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ELECTROMAGNETIC MASS, & THE INERTIAL PROPERTIES OF NUCLEI

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ABSTRACT

According to classical electrodynamics, the inertial mass Δm belonging to the electrostatic potential energy, ΔW, of two charges, q, separated by a fixed distance, r, with an angle, ψ, between $\vec{r}$ and the acceleration, $\vec{a}$, is given by,

$$\Delta m = (\Delta W/c^2)(1 + \cos^2\psi).$$

In a rigid non-spherical structure composed of charged particles, this intrinsic asymmetry generates an asymmetry in the electromagnetic part of the inertia. In a spherically-symmetrical structure, it generates a contribution to the inertial mass of amount $(4/3)\Delta W/c^2$, where ΔW is the total electrostatic potential energy. According to the theory of relativity, however, (1) the inertial mass of any physical system should be a scalar quantity (no matter how distorted its electromagnetic structure) and (2) the "excess" inertial mass of electromagnetic origin, $(1/3)\Delta W/c^2$, should not be observable. We have examined the experimental evidence on both these points with respect to nuclei. Nuclei are particularly significant for this test because (1) they are the only structures (excepting the elementary particles) which possess an appreciable fraction of their net mass in the form of electrostatic energy, (2) they are the only structures which are formed as a result of the equilibrium between two very different types of known forces (electromagnetic,

and nuclear), (3) their dimensions are large enough that there is reason to trust the validity of the electromagnetic laws.

We find that the inertial mass of a distorted nucleus, as measured by a mass spectrometer, has no observable asymmetry to an accuracy of 1 part in 100 of the asymmetry which is calculated to exist as a consequence of the electromagnetic energy. Also, we find that a comparison of nuclear mass differences (as measured by the mass spectrometer and by nuclear reactions) shows that the "excess" electromagnetic inertial mass $(1/3)\Delta W/c^2$ is not observable, to an accuracy of 1 part in 600.

1. Introduction

In the special theory of relativity, the inertial mass, m, of a particle is assumed to be a scalar function of the magnitude of the velocity, and it is found that m is related to the total energy, W, of the particle by the equation,

$$m = W/c^2. \qquad (1)$$

However, it is only the total energy of a stable structure, which obeys (1). It is not necessary that each energy source which contributes to W have an inertial mass which obeys this law. Indeed, H. A. Lorentz showed that the inertial property of the electrostatic energy of system is not given by (1).[1,2] He showed that although a spherical shell of charge e and radius r has the electrostatic energy $W_0 = e^2/8\pi\epsilon_0 r$ joules (rationalized mks units), when this structure is accelerated so that it attains a velocity $\vec{v}$, the momentum $\vec{G}$ (which may be computed by integrating $\vec{E} \times \vec{H}/c^2$ over all space) turns out to be

$$\vec{G} = (4/3)(W_0/c^2)\vec{v} \qquad (2)$$

Thus, when a spherical object contains the electrostatic energy W_0, classical electromagnetic theory requires that the inertial mass should have the value $(4/3)W_0/c^2$, not W_0/c^2 as one might expect from (1). However, any structure composed only of

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1. H. A. Lorentz, "The Theory of Electrons", 2nd Ed. Teubner 1908, (reprinted by Dover Publications, Inc., New York)(1952) p. 39.

2. See also, for example, W. K. H. Panofsky and M. Phillips, "Classical Electricity and Magnetism", Addison-Wesley Publishing Co. Inc., Reading, Massachusetts (1955) p. 317.

charges of one sign can not be held together by the classical electromagnetic forces alone, so that one customarily assumes that any physically realizable stable structure contains attractive forces (Poincare stresses). The negative energy associated with the fields of these attractive forces is assumed to have an excess negative mass of magnitude $(1/3)W_0/c^2$, so that the structure as a whole obeys (1). To our knowledge a careful test of the precise cancellation of this excess mass has never been made prior to the one reported in this paper.

