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A PROLEGOMENON TO THE FORECASTING OF TRANSPORTATION DEVELOPMENT

FINAL REPORT

By

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August 1965

U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA

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ABSTRACT

The analysis proceeds with a review of the problem of forecasting transportation requirements. Several idiosyncratic models are examined and their properties specified. The forecasting of transportation stocks and commodity flows is discussed. Models of the demand for transportation services are implemented with empirical data. Graph theory as an interpretative frame of reference is introduced and the analysis moves to a detailed examination of network structure. Inter- and intra-nation network structures are compared. Codified route and commodity networks are factor analyzed and fundamental structures isolated. An attempt is made to simulate the morphological characteristics of transportation networks. Simulations of the Northern Ireland railway network are undertaken using nearest neighbor methods. Finally, the problems of planning and forecasting are reviewed in the light of the foregoing analyses. Criteria and objectives are discussed. An overall evaluation of available planning and forecasting capabilities is presented.

PREFACE

This is the Final Report of an investigation of problems of forecasting developments of transportation. The studies were performed at the Transportation Center at Northwestern University and were supported by the U.S. Army Aviation Materiel Laboratories, Fort Eustis, Virginia. The first study in the sequence of studies was supported under contract DA 44-177-TC-574. The subsequent studies were supported under contract DA 44-177-TC-685 (see references 1-18, at end).

This Final Report provides overviews and summaries of these studies, as well as a general synthesis. Some materials not reported previously have also been included.

The studies should be viewed as having produced several varieties of forecasting capability, rather than a single, composite predictive system. A collection of tools has been investigated, and that collection provides the capabilities. It is expected that researchers desiring to use or extend our work would select from the collection of tools those appropriate to their tasks. Our emphasis is upon capabilities, for we feel that there are many problems for which at least partial capabilities are available from our work. We have not completely solved any unique forecasting problem, of course.

The content of this Final Report represents a compromise. It was desired to review in detail the entire plan and substance of the work; yet it was also desired to highlight the structure of the work. These two objectives are somewhat conflicting, for emphasis upon structure skimps on detail and detail obscures structure. Another point of compromise was the desire to present materials on forecasting using historical trends as against the desire to discuss forecasting based upon likely planning. It is felt that both statements of the forecasting problem have merit. Forecasting using historical trends would seem to be of greatest short run value. In the long run, however, the planning emphasis should have greater value.

Research tasks were undertaken by the authors of the reports cited above. Credit for such values as the research may have goes to these individuals.

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I. INTRODUCTION, BACKGROUND, AND REVIEW

General Discussion

The materials in this chapter provide an introduction to research which has been recently undertaken in order to develop a forecasting capability for transportation developments on both national and regional levels. (Regions in this case may be either smaller than nations, or they may consist of sets of nations.) Despite the large number of forecasting studies available and, in particular, studies which could be classified as forecasts of transportation developments -- for instance, the work with highway needs studies in the United States -- the present study is broad and appears to be unique. Consequently, it would seem appropriate to provide a general background discussion prior to the review of work completed or underway. Some of the general topics which require discussion are: (1) the needs for and cases of forecasts, (2) the properties of transport systems that must be reflected in forecasting models, (3) the kinds of forecasts that may be undertaken, and (4) the close articulation which exists between forecasting work and planning, and between transportation development and the development of other aspects of the economy.

A review of the research work completed follows a general background discussion. The review provides orientation to the several studies that were undertaken as subtasks of the larger effort of developing a forecasting capability. There is no intent in this review to give detailed discussions of the several subtasks, for separate documents have been prepared from those studies. Also, portions of certain of those studies have been adapted for use in this present document. The review discussion is intended as an overview of the entire research effort and it is written in a style and at a level designed to complement the prior general background discussion.

Nature of Transportation Forecasts: Some Examples

An example can be used to introduce some of the problems encountered in attempting to forecast transportation developments and changes in the environment within which developments take place, and the example will introduce the self-fulfilling character of certain classes of forecasts. A highway development organization --

say, a state highway department that is charged with land acquisition, construction of pavement and structures, and highway maintenance -- may need projections of expected numbers of persons in the population, as well as the number of vehicles and how they will be used, in order to develop notions of needs for highway construction. The forecast in this instance is tied to certain planning considerations, and it is based upon parallel forecasts or projections of specific determinants of transport development. The transportation forecast produced here is a contingency forecast, "If population and number of vehicles are..., then needs for highway construction will be...". This example also reveals the presence of forecasting assumptions. It is assumed that the numbers of persons in the population, the numbers and uses of their vehicles, and needs for highway construction are related in certain stable ways. The notions of contingency and relationships illustrated here are central to the forecasting problem and will be elaborated upon later.

Some additional properties of forecasts may be illustrated through the examination of other forecasting situations. Those examined range from situations where there is little or no control possible over the outcome of the forecasts to those situations in which the forecasts become virtually self-fulfilling because the forecasting agency uses these forecasts as the basis of actual operating plans. The examples which follow are, of course, intended only to illustrate these points and not to provide an exhaustive listing of possible situations.

One specific situation which has resulted in a great deal of forecasting activity is that where equipment manufacturers require forecasts of the level of demand for certain types of transportation equipment. A case in point was the potential market for jet aircraft. This market was extensively explored by various aircraft manufacturers who made studies of travel and attempted to judge how, in the presence of certain economies resulting from jet operation, travel might be diverted from existing equipment to jet aircraft. In this type of forecasting, the forecasting agent is the equipment builder and the contingencies in the forecast are those which arise from the realization of certain economies resulting from specific levels of equipment utilization. It should be noted that the pricing and sales efforts of equipment manufacturers may affect the extent to which the forecasts hold.

Forecasts of investments in routes and other facilities provide still another example. In this instance, the forecasting agency may be a national highway department, or perhaps a national railroad or airways agency, and the problem is that of anticipating needs for route development. The forecasting agency might be a private organization -- say, an agency developing its construction schedule or estimating construction by competitors, or an ocean transportation firm attempting to anticipate demand for and the development of an ocean transport service. Investment in pipelines provides still another instance. The examples cited here are similar to the instances mentioned previously in that the forecasts provide a measure of the demand for transportation services, and the forecasts which are made of transport developments are to a certain extent self-fulfilling. A forecast is made and transportation facilities are estimated in the forecast.

Still another instance may be provided by consideration of a state or regional planning agency where, again, forecasts are undertaken by a development authority and the separation is not clear between forecasting to determine what developments may occur and forecasting to establish needs and consequent investment programs.

Nature of Forecasts

It should be clear from the examples above that it is difficult to speak of forecasts in the present context without introducing certain questions of the demand for transportation, and that forecasts may well be self-fulfilling in that the forecasting and development groups may be one and the same. Existing forecasting devices vary in the degree to which demand and supply relations are dealt with explicitly, forecasts and estimations of need are separated, and forecasts are self-fulfilling. This variability among forecasting situations makes a general discussion of forecasting difficult.

Forecasting the Decisions of Development Agencies

It was remarked earlier that the forecasting work represented by the present study is relatively unique. In contrast to the examples noted above, the present investigation is much more general in scope. It is intended to provide a general forecast capability which will be applicable to developments in small areas as well as nations and large regional groupings, and a capability that is not specialized in terms of a particular mode of transportation. There is still another difference between the present investigation and the work of other transportation forecast activities. Forecasts in the example above were made by development agencies, and contingencies in the forecasts related to matters such as expected rates of population growth. The broader capability which is under development here must include this type of contingency, but it must also include as contingencies questions of just what various development agencies might do in specific circumstances.

Constraints on Forecasts

It is clear that a number of constraints act to affect the quality of a forecast. Lack of data is, of course, a very real problem, especially for intercountry comparisons. The contingent character of many transportation forecasts provides still another problem. It is true that all forecasts are subject to contingencies.

But because of the interdependence between transportation and other sectors of the economy, the contingencies encountered in transport forecasting seem to be more difficult to handle than those in many other types of forecasts. One might estimate, for example, that if a certain level of output was reached, then transportation would expand to a certain related level. A difficulty is that expansion of transportation might be required to raise level of output, so the simple contingency is inadequate. One other practical constraint upon transport forecasts arises out of institutional structures because the possible courses of action of various development agencies must be taken into consideration and development agencies vary quite widely in purpose, structure, and degree of control exerted upon the situation. Furthermore, forecasts must range from the highly aggregative to the very detailed. Possible considerations range, for instance, from gross yearly expenditures on transportation services within a national economy through estimates of the demand for weekday passenger movement via air between two distinct cities.

As was mentioned before, an overriding constraint on what may be anticipated from forecasting work arises from the high degree of interdependence which exists between transport development and developments in other sectors of the economy. Current ability to deal with these high levels of interdependence is somewhat limited. This point may require some elaboration because what constitutes a successful forecast seems to depend upon a complex set of relations between the cost of making the forecast and the cost of errors arising from incorrect forecasts, as well as the extent to which these errors could be reduced through further expenditures on forecasting operations and data gathering.

When transportation forecasting is considered, the following statements seem to be reasonable. While something is known about the cost of forecasting, there is little or no knowledge of the costs induced by errors in forecasting or of the degree to which these errors might be reduced by increasing the amount of resources devoted to the forecasting activity. It is suspected that costs due to errors of forecasting are very high and also that improvements in the quality of the forecast would be difficult to obtain. This latter is due in part to the cost of obtaining improved data for forecasting. But even if better data were available, it might be difficult to utilize these in a more effective fashion.

Again, an important constraint is that of understanding and formally conceptualizing the high levels of interdependence which exists between transportation and the other sectors of the economy. In other words, even if better data were available, it would be difficult to use this information effectively since it is currently quite difficult to handle the complex problem of interrelationships in an efficient and effective fashion.

The problem is somewhat further complicated since it is not clear what degree of accuracy is needed or desired for the various kinds of forecasts, though it seems apparent that increased accuracy pays off in the long run. It is also not clear just how increased accuracy can be achieved in any given situation, given the difficult data and conceptual problems associated with the forecasting activity.

Forecasts as Self-Fulfilling Prophecies

It was pointed out earlier that forecasts are frequently tied into some type of action program. Forecast results are frequently given some sort of "needs" connotation, and the agency then proceeds to meet these needs via various planning and action programs. The question of how accurate the forecast is may not be answered satisfactorily under these circumstances. This point will be discussed again in the ensuing section which examines the nature of forecast models.

Forecasting Devices

Continuing in the vein of the first portion of this discussion, which was explanatory and expository, the problem of developing forecasting devices, or models, will now be examined.

Properties of Forecasting Models

It is easy to set down in principle those properties that the ideal forecasting model should possess, and these will be listed (c.f., Reference 19, pp. 11-31):

(1) Forecasting models should display the pertinent behavioral relations of the system under study. If it can be established, for instance, that transportation development in a region is some function of the rate and character of resource development, then such a functional relationship should explicitly appear in the forecast model. This property of the model specifies the type of data that will be used, the theoretical relationships that are assumed, and the kind of forecast produced.

(2) The forecast model should give accurate results. This requirement of accuracy presumes some method for verifying the accuracy of the model. The model which produces the statement "At some time there will be so many automobiles that all the world will be paved" is one that cannot be considered to be verifiable because there clearly is no way in which the accuracy of the forecast can be checked.

(3) The model should be as simple as possible. This third property seems to be quite reasonable, and is one that it

is easy to defend. Where two given models are of equal power but of different simplicity, then the simplest should be selected.

One difficulty that occurs in attempting to make use of these desirable properties of forecast models is that of trade-offs among the various properties. It is easy to say that given two models of equal efficacy, select the simpler. Suppose the models are not of equal efficacy and they are not of equal simplicity, and suppose further that one model is simpler than the other but also has less efficacy. Whether or not this model is more desirable than the more complex model of greater efficacy depends largely upon willingness to engage in a trade-off of simplicity against efficacy. This trade-off requires a high level of knowledge on the part of the decision maker regarding the benefits which may be expected from different levels of efficacy and the cost of increased levels of complexity. While it is possible to agree in principle to the trade-off notion, it is equally easy to argue that knowledge upon which to base the trade-offs will be very difficult to obtain and use.

What has been said in the previous paragraph points out an additional and rather obvious property of the forecasting situation. The purposes of forecasts must be known in advance and the decision situations based upon the forecasts must be clearly understood. Again, this is a property of the forecasting situation that is easy to describe in general but one which is difficult to analyze efficiently.

Assumptions

Two points about assumptions in transportation forecasting need to be emphasized at this time. One assumption is the usual one which is made in most forecasting situations and pertains to the structure of the forecasting problem. It is assumed that the characteristic of the transportation system which is under consideration in the forecast is some function of a given set of variables. That is, given known levels of certain variables, forecasts of the levels of transportation variables may be made through the use of functional relationships which are presumed to exist.

Since this is a far from perfect world and functional relationships are never exact, some companion assumptions must be made regarding the nature of forecasting errors. It might be assumed at the outset, for instance, that forecast errors are randomly distributed with constant variance and mean of zero. This may or may not be the case in any given situation, but it serves to exemplify a type of assumption which may be necessary.

Ideally, the functional relationships selected would completely describe <u>all</u> of the systematic influences operating upon the transportation variables, and the forecast errors would then consist of random variations arising out of measurement problems and the operation of strictly random influences. In practice, however, the functional relationships identified can do no more than specify some of the systematic influences operating upon the objects it is desired to forecast. Consequently, systematic bias frequently occurs in the errors of the forecast. The extent to which errors are systematic, as opposed to random, and their magnitude cannot be determined strictly on logical grounds. Empirical work is required to provide estimates of the nature and magnitude of these errors.

The functional relationships within the forecasting model may not be simple ones and since there may be more than one variable whose level is to be estimated, error relationships, in turn, may be quite complex. For instance, it is difficult to think of variables which are strictly independent of the level of transportation activity. Variables that might be thought of as conditioning levels of transportation, and thus entering into functional relationships as predetermined, include levels of population and economic activities. As was mentioned before, however, the distribution of population and the level and distribution of economic activity are, in turn, very definite functions of transportation availability. These variables also have relations extending to distribution of resources, capital accumulation, and still other factors. In other words, either conditional forecasts must be made or it becomes necessary to attempt the construction of a model which will simultaneously forecast the levels of the several factors involved (for example, transport activity, spatial structure of the economy, demographic structure, etc.).

It should be emphasized that there are some ways around the conditional forecasting dilemma. The conditional forecast may be of little utility if it uses as its input some variable which in itself is difficult to forecast. On the other hand, the possibility that a conditional forecast can be made using variables which may be relatively independent and simple to forecast does exist. In the case of income, for example, national policy may set targets regarding the growth of national income and the forecaster may have strong reasons to believe that these targets will be met. As income goals are met from year to year, the conditional statement would hold and hence transportation forecasts would also hold. It is also possible that the forecast may not require that the time dimension be explicitly stated since the area of interest may lie in what level of transportation activity is anticipated at that time (whatever it may be) when national income reaches some specified level. A second way of attacking the problem of the conditional forecast is, as mentioned before, to select variables that themselves are relatively simple to forecast. Population is often put forth as an example of such a variable. If relationships can be established between population and the level of transportation activity, then the presumed ease of forecasting population provides a simple way of determining expected levels of transportation development.

Cross-Section Versus Time-Series Models

It was mentioned previously that the forecaster has essentially two types of data available to him. He may use a set of observations over time periods (for instance, he might trace out railroad expansion versus population expansion for a 100-year period), or he may make comparisons among different areas at the same time (for instance, he might observe miles of paved road and size of county for the counties of the state of Illinois). The forecaster may also in a single study attempt to combine cross-section and time-series materials.

It is appropriate to consider at this point the differences in the type of information that may be obtained from cross-section and time-series material. The first thing to note is that in both approaches there is some practicable limit to the number of items that may be measured and utilized as variables. In the interest of simplicity and economy, population and miles of railroad, say, might be observed in a number of instances, rather than observing population, miles of railroad, size of area, income, bushels of wheat grown, number of locomotives, towns, and so forth, in a smaller number of instances. This is a case of trading off richness in individual observations versus an opportunity to make observations in many different instances, as well as trading off richness in individual observations versus the desire to obtain a relatively simple model. While no general rule can be laid down that is applicable to every study, it generally seems advisable to find samples for many different instances, rather than attempt to put the bulk of the effort into obtaining a great variety of information within individual observations. The fact that much information in an observation may turn out to be redundant mitigates against placing much effort in obtaining a great deal of information about an individual observation, and the presence of a great deal of redundant information in an individual observation presumes a certain capability to utilize that information. By redundancy we mean, of course, that the variables observed are saying essentially the same thing because they are highly intercorrelated. For instance, it may not be useful to observe both miles of double-tracked railroad and miles of high quality roadbed since these two characteristics are quite frequently found together.

The rather lengthy discussion in the preceding paragraph has some direct relevance to the question of developing models which make use of cross-section or times-series information. Generally, in forecasting situations of the present type, many observations are preferred to a few observations where each individual observation contains a great deal of information. With respect to time-series analysis, in many instances the time span over which observations can be made may be so short that the sample to be used for forecasting lacks necessary variability. This might be true, for instance, if the forecaster were working with the variable "miles of paved road" in a nation where this information was available only at 10-year intervals. Certainly for most areas in the world there have not been very many 10-year intervals since paved roads were first developed.

Problems in the use of time-series analysis may be discussed at a slightly more general level than in the paragraphs above. It was pointed out earlier that forecasting involves questions of what is demanded and what is supplied and that transportation forecasting by and large is conditional forecasting. Over time the nature of transportation demand changes as does the nature of the supply. Observations trace over time the resultant behavior from these parameters rather than the parameters themselves. Observed relationships are, then, the result of supply and demand interactions and are not observations of those parameters as such. A great deal of effort has gone into the examination of the problem of tracing out supply and demand relationships from time-series data (for example, Reference 20). Suffice it to say that this is a vast subject and one that merits the careful attention of the forecaster.

There are also very real problems in connection with cross-section analyses. Speaking in the supply and demand framework, it might appear at first glance that the supply side of the problem has been simplified. For instance, the forecaster might suspect that transportation technology is truly international, so that from nation to nation or from one area to another at any given time it may be assumed that there is no shift in the supply parameter, and what is then observed are shifts in demand structures. Consequently, a series of cross-section observations may then be used to trace out the supply curve. While the claim of a universal transportation technology has some merit, it also should be realized that areas vary in terms of topographic, climatic, and other conditions that affect the ability to supply transportation, so one should not leap too quickly to the conclusion that supply conditions remain constant from area to area. Also, lack of foreign exchange for purchase of transportation equipment, tariffs and other barriers to national trade, and area to area variations in technical know-how may well affect ability to supply.

It should not be concluded from the above discussion that a cross-section model is in some sense better than a time-series model or that the reverse is true, because the choice of model and the data utilized clearly depend upon the circumstances of the research. Also, it should not be concluded that there are insurmountable difficulties in the development and operation of either a cross-section or a time-series model or some combination of the two. The problems discussed above are among those central to model development and forecasting. There is a large and viable literature dealing with them.

