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TECHNICAL REPORT
ME-4

THE BIAxIAL STRESS-STRAIN BEHAVIOR OF FABRICS

by

Constantin J. Monego
MECHANICAL ENGINEERING DIVISION

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November 1965

U. S. Army Materiel Command
U. S. ARMY NATICK LABORATORIES
Natick, Massachusetts



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1MG624101D503

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FOREWORD

This report was prepared by the Tentage and Equipage Branch, Mechanical Engineering Division, U. S. Army Natick Laboratories, Natick, Mass. The work was performed as part of its exploratory development program, Task 01, Project Number 1MG624101D503.

This study was initiated to determine the scale for the aerodynamic parameters for fabrics in model tents tested in the wind tunnel. The work was later extended to include all of the vinyl-coated fabrics used by the Army for air-supported tents used in Continental United States. The study was started in July 1963 and completed in November 1964.

The paper was presented before The Fiber Society at their Fall meeting held in Wilmington, Delaware, on 8 October 1965.

The work reported represents a joint effort among the Tentage and Equipage Branch and the Engineering Laboratory, Mechanical Engineering Division, for testing to obtain the data, and the Clothing and Organic Materials Division, which cooperated by furnishing the fabrics for test.

The author wishes to acknowledge Mr. W. C. Whittlesey and Mr. C. W. Weikert of the Mechanical Engineering Division, U. S. Army Natick Laboratories, for their encouragement and support of this work. Acknowledgement is also accorded to personnel of the Clothing and Organic Materials Division for furnishing the fabrics for test; to the Tentage Shop for preparing the models for tests; to the personnel of the Engineering Laboratory for their assistance in setting up the tests and obtaining test data; and Mr. Thomas C. Strain and Mrs. Elda B. Key for their assistance in preparing this report for publication.

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NOMENCLATURES

S_t = Tangential stress (lbs/in)

S_m = Meridional stress (lbs/in)

R_t = Tangential radius (in)

R_m = Meridional radius (in)

ϵ_t = Tangential strain

ϵ_m = Meridional strain

E = Modulus of elasticity (lbs/in²)

μ = Poisson's ratio

P = Pressure (lbs/in²)

T = Thickness (in)

γ_{xy} = Shear strain

τ_{xy} = Shear stress (lbs/in²)

G = Shear modulus (lbs/in²)

CI = Crimp interchange

ϵ = Strain

η = Coefficient of viscosity (Poise = $\frac{\text{dyne-sec}}{C_m^2}$)

ABSTRACT

A comparison is made of the stress-strain response of a fabric in uniaxial stress and three loading conditions in biaxial stress. The results of the stress-strain tests were also compared with strain measurements made on a full-size tent. The results of the biaxial tests were found in better agreement with the stress-strain response in the full-size tent than the uniaxial tests. A suggestion is made for the development of a theory for predicting the biaxial stress-strain behavior of a fabric. This development will include the selection of a governing equation for the perfectly elastic state and finding the expressions needed to define the variables of the equation for a non-elastic material.

THE BIAXIAL STRESS-STRAIN BEHAVIOR OF FABRICS

1. Introduction

The increasing use of fabrics for engineering materials in structures, where weight, durability, and reliability are important, has emphasized the importance of fabric stress analysis. The earliest work of importance in this area was published in 1912. Since that time, and particularly in recent years, studies on the mechanical behavior of fabrics have accelerated^(3,7,8,10,11). The studies listed can be grouped into three categories:

a. The effect of the geometry on the physical properties of a fabric^(1,6).

b. The development and use of instruments for measuring the biaxial stress-strain properties of a fabric^(2,5,9).

c. The biaxial stress-strain behavior of fabrics^(3,4,7,8,10,11).

Prior studies have indicated that the full potential of lightness in weight, durability, and reliability of a structure cannot be realized without knowing the stress behavior of the material under use conditions. The work also provided a better understanding of the mechanical behavior of fabrics under stress; however, to progress in this area at a more rapid rate, and to derive more benefit in the use of textiles for lightweight, flexible structures, past work must be extended. A more comprehensive and accurate theory of the mechanism of fabric stress behavior at low loads and at increasing loads to rupture would be of considerable value in accelerating the development of mechanical fabrics.

This paper will be limited to the discussion of the comparative results of uniaxial and biaxial stress-strain tests on fabrics of interest for air-supported tents. The discussion will be followed by a suggestion for future work leading to the development of a comprehensive theory for the stress-strain behavior of fabrics in an air-supported tent.

2. Theoretical Background

The structural shapes of interest are spheres and cylinders with spherical ends. Therefore, the principal equation governing stress and internal pressure is as follows:

$$\frac{S_m}{R_m} + \frac{S_t}{R_t} = P/t \quad (1) *$$

with stress expressed in terms of pounds per square inch. However, when fabrics are used for the envelope of the material, it is customary to neglect the thickness and the equation then becomes:

$$\frac{S_m}{R_m} + \frac{S_t}{R_t} = P \quad (2)$$

with stress expressed in terms of pounds per inch.

