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THE STRUCTURE OF THE FIELDS OF WIND-VELOCITY AND TEMPERATURE IN THE SURFACE LAYER OF THE ATMOSPHERE

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THE STRUCTURE OF THE FIELDS OF WIND-VELOCITY AND TEMPERATURE IN THE SURFACE LAYER OF THE ATMOSPHERE

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The study of turbulence in the surface layer of the atmosphere is of primary practical importance in estimating the wind resistance of structures, the diffusion of atmospheric pollution, the propagation of radio waves, the evaporation of moisture from the surface of the earth and reservoirs, the transformation of air masses, etc. At the same time the surface layer of the atmosphere provides an excellent laboratory for studying the general properties of turbulence, and particularly so since in the surface layer it is possible to study not only the dynamical factors that influence turbulence (and determined primarily by the Reynolds number) but also the influence of the stratification of the medium (characterized by the Richardson number). In addition the surface layer is readily accessible for direct observations.

This review considers the currently available theoretical and empirical knowledge concerning the wind velocity and temperature fields in the surface layer of the atmosphere. Attention is concentrated on those characteristics that can be measured directly.

1. CHARACTERIZATION OF THE SURFACE LAYER

The surface layer will denote that lower layer of the atmosphere -- having a thickness of a few tens of meters -- in which the effect of the Coriolis force can be neglected. We shall assume that the following two conditions are satisfied in this layer:

1) the terrain is flat and the extensive surface sufficiently uniform so that the fields of wind velocity and temperature are statistically uniform horizontally

2) no marked weather changes occur and during a time interval such that the normal diurnal change of weather conditions can be neglected, the fields of wind velocity and temperature are statistically stationary. Under these conditions the statistical properties of the meteorological fields are independent of the horizontal position of the point of measurement and of the time, and can depend only on the height z of measurement.

2. PRACTICAL LIMITATIONS OF MEASUREMENTS

Existing measuring devices make it possible to record directly the changes with time of the components of the wind and the temperature at fixed positions in space. The measurements are subject primarily to a time-averageing over an interval τ_0 that is determined by the inertia of the device (in the best of existing devices $\tau_0 \sim 10^{-2}$ sec), and to averaging over a cylindrical region of space having a length $\bar{u}\,\tau_0$ the direction of the wind ($\bar{u}$ =wind velocity) and a cross section determined by the dimension l_0 of the sensing element (in devices of the Institute of Atmospheric Physics $l_0 \sim$ 2cm.)

In the surface layer of the atmosphere, the microscale of turbulence $\lambda = \nu^{3/4}\,\epsilon^{-1/4}$ (where ϵ = rate of turbulent energy dissipation, ν =coefficient of viscosity) has a value of the order of 1 cm, the corresponding period of fluctuations at a fixed point in space $\tau_\lambda = \lambda / \bar{u}$ is of the order 10^{-2} sec and the angular frequency $\omega_\lambda = 2\pi / \tau_\lambda$ is of order 10^3 radian.Hertz. The parameters of existing measuring devices are still inadequate for measuring the structure of atmospheric turbulence in the region of the microscale.

3. THE STABILITY OF MEAN VALUES

Using data from measurements, the mean values of the fields of wind velocity and temperature may be determined for an averaging interval τ. If the period τ is small then it is found that the mean values are unstable -- they change appreciably under the influence of those components of turbulence having characteristic times that are not small compared with τ. But if $\tau \gg L_0 / \bar{u}$ where L_0 is the external horizontal scale of turbulence, then the mean values will be stable, since those components of the turbulence having scales that are large compared with L_0 will very rarely have large amplitudes.

Observation shows that to obtain stable mean values of wind velocity and temperature it is sufficient to base the averaging on a time interval of the order 10 min. To obtain stable mean values of characteristics of the small scale components of the turbulence the interval of averaging may be much less, although it must be borne in mind that such characteristics may also depend on some parameters of the large-scale motion (of which, in particular, ϵ is regarded). Thus, for example, for verification of the "two-thirds law"

$$\overline{\left[u_n(M) - u_n(M')\right]^2} = C^2 (\epsilon r)^{2/3}$$

(r =the distance between points M and M') it is sufficient to average using a time interval τ =30 secs, although the value of ϵ so determined will change in repeated measurements, as well as the mean velocity $\bar{u}$.

4. STRUCTURAL CHARACTERISTICS

Those characteristics of the structure of the fields of wind velocity and temperature that can be measured directly are conveniently divided into four groups:

1. Average profiles of wind velocity and temperature $\bar{u}(z)$ and $\bar{T}(z)$

2. Probability distributions of the fluctuations u', v', w' and T' at a fixed point of space, and primarily the second moments of these distributions

$$\overline{u'^2} = \sigma_u^2 ; \quad \overline{v'^2} = \sigma_v^2 ; \quad \overline{w'^2} = \sigma_w^2 ; \quad \overline{T'^2} = \sigma_T^2$$
$$\overline{u'w'} = -v_*^2 ; \quad \overline{w'T'} = q/c_p\rho \qquad -(1)$$

(v_* =friction velocity, q =average value of the vertical component of the turbulent heat flux, C_p =specific heat at constant pressure, ρ =density). In particular, of immediate interest are the correlation coefficients

$$r_{uw} = \frac{\overline{u'w'}}{\sigma_u \sigma_w} = -\left(\frac{\sigma_u}{v_*} \cdot \frac{\sigma_w}{v_*}\right)^{-1} ; \quad r_{wT} = \frac{\overline{w'T'}}{\sigma_w \sigma_T} = \left(\frac{\sigma_w}{v_*} \cdot \frac{\sigma_T}{\frac{1}{v_*}\frac{q}{c_p\rho}}\right)^{-1} \qquad -(2)$$

The remaining second moments ($\overline{u'v'}$, $\overline{u'T'}$, $\overline{v'w'}$, $\overline{v'T'}$) are zero. Also of known interest are the third moments

$$\overline{w'^3} ; \quad \overline{T'^3} ; \quad \overline{w'(u'^2 + v'^2 + w'^2)} \qquad -(\ )$$

The first two of these characterize the asymetry of the probability distributions of w' and T' while the third expresses the vertical diffusion of turbulent energy.

