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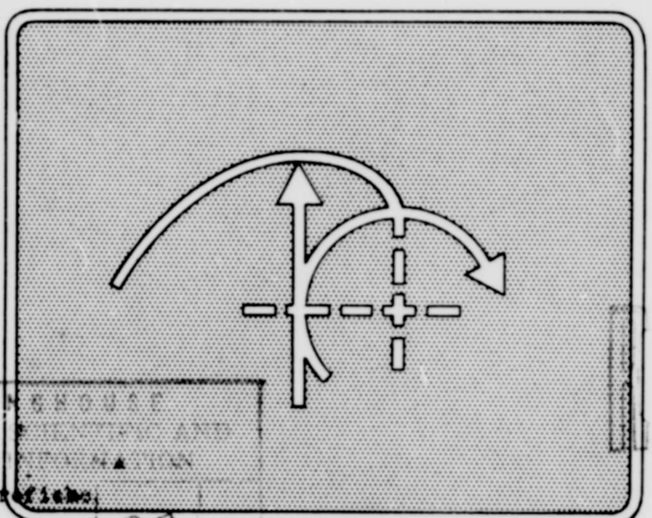
THE INVOLUTE CAM ROTARY ACTUATOR FOR HYDRAULIC SERVOMECHANISMS

by

L. H. Schwerdtfeger



THE JOHNS HOPKINS UNIVERSITY
APPLIED PHYSICS LABORATORY
SILVER SPRING, MARYLAND

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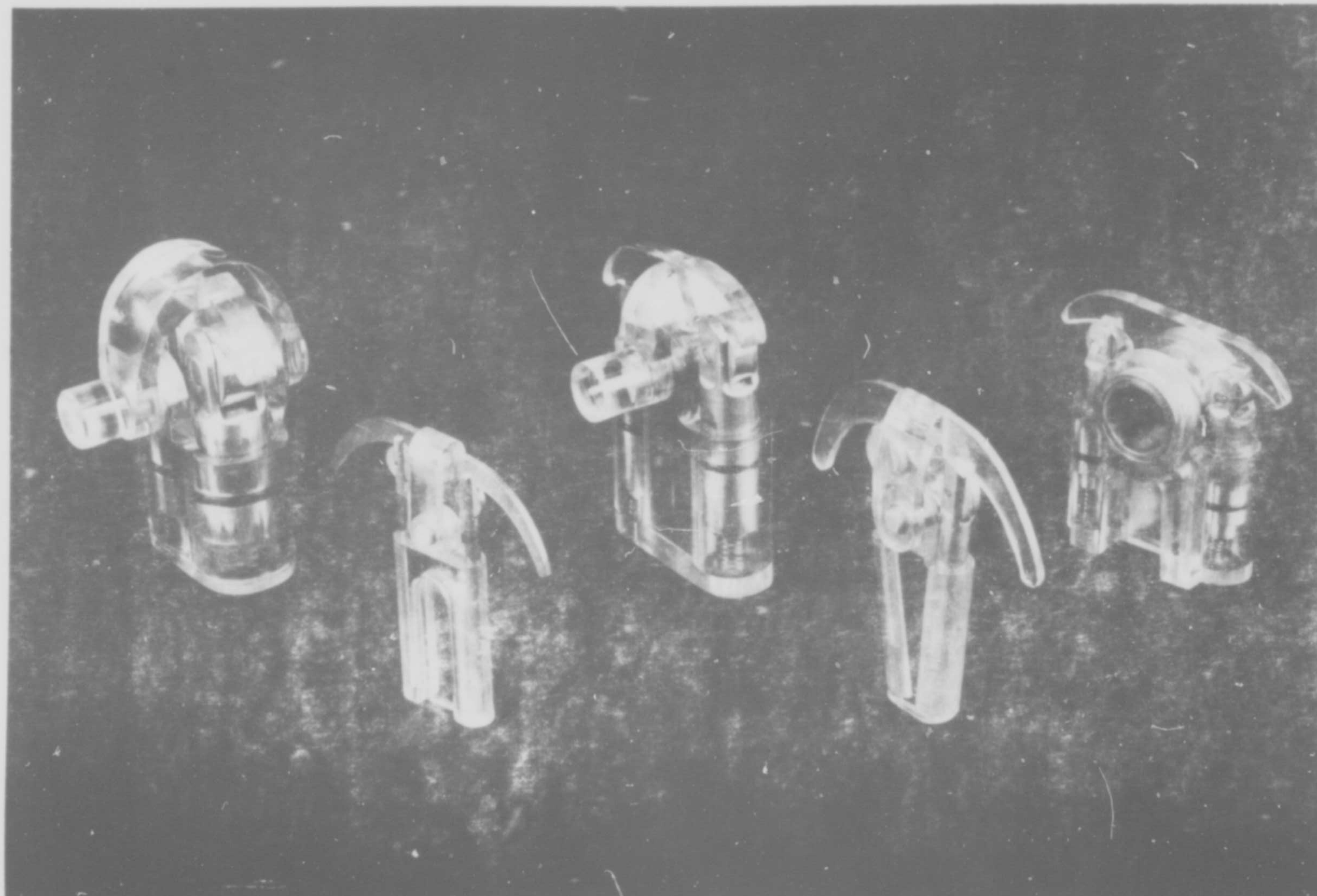
TABLE OF CONTENTS



INVOLUTE CAM ROTARY ACTUATOR BASIC TYPES

APPLIED PHYSICS LABORATORY
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THE INVOLUTE CAM ROTARY ACTUATOR FOR HYDRAULIC SERVOMECHANISMS

I. Introduction

A rotary hydraulic actuator featuring two single-acting pistons and a symmetrical involute cam rocker arm is being successfully used in several missile servomechanism applications. Figures (1) and (2) illustrate existing applications. Presented here is a discussion of its general type and design procedure for five basic types.

This actuator design is adaptable to any hydraulically powered mechanism requiring an efficient transmission of torque through an oscillation of less than one-half revolution. It eliminates all backlash, the need for rotary high pressure seals and many of the other shortcomings of a conventional rotary actuator and offers at least three unique advantages, i.e.:

1. The pressure angle is zero throughout the full range of motion. The point of contact on the cam follower is always in line with the axis of force application. Thus there is no side-loading of the pistons; short strokes do not require long pistons and seals wear evenly. Most important, of course, is the fact that large force applications do not mean correspondingly large cam sizes to keep the pressure angle to a reasonable figure. With this design, the pressure angle is ideal.

2. (a) Angular motion is directly proportional to linear motion at all times. When the piston extends one unit, the resulting cam rotation is one unit (angular). Whether at the beginning of stroke, or at the extreme end of stroke, this relationship is constant. In a closed-loop control system this feature allows a simple rectilinear potentiometer to be attached to the cam follower shaft to feed back rotational position of the rocker arm.

   (b) The torque is constant throughout the full range of motion. The same push is required to produce rotation for any position of the rocker arm. Unlike a crank and connecting-rod, the effective moment arm is unchanging and exactly the same force is available to overcome inertia for starting or reversing as that used in any part of the stroke.

3. The design is inherently simple, and therefore much less expensive to produce than many conventional types. Extremely close tolerances and/or metal-to-metal seals are unnecessary. Its simplicity assures a high order of reliability.

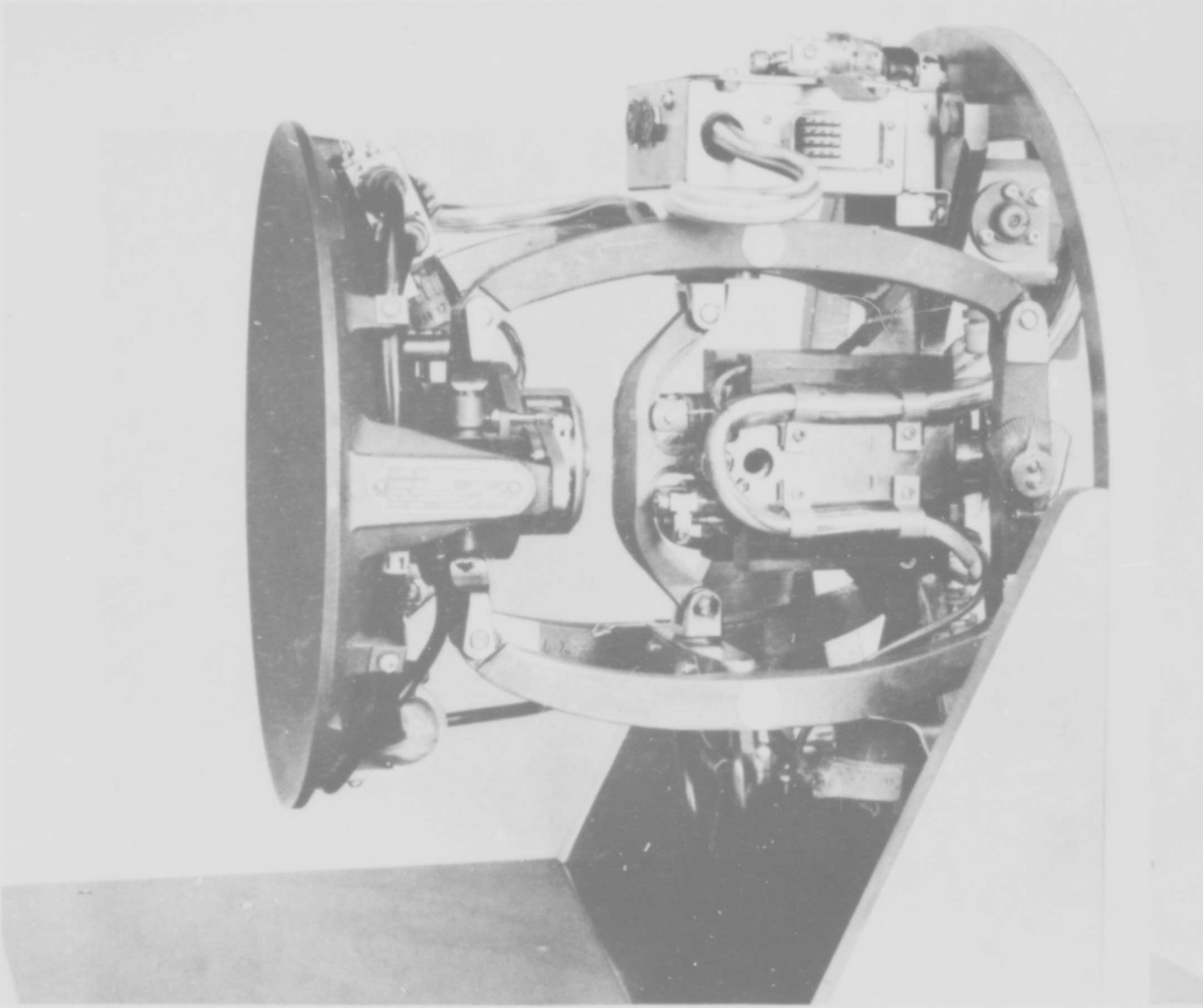


Fig. (1)