It is recognized that the controlling factors producing stress patterns are three dimensional; however, when fabrics which possess only two principal directions are considered, most stress applications can be geometrically resolved in two orthogonal components. Hence, the thickness of the fabric is not normally considered as a fabric stress situation. The stresses themselves are normally considered to act in the plane of the fabric. In order to obtain a better understanding of the test results, a brief review of the behavior of fabrics under stress is in order. Dr. Haas⁽⁴⁾ considered the deformation of a plain weave fabric to be the result of three distinct but mutually interacting mechanisms. The first of these is thread shear, where the mutually perpendicular warp and filling yarns rotate changing the angle between the yarns; the second mechanism is termed thread straightening and results from the over and under characteristics of the plain weave, each set of yarns bending over the other set. This bending is also known as crimp. When loads are applied to the two yarn systems, the system under the higher stress will tend to straighten, transferring a part of its crimp to the other set of yarns. This mechanism is termed crimp interchange. The third mechanism is that of yarn extension within the weave. Peirce⁽⁶⁾ and others have identified a fourth mechanism which will influence the stress-strain behavior of a plain weave fabric. This is concerned with the compressive properties of the yarn and its bending stiffness. Each yarn is subjected to both lateral compression and bending at every thread crossing. Lateral compression causes the yarn to flatten under load and allows the weave to extend, and bending rigidity results in increased resistance to extension of the weave.

*Numbers following equations refer to sequential order in which the equations appear.

The sequence in which the interacting mechanisms operate is assumed to be as follows. When the load is first applied, the mechanisms of shear and crimp interchange predominate. The two mechanisms operate by a geometric rearrangement of the yarns in the weave rather than by yarn extensions. Thus, the results of the initial fabric deformation under load is independent of the rheological properties of the fibers. As the loads or stresses are increased, the strain due to shear and crimp interchange reaches a limiting value which is governed by the limiting extension of the fabric. This limiting value of strain is reached sooner in densely woven fabrics than in fabrics of loose construction. Increasing the stress at this point will lead to yarn extension and yarn flattening, the latter two mechanisms predominant, as the stress applied approaches the rupture load. It should also be pointed out that textile fibers are visco-elastic materials; hence, where fabric loads reach a level where yarn extension within the fabric occurs, the results of strain become time dependent, and thus strain results can vary with the rate of loading of the material. This is particularly important when rupture strain is considered. If the rate of increase of loading is slow, there is more time for creep to occur and breaking strain can be reached at a lower load.

From the above, it is evident that the stress-strain response of a fabric can be highly influenced by the modes with which the loads are applied and the time rate of loading. For additional information relative to fabric deformation as a result of stress, the reader is referred to the selected references^(1,3,4,6,7,8,11).

3. Testing

The uniaxial stress-strain response referred to in this paper is where the load is applied to the fabric in one direction and measured in one direction. An average lapse time of 10 minutes was required to complete the test.

The biaxial stress-strain response referred to in this paper is where the warp and filling yarns are loaded simultaneously and measurements made in two orthogonal directions. The test apparatus is shown in Figures 1 and 2. A lapse time of 30 minutes was required to complete the test.

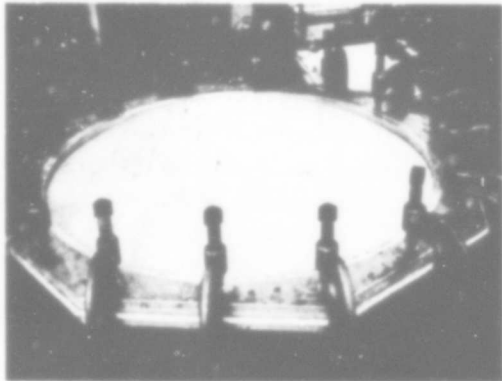


Figure 1. Diaphragm Biaxial Test
(1-1 warp to filling load ratio)

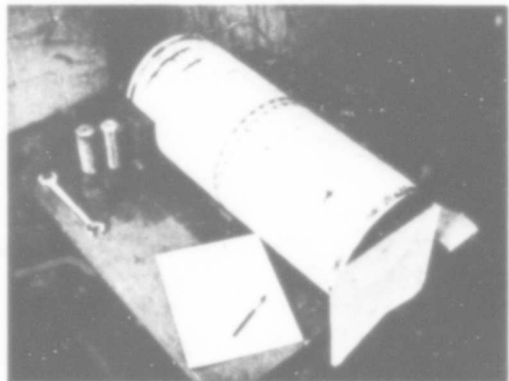


Figure 2. Cylinder Biaxial Test
(1-2 and 2-1 warp to filling load ratio)

Figure 3 illustrates the location of squares on the tent.

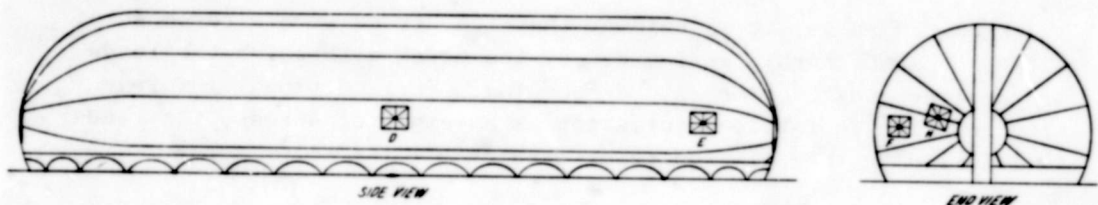


Figure 3. Location of Squares in Full-Size Tent

4. Test Results

Uniaxial as well as biaxial tensile tests were performed. The stress-strain curves for four tent fabrics were compared and selections from these curves were made and presented in graphical form to illustrate the points made in the discussion of the results which follow.

