FINAL REPORT—PART I

FINITE MEMORY NONLINEAR FILTERING OF DIGITAL SEQUENCES

Jack K. Wolf
Principal Investigator

October 1, 1965

Prepared for
U.S. ARMY RESEARCH OFFICE—DURHAM
BOX CM, DUKE STATION
DURHAM, NORTH CAROLINA 27706

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LABORATORY FOR ELECTROSCIENCE RESEARCH

DEPARTMENT OF ELECTRICAL ENGINEERING

SCHOOL OF ENGINEERING AND SCIENCE

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ABSTRACT

The autocorrelation function of sequences generated by the non-linear filtering of binary maximal length sequences are derived. It is shown that the autocorrelation function of the filtered sequences, which are recognized as interleaved sequences, are similar to the autocorrelation function of the original maximal length sequence except for the addition of minor peaks. Other problems considered are the cross-correlation functions of interleaved two-level sequences, binary and non-binary sequences, binary and non-binary sequences orthogonal for all cyclic shift, the performance of an asynchronous multiple-access system and the hard limiting of non-binary sequences.

I. INTRODUCTION

This is the final report for Contract No. DA-31-124-ARO-D-253, which was undertaken with the U.S. Army Research Office (Durham) and is entitled "Finite Memory Nonlinear Filtering of Digital Sequences." The research was primarily concerned with the autocorrelation and cross-correlation properties of certain classes of binary and nonbinary digital sequences. The applications of these sequences to synchronization problems and the multiple access problem were also considered.

In Part I of the final report, a brief summary of the aims and results of this research is presented. The details of this investigation, however, are presented in Part II of the final report, which has also been submitted by Mr. Karl N. Levitt as a doctoral dissertation. Part I also contains a list of scientific personnel who contributed to this project, and a summary of a paper accepted for publication.

II. FACTUAL DATA

The initial aim of the research was to examine the effects of filtering a binary maximal length linear-shift register sequence of period L with the nonlinear filter shown in Figure 1. The parameter, γ, defined as the ratio of the length of the delay to the pulse duration T of the input sequence, was assumed to be a positive, rational number.

Prior to the initiation of this investigation, a computer simulation of this problem indicated that the autocorrelation function of the output sequences were consistently of a particular form. For example,

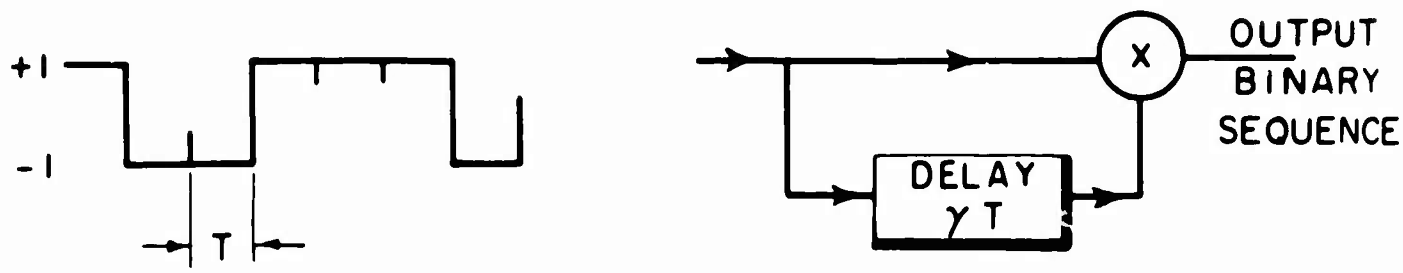


FIG. 1

FIG. 2

for delays of the form $\gamma = p + \frac{1}{2}$, $p = 1,2, \ldots, \frac{L-3}{2}$, the autocorrelation was always of the form shown in Figure 2, with two intermediate peaks whose heights depended only upon the period of the input sequence (and not upon the delay), but whose location depended both upon the delay and upon the generator polynomial of the input maximal length sequence.

Initially, we were concerned with analytically deriving the form of the autocorrelation function of the output sequence. Very early in this work, it was realized that owing to the "shift and add" property of maximal length binary sequences, this problem was a special case of the more general problem of the arbitrary n-fold interleaving of the same binary maximal length sequence[1] of length L. For this general problem, it was shown that the autocorrelation function is similar to that of the original sequence, except for the addition of n(n-1) intermediate peaks of height $\frac{L-n+1}{nL}$. A number of these intermediate peaks can occur at the same location to produce fewer peaks of larger amplitude. A formula for calculating the location of these minor peaks was also derived.[2]

If the normalized delay, γ, of the filter in Figure 1 is written in the form

$$\gamma = p + \frac{i}{n} \quad ,$$

where p, i, and n are integers and $0 \leq i \leq n-1$, then n is the number of interleavings required in the more general analysis. Thus, for $\gamma = p + \frac{1}{2}$, we have a 2-fold interleaving of the maximal length sequence leading to $n(n-1) = 2$ intermediate peaks of height $\frac{L-n+1}{nL} = \frac{1}{2} - \frac{1}{2L}$

as found in the computer simulation. The formula for peak locations also checked with the computer results.

