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ON THE CALCULATION OF THE SIMPLEST RAMJET-TYPE COMBUSTOR

By

K. P. Vlasov, Candidate of Technical Sciences

Translation

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This report is a translation of the following article:

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The calculation of the processes taking place in ramjet combustors is a complex problem.

Both here ([5] and [7]) and abroad ([1], [12], and [13]) attempts have been made to develop such a calculation method.

Usually, the position of the flame, the pressure drop, and the change in completeness of combustion along a combustor with a flame propagating from a point-ignition source are determined. In a number of works (e.g., [7]), this problem is worked out for a flame front of finite width.

The most precise mathematical solution has been obtained by A. V. Talantov, who worked out the problem by introducing, as did other authors, a number of simplifications (a homogeneous mixture, point-ignition source, constant parameters of turbulence across and along the combustor, etc.). However, these simplifications do not make it possible completely to overcome the following difficulties.

(1) Closing the system of basic equations, which is necessary to obtain the solution, requires the application of auxiliary conditions. These are not always chosen on sufficiently substantiated grounds.

(2) The precise dependence of the velocity of turbulent flame propagation on the initial parameters (u', u_H, etc.) is not known.

(3) There is no uniform view about the way in which the width of the combustion zone of a turbulent flame (or combustion time) should be determined. Experimental data on the determination of the width of the combustion zone are few and contradictory.

In this connection, it is felt that the correct solution of the problem must be found. This is the purpose of the present work.

1. Survey of the Literature

The solution of the problem of flame propagation in a cylindrical tube with ignition from a point source was attempted by Ya. B. Zel'dovich, G. Tzyan [H. Tsien?], G. I. Taganov, A. C. Scurlock, J. Fabry, etc. These authors attempted to solve the problem for a laminar flame (the thickness of flame front being infinitesimal and the velocity of flame propagation low in comparison with that of the incident flow). In other words, the flame was considered as a zone of combustion products separated from the fresh mixture by an infinitely thin surface (the flame front) at which heat is momentarily emitted as a result of the combustion reaction; the temperature then rises abruptly by several times.

Such a simplification is too unreal. In practice the width of the combustion zone may be substantial and must not be disregarded (see [4] and [8]).

Such a problem was worked out for a combustion zone of finite width by B. P. Skotnikov and, later, by A. V. Talantov [7].

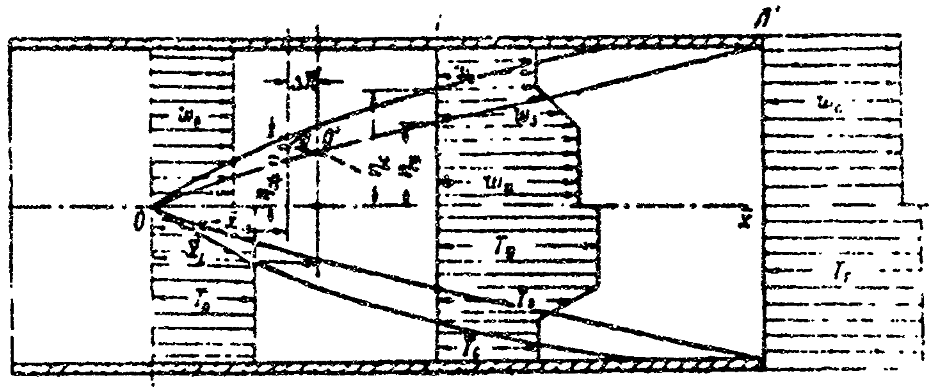

Fig. 1. Diagram of a flame in a tube

In calculating a combustor the problem is usually stated as follows. The combustor has a constant cross section. The mixture is ignited at a point on the axis of the flow. Before entering the combustor, the flow is one-dimensional. The static pressure at the combustor's cross section is assumed to be constant (Fig. 1). The laws governing changes of temperature and velocity in the combustion zone in the transverse direction are considered to be given. It is necessary to find a method for calculating all the parameters of the flow in an arbitrary cross section i. The following values are determined: the velocities of the fresh mixture (w_{c}) and of the combustion products ($w_{\text{п}}$), the temperatures of the fresh mixture (T_{c}) and of the combustion products ($T_{\text{п}}$), pressure (p_x), the ordinate of the initial flame front (y_{c}), and the ordinate of the end of combustion zone ($y_{\text{п}}$). The equations given below are used for the solution*.

The energy equation for a fresh-mixture flow is

$$c_pT_0 + A\frac{w_0^2}{2g} = c_pT_c + A\frac{w_c^2}{2g},$$

where T_0 and w_0 are the temperature and flow velocity at the combustor

*All the basic equations absolutely necessary for any calculation of a combustor are given below.

entrance, respectively; or, after transformation, it is

$$\tau_c = 1 - \frac{k-1}{2} M_0^2 (u_c^2 - 1),$$

where

$$\tau_c = \frac{T_c}{T_0}; \quad u_c = \frac{w_c}{w_0};$$

M_0 is the Mach number at combustor entrance.

The equation for the adiabatic process is

$$\frac{T_c}{T_0} = \left(\frac{p_z}{p_0}\right)^{\frac{k-1}{k}},$$

or

$$\tau_c = \pi^{\frac{k-1}{k}}, \qquad (2)$$

where

$$\pi = \frac{p_z}{p_0}.$$

The energy equation for the flow of combustion products is

$$c_p T_0 + A\frac{w_0^2}{2g} + q = c_p T_n + A\frac{w_n^2}{2g},$$

or

$$\tau_n = \lambda_n - \frac{k-1}{2} M_0^2 (u_n^2 - 1). \qquad (3)$$

where

$$\tau_n = \frac{T_n}{T_0}; \quad u_n = \frac{w_n}{w_0}; \quad \lambda_n = 1 + \frac{q}{c_p T_0}.$$

The equation of conservation of mass for the entire flow is

$$\rho_0 w_0 F_0 = \rho_c w_c (F_0 - F_c) + \rho_n w_n F_n + \int_{F_n}^{F_c} \rho_s w_s \, dF, \qquad (4)$$

where ρ_0, ρ_C and ρ_Π are the densities of the fresh mixture and the combustion products, respectively [sic], in the initial cross section; ρ_s and w_s are the density and flow velocity of the mixture in the combustion zone; F_0 and F_Π are the cross-section areas of the combustor and of the flow of combustion products; and $F_C = F_0 - (F_\Pi + F_s)$ represents the area of the entire flow minus the flow of fresh mixture;

or

$$1 = \frac{\pi}{\tau_c} u_c (1 - f_c) + \frac{\pi}{\tau_n} u_n f_n + \pi \int_{f_n}^{f_c} \frac{u_3}{\tau_3} df, \qquad (5)$$

where

$$f_c = \frac{F_c}{F_0}; \quad f_n = \frac{F_n}{F_0}; \quad u_3 = \frac{w_3}{w_0}; \quad \tau_3 = \frac{T_3}{T_0}.$$

