies dynamic soil reactions arising from inertia and/or viscous effects. In terms of distances, the ratio implies essentially static material reactions, determined primarily by displacement and but little influenced by time-rate of displacement.

Factors relating to
the soil bed or snow pack

10. $h =$ total depth of material of interest. Boundaries of significant layers, and hence their thicknesses as well, may be specified by measurements from the surface, $_1h$, $_2h$, etc. The ratios $_1h/h$, $_2h/h$, etc., are presumed equal under the assumption of geometric similarity which extends to the geometry of the terrain as well as of the vehicle.

11. $_nc_s =$ before-collapse, or structural cohesion of the material at depth $_nh$. This will in general vary at a given spot and time from layer

to layer, and hence with depth, i.e. $_nc_s = F\left(_nh\right)$.

12. $_n\tau_o =$ the full, unit after-collapse or <u>dynamic shearing resistance</u> of the material originally at depth $_nh$. For any depth, it may be approximated by Coulomb's assumption:

$$_n\tau_o = {_nc_t} + \sigma\tan{_n\phi}$$

where $\sigma =$ unit normal loading on the shear plane. On this basis, τ_o may be replaced by $_nc_t$ and $\tan{_n\phi}$, where these are, respectively, the effective cohesion and the tangent of the effective angle of internal friction of the collapsed material originally at depth $_nh$.

13. $_n\gamma =$ the before-collapse bulk specific weight of the material at depth $_nh$.

14. $_n\beta =$ plastic kinematic viscosity at depth h .

15. $A =$ slope of the surface of the material.

16. $_nf_i =$ coefficient of friction of soils or snows at depth $_nh$ on the material of part i of the vehicle. The use simply of the factor f in the analysis will signify this entire set of frictional phenomena.

17. $_nB =$ shear stress-strain parameter characterizing dynamic shearing resistance in layer n . Dynamic shearing resistance develops only after some consolidation and reorientation of the grains of the collapsed material. The typical relation in weak, loose materials between unit shearing resistance and shear travel may be approximated by the equation:

$$\tau = \tau_o \left(1 - e^{-j/B}\right)$$

Because j/B must be dimensionless, the dimension of the parameter B is L .

18. The list might be further lengthened by the inclusion of soil or snow elastic properties, but the phenomena of interest regularly involve large, permanent deformations of the material, so that elastic forces may probably be neglected from the onset, at least until experimental evidence of their importance dictates otherwise.

Factors relating to
the system as a whole

19. $g =$ acceleration of gravity.

Dependent Variables

20. z = sinkage of vehicle

21. α = trim of vehicle

22. D = drawbar pull, or measurable margin of tractive capacity over external motion resistance. Drawbar pull of a given material is largely influenced by slip ratio S and/or grouser travel j. Either drawbar pull or slippage may be taken as the independent variable, and the remaining one as dependent. In testing, it is convenient to control slippage and to measure drawbar output. In practice, the drawbar load is fixed, at any given moment, by terrain and towed load, and slippage becomes the dependent variable.

Resulting Equations

23. From the independent parameters and dependent variables listed above, the dimensional equations can be written:

$$z = d\theta(W/c_s d^2, \ c_t/c_s, \ \emptyset, \ W/\gamma d^3, \ A, \ v^2/gd, \ vd/\beta, \ f, \ S, \ j/B, \ h/d) \tag{A-1a}$$

$$D = W\theta'(W/c_s d^2, \ c_t/c_s, \ \emptyset, \ W/\gamma d^3, \ A, \ v^2/gd, \ vd/\beta, \ f, \ S, \ j/B, \ h/d) \tag{A-1b}$$

$$\alpha = \theta''(W/c_s d^2, \ c_t/c_s, \ \emptyset, \ W/\gamma d^3, \ A, \ v^2/gd, \ vd/\beta, \ f, \ S, \ j/B, \ h/d) \tag{A-1c}$$

Some Possible Simplifications

24. It is evident from the equations that unless some factors are eliminated, preferably on the basis of sound experimental data, the analysis does little to clarify vehicle-soil relations, and makes the possibilities for the use of scale models appear remote indeed. Some of the more interesting possibilities for simplication, from a practical viewpoint, are discussed below. Only equation A-1a is modified, it being understood that the form of equations for α and D follow accordingly.