Another point of comparison between time-series and cross-section models results from the formal nature of the model. The model may take the form, say, $Y_{ij} = \beta_o + \beta_1 X_{ij} + U_{ij}$. In a cross-section model, the Y_{ij} and X_{ij} are respectively the observed value of the dependent variable for the i^{th} area at time, j. j, or time, is held constant, so the only variability is among the areas, that is, over the the i subscript. The U_{ij} represents a random disturbance or error term. The relationship given in the equation above states that, apart from the error term, different values of Y arise because of variations in the values of X. This implies that a certain basic homogeneity exists among the areas examined in the study. If the sample model were applied to a time series, then i would be held constant and variability would exist over the j's, where the different values of j indicate different time periods.

As a general rule, it is easier to assume homogeneity for a given area over a period of time with respect to some relationship than it is to assume homogeneity from area to area with respect to a given relationship (Reference 20,p. 212). In a cross-section study, it may be necessary to make use of additional independent variables in order to assure homogeneity among the areas studied with respect to the functional relationship under examination. The introduction of additional variables is generally less essential in time-series work.

The identification of a systematic relation between Y and X may be difficult because of systematic relations among errors. Strong systematic relationships among the error terms, the U_{ij}'s, are likely to be associated with time-series and with cross-section observations. It would appear unwise to make a general statement of which is more likely, although it can be remarked that it may be easier to detect such systematic relationships with time-series than with cross-section data.

The Present Study

The fairly lengthy and general discussion of transportation forecasting given in the several sections above sets the stage for a review and discussion of the nature of the forecasts and forecasting devices treated in the present study. A short statement with respect to the overall nature of the effort in the current study will be made prior to presentation of the review.

One theme in the introductory sections of this paper has been that of the varied character of transportation forecasting. The present study is not directed to examination of specific instances. Rather it is addressed to the problem of generating a general forecasting capability. In scale it lies between a highly abstract study of the nature of forecasting and the production of a particular transportation forecast. It has been stressed that a knowledge of the relationships among variables that influence the levels of transportation activity provides the functional relationships which are at the core of the forecasting models. The present study is addressed particularly to the examination of these relationships. It is felt that the more exact identification of these relationships will prove to be of great assistance in any specific forecasting effort.

A number of relationships have been studied in different formats. Actual production of the forecasting capability, then, has produced a whole series of experimental forecasting studies. These studies are articulated to the extent that they complement each other as examples of forecasting abilities. The forecaster with a particular problem at hand might find one or more of the models used in this study suitable for his purpose, in spite of the fact that the chief attention here has been on the development of a better knowledge of functional relationships and the production of a general forecasting capability, rather than upon the detailed examination of specific forecasting situations.

Review of the Forecasting Models

Table 1 contains abstracts of most of the models developed during the course of the research. The second column, "Type", indicates that five of these models are basically statistical in nature and that two are mathematical in nature. The models which are described as statistical involve certain probability considerations while those classed as mathematical involve more in the way of mathematical manipulation. In a sense, models 6 and 7 represent an extension and augmentation of the work represented by models 1 through 5. This is to say that the statistical models tend to exhaust what currently may be learned through examination of available data on transportation system development, and the mathematical models represent attempts to augment the types of findings that may be made from the examination of data.

The column "References" indicates where the basic work with these models has been published. The extent to which the individual models are reviewed in the present document is variable, depending upon availability of new materials and the extent to which review is required to give an articulated overview of the research effort. Some materials related to model 7 are included within this present document, but no separate reference is available for this material. No separate statement has been prepared largely because this work parallels materials in the journal literature.

The Cross-Section Model Applied to Stock Aggregates, Model 1

This model was constructed with the objective of developing a method of estimating the quantity of transportation services in different countries (See Table 1 for references). Lack of availability of direct measurements upon "quantity of service" for individual nations made it necessary to select variables approximating those quantities, and this is why the study dealt with stock aggregates, namely: number of cars, number of busses, trucks, rail freight cars, and rail passenger cars. Miles of road and railroad were also included as measures of the quantity of service. Transportation is closely related to the structure of production and the quantity of service is influenced by the extent to which the service is used in intermediate production and the extent to which

TABLE 1

MODELS STUDIED

No.	Type	Applied to	For
1	Statistical (Cross-section)	Stock Aggregates	International Comparisons
2	Statistical (Cross-section)	Properties of Transport Systems	International Comparisons
3	Statistical (Time-series)	Properties of Transport Systems and Stock Aggregates	Individual Nations
4	Statistical (Simple Simulation)	Development of Transport Systems	Individual Nations
5	Statistical (Factor-analysis)	Maps of Transport Systems	Individual Nations
6	Mathematical	Location of Routes	Place within Nations
7	Mathematical	Addition of Capacity and Allocation of Flows	Routes within Nations

TABLE 1 -- Continued

Comments	References
Study of relationships between quantities such as numbers of automobiles or rail freight cars and population density and gross national product.	1,2
System properties such as "connectiveness" of systems related to variables such as level of technological development.	4,7,15
To augment Models 1 and 2 above .	1,2,8,13,16
Establish development rules for system expansion.	4,14
Method of breaking complex systems into subsystems; establishment of relations between subsystems and variables such as size of cities .	4,11,17
Mathematical treatment of the location of transport routes, given flows between places.	6,9,12
Mathematical treatment of the problem of adding capacity to established transport systems.	34

the transportation services are directly consumed as end products. Resources were not available to construct a model using this notion, so relationships were defined more grossly. Variables thought of as explaining the quantity of transportation service were divided into two groups. Climate, topographic conditions, and size of nations are relatively fixed quantities that directly influence the quantity of transportation service demanded. Income and population levels also influence the quantity of transportation services, and they are variables which normally change over time. Also, income and population relations determine the structure of demand for commodities and, thus, for the quantities of transportation required in intermediate production versus the quantity of transportation which is demanded as a final product in the economy.

A series of 115 regressions was fitted in order to investigate statistically the functional relationships among the variables. The results pointed up the very strong importance of income and population considerations, as would be expected. Insofar as the statistical relationships were concerned, these were found to be relatively simple in character. However, the results of the regressions require rather subtle interpretation with respect to economic interrelationships, and the identification of these relationships is handicapped by the fact that it was necessary to make use of data which only approximated the quantity of service measurements.

This model serves as a good illustration of the notion of a conditional forecast model. It was found that the quantity of transportation service was directly related to levels of population and economic activity, and in order to project the quantity of transportation service into the future, it is necessary to project the measures of population and economic activity.

System Properties, Model 2

It is obvious that any forecasting model must be constructed from hypothetical but relevant relationships, together with data which are meaningful within the context of those relationships. This remark is made because one of the major problems involved in forecasting transportation developments arises from the fact that transportation generally is part of a highly interrelated system. For one thing, the several modes of transportation may and frequently do carry out overlapping functions, so that the problem of allocating functions among modes arises in many cases. For another thing, the several modes operate on complex networks, and the expansion or contraction of a transportation network represents the expansion and contraction of a complex system of interrelated routes. The study of stocks referred to above incorporated work on the difficult problem of the split of functions among modes; the other studies described in the present section are concerned with the problems of the expansion and contraction of networks.

The problem of the recognition of "significant" variables was dealt with by creating measures which succinctly describe basic network properties. For instance, the notion of route density in relation to places served was codified by formal definition of the degree of the connectiveness of a network. The minimum number of routes or links needed to connect a system with n nodes is (n-1) routes. With any number of routes less than (n-1), the transportation network would be split into parts which are not connected with each other. With any number of routes greater than (n-1), alternate routes might be provided for movements on the network.

In a first study, a number of system properties, such as connectiveness which was mentioned above, were related to such determining factors as the level of technological development and the size and shape of nations. A series of regression models was used, and it was found that strong functional relations existed between the investigated variables. This study suggested a number of alternate measures and relationships that appeared to merit further investigation. Consequently, work was continued on this model. The model was formulated in several complementary ways, and studies were made at the within-nation level in addition to the between-nation comparisons. (Again, references are given in Table 1.)

Time-Series Analysis, Model 3

The studies of stocks and indices discussed above were essentially cross-section in character and dealt largely with comparisons among nations. The relationships studied worked rather well in the sense that a large amount of the variability of stocks and structural properties of transport networks among nations can be related to variables such as the level of economic development. The expression "worked well", is, of course, a relative one. The forecasting devices worked well considering the tremendous amount of variability encountered among national units. However, estimates for individual nations may deviate from actual levels by significant amounts.

Time-series studies were made to augment the cross-section models and improve knowledge on shifts over time in the mix of transportations used. These studies were undertaken for regions, for nations and among nations and were addressed mainly to shifts in output among modes. These models complement the cross-section studies. The cross-section studies identify the broad controls over levels of stocks and network expansion, while the time-series studies are thought of as investigating distinctive time paths of change for particular nations, subject to the actions of these broad controlling influences. Put another way, the time-series study provides a capability to apply gross parameters of change to particular sets of circumstances (for example, a specific national or regional unit).

Simple sets of relationships were fitted to changes over time in the use of transportation (amount and mix) in the case of the United States, and comparisons were made among a set of nations -- Spain, Finland, Italy, Canada, and France.

Network Simulation, Model 4

Work also was done on simulation of the areal expansion of transportation networks. The objective of this work was that of finding a set of operating rules which would describe, subject to the kinds of determining parameters discussed above, the expansion (and contraction) of transportation networks. In relation to the topics discussed in the three previous sections of this portion, this represented an attempt to make the forecasting specific with respect to particular places and times, in those cases where expansion might take place in the transportation network.

At first, considerable difficulty was experienced in developing a suitable simulation model, and it was found that simple "all or none" mathematical rules would specify the mappable pattern of the network at least as well as would the probability-based simulation rules. Continued work and increased experience gave more interesting results. This is judged to be an approach with much promise.

Factor Analysis of Network Structure, Model 5

Modes of transportation may serve overlapping and complementary purposes, and networks may consist of a complex of interdependent systems. In response to these notions, attempts had been made to develop certain indices which would display basic network properties and commodity flow properties. A factor analytic approach to structure was used in attempts to discover basic structural properties via statistical analysis of networks. For instance, systems of routes connecting points were represented in terms of a unique matrix (the connection matrix). From a technical point of view, the factor analytic approach is one which approximates the connection matrix with a matrix of lower rank. Experience indicated that the connection matrices representing transportation networks could be interpreted rather simply. For instance, it was found that certain aspects of networks represent field effects exerted by major urban centers.

In addition to the work on connection matrices, work has been done on commodity flows. This approach appears to be useful for identifying properties of networks it is desired to study on an intensive basis.

Route Location, Model 6

Experience with the various statistical models indicated the questionability of developing a forecast capability based on "forecasting" and ignoring planning. (This notion is more extensively developed in a later section of this chapter.) Also, the statistical models were gross, and we desired methods of estimating transportation networks details. In order to provide a capability in these areas, a mathematical model was devised that attempts to replicate the decisions made when a particular network configuration is decided upon. (Actually, this is not a single model. It is a set of computational techniques.) Enough work has been done with this model to indicate that it can be applied well in limited cases, but that when general cases are considered the model provides only a guide. Our work began with the simple case of connecting three points with a transportation network when the flows to (or from) each point were known and a function describing the fixed and variable cost of constructing and operating the transportation system was given. Let that function be $(a + bf_i)D_i$, where a is the fixed cost of route development per mile, f_i is the flow to be accomodated on the i^{th} link of the network, b is the cost per mile of moving a unit of that flow, and D_i is the length of the i^{th} link. The problem is that of constructing a network in such a way as to minimize $\sum_i (a + bf_i)D_i$, that is, the problem of finding the route configuration that makes the total transport cost a minimum. Put in other words, the model attempted to replicate those decisions made by transportation development authorities when it is decided how to build routes to accommodate specific traffic development goals.

A second problem that was considered was the problem of connecting two points with a route of minimal cost. Costs in this instance were functions of the distance travelled and of the type of terrain being crossed. Several methods were developed to simplify this problem for solution.

A General Mathematical Model, Model 7

The discussion in the two paragraphs above was with reference to a special case of the connections between three points where flows are fixed and variable costs were given. A completely general case, which presently seems to be intractable, would treat the connections of n points into a network. Flows would not be given but would depend upon:

(1) The configuration of the network, and
(2) The cost of transportation resulting from the chosen network configuration.

Flows would also be determined by supply and demand parameters which would be determined externally to the transport considerations. Fixed and variable costs would also be treated as variables to be determined in the general case. The problem is that of simultaneously determining as efficient configuration of the network and determining the fixed and variable costs that result from and determine that configuration. In spite of the fact that this general problem is not tractable at this time, there appear to be a number of ways of approaching it that may prove to be fruitful in the context of the development of a forecasting capability. Examples of approaches include studies of the effects of changes in fixed and variable cost ratios on network configurations using sensitivity analysis on computers (this has been done in connection with Model 6); developing ways to build up networks by considering a series of special cases, for instance, building up a total network by considering only three points at any one time; treating flows on the network as variables and using various devices to assign flows at the time network configuration considerations are made; and consideration of the problem of adding capacity to existing links.

If the capability to forecast route location is needed at a relatively microscopic level, then the effect of interrelations of costs and route orientation must be explicitly considered. If the capability is designed to deal more with the broad scale features of network development, then these local cost variations may be assumed to be of little significance.

Several kinds of exploratory work have been done with the general mathematical model. Ways have been found to treat the flows through points on a network as variables rather than fixed numbers, and ways have been found to treat the problem of the additions of capacity to a given network. Progress and problems are reported in the last chapter of this report.

The Forecasting Capability

This present document is a review of the component parts of a study designed to provide a capability for forecasting transportation developments. Some basic properties of transportation forecasting have been outlined, and models which bear on the forecasting problem have been briefly reviewed. This final section of the first chapter provides a summary with respect to the forecasting capability.

The forecasting capability which has been developed provides ways in which to state how gross parameters, such as the level of economic development, are related to transportation development (models 1 and 2). At a somewhat less gross level it also provides ways to relate these parameters to year to year changes, network expansion and contraction, and related topics (models 3-7). It would be easy to list a long series of objections to these models and the data which they utilize. Indeed, introductory sections of this document anticipated some of the classes within which such objections might fall. In the second paragraph below, what appears to be the really chief criticism of the study's methods and what a broad forecast capability truly requires are pointed out.

In working toward a transport forecast capability, the investigators have, in a real sense, assumed the worst. Our international comparisons presume only a general knowledge of the situation and the existence of rather gross levels of information; these presumptions continue to appear, although to a lesser extent, in the studies of simple networks and individual national and regional systems. No forecaster would be charged with a task so general as this because no one forecaster has to be prepared to forecast many different types of information from only gross data levels. The present capability is intended as a general forecasting scheme wherein a forecaster with a specific problem would be able to evolve a specific device which would adequately handle his problem. Generally, the specific problem would be more constrained than those explicitly considered here, and the results obtained would be considerably improved.

On the debit side, the investigators must admit the difficulty of submitting the evolving capability to precise empirical checks. It is true that both the cross-section models and the time-series models may be submitted to goodness of fit tests. But these are fits of the models to current and past data and provide little firm information on developments in the future. This, of course, is true of all forecasting devices. A particular weakness results from an inability to handle possible changes which might occur in transport technology. These changes might arise from equipment improvements or they might result from improved techniques with respect to decision making about transportation developments. To an extent, our work with mathematical models of development decisions anticipates the latter type of change.

A choice available to certain decision makers is in the choice between forecasting and planning. A national agency, for instance, might forecast population spread and income and then project transportation developments. It may have the alternate, however, of planning transportation expansion (or contraction) in such a way that desired income and population changes obtain. Planned solutions may differ sharply from solutions that might be forecast based upon historical relationships. These observations increase the scope of the forecasting problem. The forecasting capability must extend to forecasting methods of planning, including goals, techniques, and timing.

It appears that a number of inter- and extra-city transportation systems are now available, each with a different mix of costs and outputs. Consequently, technology may now permit choices from systems with rather different characteristics. The planning problem is not that of developing the system that meets performance requirements at least cost because in most situations there are mixes of noncomparable costs and outputs. These sentences point up the need for analysis of systems alternates in planning and the consequent need for that type of analysis in forecasting studies. This type of study is a large undertaking. It has not been attempted here.

Remaining Chapters

Ensuing chapters provide overviews and summaries of some of the main areas of our research. Much of the material in this chapter has been presented previously in publications referred to in Table 1. However, there is some new material. In preparing this present document, our objective has been to present the main features of the research without the details. Basic documents should be consulted when details are desired.

II. SOME FORECASTS OF AGGREGATES

An aggregate is a total or index number of a variable. For an example of an analysis of aggregates, consider a study in which the populations of cities were related to the tons of foodstuffs shipped into those cities. Here, attention would be on sums of populations and tons of food per city. Aggregations of particular interest in transportation forecasting problems are national aggregates, the miles of railroad, number of automobiles, etc., within a country. National units are quite meaningful when the study of economic activities such as transportation are undertaken. These activities are subject to national economic policy, national goals, nature of the national product mix, etc.

Comparisons of Transportation Stocks

In the Introduction to the first report from our studies (Reference 1) and in the Preface of another (Reference 5), it was pointed out that the surface transportation development of any country depends on:

- General economic development.
- Natural environment.
- Location of activities.
- Available technology and relative cost structure.
- The interests and preferences of those who make decisions affecting transportation.
- Military and political influences.
- The historical pattern of development and outlook for the future.

A list such as this one is by no means all inclusive.

One might also list aggregate measures of the transportation of a country, say:

- Number of automobiles.
- Miles of road.
- Miles of railroad.
- Ton miles hauled.
- Number of railroad cars.
- Cost of transportation.
- Number of workers.
- Fuel used in transportation.
- Number of trucks.
- Transportation industry profits.

And this list is also somewhat less than all inclusive.

These two lists and the notion "depends on" serve to introduce the major ideas within the study of transportation stock aggregates. One central notion was that of the manner in which stock aggregates depend on such matters as the level of economic development. Data had to be examined so that the nature of the functional relationships could be established. At a practical level there was the problem of what data. Behind this question was the question of what variables.

The choice of variables used to explain inter-nation variations in transportation stock aggregates involved considerable compromise. This compromise resulted from the attempt to develop a forecast methodology as well as an understanding of the basic factors influencing the development of transportation stock aggregates. As regards the forecasting of the number of motor cars, for example, if one were supplied with the number of gasoline service stations within a country, he could likely make good estimates of the number of cars. The difficulty is that data on the number of service stations would be as difficult to estimate as the data on cars, one object of the study. Also, knowledge of the number of gasoline stations would be of little interest from an explanatory viewpoint. Surely both numbers of cars and numbers of gasoline stations result from common explanatory relationships.

From an ease of forecasting point of view some unchanging variable, such as the amount of rainfall, might be valuable since it is relatively easy to estimate. On the other hand, there is little reason to expect a close relation between the amount of rainfall and the number of cars. What is desired is a series of variables that (1) can be thought of as explanatory and (2) are at hand for purposes of making estimates. The problem is that data that are explanatory are rarely easy to come by and data that are easy to come by are rarely explanatory.

The Problem

A compromise list of explanatory or independent variables was achieved. These included population, area, gross domestic production, per capita income, population density, slope, and rainfall. Certain of these variables are relatively unchanging. Others, such as population, do change over time. However, these latter were suspected to be of prime importance in explaining the level of transportation stock aggregates and thus essential in the calculations. Also, they are relatively easy to project in an aggregate manner. Income and production more and more are tending to be planned, so, providing that planning goals can be met, their levels at future time periods should be establishable.