The equation of quantity of motion is

$$p_0 F_0 + \rho_0 w_0^2 F_0 = p_1 F_0 + \rho_c w_c^2 (F_0 - F_c) + \rho_n w_n^2 F_n + \int_{F_n}^{F_c} \rho_3 w_3^2 \, dF, \qquad (6)$$

or, after making simple transformations,

$$\frac{z}{2} + 1 = \frac{\pi}{\tau_c} u_c^2 (1 - f_c) + \frac{\pi}{\tau_n} u_n^2 f_n + \pi \int_{f_n}^{f_c} \frac{u_3^2}{\tau_3} df, \qquad (7)$$

where

$$z = \frac{p_0 - p_1}{\frac{\rho_0 w_0^2}{2}} = \frac{2}{k M_0^2} (1 - \pi).$$

The equation of conservation of mass of the fresh mixture is

$$\rho_{c(i-1)} (F_0 - F_{c(i-1)}) w_{c(i-1)} = \rho_{ci} (F_0 - F_{ci}) w_{ci} + \rho_{cp} u_T \Delta S, \qquad (8)$$

where ρ_{cp} is the average density in a portion between neighboring cross sections, and ΔS is the area of the flame front between those sections, or, in a dimensionless form,

$$\pi_{c(i-1)} \frac{u_{c(i-1)}}{\tau_{c(i-1)}} (1 - f_{c(i-1)}) = \pi_{ci} \frac{u_{ci}}{\tau_{ci}} (1 - f_{ci}) + \frac{1}{2} \left[\left(\frac{\pi}{\tau_s} \right)_i + \left(\frac{\pi_c}{\tau_s} \right)_i \right] \bar{u}_T \Delta \bar{S},$$ where

$\bar{u}_T = \frac{u_T}{w_0}$ и $\Delta \bar{S} = \frac{\Delta S}{F_0}$.

In order to solve this system of equations it is necessary to introduce additional conditions. In works [7] and [11] these conditions are not similar and, therefore, the final results differ.

In A. V. Talantov's work the equations needed are found from the condition of "equality of the required and available time for the streamtube." The available time, when the motion is along the line of flow OO' (see Fig. 1), is

$$\Delta t_p = \frac{\Delta x}{w_{cp}}.$$

The time required to complete the combustion process can be found when the law of burnout of mixture in the streamtube is known in terms of time:

$$\Delta t_n = t_n - t_{3(i-1)},$$

or

$$\frac{\Delta x}{w_{cp}} = t_n - t_{3(i-1)}, \qquad (10)$$

where Δx is the distance between neighboring cross sections, w_{cp} is the average speed of motion of the mixture between those sections, t_n is the total time of combustion of the mixture in the streamtube, and $t_{3}(i-1)$ is the time during which the mixture remains in the zone, beginning with the moment it crosses the initial boundary of the flame.

In addition, the author makes use of an auxiliary approximate condition first introduced by G. Tzyan [9]. Equations (1) and (3) are employed:

$$\frac{\tau_в}{\tau_c} = \frac{\lambda_в - \frac{k-1}{2} M_0^2 (\omega_p^2 - 1)}{1 - \frac{k-1}{2} M_0^2 (\omega_p^2 - 1)},$$

or approximately

$$\frac{\tau_в}{\tau_c} = \lambda_в. \qquad (11)$$

Finally, the portion Q_B of the heat liberated at any cross section as related to the total heat Q_H available in the mixture (burnout) can be found from the relation

$$r = \frac{Q_в}{Q_н} = \frac{\Sigma I_i - I_0}{Q_н}, \qquad (12)$$

where ΣI_i and I_0 are the total heat content in the mixture at cross section i and at the initial cross section, respectively.

After transformation, one obtains

$$r=\frac{1}{\lambda_z-1}\left\{\left[\frac{\pi}{\tau_c}u_c(1-f_c)+\frac{\pi}{\tau_n}u_n f_n-1\right]\left(1+\frac{k-1}{2}M_0^2\right)+\right.$$
$$\left.+\frac{\pi}{\tau_n}u_n f_n(\lambda_\kappa-1)+\pi\int_{f_n}^{f_c}u_z\,df+\frac{k-1}{2}M_0^2\pi\int_{f_n}^{f_c}\frac{u_z^3}{\tau_z}\,df\right\}. \qquad (13)$$

The system is closed when the laws of change of the parameters of the combustion zone are known and the integrals in the equations (5), (7), and (13) are evaluated. Talantov assumes that the law of change of velocity and temperature in the zone is linear.

The most questionable point in Talantov's calculation method is his introduction of a characteristic combustion time in a turbulent flow; the combustion time is selected on the basis of the author's own data. [8]

The accuracy of both the methods and the results of measuring $\tau_{гр}$ are debatable. This matter is discussed in detail below.

Virtually the same equations [(1), (4), (6), and (8)] are considered by B. P. Skotnikov.

The following additional conditions were used to solve the equations:

(1) The connection between u_c and $u_{п0}$ (where $u_{п0}$ is the velocity of combustion products along the combustion axis) was determined in the form of the relation

$$u_{п0}=\sqrt{1+\tau(u_c^2-1)},$$

where τ is the degree of preheating,

$$\frac{\rho_c}{\rho_п}=\tau;$$

(2) The law of change of the velocity profile along the cross section of combustor (and of the density profile) was defined as parabolic.

(3) The dependence $\delta_T = \eta_c - \eta_\Pi$ of the width of the combustion zone in a radial direction was taken in the form $\delta_T = f(\eta_c)$, for which experimental data were utilized.

Here η_c = the radius of the initial boundary of the flame;
η_Π = the radius of the boundary of the flame combustion products.

The calculation by B. P. Skotnikov is also not entirely correct. The definition $\delta_T = f(\eta_c)$ is incorrect, since δ_T depends not on η_c but on $\bar{x}$, which is the distance from the point of ignition of the fuel mixture. Therefore, the author's calculation data are true only for the single case where w, a, ε, and $\bar{x}$ are constant; changes in these or other parameters would also change the dependence $\delta_T = f(\eta_c)$.

In Yu. A. Shcherbina's work [11], the calculation of burnout behind one or several ignition sources is presented. On the basis of that calculation, it is possible to construct temperature fields at any distance from ignition sources. It is assumed that the flame front is oscillating irregularly in relation to some average position α, with some mean square deviation σ from that average position. The value of σ can be found by means of Taylor equations for given l, ε, and x.

At present, the value of u_T cannot be determined analytically, and, therefore, the author has found an empirical dependence, $u_T = f(u_H, w, \sigma)$. When u_T is known, it is possible to find the value $\alpha = f(x)$ and consequently to calculate the temperature profile and the completeness of combustion in a given cross section of the combustor. The following observations may be made regarding [Shcherbina's] work [11].