One can readily see that ordinary macroscopic structures will not permit a satisfactory test, since even objects with the largest possible excess electrostatic charge still have a completely negligible fraction of their total mass in the form of the electrostatic field, and in addition, the resulting electromagnetic stresses can be calculated to produce just the correct compensation. The nucleus, however, is an ideal structure in which to test the relativistic prediction that the peculiar inertial behavior of the electrostatic energy is always compensated exactly by the energy of the attractive force field. Only nuclei have a significant fraction of their total mass in the electrostatic form. (For the heavier nuclei, this fraction is about 1 part in 300.) In addition, the nucleus is still large enough that one expects the laws of electricity to correctly predict the electrical component of the force between the

nucleons--a situation which is not necessarily true if one considers the interior of smaller particles, such as the nucleons themselves. The nuclear component of these forces obeys entirely different laws, and it would appear remarkable if it exactly cancelled the mass excess and the mass asymmetry due to the electromagnetic forces. Finally, some nuclei are known to have highly distorted electrical structures (they have large electric quadrupole moments) so that one is not restricted to observations on simple spherical structures.

2. The Calculation of the Inertial Mass of the Electrostatic Potential Energy

To illustrate in a graphic manner how sharply the inertial properties of the electrostatic energy departs from ordinary mass, we concentrate our attention on the potential energy of two point objects each of which has a rest mass m_0 and a charge q. We assume that they are maintained at a fixed separation, r, as would occur if the charges formed part of a highly rigid structure, such as a nucleus. (See Figure 1.) The electrostatic energy ΔW is $(1/4\pi\epsilon_0)(q^2/r)$. We now assume that an external force (such as would be produced by a uniform external electric field) causes this structure to accelerate uniformly with the acceleration $\vec{a}$ in the +y-direction. As shown in the figure, $\vec{a}$ makes the angle ψ with respect to r. We will calculate the dynamical behavior of this structure under the

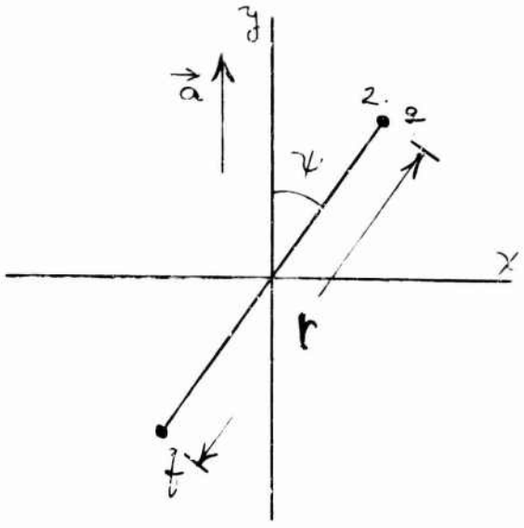

Figure 1. Two equal charges q with a fixed separation r are accelerated together in y-direction ($v \ll c$).

specified acceleration by using the solution to Maxwell's equations which applies throughout the region external to a source charge q whose state of motion is given. For the case where $v \ll c$, a source charge, q, produces an electric field, $\vec{E}$, at the field point, P, located a distance, $\vec{r}$, from the charge,

$$\vec{E} = \frac{q}{4\pi\epsilon_o}\left[\frac{\hat{r}}{r^2} + \frac{(3\vec{v}\cdot\hat{r})\hat{r}-\vec{v}}{r^2c} + \frac{\hat{r}\times(\hat{r}\times\vec{a})}{rc^2}\right] \quad (3)^3$$

The unit vector, $\hat{r} \equiv \vec{r}/r$, always points from the source charge toward the field point P. If the position, the velocity, $\vec{v}$, and the acceleration, $\vec{a}$, of the source charge are specified at $t = 0$, then the field, $\vec{E}$, as calculated by (3) will exist at the field point P at the later time, $t = r/c$.