These variables should be compared to the list given earlier of factors upon which levels of transportation stocks might depend. Again, the variables represent a compromise between the explanatory desiderata and usable data.

Data developed to serve as measures of transportation stocks are listed below. These were regarded as the dependent variables.

Cars, January 1, 1957
Buses, January 1, 1957
Trucks, January 1, 1957
All Roads, 1956
Paved Roads, 1958
All Roads, 1958
Rail Tracks, 1956
Rail Passenger Cars, 1956
Locomotives, 1956
Freight Train Cars, 1956
Rate of change in Number of Cars
Rate of change in Number of Buses
Rate of change in Number of Trucks

The complete data matrix contained data on the variables listed above for 71 nations. However, there were certain instances in which data were not available. A large effort was required in developing the data matrix, and one consequence of this was some later survey work on the availability of data of this type (Reference 10). One conclusion from our work is that a considerable effort will be required to develop and maintain those data systems that would form the basis for forecasts and model estimation of the type studied here.

Table 2 provides examples of correlations among the dependent variables. The entry .78, reading across the line beginning "1. Cars, 1/1/57", indicates a correlation between numbers of cars and variable number 2 (Buses 1/1/57) of the magnitude .78. These examples are given here in order to provide a rough idea of some of the redundancies in the measures of stocks. It is not unexpected that nations with many cars would have many buses and trucks and would have considerable milages of paved roads and railroads.

TABLE 2

SIMPLE CORRELATION COEFFICIENTS

BETWEEN SELECTED DEPENDENT VARIABLES

	1	2	3	4	5
1. Cars, 1/1/57	1	.78	.94	.89	.75
2. Buses, 1/1/57		1	.87	.74	.77
3. Trucks, 1/1/57			1	.84	.78
4. Paved Roads, 1958				1	.64
5. Rail Tracks, 1956					1

Table 3 provides a sample of the correlations among certain of the independent or explanatory variables. Although this sample is small, it show properties of the relationships among the independent variables. The physical variables, such as slope, tended to have low correlations with the other independent variables, while economic variables were intercorrelated with each other. The significance of this point to the analysis will be discussed in a later section of this chapter.

TABLE 3

SIMPLE CORRELATIONS

BETWEEN SELECTED INDEPENDENT VARIABLES

	National Income	Slope
Population Density	.12	.15
Income Per Capita*	.54	.16
Gross Domestic Produce*	.96	-.06
Area	.47	
Population	.76	

*Measured at free exchange rates.

Computations

The question was then asked: how does each of the measures of transportation stocks relate to the independent variables? A number of considerations went into answering this question. For one, when values of the variables were plotted on graph paper, it was noted that relationships did not follow a straight line. Also, rates of change seemed important. For these reasons, nonlinear relations were studied. This was done by fitting the equation

$$x_0 = ax_1^{b_1} x_2^{b_2} \dots x_n^{b_n}$$

in logarithmic form; namely,

$$\log x_0 = \log a + b_1 \log x_1 + b_2 \log x_2 + \dots + b_n \log x_n$$

where:

x_0 is the value of a transportation stock

x_1 through x_n are the values of the various independent variables

b_1 through b_n are the values of the partial regression coefficients

The b's, the multiple regression coefficients, were estimated using well-known methods of linear regression. Estimates of error for each b and partial correlation coefficients were also estimated. The square of the partial correlation coefficient is known as the partial coefficient of determination. The latter may be interpreted as the percentage of the variability in the dependent variable associated with concommitant variation in the particular independent variable.

In order to investigate all of the combinations of variables that seemed promising, 115 regressions were calculated.

Interpretations

Many calculations were made. All that can be done here is to give some examples of results from these calculations and provide general statements on research of this type.

Table 4 presents one set of results from the computations. The regression coefficients shown on lines 1 and 4 may be interpreted by reference to the equation given in the section above. Of interest to the present discussion are the partial coefficients of determination given on lines 3 and 6. These coefficients have a very simple interpretation -- they show the percentage of the variation in a dependent variable associated with variations in an independent variable. It is easy to see that income is a much more meaningful independent variable than is income per capita, and it is also easy to see that the importance of income varies greatly among stocks.

Supplemental information is provided in Table 5. The table is with reference to three of the sets of independent variables used in the regressions and with reference to seven of the dependent variables. These combinations were 21 of the 115 regressions.

The multiple coefficients of determination indicate the percentage of variation in the dependent variable associated with the independent variables, taken all together. For instance, 79 percent of the variability in number of motor cars, using data for January 1, 1957, is associated with variation in gross national product, income per capita, and population density.

Data in Tables 4 and 5 illustrate findings that permit a very simple interpretation. The relationships between individual independent variables and the dependent variables are small. Sets of independent variables give stronger and fairly uniform results from dependent variable to dependent variable. Results are improved when a dependent variable is used in the role of an independent variable, that is, as shown in line 3 of Table 5.

Table 6 gives information similar to that in Table 3 on page 40. These data illustrate ways in which supplies of transportation stocks are related to each other. Here the interpretation is that levels of stocks are determined by common explanatory relationships. Another important kind of relationship derives from the substitution of one stock for another. An example of the former interpretation is the notion that nations with high levels of income and large populations

TABLE 4

PARTIAL REGRESSION COEFFICIENTS, THEIR STANDARD ERRORS AND PARTIAL COEFFICIENTS OF DETERMINATION OF INCOME PER CAPITA AND INCOME

(Variables Transformed into Logarithms)

	1 January 1957		
	Cars	Buses	Trucks
Income Per Capita			
1. Partial Regression Coefficient	.505	-.332	.043
2. Standard Error	.138	.134	.126
3. Partial Coefficient of Determination	.25	(-).13	.00
Income			
4. Partial Regression Coefficient	1.439	.493	1.009
5. Standard Error	.165	.141	.140
6. Partial Coefficient of Determination	.66	.23	.56

TABLE 4 -- Continued

	1956			1958	1956
	Rail Passenger Cars	Rail Freight Cars	All Roads	Paved Roads	Rail Tracks
Income Per Capita					
	-.064	.065	-.147	.233	-.260
	.320	.260	.145	.191	.239
	.00	.00	(-).03	.04	(-).03
Income					
	1.180	1.178	.847	1.061	.843
	.338	.286	.160	.214	.251
	.23	.30	.41	.38	.22

TABLE 5

MULTIPLE COEFFICIENTS OF DETERMINATION OF THREE SETS OF REGRESSIONS

	1 January 1957		
	Cars	Buses	Trucks
Independent Variables			
1. G. D. P., Income Per Capita, Density	.79	.76	.83
2. G. D. P., Income Per Capita, Density, Average Slope, Average Rainfall	.82	.76	.86
3. G. D. P., Income Per Capita, Density and Another Mode of Transportation (in brackets)	.86 (Roads)	.82 (Cars)	.87 (Roads)

TABLE 5 -- Continued

	1956			
	Rail Passenger Cars	Rail Freight Cars	All Roads	Rail Tracks
Independent Variables				
	.68	.69	.83	.69
	.66	.65	.81	.65
	.91 (Rail Tracks)	.96 (Rail Tracks)	.87 (Rail Tracks)	.97 (Rail Freight Cars)

TABLE 6

MATRIX OF SIMPLE SQUARE CORRELATIONS BETWEEN THE LOGARITHM OF THE VARIABLES[1]

	1 January 1957		
	Cars	Buses	Trucks
Cars, 1 January 1957	1	.610	.885
Buses, 1 January 1957		1	.755
Trucks, 1 January			1
1956 Roads			
1958 Paved			
Rail Tracks, 1956			
Rail Passenger Cars, 1956			
Train Freight Cars, 1956			

[1] All the correlations are positive.

TABLE 6 -- Continued

1956	1958	1956		1956
All Roads	Paved Roads	Rail Track	Rail Passenger Cars	Train Freight C
.766	.787	.566	.613	.648
.579	.555	.591	.621	.594
.805	.707	.604	.619	.632
1	.576	.702	.661	.672
	1	.407	.521	.513
		1	.810	.903
			1	.880
				1

would be expected to consume much transportation and thus have large stocks. An example of the latter relationship is the notion that a nation with many trucks would need few rail freight cars. Trucks may be substituted for cars.

Related Work

There is no way to say whether the results of this work were good or bad. Certain relationships were found, and these illustrate the results that might be expected from work with actual problems where this method and this or a similar set of data were used. The nature of the findings did supply one motive for some work on our part using within-nation (References 8, 16 and 18) and among-nation (Reference 13) comparisons. In addition, some within-nation models were investigated at the time the study just reviewed was completed (Reference 1). A second motive for these additional studies was the desire to investigate the roles of time trends in transportation utilization. A third motive was the desire to study substitution relations among transportation modes, the kinds of relationships mentioned in the paragraph above.

Comparisons Among Nations

The comparisons among nations (Reference 13) were made by computing a single model several times for railroad passenger movements and for railroad freight movements for

France
Finland

Canada
Spain

Italy

The model utilized was of the form

$$\text{Log } Q = \text{Log } A + b_1 \text{Log } P + b_2 \text{Log } Y + b_3 T$$

where

Log Q = log of passenger (ton) miles (km.) per capita,
Log P = log of passenger (ton) revenue per mile (km.), both in nominal and real terms,
Log Y = log of national income per capita, in real terms,
T = linear time trend (1948-1960).

The coefficients, the b's, were the parameters that were estimated. The coefficient of Log P represented the price elasticity of demand and the coefficient of Log Y was the income elasticity of demand. The coefficient of T gave the percentage trend in consumption of the dependent variable Log Q. Table 7 presents a sample of the results of this estimation process. The table is divided into an upper half for passenger analysis and a lower half for freight analysis. Each regression was estimated under two alternatives; the first included the price variable in nominal terms and the second included price in real terms. Standard errors of the coefficients are listed in parenthesis, and partial correlations are listed under the standard errors.

General Remarks

The study now under discussion emphasized the impact of time trends on transportation utilization and the substitution of one type of transportation for another as nations develop and as price relationships change. Again, data were processed for each of five nations, and conclusions could be reached for each nation as well as from comparisons among the nations. The table presented here, Table 7, provides only a sample of the results.

From general knowledge one would expect that as nations develop more advanced economies, larger numbers of substitutable products and services would appear in the market. This increases the competitive milieu of any one such "good". From the point of view of price relationships, this developmental process serves to push price elasticities from relative inelastic positions to greater elasticities. Similarly, one would expect that consumption of most services would increase with rising incomes. Thus, one might expect income elasticities to be higher for the more developed nations. The income elasticity notion may or may not pertain, depending upon whether the service under question represents a superior or an inferior good. If the former, elasticities will rise with national income; if the latter, elasticities will decline. These, of course, are *a priori* notions derived from general ideas and knowledge. However, they provide the background for the study of railroad services.

Results

Twenty regressions were run and 16 of these had coefficients of determination (R^2) of greater than .75. Table 7 is an example of one of the poorer results, the passenger services in Spain. Trend effects were small in all cases except for Canada. Both passenger and freight services displayed negative trends for all countries, except passenger service in Italy. A large negative trend found for Canada (3.5 percent) is not surprising; it parallels the United States experience of declining rail consumption in the face of sharply rising motor carrier consumption. European experience indicated trend terms clustering about zero, or the lack of any obvious trend. It is difficult to make inferences regarding the availability and influence of substitutable services in light of such evidence. Yet, the analysis seemed to indicate that the portrayal of increased European railroad consumption in absolute units conveys a stilted view of the situation. The growth of railroad consumption during the post-war period seems to have been offset by population growth.

TABLE 7

SPAIN: PARTIAL REGRESSION COEFFICIENTS, STANDARD ERRORS (in Parenthesis), AND PARTIAL CORRELATION COEFFICIENTS FOR TRANSPORTATION RELATIONSHIPS.

1. Dependent Variable Log Rail Passenger Kilometers per Capita, 1948-60.

Intercept Standard Error	Log Income per Capita $\frac{\text{(Peseta)}}{\text{C.P.I.}}$	Log Rail Price (pes./pass. km.)
1.2842	0.2112	-0.3867
0.4278	(0.1251)	(0.1426)
	0.4903	-0.6705
1.0952	0.2782	
0.4708	(0.1326)	
	0.5729	

2. Dependent Variable Log Rail Ton Kilometers per Capita, 1948-60.

Intercept Standard Error	Log Income per Capita $\frac{\text{(Peseta)}}{\text{C.P.I.}}$	Log Rail Price (pes./pass. km.)
-1.0389	0.9219	0.0470
0.6221	(0.1702)	(0.1339)
	0.8747	0.1163
-0.9971	0.9058	
0.6218	(0.1708)	
	0.8701	

TABLE 7 -- Continued

Log Rail Price $\frac{\text{(pes./pass. km.)}}{\text{C.P.I.}}$	Time (0-12)	R^2
1. Dependent Variable Log Rail Passenger Kilometers per Capita, 1948-60.		
	0.0106 (0.0071) 0.4457	.6878
-0.3620 (0.1822) -0.5521	-0.0005 (0.0045) -0.0429	.6056
2. Dependent Variable Log Rail Ton Kilometers per Capita, 1948-60.		
	-0.0147 (0.0067) -0.5866	.8527
0.0106 (0.2104) 0.0168	-0.0130 (0.0049) -0.6620	.8507

All of the price elasticities of demand had the correct sign, except for freight service in Spain, as shown in Table 7. The Spanish elasticity was small and regarded as zero. The freight price elasticities displayed a range of progression according to rankings of economic development level. That is, the least developed nation, Spain, has the smallest price elasticity while the most developed nation, France, has the greatest price elasticity. This observation fits the general notion about the movement of price elasticities and economic development. With regard to passenger services, the correlation between price coefficients and development was not so pronounced.

Income coefficients displayed a more complex pattern of variation. Passenger income elasticities varied directly with development, except for Canada and Italy. Income elasticities for freight consumption clustered in the range 0.75-0.90, except for Canada and France; the former was appreciably greater and the latter was appreciably less. The very large Canadian elasticity (2.197) was quite unexpected, but the relatively large negative trend associated with it may add further support to the existence of an even larger income effect for motor carrier transport in Canada. Conversely, the extremely small freight elasticity for France (.0103) was taken to be zero or possibly even negative. If negative, then freight service by rail would be considered an 'inferior' good. Such a conclusion would not preclude, nor run counter to, the medium-sized negative trend associated with French freight consumption.

Summary and Interpretations

The analyses centered upon railroad services and functions were fitted to 1948-50 data for five nations. The nations represented a broad spectrum of economic development. With exceptions, the fits of the functions to the data were quite good. Time trends were found to be generally small, and price and income effects could be identified and interpreted, although interpretations were rarely simple ones. It would appear that this approach is a useful one for data of this type.

Regional and Community Studies in the United States

Previous sections of this chapter have reviewed comparisons among nations based upon cross-section information and comparisons within and between nations based upon time-series data for nations. Studies (References 8, 16, and 18) of the demand for freight transportation in the United States will now be reviewed. Reference is to motor carriers and railroads during the 1956-60 interval. The order of analysis was (1) aggregate transport demand for all commodities in the continental United States, (2) transport demand for the nation by individual commodities, (3) regional transport demand for all commodities, and (4) individual region-commodity combinations. Thus, the studies proceeded from greater to lesser levels of aggregation. This is true for these studies as well as for all the studies reviewed in this chapter.

Data

Data problems in transportation research are great. While in the United States, due to governmental regulation, there is a rich and abundant stock of railroad information which pertains to almost all railroad movements, commodity origin data are reported in terms of a particular regionalization of the nation (9 regions) or in terms of state-to-state movements. On the other hand, motor carriers have not been regulated for as long a period or as completely as have the railroads. Therefore, there is less available information and it is less complete than for the railroads. Furthermore, motor carrier commodity origins are reported in terms of a different set of regions than are the railroads (9 different regions, except for New England). The disparity between commodity information for these two modes is clear when one realizes that over 95 percent of all commodity movements by rail are regulated and reported, while only one-fourth of the intercity commodity movements for motor carriers are regulated and reported.

Consequently, in order to ensure comparability it was necessary to assemble railroad information on tons originated and total freight revenues for each commodity from the state-to-state waybill series and aggregate these into the nine motor carrier regions. The commodity classifications utilized for the two transport modes were the same, and an aggregate classification was used composed of five classes: (1) products of agriculture, (2) animals and products, (3) products of mines, (4) products of forests, and (5) manufactures and miscellaneous.

From the data, attempts were made to construct comparable series for both modes, but these were neither composed of data in the desired form nor were they strictly comparable. The railroad data were from a 1 percent sample of a well-defined population; the motor carrier data were from an enumeration of a portion of the motor carrier population. It was necessary to use tons rather than ton-miles as a measure of output because of limitations on the motor carrier data. Revenue per ton was used as a surrogate for price data, because of a lack of actual price data.

Analysis

Given 9 regions, 5 commodity groups, and 5 years of comparable data, there was a total of 225 observations available for analysis. The most aggregative level of analysis treated all observations together in a national demand model. The second level of analysis treated each of the 5 commodity groups individually on a national basis. The 225 observations were divided into 5 groups of 45 observations each. Since there were 9 regions, the 225 observations were divided into 9 groups of 25 observations each for the regional analysis. Lastly, the most disaggregative level of analysis treated each commodity and region combination. Since there were 9 regions and 5 commodity groups, there was a total of 45 combinations, where each combination was composed of 5 observations, the years 1956-1960.

Thirty-three separate equations or models were estimated and, as was the case with the cross-section studies, it is not practicable to review all of these results. It may be mentioned, however, that the models could be grouped into two classes. Certain models took the form of simple demand expressions; the quantity of the i^{th} mode that is used is a function of the price for that mode and the price of j^{th} mode. Here, the parameters estimated were price elasticities. Other models related quantity and price ratios; for example, the ratio of motor carrier use to railroad uses is a function of their prices. Here, the estimated parameters were elasticities of intermodal substitution.

Results and Interpretations

Again, all that can be given here are some examples of results and interpretations. One model estimated was (Reference 6, p. 51)

$$\text{Log } (Qm/Qr) = a + b \text{ Log } (Pm/Pr)$$

where the subscripts m and r referred to motor carriers and railroads and Q and P referred to quantities and prices respectively. The notion here was that the relative share of the freight market was a function of relative prices. When fitted to nation freight data, the results shown in Table 8 were obtained.

TABLE 8

RESULTS FROM NATIONAL TRANSPORTATION DEMAND ESTIMATION

Item	Estimate
Intercept (a)	1.09
Elasticity of Substitution (b)	-1.87
Standard Error of b	.14
Coefficient of Determination (R^2)	.43

These results (Table 8) were not felt to be of special significance. The goodness of fit (R^2) was not high, and the empirical experience would indicate that the elasticity of substitution was too high. Furthermore, when the model was rewritten in a manner to highlight regional and commodity effects, more reasonable results were obtained (Reference 18, p.53). A higher coefficient of determination (R^2) was achieved, and the elasticity of substitution was approximately one (Table 9).

