TRANSLATION

LOW FREQUENCY INSTABILITY OF THE NOMINAL REGIME OF A LIQUID ROCKET ENGINE

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LOW FREQUENCY INSTABILITY OF THE NOMINAL REGIME OF A LIQUID ROCKET ENGINE

K. S. Kolesnikov

To analyze slow processes, the characteristic time of which is incommensurable for a longer period of time during the propagation of pressure wave over the chamber, the working model of the liquid rocket engine chamber (ZHRD) is characterized by the fact that all fuel particles after injection into the chamber within a certain time, necessary for dispersion, heating, evaporation, mixing, chemical reactions, transforms into final combustion products. This time, identical in the given model for all fuel particles, is called conversion time (lagging of combustion). In a simpler variant of the model the conversion time does not depend upon the pressure in the combustion chamber.

Several mechanisms of unstable combustion in the chamber were introduced. One of the first mechanisms was shown by Karman (1,2). It is based on the fact, that the rate of fuel injection into the chamber with respect to the pressure drop on the sprayers has a lagging of the relaxation time order of the fuel feeding line, and the combustion follows the rate of injection with a delay, equal to the conversion time. The oscillatory semi-period is approximately equal to the time of conversion and time of the relaxation system.

Another mechanism of origination of low frequency oscillations, which does not depend upon the fuel injection process into the chamber and is called therefore intra-chamber instability, was introduced in report (3). It is based on the fact, that the combustion lagging time in the oscillatory condition also appears to be an oscillatory value. The rate of combustion reaches maximum at maximum rate of reducing lagging time and minimum at maximum rate of increase in lagging time. If these oscillations coincide in phase with pressure oscillations in the chamber, then favorable conditions for self excitation are created.

In monography (4) is given a detailed analysis of simultaneous action of both excitation mechanisms.

Analysis of working stability of the engine installation with consideration of liquid of compressibility of the liquid in the pipeline for the simple mono-component system shows, that the critical lagging time depends upon the length of the pipe, whereby with the increase in frequency of wave oscillations the liquid in the tube decreases within a critical lagging time (5). The best conditions for self-excitation are created in the case, when the oscillations frequency in the pipe is close to one of the natural oscillation frequency in the pipe is close to one of the natural oscillation frequency of the gases in the chamber. Such oscillations are being called high frequency oscillations. For the case of purely longitudinal oscillations of gases the mechanism of high frequency instability at constant rate of fuel injection into the chamber, was discussed in monography (4).

Stability of liquid rocket engine work depends upon other properties of the body of the flying apparatus, on which the engine is mounted. Through an elastic body is formed a power full additional feedback between combustion chamber and fuel lines. This problem was investigated on the example of a monocomponent system with incompressible fuel in operation (6).

In practice can be found cases, when the natural frequencies of wave processes in fuel feeding lines have the very same order or are below the oscillation frequency, determinable by transformation time. They acquire a special importance, if the characteristic frequencies of elastic oscillations of the body of the flying machine or stand, on which the chamber is placed, are close to frequencies of wave processes in the lines. This report is devoted to the examination of such processes.

In the report is given a further broadening of the concept low frequency instability of ZHRD. The closed system can be unstable, even if the lagging combustion time in the chamber equals zero. The frequency, distinguishing this type of instability, is determined by the time of propagation of elastic wave in the line and in the body and can change in broad limits. The lagging time in the chamber changes the phase ratios and can serve as an additional cause of exciting oscillations.

The dynamic scheme consists of individual links, united on the basis of boundary conditions. Determination of link parameters represents, generally speaking, an independent and highly complex problem. The closed system is presented by a block diagram and appears to be poly-contour. The transmission functions of fuel lines were obtained in such a form, that their conjugation with the body would be full. Solution was made in linear installation, properties of transmission functions of individual links and of the entire system are analyzed with the aid of frequency method of the theory of automatic control.

1. We are utilizing the assumption that for low frequency oscillations (a) gas pressure p3 at each moment of time is practically identical over the entire chamber, (b) time of conversion τ_0 has identical values for all fuel particles, (c) stream of gases through the nozzle is quasi-stationary; in report (3) was obtained a linearized equation of the balance of gas mass in the chamber

$$\frac{d\beta}{dz} + (1-\nu)\beta + \nu\beta(z-\tau^*) = A[\mu_1(z-\tau^*-1) - \mu_2(z-\tau^*-1)] + \\ + (1/2 - H - 2A)\mu_1(z-\tau^*) + (1/2 + H + 2A)\mu_2(z-\tau^*) \qquad (1.1)$$

here

$$\beta = \frac{p_3 - p_3^\circ}{p_3^\circ}, \quad \mu_1 = \frac{m_1 - m_1^\circ}{m_1^\circ}, \quad \mu_2 = \frac{m_2 - m_2^\circ}{m_2^\circ}$$

$$H = \frac{1}{2}\frac{r-1}{r+1}, \quad r = \frac{m_2^\circ}{m_1^\circ}, \quad z = \frac{t}{\theta}, \quad \tau^* = \frac{\tau^*}{\theta} \qquad (1.2)$$

m_2, m_1 - rate of injection into the chamber of oxidizer and fuel mass; p_3°, m_2°, m_1° - pressure in the chamber and rates of injecting the mass of oxidizer and fuel in imperturbed condition; Θ - chamber relaxation time; ν - index of interaction between the combustion process and oscillations in the combustion chamber.

Coefficient A takes into consideration the effect of temperature oscillations. He characterizes the selected fuel components and suspends their consumptions in imperturbed state upon the ratios r and upon the pressure in the combustion chamber. For ordinary di-component engines the coefficient A - has a very low positive value.

To analyze the low frequency instability of the closed system it is interesting in first approximation to examine a more simple model of the combustion chamber, in which the conversion time τ_0 does not depend upon pressure (v = 0), and the temperature of the gas in the chamber is constantly independent upon the oscillation pressure and the ratios of fuel components (A = 0)

$$\frac{d\beta}{dz} + \beta = (1/2 - H)\mu_1(z - \tau^*) + (1/2 + H)\mu_2(z - \tau^*) \qquad (1.3)$$

This model is analogous to the model of a monocomponent engine, but, if the properties of fuel feeding lines of oxidizer and fuel will be different then it should be expected, that the dicomponent system will always be more complex and monocomponent.