First, we regard charge 1 in Figure 1 as the source charge, with its position and motion specified at $t = 0$. We then com-

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3. W. K. H. Panofsky and M. Phillips, "Classical Electricity and Magnetism", Addison-Wesley Publishing Co.Inc. Reading, Massachusetts (1955), p. 299.

pute the electric force on charge 2 at the later time, t = r/c. Second, we regard charge 2 as the source charge at t = 0, and compute the electric force on charge 1 at the later time, t = r/c. Third, at t = r/c, we take the vector sum $\vec{F}$ of these forces. As is characteristic of the dynamical forces between moving charges, $\vec{F}$ is not zero. Its component in the y-direction is

$$F_{\parallel} = -(q^2/4\pi\epsilon_o c^2 r)\,(1 + \cos^2\psi)a \qquad (4a)$$

and is directed opposite to the acceleration, $\vec{a}$. There is also a component of $\vec{F}$ normal to the acceleration,

$$F_{\perp} = -(q^2/4\pi\epsilon_o c^2 r)\,(\sin\psi\cos\psi)a \qquad (4b)$$

which is directed in the -x-direction for the charges in Fig. 1.[4] If the charges are rotated in position shown by the dashed line (reversing the sign of ψ), then $F_{\perp}$ reverses. Thus, when a structure is composed of charges which are symmetrically disposed about the y-axis, $F_{\perp}$ adds up to zero. The "self-force" $F_{\parallel}$, however, is always directed opposite to the acceleration. Since it opposes the external force, it causes the structure to accelerate more slowly than it otherwise would. Thus, the structure behaves as if it had excess inertial mass. The transverse "self-force" $F_{\perp}$ illustrates graphically the tensor character of electromagnetic inertia. We shall be concerned, however, only with structures which either are symmetrical about $\vec{a}$, or which are precessing in such a manner that $F_{\perp}$ averages to zero.

If we explain the reduced acceleration in the y-direction by assuming that the structure has an

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4. Equations (4) are most simply derived from (3) by assuming that v = 0 at T = 0, since for v<<c, $\vec{F}$ turns out to be independent of v. It is interesting to note that the velocity dependent term in (3), $\left[(\vec{3v}\cdot\hat{r})\hat{r} - \vec{v}\right]/r^2c$, performs the function of exactly cancelling the consequences of the retardation in the electrostatic field, $\hat{r}/r^2$ which arise from the velocity of the source charge. If this velocity-dependent electric field term were not present, the electrostatic inertia would depend on the first power of the velocity, and could under no conditions be called inertia. Thus, this velocity-dependent term is essential if the electrostatic energy is to have true inertial properties.

Since v << c, any magnetic forces between the two charges are of negligible consequences.

excess mass, we find that the structure acts as if it possessed the total inertial mass of amount $m = 2m_0 + \Delta m$, where

$$\Delta m = (\Delta W/c^2)(1 + \cos^2\psi) \qquad (5)$$

The quantity $\Delta W = q^2/4\pi\epsilon_o r$ is the electrostatic part of the energy associated with the "bond" between the two particles. (5) clearly illustrates the large intrinsic asymmetry of the electromagnetic mass. Thus, when $\psi = \pi/2$ (the acceleration is perpendicular to the "bond"), the electromagnetic inertial mass has the normal relativistic value, $\Delta W/c^2$, but when $\psi = 0$ (the accleration is parallel to the "bond"), the electromagnetic inertial mass is $2\Delta W/c^2$, which is <u>twice</u> the normal relativistic value. This large intrinsic asymmetry is the source of the factor (4/3) which appears when one calculates the total (electromagnetic) inertia of any spherically symmetrical structure, such as a shell, or a sphere of uniform charge density, since for this case, one must average (5) with uniform weighting over all elements of solid angle.