The quantities (1), (2) and (3) are statistically stable providing an averaging is used that is based on a sufficiently long interval of time (of order 10 min). In this same sense it is possible to determine the moments of connection between values u_i' and T', at various instants of time and at various points of space and, in particular, the time correlation functions

$$\begin{aligned} \overline{u_i'(x,t+\tau)\,u_j'(x,t)} &= b_{ij}(\tau,z) \\ \overline{u_i'(x,t+\tau)\,T'(x,t)} &= b_{iT}(\tau,z) \\ \overline{T'(x,t+\tau)\,T'(x,t)} &= b_{TT}(\tau,z) \end{aligned} \qquad -(4)$$

of which (1) are the values for $\tau = 0$.

3. Probability distributions for the local characteristics of the turbulent fields i.e., space and time differences of the wind velocity and temperature.

$$\begin{aligned} \delta_r u_i &= u_i(x+r,t) - u_i(x,t) \\ \delta_r T &= T(x+r,t) - T(x,t) \\ \delta_\tau u_i &= u_i(x,t+\tau) - u_i(x,t) \\ \delta_\tau T &= T(x,t+\tau) - T(x,t) \end{aligned} \qquad -(5)$$

and primarily the second moments of these distributions which are the space and time structural functions

$$\begin{aligned} \overline{(\delta_r u_i)(\delta_r u_j)} &= D_{ij}(r,z)\,; \quad \overline{(\delta_r T)^2} = D_{TT}(r,z) \\ \overline{(\delta_\tau u_i)(\delta_\tau u_j)} &= D_{ij}(r,z)\,; \quad \overline{(\delta_\tau T)^2} = d_{TT}(r,z) \end{aligned} \qquad -(6)$$

The spatial structural function $\overline{(\delta_r u_i)(\delta_r T)}$ of the wind velocity and temperature fields is equal to zero under the hypothesis of local isotropy and incompressibility. Likewise with the hypothesis of "frozen turbulence" (see below) the corresponding temporal structural function is also zero.

Also of known interest are the third moments of the distributions for space differences

$$\overline{(\delta_r u_\ell)^3} = D_{\ell\ell\ell}(r, z); \quad \overline{(\delta_r u_\ell)(\delta_r T)^2} = D_{\ell TT}(r, z) \qquad -(7)$$

which enter into the dynamical equations for the structural functions of locally isotropic turbulence, and the dimensionless quantities related to them

$$S = D_{\ell\ell\ell} D_{\ell\ell}^{-3/2}; \quad F = D_{\ell TT} D_{\ell\ell}^{-1/2} D_{TT}^{-1} \qquad -(8)$$

where the index ℓ denotes the direction parallel to the vector r, so that u_ℓ is the component of the wind along the direction of the vector r.

4. Probability distributions of the spectral components of the fields of wind velocity and temperature $u_i(\delta_\omega)$ and $T(\delta_\omega)$, corresponding to specified intervals of the frequency δ_ω in the time spectra (in particular, the semi-infinite $\delta_\omega = (\omega, \infty)$), and primarily the second moments of the spectral components corresponding to very small δ_ω, that is, time spectral functions.

$$S_{jk}(\omega, z) = \frac{1}{2\delta_\omega}\left[\overline{u_j(\delta_\omega) u_k(\delta_\omega)} + i\,\overline{u_j(\delta_\omega) u_k^+(\delta_\omega)}\right]$$

$$S_{jT}(\omega, z) = \frac{1}{2\delta_\omega}\left[\overline{u_j(\delta_\omega) T(\delta_\omega)} + i\,\overline{u_j(\delta_\omega) T^+(\delta_\omega)}\right] \qquad -(9)$$

$$S_{TT}(\omega, z) = \frac{1}{2\delta_\omega}\overline{T^2(\delta_\omega)}$$

where the plus sign indicates that the given spectral component is taken out of phase in the frequency ω by an amount $+\pi/2$.

The spectral functions (9) are connected with the time correlation functions (4) by relations of the form

$$b_{jk}(\tau,z) = 2\int_0^\infty \left[\cos\omega\tau \, Re\, s_{jk}(\omega,z) - \sin\omega\tau \, Im\, s_{jk}(\omega,z)\right] d\omega$$

$$s_{jk}(\omega,z) = \frac{1}{\pi}\int_0^\infty \left[\cos\omega\tau \, \frac{b_{jk}(\tau,z)+b_{kj}(\tau,z)}{2} + i\sin\omega\tau \, \frac{b_{jk}(\tau,z)-b_{kj}(\tau,z)}{i}\right] d\tau \qquad -(10)$$

The spectral functions may be determined either by the formulas (9) on the basis of direct measurements of the spectral components of the fluctuations of wind velocity and temperature, or by use of the second formulae (10) based on measured time correlation functions. In particular, from the first of formulae (10) we obtain

$$\overline{u'w'} = 2\int_0^\infty Re\; S_{uw}(\omega,z)\,d\omega$$

$$\overline{w'T'} = 2\int_0^\infty Re\; S_{wT}(\omega,z)\,d\omega$$

Thus the quantity $-2\,Re\;S_{uw}(\omega,z) = -\overline{u(\delta\omega)\,w(\delta\omega)}/\delta\omega$ is the spectral density of the shear stress v_*^2 , and $2\,Re\;S_{wT}(\omega,z) = \overline{w(\delta\omega)T(\delta\omega)}/\delta\omega$ is the spectral density of the turbulent heat flux $q/c_p\rho$.

5. FROZEN-IN TURBULENCE

As pointed out by G. I. Taylor, the variations in time of the hydrodynamic characteristics at a given point x can be approximately explained by regarding them as resulting from a pure translatory motion of a field of turbulent eddies across the point with the speed of the mean wind and along straight line $\mathcal{L}$ which passes through x parallel to the direction of the mean wind. This implies that for a sensing system that is moving with the velocity of the mean wind the distribution of hydrodynamic characteristics along the line $\mathcal{L}$ does not change with time, i.e., is "frozen". Actually, of course, the field of turbulence is not "frozen" and its configuration changes with the passage of time. However, if the energy of the turbulent motion is much less than the energy $\frac{1}{2}\rho\bar{u}^2$ of the mean motion, then the error from using the hypothesis of "frozen turbulence" to describe the structure of turbulence on sufficiently small scales will not be great.