As a matter of fact, the author does not in any way take into account in her calculation the effect of the combustor walls on the development of combustion and the formation of the flame. Therefore, the calculation is true only for some particular cases for which the dependence $\alpha = f(\bar{x})$ was found.

Satisfactory agreement of calculation results with the data from experiments by other authors was obtained. However, all such experiments apply either to conditions in half-open combustors or to comparatively short combustors, where the wall effect is insignificant. Such agreement is not surprising, since the author employs the empirical dependence, $u_T = f(u_H, w, \sigma)$, derived from these same experiments. Actually, she considered the position of the flame in the combustor, $\eta_c = f(x)$, as being given, when in fact it must be found as a result of the calculation.

2. Preliminary Observations

Before turning to the description of the proposed method of calculating the combustor, we will discuss in more detail some characteristic features without which no calculation of any combustor could be carried out. Two such basic features are the velocity u_T of turbulent-flame propagation and the width δ_T of the combustion zone of a turbulent flame (or the value τ_{cr}, i.e., combustion time, analogous in meaning to δ_T).

At present, it is not possible to calculate theoretically the value of u_T. In a number of investigations, attempts have been made to determine experimentally the value of u_T in an open flow of homogeneous fuel mixtures. The question of how to determine u_T is a debatable matter (see [2], [3], [9], and [10]), and since the results of experimental determination of u_T depend on the method chosen they differ considerably with different authors. In cases where the dependences $u_T = f(u', u_H\ p)$, as found by various authors, are more or less similar [3], the absolute values of u_T differ considerably (thus u_T (1 to 6)u'). A change in the absolute value of u_T affects considerably the results of calculation of η_c and $r = f(\bar{x})$. Moreover, no one has ever measured u_T in any combustor. Therefore, in order to solve this problem, we made an attempt to determine the values of $\bar{u}_T$ by utilizing the results of the processing of experimental investigations of burnout in the combustor.

The matter of determining the δ or τ_{cr} is still more controversial.

In Talantov's work [8], there is a derivation of the theoretical dependence $\tau_{cr} = f(\ , w, w', u_H)$, and a comparison between this dependence [ic] and experimental results. The derivation of these theoretical dependences, as carried out by the author, is far from being self-evident, and the experimental determination of τ_{cr} gives rise to a number of objections.

The author calculates the combustion time in this way. He considers the combustion of an individual nucleus of fresh mixture, cut off by turbulent pulsation from the initial flame surface and surrounded by combustion products. This assumption is not obvious. Further, Talantov assumes that such a nucleus of fresh mixture starts burning on the surface with a speed at the initial moment of

$$u_u = u_n + K_{\mu 0} w'_0 \frac{l}{l_0},$$

where K_{μ_0} is a coefficient in which the effectiveness of pulsation-speed action is taken into account (the value of this coefficient is not determined), w'_0 and l_0 are the initial values of the pulsation speed and of the dimension of the source, respectively, and l is the running dimension of the nucleus, which burns with the speed $u_{н} = u_{н}$.

Also, no allowance is made for the increase in normal velocity due to the small dimensions of the source, i.e., the sharp curvature of its surface.

The author writes the equation of combustion in the form

$$dl = u_{н} d\tau$$

and after integration he arrives at the final form of the equation:

$$\tau_{н} = \frac{l_0}{w'_0} \ln\left(1 + \frac{w'_0}{u_{н}}\right);$$

it is assumed that $K_{\mu_0} = 1$.

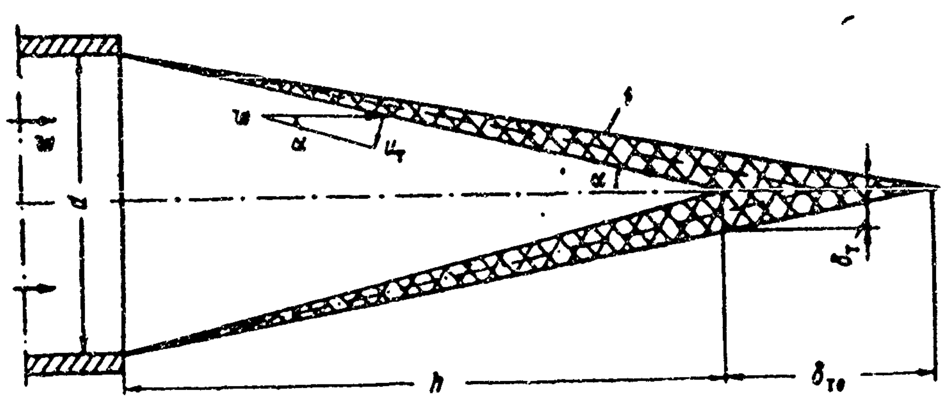

Fig. 2. Diagram of right circular cone of flame

1 - combustion zone.

The entire derivation is based on many *a priori* assumptions, and therefore the results cannot be considered as proved.

In order to determine τ_{cr}, the width of combustion zone δ_{T_0} was measured along the axis of the right circular cone of the flame. In a number of works (e.g., [4]), it is shown that the value of δ_{T_0} depends on the distance $\bar{x}$ to the ignition source; this is in no way accounted for by Talantov. Such a result can be determined from an elementary analysis (see Fig. 2), thus:

by definition $$\tau_{cr} = \frac{\delta_T}{w};$$

it is possible to consider approximately

$$\delta_T \approx l_t \frac{w}{u_t} \text{ and } \frac{u_t}{w} \approx \frac{d}{2x},$$

where x is the distance from the flame-cone apex to the base of the burner, d is the diameter of the burner, and δ_T is the radial width of the combustion zone. From this

$$\tau_{cr} \approx \frac{l_t}{u_t}.$$

When D = const. = u' l',

$$l_t = Ax \quad \text{for small } x; \qquad (*)$$
$$l_t = A\sqrt{2l_0 x} \quad \text{for large } x. \qquad (**)$$

Since u_T does not depend on x the value of τ_{cr} in both cases [(*) and (**)] depends on the distance to the ignition source. This point is not considered at all in Talantov's work, not to mention the fact that, generally, the characteristic of τ_{cr} is quite relative.

In reality, the combustion time in a turbulent flame is determined in the same way as that in a laminar flame:

$$\tau_{cr} = \frac{l}{u_H}.$$

According to the modern views regarding turbulent combustion, stated for example in [14], [15], and other works based on Shchelkin's physical model, the width of the combustion zone of a turbulent flame is defined by the depth of curvatures in the flame front under the action of turbulent pulsation. The curvatures and also the pulsation of the flame front are determined by the statistical value of the mean statistical displacement of the flame element $\sqrt{\overline{Y^2}}$, which, according to what was said above, results in the equality $\delta_T \simeq \sqrt{\overline{Y^2}}$.