Dimensionless variations μ_1, μ_2, should be determined from equations of fuel and oxidizer line dynamics.

Dynamic properties of the combustion chamber will be expressed through complex transmission numbers. Since the variation of fuel injection into the chamber is assumed to be taking place by the harmonic law, then with consideration of the lagging arguments z - τ^{-} and z - τ^* - 1 we have (7)

$$\mu_1(z-\tau^*) = \mu_1(z)\,e^{-is\tau^*} = \mu_1 e^{is(z-\tau^*)} \qquad (sz = \Omega t,\ s = \Omega\theta)$$
$$\mu_1(z-\tau^*-1) = \mu_1 e^{is(z-\tau^*-1)}$$

Here Ω - frequency of oscillations. Solution of equation (1.1) is sought in form

$$\beta(z) = \beta e^{isz}, \qquad \beta(z-\tau^*) = \beta e^{is(z-\tau^*)}$$

Then the complex transition numbers, determining the ratios of output coordinates to the input, will be found from expression

$$K[\beta, \mu_1] = \beta/\mu_1 = d_1/d, \qquad K[\beta, \mu_2] = d_2/d$$
$$d = \nu + (is + 1 - \nu)\,e^{is\tau^*}, \qquad d_1 = 1/2 - H - 2A + Ae^{-is} \tag{1.4}$$
$$d_2 = 1/2 + H + 2A + Ae^{-is}$$

Hodographs of vectors K (β, μ_1), K (β, μ_2), on the complex plane Z = U + iV in interval 0 $\leq$ a $\leq \infty$ represent coagulating spiral to the beginning of coordinates. The smaller τ^*, the more compressed becomes the spiral and a smaller phase lag φ originates for one and the same values s. These are typical aperiodic links of first order with lagging.

Vector hodograph graphs K (β, μ_2) for H = 0.214 are shown in Fig. 1 and 2. Curves 1 and 2 were formed at A = ν = 0, curve 1 corresponds to τ^* = 0.5, curve 2 - for τ^* = 1.5. In Fig. 2 is shown the effect of coefficient A. Graphs here are depicted for τ^* = 0.5

and $\gamma = 0$, whereby curve 3 - for $A = 0$, curve 4 - for $A = 0.05$, curve 5 - for $A = 0.1$. If, it is assumed, for example, $A = 0$, $\gamma = 0.2$ then the graph practially coincides with curve 3.

For small values s the coefficient γ almost exerts no effect neither in the modulus nor on the argument of the complex transmission number, and increase in coefficient A leads to a rise in modulus and practically changes not the argument.

2. Pressure variation in the combustion chamber causes an acceleration variation of the flying apparatus or, if ZHRD is mounted on a nonrigid stand - change in deformed state of the stand and connected with it fuel lines. In both instances the movement of the body and connected with it fuel lines leads to a change in pressure before the sprayers of the combustion chamber.

We will examine a perturbed movement of the body. Under imperturbed (nominal) condition of the system we will comprehend such a state, when the supply of fuel into the combustion chamber and the thrust of the engine appears to be constant, and the flying apparatus executes a rectilinear flight. Since the mass of the flying apparatus as result of fuel consumption at the time of tenths of periods of the investigated oscillations does practically not change, then for investigation of equations of motion we will "freeze" the equation coefficients, i.e., levels of filling up the fuel tanks, mass of flying apparatus, frequencies and forms of natural elastic oscillations of the body within a small section of time will be considered unchanged.

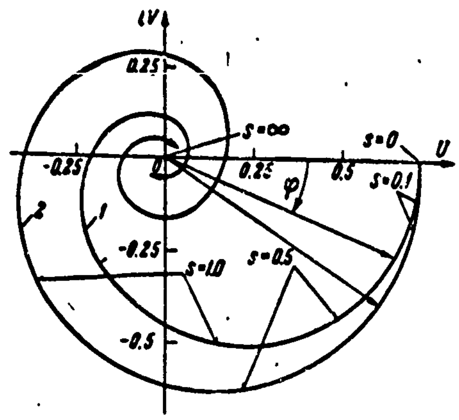

Fig. 1.

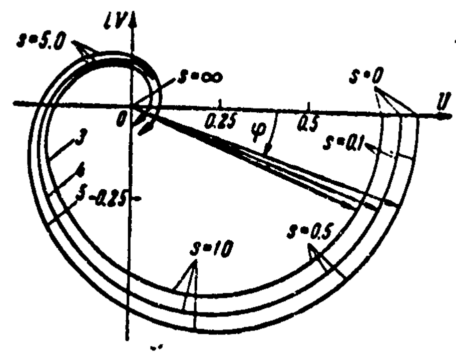

Fig. 2.

Displacement variation of any given transverse section of the body in direction of its longitudinal axis will be presented in form of a sum

$$q(x, t) = q_c(t) + \sum_{n=1}^{\infty} f_n(x) q_n(t) \qquad (2.1)$$

Here $q_c(t)$ - displacement variation of center of mass of the flying apparatus, $f_n(x)$ - form of n-normal tone of longitduinal oscillations of the body, $q_n(t)$ - displacement of relative center of mass at oscillations of n-tone of this section of the body, for which $f_n(x) = 1$.

The problem of determining natural frequencies Ω_n and natural functions $f_n(x)$ for an elastic body represents an individual complex problem and it is not evaluated here. We will consider them as unknown, whereby the beginning of the reading will be selected so, as to fulfill the ratio

$$\int_0^l m(x) f_n(x)\,dx + \sum_{k=1}^{N} m_k f_{nk} = 0 \qquad (k = 1, 2, \ldots, N) \qquad (2.2)$$

in which l, m (x) = length and mass of unit of body length m_k - concentration of mass, Such masses in first approximation can be considered the mass of the engine, fuel pumps, fuel in tanks, etc., whereby with the walls of the body, as a rule, they are elastically connected.

Oscillations in fuel lines practically cannot cause longitudinal body oscillations and that is why the latter takes place only under the effect of engine thrust variations.