The hypothesis of "frozen turbulence" makes it possible to identify the sinusoidal component of the distribution of any hydrodynamic characteristic along the straight line $\mathcal{L}$ that has a wave length ℓ (and wave number $k = 2\pi/\ell$), with the sinusoidal component of the temporal variations of this characteristic at the point x having the period $\tau = \ell/\bar{u}$ (and angular frequency $\omega = 2\pi\bar{u}/\ell = \bar{u}k$). From this it is apparent that the periods τ and frequencies ω of the fluctuations in time of the hydrodynamic property at a fixed point of space are not representative magnitudes of the turbulence, since they depend on the wind velocity $\bar{u}$. Hence when comparing the time correlation or structural (or spectral) functions obtained with various $\bar{u}$, the comparisons should not be made at fixed τ (or ω) but rather for a fixed $\bar{u}\tau$ (or $\omega/\bar{u}$).

When using the hypothesis of "frozen turbulence", the space correlation function $B(r, z)$ corresponding to a vector r parallel to the mean wind and the time correlation function $b(\tau, z)$ are connected by the relation

$$b(\tau, z) = B(\bar{u}\tau, z) \qquad -(11)$$

An analogous relation is true for structural functions. Further, suppose that

$S(k, z)$ is the one-dimensional spatial spectral density corresponding to the line $\mathcal{L}$ (that is the Fourier transform of the function $B(r, z)$) and that $s(\omega, z)$ is the temporal spectral function. Then when using the hypothesis of "frozen turbulence" we have

$$s(\omega, z) = \frac{1}{\bar{u}} S\left(\frac{\omega}{\bar{u}}, z\right) \qquad -(12)$$

6. SPECTRUM OF TURBULENCE

The spectrum of wave-numbers for the motion of a fluid (or in the case of the temporal spectrum, of values $\omega/\bar{u}$) can be somewhat arbitrarily divided into four regions:

1. the range of small wave-numbers, corresponding to the macrocomponent or the averaged fields of wind velocity and temperature.

2. the range of macrostructural elements of the turbulence that contain almost all of the energy of the turbulence.

3. the so-called inertial range in which the turbulent motion is determined by the dominating influence of inertia forces.

4. the dissipation range, that is, the range of large wave numbers in which occurs almost all of the dissipation of turbulent energy due to the action of molecular forces.

The 3rd and 4th regions together form an equilibrium range of the spectrum in which, according to the theory of locally isotropic turbulence, the forces of inertia and of viscosity are found in equilibrium.

7. SIMILARITY THEORIES FOR ATMOSPHERIC TURBULENCE

Generally speaking, the relative importance of the different characteristics of the turbulence depends upon the weather conditions. To a considerable degree the weather conditions are determined by the wind velocity and the vertical temperature gradient (which characterizes the degree of stability associated with the thermal stratification of the atmosphere). It is known that with a strong wind and an unstable stratification, the turbulence is much more marked than with a light wind and stable stratification. However, such a qualitative specification of the dependency of turbulence on weather conditions is, of course, quite inadequate and we need to establish a quantitative dependency on weather conditions for all the structural characteristics of the fields of wind velocity and temperature that are listed in Section 4.

The characteristics of turbulence depend on the weather conditions through several "external parameters". Generally speaking, the choice of these parameters turns out to be different for different regions of the spectrum of turbulence.

There are two theories of similarity for atmospheric turbulence that indicate how the "external parameters" are to be selected and how the characteristics of the turbulence depend on these parameters. One of these is A. N. Kolmogorov's theory of similarity for locally isotropic turbulence, which applies to the equilibrium range of the spectrum. The second is the theory of similarity for the turbulent regime in the surface layer of the atmosphere, as developed in the works of A. M. Obukhov and A. S. Monin; this theory is applicable to the entire turbulence spectrum except the dissipation range. Both of these theories of similarity are applicable to the inertial range.

The collection of experimental data relating to the structure of the fields of wind velocity and temperature should aim at determining those universal characteristics of the structure, the existence of which is indicated by the theories of similarity.

8. THEORY OF SIMILARITY FOR THE SURFACE LAYER OF THE ATMOSPHERE

This theory stems from the fact that, except in the dissipation range, the turbulent regime in all portions of the spectrum is completely determined by the three dimensional parameters

$$v_* = \left(-\overline{u'w'}\right)^{\frac{1}{2}} ; \quad q/c_p\rho = \overline{w'T'} \quad ; \quad g/T_0 \qquad -(13)$$

where v_* = friction velocity, q = vertical turbulent heat flux, c_p = specific heat at constant pressure, ρ = air density, g = gravitational acceleration, T_0 = standard mean temperature of the surface layer.

The dynamical equations (that is the equations of motion, heat transfer and continuity) for any individual realization of the turbulent regime in the surface layer of the atmosphere (where the Coriolis force can be neglected and the only changes of density considered are those related to changes of temperature but not pressure) contain only the parameter g/T_0, the coefficients of molecular viscosity and heat conduction (which drop out if only those motions are considered that have scales outside of the dissipation range), and maybe those parameters that characterize the radiative heat transfer (which we shall disregard for reasons given below). Under these circumstances the averaged dynamical equations (which are analogous to the Reynolds equations) take the form $\overline{u'w'}$ = constant, $\overline{w'T'}$ = constant. Thus the individual and averaged equations only contain the three parameters of (13).

Strictly speaking, even in the case under consideration of a horizontally homogeneous stationary surface layer of the atmosphere, the turbulent heat flux is not necessarily constant with height, but only the sum $q + q_1$, where q_1 is the radiative heat flux. If the radiative flux q_1 changes appreciably with height then the theory of similarity as stated should be modified. However, it may be shown that q_1 apparently changes appreciably only in the very thin layer near the surface of the earth (of thickness 1-2 meters) where as will be shown below, the influence of thermal stratification on the turbulent regime is still not detectable.

From the parameters of (13) it is possible to define in a unique manner (to within numerical multipliers) a scale of length L , velocity V , and temperature T_* :

$$L = -\frac{v_*^3}{\kappa \frac{g}{T_0} \frac{q}{c_p \rho}} ; \quad V = \frac{v_*}{\kappa} ; \quad T_* = -\frac{1}{\kappa v_*} \frac{q}{c_p \rho} \qquad -(14)$$

Here κ is the numerical constant of von Karman, and is introduced for the convenience of the subsequent calculations.

The dimensionless characteristics of the structure of the fields of wind velocity and temperature as determined by means of the scales of (14), will be universal functions of the dimensionless height $\zeta = z/L$. Here a negative value of ζ signifies unstable stratification ($q > 0$) while a positive ζ means stable stratification ($q < 0$).