Airborne Radar Support and drive developed by APL/JHU

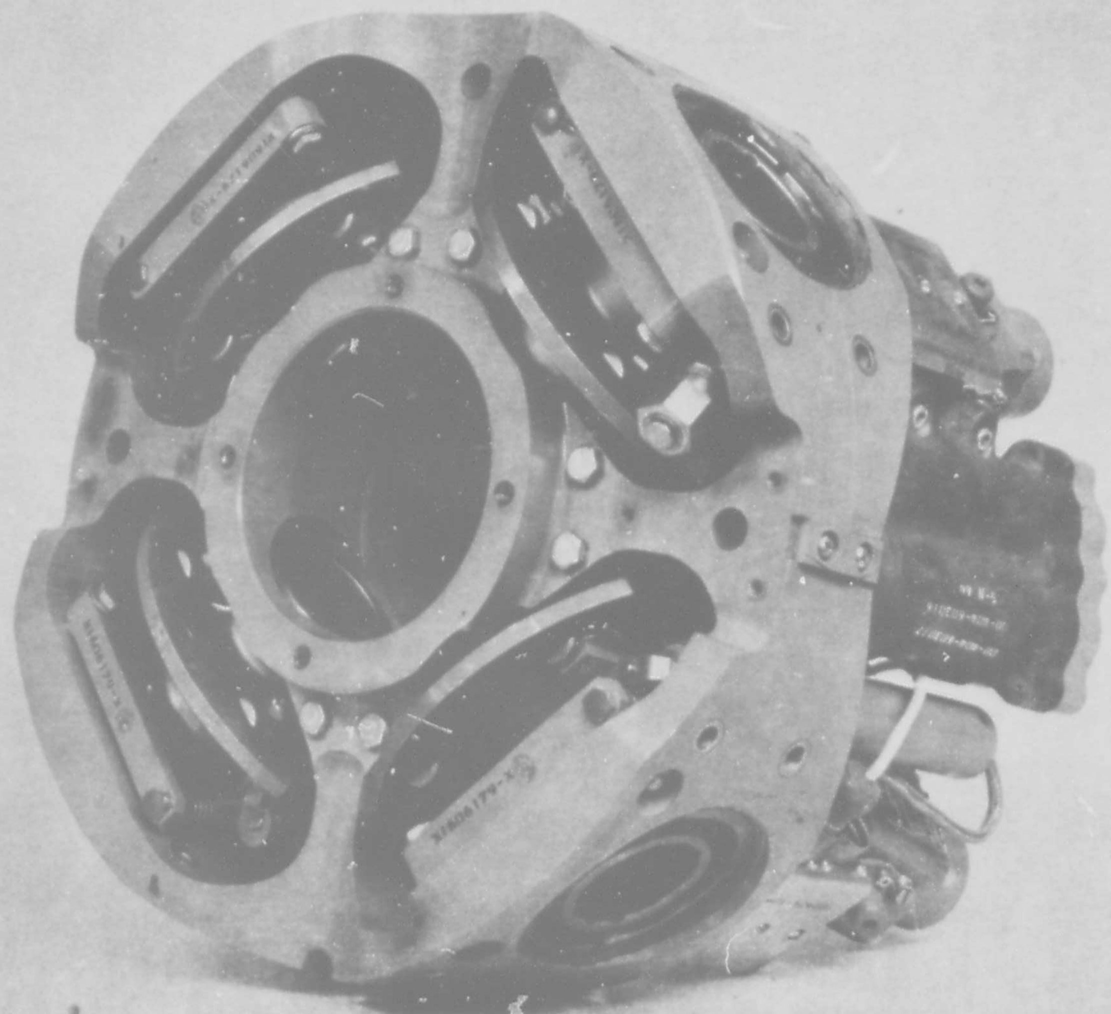


FIG. (2)

Control Surface Actuator Assembly
built by Convair, A Division of General Dynamics (Pomona, Calif.)

II. The Involute

These useful features are all available because of the involute shape of the cam surfaces. An involute curve is defined as the path traced by the end of a taut string as it is unwound from a cylinder, as graphically presented in Figure (3). It is a special form of cycloid wherein the evolute, or locus of the centers of curvature, is a circle of the same diameter as the cylinder with the string around it. Figure (3a) shows the curve for one complete revolution of the end of the string, and the length of the free part of the string is given for each quarter turn. Figure (3b) is an enlargement of the initial portion which is of most interest in this discussion. The equation for an involute is given for both rectangular and polar co-ordinate systems.

It can be seen that if the direction of a force coincides with the free portion of the string, it will always be perpendicular to the curve at any point (instantaneously) because the slope of a curve is always perpendicular to the radius of curvature (the free part of the string). Thus the statement about zero pressure angle can be made. It is also evident that the involute cam surface of the rocker arm could just as well be convex as concave, and another type of involute actuator would result.

It is also obvious from the curve of Figure (3b) that if the stroke of a piston is the same as the length of the free portion of string BP, it will also be a simple linear function of the angle from the x-axis to the end of the string that is tangent to the cylinder, angle ψ or AOB. Hence the second statement claiming that angular and linear motions of the actuator were always directly proportional is true. For example, when the angle is one radian, the length of free string is the same as the radius of the cylinder, ... two radians, ... the diameter, etc. Close examination of the curve will reveal that the angular motion of the type of actuator under disucssion is limited to something near $3\pi/4$ or 135°, otherwise one end of the rocker arm will begin to pass through the cylinder of the retracted piston, as will be explained later.

III. Basic Actuator Types

Variations in application create at least five basic actuator types, as sketched in Figures (4) through (8).

Figure (4) -- Type A is an actuator with a straight-through cam pivot shaft having bearings in the cylinder block outside the plane of the involute. Torque is not transmitted by the cam shaft -- instead the rocker arm becomes one member of a linkage, as in Figure (1).

Figure (5) -- Type B features a socket drive instead of a simple shaft, but with outboard bearings in the cylinder block similar to Type A.

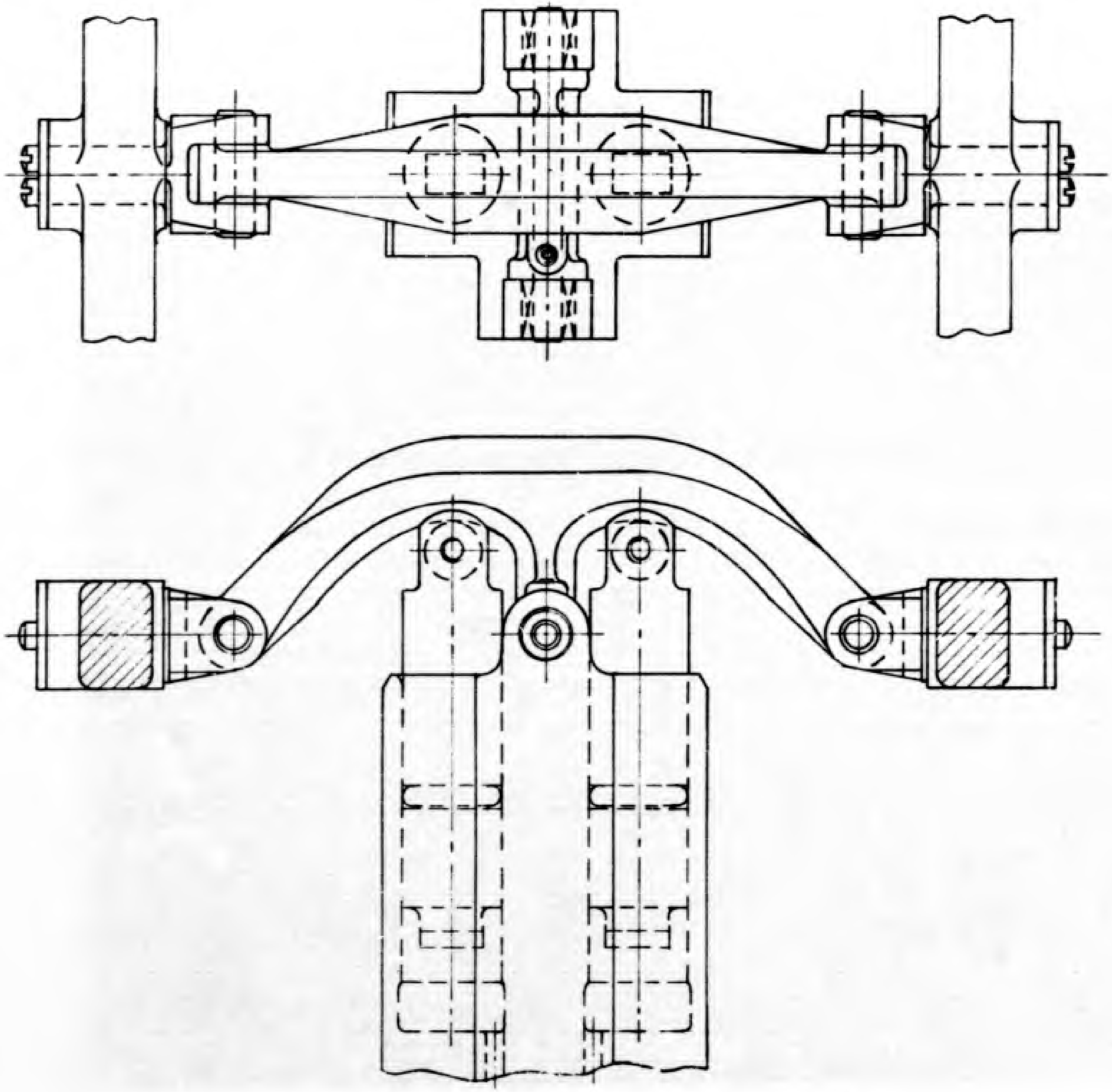
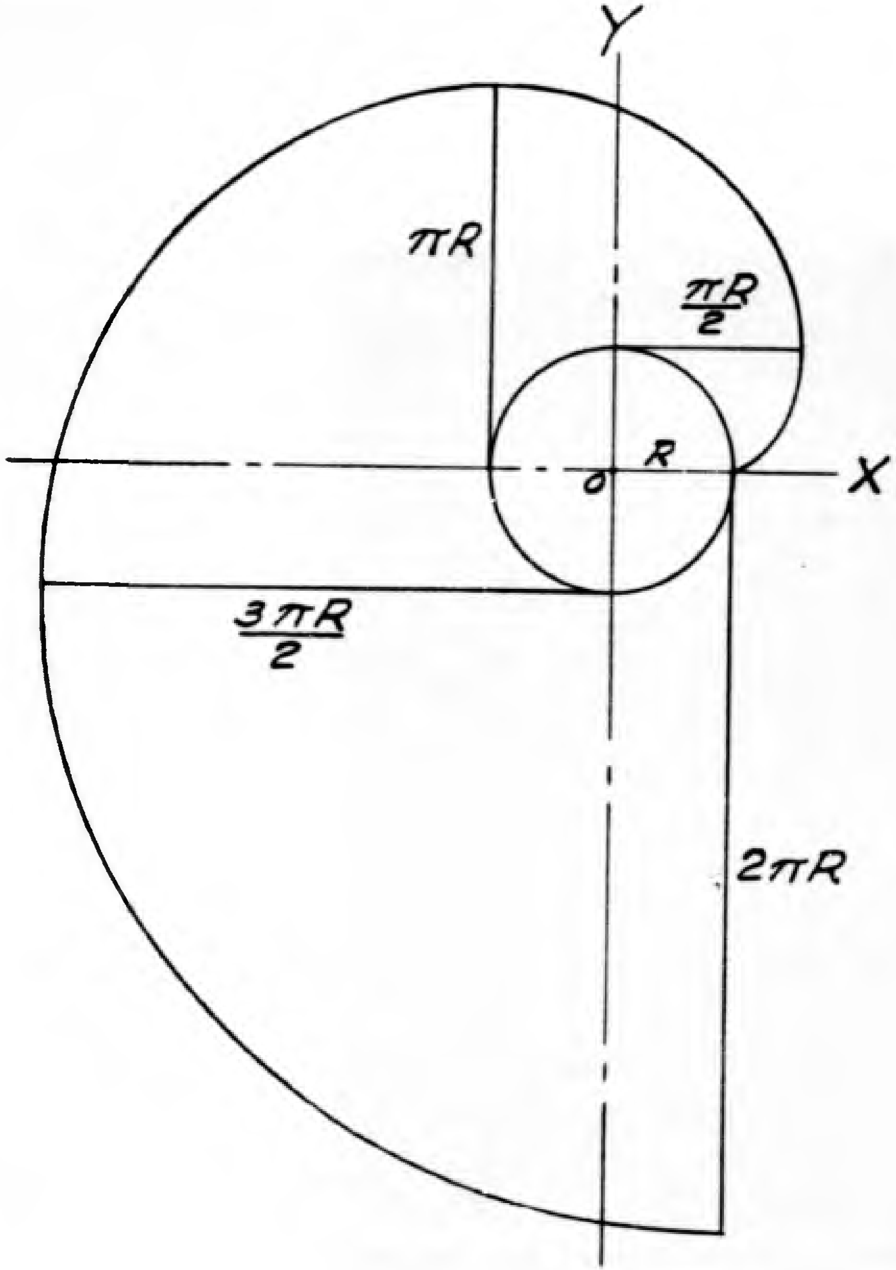
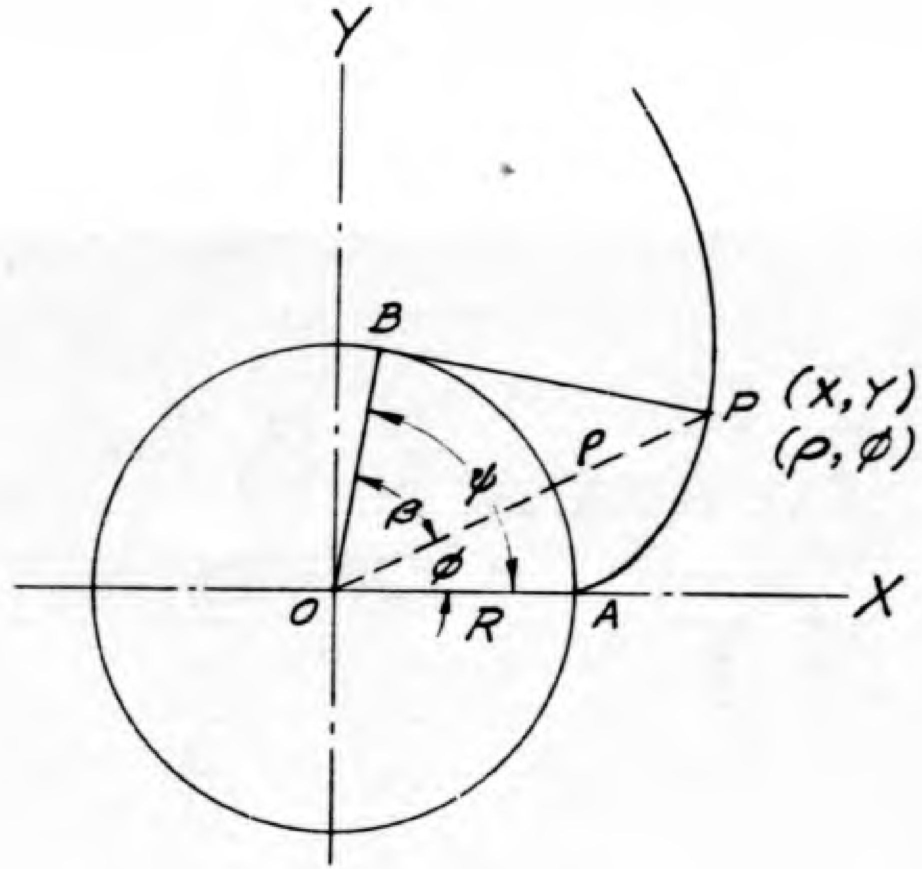