We will designate by m the mass of the flying apparatus, Ω- frequency of natural elastic longitudinal oscillations of the body, k^{o}-proportionality coefficient between pressure variation in the combustionchamber and thrust force variation of the engine; we are assuming, that the direction of the thrust force vector coincides with the direction of longitduinal axis of the body. Then with consideration (2.1) and (2.2) the linearized per turbation of motion equation of the center of mass of the flying apparatus and elastic body oscillations with respect to the center of mass will be presented in form

$$\frac{d^2q_c(t)}{dt^2} = \frac{k^\circ p_3^\circ}{m}\beta, \qquad \frac{d^2q_n(t)}{dt^2} + \Omega_n^2 q_n(t) = \frac{k^\circ p_3^\circ f_{nr}}{m_n}\beta \tag{2.3}$$

Here m_n - given body mass, f_{nv}-value of natural function in that ...tion, in which the pressure in the combustion chamber is transformed into thrust fo... f the engine. For determinability this section will be combined with the spraying head of the chamber.

$$m_n = \int_0^l m(x) f_n^2(x)\, dx + \sum_{k=1}^{N} m_k f_{nk}^2$$

We will introduce dimensionless displacements and time

$$\eta_c = \frac{q_c}{l}, \quad \eta_n = \frac{q_n}{l}, \quad \sigma = t\frac{a_0}{l}$$

where a_0 - given rate of sound in elastic body.

Taking into attention, that $\beta(z) = \beta \exp isz$, $sz = \Omega = \omega\sigma$ solution of equation (2.3) will be presented in form

$$\eta_c(\sigma) = \eta_c e^{i\omega\sigma}, \qquad \eta_n(\sigma) = \eta_n e^{i\omega\sigma}$$

Ratios of coordinates η_c, η_n to variation β are expressed by transition numbers

$$K[\eta_c, \beta] = -\frac{k^*}{\omega^2}, \qquad K[\eta_n, \beta] = \frac{k_n}{\omega_n^2 - \omega^2} \qquad (2.4)$$

The dimensionless natural frequency ω_n and amplification coefficients k^*, k_n are determined by formulas

$$\omega_n = \Omega \frac{l}{a_0}, \qquad k^* = \frac{k^\circ p_3{}^\circ l}{m a_0^2}, \qquad k_n = k^* \frac{m}{m_n} f_{n0} \qquad (2.5)$$

When the frequency is changed within limits $0 \leq \omega \leq +\infty$ the vector hodographs $K(\eta_c, \beta)$ $K(\eta_n, \beta)$ on the complex surface $Z = U + iV$ will be direct (straight) lines, coinciding with actual axis.

As result of energy diffusion in the material and body connections natural oscillations of the latter always dampen, and therefore an infinitely great value of vector $K(\eta_n, \beta)$ at $\omega = \omega_n$ have no physical sense. When it is necessary to determine oscillations at resonance, in practical calculations into elastic oscillations equations are introduced resistance forces, proportional to the first degree of velocity.

The transition number $K(\eta_n, \beta)$ in this case will be complex

$$K[\eta_n, \beta] = k_n / D_n, \qquad D_n = \omega_n^2 - \omega^2 + 2\varepsilon_n i\omega \qquad (2.6)$$

where ε_n - damping coefficient of natural oscillations.

Stability analysis of the system is simplified, if forced oscillations of the body are not broken down into series by the natural functions $f_n(x)$, but are presented in the form of

$$q(x, t) = \beta(t)\, l f\left(\frac{x}{l}, \omega\right) = \beta l f(\xi, \omega)\, e^{i\omega t} \qquad (2.7)$$

where $f(\xi, \omega)$ – form of forced oscillations of the body. For the calculated scheme of a heterogeneous rod with gradual change in mass and rigidity along the length even in the presence of elasticity suspended concentrated masses of functions $f(\xi, \omega)$ without consideration of dissipation of energy, the energies have a substantial value and appear to be relatively simple. During their determination no principal difficulties do originate, and therefore for such calculated schemes forced body oscillations can instead of (2.1) be presented in form of (2.7). With consideration of energy dissipation the functions $f(\xi \omega)$ will be complex.

3. For analysis we will adopt main lines, in which the feeding of components from the tank into the combustion chamber is done by pumps. Such systems appear to be mostly spread for engines of greater thrust, and as a special case these can be main lines, over which the feeding of components is realized with compressed gas. Liquid components in the feeding line are not absolutely rigid, and possess a certain elasticity, which may exert a greater effect in the oscillatory processes in the main line.

Fuel line schemes for oxidizer and fuel have an identical structure. Each scheme consists of three in series connected physical links: pipe for feeding the component from the tank to the pump, pump and pipe between pump and sprayers (8). The first pipe in many instances is homogeneous and straight. The second pipe usually has a complex construction. It includes a valve capacity of engine head, and for the cooling component – an additional system of branched out pipes and interjacket slotted tract. There are no properly geometrical forms here, allowing to make an accurate solution of the problem about combined oscillations of elastic capacity with liquid. But, as result of elasticity of fuel lines their volume changes during pressure oscillations, and therefore the variation of fuel injection into the combustion chamber does not coincide with consumption variation through the pump. Quantitative differences depend upon the volume of the pipeline and upon the ratio of oscillation frequencies.

To solve the hydrodynamic problem the fuel line, connecting pump with chamber, will be replaced by a model in form of direct constant section pipe. The sprayer head and internal walls of the combustion chamber are accepted as rigidi in first approximation. Then the pressure variation in the combustion chamber causes no changes in volume of the pipeline. This volume will change only on account of elasticity of outer walls. Actually the internal walls of the chamber and the sprayer head possess a certain pliability and if the latter is measureable with the pliancy of outer walls or with a pliancy of liquid, then it should be considered.

The calculated scheme for fuel main lines of oxidizer and fuel, consists in such a way, of two direct uniform pipes, between which is built in a fuel pump. At the pipe ends can be placed compensators (bellows), having a negligibly small rigidity in axial direction (Fig. 3). In the calculated scheme the sylphon is used as a volume, pressure variation in any given point of the volume of which is identical, and the variation of the volume is proportional to pressure variation and to the change in distance between the end sections.

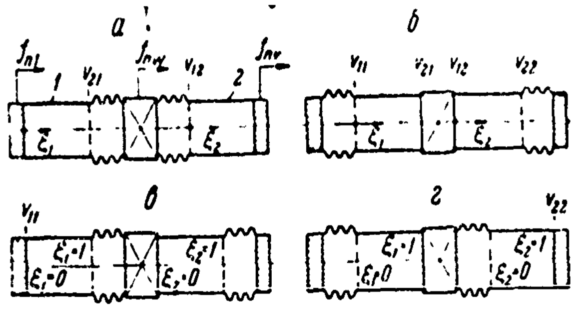

Fig. 3.