25. There has been no indication in the data for the past several years of any marked effect of vehicle speed, per se, upon vehicle performance in soils or snows over the range of rather slow speeds at which

quantitative tests have been run. On the basis of this experience, it is possible to eliminate (for similar tests) all terms related to speed, except the slip ratio. The apparent influence of slip ratio upon vehicle performance at slow speed is undoubtedly due to its geometric meaning, which remains valid regardless of any lack of actual speed effects.

26. Further, in relation to the vehicle and test procedures utilized in the present program, the factors A (equals zero for tests in level terrains) and f may be considered constant. On these bases, eliminating all terms relating to V, A, and F, equation (A-1a) becomes:

$$z = d\theta^i \ (W/c_s d^2, \ c_t/c_s, \ \phi, \ W/\gamma d^3, \ S, \ j/B, \ h/d) \tag{A-2}$$

27. If, as past experience in nonplastic soils and dry snows indicates, body forces may be considered negligible, factors related to γ may also be neglected. Accordingly, equation A-2 may be further simplified:

$$z = d\theta^{ii} \ (W/c_s d^2, \ c_t/c_s, \ \phi, \ S, \ j/B, \ h/d) \tag{A-3}$$

28. Finally, if the unit dynamic shearing resistance of a material (such as sands and disaggregated snows) is considered to be entirely frictional and constant with depth, equation A-3 becomes:

$$z = d\theta^{iii} \ (W/c_s d^2, \ \phi, \ S, \ j/B, \ h/d) \tag{A-4}$$

29. It has been found in practice that both the above assumptions are reasonably factual for sands and dry snows, at least insofar as these may be ascertained by means of shear vane tests, although some small cohesion (usually less than 0.1 psi) is often registered.

30. There remains the question, at low slippages, of whether expansion of model data to full-size predictions should be made on the basis of the slip ratio S or of the numeric j/B . In order to satisfy both conditions simultaneously,

$$B_m = \lambda B_f \tag{A-5}$$

i.e., barring an unlikely, relatively steady decrease in material stiffness with depth (measured from the surface), both conditions cannot be satisfied by tests of model and prototype in the same terrain.

A5

31. At this moment, little is known of the structure or behavior of the parameter B . It is not known how, or even whether, it varies with normal and/or shear stress in a given soil or snow, to what extent it is a property independent of size of the test sample or test apparatus, etc. The importance of at least one parameter, whatever its final form, must be recognized, however, for it offers the only acceptable explanation yet advanced for the characteristic shape of curves of drawbar pull versus slippage for all types of vehicles. This basic explanation was first offered and discussed by Bekker in 1954.[14] On the basis of experience to date, however, it is (somewhat arbitrarily) considered that present use of scale models should utilize expansion with the slip ratio as the controlling numeric, rather than j/B . Accordingly, equation A-3, upon which practical vehicle mobility tests of vehicles using models in any type of terrain should be planned and interpreted, becomes:

$$z = d\theta^{iv} (W/c_s d^2, c_t/c_s, \phi, S, h/d) \qquad (A-6)$$

32. Thus, in a given terrain (ϕ relatively constant) dimensionless sinkage z/d (and likewise drawbar pull, trim, etc.) of a given vehicle and its scale model is a function primarily of the loading $W/c_s d^2$, the slip ratio S , cohesion ratio c_t/c_s , and, where depths are such that hardpan or ground-support comes into play, the dimensionless depth h/d . All may readily be adjusted and/or controlled if means are available to measure c_s and c_t , or relative values thereof, in relation to some arbitrary standard. In materials which can be considered purely frictional, the cohesion ratio, of course, also may be dropped; i.e., finally,