5. Test to Rupture

The stress-strain behavior of the 2-oz/yd² polyurethane-coated nylon fabric is shown in Figures 4 and 5. Figure 4 illustrates the

behavior of the warp yarns and Figure 5 that of the filling yarns. The curve designated as A in both figures represents the uniaxial stress-strain response for the fabric. It should be noted that the curve for the warp direction (Figure 4) is steeper than the filling curve (Figure 5). Since uniaxial tests represent a straightening of the yarn systems under load, this indicates that the warp yarns have less crimp; hence, they appear stiffer under load than the filling yarns and require less extension to straighten the crimp before fiber elongation sets in. Since the uniaxial tests represent primarily a straightening of the yarns under load, this test will be taken as a point of reference for the results of the biaxial tests. Curve B, Figures 4 and 5, represents the results of the diaphragm biaxial stress-strain test, 1-1 warp to filling load ratio. In Figure 4, curve B indicates that the warp direction is not as stiff at low loads and somewhat stiffer at rupture elongation than the uniaxial case. The greater elongation at low loads of the warp yarns is due to the mechanism of crimp interaction between the warp and filling yarns. This same mechanism shows the filling yarns which have the highest crimp (curve B, Figure 5) to be markedly stiffer than the uniaxial case. The increased stiffness shown by the filling yarns happens because both yarn systems are loaded at the same rate and both yarn systems mutually resist the yarn straightening action imposed by their respective loads. This is a good example of the effect of crimp interchange.

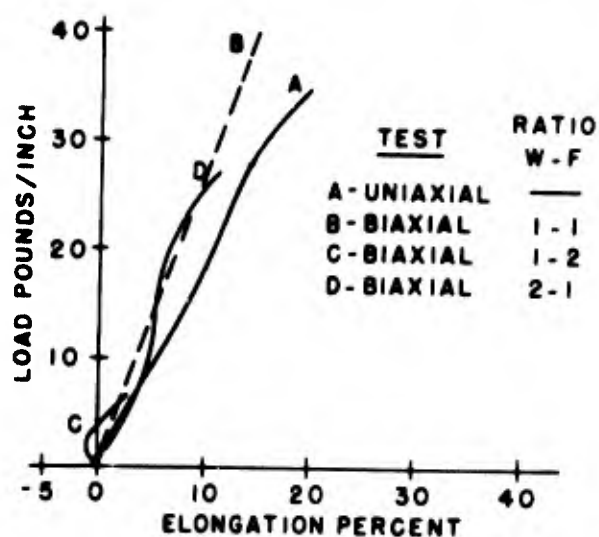


Figure 4. Stress-Strain Response for 2-oz/yd² Polyurethane-Coated Nylon

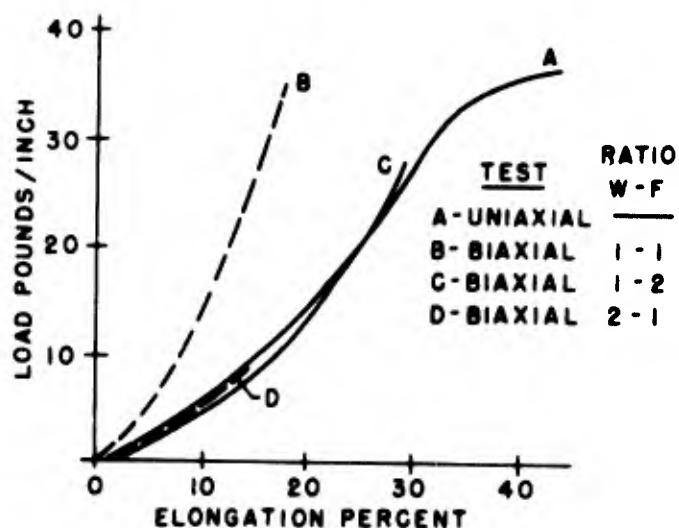


Figure 5. Stress-Strain Response for 2-oz/yd² Polyurethane-Coated Nylon

Another interesting example of the mechanism of fabric deformation is by curve C, Figures 4 and 5. Curve C represents the cylinder biaxial test with the filling in the hoop direction. In this case, the filling yarn system with the highest crimp is loaded at twice the rate of that applied to the warp yarn. The straightening of the filling yarns caused the warp yarns to bend, showing a contraction in cylinder length, negative elongation (curve C, Figure 4). This contraction continued until the limiting extension was reached when, with added stress, the warp yarn began to extend in the fabric causing the curve to reverse itself and showing an elongation of the warp yarns at the conclusion of the test. The filling yarns (curve C, Figure 5) under load appeared to show the same stiffness behavior as the uniaxial case, with a slight increase in stiffness at rupture elongation. Curve D, Figures 4 and 5, shows the stress-strain behavior of the cylinder biaxial test with the warp yarns in the hoop direction. In examining curve D, Figures 4 and 5, it should be noted that the warp yarns show a stiffer response to loading than in the curve C biaxial test, and that the filling yarns do not contract at low loads the way the warp yarns did in this position (curve C). This situation indicates a very low level of crimp interchange between the yarn system. It should also be noted that the breaking load in the biaxial tests is lower than the uniaxial tests. In the case of the diaphragm biaxial test, this difference is small and could probably be accounted for by the longer time required to run the test. The breaking load for the cylinder biaxial tests is nearly 25% lower than the uniaxial test. Only part of this difference can be accounted for by the additional time required to run the test, but can offer no explanation at this time to account for all of the loss in breaking load.