Structurally the fuel lines are connected with the body: the forward end of the first pipe is connected with the body of the tank, the rear end of the second pipe with the sprayer head of the combustion chamber, the pump is suspended from the body on a frame or is fastened in the combustion chamber(8). The fuel lines execute forced oscillations the law of which is determined by the movement of the corresponding body sections. The presence of sylphons allows the pipes and the pump to execute oscillations with various amplitudes.

We will designate the value of natural functions: f_{nl} - for tanks bottom flange, f_{nw} - pump, f_{nv}-sprayer head. Since on the bottom of the tank and engine frame possess a certain elasticity, the values of functions f_{nl}, f_{nw}, f_{nv}, do not coincide with values of functions for corresponding body bulkheads. This difference is the greater, the closer the natural frequency of oscillations of the body and partial oscillation frequencies of tank bottom with the fuel, engine, pump are. Functional values f_{nl}, f_{nw}, f_{nv} can be changed in certain limits by structural measures, and consequently they can be referred to the number of variating parameters of the system.

The hydrodynamic problem for fuel lines lies in the fact, to determine variation of fuel injection into the chamber in dependence upon small longitudinal body oscillations and pressure variations in the combustion chamber.

We will assume, that the fuel components represent an ideal liquid, the imperturbed stream in the pipes in homogeneous - rate v_0, pressure p_0 and density ρ_0 is constant; elasticity of pipes will be considered through the given rate of sound.

Since the equivalent pipes are assumed to be rigid, and the friction between their walls and liquid is not taken into account, then the small pipe movements in axial direction does not lead to a change in rate of flow.

The effect of sprayer head movements on the liquid flow, pump, tank bottom, and also the change of sylphon volumes will be pertained to the perturbed boundary conditions of the flow. With consideration of the above adopted assumptions about the quasistationary nonperturbed movement the hydrodynamic problem can be solved so as if the pipes would be stationary.

To the obtained calculation scheme we will adopt a solving method, explained in report (13). We will introduce analogous designations. We will designate the dimensional variation values of pressure and speed in the j-pipe by p_{xj}, coordinate of transverse section of stream in j-tube - x_j, time - t, we will adopt the following ratios between dimensional and dimensionless values

$$v_j=\frac{v_{xj}}{a_{0j}},\quad p_j=\frac{p_{xj}}{\rho_{0j}a_{0j}^2},\quad s_j=\Omega\frac{l_j}{a_{0j}},\quad \tau_j=t\frac{a_{0j}}{l_j},\quad \xi_j=\frac{x_j}{l_j}$$

where a_{0j} - speed of sound in imperturbed stream, l_j - length of j-pipe. For pipe of pump j = 1, after the pump j = 2.

We shall discuss the boundary conditions. If the pressure of gases over the liquid mirror in the tank at body oscillations remains unchanged, then pressure perturbation at the output from the tank into the pipe originates only as result of tank bottom oscillations. Neglecting wave formation on the free surface in the tank, we can write

$$p_x = \rho_{01} h_1 \frac{d^2}{dt^2}\left[q_c(t) + \sum_{n=1}^{\infty} x_n f_{n1} q_n(t)\right]$$

Here ρ_{01}, h_1 - density and height of liquid column in the tank; x_n-a certain coefficient depending upon the ratio of diameters of tank and pipe, forms of the bottom and output conditions of the liquid from the tank into the pipe. We will refer this variation to $\rho_{01}\ a_{01}^2$ - to parameters of imperturbed stream in the pipe in front of the pump. We will obtain

$$p^* = -\omega^2 (N_c \eta_c + \sum_{n=1}^{\infty} V_n \eta_n), \qquad p^* = \frac{p_x}{\rho_{01} a_{01}^2}, \qquad N_c = \frac{h_1 a_0^2}{l\ a_{01}^2}, \qquad N_n = N_c x_n f_{n1} \ (3.1)$$

Velocity variations at the ends of the pipes are determined from conditions of flow inseparability in the main lines. They depend upon the properties of the sylphon and places of their arrangement.

Sylphon properties will be expressed by two independent dimensionless characteristics. The geometrical characteristic λ_j will consider the change in the geometry of corrugations and the area of the lateral section of the sylphon

$$\lambda_j = \frac{1}{F_{0j}} \left(\frac{\partial V_{xj}}{\partial x}\right)_{x=x_{0j}} - 1$$

Here F_{0j}-area of passing section of j-pipe, V_{xj} sylphon volume, x_{0j}-distance between sylphon fronts in conditions of imperturbed regime.

The elastic characteristic r_j will take into consideration the change in compensator volume on account of pressure

$$r_j = r_{xj}\frac{a_{0j}^2 p_{0j}}{F_{0j} l_j} \qquad \left(r_{xj} = \left[\frac{\partial V_{xj}}{\partial p_x}\right]_{p_x = p_{0j}}\right)$$

The geometry of sylphon corrugations changes proportionally to the value difference $f_{nw}-f_{nl}$ and $f_{nv}-f_{nw}$. For characteristics of the sylphon, pressures, velocities at the end of the pipes we will introduce additional indices: 1-for entry into the pipe; 2-for exit from the pipe.

As a scale of dimensionless variation ω of stream velocities through pump, we will accept α_{02}, dimensionless variations of generalized speeds of the body, we will designate by

$$u_c = -\frac{1}{a_{02}}\frac{dq_c(t)}{dt} = -i\omega\frac{a_0}{a_{02}}\eta_c, \qquad u_n = -\frac{1}{a_{02}}\frac{dq_n(t)}{dt} = -i\omega\frac{a_0}{a_{02}}\eta_n \quad (3.2)$$

Since as the positive body displacements q (x,t) was adopted a displacement in direction, opposite the positive direction of velocity v_j of the liquid stream in the pipes, then for conjugation of coordinates q (x,t) and v_j in formulas (3.2) are introduced "minus" signs.