If no allowance is made for the effect of normal velocity and autoturbulization (as a result of smoothing effect, normal velocity reduces the quantity $(\overline{Y^2})^{1/2}$; this turns out to be justified in a number of cases), the quantity $(\overline{Y^2})^{1/2}$, can be calculated theoretically:

$$(\overline{Y^2})^{1/2} = \varepsilon x \text{ for } x \leq \frac{l_0}{\varepsilon}. \qquad (14)$$

$$(\overline{Y^2})^{1/2} = \sqrt{2 l_0 x} \text{ for } x > \frac{l_0}{\varepsilon}. \qquad (15)$$

Thus, to determine the value $\delta_T \sim (\overline{Y^2})^{1/2}$, statistical processing of a great number of instantaneous positions of the flame front is required.

In practice, however, the determination of the width of the combustion zone is usually carried out by measuring flame temperature with thermocouples or by other inertial measurement methods.

The data obtained by means of ionization-type pickups along the width of the combustion zone may be of definite interest because such a measurement method is quite sensitive and makes it possible to register individual flame oscillations.

In [4], the width of the combustion zone was determined by ionization-type pickups rapidly moving across the flame (Fig. 3) in an open flow and in a cylindrical combustor. The speed with which the flame was crossed amounted to about 0.5 m/sec, while the velocity of the incident flow was $\geq$ 50 m/sec. Actually, not an instantaneous, but an average-time picture of the distribution of ionization current in the cross section of the flame was taken in this way.

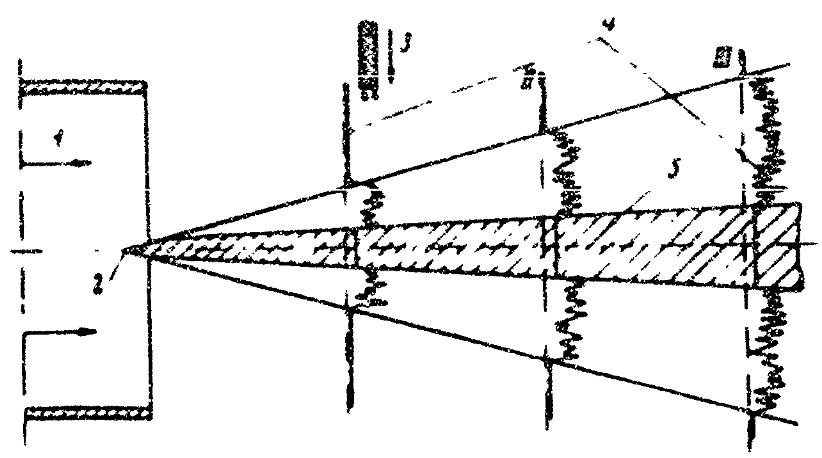

Fig. 3. Determination of flame boundaries by ionization pickups

1 - homogeneous gasoline-air mixture; 2 - mixture-igniting source; 3 - direction of motion of an ionization pickup across the flame, 4 - typical pictures of the distribution of ionization current, 5 - products of complete combustion of the mixture

It is known that in the region of chemical transformations, e.g., of the laminar flame, an increased ionization can be observed; it is just this phenomenon which makes it possible to separate the region of chemical transformations from that of the products of complete combustion in a turbulent flame.

The width δ_{T_u} of the combustion zone (along the radius of flame), determined in [4] on the basis of relations from the probability theory, should (allowing ~1% measurement error) amount to

$$\delta_{T_u} \approx 4.6\left(\overline{Y^2}\right)^{1/2}. \qquad (16)$$

In order to verify the correctness of such a relation, the results of experiments from [4] were processed in the form of the relation

$$\bar{\delta}_T = f(\bar{x}), \text{ where}$$

$$\bar{\delta}_T = \frac{\delta_{T_u}}{4.6}, \text{ a } \bar{x} = \frac{2x}{d}.$$

On the other hand, relations (14) and (15) are known.

As seen from Fig. 4, the quantity $\delta_T/4.6$ found and the values of $(\bar{y})^{1/2}$ calculated from equations (14) and (15) practically coincide. The velocity of the incident flow, the fuel-mixture composition, and therefore u_H, did not affect the value of δ_T. Also, the value δ_T did not change as a result of the specific conditions under which it was determined — in an open flow or in a cylindrical combustor. The results obtained have also been confirmed by the data in [11].

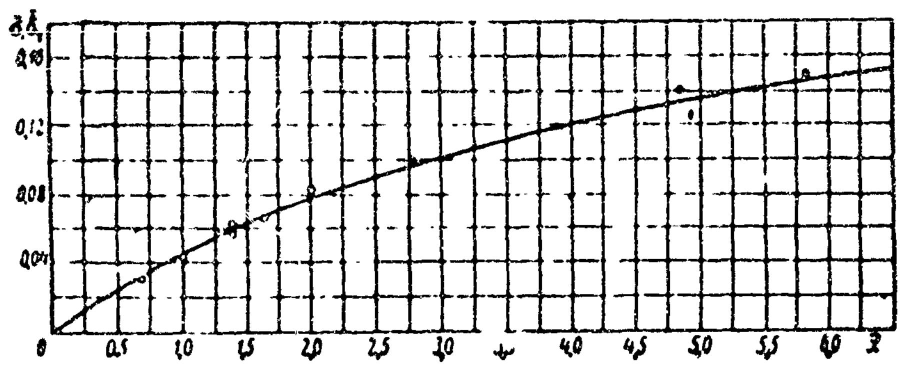

Fig. 4. The dependence of the value of the mean square displacement of the flame on the relative distance to the ignition source

——— - calculated values of the mean square displacement σ; O - experimental values of $\bar{\delta}_T$ found in open-flow installations (W = 35 to 50 m/sec and α = 0.5 to 1.3); ● - values of $\bar{\delta}_T$ in cylindrical combustor 145 mm in diameter (w = 35 to 75 m/sec and α = 0.7 to 1.3)

3. Proposed Calculation Method

The foregoing discussion indicates that, in principle, the calculation of a combustor is feasible. As was done in a number of the works discussed above, the equations (1), (4), (6), and (8) can be used as a basis. Also, G. Tzyan's condition [9] can be used, as was done by Talantov, to close the system of equations. Finally, the last equation needed for the calculation is the relation

$$\delta_\tau \sim (\overline{Y^2}) = f(\overline{x})$$

(see the equations (14) and (15)).