6. Ratio of Warp to Filling Yarn Extensions

It can quickly be seen from the following table that the ratio of warp to filling yarn extensions is not constant but differs according to the mode of loading the fabric. This effect is analogous to Poisson's ratio over stress levels tested.

TABLE I

RATIO OF WARP AND FILLING YARN EXTENSIONS

<u>Stress</u> (lbs/in)	<u>Uniaxial</u> --	<u>Biaxial</u> 1-1	<u>Biaxial</u> 2-1	<u>Biaxial</u> 1-2
2	.33	.74	0.04	-0.14
8	.36	.81	.28	.42
15	.39	.86	--	--
20	.45	.88	--	--

7. Test to 10% Breaking Load

Another illustration of the effect of crimp interchange in a biaxial stress-strain situation is made with two specimens of 18-oz vinyl-coated nylon fabric, both conforming to the requirements of MIL-C-20696. The two fabrics are stressed to approximately 10% of their breaking load. The uniaxial stress-strain response of the two fabrics is shown in Figure 6. The stress-strain response of the warp yarns A and B appears to be the same for both fabrics, but the response of the filling yarns differs. Fabric A shows a lower stiffness in the filling direction than fabric B. The lower stiffness indicates a higher amount of crimp in the filling direction. The diaphragm biaxial stress-strain response (1-1 warp to filling load ratio) of the same two 18-oz/yd² vinyl-coated nylon fabrics is shown in Figure 7. The behavior of the warp yarns under load does not differ appreciably from the uniaxial test case (Figure 6); however, the behavior of the filling yarns is markedly different. The filling yarns in fabric A, due to their higher crimp, stiffen considerably under load--in fact becoming stiffer than the warp yarn. The filling yarns in the B fabric, with the lower crimp, show only a slight stiffening action when compared with the uniaxial case in Figure 6. The cylinder

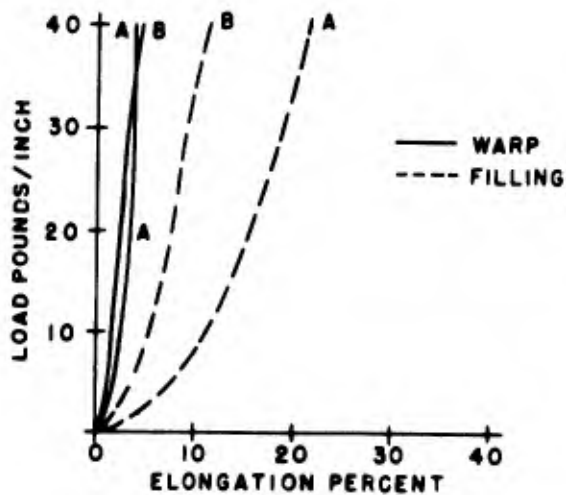


Figure 6. Uniaxial Stress-Strain Response for 18-oz/yd² Vinyl-Coated Nylon

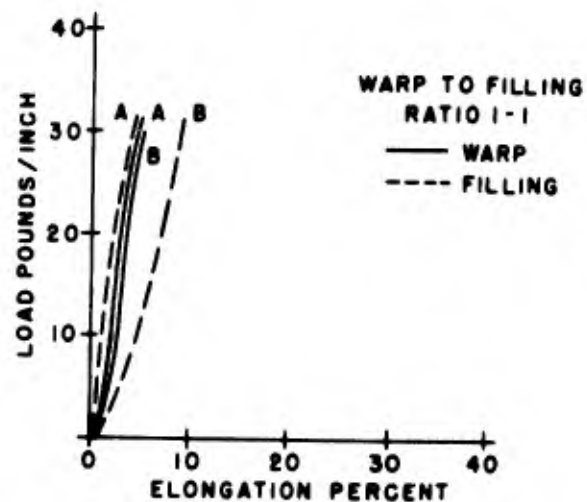


Figure 7. Biaxial Stress-Strain Response for 18-oz/yd² Vinyl-Coated Nylon

biaxial stress-strain response of the 18-oz/yd² vinyl-coated nylon fabrics with the filling in the hoop direction is shown in Figure 8. The fabrics were loaded to one tenth the breaking strength. The higher rate of loading in the filling direction causes the filling yarns to straighten and the warp yarns to crimp.

8. Effect on Elastic Stiffness of Fabric

In wind tunnel studies, with reduced size model tents, it is necessary to match the elastic stiffness of the fabric for the model and the full-size tent. The matching of the elastic stiffness of the two fabrics is necessary if realistic observations are to be made of the response of the model tent to wind loads. In Figure 9, a comparison is made between the uniaxial stress-strain response of an 18-oz/yd² vinyl-coated nylon (MIL-C-20696) and a 6-oz/yd² vinyl-coated nylon (MIL-C-40039). As expected, the 18-oz/yd² fabric A, with less stiffness in the filling direction, showed a contraction, negative elongation, in the warp direction due to crimp interchange between the warp and filling. The 6-oz/yd² fabric B showed an elongation in both warp and filling directions. The biaxial stress-strain behavior of a third fabric is an 18-oz/yd² vinyl-coated fabric C, shown for comparison to indicate again the wide difference which can be expected as a result of crimp interchange on the stress-strain response of two fabrics equal in weight.