Having adopted $p_j(\xi_j, \tau_j) = p_j(\xi_j)$ exp is j τ j and having deisnga designated through v exp is $_2$ τ $_2$ r ferred to α_{02} the velocity variation of liquid injection into the chamber $h = \alpha_{02}/\alpha_{01}$ we will compile discontinuity equations. In order to accept in a greater degree the solution results (9), we present them in form

$$\begin{aligned} v_{21} &= wh + huf_{w2} + is_1 r_{21} p_{21} \\ v_{21} &= w + uf_{w1} - is_2 r_{12} p_{12} \\ v_{22} &= v + uf_v + is_2 r_{22} p_{22} \end{aligned} \qquad (3.3)$$

where uf_{ω_2}, uf_{ω_1}, uf_V - speeds, called motional characteristics of fuel line sections together with the body.

In the insuction part of the pump as result of cavitational phenomena may be situated certain gas volumes (10). If, it is assumed, that the variation values of these volumes are inversely proportional to pressure variations in front of the pump, then the nonseparation equations of the stream in the main lines will have the very same form, if under r_{21} is understood a generalized elastic characteristic of the sylphon together with gas volumes.

Possible schemes of sylphon arrangement in the fuel line are presented in Fig. 3. For Fig. 3a

$$\begin{aligned} uf_{w2} &= u_c + \sum_{n-1}^{\infty} [f_{nw}(1+\lambda_{21}) - \lambda_{21} f_{n1}]\, u_n \\ uf_{w1} &= u_c + \sum_{n=1}^{\infty} [f_{nw}(1+\lambda_{12}) - \lambda_{12} f_{nv}]\, u_n \end{aligned} \tag{3.4}$$

For Fig. 3, b, v

$$uf_v = u_c + \sum_{n=1}^{\infty} [f_{nv}(1+\lambda_{22}) - \lambda_{22} f_{nw}]\, u_n \tag{3.5}$$

Expressions uf ω_2 for Fig. 3, b, g, uf ω_1-for Fig. 3, b, v, and uf_v-for Fig. 3, α, g, can be obtained from formulas (3.4), (3.5), if the coefficients λ_{21}, λ_{12}, λ_{22} respectively equal zero are placed in them. In equalities (3.3) is necessary in this case correspondingly to adopt coefficients r_{21}, r_{12}, r_{22}, equal to zero.

In case the sylphon is arranged between pipe and tank (Fig. 3, b, g) stream velocity variations at output from tank will differ from variation v_{11} of the input speed into the tank, and consequently the variation of pressure drop p^*-p_{11} will be expressed by the dependence

$$p^* - p_{11} = \psi_1 M_1 \left\{ v_{11} - \sum_{n=1}^{\infty} [f_{n1}(1 + \lambda_{11}) - f_{nw}\lambda_{11}] + is_1 r_{11} p_{11} \right\} \qquad (3.6)$$

where ψ_1 - referred to velocity resistance coefficient at output of the liquid from the tank.

For Fig. 3, a, and 3, v

$$p^* - p_{11} = \psi_1 M_1 v_{11} \quad \left(M_1 = \frac{v_{01}}{a_{01}} \right)$$

Formulas (3.4) and (3.5) can also be adopted in case, when the movement of the body is presented in form (2.7). It should be written in the $u_C=0$ and instead of infinite sums of natural functions, it is necessary to take simply the difference of forms of perturbed oscillations

$$\begin{aligned} uf_{w2} &= -i\omega \frac{a_0}{a_{01}} \beta [f(\xi_w, \omega)(1 + \lambda_{21}) - f(\xi_1, \omega)\lambda_{21}] \\ uf_{w1} &= -i\omega \frac{a_0}{a_{02}} \beta [f(\xi_w, \omega)(1 + \lambda_{12}) - f(\xi_v, \omega)\lambda_{12}] \qquad (3.7) \\ uf_v &= -i\omega \frac{a_0}{a_{02}} \beta [f(\xi_v, \omega)(1 + \lambda_{22}) - f(\xi_w, \omega)\lambda_{22}] \end{aligned}$$

where $f(\xi_1, \omega)$, $f(\xi_w, \omega)$-forms of forced oscillations of tank bottom flange, pump, sprayer head.

The scheme of the fuel line and the boundary conditions of the problem by means of introducing summation links $uf_{\omega 2}$, $uf_{\omega 1}$, uf_v are brought to the structure, which like the one investigated in report (9). Here are used results of this work without their reproduction. Formulas for complex transition numbers : $K[p_{21}, p^*]$, $K[p_{21}, w]$, $K[v, p_{12}]$, $K[v, p]$, $K[w, p]$, $K[w, p_{12}]$ retain their previous form, formulas for $K[p_{21}, uf_{w2}]$, $K[v, uf_v]$, $K[w, uf_{w1}]$, $K[w, uf_v]$ can be obtained from expressions $K[p_{21}, u]$, $K[v, u]$, $K[w, u]$, presented in (9), by placing in them $f_{\omega 1} = f_{\omega 2} = f_v=1$. Instead of 1 K ($\omega$, u) block here in the scheme will be two parallel blocks with transition numbers K (ω, uf_v)

and K (ω , uf$_{\omega 1}$) = - 1. The latter differences appear to be the result of the fact, that the change in sylphon geometry of the second pipe takes place in general case simultaneously from two displacements - pump and sprayer head.

Transmission number for the first pipe will be expressed by other formulas, if the sylphon is situated between pipe and tank (Fig. 3, b, g). They can be obtained with consideration (3.6) by the method, explained in (9). The effect of sylphon disposition place on the dynamic properties of the line becomes noticeable, if the coefficients r_{11} and r_{21} of sylphon adaptability are considerable or the geometric characteristics $\lambda_{11} \gg 0$ $\lambda_{21} \gg 0$.

4. The block diagram, composed of physical links is expressed in Fig. 4. Here 1-chamber; 2-body of flying apparatus (stand); 3-fuel main line; 4-oxidizer line. Perturbation of body movement causes pressure variations in the fuel feeding lines and, consequently, fuel injection variation into the chamber. In the combustion chamber originates a pressure variation, which affects the fuel line and the movement of the body. The system, in this way, appears to be closed and has in addition, supplementary feedbacks.

In Fig. 5 is shown the developed block diagram, in which only one fuel line is connected. The second fuel line should be connected according to Fig. 4. The movement of the body was adopted in form (2.1). Here is depicted only one link for the n-tome of body oscillations. The fact is n = 1,2,3..., whereby the links are arranged parallel, what is shown by the dotted line.