Equation (5) permits the calculation of the electromagnetic inertial mass of a distorted nucleus provided, of course, that the classical theory is applicable for structures of these dimensions. In this case, the "bonds" are not oriented uniformly over all directions. In a prolate spheroid, for example, the "bonds" are aligned preferentially along the axis of symmetry. If the structure is accelerated parallel to this

axis, the electrostatic inertia will be augmented (over the normal spherical value of $(4/3)\Delta W/c^2$) since the "bonds" parallel to the acceleration have higher inertia. Similarly, if the prolate spheroid is accelerated in a direction perpendicular to its axis of symmetry, the electrostatic inertia will fall below its normal or spherical value. These same results, of course, may be calculated from the electric and magnetic fields of moving ellipsoids of uniform charge density.

3. Two Proposed Experimental Tests

We have asked two specific questions: (1) Is there any experimental evidence that distorted nuclei have asymmetrical inertia? and (2) What is the experimental evidence that the "excess" electrostatic inertial mass $(1/3)(\Delta W/c^2)$ is not observable in nuclei, spherical, or otherwise? We believe as a matter of principle that these questions should be asked, in spite of the fact that there are good reasons to expect that the inertial mass of a particle is an exact scalar, and also that the electromagnetic part of the mass does not produce any anomolous inertial mass in the nucleus. No aspect of any theory, no matter how well established, should be left untested to the maximum practical accuracy. Entirely aside from these considerations, however, and assuming that nuclei do not show any anomolous inertial behavior, we will find that by directing our attention to the calculated asymmetry in the electrostatic inertia, we are induced to ask some significant questions

regarding the nature of inertia: (a) Are the electromagnetic calculations trustworthy?and (b) Given that the calculations are correct, by what detailed means could the nuclear field energy always exactly compensate for the asymmetries arising from the electrostatic field energy?

With respect to (a) we note that Rohrlich[5] has recently questioned the correctness of the classical definition of electromagnetic energy and momentum as applied to the classical point electron. He states that if one starts with a covarient definition of energy and momentum, and applies one of the known renormalization methods, that the factor (4/3) reduces to 1, and there is no need for a self-stress. We do not know if this method of analysis can remove the "excess" electromagnetic inertia which is calculated to exist in structures of nuclear size (as contrasted to "point" particles) but we can not at this time rule out the possibility that electrodynamics can be modified, or perhaps, better, re-interpreted, in such a way that there is no anomolous electromagnetic inertia properties to be explained away.

If, however, one can not reasonably doubt the existence of the unsymmetrical behavior of the electromagnetic mass in nuclear structures, and if the total inertia is to be without anomaly, then it is necessary to assume the existence of a compensating asymmetry--presumably in the energy of the

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5. F. Rohrlich, Bull. Amer. Phys. Soc. Ser.II, 5, 83 (1960)

nuclear field. Thus, a distorted distribution of the protons might imply some type of compensating distortion in the spatial orientation of the nucleon-nucleon "bonds". The detailed means by which this compensation could be achieved is complicated by the great difference in form of the electromagnetic and the nuclear forces. For example, the factor of 2 in the asymmetry of the electrostatic inertial mass appears to be intimately related to the fact that the basic electric force has a $1/r^2$ dependence and also that it is transmitted with the speed of light. Nuclear forces on the other hand have a very different range dependence, and indeed, cut off so sharply with range that only nearby nucleons influence each other directly. As a result, although the form of the entire distorted nucleus affects the asymmetry of the electrostatic inertial mass, only local arrangements of nearest-neighbor nucleons might be expected to produce unsymmetrical properties of the nuclear field energy. In addition, the nuclear fields--which are assumed to be mediated by π-mesons--are not transmitted with the speed of light. Finally, the (negative) nuclear field energy is generally several times larger than the (positive) electrostatic energy. In the light of all these differences, it would seem surprising (at least from a superficial point of view) that the two different fields could always exactly compensate each other's anomolous inertial properties.