Fig.(4) Actuator Type A



(a)



$$X = R(\cos\psi + \psi\sin\psi)$$
$$Y = R(\sin\psi - \psi\cos\psi)$$

$$OP = \rho = R\sec\beta$$
$$AOP = \phi = \tan\beta - \beta$$
$$(\beta = \psi - \phi)$$

(b)

Fig.(3) The Involute Curve

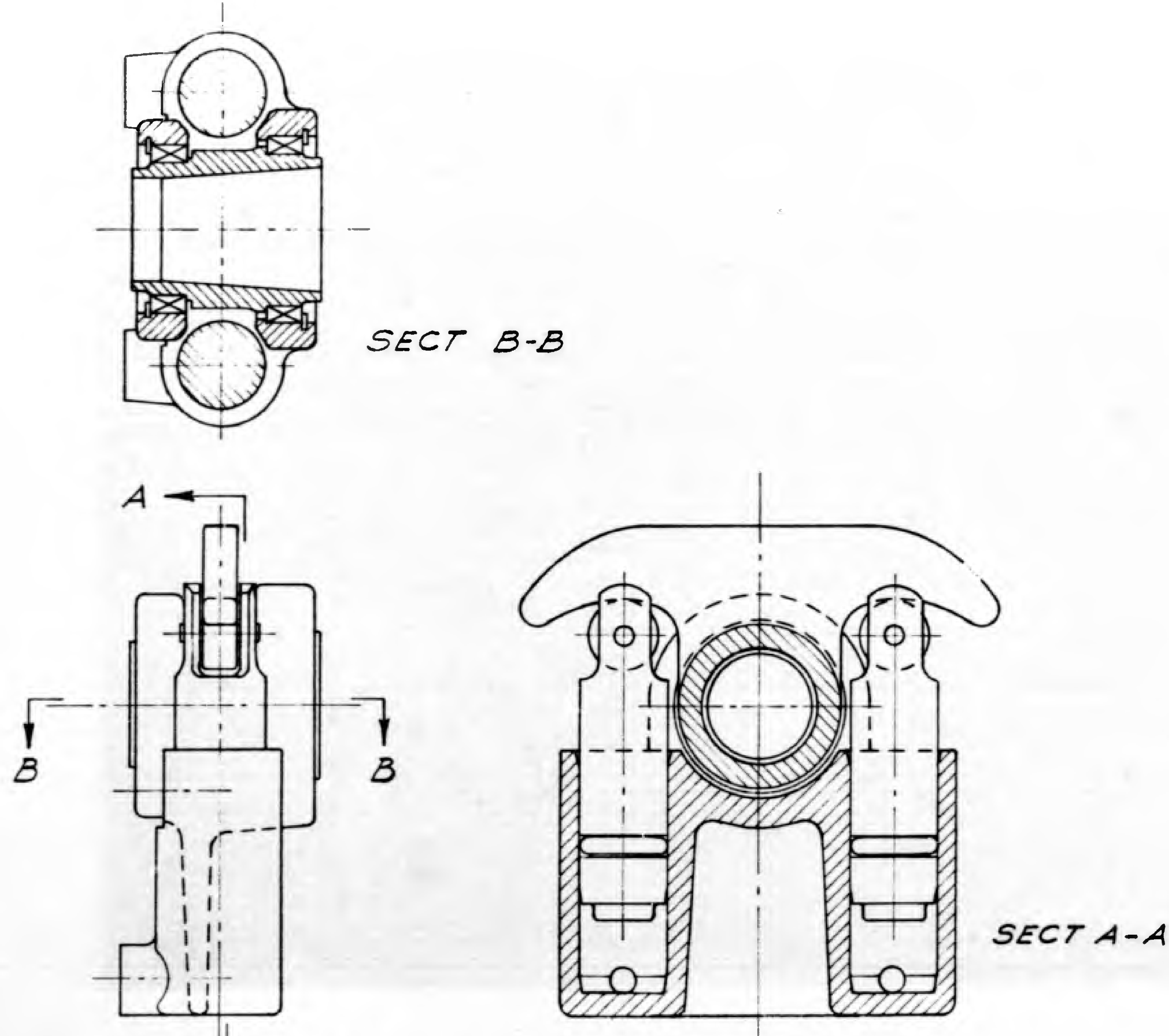


Fig. (5) Actuator Type B

Figure (6) — Type C shows a male drive shaft extension and a straight-through cam shaft. The rocker arm pivot is a clevis type, however, to allow a central bearing to be installed in the plane of the involute between the pistons.

Figure (7) — Type D has a male drive like Type C, but with an interrupted cam shaft to accommodate very large pistons and rollers.

Figure (8) — Type E shows a variation of Type D which produces maximum deflection instead of maximum torque.

Each of these useful types will be discussed subsequently at greater length, but first the design procedure for the simplest, Type A, will be presented. Procedures for the others differ only slightly, since they are mainly limiting cases for the proportions of certain elements.

IV. Design Procedure

To begin designing an actuator of any of these types, it is reasonable to assume that at least three things will be known:

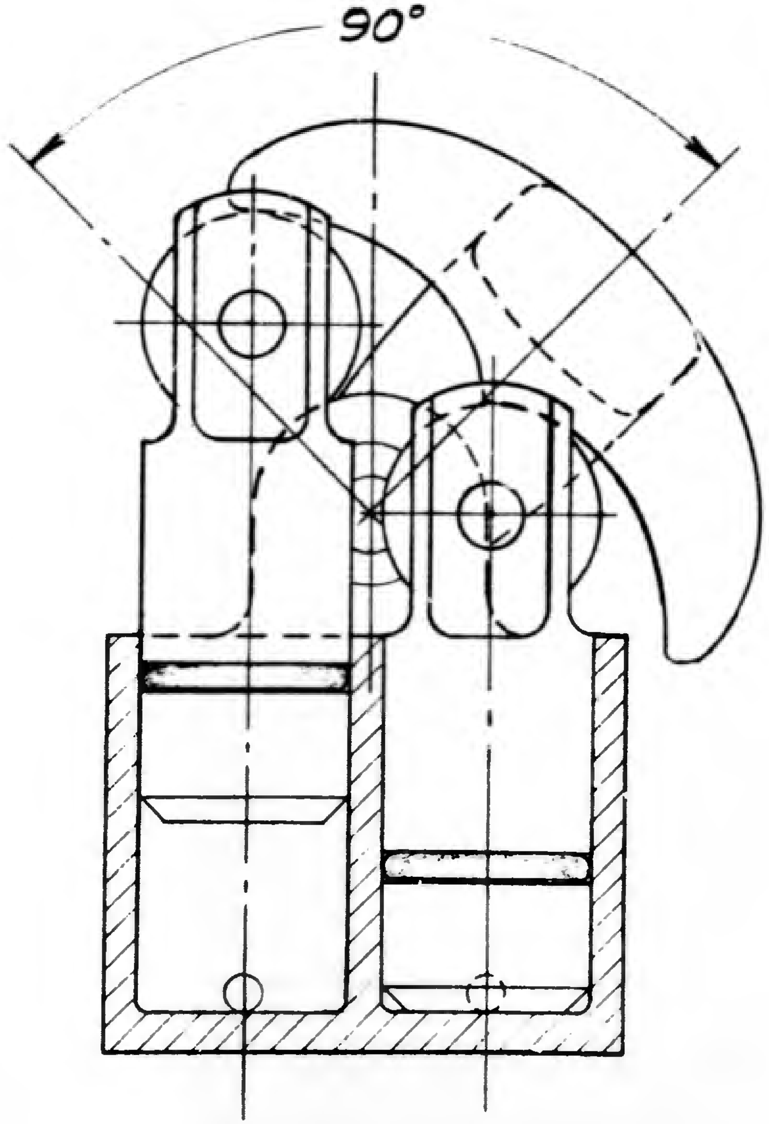
1. The hydraulic pressure differential between pistons.