The system has three internal feedbacks; between the chamber and second pipe- by the pressure p = L_2, ρ between second and first pipes - by speed , third feedback envelopes the chamber, it appeared as result of the fact, that the transmission function of the chamber is formulated by the rate of injection of fuel mass, and the output coordinate for the pipe serves the rate of fuel injection.

Since the dimensionless parameters of physical links are different, and the change in boundary conditions at the ends of each pipe is determined by displacements of two body sections, when uniting the links into a general scheme are introduced scale coefficients and summation links. The composition of the summation links uf_{w2}, uf_{w1} and uf_v for Fig. 5 is determined by formulas (3.4) and (3.5).

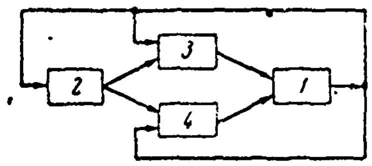

Fig. 4.

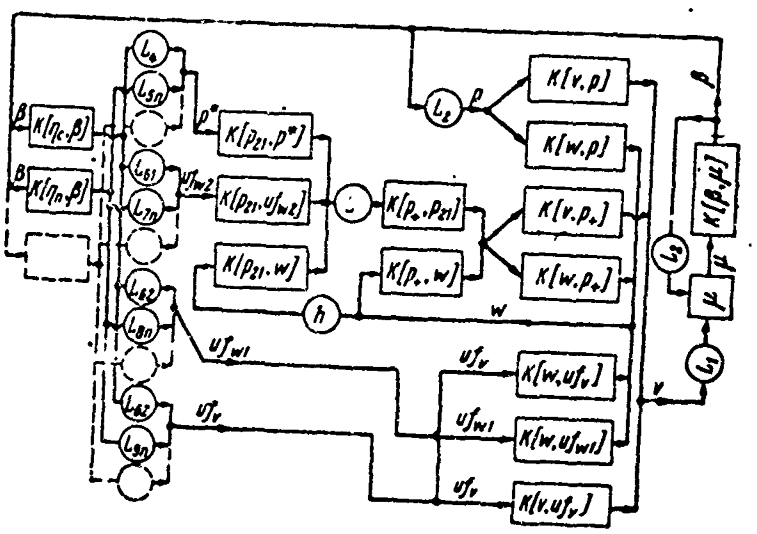

Fig. 5.

Injection rate variation of the fuel mass component into the chamber depends upon velocity variation v and density variation ρ_{22} on the right end of the second pipe. And so, for example, for oxidizer

$$m_2 = (v_0 + a_{02}v)(1 + p_{22})\rho_{02}F_{02}, \qquad m_2^\circ = v_{02}\rho_{02}F_{02}$$

Maintaining smallness of the first order only and taking into consideration, that $p_{22} = \rho_{22}$ on the basis of (1.2) we will obtain

$$\mu_2 = p_{22} + \frac{v}{M_2}$$

Pressure p_{22} can be conveniently expressed with the aid of formula (2.12) from (9) for j=2. Taking into attention, that $\rho_{02}\alpha_{02}^2 = \beta p_3^\circ$, we will have

$$\mu_2 = L_1 v + L_2\beta, \qquad L_1 = (M_2^{-1} + \psi_2 M_2), \qquad L_2 = p_3^\circ / \rho_{02}a_{02}^2 \qquad (4.1$$

On the basis of equations (3.1) we conclude, that

$$L_4 = -\omega^2 N_c, \qquad L_{5n} = -\omega^2 N_n \qquad (4.2)$$

Expressions for coefficients $L_{61}, L_{62}, L_{7n}, L_{8n}, L_{9n}$ will be found from the compilation of equations (3.2) and (3.4), (3.5)

$$\begin{aligned} L_{61} &= -i\omega a_0 / a_{01}, \qquad L_{62} = -i\omega a_0 / a_{02} \\ L_{7n} &= L_{61} = [f_{nw}(1 + \lambda_{21}) - \lambda_{21} f_{n1}] \\ L_{8n} &= L_{62}[f_{nw}(1 + \lambda_{12}) - \lambda_{12} f_{nv}] \\ L_{9n} &= L_{62}[f_{nv}(1 + \lambda_{22}) - \lambda_{22} f_{nw}] \end{aligned} \qquad (4.3)$$

Part of the block diagram, corresponding to the representation of body displacements in form of (2.7) is shown in Fig. 6. Instead of an infinitely greater number of blocks, expressing dynamic properties of the body, here all three blocks, which has an advantage during the analysis of the system. Expressions $uf_{\omega 2}$, $uf_{\omega 1}$ and uf_v are determined from equations (3.7), pressure variation p* is calculated by formula

$$p^* = L_3 f(\xi_1, \omega)\,\beta, \quad L_3 = -\omega^2 \frac{h_1 a_0^2 \varkappa(\omega)}{a_{01}^2 l} \quad (4.4)$$

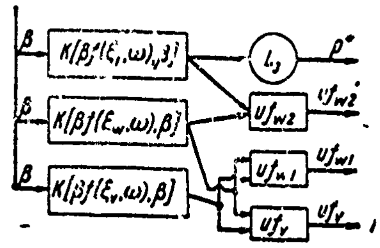

Fig. 6.

whereby the coefficient x (ω) depends upon the frequency of oscillations and it is referred to full displacement variation of tank bottom flange. The remaining part of the block diagram is the same as in Fig. 5.

If the geometric characteristics of the sylphons are such, that it is possible to adopt $\lambda_{12} = \lambda_{21} = \lambda_{22} = 0$, then on the basis of (3.7) velocities of uf_{ω_2}, uf_{ω_1}, uf_v are determined without changed part of block diagram, corresponding to this case, is presented in Fig. 7.

Here oscillations $\beta f' \xi_1, \omega)$ of the tank bottom cause only pressure variation p* at the input into the first pipe, oscillations $\beta f(\xi_\omega, \omega)$ of the pump change the rate of liquid movement at the output from first pipe and entry into the second pipe, oscillations $\beta f(\xi_v, \omega)$ of the sprayer head change rates of motion of the liquid at the output from the second pipe.