9. SIMILARITY OF AVERAGED PROFILES

When using the similarity theory of Section 8 to describe the averaged profiles $\bar{u}(z)$ and $\bar{T}(z)$, it is customary to introduce an additional hypothesis concerning the proportionality of the exchange coefficients for heat and momentum, namely

$$k_T/k = -\frac{q}{c_p \rho \frac{d\bar{T}}{dz}} \Big/ \frac{v_*^2}{\frac{d\bar{u}}{dz}} = \kappa \frac{T_*}{v_*} \frac{d\bar{u}}{dz} \Big/ \frac{d\bar{T}}{dz} = \alpha = const \qquad -(15)$$

The hypothesis α = const is equivalent to the assumption of similarity for the profiles of wind velocity and temperature

$$\bar{T}(z_2) - \bar{T}(z_1) = \frac{k T_*}{\alpha v_*}\left[\bar{u}(z_2) - \bar{u}(z_1)\right]$$

This has been checked directly against the observational data of A. V. Perepel... (unpublished work) and is confirmed, at least when the departures from neutral stratification are not large. However, additional confirmation is desirable since Swinbank (1955) presents data showing some dependence of α on the Richardson number.

In accordance with the theory of similarity above (with α = const), the wind and temperature profiles in the surface layer have the form

$$\bar{u}(z) = \frac{v_*}{k}\left[f\left(\frac{z}{L}\right) - f\left(\frac{z_0}{L}\right)\right]$$

$$\bar{T}(z) - \bar{T}(z_0) = \frac{T_*}{\alpha}\left[f\left(\frac{z}{L}\right) - f\left(\frac{z_0}{L}\right)\right] \qquad (17)$$

where $f(\xi)$ is a universal function that is determined to within an additive constant, and z_0 is the so-called roughness length for the earth's surface. In these formulae the height z should be adjusted to allow for a "zero-plane displacement" (which is essential in the case of high vegetation). The function $f(\xi)$ is connected with the Richardson number Ri by the relation

$$Ri = \frac{g}{T_0}\frac{d\bar{T}}{dz}\left(\frac{d\bar{u}}{dz}\right)^{-2} = \frac{1}{\alpha f'(\xi)}$$

which shows, in particular, that Ri and ξ are equivalent local parameters of the thermal stability.

If the stratification is very close to neutral ($|q|$ is small), then the wind profile should be described by the known logarithmic law. With strong stability ($\xi \gg 1$) large scale turbulence cannot exist (since it would consume too much energy in working against the Archimedean forces) while for small scale turbulence and heights which are not too small the presence of the earth's surface is unimportant so that Ri $\to$ const, and hence from (18) the function $f(\xi)$ should be asymptotically linear. Values of $\xi \ll 1$ (strong instability) are obtained for a fixed z as $L \to -0$ or with $v_* \to 0$; hence the asymptotic behaviour for $\xi \ll 1$ may be obtained by considering the limiting self-generated regime of free convection ($v_* = 0$). In summary, it may be shown that the function $f(\xi)$ must possess the following asymptotic properties:

$$f(\xi) \simeq \log|\xi| + \beta\xi \quad \text{for } |\xi| < 1 \quad (\text{i.e. } z < |L|)$$

$$f(\xi) \sim C_1\xi + c_1 \quad \text{for } \xi \gg 1 \quad (\text{i.e. } z \gg L \text{ in a stable stratification}) \qquad (19)$$

$$f(\xi) \sim c_2 - C_3|\xi|^{-\frac{1}{3}} \quad \text{for } \xi \ll -1 \quad (\text{i.e. } z \gg |L| \text{ in an unstable stratification})$$

Here the arbitrary constant term in the function $f(\xi)$ has been chosen so that $f(\xi) - \log|\xi| \to c \; (\xi \to 0)$.

The universal function $f(\xi)$ was constructed empirically by A. M. Obukhov and A. S. Monin (1953), and later by A. V. Perepelkina and A. S. Gurvich (see p 81 of present collection) while in the investigations of A. S. Gurvich the wind and temperature profiles and the parameters v_* and q were measured independently. The observations establish the existence of the universal function $f(\xi)$

and confirm the asymptotic properties (19). Data from Priestly (1955) show that with unstable stratification the limiting regime of (19) is actually reached at small heights of a few meters.

The formulae (17) and (19) for the profiles of wind velocity and temperature, contain a number of numerical constants: $K, \alpha, \beta, C_1, C_2, c_1$ and c_2. All of these constants may be obtained by means of gradient measurements. Thus, $K \approx 0,4$, the value of α is apparently a little less than unity, and the values of C_1 and C_2 are close to unity. By approximating the wind profiles in an interval of positive and negative Ri values by the function $f(\xi) = \log|\xi| + \beta\xi$, A. M. Obukhov and A. S. Monin obtained $\beta = 0.6$. However, a number of authors using the formula $\beta = \left(\frac{d}{d\xi}\frac{Kz}{v_*}\frac{d\bar{u}}{dz}\right)_{\xi=0}$, have obtained a value for β that is two to three times greater. Possibly this difference is explained by the fact that the two calculations of β are essentially different. We also note that, in general, the values of $\beta = \left(\frac{d}{d\xi}\frac{Kz}{v_*}\frac{d\bar{u}}{dz}\right)_{\xi=0}$ will be different for conditions of stable ($\xi = +0$) and unstable ($\xi = -0$) stratification. The value of β for unstable stratification is not too significant as, according to Priestly, in this case the transition from the purely logarithmic law $f(\xi) \approx \log|\xi|$ to the limiting law of free convection $f(\xi) \sim c_2 - C_2|\xi|^{-\frac{1}{3}}$ occurs very rapidly (in a very thin layer).