To calculate the position of the boundaries of the flame and of the burnout, one should know the values of changes in velocity and temperature in the cross section of the flame rather than the values δ_T or τ_{cr}. As the processing of experimental results shows, the temperature profile along the cross section of the flame is quite complicated (Fig. 5). In [11] it is shown that the temperature profile can be determined by means of the relationship

$$\rho_2 = \frac{\overline{T} - T_0}{T_{max} - T_0} = \frac{1}{2}[1 - \Phi(t)],$$

where

$$t = \frac{y - a}{\sigma};$$

$\Phi(t)$ is a Gaussian integral obtained from the table.

However, such a law of temperature change makes it impossible to integrate equation (6); (a numerical integration results in excessively cumbrous equations).

Analysis of the existing experimental material makes it possible to find a more simplified solution. The law of temperature change is introduced in a linear form; then, the entire change in temperature from T_0 to T_{max} takes place in the zone at the depth

$$\delta_\tau = 3\sigma \quad (\sigma = \sqrt{\overline{Y^2}}).$$

The coefficient "3" is found as a result of processing many experiments (see Fig. 4). In other works, e.g. [6], the law of temperature and velocity changes was also assumed to be linear, but at the depth of ~ 6σ).

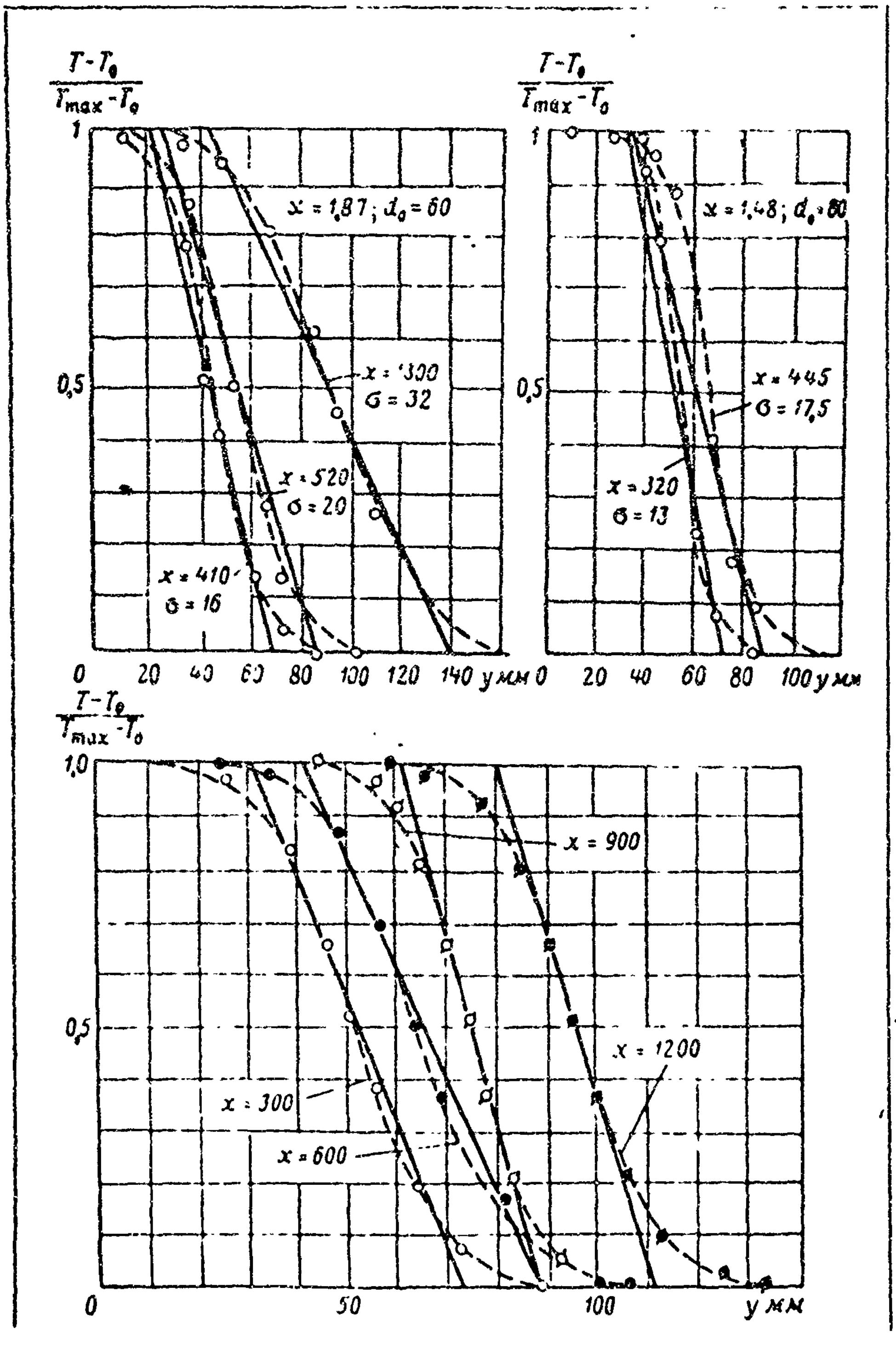

Fig. 5. Experimental and theoretical dimensionless temperature profiles behind a single stabilizer. (A linear law of temperature change is assumed in this work: solid lines.)

Thus, we assume $\tau_3 = \frac{\eta_i - \eta_n}{\eta_c - \eta_n}(\tau_c - \tau_n) + \tau_n$.

The change of velocity with δ_T is also assumed to be linear:

$$u_3 = \frac{\eta_i - \eta_n}{\eta_c - \eta_n}(u_c - u_n) + u_n.$$

After all simplifications, the system of equations for determining $\eta_c = f(\overline{x})$ appears in the form:

$$\tau_n = \lambda_n \tau_c; \qquad (17)$$

$$u_c = \sqrt{\frac{1 - \tau_c}{\frac{k-1}{2} M_0^2} + 1}; \qquad (18)$$

$$u_n = \sqrt{\frac{\lambda_n - \tau_n}{\frac{k-1}{2} M_0^2} + 1}; \qquad (19)$$

$$\pi = \tau_c^{k/(k-1)}; \qquad (20)$$

$$\delta_T = 3\sigma = f(x); \quad 3\sigma = \eta_c - \eta_n; \qquad (21)$$

$$\Delta x = \sqrt{\left\{\frac{2\left[\left(\frac{\pi}{\tau_{c(i-1)}}\right)(1 - \eta^2_{c(i-1)}) u_{c(i-1)} - \left(\frac{\pi}{\tau_{ci}}\right)(1 - \eta^2_{ci}) u_{ci}\right]}{\left[\left(\frac{\pi}{\tau_{c(i-1)}}\right) + \left(\frac{\pi}{\tau_{ci}}\right)\right](\tau_{c(i-1)} + \tau_{ci})\,\overline{u}_T}\right\}^2 - (\eta_{ci} - \eta_{c(i-1)})^2}, \qquad (22)$$

$$\eta_{ci} = -\frac{P}{2A} + \sqrt{\frac{P^2}{4A^2} - \frac{Q}{A}}, \qquad (23)$$

where

$$P = \frac{2\sigma}{\tau_c - \tau_n}\left(u_c - u_n \frac{\tau_c}{\tau_n}\right);$$

$$A = \frac{u_n}{\tau_n} - \frac{u_c}{\tau_c};$$

$$Q = \frac{u_c}{\tau_c} - \frac{1}{\pi}.$$

It is necessary to find: τ_c, τ_n, u_c, u_n, π, δ, η_c, and $\Delta\overline{x}$.