Ratios between dimensionless oscillation frequencies for different links will be established from equations

$$\Omega = \frac{s}{\theta} = \omega \frac{a_0}{l} = s_1 \frac{a_{01}}{l_1} = s_2 \frac{a_{02}}{l_2}$$

where

$$s = s_1 q, \qquad s_2 = s_1 q_2, \qquad \omega = s_1 q_1 \qquad (4.5)$$

$$q = \theta \frac{a_{01}}{l_1}, \qquad q_1 = \frac{l}{l_1} \frac{a_{01}}{a_0}, \qquad q_2 = \frac{l_2}{l_1} \frac{a_{01}}{a_{02}}$$

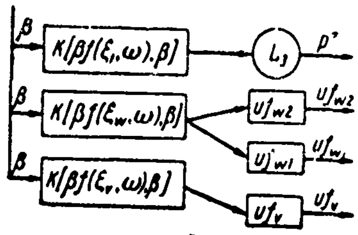

Fig. 7.

The poly-contour system presented in Fig. 5, can be simplified, by replacing links with feedback contours with equivalent to those links without feedback. An example of such a replacement is shown in Fig. 8. The equivalent link with complex transition number K* (β, v) expresses the ratio of pressure variation in the chamber to the fuel injection velocity variation

$$K^* [\beta, \nu] = \frac{K [\beta, \mu] L_1}{1 - K [\beta, \mu] L_2} \tag{4.6}$$

The vector hodograph K* (β, ν) in scale L_1 is almost no different from hodograph K (β, μ), shown in Fig. 1, 2.

Analysis of system properties shows, that the most favorable conditions for the origination of movement instability exists in cases, when natural frequencies of lower body tones and of the fuel line appear to be close. On frequencies, close to natural frequency ω_n of the body, as is evident from formulas (2.4), $\eta_n \gg \eta_c$, $\eta_n \gg \eta_m$ (n + m), and therefore in the first approximation can be written $\eta_c = \eta_m = 0$ (m = 1,2...m + n). The block diagram of the system is in this case most simplest; with consideration of one fuel line, it is presented in Fig. 9.

Complex transmission numbers K* (v, p), K* (v, p*) of equivalent links for the fuel line can be determined by scheme Fig. 5. Complex transmission numbers K* (v, p), K* (v, p*) of equivalent links for the fuel line can be determined by scheme Fig. 5. Complex transmission number K* (v, u) is more complex. It can be obtained either by scheme Fig. 5, by uniting actions uf_{w2}, uf_{w1}, uf_v with consideration of formulas (3.4), (3.5), (4.3) or by using formulas (9) having adopted on the basis of (3.2), (3.5)

$$f_{w2} = f_{nw}(1 + \lambda_{21}) - \lambda_{21} f_{n1}, \qquad f_{w1} = f_{nw}(1 + \lambda_{12}) - \lambda_{12} f_{nv}$$
$$f_v = f_{nv}(1 + \lambda_{22}) - \lambda_{22} f_{nw}$$

A further simplification of the system will be obtained by replacing links with complex transmission numbers K* (β, v) K (v, p) with an equivalent link (Fig. 10) whereby

$$K^{**}[\beta, v] = \frac{K^*[\beta, v]}{1 - K^*[\beta, v]\, K^*[v, p]\, L_2} \tag{4.7}$$

The complex transition number K** (β, v) expresses the variation ratio of the pressure in the chamber to the variation v of fuel injection velocity with consideration of reaction of the fuel line to this pressure. Now variation v is caused only by pressure variation p* at fuel entry into the line and by movements of the pump and sprayer head relative to the imperturbed stream, symbolically designated on the velocity variation scheme u.

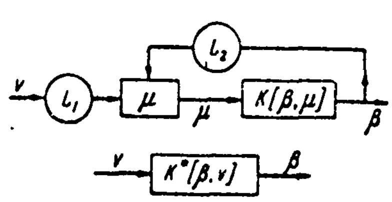

Fig. 8.

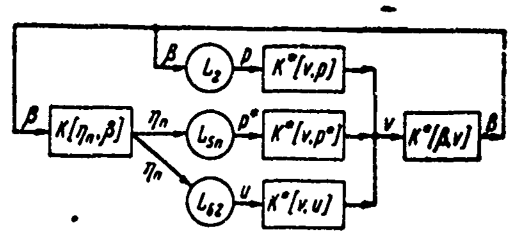

Fig. 9.

5. Dynamic schemes, presented in Fig. 4 -10 give the possibility for solving various problems: to investigate the effect of parameters and their combinations on the stability of the rated regime of the system, to mark such ratios of parameters, that the system would be stable, to select means and stabilization methods (including the use of automatic devices to control the fuel feed into the chamber) and formulate requirements to same[11,12], to determine permissible change areas of certain parameters, taking into consideration the values of remaining parameters; and finally to take into account fuel injection oscillations during the study of high frequency vibrations in the chamber.

Properties of the system depend upon a considerable number of physical parameters, many of which are closely interconnected. For example, due to fuel consumption at the time of the flight h_1, m are reduced, Ω_n increases, values f_{n1}, $f_{n\mu}$, f_{nv} change. An increase or reduction in bottom thickness of the fuel tank or a change in suspension rigidity of the engine leads to a change in values Ω_n, f_{n1}, $f_{n\mu}$, f_{nv}. Analysis of the system in general case becomes therefore very complex. In practical cases, it simplifies somewhat because it is necessary to deal with a concrete flying apparatus and ZHRD, the parameters of which are known, or permit variation on larger scales.

We will mention certain general properties of the system, leaving on side the features, connected with the change of many parameters within wide limits. Analysis of equations

$$1 - \bar{K}^*[\beta, v]\, K^*[v, p]\, L_2 = 0$$

which appear characteristic for link (4.7) shows, that its complex roots at certain chamber parameter ratios and fuel line ratios are situated in the right semi-surface, and therefore link (4.7) can be unstable. This corresponds to conclusions [5].