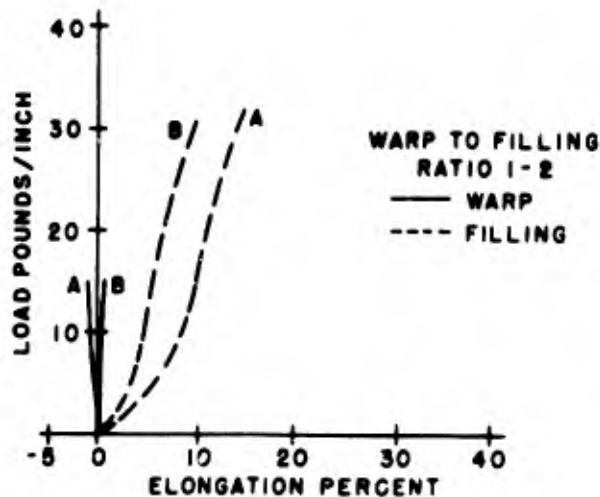


Figure 8. Biaxial Stress-Strain Response for 18-oz/yd² Vinyl-Coated Nylon

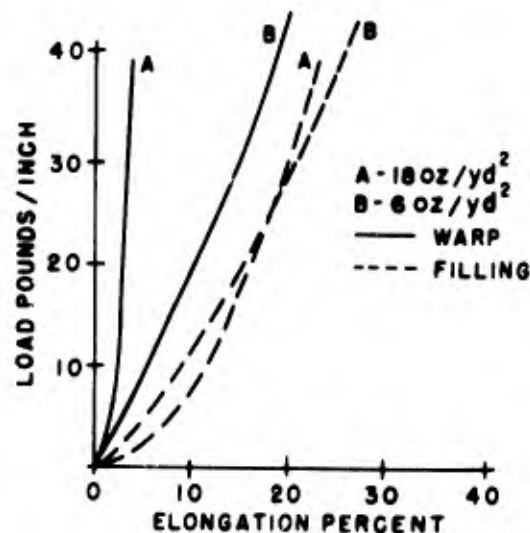


Figure 9. Uniaxial Stress-Strain Response for Vinyl-Coated Nylon

9. Effect of Mode on Loading

It has been shown above that the stress-strain response of a fabric differs with the mode of loading, and that the stiffness response of the fabric is highly influenced by the relative amount of crimp in the two yarn systems. One additional comparison is deemed of interest and that is the effect of prior loading. Figure 10 illustrates the stress-strain response for vinyl-coated nylon. Figure 11 illustrates the stress-strain response of the uniaxial test of two 18-oz/yd² fabrics. Fabric A was taken from a sample of cloth not previously loaded, while fabric B was taken from a tent which was fabricated in 1960 and as near as can be determined, it was inflated and used for a period of two years. Both fabrics were manufactured by the same firm and presumably had similar stress-strain characteristics when new. The breaking strength of the two fabrics is nearly alike--fabric A, 350 lbs/in warp strength, 255 lbs/in filling, and fabric B, 320 lbs/in warp strength, 215 lbs/in filling. The tent was used by the U. S. Army Natick Laboratories to obtain the strain measurements which will be described later.

