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A METHOD TO COMPUTE THE CHARACTERISTICS OF LUMPED ELECTROMAGNETIC DELAY LINES

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-628

DECEMBER 1965

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H. B. Opladen

Prepared for

DIRECTORATE OF COMPUTERS
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 708

Prepared by

THE MITRE CORPORATION
Bedford, Massachusetts
Contract AF 19(628)-2390

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ABSTRACT

This document presents a method of computing the coefficients of the transfer function of iterated or noniterated lumped electromagnetic delay lines. The method shows that the coefficients can be found by using Pascal's triangle for iterated delay lines.

REVIEW AND APPROVAL

This technical documentary report has been reviewed and is approved.

SEYMOUR JEFFERY
Major, USAF
Chief, Computer Division
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SECTION I

INTRODUCTION

A method is developed which enables one to determine the frequency response and phase shift of nonhomogenous lumped delay lines. From this general relationship between input and output signals, the special case of a homogenous lumped delay line can be treated.

SECTION II

NONHOMOGENOUS LUMPED DELAY LINES

Every section of the nonhomogenous delay line is considered as a half π-section. The sequence of these elementary sections is shown in Fig. 1. A series element impedance is expressed by the symbol a, and a shunt element impedance by the symbol b. Every one of these impedances has an associated frequency operator containing resistance, capacitance, or inductance. For example, an inductance with loss is written in the operator form

$$a_i(P) = R_i + PL_i \quad ,$$

or a lossless capacitor in the formulation

$$b_i(P) = \frac{1}{PC_i} \quad .$$

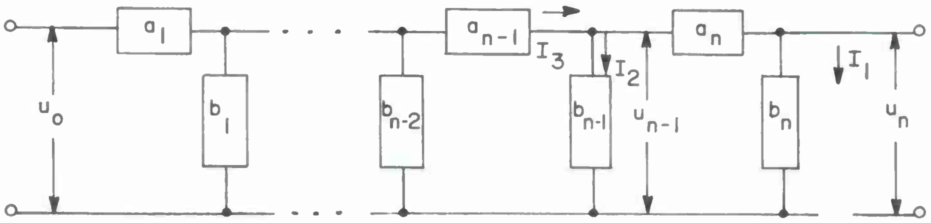
In general, each elemental series or shunt impedance can be expressed by

$$a_i = a_i(P)$$

or

$$b_i = b_i(P) \quad .$$

In the following analysis, the operator variable (P) is eliminated for simplification.



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The frequency response is expressed by the ratio between input and output voltages

$$F(P) = \frac{u_n}{u_o} \quad . \tag{1}$$

The input u_n is calculated from the voltage u_{n-1} across the impedance b_{n-1} :

$$u_n = I_1 b_n \quad ,$$

$$u_{n-1} = I_1 \left(a_n + b_n\right) \quad ,$$

$$u_{n-1} = I_1 c_n \quad ,$$

or

$$u_n = \frac{b_n}{c_n} u_{n-1} , \tag{2}$$

and

$$u_{n-1} = \frac{b_{n-1} c_n}{b_{n-1} + c_n} I_3 = \frac{b_{n-1} c_n}{b_{n-1} + c_n} \frac{u_{n-2}}{c_{n-1}} . \tag{3}$$

By substitution of Eq. (3) into Eq. (2), it follows that

$$u_n = \frac{b_n}{c_n} \frac{b_{n-1} c_n}{b_{n-1} + c_n} \frac{u_{n-2}}{c_{n-1}} . \tag{4}$$

Thus, one can write

$$\frac{u_n}{u_{n-2}} = \frac{b_n}{c_n} \frac{b_{n-1} c_n}{b_{n-1} + c_n} \frac{1}{c_{n-1}} . \tag{5}$$

Therefore, the ratio between input and output voltage of the whole transmission line can be written as

$$F(P) = \frac{u_n}{u_o} = \frac{b_n}{c_n} \frac{b_{n-1} c_n}{b_{n-1} + c_n} \frac{1}{c_{n-1}} \frac{b_{n-2} c_{n-1}}{b_{n-2} + c_{n-1}} \frac{1}{c_{n-2}} \cdots \frac{1}{c_2} \frac{b_1 c_2}{b_1 + c_2} \frac{1}{c_1} , \tag{6}$$

or

$$F(P) = \frac{\prod_{i=1}^{i=n} b_i}{c_1 \prod_{i=1}^{i=n-1} (b_i + c_{i+1})} , \tag{7}$$

as can be seen from Fig. 1.