10. DETERMINATION OF v_* AND q

In the theory of similarity for the surface layer of the atmosphere, the dependence of the turbulent regime on the weather conditions is expressed through the "external parameters" v_* and q. These parameters occur in the formulae (14) and (17) for the profiles of wind velocity and temperature. The reverse problem can also be solved, namely the determination of v_* and q based on measurements of the profiles of wind velocity and temperature. If the roughness length z_0 is known, then it is sufficient to measure the wind velocity at one height

$z = H$, and the difference of temperatures at two heights, for example, $z = 2H$ and $z = H/2$. Let

$$u = \bar{u}(H); \quad \delta T = \bar{T}(2H) - \bar{T}(H/2); \quad B = \frac{\alpha g H}{T_0} \frac{\delta T}{u^2}; \quad \xi_0 = \frac{z_0}{H}$$

Then, according to the theory of similarity

$$\frac{v_*}{u} = F_1(B, \xi_0); \quad \left| \frac{\frac{q}{c_p \rho}}{\alpha u \delta T} \right| = F_2(B, \xi_0); \quad \frac{L}{H} = F_3(B, \xi_0) \qquad -(20)$$

The functions F_1, F_2, F_3 , may either be constructed empirically from independent simultaneous measurements of the values u, δT, v_* and q , or theoretically with the help of interpolation formulae for the universal function $f(\xi)$ satisfying the conditions (19). The latter method has been used by A. B. Kasanski and A. S. Monin who have published nomograms for the functions (20) in both stable and unstable stratification. The following properties of the functions (20) may be noted. For near-neutral stratification (small $|\beta|$) we have

$$F_1 = \frac{v_*}{u} \approx \frac{K}{\log \frac{1}{\xi_0}}; \quad F_2 = \left| \frac{\frac{q}{c_p \rho}}{\alpha u \delta T} \right| \approx \frac{K^2}{\log 4 \log \frac{1}{\xi_0}}$$

$$F_3 = \frac{L}{H} \approx \frac{\log 4}{\left(\log \frac{1}{\xi_0}\right)^2} \cdot \frac{1}{B} \qquad -(21)$$

With increasing $B > 0$ (stable stratification) the values of F_1, F_2 and F_3 decrease, while for increasing absolute value of $B < 0$ (unstable stratification) the values of F_1 and F_2 increase, and $|F_3|$ decreases.

To determine the turbulent heat flux under conditions of unstable stratification and for $|B|$ not too small, it is possible to use the limiting relation for free convection, according to which

$$q = AH^{\frac{1}{2}} |\delta T|^{3/2} \qquad -(22)$$

where A is a constant. If H is expressed in meters, δT in degrees, and q in cal/cm^2·min, then A has a value of about 0.2.

The nomograms of the functions (20) are the most convenient means in practice for determining the parameters v_*, q and L (in cases when z_0 is known). However, since the theoretical basis of the nomograms depends on the use of these or other interpolation formulae, it is desirable to have some means for experimental verification of the nomograms through direct measurements of the quantities v_* and q.

One method of measuring these quantities is using the fluctuation method based on the formulae (13). In this we encounter two difficulties (a) the response parameters of the measuring devices must permit the recording, without any distortion of the fluctuations, of all those frequencies (both low and high) that contribute significantly to v_* and q. (b) the processing of the fluctuations measurements, that is, the computation of mean products $\overline{u'w'}$ and $\overline{w'T'}$ should be sufficiently rapid. In overcoming (a) it is first of all desirable to have information on the spectra of the turbulent shearing stress and heat flux (see end of Section 4). For (b) it is desirable to use linear data devices (such as acoustic sensors) and equipment for obtaining the mean products automatically.

Another method of direct measurement for the shearing stress is the dynamometric method first used by Sheppard, and later by Rider, and in an improved form by A. S. Gurvich. Finally the turbulent heat flux may be determined from the equation of thermal equilibrium of the earth's surface, providing that all the remaining components of the thermal balance are measured (we note, however, that this necessitates direct measurements of the rate of evaporation, which is a more difficult problem than measuring the turbulent heat flux).

11. SIMILARITY OF FLUCTUATIONS

On the basis of similarity theory (Section 8) it can be shown that the probability distribution for the dimensionless fluctuations u'/v_*, v'/v_*, w'/v_* and T'/T_* is a universal function of the stratification parameter $\xi = z/L$ (or of the Richardson number). The most important characteristics of this distribution are its second moments as given by (1). The dependence of these on the stratification parameter ξ may be described by the functions

$$\frac{\sigma_u}{v_*} = f_1(\xi); \quad \frac{\sigma_v}{v_*} = f_2(\xi); \quad \frac{\sigma_w}{v_*} = f_3(\xi); \quad \frac{\sigma_T}{T_*} = f_4(\xi) \qquad -(23)$$

Also of interest are the coefficients of anisotropy and the correlation coefficients

$$\frac{\sigma_v}{\sigma_u} = \frac{f_2}{f_1}; \quad \frac{\sigma_w}{\sigma_u} = \frac{f_3}{f_1}; \quad r_{uw} = -(f_1 f_3)^{-1}; \quad r_{wT} = \pm K(f_3 f_4)^{-1} \qquad -(24)$$

Some deductions can be made concerning the dependence of the dimensionless quantities (23), (24) on the stability parameter ξ, in cases of strong stability ($\xi \gg 1$) and strong instability ($\xi \ll -1$). Thus in strong stability the fluctuations acquire a local character, so that their properties cease to depend on the distance z from the earth's surface, and hence z should not appear in the formulae derived from considerations of similarity. With strong instability the quantity v_* should not occur in the formulae. From these considerations it follows that with strong stability ($\xi \gg 1$) all the quantities appearing in formulae (23), (24) should be independent of ζ and should tend to constant values; with an increase of instability ($\zeta \ll -1$) the functions f_1, f_2 and f_3 should increase asymptotically like $|\zeta|^{\frac{1}{3}}$, the function f_4 asymptotically tend to zero like $|\zeta|^{-\frac{1}{3}}$ and the functions (24) approach constant values.

Data from measurements by Swinbank and A. V. Perepelkina and comprehensive data from A. S. Gurvich and L. R. Zvang, support these predictions. They show that in a neutral stratification $\sigma_u/v_* \approx 2.3$, $\sigma_w/v_* \approx 0.7$, $\sigma_T/|T_*| \approx 1$ (strictly speaking in this case $T_* = 0$ and σ_T should be very small). With an increase in instability the values σ_u/v_* and σ_w/v_* increase, and $\sigma_T/|T_*|$ apparently decreases. With an increase of stability σ_u/v_* and σ_w/v_* apparently decrease, and $\sigma_T/|T_*|$ also decreases (and much more rapidly than for an increase of instability). The quantity σ_w/σ_u increases slowly in unstable stratification. The correlation coefficient r_{uw} in neutral stratification is somewhat less than -0.5, and with an increase of instability its absolute value is slightly decreased; on the other hand the absolute value of the correlation coefficient r_{wT} in near-neutral stratification is less than 0.5 and apparently approaches 0.5 as the stratification departs from neutral.