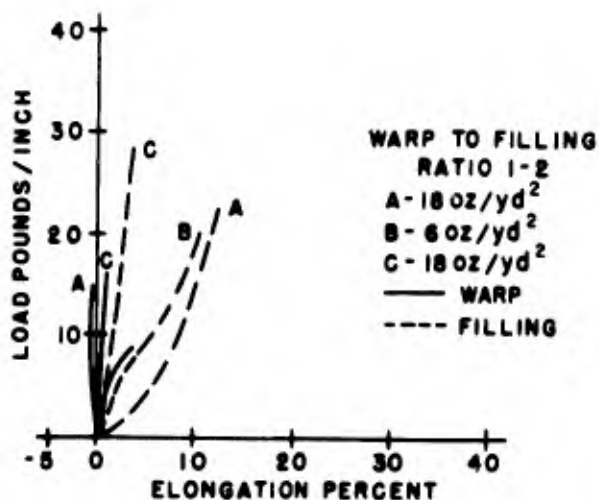


Figure 10. Biaxial Stress-Strain Response for Vinyl-Coated Nylon

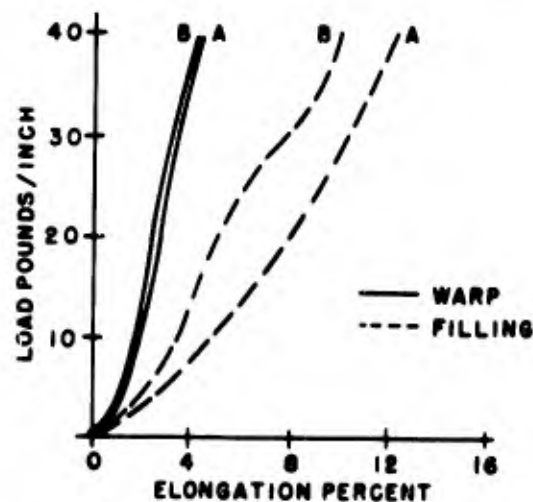


Figure 11. Uniaxial Stress-Strain Response for 18-oz/yd² Vinyl-Coated Nylon

Figure 12 illustrates the diaphragm biaxial stress-strain response for the two 18-oz/yd² fabrics. It was found that fabric B which was used for two years had a stiffer response to the biaxial stresses imposed on it than fabric A. The gain in stiffness response of the fabric after prolonged use could be due to a straightening of the filling threads after prolonged stress at low load. In Figure 13, for the uniaxial test, the stress-strain response of the warp yarns for fabrics A and B appears to be the same, while the response of the filling yarns differs. The filling yarn system of the A fabric shows a lower stiffness than that of the B fabric.

10. Comparison of Laboratory Test with Full-Scale Tent

The laboratory stress-strain characteristics of the material were obtained from the fabric specimen removed from the tent. The strain measurements were made on the tent by marking off 20-inch squares at different locations as shown in Figure 3. The strain measurements were made at different inflation pressures representing different fabric stresses. The highest stress found represented approximately 10% of the breaking load for the fabric. Two situations will be described--one for the cylindrical portion of the tent (Figure 3, square marked "C"), and the other for the spherical end (Figure 3,

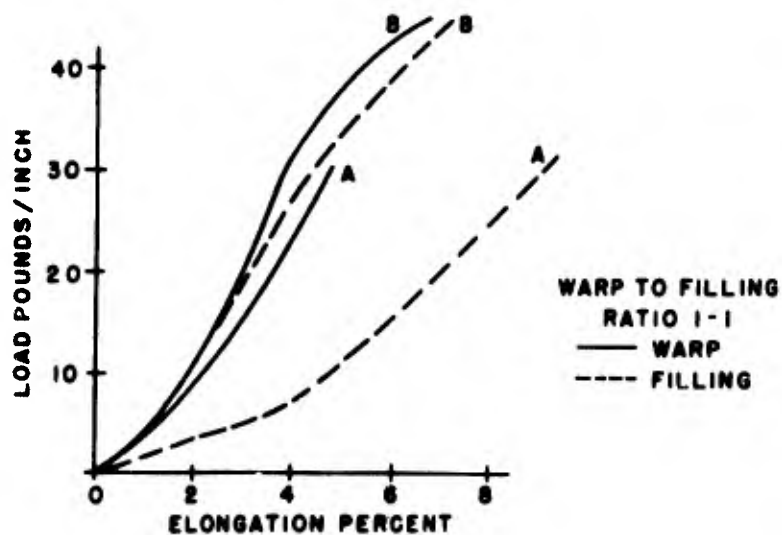


Figure 12. Biaxial Stress-Strain Response for 18-oz/yd² Vinyl-Coated Nylon

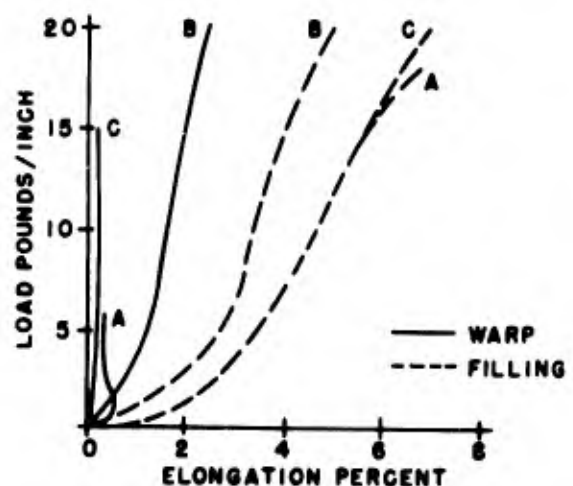


Figure 13. Stress-Strain Response
A-Tent Cylindrical
B-Uniaxial
C-Biaxial 1-2 Warp to Filling Ratio

square marked "F"). The stress-strain response for the C square is shown in Figure 13. A comparison is made with the uniaxial stress-strain response on the 18-oz/yd² fabric B taken from the tent. Also included in Figure 13 is the cylinder biaxial response of the tent fabric, with the filling in the hoop direction. This orientation of the fabric on the cylinder reflects the fabric orientation on the tent. It should be noted that the uniaxial stress-strain response, fabric B, is in poor agreement with the stress-strain measurements made on the tent. A much better agreement is found between the curve A tent and the cylinder biaxial stress-strain response, fabric C. In fact, the filling stress-strain response in both the tent test and cylinder test is identical, the curves superimpose. This agreement emphasizes the importance of assessing the stress-strain characteristics of a fabric under end use conditions.