$$c_{n-1} = a_{n-1} + \frac{b_{n-1} \quad c_n}{b_{n-1} + c_n} \tag{8}$$

or, in general,

$$c_i = a_i + \frac{b_i \quad c_{i+1}}{b_i + c_{i+1}} . \tag{9}$$

In order to eliminate the sum-product in the denominator of Eq. (7), the second factor in the denominator of Eq. (6) is rearranged as

$$b_{n-2} + c_{n-1} = b_{n-2} + a_{n-1} + \frac{b_{n-1} \quad c_n}{b_{n-1} + c_n} , \tag{10}$$

and the first factor in the denominator of Eq. (6) is defined as

$$N_{n-1} = b_{n-1} + c_n \quad . \qquad (11)$$

Substituting into Eq. (10), one obtains

$$b_{n-2} + c_{n-1} = b_{n-2} + a_{n-1} + \frac{b_{n-1}\, c_n}{N_{n-1}} \quad , \qquad (12)$$

or

$$b_{n-2} + c_{n-1} = b_{n-2} + \frac{a_{n-1}\, N_{n-1} + b_{n-1}\, c_n}{N_{n-1}} \quad . \qquad (13)$$

Again, defining

$$Z_{n-2} = a_{n-1}\, N_{n-1} + b_{n-1}\, c_n \quad , \qquad (14)$$

Eq. (13) can be rewritten as

$$b_{n-2} + c_{n-1} = b_{n-2} + \frac{Z_{n-2}}{N_{n-1}} \quad . \qquad (15)$$

From Eq. (15) it follows that

$$c_{n-1} = \frac{Z_{n-2}}{N_{n-1}} \quad , \tag{16a}$$

and also

$$c_n = \frac{Z_{n-1}}{N_n} \quad . \tag{16b}$$

If it is assumed that

$$c_n = Z_{n-1} \quad , \tag{17}$$

it follows that $N_n = 1$. Substituting the value of c_n in Eq. 14, one obtains

$$Z_{n-2} = a_{n-1} N_{n-1} + b_{n-1} Z_{n-1} \quad , \tag{18}$$

or, in general,

$$Z_i = a_{i+1} N_{i+1} + b_{i+1} Z_{i+1} \quad . \tag{19}$$

From $N_{n-1} = b_{n-1} + c_n$, one can write

$$N_{n-1} = b_{n-1} + \frac{Z_{n-1}}{N_n}$$

and

$$N_{n-1} = \frac{b_{n-1} N_n + Z_{n-1}}{N_n} \quad .$$

Since $N_n = 1$,

$$N_{n-1} = b_{n-1} N_n + Z_{n-1} \quad ,$$

or

$$N_{n-2} = b_{n-2} N_{n-1} + Z_{n-2} \quad ,$$

and, in general,

$$N_i = b_i N_{i+1} + Z_i \quad . \tag{20}$$

Substituting Eqs. (19) and (20) into Eq. (15), it follows that

$$b_i + C_{i+1} = b_i + \frac{Z_i}{N_{i+1}} = \frac{b_i N_{i+1} + Z_i}{N_{i+1}} \quad , \tag{21}$$

with Eq. (20), which gives the simple expression,

$$b_i + c_{i+1} = \frac{N_i}{N_{i+1}} \quad . \tag{22}$$

Thus, the denominator product of Eq. (7), can be written as

$$\begin{aligned} c_1 \prod_{i=1}^{i=n-1} \left(b_i + c_{i+1}\right) &= c_1 \; \frac{N_1}{N_2} \; \frac{N_2}{N_3} \; \frac{N_3}{N_4} \\ &\quad \cdots \; \frac{N_{n-2}}{N_{n-1}} \; \frac{N_{n-1}}{N_n} = c_1 \; \frac{N_1}{N_n} \quad , \end{aligned} \tag{23}$$

with $N_n = 1$, and from Eq. (16) one can write

$$c_1 = \frac{Z_o}{N_1}$$

and

$$Z_o = c_1 \; \frac{N_1}{N_n} \quad .$$

But Z_o is, according to Eq. (19),

$$Z_o = a_1 N_1 + b_1 Z_1 \quad . \tag{24}$$

Finally, one obtains

$$F(P) = \frac{\prod_{i=1}^{i=n} b_i}{a_1 N_1 + b_1 Z_1} \quad . \tag{25}$$

According to Eqs. (19) and (20) the parameters, Z_1 and N_1, for the first section of the transmission line are

$$Z_1 = a_2 N_2 + b_2 Z_2 \quad , \tag{26}$$

and

$$N_1 = b_1 N_2 + Z_1 \quad , \tag{27}$$

and, for the last, i. e., n^{th}-section of the transmission line, with $i = n\text{-}1$,