The known values are: M_0, λ_n, k, $\overline{u}_T$, w_0, and u_n.

There are eight unknown values and seven equations; however, one of the unknowns is an argument (e.g., τ_c or $\Delta\bar{x}$). Therefore, there is a sufficient number of equations to find the solution.

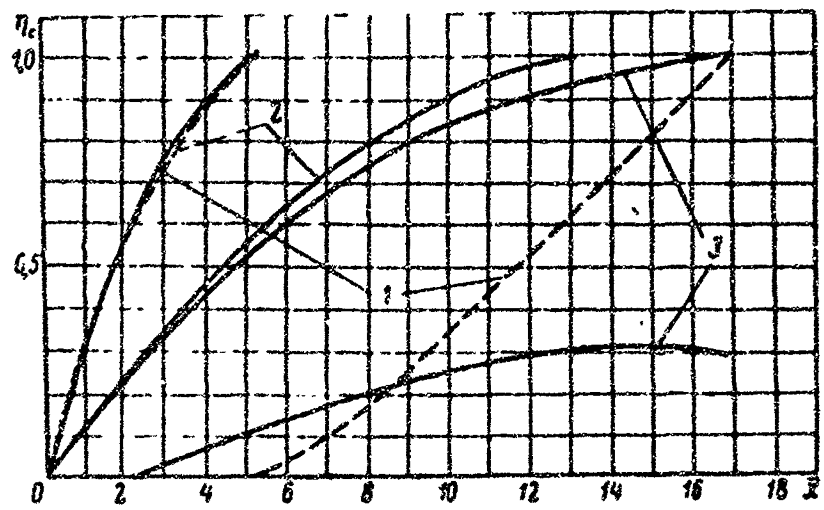

Fig. 6. Positions of flame boundaries — according to Talantov's calculation (1) and according to the calculation by the proposed method (2). [3 - calculation results by the proposed method with u_T = 2.26 m/sec; (the curves "2" were found for w = 50 m/sec, u_T = 10 m/sec, and T_0 = 288° abs.)]

These equations make it possible to carry out the calculation in stages, from one cross section to the next, by the method of successive approximations.

To compare this calculation method with that of Talantov, the example given in [9] was calculated for the following initial conditions: w = 50 m/sec, u_T = 10 m/sec, and T_0 = 288° abs. It should be noted that the value chosen for u_T in the example is considerably elevated. Under ordinary conditions at ε = 0.05, the values of u_T are, as will become evident, noticeably smaller than those given. The results of such a calculation are shown in Fig. 6. It can be seen that, as it was to be expected, the initial positions of the flame front coincide quite closely for both calculation methods; however, the final position of the front differs considerably from the data in Talantov's calculation. For a more correct comparison of both methods, the fact that the values of u_T used by Talantov differ considerably from the real one should be taken into account. This is due to the fact that u_T was determined on the basis of the front boundary of the flame and that such a determination is incorrect (see [2]). In reality, as follows from the calculation based on our experimental data (which should give a true amount for u_T), u_T = 2.26 m/sec for ε = 0.05 and α = 1.2. Under the same conditions and even for somewhat smaller values of u_H, Talantov's equation [7] gives

$$u_T = 5.3(u')^{0.7}(u_H)^{0.3} = 7.7 \text{ m/sec}.$$

When the value u_T = 2.26 m/sec is used and the position of flame boundaries is calculated by the proposed method and then compared with the calculation according to Talantov, a considerable discrepancy in the results is seen (see Fig. 6).

In order to calculate the total length of the combustor, it is necessary to find a method for determining the coordinates of n_Π in the combustor cross sections where $\eta_C = 0$.

In this case, it is recommended that the following self-evident relation be used:

$$L_{з.г} = \delta_т \frac{u_{cp}}{u_т} = 3\sigma \frac{u_{cp}^2}{u_т u_0},$$

where $L_{3.\Gamma}$ is the width of the combustion zone along the line of current and u_{cp} is the average flow velocity in the center of the combustion zone $L_{3.\Gamma}$;

$$u_{cp} = \frac{u_{c0} + u_{n.к}}{2}.$$

In many cases, it is possible to assume approximately $u_{cp} \approx u_H$; then, the final expression is

$$L_{з.г} = 3\sigma \frac{u_н^2}{u_т u_0}.$$

In a number of approximate calculations, when the shape of the curve $\eta_C = f(x)$ can be disregarded, the calculation of the position of flame boundaries [$\eta_C = f(x)$] becomes substantially simpler. In such a case the total length of the cold portion of the combustor (to the point where the flame touches the combustor walls, i.e., at $\eta_C = 1.0$) remains practically constant. In this case, the entire calculation can be carried out [by] assuming the flame front to be infinitely thin, i.e., by finding the curve $\eta_C = f(x)$, and then finding the value $\delta_T = 3\sigma$ for each running value of x. The calculation is carried out as follows.

Given: M_0, k, λ_Π, τ
To be found: τ_Π, u_C, u_Π, η_C

$$\tau_п = \lambda_п \tau_c; \quad \lambda_п = 1 + \frac{q_п}{c_p T_0};$$

$$u_c = \sqrt{\frac{1 - \tau_c}{\frac{k-1}{2} M_0^2} + 1}; \quad u_п = \sqrt{\frac{\lambda_п - \tau_п}{\frac{k-1}{2} M_0^2} + 1};$$

$$\pi = \tau_c^{k/k-1}; \quad \eta_c = \sqrt{\frac{1 - \pi/\tau_c \cdot u_c}{\pi(u_п/\tau_п - u_c/\tau_c)}}.$$

In connection with the fact that the data on absolute values of u_T are of great interest, it is interesting to calculate the u_T values by using the usual research results regarding combustors (e.g., on the basis of the change in static pressure along the combustor, $z = f(x)$). It is not difficult to construct such a calculation system by using the equations given above. The calculation sequence is as follows.