2. The required torque output of the rotary actuator.

3. The angle of oscillation throughout which this torque must be applied.

These numbers can vary widely; for instance in an aerodynamic control surface actuator with restricted packaging envelope, the angle of oscillation required might be relatively small, while the torque required might be quite large, and the pressure could be the same as the main system used throughout the vehicle. Or in an airborne radar actuator requiring total deflection of one hundred degrees or so, the torque output for sufficient stiffness might be relatively small, and the hydraulic pressure in the medium range provided by a tiny auxiliary power supply. In a piece of earth-moving machinery all the numbers might be rather large, while in an instrument or indicating device, maximum osciallation with minimum power might be desired.

From these three conditions it is possible to create a whole family of actuator configurations each satisfying the same requirements, but with all the dimensions expressed in terms of a single parameter -- the area of one of the twin hydraulic pistons. Figure (9) is a composite diagram of an actuator of Type A, in which the left side of the symmetrical rocker arm shows the geometry involved, while the right side represents one of the twin pistons with the cam follower attached, and reciprocating in a cross-sectioned cylinder.



SECT A-A
(With arm rotated 45°)

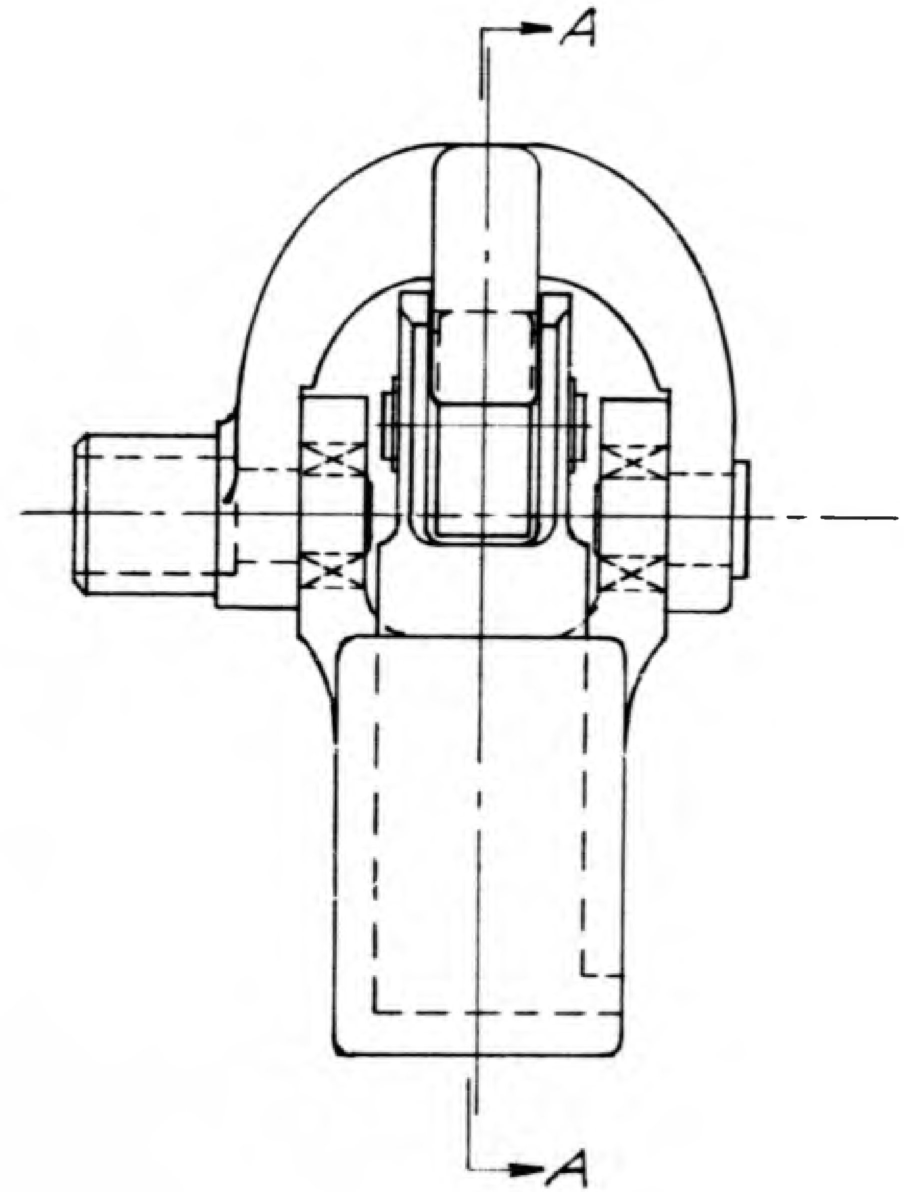


Fig.(7) Actuator Type D

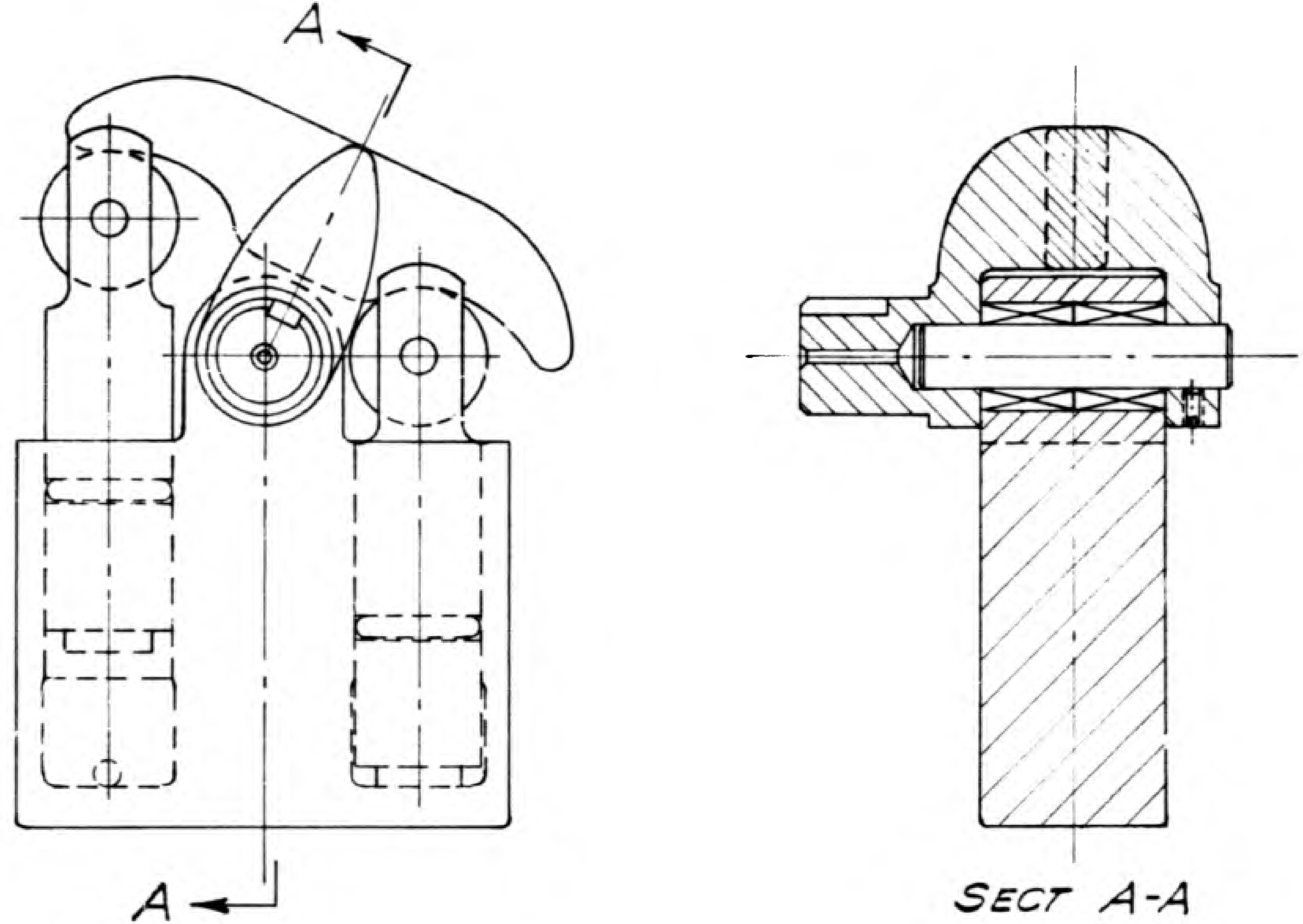
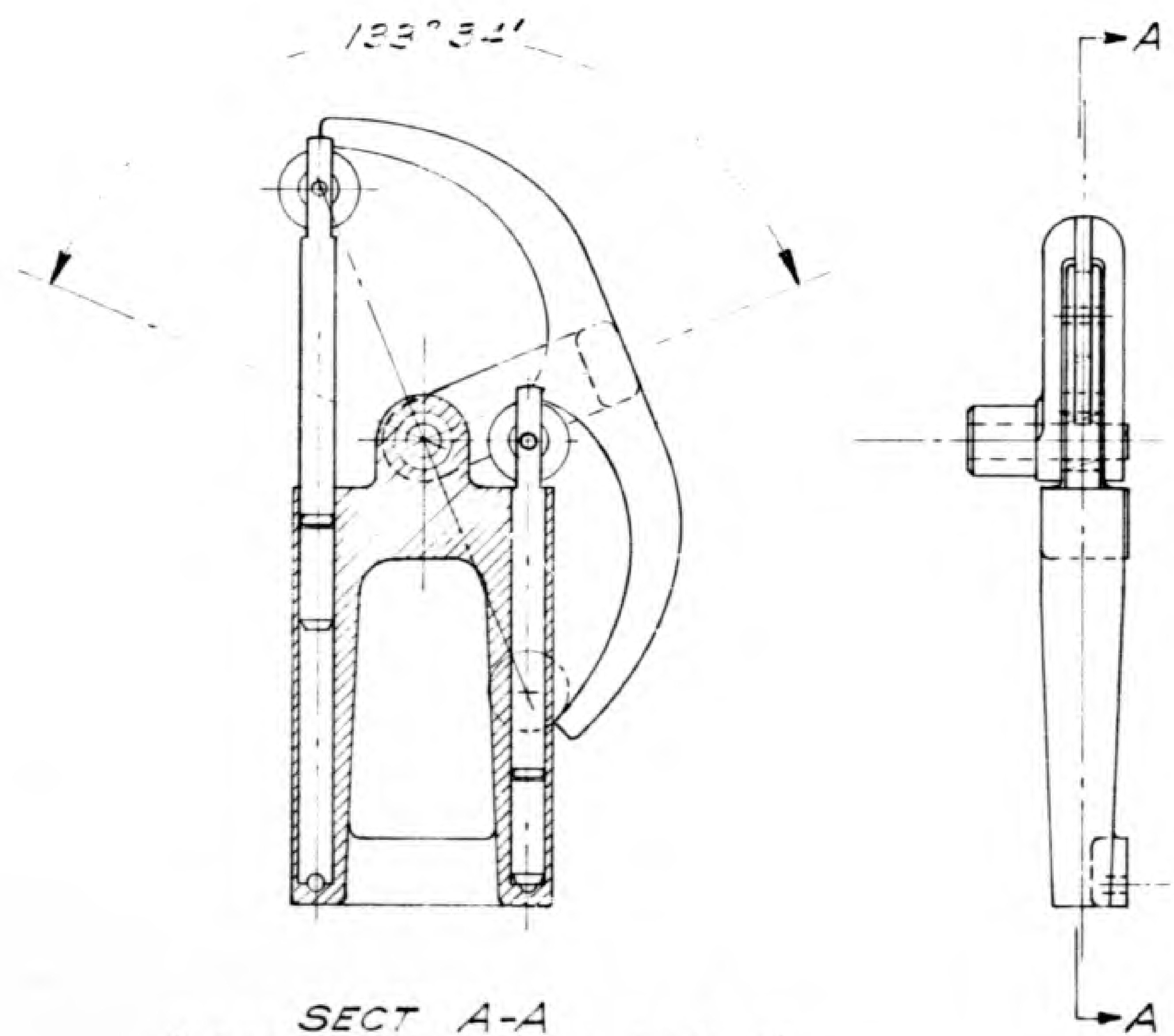