TABLE 9

SOME RESULTS FROM DISAGGREGATED TRANSPORTATION DEMAND ESTIMATION*

Item	Estimate
Intercept (a)	1.54
Elasticity of Substitution (b)	-.92
Standard Error of b	.13
Coefficient of Determination (R^2)	.85

*Coefficients were also obtained for dummy variables representing regions, commodities, and years.

Work with this and alternate forms of these models indicated the importance of the commodity variability component in national transportation demand and differences among commodities in sensitivity to price variations. The study of transportation demand by regions indicated elastic responses in some and inelastic responses in others. Work at the least aggregated level of analysis revealed much variability in responses among region-commodity combinations. The latter is exactly what would be expected, for the national demand picture is known to be composed of many regional-commodity sets of relationships acting in a highly varied manner.

Summary Comments

In this chapter we have presented results of certain of our studies of aggregate measures made upon transportation. Nearly all of these measures were "economic" in the sense of being measures of "economic things". Some of the measures were "economic" in the more limited sense of measures of economic relationships.

The work discussed thus far provides a yes answer to questions of whether or not models of conceptual interest can be fitted to transport data. Whether or not the models prove useful for forecasting depends upon the particular forecasting situation, of course.

III. FORECASTING STRUCTURAL CHARACTERISTICS

Networks possess many different structural properties. At the most simple level of conceptualization, a network may be thought of as composed of points and lines. At higher levels of complexity, the notions of distance, capacity, angles between lines, and so on, may be introduced. It is possible also to consider networks at different levels of aggregation, either by (1) using measures of the characteristics of entire networks or (2) using measures of relationships among links (or nodes) on the network. There is another alternative of course. It is to study individual links (or nodes) without reference to other links (or nodes) on the system. The latter is not a viable alternate at the level of generalization at which the present research takes place.

Figure 1 shows the internal airline routes of Guatemala,and Figure 2 shows the internal airline routes of Honduras. The maps are minimal in the sense that they show only the existence (or non-existence) of routes, and the location of terminals. The maps also show the lengths of routes, of course, but map information on length of route is somewhat difficult to interpret. Length of route in combination with information on amount hauled over the route provides a metric of the tie between two places on the network. Information is not available on simple maps, such as the sample map, to indicate amount of traffic, so length of route does not show the strength of ties between places. Also, it is widely known that cost of transportation is a nonlinear function of distance. Put another way, the cost of moving the first mile on a trip between two points is not the same as the cost of moving the second mile. It is generally much more costly to move the first mile than it is to move each succeeding mile. Still a further observation might be made regarding the distance measure. In the case of air transportation, the distance between two points may be variable, it depends upon the choice of route by the pilot. In the case of North Atlantic routes, for instance, flight path distances may vary greatly from day to day. For similar reasons the time required to travel between two points is a variable,and the cost of moving between two points is a variable. The interpretation of distance on a map, then, would seem to require transforming the distance onto some sort of a linear scale. The transformation would depend upon whether or not distance is given a cost interpretation, and it might require regarding the distance as a variable and recording its mean and variance.

The more elementary consideration of the existence or nonexistence of terminals and the existence or nonexistence of routes seems somewhat easier to interpret than the questions of route length. Albeit, terminals may vary in facilities as may routes. For the moment, attention will be given to questions of the layout of transportation networks viewed in the simplest manner -- the existence of terminals

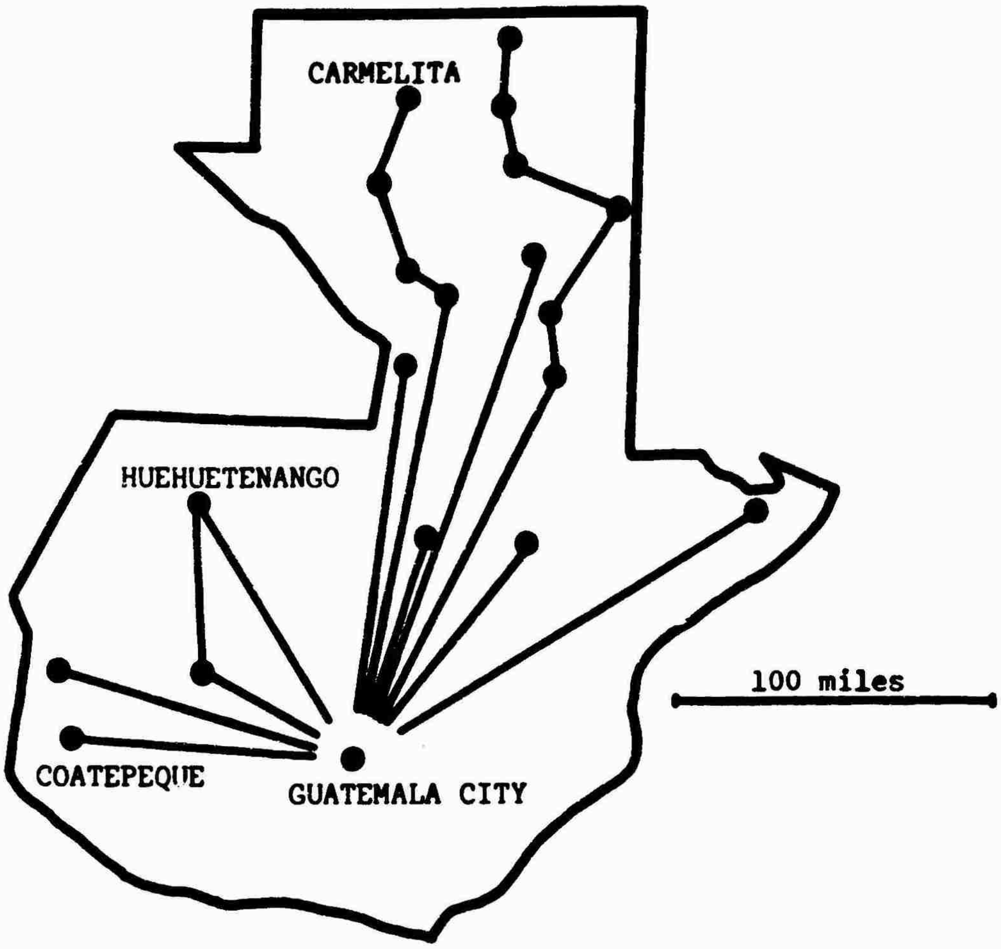


Figure 1: Internal Airline Routes of Guatemala

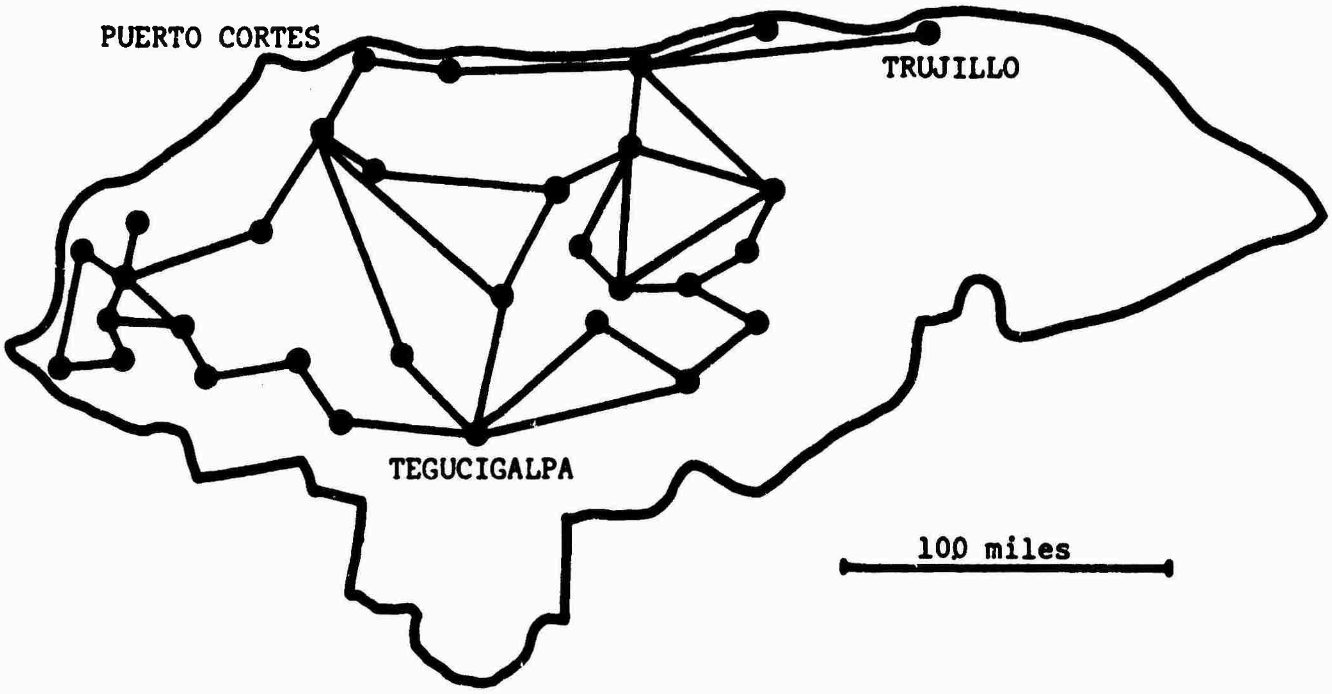


Figure 2: Internal Airline Routes of Honduras

and the existence or nonexistence of routes.

Surely any analysts would agree that the sample networks (Figures 1 and 2) differ in layout. It is equally clear that different analysts would have difficulty communicating just what they meant by statements that the systems differ. These statements reveal one of the central problems in this research; that is, the establishment of meaningful ways to codify structures of transportation networks. The remainder of this chapter details the work which was undertaken in this general area.

Graph Theoretic Concepts

The purpose of this section is to provide a brief introduction to some of the graph-theoretic concepts which will be utilized in the subsequent empirical studies. The reader who has some acquaintance with this topic, derived perhaps from a study of Berge(Reference 21), Ore (Reference 22), or Seshu and Reed (Reference 23),may wish to proceed directly to the material dealing with the substantive investigations of network structure.

Primitive Notions

Basically, a linear graph is a collection, or set, of line segments and points, The line segments are commonly known as edges, while the points which form the other basic element of the graph are normally know as vertices. The two primitive concepts, edges and vertices, are combined to form what is called a linear graph. Collections or sets of this nature may be either finite or infinite depending upon the number of elements that they contain. In the following discussion only finite graphs will be considered.

Classification

Graphs may be broken down into several different classes. One of the most basic breakdowns is that of nonoriented and oriented graphs (See Figure 3).

(a)
Non-Oriented Graph

(b)
Oriented Graph

Figure 3

In nonoriented graphs, the only operational concept is that of incidence : the notion that the end points of one or more edges may coincide with a vertex (that is, the edges are incident upon that vertex). The oriented, or directed, graph on the other hand also recognizes a sense of direction of the edges. In this case it is recognized that an edge is incident upon two vertices and also that a sense of direction is implied from one vertex to the other. So far, operations have been discussed in terms of a very simple set of concepts. Measurement has dealt only with binary relations such as existence or nonexistence, and incidence or nonincidence, with no introduction of notions of metrization. It is possible to introduce into the system certain metrics so that a specific numerical value is associated with each edge and/or vertex. For instance, distances between urban centers on a transport network might be associated with the edges of the graph of that network. Each edge in the graph would then have associated with it a specific numerical value, or weight, and the resulting graph would be a weighted graph, or net. (See the discussion by Hohn, Seshu, and Aufenkamp (Reference 24).) In an even more complex case, it would be possible to assign weights to the nodes or vertices of the graph, as well as the edges. When this is done, the system's configuration more closely resembles that of a stochastic process (see Bartlett, Reference 25), rather than a linear graph.

A third important classification deals with mapping the graph onto the plane. A graph which can be mapped onto a plane, such that no two edges have a point in common that is not a vertex, is known as a planar graph. Graphs which cannot be so mapped are known as non-planar graphs. This distinction is important in the study of transportation networks due to the problem of involuntary intersections; that is, intersections created by the physical crossing of two or more routes connecting nodes in the system. In general, planar graphs correspond to those systems which may be constructed without creating involuntary intersections. Empirical examination of transportation systems indicates that surface routes, rail and highway, tend to have the characteristics of planar graphs, while airline routes appear more like non-planar graphs.

If the transportation network is regarded as a graph, it becomes useful to develop certain summary indices which relate to the structure of the network. Some information has been lost in passing from the actual system to its graph or matrix representation, and further loss becomes necessary in order to assist in information handling. The problem here is similar to one encountered in most forms of statistics where data has been gathered in the form of a frequency distribution. In order to obtain a readily comprehensible summary index of the distribution, and in order to be able to distinguish between different distributions, certain summary measures such as mean, variance, etc., have been developed. The problem at hand in the present case is to compile a set of analogous measures pertaining to the structure of the graph.

Isomorphisms

It is easy to see that a given linear graph may be structured in several different ways, for example, relabeling the nodes and edges. In such a case it would be useful to have some precise way of recognizing that the graphs are really identical even though they may be arranged differently and that their vertices and edges may bear different labels. This situation represents what is known mathematically as an isomorphism. Two graphs, say G and G*,can be said to be isomorphic if there is a one-to-one correspondence between the vertices of G and G*, and a one-to-one correspondence between the edges of G and G*, which preserves the incidence relationships. (See Figure 4.)

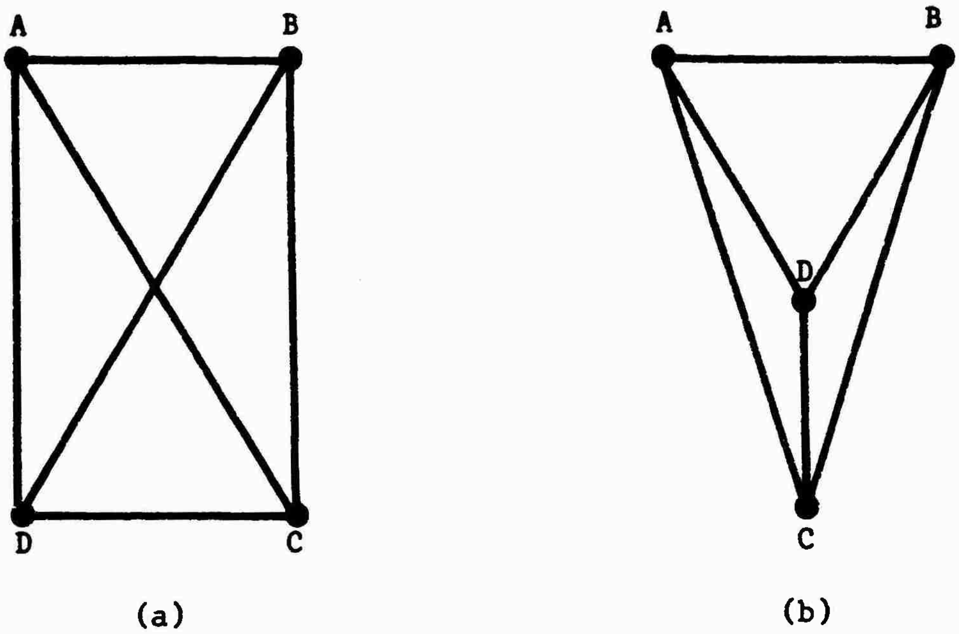


Figure 4: Two Isomorphic Graphs

Connectivity

It is possible to count the number of edges that are incident at a particular vertex, and this number is known as the degree of the vertex. A path is a collection of vertices and a subset of their incident edges such that the degree of each internal vertex is two and the degree of each terminal vertex is one. A circuit is a closed path where all vertices are of degree two.

Now, using these concepts, the very important notion of connectivity may be introduced. A graph G is said to be connected if there exists a path between any pair of vertices in the graph. Thus, from an intuitive standpoint, we may feel that a graph is connected if it is in "one piece". Suppose the graph is not connected; then this means that there are pairs of points or vertices in the system which cannot be joined in a path. The graph is then unconnected and it is intuitively obvious that it must consist of a number of "connected pieces". These "pieces" of the larger graph are subgraphs and are known as maximally connected subgraphs. The number of these maximal connected subgraphs in any finite graph, G, is denoted by p and, as a consequence, p=1 for a graph G if, and only if, G is connected. This count of the number of maximal connected subgraphs present represents one of the simplest descriptions of the structure of a graph and provides an index that remains invariant under all isomorphic transformations. The technical name for this index is the zeroth Betti number.

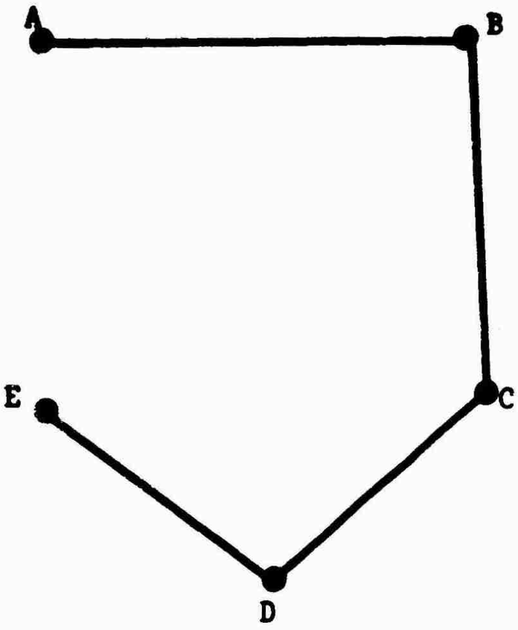
Trees and Fundamental Circuits

One notion commonly encountered in graph theory is that of a tree. A tree is defined as a connected subgraph of a connected graph which contains all the vertices of the graph but which does not contain any circuits. A given finite graph is a tree if, and only if , there exists exactly one path between any two vertices of the graph. It can be shown that if a tree contains v vertices, it contains v-1 edges. For instance, in a transport system the smallest number of routes that will completely connect five urban places is four. Conversely, it can be shown that the maximum number of routes between n points is $\frac{n(n-1)}{2}$; that is, ten transport routes would be required to connect completely a system containing five urban places. (See Figure 5.)

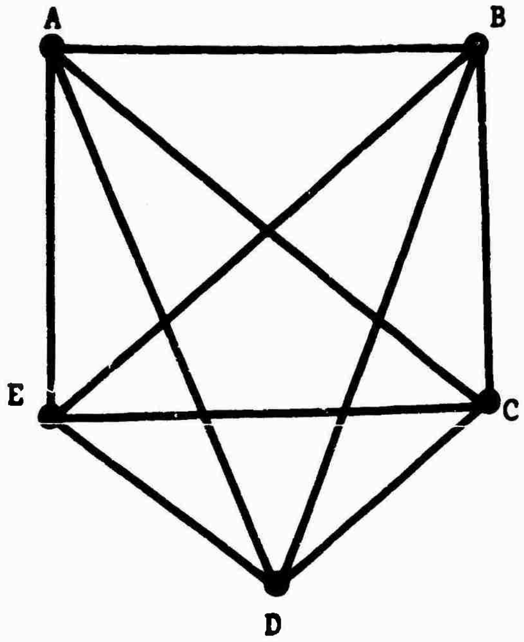
Fundamental Circuits

For a given graph and a given tree defined on the graph, elements of the graph may be divided into two classes, branches and cords. Branches are those elements which are contained in the tree whereas cords are elements that are not in the tree and are therefore in its complement (or co-tree).It also may be shown that a connected graph consisting of v vertices and e edges contains v-1 branches and e-v+1 cords. If one cord should be added to a tree, a graph is obtained that is no longer a tree. The cord and the path in the tree between the vertices of the cord constitutes a circuit. This is, however, a unique circuit and the only circuit of the resulting graph.