$$\pi = 1 - \frac{k\sigma M_0^2}{2}; \qquad (24)$$

$$\tau_c = \pi^{\frac{k-1}{k}}; \qquad (25)$$

$$\tau_n = \lambda_n \tau_c; \qquad (26)$$

$$u_c = \sqrt{\frac{1 - \tau_c}{\frac{k-1}{2} M_0^2} + 1}; \qquad (27)$$

$$u_n = \sqrt{\frac{\lambda_n - \tau_n}{\frac{k-1}{2} M_0^2} + 1}; \qquad (28)$$

$$\bar{u}_T = \sqrt{\frac{\left\{2\left[\left(\frac{\pi}{\tau_{c1}}\right)\left(1 - \eta_{c1}^2\right) u_{c1} - \left(\frac{\pi}{\tau_{c2}}\right)\left(1 - \eta_{c2}^2\right) u_{c2}\right]\right\}^2}{\left\{\left[\left(\frac{\pi}{\tau_{c1}}\right) + \left(\frac{\pi}{\tau_{c2}}\right)\right]\right.\left(\eta_{c2} + \eta_{c1}\right)^2\left[\Delta x^2 + \left(\eta_{c2} - \eta_{c1}\right)^2\right]}}; \qquad (29)$$

$$\eta_c = -3\sigma p + \sqrt{9\sigma^2 p^2 Q}; \qquad (30)$$

$$p = \frac{u_c - u_n \frac{\tau_c}{\tau_n}}{(\tau_c - \tau_n)\left(\frac{u_n}{\tau_n} - \frac{u_c}{\tau_c}\right)};$$

$$Q = \frac{\pi \frac{u_c}{\tau} - 1}{\pi\left(\frac{u_n}{\tau_n} - \frac{u_c}{\tau_c}\right)}.$$

Let us now turn to the examination of experimental results of research on burnout in combustors.

In the present work, a combustor of the simplest type, 150 mm in diameter and with its hot portion 720 mm in length, was investigated. A conic stabilizer 12 mm in diameter was used as the ignition source and for stabilization. Static pressures were picked up along the combustor (see Fig. 7).

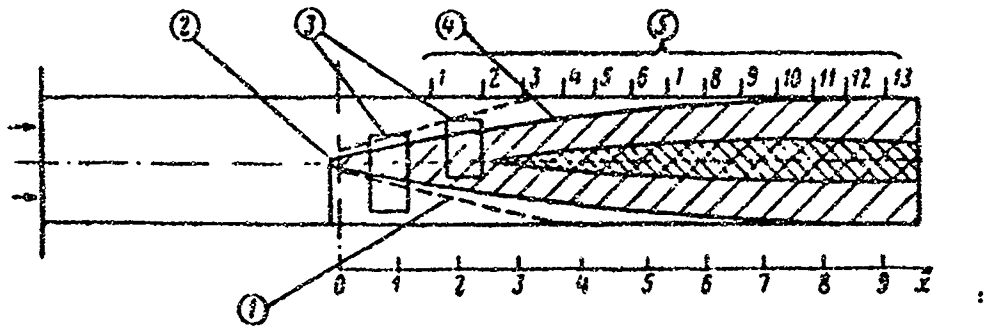

Fig. 7. Diagram of combustor measurement system and arrangement. 1 - Position of turbulization grating; 2 - flame stabilizer; 3 - transparent windows; 4 - flame; 5 - static pressure pickup points

The following conditions were made variable: the velocity of incident flow and mixture composition α. The experimental results were found in the form of the function $z = f(\bar{x})$ for various initial conditions.

Since the value

$$z = \frac{\Delta p}{\frac{\rho_0 u_e^2}{2}}$$

in a given cross section of combustor is known from experiments, it is possible to determine the running values of η_c, η_Π, u_T, etc. The values of Δx and σ are known (in this work, it was assumed that $\varepsilon = 0.05$; for these conditions, the value of σ is determined from Fig. 4).

The values of $\bar{u}_T$ were determined for individual portions Δx of the combustor, after which the mean value was calculated. The results of the calculations are shown in Figs. 8 and 9.

The values found for u_T were of the order 0.5 to 1.5 $\bar{u}'$; this agrees well with modern views about turbulent combustion (see [14] and [15]). The relationships found, $u_T \approx (u')^{o8 \text{ to } o8}$ and $u_T \approx u_H$ (in the range $\alpha \geqslant 0.9$), are in satisfactory agreement with known experimental results, which give $u_T = f(u', u_H)$ ([15]), taking into account calculation errors.

The fact that the effect of u' (through the influence of w) and u_H on the value of u_T is somewhat stronger than that in [15] can be explained by the effect of the ignition source. In the case of a small ignition source, the completeness of combustion close to the ignition source in the axis of flow, as is known, may be considerably less than

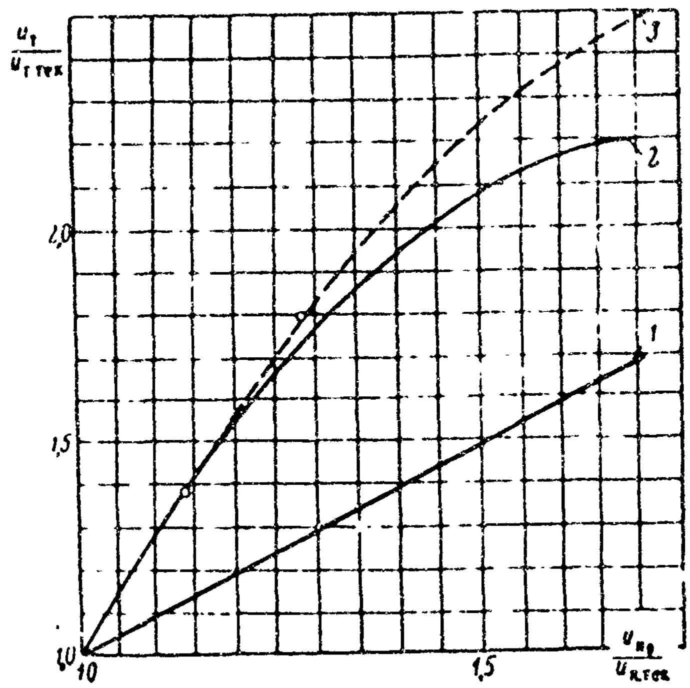

Fig. 8. Relative change in the velocity of turbulent combustion in relation to that of normal combustion

1 - Dependence $\frac{u_T}{u_{T.тек}} = \frac{u_H}{u_{H.тек}}$; 2 - dependence $\frac{u_T}{u_{T.тек}} = \frac{u_H}{u_{H.тек}} \times \frac{T_2}{T_{2тек}}$; 3 - dependence - calculation results from experimental data for w_o = 50 m/sec, α = 1 1.6 [TEK = running]

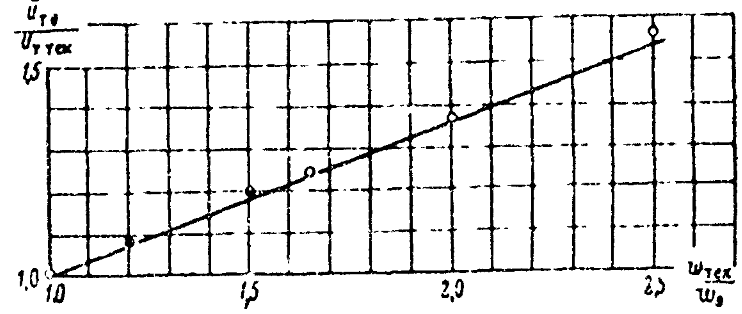

Fig. 9. Relative change in the velocity of turbulent combustion with increasing flow velocity

$u_T \sim w^n$; $\frac{u_{T w_0}}{u_{T w_{тек}}} = \left(\frac{w_{тек}}{w_0}\right)^{1-n}$, n 0,55, w_0 30 м/сек, $\bar{u}_{T w_0}$ = 0,390 the usual 0.95 to 0.98. In this case, the ignition of the fuel mixture does not start close to the stabilizer edges but further down the flow; this is not taken into account in the calculation.