The third moments (3) should have the form

$$\frac{\overline{w'^3}}{v_*^3} = f_5(\zeta); \quad \frac{\overline{w'(u'^2+v'^2+w'^2)}}{v_*} = f_6(\zeta); \quad \frac{\overline{T'^3}}{|T_*|^3} = f_7(\zeta) \qquad (25)$$

In a stable stratification ($\zeta \gg 1$) these quantities should tend to constant values, and with increase of instability ($\zeta \ll -1$)the functions f_5 and f_6 should increase asymptotically like $|\zeta|$ and f_7 decrease asymptotically like $|\zeta|^{-1}$. Apparently, the functions f_5 and f_7 should be positive. The coefficients of asymmetry $\overline{w'^3}/\sigma_w^3$ and $\overline{T'^3}/\sigma_T^3$ should tend to a constant value, both for increase of stability and increase of instability. According to the date of Deacon and A. S. Gurvich, with an increase of instability the coefficient of asymmetry $\overline{w'^3}/\sigma_w^3$ increases.

The fact that the correlation coefficients r_{uw}, r_{wT} and the coefficient of asymmetry $\overline{w'^3}/\sigma_w^3$ all vary with ξ indicates that the probability distribution for the dimensionless fluctuations u'/v_*, v'/v_*, w'/v_* and T'/T_* depends on the stratification parameter ξ, but not solely because of the dependence on ξ of the standard deviations of (23). In other words, the probability distribution for the quantities u'/σ_u, v'/σ_v, w'/σ_w and T'/σ_T is not a universal function but varies with ξ. Apparently joint distributions for u'/σ_u and v'/σ_v are universal (Deacon showed that their third and fourth moments are the same as in the Gaussian distribution). However, joint distributions for w'/σ_w and T'/σ_T probably vary with ξ.

The dependency on the external parameters of the time correlation functions of formulae (4) has the form

$$\overline{u_i'(x,t+\tau)u_j'(x,t)} = v_*^2\,\beta_{ij}\left(\frac{\bar{u}\tau}{z},\xi\right)$$

$$\overline{u_i'(x,t+\tau)T'(x,t)} = v_* T_*\,\beta_{iT}\left(\frac{\bar{u}\tau}{z},\xi\right) \qquad -(26)$$

$$\overline{T'(x,t+\tau)T'(x,t)} = T_*^2\,\beta_{TT}\left(\frac{\bar{u}\tau}{z},\xi\right)$$

where β_{ij}, β_{iT} and β_{TT} are certain universal functions of $\theta = \bar{u}\tau/z$ and ξ. The appearance in formulae (26) of the velocity $\bar{u}$, which itself depends on the same external parameters, is not inconsistent with the theory of similarity and permits the use as representative magnitudes, of the lengths $\bar{u}\tau$ along a straight line $\mathcal{L}$ parallel to the direction of the wind (see formula (11)), instead of the intervals of time τ which are not themselves representative for the turbulence.

Analogous conclusions are obtained from the similarity theory of Section 8, for the space and time differences of wind velocity and temperature of (5). Thus the probability distribution for the dimensionless space differences $\delta_r u_i / v_*$ and $\delta_r T / |T_*|$ is a function of r/z and ξ, and the distribution for the dimensionless time differences $\delta_\tau u_i / v_*$ and $\delta_\tau T / |T_*|$ is a function of $\theta = \bar{u}\tau / z$ and ξ. In particular, for the structural functions of (6), we obtain

$$\overline{(\delta_r u_i)(\delta_r u_j)} = v_*^2 D_{ij}\left(\frac{r}{z}, \xi\right); \quad \overline{(\delta_r T)^2} = T_*^2 D_{TT}\left(\frac{r}{z}, \xi\right)$$

$$\overline{(\delta_\tau u_i)(\delta_\tau u_j)} = v_*^2 d_{ij}\left(\frac{\bar{u}\tau}{z}, \xi\right); \quad \overline{(\delta_\tau T)^2} = T_*^2 d_{TT}\left(\frac{\bar{u}\tau}{z}, \xi\right) \qquad -(27)$$

where D and d are certain universal functions of the two arguments (in contrast to the dimensional values of (6), although we have used the same symbols).

With strong instability ($\xi \ll -1$) the dependence on ξ of the functions β_{ij}, D_{ij} and d_{ij} becomes one of proportionality to $|\xi|^{2/3}$, the function β_{iT} tends to a limit which is independent of ξ, and the functions β_{TT}, D_{TT} and d_{TT} become proportional to $|\xi|^{-2/3}$. If $\xi \ll -1$, the dimensionless coefficients S and F of formulae (8) are independent of . With strong stability ($\xi \gg 1$) all the functions of the two arguments that appear in (26), (27), become dependent only on the product of the arguments, that is, on r/L (or $\bar{u}\tau/L$); this is also true for the coefficients S and F.

12. SIMILARITY OF TIME SPECTRA

Analogous to the formulae (26), (27), the theory of similarity of Section 8, leads to the following formulae for the time spectral functions of (9):

$$S_{jk}(\omega, z) = \frac{v_*^2 z}{\bar{u}} \sigma_{jk}\left(\frac{\omega z}{\bar{u}}, \xi\right)$$

$$S_{jT}(\omega, z) = \frac{v_* |T_*| z}{\bar{u}} \sigma_{jT}\left(\frac{\omega z}{\bar{u}}, \xi\right) \qquad -(28)$$

$$S_{TT}(\omega, z) = \frac{T_*^2 z}{\bar{u}} \sigma_{TT}\left(\frac{\omega z}{\bar{u}}, \xi\right)$$

where σ_{jk}, σ_{jT} and σ_{TT} are universal functions of the two variables. The wind velocity $\bar{u}$ is introduced into these formulae in accord with the equation (12) that results from the hypothesis of "frozen turbulence". By using this hypothesis we can interpret $\omega/\bar{u}$ as a wave-number, and the functions σ_{jk}, σ_{jT} and σ_{TT} as dimensionless, space spectral functions in one dimenion. At the same time the introduction of $\bar{u}$ into the formulae (28) does not mean that the hypothesis of "frozen turbulence" is accepted, and nothing restricts the generality of these formulae from the viewpoint of the theory of similarity of Section 8, since the quantity $\bar{u}/v_*$ is itself a universal function of ξ (and of $\xi_0 = z_0/L$).