The fundamental circuits of a connected graph G for a tree T are the e-v+1 circuits consisting of each cord and its unique tree path. In a more general sense, this number is given by $\mu=e-v+p$, where v is the number of vertices, e the number of edges, and p the number of maximal connected subgraphs. The index μ is invariant under isomorphic transformations and is known as nullity, cyclomatic number, or first Betti number.



(a)
Minimal Network (Tree)
($v = 5$, $e = 4$)



(b)
Completely Connected Network
($v = 5$, $e = 10$)

Figure 5

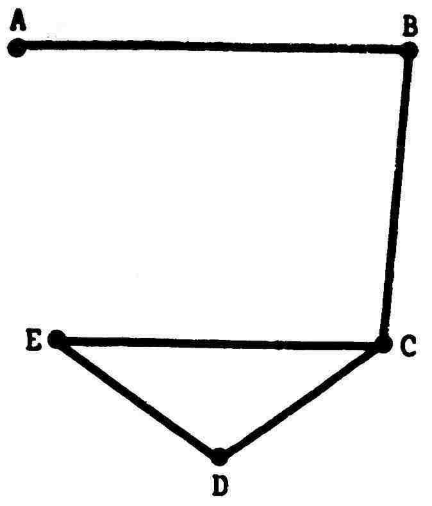
Matrix Representation

It has been observed that the most fundamental characteristic of a graph is the relationship between the edges and the vertices. The graph is completely specified as soon as it is known which edges are incident upon which vertices. Such a specification can be made through a simple diagram, such as has been used in the preceding discussion, or even more compactly by means of a matrix.

The matrix representation which has proven most useful in many types of network analysis is known as the connection matrix. In a graph with v vertices, the connection matrix is a v x v matrix where each row and each column correspond to a specific vertex in the graph. The elements of the matrix are zero or one depending upon the existence or nonexistence of an edge directly connecting the two vertices. That is, $c_{ij}=1$ if there is an edge which is incident at one end upon vertex i and at the other upon vertex j. The element $c_{ij}=0$ if no such direct connection exists. The elements upon the principal diagonal, the c_{ii}, which represent internal or self linkages are usually defined as either all zeros or all ones, depending upon the structure of the problem being investigated. (See Figure 6.)

In a system that is not completely connected, there will be many places between which no direct link will exist. However, it is quite possible that these places may be reached by moving through one or more intermediate vertices, that is, via some indirect route. Given the connection matrix, it is possible to determine how many indirect routes of any given length connect any two vertices in the system. For instance, if it is desired to know the number of two-link routes that exist between two vertices in the system say i and j, we may do so by finding the square of the original connection matrix. The ij-th element of the matrix C^2 is then interpreted as the number of two-link routes connecting vertex i and vertex j. (See Figure 7.) A similar procedure is followed for routes with greater numbers of links. C^5, for instance, will indicate how many five-link routes exist between each vertex and every other vertex. The entries in these cells, however, contain an unknown number of redundant paths since many paths have been counted which contain edges with a multiplicity greater than one. However, it is possible to calculate the number of non-redundant paths (that is, those containing only edges of multiplicity one) of any given length by means of a relatively complex mathematical manipulation. (See the work by Ross and Harary, Reference 26.)

We may imagine two vertices in the system which are quite "remote" from each other. That is, there are no direct links between them, no two-stage, no three-stage, no four-stage, etc., links. In this case, the corresponding element of successive powers of the connection matrix will remain at zero. Eventually, the entry in this cell will change from zero to some non-zero number if the graph is connected. If the two vertices are the "most remote" on the system, it can be seen that the matrix will now contain no zeros. The power to which the original connection matrix has been raised to obtain this situation, is known as the solution time of the network, or the diameter of the



$$C = \begin{array}{c|ccccc} & A & B & C & D & E \\ A & 0 & 1 & 0 & 0 & 0 \\ B & 1 & 0 & 1 & 0 & 0 \\ C & 0 & 1 & 0 & 1 & 1 \\ D & 0 & 0 & 1 & 0 & 1 \\ E & 0 & 0 & 1 & 1 & 0 \end{array}$$

Figure 6: A Graph and its Connection Matrix

$$C^2 = \begin{array}{c|ccccc} & A & B & C & D & E \\ A & 1 & 0 & 1 & 0 & 0 \\ B & 0 & 2 & 0 & 1 & 1 \\ C & 1 & 0 & 3 & 1 & 1 \\ D & 0 & 1 & 1 & 2 & 1 \\ E & 0 & 1 & 1 & 1 & 2 \end{array}$$

Figure 7: Square of the Matrix C

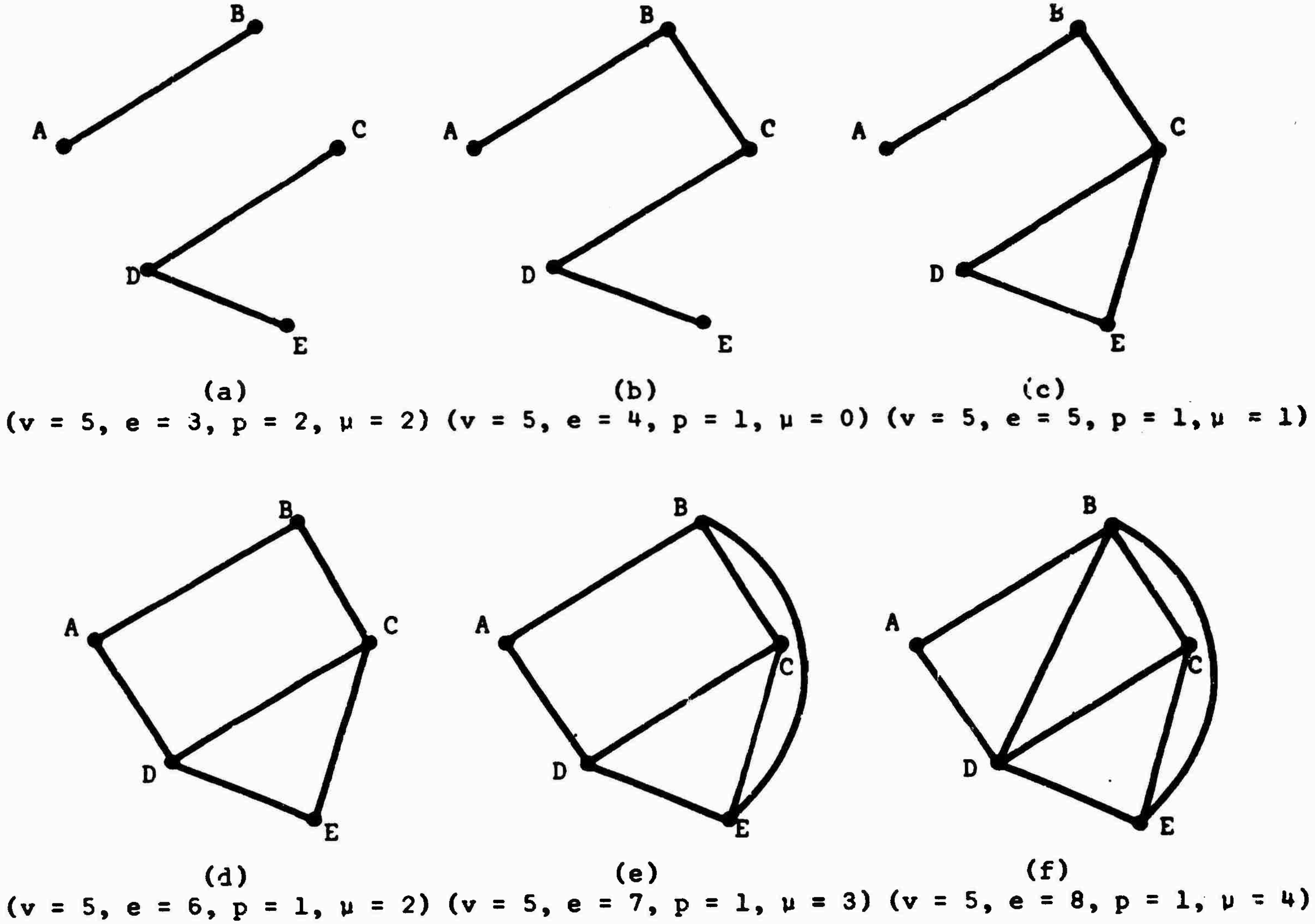
system. Put another way, the diameter or solution time of the system may be found by listing the number of links in the shortest path between each pair of nodes and selecting the largest of these numbers.

The Development of Structural Indices

If the transportation network is viewed as a graph, it becomes useful to develop certain summary indices which relate to the structure of the network. A certain amount of information has been lost in passing from the actual system to its graph or matrix representation, and further loss becomes necessary in order to assist in information handling. The problem here is similar to one encountered in most forms of statistics where data has been gathered in the form of a frequency distribution. In order to obtain a readily comprehensible summary index of the distribution, and in order to be able to distinguish between different distributions, certain summary measures such as mean, variance, etc., have been developed. The problem at hand in the present case is to compile a set of similar measures pertaining to the structure of the graph.

The Betti Numbers

In the study of the theory of linear graphs, mathematicians have developed certain indices which are regarded as invariant; that is, their values are not changed by isomorphic transformations of the graph. Perhaps the easiest to comprehend of these are the 0^{th} Betti numbers, which is a count of the number of disconnected parts of the network, and the 1^{st} Betti number, or cyclomatic number as it is commonly known, which presents a somewhat more sophisticated index pertaining to network structure. If one cord is added to a tree, a graph is obtained with a unique single circuit known as a fundamental circuit. If this operation of cord addition is repeated for a graph with v vertices, there are e-v+1 circuits consisting of each cord and its unique tree path. In a more general fashion, if the graph is not connected, it consists of maximal connected subgraphs. A tree can be defined for each subgraph and a set of these trees is called a forest of G. It follows that there are v-p elements in the forest and e-v+p elements not in the forest. This number is μ, the cyclomatic number, and it is a count of the number of fundamental circuits existing in the graph. In one sense, the cyclomatic number may be considered to be a measure of redundancy in the system. Since it was noted that a tree provides one, and only one, path between any pair of points, it can be seen that additional paths provided by circuits are redundant and that the total number of circuits present in the graph may be considered as a crude measure of the redundancy of the system. As may be seen from the structure of this index, any tree or disconnected graph has a cyclomatic number of 0, whereas as the graph moves closer and closer to the completely connected state, the cyclomatic number increases. (See Figure 8.) Applying this notion to the structure of transportation networks, it might be hypothesized that the magnitude of the cyclomatic number



(a) (v = 5, e = 3, p = 2, μ = 2) (b) (v = 5, e = 4, p = 1, μ = 0) (c) (v = 5, e = 5, p = 1, μ = 1)

(d) (v = 5, e = 6, p = 1, μ = 2) (e) (v = 5, e = 7, p = 1, μ = 3) (f) (v = 5, e = 8, p = 1, μ = 4)

Figure 8: The Relation of the Cyclomatic Number to Network Structure

which characterizes a nation's transportation system would bear a direct relationship to the level of social and economic development of the nation.

The Alpha and Gamma Indices

While the cyclomatic number does provide an index of network structure that is invariant under isomorphic transformations, it does not provide a readily intelligible measure of structure since it is bounded below by zero and bounded above only by some number which is a function of the number of nodes in the system. Ideally, it would be desirable to transform this index in such a manner that has common upper and lower bounds for all networks. Two additional measures are suggested which have this property, and which also remain invariant under isomorphic transformations. They will be defined as the alpha and gamma indices.

The gamma index for a planar network with e edges and v vertices is defined as $\alpha = \frac{e}{3(v-2)}$. This is the ratio of the observed number of edges, e, to the maximum number of edges in a planar graph. Obviously, this index would have a slightly different structure where non-planar graphs were under consideration (for example, airline routes). In any network, the maximum number of direct connections is strictly a function of the number of nodes present. For a given network, as the number of edges in the system decreases, the gamma index will approach one as an upper limit. It appears to be most convenient to express this index as a percentage and it is therefore multiplied by 100, giving it a range from 0 to 100, and it is then interpreted as percent connected.

The alpha index is somewhat similar, consisting of the ratio of the observed number of fundamental circuits to the maximum number of fundamental circuits which may exist in the system. The observed number of fundamental circuits is, of course, the cyclomatic number. Whereas the maximum number of fundamental circuits is equal to the number of edges present in a completely connected planar graph minus the number of edges contained in the complete tree. Therefore, for a planar graph, the alpha index will have the form $\alpha = \frac{\mu}{2v-5}$. This index is also multiplied by 100 to give it a range from 0 to 100 and an interpretation as percent redundant.

The two indices provide two percentage measures of the structure of a network. The alpha index may be interpreted as a percent redundant with a tree having zero redundancy and a maximally connected network having 100 percent redundancy. The gamma index, on the other hand, may be interpreted as percent connected with a completely unconnected system having a zero value and a completely connected system having a value of 100 percent.

The indices which have been suggested here are far from exhaustive and, in their present form, are far from adequate. Perhaps their major deficiency, together with that incurred in the general use of graph-theoretic models, is the lack of a precise statement pertaining to the angular structure of the network. The work by Beckmann (Reference 27) has shown how important this item is in the analysis of transportation systems. However the incorporation of such a measure would be extremely difficult, and it would appear wise to evaluate the empirical performance of the indices proposed here before embarking upon a more sophisticated analysis.

International Comparisons Of Indices Of Network Structure

The previous sections have discussed how the structure of transportation networks may be viewed from a rather simple mathematical point of view and have presented a number of network measures. The measures represent succinct, though perhaps unfamiliar, ways to summarize the structure of transportation networks. The present section reports the use of regression studies of these measures to answer the question "Can the structures of transportation systems be related to the features of the areas within which they are located?" In addition to the measures developed in a previous chapter, certain other measures developed for other aspects of the research were used in the regression studies. This chapter presents, in turn, the measurements used in the regression analyses, the computations made, and the results of the computations.

Inputs: The Independent Variables

The characteristics of areas containing transportation networks were represented by independent or explanatory variables. These variables fall into two categories -- characteristics of areas that are functions of (1) the level and nature of economic, social, and resource development, and (2) the physical makeup of the area. The former may be described as development variables and the latter as physical variables.

Development Variables

There is widespread agreement that transportation development is closely correlated with the level of national development. In the preceding section, certain indices were developed to measure the structure of transportation development. Before the relation between the level of national development and these transportation measures can be investigated, measures must be established for the notion "national development". Much work has been done on the notion of development, and the problem of measurement has been solved elsewhere to a degree sufficient for the current needs of this research. The ensuing discussion takes advantage of a detailed statistical analysis of the development measurement question by Brian J.L. Berry (Reference 28).

Statistics are available for many nations treating such matters as the value of foreign trade, value of imports, development of energy resources, population density, and newspaper circulation. Berry found some 43 such measures for some 95 nations. Any one of these might serve as an index of development in an approximate fashion. The notion is tempting that if one measure is a useful one, two or more indices should be better. However, an inspection of the set of statistics will reveal quickly that various measures are redundant on each other. Berry's contribution was the combining of these statistics in a manner that established the basic factors that underlie variations of the statistics. These results were obtained from a direct factor analysis of a table showing the ranks of 95 countries on the 43 available statistical measures. Nations were ranked from 1 to 95. The nation with a score of 1 would be that nation with the highest value. The nation with the score of 95 was that nation with the lowest value.

Berry's work revealed that four basic factors underlie variations in measures of degrees of development. These factors were: technological level, demographic level, income and external relations level, and size level. Nearly all of the variability that occurred in the 43 statistics could be attributed to the first two of the factors. The remaining two factors were relatively unimportant. Technological level takes account of various measures that may be made on the degree of urbanization, industrialization, transportation, trade, income, and the like. Demographic level reflects largely birth and death rates, population densities, population per unit of cultivated land, and similar measures.

In summary, development may be measured by synthesizing certain available statistics. The study of these statistics has revealed that the development of a nation may be measured on two scales: a technological scale and a demographic scale. Also, it was noted that although these scales were derived from a rather complex set of operations on a large number of statistics, it is possible to assign values appropriate to individual areas using simple information and a relatively simple technique. Thus, a relatively simple set of information will yield measures of development interpretable in terms of a large set of statistical indices.

Physical Variables

The nature of the transportation network may depend upon the physical properties of the area it traverses. Three physical properties of areas were measured -- size, shape, and relief.

Information on the size of areas is available in a number of places. The data used here were taken from tables in the Encyclopedia Britannica and rounded by two decimal points. For example, Tunisia was reported as 48,332 square miles in size. This was recorded as 483 for purposes of this study.

Shape was measured on maps. The longest axis across each nation was determined by inspection and a perpendicular constructed across the nation at the midpoint of the longest axis. The measure of shape was obtained by dividing the airline distance along the longest axis by the airline distance along the perpendicular. It might be noted that this is not a completely satisfactory measure of shape. A nation shaped like a rectangle may have the same measure of shape as a nation shaped like an ellipse if the ratio is the same for the two areas. It might also be noted that this measure of shape is a pure number and, so far as the measure is concerned, is independent of size.

As is true of shape, there is no entirely satisfactory measure of the relief of areas. The measure used here was constructed in an ad hoc manner and proved to be suitable to the study. Three lines were drawn at random across each area of study, and the airline length of each line was measured. The distance along each route was also measured along the surface using profiles in the Times Atlas. These surface routes, of course, were greater than or equal to the airline distances. The airline route was taken to be 100 percent and the surface route a percentage larger than 100 percent. For each country, the "percent larger" sums were added and divided by three, and the resulting value was used to express the relief of the area of study.

The Data

Transportation networks within 25 nations were selected for analysis and values of the 5 independent variables described above were computed for each nation. Table 10 presents the list of nations studied and the values of the independent variables. Preliminary investigation of the sizes of areas and the characteristics of their transportation systems revealed existence of nonlinear relationships. Before computations were made, the raw data on size were converted into natural logarithms. Table 11 displays the associations among the independent variables in terms of the correlation coefficients between variables taken two at a time.

The Dependent Variables

The rationale underlying this study is that transportation structure is dependent upon the characteristics of the area containing the network. The characteristics of areas have been summarized in terms of the independent variables just discussed. The dependent variables to be discussed now are those measures of transportation that are pertinent to the notion of transportation structure. These measures fall into two categories: (1) the measures based on graph-theoretic considerations and (2) measures based on certain other work.