$$z = d\theta (W_1/c_s d^2, \phi, S, h/d) \qquad (2)$$

$$D = W\theta' (W_1/c_s d^2, \phi, S, h/d) \qquad (3)$$

$$\alpha = \theta'' (W_1/c_s d^2, \phi, S, h/d)$$

APPENDIX B: VARIATION OF SOIL STRENGTH WITHIN THE
LENGTH OF THE YUMA SAND TEST CARS

1. A brief study has been conducted to determine how much the 0- to
6-in. average cone index readings for individual penetrations in the Yuma
sand test cars varied from the average of all of the penetrations. Normally,
three penetrations were made at approximately equal intervals along the
54-ft length of the test section. This study was concerned with the uni-
formity of soil preparation only; therefore, it was based upon the 0-pass,
or before-traffic, penetrations. The penetrations were made in the traffic
lanes so that, in many cases, two sets of readings were taken in the same
test cars and both sets of readings have been plotted without distinction.
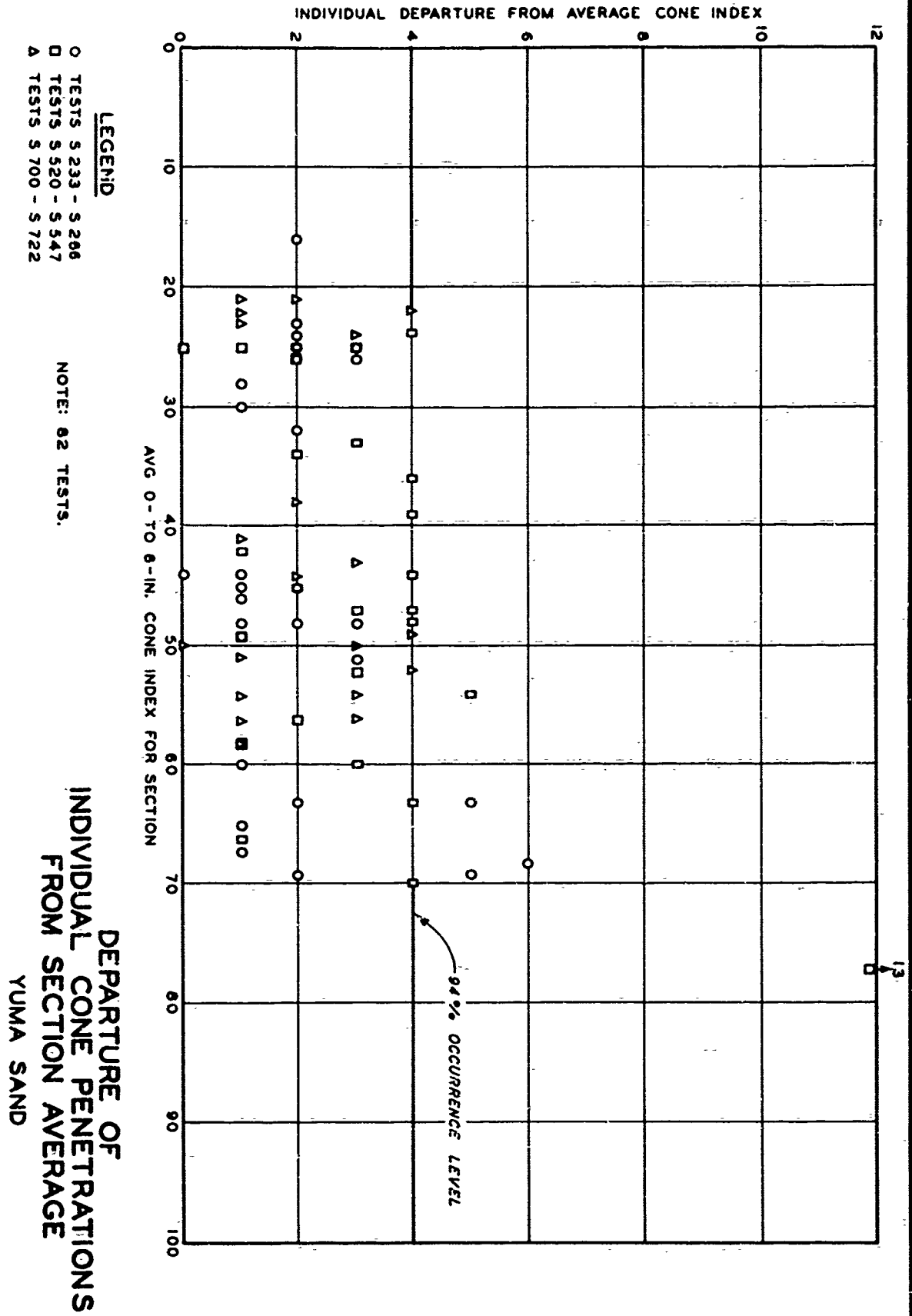
2. Eighty-two tests were chosen at random for the data sample and
should be representative of the entire test program. Three groups, each
consisting of approximately the same number of tests, were chosen to reflect
conditions in the early, intermediate, and late stages of the program. The
tests in the latter group include soil strength profiles that varied more
uniformly with depth than those in the other two groups. Each group was
large enough to include a reasonable number of tests at all strength levels.