The comparison among the strain measurements made on the spherical segment of the tent and uniaxial and biaxial stress-strain response of the fabric is shown in Figure 14. The strain measurements were made on the spherical portion of the tent (Figure 3, square F). The stress-strain response of square F is the A tent, Figure 14. The uniaxial stress-strain response for the fabric is shown as fabric B, a poor fit. It was expected that the diaphragm biaxial test, fabric C, would show a good fit; however, it can be seen in Figure 14 that this is not the case. Curiously enough, the stress-strain response for a cylinder

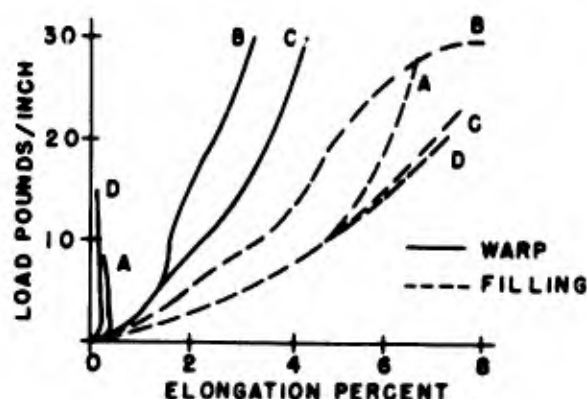


Figure 14. Stress-Strain Response

A-Tent Spherical
 B-Uniaxial
 C-Biaxial 1-1 Warp to Filling Ratio
 D-Biaxial 1-2 Warp to Filling Ratio

biaxial test (fabric D) shows a better agreement with the strain measurements made on square F. This effect was not expected and a reassessment is being made regarding the stress distribution in the spherical end of the tent. An examination of the stress picture (Figure 15) for the cylindrical portion of the tent (square C) shows that this square elongates in the vertical direction while it nearly maintains its width with increasing stress, about 0.2% elongation according to Figure 13, fabric C warp. The corners remain square as a well-behaved square or rectangle should to conform to the theoretical behavior of a cylinder. Now the examination of the square F (Figure 16) in the spherical portion of the tent does not show this behavior, but the sides go askew, the corners are no longer square (indicating an uneven distribution of stress). Incidentally, this also shows the ability of the fabric to accommodate an uneven stress pattern which is fine for the tent but it is not good for our theoretical prediction of the stress pattern in the spherical portion of the tent.

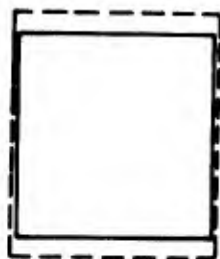


Figure 15. Square "C" Cylindrical Portion Deformation Under Load (Not to Scale)

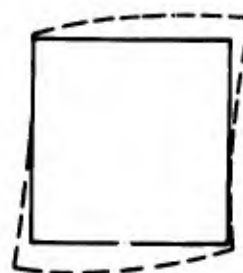


Figure 16. Square "F" Spherical Portion Deformation Under Load (Not to Scale)

11. Discussion

It has been demonstrated that the stress-strain behavior of a fabric is influenced by the mode of loading, the uniaxial case differing from the biaxial stress situations. From this, it is evident that the geometry of a structure will influence the rate of loading of the fabric from which it is made. Therefore, in order to properly assess the stress-strain behavior of a fabric, future studies of fabric stresses must be made in relation to the geometric shape of the desired structure.

12. Fabric Behavior

A suggested approach to future studies is to develop the theory for the biaxial stress-strain behavior of a fabric in relation to the geometric shape of the structure in which the fabric is used. This approach consists of considering membrane-type stresses and developing the equations for the perfectly elastic situation--then, finding the expressions which will define the variables in this equation (after the variables are established and boundary conditions defined, the approximate solutions to the equations will be sought to predict the stress-strain behavior of the fabric).

This theoretical development using the theory of elasticity is not new. Haas⁽⁴⁾ started this in 1912. More recently, Davidson⁽³⁾, Klein⁽⁵⁾, Popper^(7,8), Reichardt⁽⁹⁾, Topping⁽¹¹⁾, and Stein and Hedgepeth⁽¹⁰⁾, have all published information relative to biaxial stress-strain behavior of fabrics. These studies, together with studies of the effect of fabric geometry, have been used with gratifying results in extending the theoretical stress-strain behavior of fabrics in biaxial situations. However, most theories proposed to date are limited by simplifying assumptions in order to solve the equations. The simplifying assumptions eliminate the mechanical properties of the threads comprising the weave. If the exacting modern-day requirement for mechanical fabrics is to be met with a minimum weight and bulk, the modern theories must be expanded to include the mechanical properties of the threads in weave.

A good start in the development of governing equations for flat membrane structures is provided by Stein and Hedgepeth in their analysis of partial wrinkled membranes⁽¹⁰⁾. The equations proposed for rectangular structures follow the general order for plane stress and include (a) geometric relationship, (b) constitution relationship (Hooke's Law), and (c) force equilibrium. These equations are needed to give physical significance to the solutions for stress in any given situation. Stein and Hedgepeth⁽¹⁰⁾ have developed compatibility equations for rectangular plates and cylinders which need not be repeated here. It suffices for our purpose to consider the surfaces of revolutions, cylinders and spheres. The general equation for stresses in the membrane case is:

$$\frac{S_m}{R_m} + \frac{S_t}{R_t} = P/t \quad (1)$$

Once the stresses are obtained, the strains can be found by using Hooke's Law for the elastic case:

$$\epsilon_m = \frac{1}{E} (S_m - \mu S_t) \quad (3)$$

$$\epsilon_t = \frac{1}{E} (S_t - \mu S_m) \quad (4)$$

It has been shown that structural shape can cause shear; therefore, one more expression must be added to the above and that is the shear strain:

$$\gamma_{xy} = \frac{1}{G} \tau_{xy} \quad (5)$$

The parameters in the above equations, which must be further defined to fit the textile situation, are the elastic modulus E , Poisson's ratio μ , and the shear modulus G .