From Section 4 and the formulae (28) it follows that the contributions from the spectral interval $\delta\omega$ (near frequency ω) to the values of v_*^2 and q are, respectively,

$$\frac{-2\,\delta\omega}{v_*^2} \operatorname{Re} S_{uw}(\omega, z) = \frac{\overline{u(\delta\omega)w(\delta\omega)}}{v_*^2} = \frac{\delta\omega, z}{\bar{u}} \Phi_1\left(\frac{\omega z}{\bar{u}}, \xi\right)$$

$$-(29)$$

$$-\frac{2\,\delta\omega}{K v_* T_*} \operatorname{Re} S_{wT}(\omega, z) = -\frac{\overline{w(\delta\omega)T(\delta\omega)}}{K v_* T_*} = -\frac{\delta\omega, z}{\bar{u}} \Phi_2\left(\frac{\omega z}{\bar{u}}, \xi\right)$$

where Φ_1 and Φ_2 are certain universal dimensionless functions. From (28) it is also evident that the abscissa (frequency) of each characteristic point of the spectral function (maximum, point of inflection, etc.) will depend on the external parameters through the law $\omega = \frac{\bar{u}}{z}\phi(\xi)$.

Using the arguments of the previous Section it can be shown that, in conditions of strong instability ($\xi \ll -1$), the dependence of the dimensionless functions $\sigma_{\zeta k}$, $\sigma_{\zeta T}$ and σ_{TT} on the stability parameter ξ should be as follows: $\sigma_{\zeta k}$ increases proportionally to $|\xi|^{2/3}$, $\sigma_{\zeta T}$ is independent of ξ, and σ_{TT} decreases as $|\xi|^{-2/3}$. For conditions of strong stability ($\xi \gg 1$) each of these functions assumes the form

$$\sigma(\tilde{\omega}, \xi) = \frac{1}{\xi}\,\sigma_1\left(\frac{\tilde{\omega}}{\xi}\right) \qquad -(30)$$

The abscissae $\omega = \frac{\bar{u}}{z}\phi(\xi)$ of the characteristic points of the spectra do not depend on ξ for conditions of strong instability, while for strong stability they take on the form ω = const. $\bar{u}/L$, that is, they increase with increase of stability (since L then decreases).

We will consider in more detail the spectrum of the temperature field under conditions of strong instability. Assuming $\sigma_{TT}(\tilde{\omega}, \xi) \sim |\xi|^{-2/3}\psi_{TT}(\tilde{\omega})$, we obtain

$$S_{TT}(\omega, z) = \frac{1}{\bar{u}}\left(\frac{q}{c_p\rho}\right)^{4/3}\left(\frac{g}{T_0}\right)^{-2/3} z^{1/3}\,\psi_{TT}\left(\frac{\omega z}{\bar{u}}\right) \qquad -(31)$$

In the event that the mean wind $\bar{u} = 0$, the characteristic quantity having the dimensions of velocity will be $\left(\frac{q}{c_p\rho}\cdot\frac{gz}{T_0}\right)^{\frac{1}{3}}$. Substituting this value in (31) in place of $\bar{u}$, we obtain

$$S_{TT}(\omega, z) = \frac{q}{c_p\rho}\left(\frac{g}{T_0}\right)^{-1} \psi_{TT}\left[\left(\frac{q}{c_p\rho}\cdot\frac{g}{T_0}\right)^{-\frac{1}{3}} z^{2/3}\omega\right] \qquad -(32)$$

From this it is apparent that the maximum of the spectrum corresponds to a frequency $\omega \sim z^{-2/3}$ that decreases with increasing height.

The measurements of S_{ww} and S_{TT} obtained by A. S. Gurvich and L. R. Zvang, show that the predictions concerning spectral similarity under various meteorological conditions and expressed by formulae (28), are in satisfactory agreement with the observational data. These measurements were made in the range of dimensionless frequencies $0.01 \leq \frac{\omega z}{2\pi \bar{u}} \leq 100$ (ω = angular frequency measured in rad·Hertz). Figure 19 of A. S. Gurvich's work (see p 129) shows the dependence of the dimensionless spectral function σ_{ww} on the stability parameter ξ. The graph shows that with increase of ξ at fixed $\tilde{\omega}$ the function $\sigma_{ww}(\tilde{\omega}, \xi)$ decreases, and the maximum of this function shifts to the side of high frequencies.

According to the data of A. S. Gurvich, with a neutral stratification the maximum σ_{ww} occurs with $\omega z/2\pi\bar{u} \approx 0.01$-$0.02$, and has a value of about 10 (the wave-length of the turbulent heterogeneities $\ell = \frac{2\pi\bar{u}}{\omega} \approx 50$-$100\ m$, corresponding to such a frequency of the maximum occurs at the height $z = 1m.$).

13. SIMILARITY THEORY FOR THE INERTIAL RANGE OF THE SPECTRUM

According to the theory of similarity for locally isotropic turbulence, as developed by A. N. Kolmogorov, the statistical characteristics of the velocity field associated with the equilibrium range of the spectrum of turbulence are uniquely determined by the two (dimensional) parameters ϵ and ν in a temperature-homogeneous medium. A. M. Obukhov pointed out that the statistical properties of the turbulence will depend not only on the velocity field but also on the temperature field, and in the equilibrium range of the spectrum may therefore additionally depend on the parameter $N = \chi \overline{(\nabla T)^2}$, that characterizes the rate of equalization of temperature heterogeneities under the influence of molecular thermal conductivity, and on the coefficient of thermal conductivity χ. A. M. Obukhov also pointed out that in the case of a temperature-heterogeneous medium it is necessary to add to the above parameters, a parameter g/T_0 which characterizes the influence of Archimedean forces on fluid elements having a density different from that of the surrounding medium.

Inclusion of the parameters ν and χ will be important only at the high frequency end of the equilibrium range, in the range of dissipation. By restricting attention to the inertial interval of the spectrum we may dispense with ν and χ. Thus the regime of turbulent motion having scales in the inertial range of the spectrum will depend on three external parameters ϵ, N, and g/T_0.