The feature of the dynamic system consists in the fact, that it can be unstable, even if the combustion chamber is considered an ideal link $\cancel{\theta} = \tau = v = A = 0$. Formula (4.7) in this case will be simple

$$K^{**}[3, v] = \frac{k_0}{1 - K^{*}[v, p] k_0 L_2} \tag{5.1}$$

We will establish an important property of the complex transmission number $K^{**}(\beta, v)$ which for graphic purposes will be formulated for the case, when $K^{*}(v, p) = K(v, p)$. We will assume, that on both ends of the pipe exist resistances, corresponding to conditions $\psi_1 M_1 < 1$, $\psi_2 M_1 < 1$ (in practical cases these conditions are always fulfilled). The dimensionless natural frequency of liquid stream oscillations in the pipe is the same as for a pipe open on both ends of the pipe[13]; it equals $(1-M_1^2)\, n\pi$ $(n = 1, 2\ldots.)$.

Since the vector hodograph $K(v, p)$ is situated in left semi-space $Z = U + iV$ and the modulus of the vector $K(v, p)$ has a minimum value at $s_{1n} = (1 - \ \ 1^2)(2n - 1)\,\pi/2$ $(n = 1, 2\ldots.)$ then from (5.1) can be established that the vector hodograph $K^{**}(\beta, v)$ will be situated in the right semisurface Z (Fig. 11,a);

and the modulus of this vector reaches maximum value at $s_1 = s_{1n}$. To that value is equal the stream frequency in the pipe, covered from one end. In this way, the presence of feedback in the chamber in form of a pipe, "open" from both ends, forms the equivalent link, the natural frequency of which is as if equal to pipe frequency, covered from one side. The first natural frequency of the equivalent link (5.1) is double lower than the first natural frequency of the liquid stream in the pipe between tank and chamber.

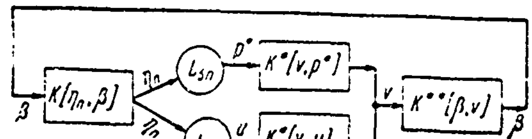

Fig. 10.

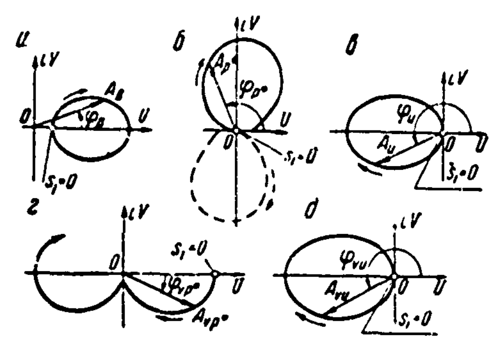

Fig. 11.

Analysis of the stability of a closed system presented in Fig. 10, is convenient to make with the aid of amplitude-phase criteria (7). In Fig. 11 is shown the approximate form of amplitude-phase frequency characteristics of system links applicable to a simpler case, when the line consists of a homogeneous pipe K* (v, p*) = K (v, p*), K* (v, u) = K (v, u) and the combustion chamber appears to be an ideal link. Curves b, v, g, d represent the vector hodographs respectively

$$K[\eta_n, \beta] L_{5n} = A_{p^*} e^{i\varphi_{p^*}}$$
$$K[\eta_n, \beta] L_{62} = A_u e^{i\varphi_u}$$
$$K[v, p^*] = A_{vp^*} e^{i\varphi_{vp^*}}$$
$$K[v, u] = A_{vu} e^{i\varphi_{vu}}$$
$$K^{**}[\beta, v] = A_\beta e^{i\beta}$$

Each individually taken link of the scheme in Fig. 10 is stable and therefore the closed system will be unstable only in the case, when $0 \leq s_1 \leq \infty$ is the amplitude phase characteristic of the open chain

$$[A_{p^*}A_{vp^*} \exp i(\varphi_{p^*} + \varphi_{vp^*}) + A_u A_{vu} \exp i(\varphi_u + \varphi_{vu})] A_\beta \exp i\beta \qquad (5.2)$$

on the plane Z = U + iV will envelop point with (1, i0).

On the basis of formulas (2.5), (2.6), (3.1) and (3.2) from [9] it can be concluded that moduli of complex transmission numbers $A_{p*} A_{vp*}$, $A_u A_{vu}$ are proportional respectively to $n_x f_{nl} f_{nv} hl/l$, $n_x f_{nv}^2$, where n_x - coefficient of axial overload of the flying apparatus. If $f_{nv} = 0$, the chain breaks, and the closed system does not exist.

When $f_{nl}=0$ ($p^* = 0$), the system remains closed, variation of fuel injection into the chamber is caused by the movement of the sprayer head. As is evident from Fig. 11, a, v, d, in the interval $0 \leq s_1 \leq \infty$ will be found $s_1 = s_1^0$ such, that $\varphi_u + \varphi_{vu} + \varphi_\beta = 0$. The system will be unstable, if at $s_1 = s_1^0$ we have $A_u A_{vu} A_\beta > 1$. The best conditions for such a situation originate, when $\omega_h \approx s_{ln}$.

Mostly $f_{nl} \neq 0$, $f_{nv} \neq 0$ and the fuel injection variation into the chamber is caused by the movement of tank bottom and spraying head. Two possibilities do exist: $f_{nl}f_{nv} < 0$ - solid curve and $f_{nl} f_{n\omega} > 0$, dotted curve in Fig. 11, b. From the analysis of expression (5.2) and curves in Fig. 11 can be established, that in case of $f_{nl}f_{n\omega} > 0$ the possibility of loosing stability at lower natural frequencies increases, and at $f_{nl}f_{n\omega} < 0$, it decreases in comparison with the fact, when $f_{nl} = 0$. At other equal conditions the possibility of origination of instability increases with the increase in n_x, and for $f_{nl}f_{nv} > 0$, it is greater, the greater h_l/l. Damping of the body (ξ_n) and resistance in the pipeline (ψM_l, $\psi_2 M_l$) promotes stability of motion. The greater the pressure drop on the sprayers, the more stable is the system.

Stability criteria of the system are less apparent, when the chamber cannot be considered as an ideal link, and the fuel conducting line has a pump. They become less reviewable for a di-component system. For many constructions of flying apparatuses with ZHRD (especially with liquid gas generator, working on basic components) the dynamic scheme is more complex and can include automatic devices to control fuel feeding into the chamber [4].

By linearized equations can be obtained a conclusion only about the stability or instability of the system. If the system is unstable and the accidently originated oscillations will grow, then the assumption about linearity becomes incorrect. Nonlinearity in equations for combustion chamber, possibility of origination of cavitational phenomena in the mainlines etc., lead to a change in dynamic properties of the system. In the system can be established an autooscillatory regime.

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