Fig.(6) Actuator Type C



SECT A-A
(With arm rotated 66°47')

Fig.(8) Actuator Type E

The rocker arm is shown centered with the piston stroke equal in each direction. Phantom circles show the extreme cam follower positions and the numerals I, II and III identify the retracted, middle and extended centers, respectively. The notation is as follows:

P = Hydraulic differential pressure at actuator, psi.

T = Torque about the cam pivot, inch-lbs.

Θ = Maximum angular motion (±) from the mid-position, degrees.

D = Piston diameter, inches.

F = Piston force (single), lbs.

R = Radius of the generating circle (Evolute), or half distance between pistons, inches.

L_{max} = Total actuator stroke, inches.

a = Piston area (single), sq. in.

r = Cam follower roller radius, inches.

r_1 = Radius of rocker arm at pivot, inches.

s = Arc length of involute surface, inches.

Q = Hydraulic displacement per degree of oscillation, cu. in./degree

K = Theoretical lead of cam, inches per revolution.

The type of rocker arm shown utilizes a continuous, or straight-through cam-shaft which limits the maximum diameter of the pistons, since the cam pivot axis intersects the plane of the cylinders. In other words, some space must be allowed for the cam-shaft between the pistons. It will be shown how this maximum piston diameter can be expressed entirely in terms of the known values of pressure, torque and angle.

First, let us examine Figure (9) closely. From the obvious fact that the torque will be a product of the actuator force and the radius of the evolute, we have the equations:

$$F = Pa \tag{1}$$

and $T = RPa$,

$$\text{or } R = \frac{T}{Pa} \tag{2}$$

$$\text{and } a = \frac{T}{RP} \tag{3}$$

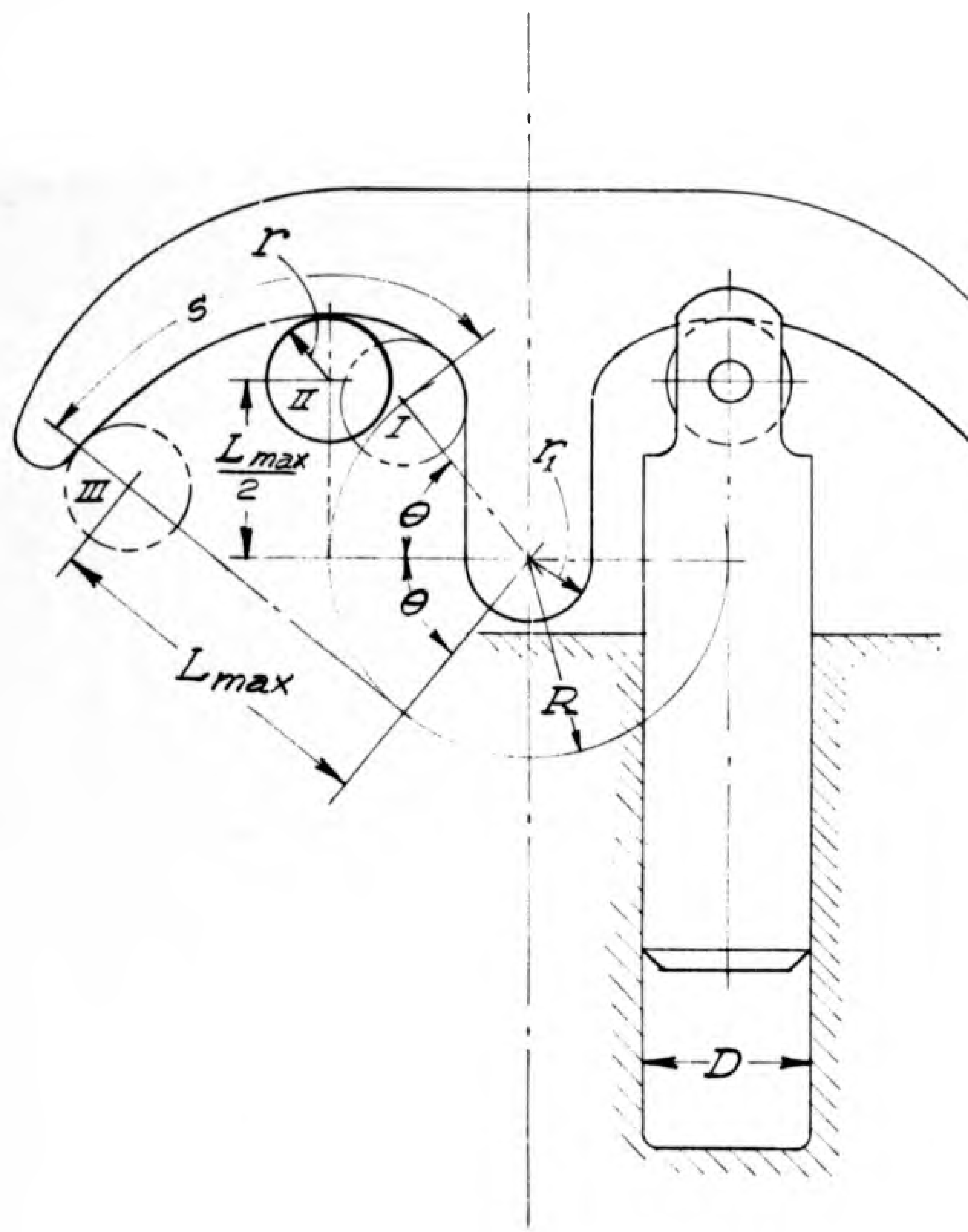


Fig.(9) Actuator Diagram

If θ is measured in degrees

$$\frac{L_{max}}{2} = \pi 2R \frac{\theta}{360} = \frac{\pi T\theta}{180\ Pa} \tag{4}$$

and $$L_{max} = \frac{\pi T\theta}{90\ Pa} \text{ inches.} \tag{5}$$

Of course, for θ measured in radians, we may say simply

$$\frac{L_{max}}{2} = \frac{T\theta}{Pa} \tag{4a}$$

and $$L_{max} = \frac{2T\theta}{Pa} \text{ inches.} \tag{5a}$$

Because work is defined when torque and angular displacement are specified, the following equation for hydraulic displacement per degree of deflection can be derived:

$$Q = \frac{(a)\frac{L_{max}}{2}}{\theta}$$

$$= \frac{\frac{\pi T\theta a}{180\ Pa}}{\theta}$$

$$= \frac{\pi T}{180\ P} \text{ cubic inches per degree} \tag{6}$$

If the involute shape is to be generated on a cam milling machine, it may be desired to compare a number of values for the cam lead, or advance per revolution.

$$K = 2\pi R = \frac{2\pi T}{Pa} \tag{7}$$

It is interesting to note that while piston stroke per degree of angular motion is a function of piston area, hydraulic displacement is a constant for any set of values of pressure, torque and angle.

Piston stroke per degree, among other things, is useful in comparing the effect on angular motion error of a standard manufacturing tolerance for piston assembly length and cylinder depth. Depending on class of work available and precision desired, it is usually safer to choose a system with a value comfortably large for the following equation:

$$\frac{L_{max}}{2\theta} = \frac{\frac{\pi T\theta}{90\ Pa}}{2\theta} = \frac{\pi T}{180\ Pa}, \quad \left(= \frac{\pi R}{180} \right) \tag{8}$$

For a uniform cross-section in the center of the rocker arm, as shown, we observe the position of the roller when its piston is fully retracted (position I).

$$r + r_1 = R \cos \theta$$

$$R = \frac{r + r_1}{\cos \theta} = \frac{T}{Pa}, \text{ from (2)}$$

$$\text{then } r + r_1 = \frac{T \cos \theta}{Pa} \tag{9}$$

In the course of subsequent stress analysis it very often becomes possible to allow the radius at the cam pivot (r_1) to be identical to the radius of the roller (r). Equation (9) can then be simplified to:

$$r = \frac{T \cos \theta}{2\ Pa} \qquad (\text{for } r_1 = r) \tag{9a}$$

As stated earlier, the maximum piston diameter is limited because of the particular actuator type under discussion. The cam shaft intersects the plane of the involute (and hydraulic cylinders). The limitations can be expressed as follows:

$$R \geqq r_1 + \frac{D}{2}$$

$$\text{or, } D_{max} = 2\ (R - r_1) \tag{10}$$

Substituting the parametric expressions for evolute radius and cam follower radius, Equations (2), and (9a) which is the same as the cam pivot radius, gives an expression in terms of the piston area.

$$D_{max} = 2 \left(\frac{T}{Pa} - \frac{T \cos \theta}{2\ Pa} \right)$$

$$= \frac{T\ (2 - \cos \theta)}{Pa}$$

$$\text{but since } a = \frac{\pi D_{max}^2}{4},$$

$$D_{max} = \frac{4T\ (2 - \cos\theta)}{\pi D_{max}^2 P}$$

$$D_{max}^3 = \frac{4T\ (2 - \cos\theta)}{\pi P}$$

$$D_{max} = \left[\frac{4T\ (2 - \cos\theta)}{\pi P}\right]^{\frac{1}{3}} \qquad (10a)$$

The piston diameter usually is restricted to commercial seal sizes also, but this means only choosing the next smaller sixteenth of an inch, in most cases.

Example:

An actuator is desired which will produce 1200 inch-lbs. of torque over an angular deflection of plus or minus 25°. The hydraulic pressure available is 3100 psi, and the back pressure is 100 psi. (The torque should include an estimate of system friction, and the differential pressure should include an estimate of pressure drop across the directional valve.) Using Equations (1) through (10), a family of actuators can be created, each one of which satisfies the stated requirements.

The variables are found as follows, expressed as functions of the area of the piston:

From (1) $\quad F = Pa$

$$= 3000a \text{ lbs.}$$

From (2) $\quad R = \frac{T}{Pa}$

$$= \frac{1200}{3000a} = \frac{0.4}{a} \text{ inches}$$

From (5) $\quad L_{max} = \frac{\pi T\theta}{90\ Pa}$

$$= \frac{(3.1416)(1200)(25)}{(90)(3000)\ a} = \frac{\pi}{9a}$$

$$= \frac{0.34907}{a} \text{ inches}$$

$$\text{or } \frac{L_{max}}{2} = \frac{0.17453}{a} \text{ inches}$$

The displacement per degree of angular motion is the same throughout the range of possible piston diameters.