The parameter g/T_0 has a standard value, while the values of ϵ and N will vary, depending on the weather conditions. These values are characteristics of the large-scale turbulent motions, and their dependence on the weather conditions in the surface layer of the atmosphere may be determined by using the similarity theory of Section 8. We obtain

$$\epsilon = \frac{v_*^3}{k' z} \phi_\epsilon(\xi) \; ; \quad N = \frac{v_* T_*^2}{z} \phi_N(\xi) \qquad -(33)$$

where ϕ_ϵ and ϕ_N are definite universal functions. By using the equation of turbulent energy balance, and that for the balance of temperature heterogeneities $\overline{T'^2}$ in the surface layer of the atmosphere, these functions can be expressed with certain simplifications, in terms of the universal function $f(\xi)$ that describes the profiles of wind and temperature:

$$\phi_\epsilon(\xi) = \xi f'(\xi) - \xi \; ; \; \phi_N(\xi) = \alpha k \xi f'(\xi) \qquad -(34)$$

In the case of a temperature-homogeneous medium, the parameter g/T_0 can be disregarded; since it is impossible to construct any length scale from the parameters ϵ and N, the turbulent regime in the inertial range is then universal. For a temperature-heterogeneous medium the situation turns out to be more complex. As A. M. Obukhov pointed out, from the parameters ϵ, N and g/T_0 it is possible to construct the scale of length

$$L_* = \epsilon^{5/4} N^{-3/4} \left(\frac{g}{T_0}\right)^{-3/2} = k^{7/4} |L|^{3/2} z^{-\frac{1}{2}} \phi_\epsilon^{5/4}(\xi)\, \phi_N^{-3/4}(\xi) \qquad -(35)$$

which is the characteristic scale of the turbulent motion whose statistical properties depend essentially on the degree of thermal heterogeneity of the medium. For the inertial range the calculation of the scale L_* (and hence the parameter g/T_0 is essential unless this scale is very large.

If the stratification of the atmosphere is near to neutral, that is $|\xi| < 1$ and $f'(\xi) \approx 1/\xi$, then $L_*/z \approx k_\alpha^{-3/4} |\xi|^{-3/2}$ is large and the calculation of g/T_0 for the inertial range is not essential. For a strong instability $L_* \sim (3\alpha/C_2)^{3/4} kz$, and, generally speaking, it is necessary to calculate g/T_0 for the inertial range. For a strong stability $L_* \sim k(\alpha C_1)^{-3/4}(C_1 - 1)^{5/4} L$ and since $\xi \gg 1$, that is $L \ll z$, we have $L_* \ll z$, so that calculation of g/T_0 is necessary.

14. STRUCTURAL AND SPECTRAL FUNCTIONS IN THE INERTIAL RANGE

In the inertial range the spatial structural functions (6) depend only on the parameters ϵ, N and g/T_c, in addition to the argument r (that is they do not depend on z directly but only through the parameters ϵ and N).

By using dimensional considerations to construct expressions for these functions we obtain

$$\overline{(\delta_r u_j)^2} = (\epsilon r)^{2/3} \Phi_{jj}\left(\frac{r}{L_*}\right) = v_*^2 \left(\frac{r}{Kz}\right)^{2/3} \phi_\epsilon^{2/3}(\xi)\, \Phi_{jj}\left(\frac{r}{L_*}\right)$$

$$\overline{(\delta_r T)^2} = N \epsilon^{-\frac{1}{3}} r^{2/3} \Phi_{TT}\left(\frac{r}{L_*}\right) = kT_*^2 \left(\frac{r}{Kz}\right)^{2/3} \phi_N(\xi)\, \phi_\epsilon^{-1/3}(\xi)\, \Phi_{TT}\left(\frac{r}{L_*}\right) \qquad -(36)$$

where Φ_{jj} and Φ_{TT} are universal functions. For sufficiently small r/L_* these functions are close to their values at the origin, so that the structural functions of (36) become proportional to $r^{2/3}$ (the "two thirds law").

When using the hypothesis of "frozen-turbulence" the time structural functions are obtained from the spatial ones by replacing r by $\bar{u}\tau$. If we do not resort to this hypothesis then from considerations of the theory of locally isotropic turbulence, it follows only that in the inertial range, $\delta_\tau u/(\epsilon\tau)^{\frac{1}{2}}$ and $\delta_\tau T/(N\tau)^{\frac{1}{2}}$ are isotropic random functions of $u_1/(\epsilon\tau)^{\frac{1}{2}}$, that also depend on the parameter $(\epsilon\tau^3)^{\frac{1}{2}}/L_*$. Here u_1 is the value of the velocity of the fluid at the point relative to which the time differences of (5) are calculated. For the calculation of the time structural functions in the surface layer of the atmosphere it can be assumed approximately that $u_1 \approx \bar{u}$ Then

$$\overline{(\delta_\tau u_j)^2} = \bar{u}^2 \phi_{jj}\left(\frac{\epsilon\tau}{\bar{u}^2}, \frac{\bar{u}^3}{\epsilon L_*}\right)$$

$$\overline{(\delta_\tau T)^2} = \frac{N}{\epsilon} \bar{u}^2 \phi_{TT}\left(\frac{\epsilon\tau}{\bar{u}^2}, \frac{\bar{u}^3}{\epsilon L_*}\right) \qquad -(37)$$

By means of these formulae and using the relations (10), we also obtain the form of the time spectral functions:

$$S_{jj}(\omega) = \frac{\bar{u}^4}{\epsilon} \tilde{\sigma}_{jj}\left(\frac{\omega \bar{u}^2}{\epsilon}, \frac{\bar{u}^3}{\epsilon L_*}\right)$$

$$S_{TT}(\omega) = \frac{\bar{u}^4}{\epsilon} \frac{N}{\epsilon} \tilde{\sigma}_{TT}\left(\frac{\omega \bar{u}^2}{\epsilon}, \frac{\bar{u}^3}{\epsilon L_*}\right) \qquad -(38)$$

It is possible to write the last of these formulae in the form

$$S_{TT}(\omega) = N_\omega^{-2} \tilde{f}\left(\epsilon^{-\frac{1}{3}} L_*^{\frac{2}{3}} \omega, \frac{\bar{u}^3}{\epsilon L_*}\right) \qquad -(39)$$

If the mean wind vanishes and L_* is very large, we obtain $S_{TT} \sim N_\omega^{-2}$.

If we disregard the scale L_* and make use of the hypothesis of frozen turbulence, then the dependence of the spectral functions (38) on ω leads to a proportionality with $\omega^{-5/3}$. At the present time there is no data indicating that the spectral functions in the inertial range depart from this law. However, in Section 13 it was noted that real situations are possible in which it is necessary to allow for L_* , and formulae (36) - (39) will then be useful for the treatment of data relating to the structural functions and spectra.