An expression must be developed for each parameter to reflect the mechanical behavior of fabrics and the expressions should be compatible with the stress equations developed for a structure.

The simplest expression for tensile stress, reflecting the mechanical properties of a yarn in a fabric structure, would probably take the following form. At low stress, the principal mechanisms of fabric deformation are shear and crimp interchange. Consider crimp interchange first. Then the stress in a yarn is some function of crimp interchange and yarn straightening, $f(CI)$. Once the limiting extension is reached, yarn elongation in the fabric takes place, the initial phases of this mechanism are elastic; therefore, this part of fabric stress is a function of the modulus of elasticity, $f(E)$. On continued stress, the yarn extends to the yield point elastically, beyond the yield point and on to rupture. The stress-strain phenomena becomes time dependent. A suggested mechanism of deformation for this phase is to consider it as a function of viscosity and time, $f(\eta/t)$. The mechanics of the stress-strain behavior of a fabric are therefore related in some complicated way to an expression containing the following elements:

$$S = f(CI); f(E); f(\eta/t)$$

The parameter for yarn shear is considered next. As stress is applied to the fabric, yarn shear will occur; i.e., the angle between the two yarn systems will change. The initial shearing action of the yarn is related to the angle in which the loads are applied (geometry of the

structure), mechanical properties of the yarns (friction, bending stiffness, size), the weave and number of yarns per unit length. Hence, shear stress at low loads in its simplest form can be described as a function of direction of loading, yarn and fabric properties. As the load is increased, the yarns extend and the elastic modulus of the material (E) must be considered.

For the present, no consideration will be given to the effect of stress beyond the yield extension of the yarns, since with the relatively high load in which this action occurs the tensile forces predominate. The simplest expression for this must contain the following:

$$\gamma_{xy} = f(\text{load direction}); f(\text{fabric and yarn properties}); f(E)$$

The third parameter is an effect analogous to Poisson's ratio. Only two factors will be considered--crimp interchange for low loads and the elastic modulus for the higher loads when the yarns start to extend. Again, the effect of Poisson's ratio will not be considered beyond yield stress of the material. In its simplest form the Poisson ratio effect can be defined as:

$$\mu \approx f(CI); f(E)$$

Since textile materials do not represent a continuous structure, the definitions of the functions will have to be arrived at through experimentation. The full development of the expression for the stress functions of a fabric depends on the development of theory for the mechanics of rupture in yarns. Studies in this area are currently being conducted in the Textile Division, Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

The development of the expressions for shear modulus and crimp interchange will have to be arrived at by continued studies on the biaxial stress-strain behavior of fabric. A limited program in this area is currently underway at the U. S. Army Natick Laboratories.

13. Conclusions

In conclusion, it has been shown that the stress-strain behavior of a fabric is largely dependent on the mode of loading the fabric. Since the magnitude and direction of fabric stress is determined by the size and shape of the structure, the stress-strain behavior of a

fabric cannot be fully predicted without considering the geometry of the structure. Plans for future study should include the development of a comprehensive and accurate theory for the stress-strain behavior of a fabric. A plan for this study includes the development of a governing equation to determine the stress in a structure and then to design a fabric which can cope with the structural stresses in a predictable manner.

The development of a comprehensive and accurate theory for fabric stress is essential to accelerate the development of fabrics meeting the exacting mechanical requirements of all structures at a minimum weight and bulk.

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Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D		
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)		
1 ORIGINATING ACTIVITY (Corporate author) U. S. Army Natick Laboratories		2a REPORT SECURITY CLASSIFICATION Unclassified
		2b GROUP
3 REPORT TITLE The Biaxial Stress-Strain Behavior of Fabrics		
4 DESCRIPTIVE NOTES (Type of report and inclusive dates)		
5 AUTHOR(S) (Last name, first name, initial) Monego, Constantin J.		
6. REPORT DATE November 1965	7a TOTAL NO OF PAGES 18	7b. NO OF REFS 11
8a CONTRACT OR GRANT NO.	9a ORIGINATOR'S REPORT NUMBER(S) ME-4	
b. PROJECT NO. 1MG624101D503		
c.	9b OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.		
10. AVAILABILITY/LIMITATION NOTICES Distribution of this document is unlimited. Qualified requesters may obtain copies of this report from DDC and CFSTI.		
11. SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY Tentage & Equipage Branch Mechanical Engineering Division U. S. Army Natick Laboratories Natick, Massachusetts	
13. ABSTRACT A comparison is made of the stress-strain response of a fabric in uniaxial stress and three loading conditions in biaxial stress. The results of the stress-strain tests were also compared with strain measurements made on a full-size tent. The results of the biaxial tests were found in better agreement with the stress-strain response in the full-size tent than the uniaxial tests. A suggestion is made for the development of a theory for predicting the biaxial stress-strain behavior of a fabric. This development will include the selection of a governing equation for the perfectly elastic state and finding the expressions needed to define the variables of the equation for a non-elastic material.		

14	KEY WORDS	LINK A		LINK B		LINK C	
		ROLE	WT	ROLE	WT	ROLE	WT
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