From (6) $$Q = \frac{\pi}{180} \frac{T}{P}$$

$$= 0.00698 \text{ cubic inches per degree}$$

From (7) $$K = \frac{2\pi T}{Pa}$$

$$= \frac{(2)(3.1416)(1200)}{(3000)\ a}$$

$$= \frac{2.5133}{a} \text{ inches per rev}$$

From (8) $$\frac{L_{max}}{2\theta} = \frac{\pi}{180} \frac{T}{Pa}$$

$$= \frac{(3.1416)(1200)}{(180)(3000)\ a}$$

$$= \frac{0.00698}{a} \text{ inches per degree}$$

From (9a) $$r = \frac{T \cos \theta}{2\ Pa}$$

$$= \frac{(1200)(0.906)}{(2)(3000)a}$$

$$= \frac{0.1812}{a} \text{ inches } (= r_1 \text{ also})$$

The maximum piston diameter is calculated from the last equation, by substituting values for pressure, torque and angle.

From (10a) $$D_{max} = \left[\frac{4T\ (2 - \cos \theta)}{\pi P}\right]^{\frac{1}{3}}$$

$$= \left[\frac{(4)(1200)(2 - 0.906)}{(3.1416)(3000)}\right]^{\frac{1}{3}}$$

$$= \sqrt[3]{.5557}$$

$$= 0.823 \text{ inches}$$

Therefore, the largest standard piston diameter which need be considered is thirteen-sixteenths of an inch.

The values for piston diameters from one- uarter through thirteen-sixteenths of an inch are given in Table (1). The most compact design usually results from use of as large piston diameter as possible, but this is not necessarily always of prime importance. For example, as piston diameters increase, the resolution of a linear feedback potentiometer becomes increasingly important.

TABLE (1)

SAMPLE ACTUATOR DIMENSIONS

Given: 3000 psi differential hydraulic pressure; 1200 in-lbs torque; ± 25° deflection							
Piston dia., in.	Piston area, in^2	Force F, lbs.	Radius R, in.	Stroke L_{max}, in.	Rad. r & r_1, in.	Lead K, in/rev	Stroke per deg., in.
1/4	0.0491	147.3	8.1466	7.1094	3.6904	51.1874	0.14216
5/16	0.0767	230.1	5.2151	4.5511	2.3624	32.7670	0.09100
3/8	0.1105	331.5	3.6199	3.1590	1.6398	22.7448	0.06317
7/16	0.1503	450.9	2.6613	2.3225	1.2056	16.7219	0.04644
1/2	0.1964	589.2	2.0367	1.7734	0.9226	12.7968	0.03554
9/16	0.2485	745.5	1.6097	1.4047	0.7292	10.1138	0.02809
5/8	0.3068	920.4	1.3038	1.1377	0.5906	8.1920	0.02275
11/16	0.3712	1113.6	1.0776	0.9404	0.4881	6.7707	0.01880
3/4	0.4418	1325.4	0.9054	0.7901	0.4101	5.6888	0.01580
13/16	0.5185	1555.5	0.7715	0.6732	0.3495	4.8473	0.01346

V. Modified Procedure for Other Types

The calculations for other types of actuators are identical to the foregoing procedure with the exception of the determination of cam follower roller size and piston diameter.

In case of an actuator of Type B, Figure (5), the size of the socket is usually governed by one or more system conditions such as torque, bending moment of the driven shaft, or some other outside consideration. If we let r_1 be the outer radius of the socket (in the plane of the involute), then from Equation (9) we obtain:

$$r = \left| \frac{T \cos \Theta}{Pa} - r_1 \right| \tag{9b}$$

The maximum piston diameter for an actuator of Type B is independent of the rotation angle, but is instead a function of the socket outer radius, r_1. The new equation is obtained in a manner similar to that in which Equation (10) was found, except that the value for r_1 is no longer interchangeable with r.

Let $\frac{r_1}{r + r_1} = k$, then $r_1 = k\,(r + r_1) = k \left(\frac{T \cos \Theta}{Pa} \right)$, from Eq. (9).

$$D_{max} = 2\,(R - r_1), \text{ from Eq. (10)}$$

$$= 2 \left[\frac{T}{Pa} - k \left(\frac{T \cos \Theta}{Pa} \right) \right]$$

$$= \frac{2T}{Pa} (1 - k \cos \Theta)$$

but since $a = \frac{\pi D^2_{max}}{4}$,

$$D^3_{max} = \frac{8T}{P\pi} (1 - k \cos \Theta)$$

$$\text{and } D_{max} = 2 \sqrt[3]{\frac{T}{P\pi} \left(1 - \frac{r_1 \cos \Theta}{r + r_1} \right)} \tag{10b}$$

In the case of an actuator of Type C, Figure (6), the restricting feature is the size of the bearing housing. This is usually a function of twice the piston force, and therefore often bears a definite relationship to the size of the roller. Let the radius of the bearing housing be represented by r_2. Then the relationship can be expressed as:

$$r_2 = C\,(r),$$

where r is the roller radius and C is a constant. Then the equation for maximum piston diameter may be obtained as follows:

Let $\frac{C(r)}{r + C(r)} = k_1$, and then proceed as in the derivation of Eq. (10b):

$$D_{max} = 2\sqrt[3]{\frac{T}{P\pi}\left[1 - \frac{C(r)\cos\theta}{r + C(r)}\right]} \tag{10c}$$

The constant C is usually chosen to be 1.25 to 1.50 depending on the particular bearings one decides to use. Frequently it will be found that the actual maximum piston diameter (which must be a multiple of sixteenths of an inch, for standard seals) will be the same for all values of C between 1.25 and 1.50.

Type D and Type E illustrate limiting cases for very large pistons and rollers, and very large oscillation angles, respectively.

With an interrupted cam shaft such as that illustrated in Type D, Figure (7), it is possible to use piston and roller diameters nearly equal to the radius of the evolute, it being necessary only to maintain sufficient clearance for the rollers to pass each other. The cylinder block material and hydraulic pressure being used determines the thickness of the wall separating the pistons.

Since the involute profiles may now intersect at the cam centerline, the position of the roller when its piston is full retracted determines the maximum roller radius, or

$$r_{max} = R\cot\theta = \frac{T\cot\theta}{Pa} \tag{9c}$$

It should be noted that the expression for Equation (9c) above is now a function of the cotangent of the angle of oscillation, not the cosine as in the previous Equations (9), (9a) and (9b). In the case of Type D, where the radius of the roller approaches the radius of the evolute, it is evident that the angle of oscillation approaches plus or minus forty-five degrees as a limit.

A limiting case of maximum angular motion is presented in Figure (8), Type E, with the associated geometry in the diagram of Figure (10). In Figure (8) it can be seen that the maximum extension of either piston occurs when the opposite tip of the rocker arm touches the cylinder block. With the construction of Figure (10), the maximum angle of motion occurs when the x-co-ordinate value of the involute (locus of the roller center) is equal to the radius of the evolute and the y-co-ordinate value is equal to the evolute radius times the rotation angle. Then from the equation for an involute we have:

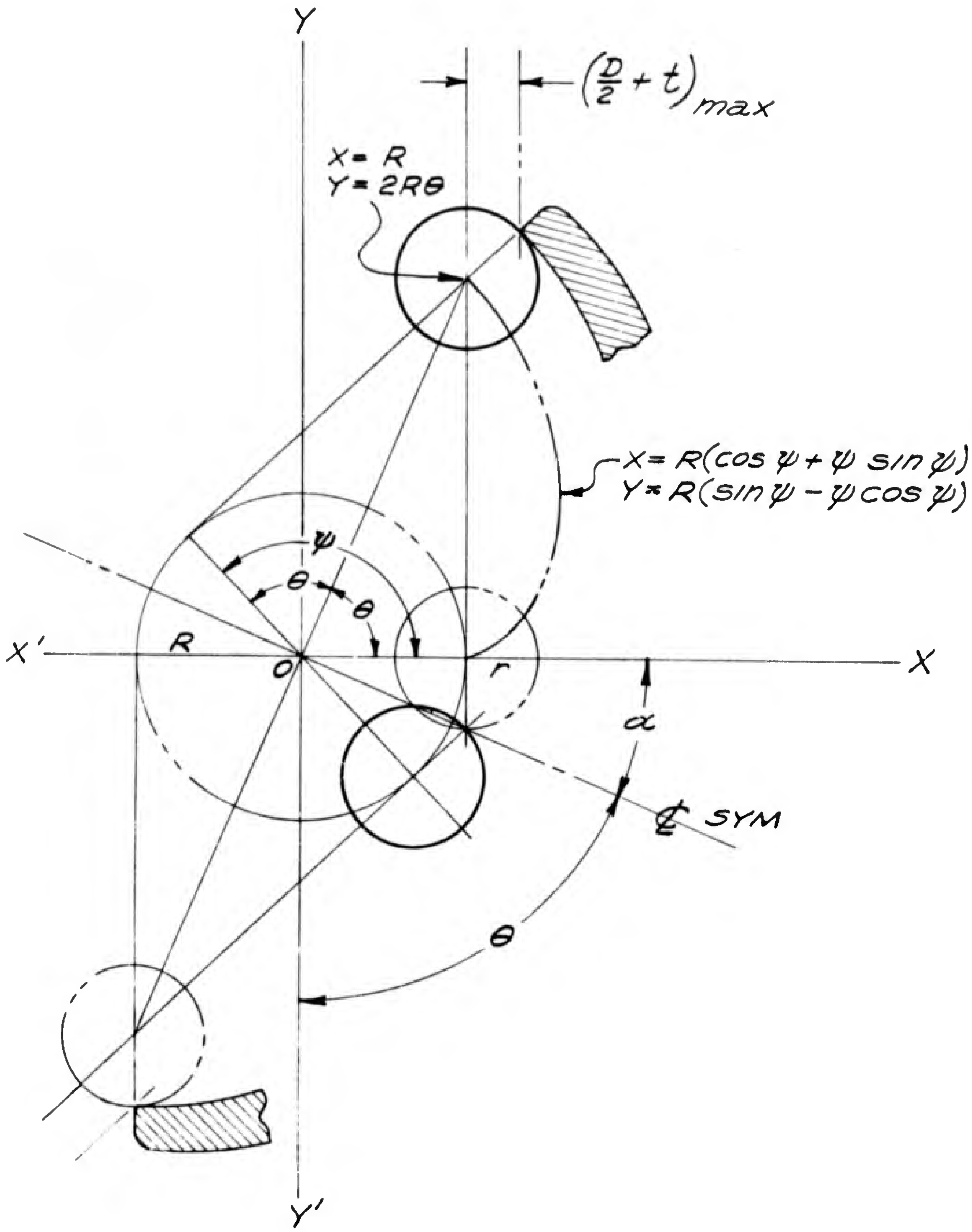


Fig. (10) Diagram for Maximum Angular Motion

$$x = R\,(\cos\psi + \psi\sin\psi)$$

and when $x = R$, $(\psi \neq 0, 2\pi)$,

$$\cos\psi + \psi\sin\psi = 1$$

$$\psi = \frac{1 - \cos\psi}{\sin\psi}$$

$$= 133^\circ\ 34', \text{ very nearly.} \qquad (11)$$

The maximum permissible roller size is determined from Equation (9c)

$$r = R\cot\theta$$

That this is so can be understood if one remembers that the point of contact between the roller and the cam surface must be in line with the axis of force application.