4. The Search for Asymmetric Inertia in a Distorted Nucleus

A mass spectrometer is an ideal instrument to make a specific test for the possible asymmetry in the inertia of a distorted nucleus. One should choose a nucleus with a large atomic number, Z, (so that the electrostatic energy forms as large a fraction of the total energy as is practical), a large electric quadrupole moment (to insure large distortion), a magnetic moment (to insure quantization in the magnetic momentum-sorting field of the instrument), a spin, J, of at least 3/2 (so that there are at least two distinctive orientations of the axis of symmetry under quantization), and a $^1S_0$ electronic ground state for the ion (so that the nuclear quantization will be unaffected by the electronic structure).

To see the general nature of the expected effect, consider a prolate spheroid which possesses an invarient electric charge. Nuclei which are quantized with their axis symmetry most nearly parallel to the magnetic field will suffer acceleration which, on the average, is approximately normal to their axis of symmetry, whereas those nuclei with the lowest possible value of $|m_J|$ will be precessing nearly in the plane of the path, and will be as often accelerated parallel to their axis of symmetry as normal to it. These latter nuclei have augmented electrostatic inertial mass compared to those which are quantized nearly parallel to the field.

Thus, if one considers only the electromagnetic inertial mass, the mass line should split into $(2J + 1)/2$ lines, not necessarily equally-spaced.

There are not many nuclei which meet all these requirements, but Lu^{175} happens to be ideal in practically every respect, so that our attention has been directed toward this particular nucleus. The first step is the computation of the contribution, Δm, of the electrostatic energy to the total nuclear mass, m. To do this, we have constructed a model of the nucleus, composed of Z protons each of charge e. These protons are arranged into highly symmetrical inner shells, and with two end caps such that the model has the experimentally determined dimensions of the nucleus being considered and also the same electric quadrupole moment. The electrostatic energy of assembly of this structure may be computed from

$$\Delta W = \frac{e^2}{4\pi\epsilon_o} \sum_{i,j} \frac{1}{r_{ij}} \qquad i < j \qquad (6)$$

where r_{ij} is the separation of the i^{th} and the j^{th} protons. Although the "normal" electromagnetic mass is expected to be $\Delta W/c^2$, when one computes the inertial behavior of the model structure, one must allow for the orientation of each bond with respect to the direction of acceleration. Thus, using (5), the electromagnetic <u>inertial</u> mass is computed to be

$$\Delta m = \frac{e^2}{4\pi\epsilon_o c^2} \sum_{i,j} \frac{1 + \cos^2\psi_{ij}}{r_{ij}} \qquad i < j \qquad (7)$$

where ψ_{ij} is the angle between r_{ij} and the direction of the acceleration of the rigid structure. (See Figure 2). As we have already noted, for the case of a distorted nucleus, the

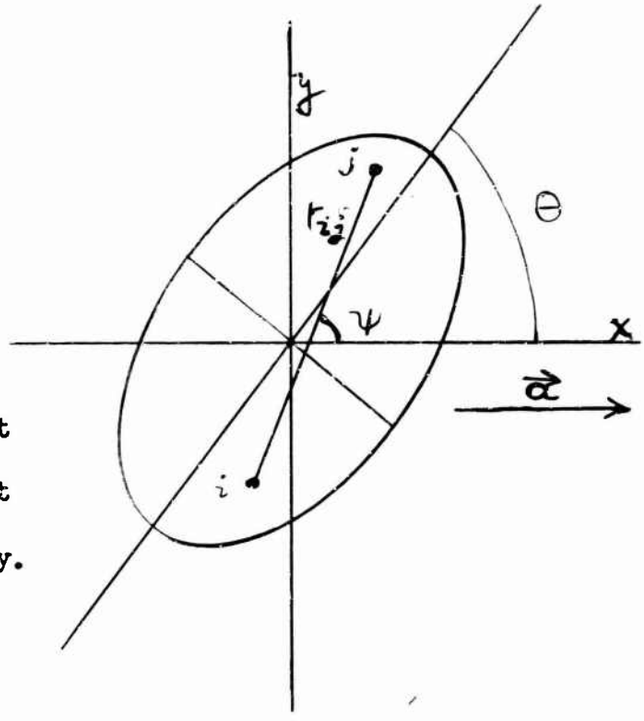

Figure 2. The calculation of the electrostatic inertia of a distorted nucleus which is accelerated at an angle θ with respect to its axis of symmetry.