The interleaving technique was utilized to synthesize sequences with other interesting autocorrelation functions. For example, a particular method of the 7-fold interleaving of a length 15 maximal length sequence led to the autocorrelation function given in Figure 3. Sequences of this type may have use for rapid synchronization in a high signal-to-noise ratio environment where the height of the side-lobe peak identifies the magnitude of the synchronization error.[3] The probability of false synchronization was evaluated for these sequences and compared to results obtained for maximal length sequences. Sequences with small side-lobes were synthesized using another interleaving method leading to "almost" two-level autocorrelation functions. An example of the resulting autocorrelation functions is given in Figure 4, which was obtained by a particular 8-fold interleaving of a length 7 maximal length sequence. Interleaved sequences can be obtained by a generalization of the nonlinear filtering originally studied where now many delay networks are required.[4]

Another major problem considered in this research is the cross-correlation of various digital sequences. This problem arises when many sequences are simultaneously transmitted over a common channel (such as in a multiple access communications system) and synchronous reception is utilized at the receiver. It was found that[5] particular sets of basis functions for the Walsh functions had the property that the cyclic cross-correlation between any pair of sequences in the set was identically zero for all time shifts.[6] The cyclic cross-correlation between other sets of

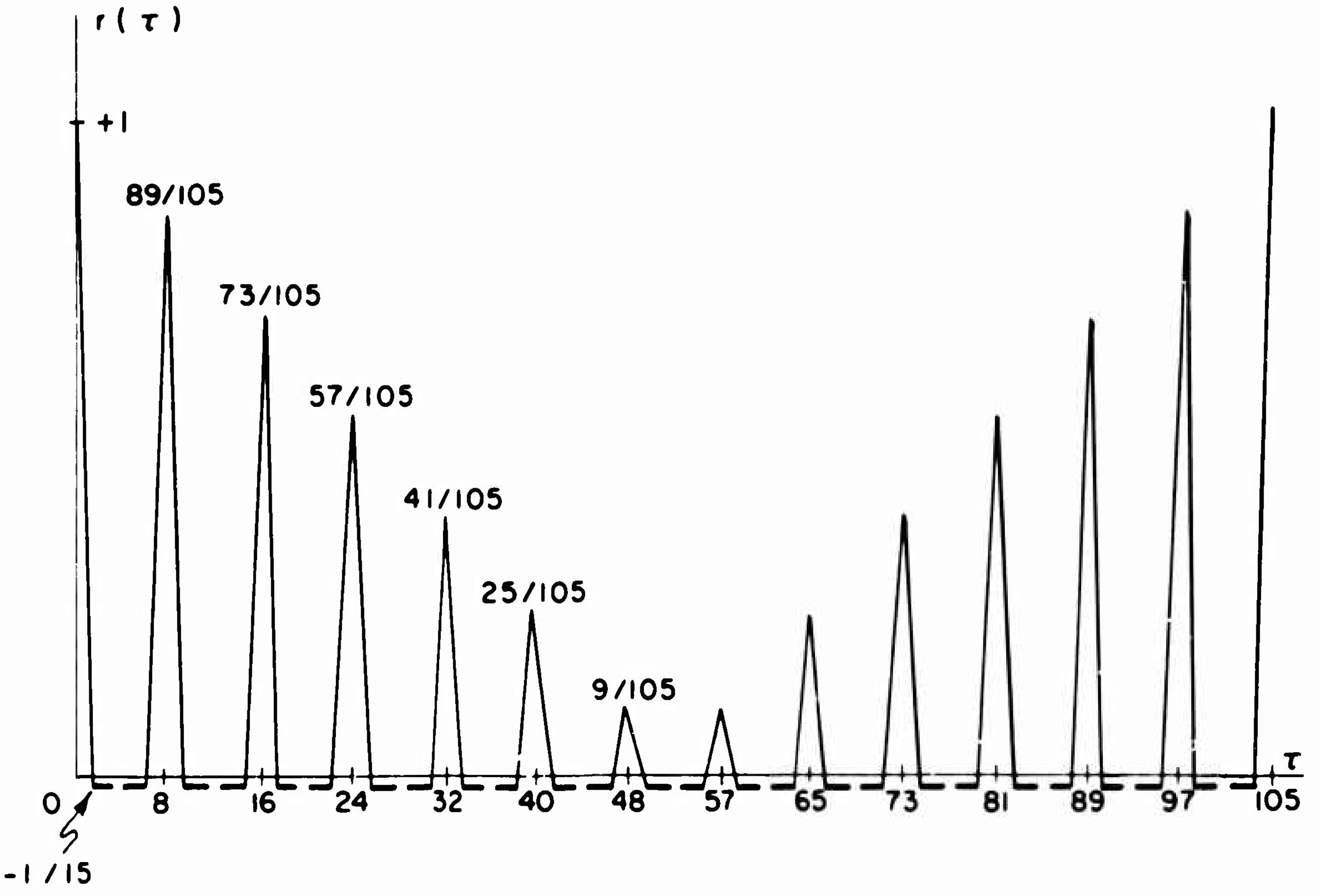


FIG. 3

DIFFERENT PEAK AUTOCORRELATION FUNCTION OF 7-FOLD INTERLEAVING OF LENGTH 15 MAXIMAL LENGTH SEQUENCE sequences were also investigated, e.g. sequences obtained by interleaving maximal length sequences. Both binary and nonbinary sequences were considered in this problem.

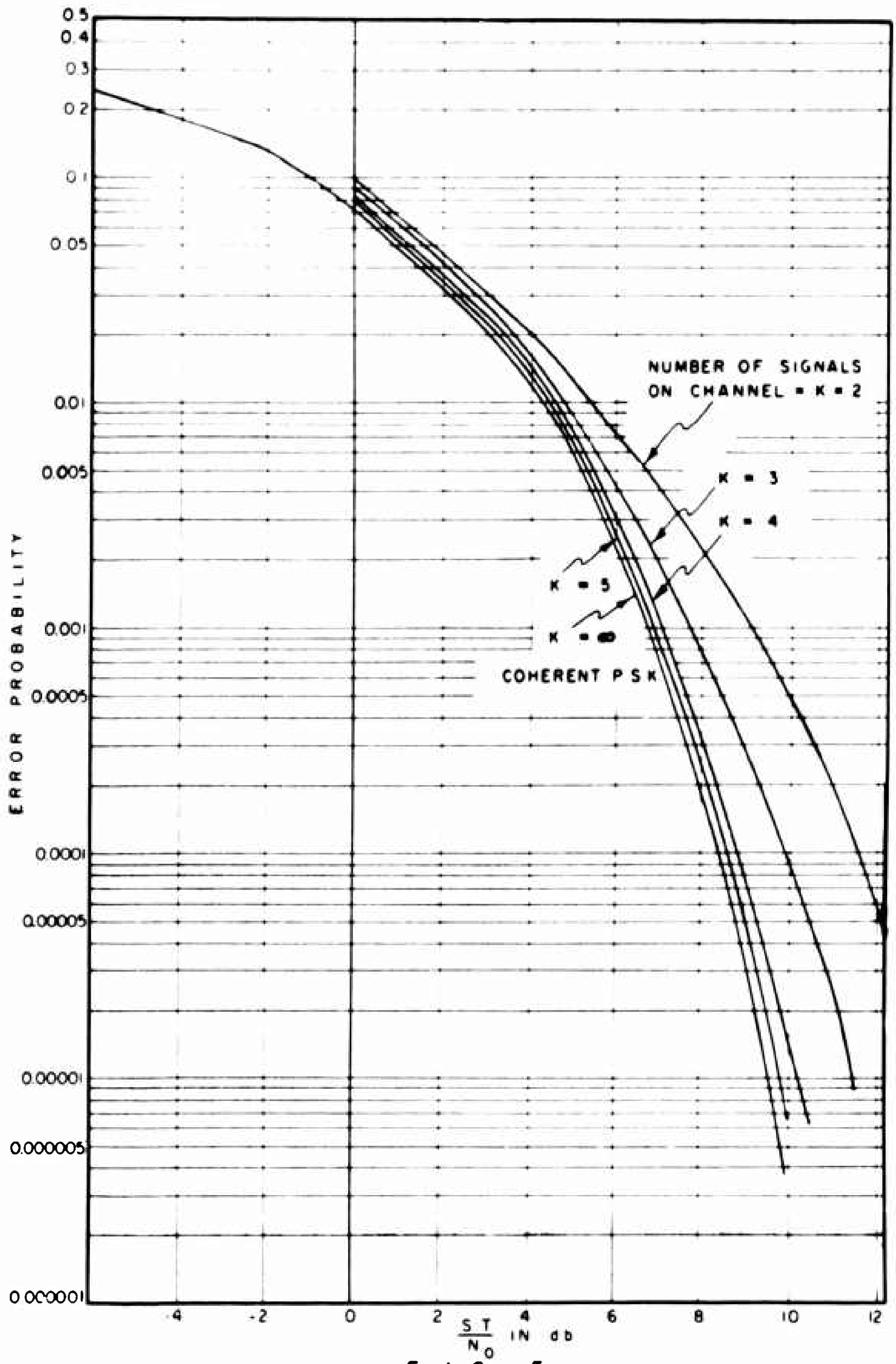
When information is to be transmitted using a binary sequence as a carrier, a common method is to transmit either the binary sequence or its complement. Thus, for the case where many carriers share a common channel, it is important to know the cross-correlation between any sequence and another sequence shifted in time where all digits prior to the time shift are complemented and all digits after the time shift are uncomplemented. The cyclic "flipped" cross-correlation has been derived for:

a. a set of basic Walsh functions
b. randomly chosen binary sequences
c. sequences based upon Bose-Chaudhuri codes
d. a set of sinusoids with frequencies which are multiples of some fixed frequency

The cyclic "flipped" cross-correlation was then used to determine the probability of error in transmission for any given channel. A typical result for the case of Walsh functions is given in Figure 5. Here the probability of error for the channel with carrier 101010 ... is given in terms of the energy per bit-to-noise power density ratio for various numbers of signals simultaneously sharing the channel.[7]

Other research topics considered during this investigation include

1. Use of cyclically P-orthogonal sequences for a plurality logic multiplexing system[8]
2. Nonbinary Hadamard matrices[9]



F I G. 5

PROBABILITY OF ERROR VS SIGNAL ENERGY (PER BIT)/
NOISE POWER DENSITY WHEN CORRELATING WITH SIGNAL S_0
(1010101010 ······)

CURVE FOR K = 2 DERIVED FROM CLOSED FORM SOLUTION
CURVE FOR K = 3 DERIVED BY COMPUTER
CURVE FOR K = 4,5 DERIVED BY ASSYMPTOTIC METHOD

It is believed that the research outlined in the original proposal has been carried to a successful conclusion. In addition, many other research problems associated with this work were also solved.

III. SCIENTIFIC PERSONNEL

J.K. Wolf (Ph.D.)	Associate Professor of Electrical Engineering Principal Investigator
K.N. Levitt (M.E.E.)	Research Assistant

Mr. Levitt has submitted a doctoral dissertation, entitled "Correlation Properties of Multi-Level Cyclic Sequences" (which makes up Part II of this Final Report), based upon research conducted under this contract.

IV. PUBLICATIONS

The following is an abstract of a paper which will be presented at the National Electronics Conference, Chicago, Illinois, on October 25, 26, 27, 1965. Other papers are presently being prepared for publication.

ON THE INTERLEAVING OF TWO LEVEL PERIODIC BINARY SEQUENCES

Karl N. Levitt and Jack K. Wolf

ABSTRACT

Periodic binary sequences are considered which are obtained by interleaving n identical, known, two-level sequences of length L. It is shown that the autocorrelation function of the resultant sequence is similar to that of the original sequence except for the addition of n(n-1) intermediate minor peaks of height $\frac{L - n + 1}{nL}$. A number of peaks can occur at the same location to produce fewer larger peaks. An algorithm for calculating the location of the minor peaks is given. Examples of interleaved sequences exhibiting autocorrelation functions which are "almost" two level, and autocorrelation functions with (n-1) peaks of different amplitudes are presented.

REFERENCES

1. By the n-fold interleaving of the sequence $a_0 a_1 \ldots a_{L-1}$, we mean a sequence of the form $a_0 a_{i_1} a_{i_2} \ldots a_{i_{n-1}} a_1 a_{i_1+1} a_{i_2+1} \ldots a_{i_{n-1}+1} \ldots a_{L-1} a_{i_1-1} a_{i_2-1} \ldots a_{i_{n-1}-1}$. The choice of the parameters $i_1, i_2, \ldots, i_{n-1}$, called the interleaving constants, determine the form of the autocorrelation of the resultant sequence.
2. The location of the n(n-1) minor peaks are given in terms of the interleaving constants in Chapter 3 of Part II.
3. See Chapter 3 of Part II. Also considered in this chapter was the interleaving of a maximal length sequence with the complement of the sequence. This leads to autocorrelation dunctions with both positive and negative peaks.
4. See Chapter 3 of Part II.
5. See Chapter 4 of Part II.
6. Interleaved sequences were also exhibited whose cyclic cross-correlations were identically zero for all time shifts.
7. See Chapter 5 of Part II for appropriate assumptions and analysis.
8. See Chapter 6 of Part II.
9. See Chapter 4 of Part II.

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