The construction of Figure (10) shows that the maximum permissible roller size yields the maximum space for cylinder plus cylinder wall.

If t = thickness of cylinder wall, inches,

$$\psi = 2\,\theta,$$

and $\alpha = 90^\circ - \theta$, then

$$\left(\frac{D}{2} + t\right)_{max} = r\sin 2\alpha$$

$$= 2r\,(\sin\alpha\cos\alpha)$$

$$= 2r\,(\cos\theta\sin\theta)$$

Substituting $r = R\cot\theta = R\,\dfrac{\cos\theta}{\sin\theta}$ from Equation (9c)

and $\theta = 133^\circ\ 34'$ from Equation (11),

$$\left(\frac{D}{2} + t\right)_{max} = 2\,R\cos^2\theta$$

$$= 0.3108\ R \qquad (12)$$

and $r_{max} = R\cot\theta$

$$= 0.4289\ R \qquad (13)$$

The imaginative reader will surely agree that these five basic actuator types hardly begin to describe the possible variation of this general type. However, the foregoing types have been purposely limited to simplify presentation of the fundamental design procedure. For example, the pistons are stepless, single-acting, non-telescoping and are arranged with parallel axes. The rollers have all been clevis-mounted and have followed cam faces of the simplest type, etc. To illustrate how some single departure may overcome a shortcoming, consider Figure (11) which shows an actuator similar to Type E of Figure (8), except that the piston axes are inclined toward each other at the base at an angle of fifteen degrees. The evolute radius and piston stroke have not been altered, but note how the ratio of piston diameter to length has been improved merely by the ability to increase the roller size. This modification does not affect the angle of motion in any way, since nothing has been changed which affects the involute path of the roller centers. These paths have merely been rotated apart the same angle as the cylinder inclination, but the slight increase this allows in roller size has provided room for pistons of twice the original diameter.

Modifying the cam surface to provide side-by-side tracks spaced an unequal distance apart from one another, and with appropriate dual rollers on each piston could allow overlapping cam surfaces across the rocker arm centerline of symmetry, and straddling of the cylinders -- which opens up altogether new limits to actuator performance. So it easily can be seen how many new types could be created as the need might arise. The designer is required to remember only the fundamentals associated with the involute as presented at the beginning, and then give free rein to his imagination.

VI. Cam Profile Dimensions and Arc Length

Drawings for involute cam rocker arm fabrication require that the profile be furnished as a table of co-ordinates to which the part, or a pattern, may be cut and inspected. Both polar and rectangular co-ordinates are useful depending on the machinery and instruments involved. The actual cam profile is an involute generated by the evolute circle, limited by the maximum angle of rotation, but complicated somewhat by the roller on the end of the piston. This profile is extremely simple to construct on a layout, but requires a considerable effort to compute compared with merely the path of roller centers.

For practical reasons it is advisable to prepare an enlarged layout which can be scaled accurately to at least three significant figures of the full-scale dimensions, and to use this drawing to check the calculated dimensions. A slight error in arithmetic can result in a costly mistake that is difficult to find in the co-ordinate table, but stands out boldly from a fair curve on a layout.

A conventional manner of dimensioning the profile is to assign Y values in arbitrary uniform increments (to ease half the workman's job) and provide the corresponding X values. Mathematically, it is extremely tedious to compute values in just that way, so an accurate enlarged layout

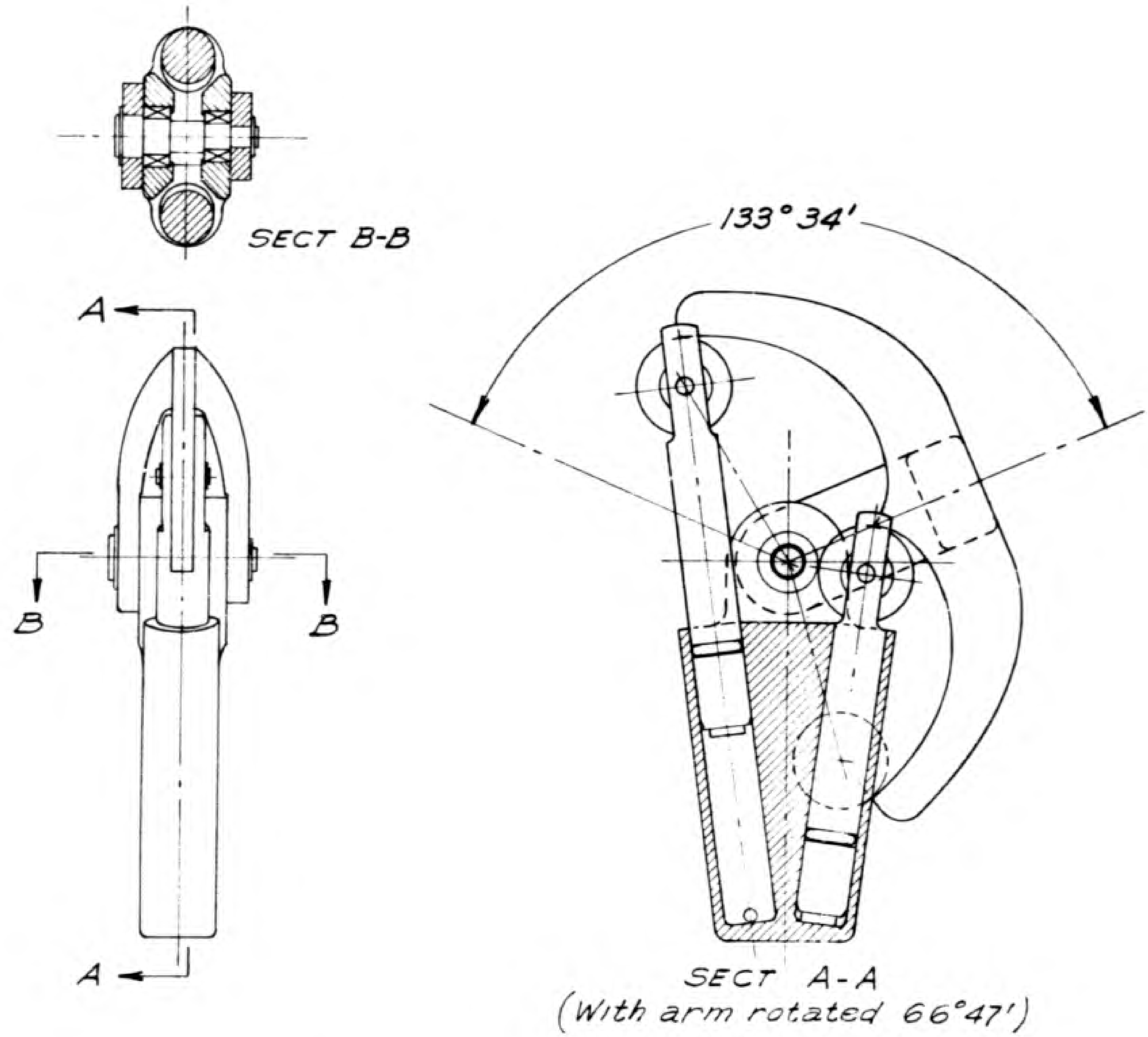


Fig.(11) Actuator Type E - Modified

can be generated and checked with more readily obtainable co-ordinates, and then laid off in one axis in steps of .050" for which matching dimensions can be scaled in the other axis.

Still another use for an enlarged layout will be found in inspection of cams with an optical comparator. If one has access to a machine with a sufficiently large screen, it is only necessary to produce the layout on a plastic sheet complete with tolerance bands, and a great many finished articles may be inspected quickly and very accurately.

The diagram of Figure (12) shows the geometry involved for one side of the symmetrical rocker arm. The roller is shown in the mid-position, which is defined as the position for equal extension of the pistons. Schematically, the figure is shown for actuator Types A, B and C, but the accompanying procedure is valid for Types D and E as well. Notation for the constants, evolute radius R, roller radius r, and maximum angular motion from mid-position $\pm\theta$, is identical to preceding figures. Notation for the dependent variables is as follows:

- L = Extension of piston from the fully retracted position.
- ψ = Angle between the $\phi = 0$ axis and the diameter of evolute which is normal to direction of piston motion.
- ϕ = Angle between the $\phi = 0$ axis and roller contact point on cam.
- α = Angle between the $\phi = 0$ axis and a line parallel to direction of piston motion at the mid-position.
- ρ, δ = Polar co-ordinates of the roller contact point on the cam profile with respect to the rocker arm axis of symmetry.
- x', y' = Rectangular co-ordinates corresponding to ρ, δ.

Figure (12) is shown with the axes rotated from preceding figures so that the co-ordinates can be reflected across the rocker arm axis of symmetry. With the roller shown in the mid-position, the $\pm\theta$ motion is measured from the Y' axis. This likewise causes the terminal side of ψ to coincide with the Y' axis. It is important to understand that this coincidence occurs <u>only</u> at the mid-position.

Because of the existence of the roller on the end of the piston, it is necessary to establish the $\phi = 0$ axis from which to measure ϕ and ψ. This auxiliary axis corresponds to the X axis of preceding figures, and is defined as a line from the origin passing through the theoretical intersection of the involute and evolute when the radius of the roller