computed value of Δm will depend upon θ, the angle between the axis of symmetry and the acceleration. For a prolate spheroid, such as the one whose outline is sketched in Figure 2, Δm will be largest when $\theta = 0$, and smallest when $\theta = \pi/2$. For a prolate nucleus such as Lu^{175} whose measured quadrupole moment is consistent with a ratio of major axis to minor axis of 1.3 (a relatively large distortion), computations based on (7) indicate that the two extreme values of Δm differ by about 1 part in 50, while the mean value of Δm (averaged over all values of θ) is very nearly $(4/3)\Delta W/c^2$. (The latter

result would hold exactly if the structure possessed spherical symmetry.)

Since, even for heavy nuclei, the electrostatic energy is only about 1 part in 300 of the total energy, the above asymmetry will only be about 1 part in 1.5×10^4 of the total mass, m, and the "excess" electrostatic inertial mass $(1/3)(\Delta W/c^2)$ will only be about 1 part in 10^3 of the total mass. Even though these effects are not very large, existing mass spectrometers are capable of observing them.

For the case of Lu^{175}, the calculations for the electromagnetic mass were performed as follows. We assumed a model composed (for reasons of symmetry) of 72 point charges, arranged into two spherically symmetrical inner shells of 6 and 48 charges respectively, and two end caps composed of 9 charges each. This structure has the mean radius given by $r = r_0 A^{1/3}$, where $r_0 = 1.1 \times 10^{-15}$m, and A = 175, and it has the same electric quadrupole moment as Lu^{157}, 5.9×10^{-24} cm^2. Using this model, we used (7) to compute the electromagnetic inertial mass Δm for each angle θ (see Figure 2). Since Lu^{175} has a spin of J = 7/2, the axis of symmetry has four distinct quantized angles with the magnetic field, (which we assume is parallel to the y-axis) and about which it precesses at a rapid rate. During the precession, the angle θ varies, so for each of the four states, $|m_J| = 7/5, 5/2, 3/2, 1/2$ it is necessary to perform a suitable average over

θ, obtaining thereby the mean value of the electromagnetic inertial mass, Δm, for the particular quantized state. These computations were performed on the University of Illinois digital computer. Each of the four different mass values were added to the non-electromagnetic mass (which was assumed to be the same for each of the four quantum states), giving four distinct values, $m_{7/2}$, $m_{5/2}$, $m_{3/2}$, and $m_{1/2}$, for the total inertial mass. The fractional separations are 5.56×10^{-5}, 3.72×10^{-5}, and 1.78×10^{-5}, respectively between adjacent lines. Thus, if the electromagnetic inertia asymmetry were not compensated, the Lu^{175} mass line would split into four lines, and even the smallest of the three splittings would be completely resolved on a spectrometer with a resolution of 1 part in 10^5.

At our request, W. A. Johnson and R. A. Damerow, who are associated with A. O. Nier at the University of Minnesota have made a careful examination of the Lu^{175} mass line using a spectrometer with a resolution of 1 part in 50,000. They conclude that if any splitting exists, it must be at least 100 times smaller than that which is predicted by the uncompensated electromagnetic mass effect.[6]

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6. W. H. Johnson Jr. and R. A. Damerow, Private Communication.