TABLE 10

OBSERVED VALUES OF INDEPENDENT VARIABLES

		Technological* Development	Demographic* Level	Size**	Shape	Relief
1.	Tunisia	351	32	4.683	2.677	5.51
2.	Ceylon	323	14	4.403	2.510	3.54
3.	Ghana	355	15	4.962	2.506	2.83
4.	Bolivia	370	18	5.627	2.135	20.00
5.	Iraq	344	25	5.234	2.376	2.21
6.	Nigeria	394	0	5.530	2.164	3.60
7.	Sudan	410	6	5.985	2.093	4.48
8.	Thailand	400	9	5.297	2.245	5.41
9.	France	125	38	5.327	1.468	12.04
10.	Mexico	222	19	5.881	2.422	20.96
11.	Yugoslavia	241	16	4.994	2.553	23.25
12.	Sweden	154	55	5.239	2.872	8.34
13.	Poland	182	25	5.080	2.155	3.15
14.	Czechoslovakia	159	38	4.693	2.706	13.93
15.	Hungary	221	29	4.555	2.521	5.94
16.	Bulgaria	279	47	4.631	2.438	12.97
17.	Finland	202	46	5.114	2.780	0.35
18.	Angola	438	28	5.682	1.809	1.42
19.	Algeria	323	26	5.963	1.230	1.52
20.	Cuba	256	37	4.645	2.979	12.78
21.	Rumania	258	23	4.962	2.135	25.01
22.	Malaya	256	17	4.705	2.159	19.51
23.	Iran	372	12	5.803	2.422	8.50
24.	Turkey	283	8	5.481	2.692	19.45
25.	Chile	239	24	5.456	3.100	66.80

*From Berry (Reference 5, p. 110), Table VIII-1, cols. 1 and 2, "Second Values". Twenty was added to each Demographic Level entry.

**Natural log of computed value.

TABLE 11

CORRELATION MATRIX, INDEPENDENT VARIABLES

	1 Technological Development	2 Demographic Level	3 Size	4 Shape	5 Relief
1.	1				
2.	-.61	1			
3.	.35	-.40	1		
4.	-.24	.21	-.43	1	
5.	-.27	-.04	.05	.36	1

Graph-Theoretic Measures

Six measures of a graph-theoretic type were made on transportation networks in each of the 25 nations selected for study. These were:

(1) The number of vertices, nodes, or places.

(2) The number of edges, links, or routes. A variety of sources were used for the vertex and edge measurements. Sources included World Railways (Reference 29) and maps published in various issues of the journal Road International. Definitions of vertices, and thus of edges, were partly topological and partly based on certain information contained on the maps. Any intersection of routes defined a vertex. Also, any place on the network deemed significant by the person who drafted the map was taken to be a vertex. End points were always treated as vertices.

(3) Alpha index. This is the cyclomatic number (see item 5 below), divided by the maximum possible number of complete circuits, or $\mu/(2v-5)$.

(4) Gamma index. This is the number of observed edges divided by the maximum possible number of edges in a planar graph with the observed number of vertices, or $e/3(v-2)$.

(5) Cyclomatic number. This is the measure of the number of circuits in the transportation system, or the number of links in the system excess to the number required to tie the vertices together in a minimal way. $\mu = e-(v-1)$.

(6) Diameter. This is a measure of the "span" of the transportation system. It is the minimum number of links that must be traversed in order to move between the two points that are the greatest distance apart on the network.

Other Dependent Variables

The above are six measures of some of the structural characteristics of transportation networks. These were supplemented by an additional measure that was adopted after extensive empirical measurements of transportation networks (Kansky, Reference 7). Measurements of the lengths of edges in miles proved practicable, and preliminary correlations indicated that such measures were significantly related to variables such as technological development. Consequently, measures were made in the nations under study of the average length of edges. Two measures were made, one for highways and one for railroads.

The Data

On the basis of preliminary graphic analysis, it was decided to transform many of the dependent variables to their natural logarithms. These transformations are listed below:

(1) Vertices, transformed to the natural logarithm of the observed number.

(2) Edges, transformed to the natural logarithm of the observed number.

(3) Cyclomatic number, one added to the observed value and the result transformed to its natural logarithm.

(4) Average highway edge length, transformed to the natural logarithm of the natural logarithm of the observed value.

(5) Average railroad edge length, transformed to the natural logarithm of the natural logarithm of the observed value.

The purpose of these transformations was to assume linearities in the regression analyses. No attempt is made to indicate the presence of these transformations in various tables that follow in this chapter.

Table 13 indicates the relationships between the dependent and independent variables taken two at a time. It may be noted, for example, that there is a correlation of -.86 between the natural

TABLE 12

OBSERVED VALUES OF THE DEPENDENT VARIABLES

	Nation	No. of Nodes	No. of Edges	Alpha Index	Gamma Index
1.	Tunisia	3.970	4.043	5.00	37.30
2.	Ceylon	3.466	3.434	0.00	34.40
3.	Ghana	3.714	3.714	1.30	35.00
4.	Bolivia	4.060	4.078	1.80	35.11
5.	Iraq	3.496	3.496	1.63	35.48
6.	Nigeria	3.989	4.159	10.70	41.00
7.	Sudan	3.296	3.296	2.04	36.00
8.	Thailand	3.989	4.007	1.94	35.25
9.	France	6.433	6.733	17.67	45.16
10.	Mexico	5.236	5.371	7.54	38.50
11.	Yugoslavia	5.553	5.727	9.78	39.90
12.	Sweden	5.771	6.094	19.30	46.20
13.	Poland	5.226	5.529	18.25	45.60
14.	Czechoslovakia	5.553	5.802	14.14	43.00
15.	Hungary	5.645	5.916	15.68	44.00
16.	Bulgaria	4.454	4.554	6.00	37.70
17.	Finland	5.118	5.170	3.00	35.60
18.	Angola	4.248	4.344	5.90	37.70
19.	Algeria	4.220	4.407	11.40	41.40
20.	Cuba	4.511	4.682	10.20	40.40
21.	Rumania	5.493	5.645	8.50	39.10
22.	Malaya	3.951	3.931	0.00	34.00
23.	Iran	3.689	3.611	0.00	32.45
24.	Turkey	4.727	4.718	0.45	33.60
25.	Chile	5.004	5.050	3.75	35.30

TABLE 12 -- Continued

Cyclomatic Number(+1)*	Diameter	Average Edge Length (Hwy)**	Average Edge Length (Rail)**
1.792	19	0.218	0.150
0.000	14	0.162	0.275
0.693	15	0.215	0.237
1.099	31	0.190	0.291
0.693	21	0.185	0.259
2.398	14	0.227	0.362
0.693	13	0.232	0.356
1.099	24	0.219	0.300
5.389	43	0.149	0.214
3.332	43	0.223	0.284
3.912	35	0.180	0.151
4.812	41	0.222	0.185
4.205	21	0.148	0.186
4.290	41	0.130	0.170
4.477	31	0.133	0.074
2.303	17	0.155	0.139
2.303	36	0.131	0.181
2.079	8	0.275	0.319
2.708	18	0.243	0.226
2.890	24	0.178	0.168
3.714	27	0.156	0.160
0.000	21	0.186	0.233
0.000	--	0.237	0.351
0.693	--	0.159	0.271
2.485	--	0.202	0.086

*Natural Log
**Natural log of natural log

logarithm of the number of vertices and the index of technological development. This indicates, as would be expected, that the more developed the area, the greater the number of nodes or vertices on the transportation system. It may also be noted that the more developed the country, the shorter is the average edge length.

TABLE 13

SUMMARY: SIMPLE COEFFICIENTS OF CORRELATION BETWEEN THE DEPENDENT AND INDEPENDENT VARIABLES

	Technological Development	Demographic Level	Size	Shape	Relief
1. Vertices	-.86	.53	-.15	.08	.30
2. Edges	-.85	.55	-.16	.05	.26
3. Alpha Index	-.64	.50	-.12	-.14	-.09
4. Gamma Index	-.61	.49	-.14	-.18	-.15
5. Cyclomatic Number	-.73	.56	-.13	-.04	.11
6. Diameter	-.79	.44	-.02	.24	.49
7. Average Edge Length (Highway)	.67	-.34	.64	-.27	-.10
8. Average Edge Length (Rail)	.66	-.62	.61	-.40	-.40

The Computations

Eight regression analyses were made. In each analysis the value of a dependent variable was assigned to the independent variables to the extent that variations in the data indicated that assignments were warranted. This section contains a discussion of the regression model used, the steps in computation, and the outputs from the regressions. It will be noted in this section that these computations were very good in the sense that much of the variability in individual dependent variables could be associated with the independent variables.

The model used in this study was the standard linear equation used in regression studies, namely:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + E .$$

y represents the value of one of the dependent variables--say, number of vertices--for a nation. x_1 through x_5 represent the values of the independent variables for that nation. These were respectively: technological development, demographic level, size, shape, and relief. Values b_0 through b_5 are the regression coefficients estimated from the data. Each of the eight regressions was done in the following way: First, estimates were made of values of the regression coefficients using least-squares methods, the standard method for regression analysis. While the necessary matrix multiplications and inversions were accomplished for the data as a whole, the regressions differed somewhat from usual computations in that the regression coefficients were determined one at a time. In the first regression, for instance, the value of the coefficient b_1 was established and certain auxiliary computations made of correlation and error. Next, the value of regression coefficient b_2 was established and the estimates of error and correlation recomputed. These computations were continued in this incremental fashion until all 5 of the regression coefficients were determined. Since there were 5 of the regression coefficients for each of the 10 regressions, some 50 regressions were actually run. It is most convenient, however, to speak of 10 regressions in all. The preliminary 4 regressions of each of the dependent variables are of greatest interest in regard to the estimates of error to be discussed below.

Outputs

The outputs from the calculations fall into two categories: (1) the regression coefficients, and (2) various estimates of error or reliability of the regressions. The regression coefficients are given in Table 14; these are numerical values of the b's in the linear regression equation given above. For the first regression, for instance, we have:

$$y = 6.30 - .008x_1 - .006x_2 = .201x_3 - .267x_4 - .007x_5 = e$$

or:

$$\left\{\begin{matrix}\text{number of}\\ \text{vertices}\\ \text{on network}\end{matrix}\right\} = 6.30 - .008 \left\{\begin{matrix}\text{index of}\\ \text{technological}\\ \text{development}\end{matrix}\right\} - .006 \left\{\begin{matrix}\text{index of}\\ \text{demograph-}\\ \text{ic level}\end{matrix}\right\}$$

$$+ .201 \left\{\begin{matrix}\text{size}\\ \text{measure}\end{matrix}\right\} - .267 \left\{\begin{matrix}\text{shape}\\ \text{measure}\end{matrix}\right\} - .007 \left\{\begin{matrix}\text{relief}\\ \text{measure}\end{matrix}\right\}$$

$$+ \text{ error of the estimate}$$

and similar equations may be written for the nine remaining regressions.

Several different measures were made of the reliability of the regressions. Table 15 presents a measure of how well the regressions work.

TABLE 14

SUMMARY: REGRESSION COEFFICIENTS OF THE EIGHT REGRESSIONS

	Technological Development	Demographic Level	Size	Shape	Relief	Constant Term
1. Vertices	-.008	-.006	.201	-.267	-.007	6.30
2. Edges	-.009	-.007	.213	-.335	-.006	6.74
3. Alpha Index	-.049	.068	1.208	-3.306	-.090	22.123
4. Gamma Index	-.031	.043	.412	-2.419	-.071	50.543
5. Cyclomatic Number	-.012	.027	.354	-.799	.001	5.278
6. Diameter	-.0918	.012	10.101	7.447	.347	-21.565
7. Average Edge Length (Highway)	.0003	.0007	.0423	.0027	.0002	-.1424
8. Average Edge Length (Rail)	.0001	-.0023	.0802	.0138	-.0025	-.1757

TABLE 15

SUMMARY OF COEFFICIENTS OF DETERMINATION OF THE 8 REGRESSIONS

	Technological Development	Demographic Level	Size	Shape	Relief
1. Vertices	.73	.73	.76	.77	.77
2. Edges	.73	.73	.75	.76	.77
3. Alpha Index	.42	.44	.46	.54	.57
4. Gamma Index	.37	.40	.41	.52	.56
5. Cyclomatic Number	.54	.56	.59	.62	.62
6. Diameter	.62	.62	.67	.75	.80
7. Average Edge Length (Highway)	.45	.46	.66	.67	.67
8. Average Edge Length (Rail)	.43	.51	.63	.65	.77

The coefficients of determination, sometimes termed the power of the model, are numerically the squares of the multiple correlation coefficients, and they may be interpreted as the percent of the variability in the dependent variable associated with variation in the independent variables. The first entry indicates that some 73 percent of nation-to-nation variation in the number of vertices may be associated with the level of technological development , and the right-hand entry of the first column indicates that some 77 percent of the variability from nation-to-nation in number of vertices may be associated with the five independent variables taken altogether.

A number of other outputs from the regression bear on the "goodness" of the regressions. The 22 to 25 nations used in this study may be viewed as a sample from a larger set of nations, though it might be somewhat difficult to decide on the number of nations in the world. The United Nations' Statistical Yearbooks list approximately 260 political divisions, but about 100 of these are subdivisions of larger units. Ginsburg (Reference 30) found it practicable to consider about 140 countries. Thus, the computations made here might be regarded as a sample from approximately 140 areas. Still a broader view might be adopted. The 140 areas might be regarded as displaying patterns from a larger universe of transportation network structures that might have developed, given the conditions that control network development. This set of possible patterns might be very large or even unlimited in number. Reasoning this way, the sample might be regarded as one from an extremely large universe of possible transportation network structures. Table 16 presents the variance ratios for the regressions. In connection with the coefficients of determination of the regressions (Table 15), it was mentioned that these may be regarded as the amount of variation in the dependent variable associated with variations in the independent variables. The variance ratios permit significance tests of the increments in the variation explained by the regressions as individual regression coefficients are added. It may be seen that the reduction in the variance associated with technological development is extremely significant and that other reductions in variance are significant on occasion.

The regression equations may be viewed as forecasting devices. Observations on the independent variables may be entered into the equations and estimates of transportation structure derived. A change in some character of an area, such as level of technological development, may be postulated and new values of transportation structure estimated. Persons using regression equations of this sort for estimating purposes should proceed with caution, of course. For one thing, estimates that extend beyond the range of variability in the original data should be made only with great care. Too, the regressions limit themselves to measures of structure and do not display exactly how a change will take place on the map. (This point is investigated later in this monograph.) In addition to the preceding points, the behavior of errors of estimation is critical where estimates are made. For this reason, a number of measures bearing

TABLE 16

VARIANCE RATIOS WITH 1 AND m-j DEGREES OF FREEDOM

j equal Variable	Technological Development	Demographic Level	Size	Shape	Relief	m No. of Observations
	1	2	3	4	5	
Vertices	63.31**	.01	2.32	.48	.76	25
Edges	61.27**	.07	2.23	.90	.39	25
Alpha Index	16.43	.75	.83	3.53	1.18	25
Gamma Index	13.73**	.83	.45	4.44*	1.67	25
Cyclomatic Number	26.62**	1.03	1.51	1.84	.00	25
Diameter	32.33**	.15	3.73	4.30*	4.21	22
Average Edge Length (Highway)	18.69**	.33	13.08**	.14	.14	25
Average Edge Length (Rail)	17.30**	3.45	7.38*	.61	10.26**	25

**Significant at the 1% level.
*Significant at the 5% level.

on errors of estimation were obtained in the course of the regression calculations.

Table 17 presents the standard errors of estimate for the eight regressions. In discussing the regression equation earlier in this section, mention was made of the error term that must be introduced into the equation in order to correct the computed structure value to the actual observed value. The standard error of the estimate is a measure of the distribution of these error terms (their means are zeros). For the first regression, for example, about 66 percent of the computed values from the regressions lie within plus or minus one standard error or plus or minus .457 of the observed values; 95 percent lie within plus or minus two standard deviations, or plus or minus .950.

TABLE 17

STANDARD ERRORS OF ESTIMATE*

Variable	Standard Error of the Estimate
1. Vertices	.457
2. Edges	.514
3. Alpha Index	4.617
4. Gamma Index	3.033
5. Cyclomatic Number	1.127
6. Diameter	5.488
7. Average Edge Length (Highway)	.026
8. Average Edge Length (Rail)	.043

*Unbiased

Results

It might be wise to repeat certain comments made earlier on the structure of this study before going on to a summary of the results of the regressions, because the results of the regressions must be judged within the overall structure of the study. Earlier the question was asked, "Can the structures of transportation systems be related to the features of the areas within which they are located?" In terms of certain measures of structure and certain ways of measuring the "characteristics of the features of the areas", the answer to this question is a definite yes. But beyond this it is desired to answer this question in the affirmative so far as the actual networks or maps of the transportation systems of areas are concerned. That is, the ultimate answer to the question requires generating the actual transportation network, given the characteristics of the area that contains the network. The first step in obtaining the ultimate answer to this question is that of generating the characteristics of network structure, so our ability to answer the question with a yes in the case of the characteristics of the network is an essential part of the objective of the overall objective of the research.

Simplicity and preciseness may always be taken to be desirable attributes of models. It might be noted again that the regression equations used in this study have these attributes. Measurement requirements for calculating values of the dependent variables are not at all demanding. Computation of size, shape, and relief are simple and straightforward. While development of the technological development and demographic level scales required a great deal of statistical work, values on these scales may be estimated from very simple information. The values of the independent variables are also very simple to measure, although they too depend upon certain mathematical considerations.

As was mentioned before, the fit of the regression model to the data may be regarded as quite good. This is a relative matter, of course. Also, it was noted that the residuals or errors were such that introduction of new data into the regressions and limited projections may be made with confidence.

It was remarked earlier that these regressions are but one stage of an effort to reproduce actual transportation networks from data on the characteristics of areas. The question of the overall pertinence of the regressions will be left open until that portion of the research is discussed. However, it is possible to make certain summary remarks on the relationships within the regressions and those remarks will be made here. Tables given earlier in this discussion give the units within which the data were measured, the regression coefficients, and the variance ratios.

The regression coefficients are subject to certain general interpretations. The table showing variance ratios (Table 16) permits identification of those regression coefficients associated with the stronger relations between independent and dependent variables. It may be seen from this table that technological development is always a major determinate of structure, with other of the independent variables important only in certain cases. Referring to the technological development column of regression coefficients in the summary of regression coefficients (Table 15), it may be seen that for the first six regressions the more developed the country (and smaller the measure of technological development) the higher the value of the dependent variable. Just the reverse is true for the last two regressions. The more highly developed the country, the smaller the average edge length and the smaller the structure index. The demographic measure is of little importance, and size is of importance only in the case of the last four measures. The greater the size, the greater the average edge length and structure index, this being more true for highways than for railroads. The shape measure is significant in relation to diameter and the gamma index, while relief has its greatest effect on the edge length for railroads.

The above statements are a mixture of observations based on the magnitudes of the regression coefficients and their relative significance in the variance ratio tests. Additional findings may be made by writing out the individual regression equations and studying their sensitivity to variations in the independent variables.

The structural indices used in this study are, with the exception of average edge length, all basically derived from information about the number of nodes and edges in the graph. Some redundancy might well be expected, and a casual examination of the regression coefficients for, say, the alpha and gamma indices (see Table 14) reveals a very similar pattern of response to variations in the level of the independent variables.