$$Z_{n-1} = a_n + b_n \quad , \tag{28a}$$

and

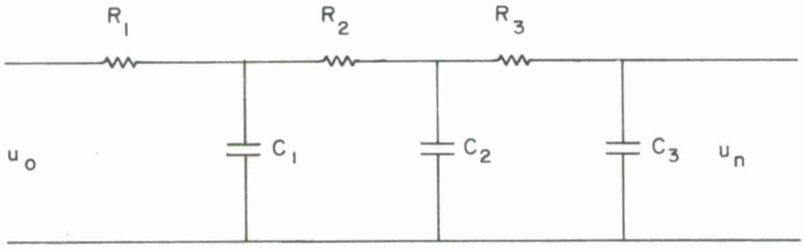
$$N_{n-1} = b_{n-1} + a_n + b_n \quad . \tag{28b}$$

The practical application of the method for a three-section line is indicated in Fig. 2; it is

$$Z_{n-1} = Z_2 = a_3 + b_3 \quad ,$$

and

$$N_{n-1} = N_2 = b_2 + a_3 + b_3 \quad ,$$



and, according to Eqs. (19) and (20),

$$Z_1 = a_2\left(b_2 + a_3 + b_3\right) + b_2\left(a_3 + b_3\right) \quad ,$$

and

$$N_1 = b_1\left(b_2 + a_3 + b_3\right) + a_2\left(b_2 + a_3 + b_3\right) + b_2\left(a_3 + b_3\right) \quad ,$$

and, finally,

$$F_{(P)} = \frac{b_1 b_2 b_3}{a_1 b_1 b_2 + a_1 b_1 a_3 + a_1 b_1 b_3 + a_1 a_2 b_2 + a_1 a_2 a_3 + a_1 a_2 b_3 + a_1 b_2 a_3 + a_1 b_2 b_3 + b_1 a_2 b_2 + b_1 a_2 a_3 + b_1 a_2 b_3 + b_1 b_2 a_3 + b_1 b_2 b_3} \quad . \tag{29}$$

If the following substitutions are made

$$a_1 = R_1 \qquad b_1 = \frac{1}{PC_1} \qquad T_1 = R_1 C_1$$

$$a_2 = R_2 \qquad b_2 = \frac{1}{PC_2} \qquad T_2 = R_2 C_2$$

$$a_3 = R_3 \qquad b_3 = \frac{1}{PC_3} \qquad T_3 = R_3 C_3$$

Eq. (11) can be written in the form:

$$F_{(P)} = \frac{1}{1 + P \underbrace{(T_1 + T_2 + T_3 + R_1 C_3 + R_1 C_2 + R_2 C_3)}_{k_1} + P^2 \underbrace{(T_1 T_2 + T_1 T_3 + T_2 T_3 + T_1 R_2 C_3 + T_3 R_1 C_2)}_{k_2} + P^3 \underbrace{(T_1 T_2 T_3)}_{k_3}} , \tag{30}$$

or

$$F(P) = \frac{1}{1 + kp + k_2 P^2 + k_3 P^3} , \tag{31}$$

where the coefficients k to k_3 are determined by the denominator of (30).

SECTION III

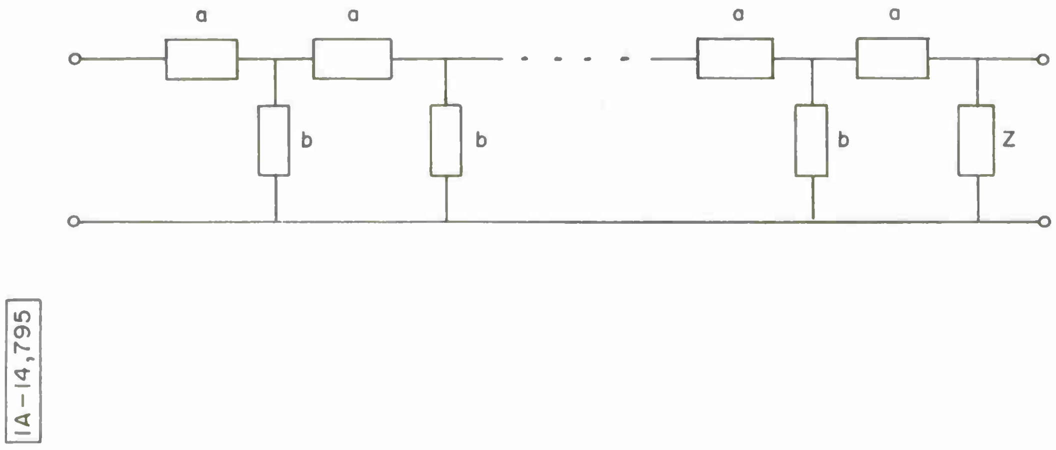
ITERATED LUMPED DELAY LINES

An iterated delay line (see Fig. 3) may have a complex termination z. Further, for simplicity, one can write:

$$\frac{a}{b} = A \tag{32a}$$

and

$$\frac{b}{z} = v \; . \tag{32b}$$



Thus,

$$\frac{a}{z} = vA \quad . \tag{33}$$

If Eq. (25) is written in the form

$$D(P) = \frac{aN_1 + bZ_1}{b^n} \quad , \tag{34}$$

with

$$D(P) = \frac{1}{F(P)} \tag{35}$$

as differential operator, the coefficients for lines with $n = 2, 3, 4$ can be written as indicated in Table I.

Table I

Coefficients for Lines with $n = 2, 3, 4$

	n=2	n=3	n=4
k_1	$(2v+1)A$	$(3v+3)A$	$(4v+6)A$
k_2	vA^2	$(4v+1)A^2$	$(10v+5)A^2$
k_3		vA^3	$(6v+1)A^3$
k_4			vA^4

To make this clear, the coefficient k_2 for $n = 3$ is calculated:

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a}{b} = A \tag{36a}$$

and

$$\frac{a_3}{b_3} = \frac{a}{b} = vA \quad . \tag{36b}$$

Thus, one finds the following expression for the coefficient k_2 of the differential operator of Eq. (35):

$$k_2 = \frac{a_1}{b_1}\frac{a_2}{b_2} + \frac{a_1}{b_1}\frac{a_3}{b_3} + \frac{a_2}{b_2}\frac{a_3}{b_3} + \frac{a_1}{b_1}\frac{a_2}{b_3} + \frac{a_3}{b_3}\frac{a_1}{b_2} \quad , \tag{37a}$$

or

$$k_2 = A^2 + A^2v + A^2v + A^2v + A^2v \quad , \tag{37b}$$

$$k_2 = (4v+1)\,A^2 \quad . \tag{37c}$$

Continuing in this way, one can construct the Table of Coefficients. From the Table, it can be seen that the coefficient of the factor v in k_1 can be

Table II

Table of Coefficients

n	1	2	3	4	5	6	7
k_1	v . 1	2	3	4	5	6	7
	+ 0	1	3	6	10	15	21
k_2	v . 0	1	4	10	20	35	56
	+ 0	0	1	5	15	35	70
k_3	v . 0	0	1	6	21	56	126
	+ 0	0	0	1	7	28	84

expressed by the second coefficient $\binom{n}{1}$ of the binomial expansion. Similarly, the second term in k_1 can be expressed by the second factor $\binom{n}{2}$ of the binomial theorem. Thus, one can write:

$$k_1 = \binom{n}{1} v + \binom{n}{2} . \tag{38}$$

The other k-coefficients can be found in an analogous manner, with

$$k_2 = \binom{n+1}{3} v + \binom{n+1}{4} , \tag{39}$$

and

$$k_3 = \binom{n+2}{5} v + \binom{n+2}{6} ,$$

or, in general, for any coefficient:

$$k_i = \binom{n+i-1}{2i-1} v + \binom{n+i-1}{2i} . \tag{40}$$

Finally, the coefficient next to the last one is

$$k_{n-1} = \binom{2n-2}{1} v + 1 . \tag{41}$$

Thus, the complete differential operator of an iterated delay line with n -sections can be written in the form

$$\begin{aligned} D_n(P) = {} & 1 + A(P) \left\{ \binom{n}{1} v + \binom{n}{2} \right\} \\ & + A^2 (P) \left\{ \binom{n+1}{3} v + \binom{n+1}{4} \right\} \\ & + A^i (P) \left\{ \binom{n+i-1}{2i-1} v + \binom{n+i-1}{2i} \right\} \\ & + A^{n-1} (P) \left\{ \binom{2n-2}{1} v + 1 \right\} \\ & + A^n (P) \, v , \end{aligned} \tag{42}$$

and the phase shift in the form

$$\operatorname{tg}\phi = \frac{A(P)\left\{\binom{n}{1} v + \binom{n}{2}\right\} \cdots\cdot \pm A(P)^{i\ (\mathrm{odd})} \left\{\binom{n+i-1}{2i-1} v + \binom{n+i-1}{2i}\right\}}{1-A^{2}(P)\left\{\binom{n+1}{3} v + \binom{n+1}{4}\right\} \cdots \pm A(P)^{i\ (\mathrm{even})} \left\{\binom{n+i-1}{2i-1} v + \binom{n+i-1}{2i}\right\}} . \qquad (43)$$

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