5. The Search for Evidence of Excessive Inertia Due to Electromagnetic Mass

To test for the presence of the "excess" $(1/3)(\Delta W/c^2)$ in the nuclear inertia due to the electrostatic energy $\Delta W/c^2$, we use the data of Scholman, Quisenberry and Nier which give the differences in mass as determined by nuclear reactions on the one hand and by the mass spectrometer on the other.[7] This set of data covers the range from B^{10} to S^{32}. We calculated the "normal" electromagnetic mass Δm from $\Delta W/c^2$, where $\Delta W = (3/5)(Z^2e^2/4\pi\epsilon_o r)$ and $r = (1.1 \times 10^{-15})A^{1/3}$ meters, where A is the atomic weight. This formula assumes that the nuclei are spheres of uniform charge density. For B^{10}, $\Delta m = 9.8 \times 10^{-3}$ M.U. (1 M.U. = 931 Mev.), and for S^{32}, $\Delta m = 67 \times 10^{-3}$ M.U. Thus, the S^{32} nucleus has 57×10^{-3} M.U. more "normal" mass of electrical origin than does B^{10}. If this added electromagnetic mass generated an excess inertial mass of amount $(1/3)(\Delta W/c^2)$, then, compared to B^{10} via a chain of nuclear reactions, the S^{32} nucleus should have an excess inertial mass (as determined by a mass spectrometer) of $(1/3)(57 \times 10^{-3}) = 19 \times 10^{-3}$ M.U. That is, if one arbitrarily sets the mass spectrometer mass value and the nuclear energy mass value to be equal at B^{10}, then the mass spectrometer mass measurement for S^{32} should be larger than the one calculated

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7. T. T. Scholman, K. S. Quisenberry and A. O. Nier, Phys. Rev. 102, 1076 (1956).

by the chain of nuclear reactions by the amount 19×10^{-3} M.U. Experimentally, however, the deviation between the two methods of measurement at S^{32} is less than 30×10^{-6} M.U. Thus, the "excess $(1/3)\Delta W/c^2$ " in the inertial mass which arises from the electrostatic energy must be compensated to an accuracy of 1 part in 600.

6. Conclusions

Thus, two different types of experiment demonstrate that, in structures of nuclear dimension, and to an accuracy of 1 part in 100 (or better), anomolous inertial properties, which are calculated by classical electrodynamics to belong to the electrostatic field energy, are unobservable. The experiments do not indicate whether the electrodynamics calculations are incorrect (when applied to small, highly rigid structures such as nuclei) so that there is actually no anomaly to be explained away, or whether the anomaly is real, and is being compensated for by the nuclear field energy. In the latter case, it is conceivable that small, but observable, asymmetries in the neutron distribution might be correlated with the existence of a highly distorted proton distribution.

There remains, however, one point which may be significant. The data compiled by Scholman, Quisenberry and Nier[8] seem to

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8. See also C. F. Giese and J. L. Benson, Phys.Rev. 110, 712,(1958).

show a systematic discrepancy between the mass values determined by nuclear reactions and those determined by the mass spectrometer. Furthermore, the deviation is such that it could be explained by an assumed failure of the nuclear fields to completely compensate for the intrinsic asymmetry in the electrostatic inertial mass. The compensation would have to fail by about 1 part in 600 if this interpretation is correct. In this event, it is possible that the mass line of a highly distorted nucleus such as Lu^{175} would show splitting if examined with a mass spectrometer whose resolution is about 1 part in 10^6 or better. Conversely, the failure of the mass line to split when examined with such resolution would be evidence that the deviations between the two types of mass measurement, whatever their cause, can not be attributed to a failure of the nuclear field energies to compensate for the asymmetry in the electromagnetic inertial mass.

7. Acknowledgements

The authors wish to thank Dr. Donald Bitzer of the Coordinated Science Laboratory for writing the program for our computations on the Illinois digital computer. We also wish to acknowledge some stimulating discussions with Dr. A. O. Nier.