Computation of the simple intercorrelations between the dependent variables in the various regressions reveals that the redundancy is quite high in almost all cases; for example, the simple correlation between the alpha and gamma indices for 22 nations is 0.998 (see Table 18). Analysis of these interdependencies by means of principal components analysis (with varimax rotation to "simple structure") reveals that the six indices may be collapsed into three factors (see Table 19). The first two of these factors accounted for nearly 98 percent of the observed communality. The first, and by far the most important, expressed network complexity as a function of the number of nodes, the number of edges, and the cyclomatic number; the second factor combines the alpha and gamma indices into a measure of network completeness, while the third factor, which may be interpreted as a size measure, related the number of nodes, the number of edges, and the diameter of the network.

This information about redundancy would appear to indicate that the eight regressions could be replaced by, say, three regressions with very little loss of information.

This section has reported the measurements used, computation, and results from eight regression studies. These regression studies established relationships between certain measurements of transportation network structure and measurements of the characteristics of the areas within which these networks lie. It was found that measures of network structure could be related rather closely to the characteristics of the areas containing the networks and that the technological development was the more important factor conditioning the character of transportation systems.

TABLE 18

CORRELATION MATRIX - DEPENDENT VARIABLES*

	No. of Nodes	No. of Edges	Alpha Index	Gamma Index	Cyclomatic No.	Diameter
1	1					
2	.98	1				
3	.73	.75	1			
4	.71	.74	.99	1		
5	.96	.97	.70	.79	1	
6	.75	.70	.48	.46	.65	1

*Based upon first 22 observations

TABLE 19

ROTATED FACTOR LOADINGS OF THE GRAPH-THEORETIC STRUCTURE INDICES

		FACTOR		
	VARIABLE	1	2	3
1	No. of nodes	0.638	0.388	0.660
2	No. of edges	0.687	0.419	0.576
3	Alpha index	0.294	0.911	0.286
4	Gamma index	0.313	0.919	0.237
5	Cyclomatic number	0.724	0.492	0.461
6	Diameter	0.283	0.208	0.740
Percent of communality*		85.6	12.0	2.4

*Over all factors

Empirical Studies of Intra-Nation Network Structure

The preceding section reviewed empirical studies of nation-to-nation variations in the structure or layout of transportation systems and their relation to varying environmental conditions. The present section is used to report upon a study of certain structural details of transportation systems within nations. It has been noted that when viewed as a group, transportation systems may be represented by their connection matrices. Connection matrices were constructed for the surface transportation networks of all the study nations mentioned in the last section, as well as for a number of others. Connection matrices for local air transport services within a number of Central and Latin American nations were also constructed to see if any basic differences existed in the patterns developed by service and air transport systems.

An examination of the connection matrices for the various surface transportation networks revealed one common factor: a heavy concentration of entries near the main diagonal of the matrix. That is, one dominant characteristic of the systems was that the routes that made up the system tended to link urban centers mainly to their near neighbors. Aside from the "neighborhood effect", the connection matrices also revealed certain minor groupings of off-diagonal elements. This tendency toward off-diagonal groupings was somewhat accentuated when connection matrices of the local service airline routes were examined. Here some organizing effect, over and above the aforementioned neighborhood effect, appeared to exist, with certain nodes or groups of nodes exerting an organizing effect on the overall pattern of routes.

Apart from these very obvious and general conclusions, it is quite difficult to make any further statements about underlying structural patterns from visual examination of the connection matrices. Analysis in greater depth must utilize more explicit and powerful tools. The most appropriate appears to be a tool of statistical analysis known as principal component analysis. This analytic tool has been used by others, see MacRae (Reference 31), in cases of a similar underlying structure.

In the development of an analytic system with the capability of forecasting the transportation network of any area, several elements need to be incorporated. Research outlined previously has examined either the properties of transport systems in the aggregate, and their relationships to the degree of economic development of the areas they serve, or the structure of networks, comprising the patterns formed by transport routes which link nodes. A further aspect of network development is that of the emergence of capacity differentials on links between nodes. Such capacity differentials emerge in response to variations in demand for movements of commodities and people between places connected by the transport system. Thus they apply differential weighting to the edges of the network in creating a spatial system in which movements between places define the connectivity, the dominance and the subdominance, and similar relations of the nodes.

As with all systems, the one formed by differential weighting of the edges may be viewed at three levels: (a) that of patterns and frameworks (for example, commodity flow mapping); (b) that of functional interdependency (for example, of the connectivity of parts defined by flow differentials); and (c) that of dynamic systems (for example, of patterns and interdependencies changing through time in secular, cyclic, rhythmic, and stochastic ways). The first of these levels has been studied in detail many times. Hence, this paper focuses upon the second and third levels. Particular questions of interest are these:

(1) Is the functional organization of a set of nodes the same when this organization is viewed in terms of (a) unweighted links, and (b) links with differential weights specified by commodity flows?

(2) How volatile is the functional organization over relatively brief time spans? Is there any definable sequence of change which may be related to degree of economic development of the areas served and to rate and nature of economic growth of these areas?

These questions are posed since one might argue that several structural components exist, and that any network can be decomposed into these (neighborhood effects, nodal organization -- " field effects" -- at regional and national levels, and complex regional interdependencies). One might further argue that these components enter in a logical sequence during the process of economic growth (neighborhood effects creating "chains", followed by field effects first at regional and then at the national levels, and finally by complex regional interdependency). Others, however, have said that when differentially-weighted links are considered, a further stage beyond "complex organization" exists -- the emergence of "high-priority main streets", along which the majority of flows are consistently channelled. These topics will be considered here.

The case examined is that of India. The data pertain to interregional commodity flows within that country; they were obtained from both official statistics and special tabulations of the original data from which the official summaries had been drawn. (See Berry, Reference 11.)

For any one year (each year for a decade is available) the data comprise sixty-three 36X36 flow matrices. Sixty-three commodities are covered, and for each commodity the flows between 36 regions and major cities (the states of India as of 1956, plus such metropolitan centers as Calcutta, Bombay, Madras, and Delhi) are recorded. All flows are in quantity terms, but value conversions are available. For the present exploratory study, 4 of the 63 commodities were selected for analysis: cattle, grain and grain products, baled raw jute, and iron and steel. These were expected to provide a spectrum both in terms of the spatial organization of the Indian economy, and changes in this organization through time due to post-independence political changes (jute) and the Indian program of planned economic development (iron and steel). Three years were examined to provide a temporal perspective for each commodity: 1956, 1959, and 1961. Twelve 36x36 commodity flow matrices are therefore the bases of the analysis reported here.

The rotated factor loadings for one commodity (cattle) for one year (1956) are presented in Table 20. Note the loadings of the 36 receivers on factor 1; they are either very high or near zero, as is consistent with normal varimax rotation to simple structure. Receivers 4, 8, 13, 15, 19, and 22 have profiles of receipts from the 36 shippers that are very similar; the scores of the 36 shippers on factor 1 reveal the shape of the profile -- the peak score associated with shipper 16 shows this to be the major source for the 6 highly correlated receivers. Similarly on factor 2, receivers 18, 31, 33, 34, and 35 form a group (Table 20), with 6 the major shipper to the group. The interpretation proceeds similarly for all other factors (see also figures 9-21).

Figure 9 shows one simple way of representing the results. Highly correlated receivers are linked to principal shippers by desire lines. The existence of organizational regions comprising groups of contiguous states receiving from specialized producing areas is immediately apparent. Similar maps can be used to summarize the results of each of the factor analyses. Each map depicts the spatial organization (interdependence of nodes) implicit in a given commodity flow matrix. The functional organization is, in turn, based upon common patterns of receipts among receivers. Since a set of relatively light flows may have the same profile as a set of very heavy flows, these patterns reveal the essential skeleton of interdependencies, and they are consistent with factor analyses of route structures presented in the previous study. Supplementary volumetric information is required if one is to consider inequalities produced by differential weighting of the links, however; so long as the profiles of shipments and receipts have the same shape, correlations will be high regardless of the relative elevation of the profiles.

Figures 10-21 summarize the complete patterns of the four commodities selected for analysis for each of the three sample years.

Note in Figures 10-12 the ascending hierarchy of what might be called "field effects." Nodal regions of varying scale apparently exist for the various agricultural commodities. In Figure 10 the areas dominated by the major metropolitan centers all have their own specialized source of cattle, but as is shown in Figure 11, there are

TABLE 20

CATTLE, 1956: ROTATED FACTOR MATRIX

FACTOR NUMBER:			1	2	3	4	5
SUM SQUARES OVER VARIABLES:			5.244	4.995	3.958	3.082	2.801
RECEIVER NO.	**NAME**	**COMMUNALITY 9 FACTORS**					
1	ANDHRA	0.951	-0.023	-0.001	-0.034	-0.026	-0.043
2	ASSAM	0.997	-0.030	-0.017	0.995	-0.041	0.013
3	BIHAR	0.656	-0.050	-0.049	-0.048	0.025	-0.029
4	BOMBAY	0.871	0.878	-0.048	-0.051	0.078	0.068
5	MADPRA	0.752	0.138	-0.045	0.116	0.603	-0.157
6	MADRAS	0.865	-0.050	-0.033	0.001	-0.029	0.032
7	ORISSA	0.994	-0.022	-0.020	0.986	0.115	0.021
8	PUNJAB	0.945	0.970	-0.019	-0.016	-0.043	0.030
9	UTTPRA	0.966	0.132	-0.017	0.001	0.964	0.087
10	W BENG	0.992	-0.028	-0.017	0.993	-0.037	0.012
11	HYDERB	0.993	-0.004	-0.031	-0.030	0.213	-0.814
12	JAMKAS	0.	0.	0.	-0.	0.	-0.
13	MADBHA	0.724	0.785	-0.034	-0.030	0.248	-0.145
14	MYSORE	0.867	-0.015	0.061	0.015	0.042	-0.140
15	PATEPN	0.959	0.968	-0.010	-0.008	0.131	-0.009
16	RAJAST	0.922	0.063	0.004	0.007	0.952	-0.004
17	SAURAS	0.969	-0.010	-0.021	-0.024	0.023	-0.981
18	TRAVCO	0.999	-0.028	0.998	-0.019	-0.036	0.015
19	AJMER	0.958	0.968	-0.014	-0.013	0.106	0.047
20	BHOPAL	0.866	0.021	-0.029	-0.025	0.015	0.054
21	COORG	0.	0.	0.	-0.	0.	-0.
22	DEHLI	0.989	0.986	-0.016	-0.016	-0.089	0.021
23	HIMPRA	0.	0.	0.	-0.	0.	-0.
24	KUTCH	0.534	-0.040	-0.051	-0.064	0.066	-0.133
25	MANPUR	0.	0.	0.	-0.	0.	-0.
26	TRIPUR	0.	0.	0.	-0.	0.	-0.
27	VINPRA	0.558	0.002	-0.028	-0.014	0.014	-0.006
28	ANDR P	0.147	-0.051	-0.038	-0.041	-0.048	0.064
29	BOMB P	0.901	0.116	-0.023	-0.013	0.830	-0.309
30	CALCUT	0.984	-0.035	-0.025	0.989	0.011	0.023
31	COCH P	1.000	-0.027	0.998	-0.017	-0.035	0.013
32	GOA	0.	0.	0.	-0.	0.	-0.
33	MADR P	0.999	-0.020	0.993	-0.016	0.088	0.018
34	OMAD P	1.000	-0.027	0.998	-0.017	-0.035	0.013
35	PONDKA	1.000	-0.026	0.998	-0.017	-0.035	0.012
36	SAUR P	0.978	-0.014	-0.021	-0.023	0.027	-0.984