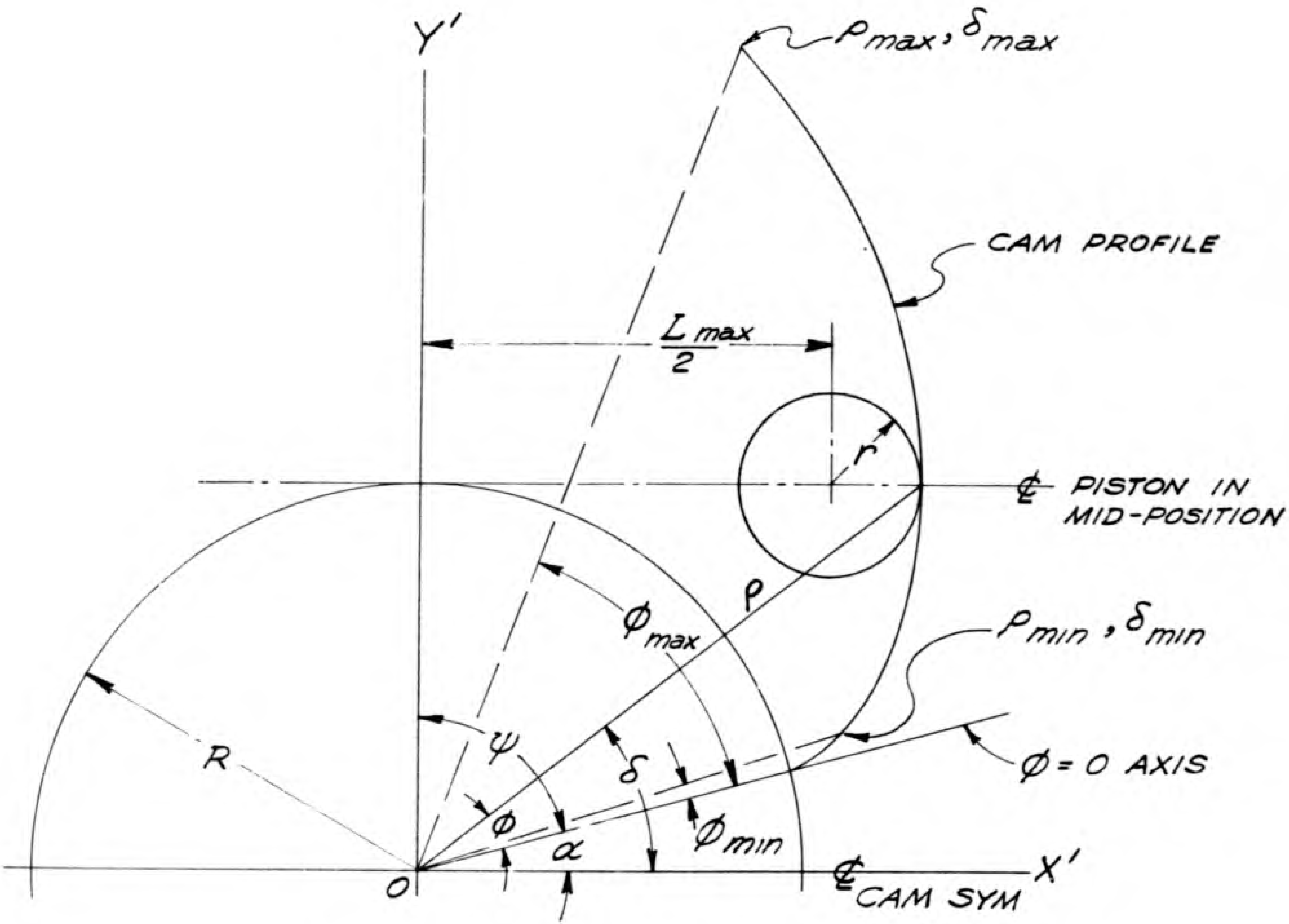


Fig.(12) Cam Profile Diagram

becomes infinitely small. For proper contact, the minimum radius of cam curvature must not be less than the radius of the roller. Hence, the minimum value of ϕ is always greater than zero, and the actual involute portion of the cam surface occurs only between the points ρ min, δ min and ρ max, δ max.

To begin developing expressions for the dependent variables, let us examine Figure (12) closely. From the diagram and the definition of an involute:

$$L + r = R\psi, \text{ or}$$

$$\psi = \frac{L + r}{R} \tag{14}$$

Next consider the right triangle formed by R, L + r, and hypotenuse ρ. The angle at the origin is $\psi - \phi$, and

$$\tan(\psi - \phi) = \frac{L + r}{R}$$

$$= \psi, \text{ from Equation (14)}$$

$$\text{or } \psi - \phi = \tan^{-1}\psi$$

$$\phi = \psi - \tan^{-1}\psi \tag{15}$$

When the piston is in the fully retracted position, then

$$L = 0$$

$$\psi = \psi_{min} = \frac{L + r}{R} = \frac{r}{R}$$

When the piston is in the fully extended position

$$L = L_{max} = 2R\theta$$

$$\psi = \psi_{max} = \frac{L + r}{R} = 2\theta + \frac{r}{R}$$

Then, for the piston in the mid-position, as shown in Figure (12)

$$L = \frac{L_{max}}{2}$$

$$\psi = \psi_{mid} = \tfrac{1}{2}(\psi_{min} + \psi_{max})$$

$$\text{or, } \psi_{mid} = \theta + \frac{r}{R}$$

Next, from the diagram, it can be seen that

$$\alpha + \psi_{mid} = \frac{\pi}{2}$$

$$\alpha = \frac{\pi}{2} - \psi_{mid}$$

$$\text{or } \alpha = \frac{\pi}{2} - \theta - \frac{r}{R} \tag{16}$$

Also from Figure (12) it is evident that

$$\delta = \phi + \alpha \tag{17}$$

Returning to the triangle from which Equation (15) was derived, we have

$$\rho^2 = (L + r)^2 + R^2$$

$$\rho = \left[(L + r)^2 + R^2\right]^{\frac{1}{2}} \tag{18}$$

Finally, converting polar co-ordinates to rectangular, we have

$$X' = \rho \cos \delta \tag{19}$$

$$Y' = \rho \sin \delta \tag{20}$$

Equations (14) through (20) are presented in Table (2) as a six-step procedure for computing cam profile dimensions, together with a simplified example using only three values of L.

Frequently it is found desirable to compute the actual arc length of the involute cam surface. For instance, this is required when calculating cam follower revolutions per minute to use in selecting an anti-friction bearing. Arc length of a curve is:

$$s = \int_a^b \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx$$

Using the equations for an involute

$$x = R\,(\cos\psi + \psi \sin\psi)$$

$$y = R\,(\sin\psi - \psi \cos\psi)$$

and assigning the limits

$$a = \psi_{min}$$

$$b = \psi_{max}$$

TABLE (2)

COMPUTING CAM PROFILE DIMENSIONS

Procedure	Example		
Given: $R = 0.5968"$; $r = 0.1875"$; $\theta = \pm 51^\circ$ (= .8901 rad)	$L = 0$ (Piston Fully Retracted)	$L = \frac{L_{max}}{2}$ (Mid-Position)	$L = L_{max} = 2R\theta$ (Piston Fully Extended)
Step One: $\psi = \frac{L + r}{R}$	0.31418 rad	1.20434 rad	2.09450 rad
Step Two: $\phi = \psi - \tan^{-1}\psi$	0.00978 rad	0.32654 rad	0.96920 rad
Step Three: $\alpha = \frac{\pi}{2} - \theta - \frac{r}{R}$	0.36652 rad	0.36652 rad	0.36652 rad
Step Four: $\delta = \phi + \alpha$	0.37630 rad	0.69306 rad	1.33572 rad
Step Five: $\rho = \left[(L + r)^2 + R^2\right]^{\frac{1}{2}}$	0.62557"	0.93422"	1.38516"
Step Six: $X' = \rho \cos \delta$ $Y' = \rho \sin \delta$	0.5818" 0.2299"	0.7188" 0.5968"	0.3226" 1.3471"

we first must determine

$$\frac{dy}{dx} = \frac{d\frac{dy}{d\psi}}{\frac{dx}{d\psi}}$$

then $\frac{dx}{d\psi} = R\,\psi\cos\psi$

and $\frac{dy}{d\psi} = R\,\psi\sin\psi$,

therefore $\frac{dy}{dx} = \tan\psi$.

Substituting in the original integral, we have

$$s = \int_{\psi min}^{\psi max} (1 + \tan^2\psi)^{\frac{1}{2}}\, R\,\psi\cos\psi\, d\psi \,.$$

Since

$$(1 + \tan^2\psi)^{\frac{1}{2}} = \sec\psi = \frac{1}{\cos\psi},$$

we have

$$s = \int_{\psi min}^{\psi max} R\,\psi\,\frac{1}{\cos\psi}\cos\psi\, d\psi$$

$$= \int_{\psi min}^{\psi max} R\,\psi\, d\psi$$

$$= \Big|_{\psi min}^{\psi max} R\,\frac{\psi^2}{2}$$

or, $s = \frac{R}{2}(\psi^2_{max} - \psi^2_{min})$ inches (21)

VII. Material Selection and Other Design Considerations

The success of any design often depends heavily on intelligent and thoughtful selection of material for each element of the device. For the parts of the rotary actuator the following considerations are basic:

1. High strength and low deflection at required temperature extremes.

2. Compatibility for resistance to wear and corrosion.

3. Ease of fabrication and dimensional stability.

4. Low inertia for those moving parts which affect frequency response.

5. Obtainability of good surface finish and low porosity where required for fluid sealing.

The first item above is of most importance for the rocker arm material. The arm must be analyzed as a non-uniform curved beam requiring a high order of stiffness. This usually means a steel alloy and the author has found a precipitation-hardening stainless to be highly satisfactory in many respects, to wit:

a. It can be investment cast to reproduce the involute curve with sufficient accuracy.

b. It can be heat-treated to high strength and hardness without distortion.

c. It does not require plating for surface protection.

One drawback is that steel does not afford an inherently low mass moment of inertia and this must be overcome with efficient design to achieve low weight.

The materials selected for the piston and cylinder block must reflect all of the basic considerations listed above. Although the involute surface theoretically eliminates side-loading of the pistons, the tendency to design them as short and compact as possible accentuates the effect of any irregularities in the cam shape. The cylinder block may often be a structural element of the machine or vehicle of which the actuator is a part, particularly in multiple-actuator designs. This in itself may dictate the choice of material for the cylinder block and force the piston to be some compatible material. For example, if the cylinder block were steel, the pistons could be anodized aluminum and the two would meet all of the five basic requirements. If the cylinder block is aluminum, magnesium or other soft material, it may be advisable to install hard steel liners in order to achieve, <u>and keep</u> good surface finish for proper sealing.

The cam follower roller is often a commercial item, or made from a standard needle bearing with a steel sleeve to enlarge the diameter. The follower shaft may be hardened to provide the inner race for the needle bearing, or may be simply a clevis bolt and thin nut. The cam pivot shaft and its bearings depend on the actuator type and application, and the materials are selected accordingly.

As happens often in the development of a somewhat unusual device, there are several incidental features which help to avoid difficulty, but are not entirely obvious from the first layouts. One of these is the advisability to extend the clevis of the piston (on either side of the roller) so that it fits the sides of the cam surface. The fit need not be very close so long as it provides some resistance to piston rotation. Theoretically, the continual curvature of the involute exerts a restoring force on the roller face, but at the outer ends of the cam this curvature decreases to the point where it becomes relatively ineffective.

Some means should be provided to maintain contact of the roller with the cam surface as each piston is alternately pushing or being pushed. Ordinarily this is provided by a definite back pressure in the hydraulic system. A mechanical spring is more effective in those cases where contact may otherwise be lost during shutdown periods, due either to gravity, load unbalance or manipulation of the driven member. Loss of contact results in a severe shock loading of the roller bearings when the hydraulic pressure is suddenly applied, and may even cause failure of the rocker arm.

It is usually advisable, and certainly more convenient, to provide internal stops to the angular motion. The pistons then bottom in the cylinders rather than the rocker arm striking the outside of the cylinder block, or meeting any other external obstruction, in order to avoid excessive and unnecessary bending loads on the rocker arm.

Design details of the actuator vary with the type of system into which it will go. The fourway directional valve may affect the actual hydraulic pressure available at the pistons, if it is a high-performance proportional type. In the usual closed-loop system of control, the actual motion of the load being driven by the actuator is electrically measured and fed back for comparison with a signal representing the desired motion. The resulting error signal is then amplified to provide the current input to the valve. The second stage of the valve being controlled by this small current features a hydraulic amplification which causes a proportional shift in the first stage valve spool. With high acceleration rates, pressure losses of one-third are not uncommon, and the designer is cautioned not to overlook them when establishing the net differential pressure value for use in the foregoing design equations.

VIII. Summary

To conclude this presentation it has been shown that a rotary hydraulic actuator, by virtue of a symmetrical involute cam rocker arm, will offer the following attractions to designers.

1. No backlash and high stiffness.
2. No rotary high pressure fluid seals, no high starting or running friction.
3. Zero pressure angle.
4. Angular motion directly proportional to linear motion.
5. Constant torque throughout full range of motion.
6. Simplicity, economy and reliability.

Through examples illustrating several basic types, the fundamental design procedure has been presented, together with instructions for calculating all pertinent dimensions. Material selection (that oft-neglected subject) has been discussed, and a few "timely hints" of practical consideration have been offered.

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