It should be noted that since under the majority of operating conditions the flame front does not touch the combustor walls ($\eta_0 < 1.0$), corrections for expansion of combustion products were introduced when computing u_T during the calculation of $u_T = f(u_H)$, as was done in Yu. A. Shchërbina's work [11].

Thus, finally, in the first approximation it is possible to take

$$\bar{u}_T = u_{T0}\left(\frac{w}{w_0}\right)\left(\frac{\varepsilon}{\varepsilon_0}\right)^{0.6}\left(\frac{u_H}{u_{H0}}\right)\left(\frac{T_{max}}{T_{max0}}\right),$$

where $u_{T0} = 0.092$, $w_0 = 30$ m/sec, $u_{H0} = 81$ cm/sec, $T_{max0} = 2400°$ abs., and $\varepsilon_0 = 0.05$.

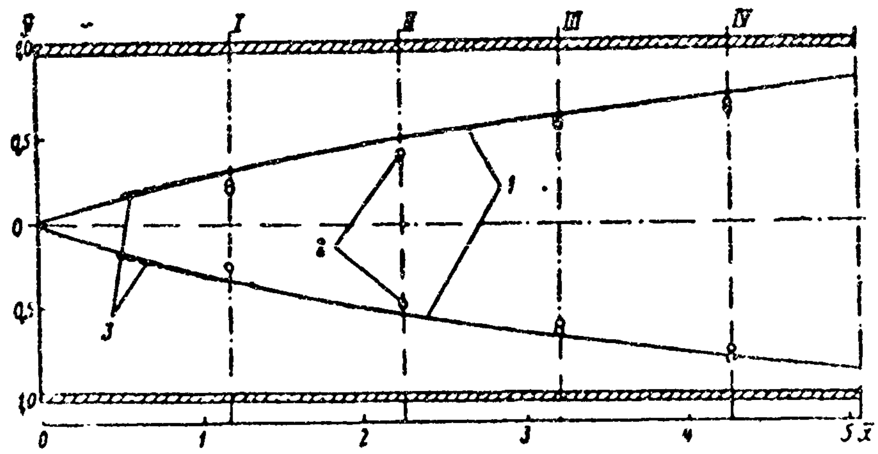

Fig. 10. Position of flame boundaries

1 - from calculation; 2(o) - experimental points determined by the ionization pickup method; 3 - boundaries found by direct photography

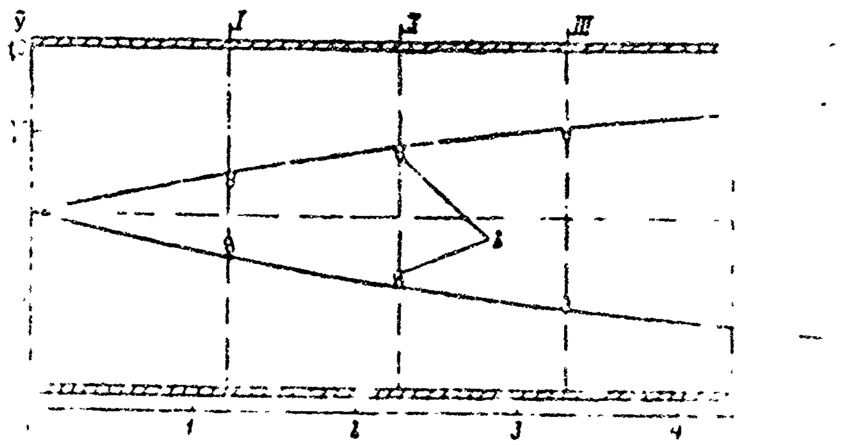

Fig. 11. Position of flame boundaries

1 - from calculation; 2(o) - experimental points determined by the ionization pickup method.

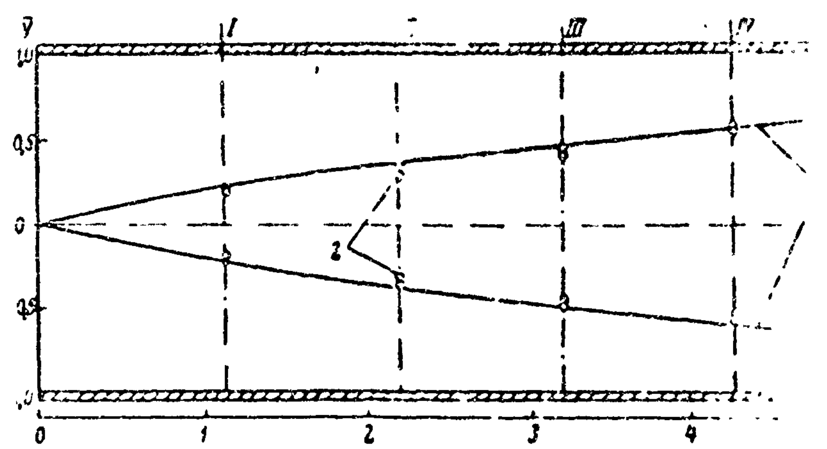

Fig. 12. Position of flame boundaries

1 - from calculation; 2(o) - experimental points determined by the ionization pickup method.

In the case of the flame touching combustion walls,

$$\frac{T_{max}}{T_{max0}} = 1.0$$

In addition, a comparison was made of the results of the determination of flame boundaries in a cylindrical combustor by the calculation method with those determined by the ionization pickup method [2], and also by photography through a window in the combustor. As seen from Figs. 10 to 12, good agreement with calculation data was found.

4. Conclusion

A calculation method for the simplest ramjet-type combustor is presented. The method consists in closing the system of the basic gas-dynamic equations (of fuel-mixture flow) within a cylindrical tube with a point-ignition source, by introducing an auxiliary condition which defines the dependence of the width of the combustion zone on the distance to the ignition source. The results of the calculation were compared with some experimental data and were found to be in good agreement with them.

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