3. Plate B1 is a plot of the numerical difference between the cone
index values for individual penetrations and the average value for the three
penetrations in the test section (four tests had only two penetrations, and
one had four) versus the average cone index for the section. There is a
marked tendency for the difference to be constant throughout the range of
strengths tested. Ninety-four percent of the differences are of a magnitude
of ± 4 cone index or less. While all of the points that lie above this level
are for strengths of 50 cone index or above, there appears to be only a
slight trend for the band of variation to spread out at the higher strengths.

4. The differences in plate B1 have been converted to percentage of
the section averages and are plotted against the section averages in
plate B2. It is apparent that the percent variation is much higher at the
lower strengths than at the higher. The line that encompasses 94 percent
of the points has been transferred directly to this plot from plate B1 and
is exponential in nature. It is interesting to note that percentages of
variation approach ± 20 percent only at the 20 cone index level and that

B1

the vast majority of the variations are of a magnitude of ± 12 percent or less.

5. From the above, it is apparent that the variation in cone index readings that can be expected within a given test section will be essentially constant regardless of the strength level of the section. The reasons for this may be inherent in the preparation procedures and controls utilized in this test program, however, and the conclusion reached above should not be applied to sand test beds in general. Insofar as this specific test program is concerned, references to likely variation in strength within a test section should probably be in terms of numerical cone index rather than percent unless the percentage figures are qualified by specifying the corresponding strength level.

DEPARTURE OF
INDIVIDUAL CONE PENETRATIONS
FROM SECTION AVERAGE
YUMA SAND

LEGEND
O TESTS S 233 - S 266
□ TESTS S 520 - S 547
△ TESTS S 700 - S 722

NOTE: 82 TESTS.

INDIVIDUAL DEPARTURE FROM AVERAGE CONE INDEX

AVG 0 - TO 6-IN. CONE INDEX FOR SECTION

94 % OCCURRENCE LEVEL

INDIVIDUAL DEPARTURE FROM AVERAGE CONE INDEX IN PERCENT

LEGEND

O TESTS S 233 - S 266
□ TESTS S 520 - S 547
△ TESTS S 700 - S 722

NOTE: 82 TESTS.

AVG. 0 - TO 6-IN. CONE INDEX FOR SECTION

94% OCCURRENCE LINE

PERCENT DEPARTURE OF
INDIVIDUAL CONE PENETRATIONS
FROM SECTION AVERAGE
YUMA SAND

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A semiempiric, dimensionless load coefficient is developed from extensive WES field and laboratory data on the performance of pneumatic tires in dry sands. The coefficient appears to collapse these diverse data to a useful degree. The physical meaning of the numeric is discussed, and its relation to some earlier work is shown. A preliminary consolidation of the data using the final form of the numeric is presented.

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