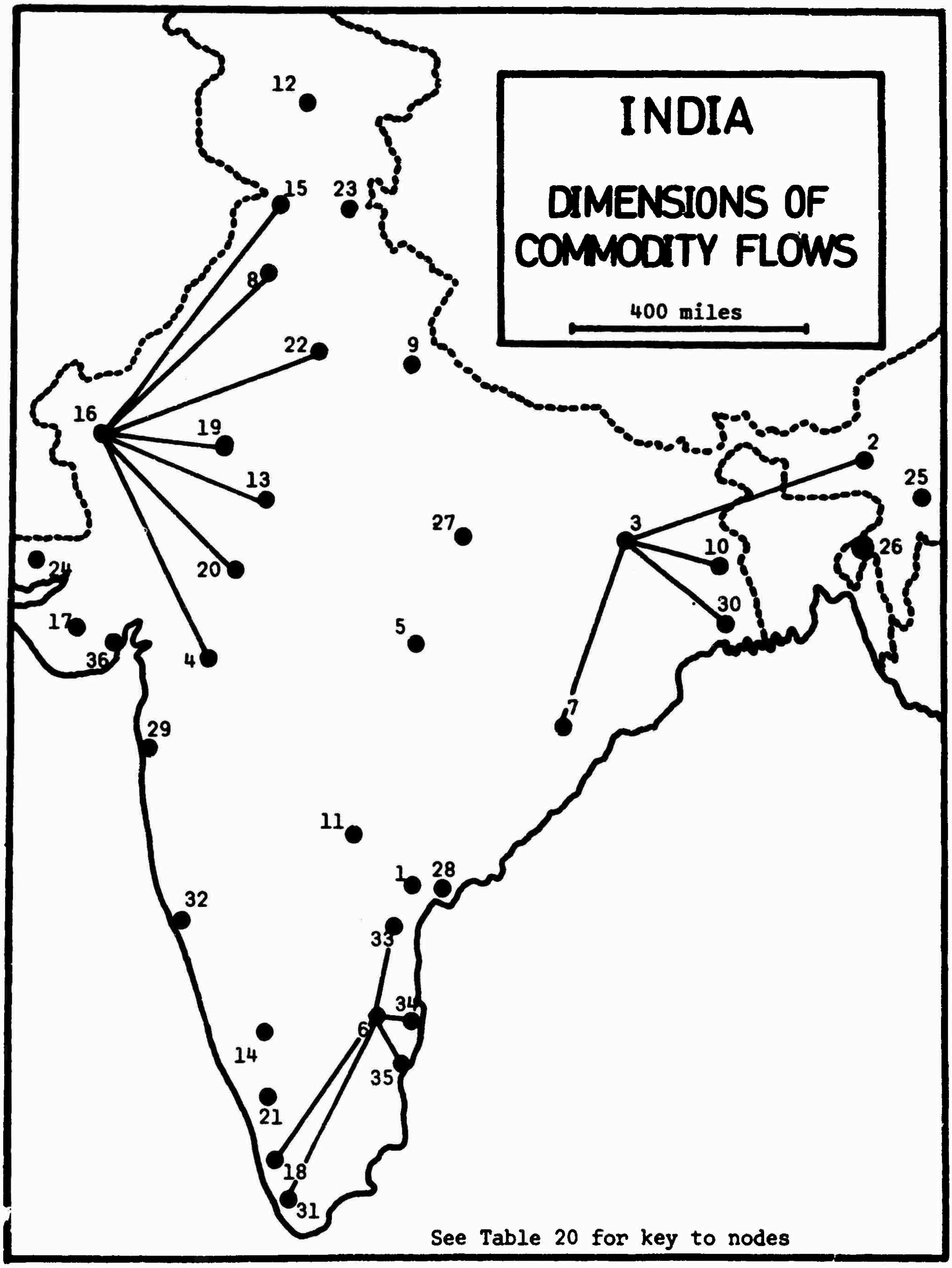


Figure 9: Cartographic Representation of Factors

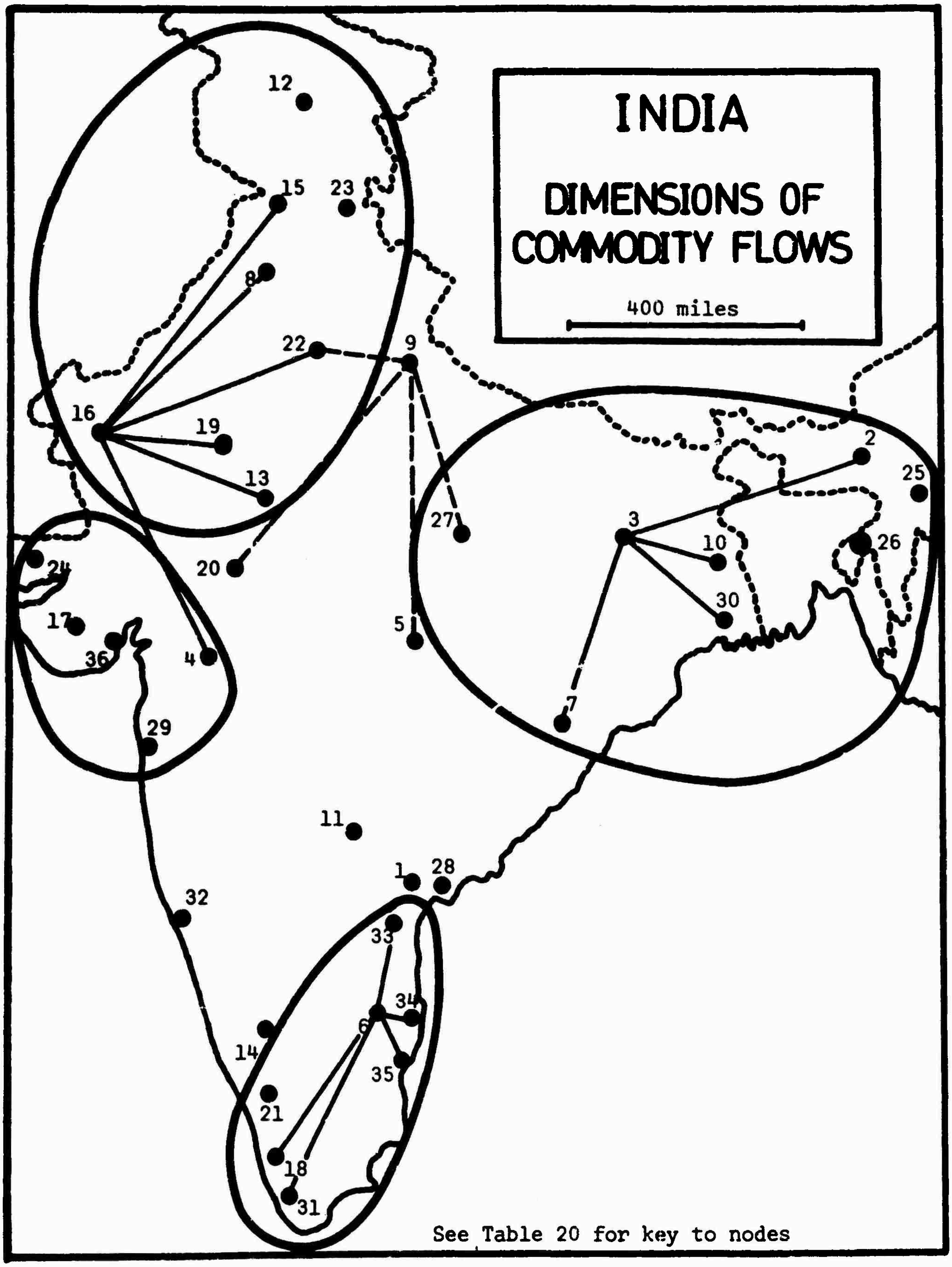


Figure 10: Cattle, 1956

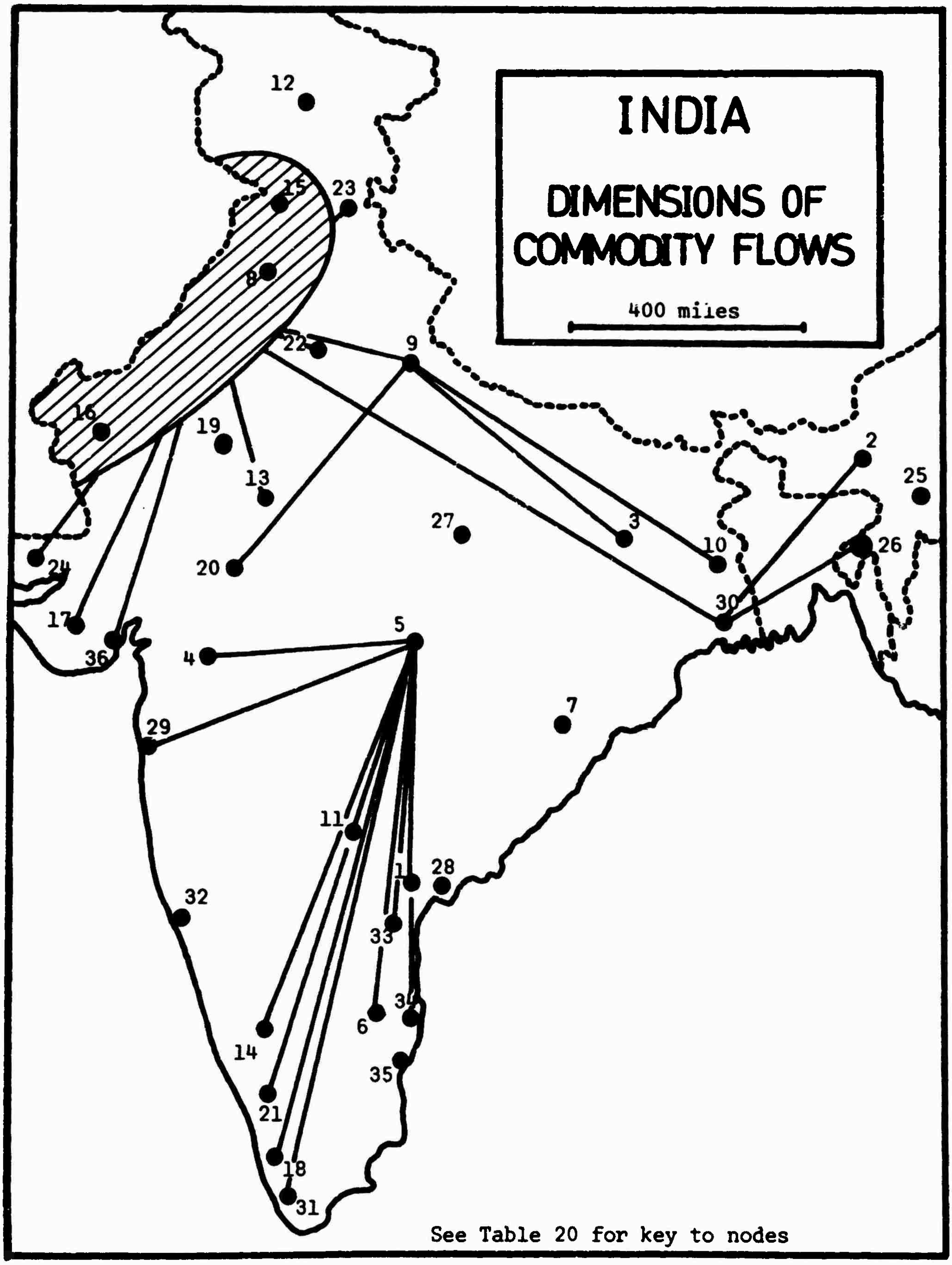
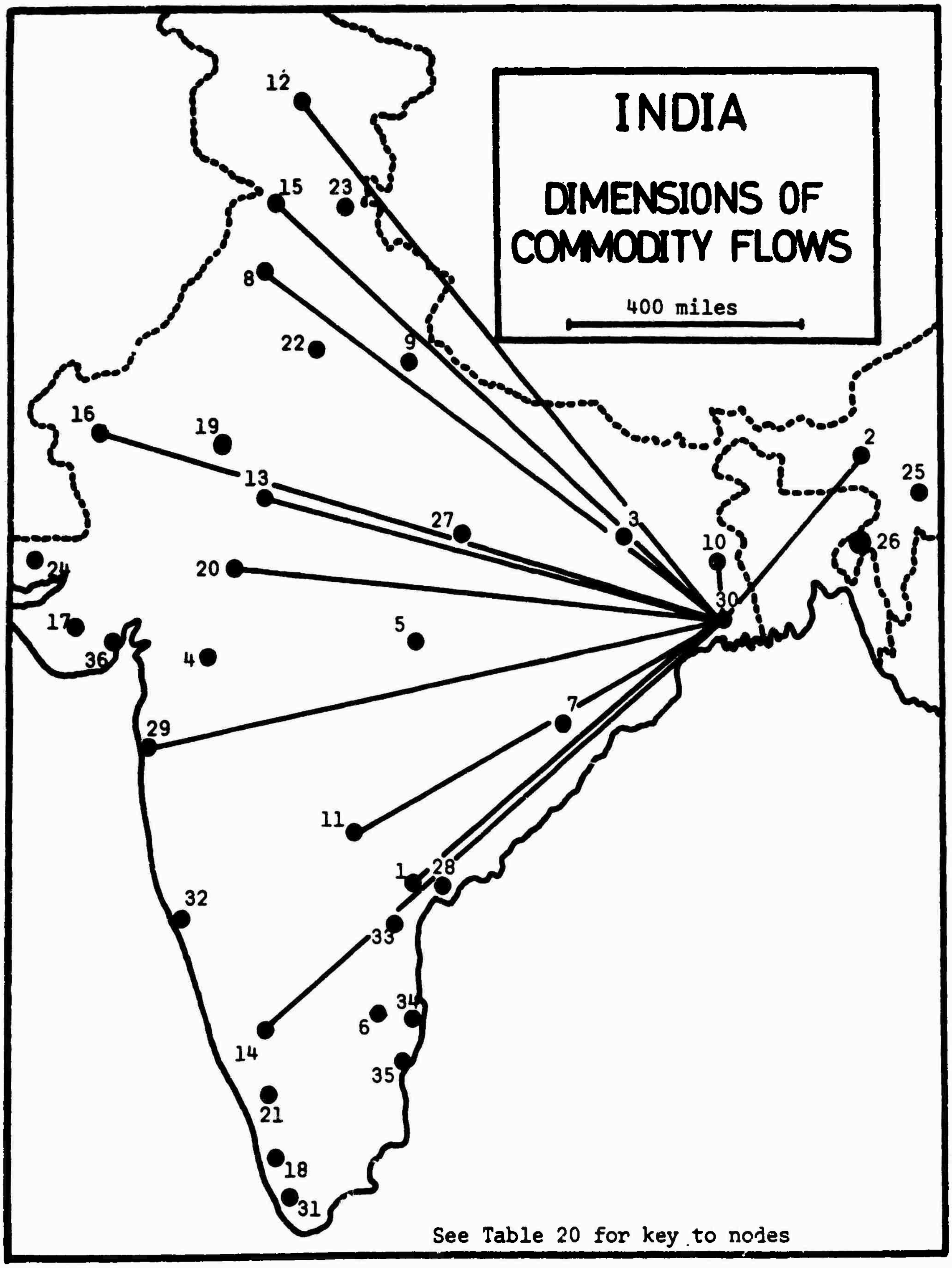
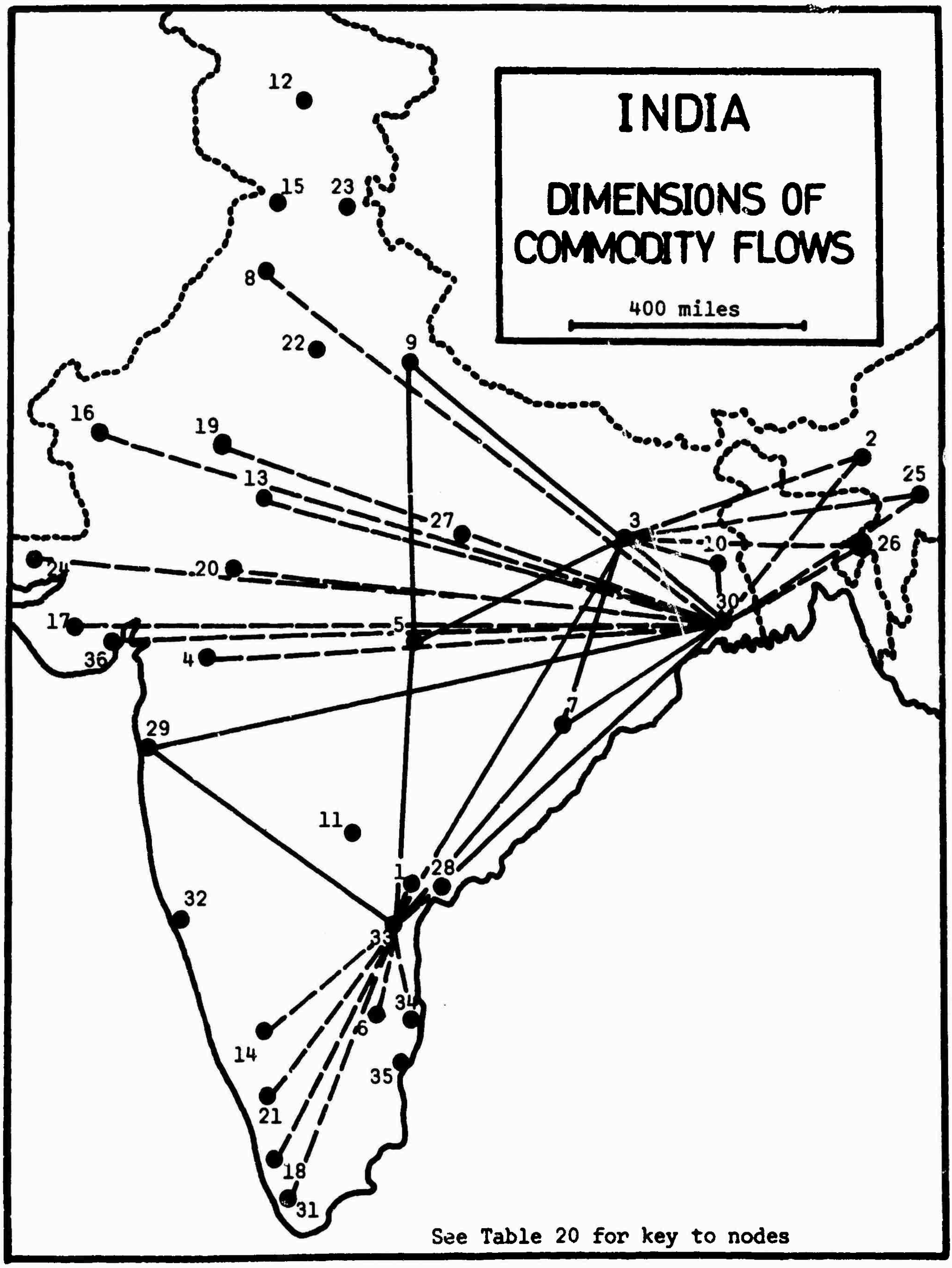


Figure 11: Gram, 1956



See Table 20 for key to nodes

Figure 12: Baled Raw Jute, 1956



See Table 20 for key to nodes

Figure 13: Iron and Steel, 1956

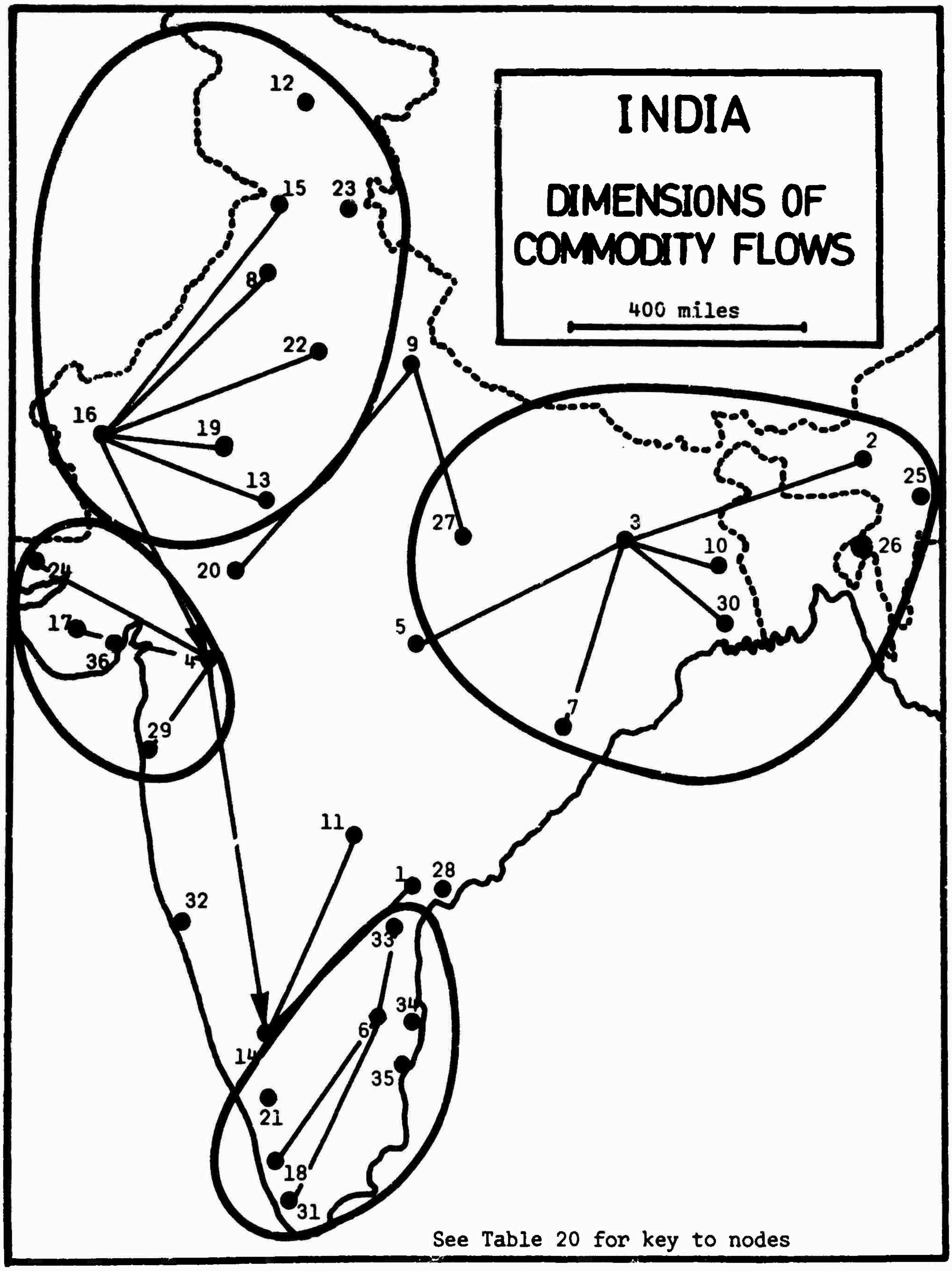


Figure 14: Cattle, 1959

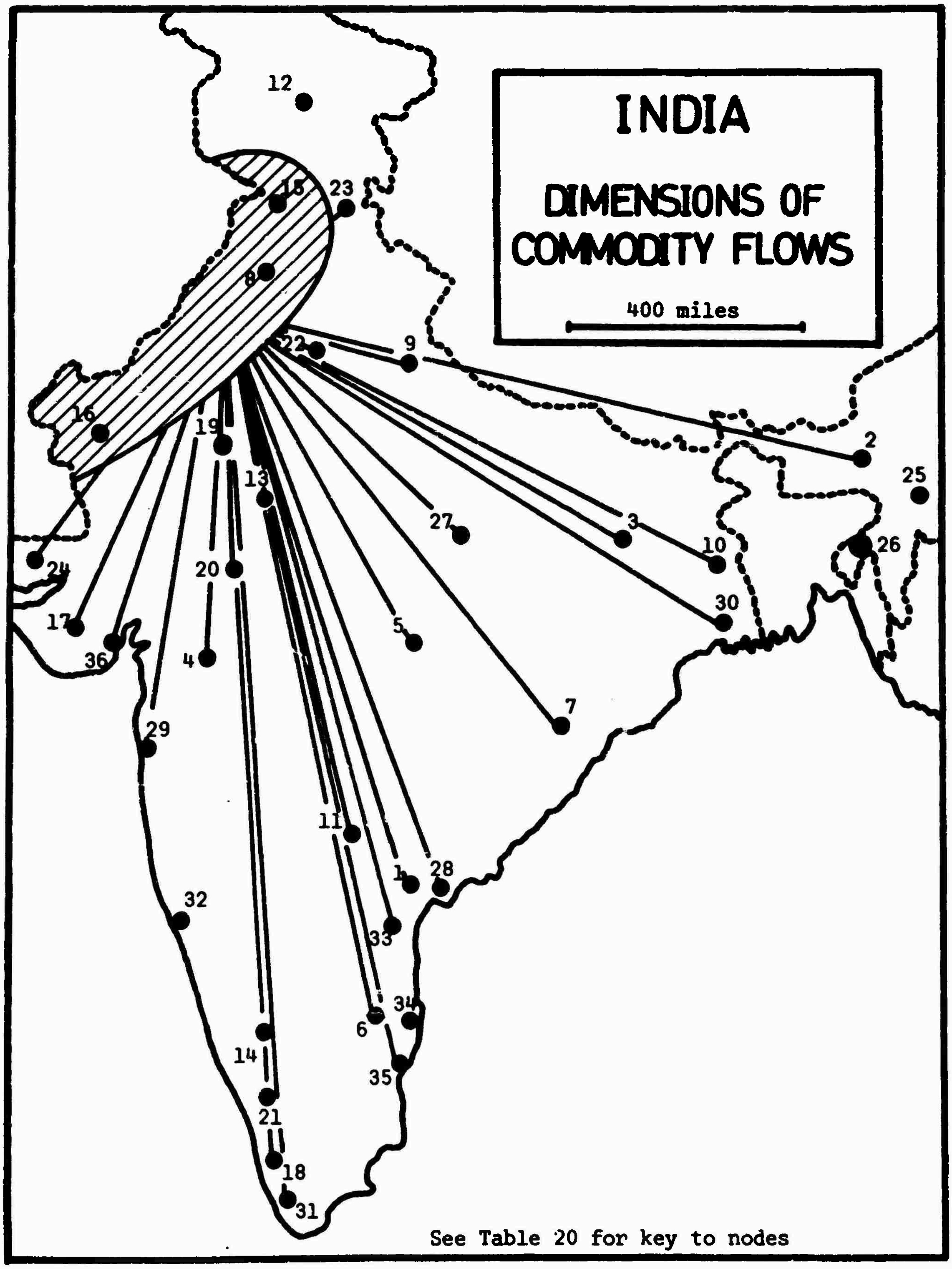
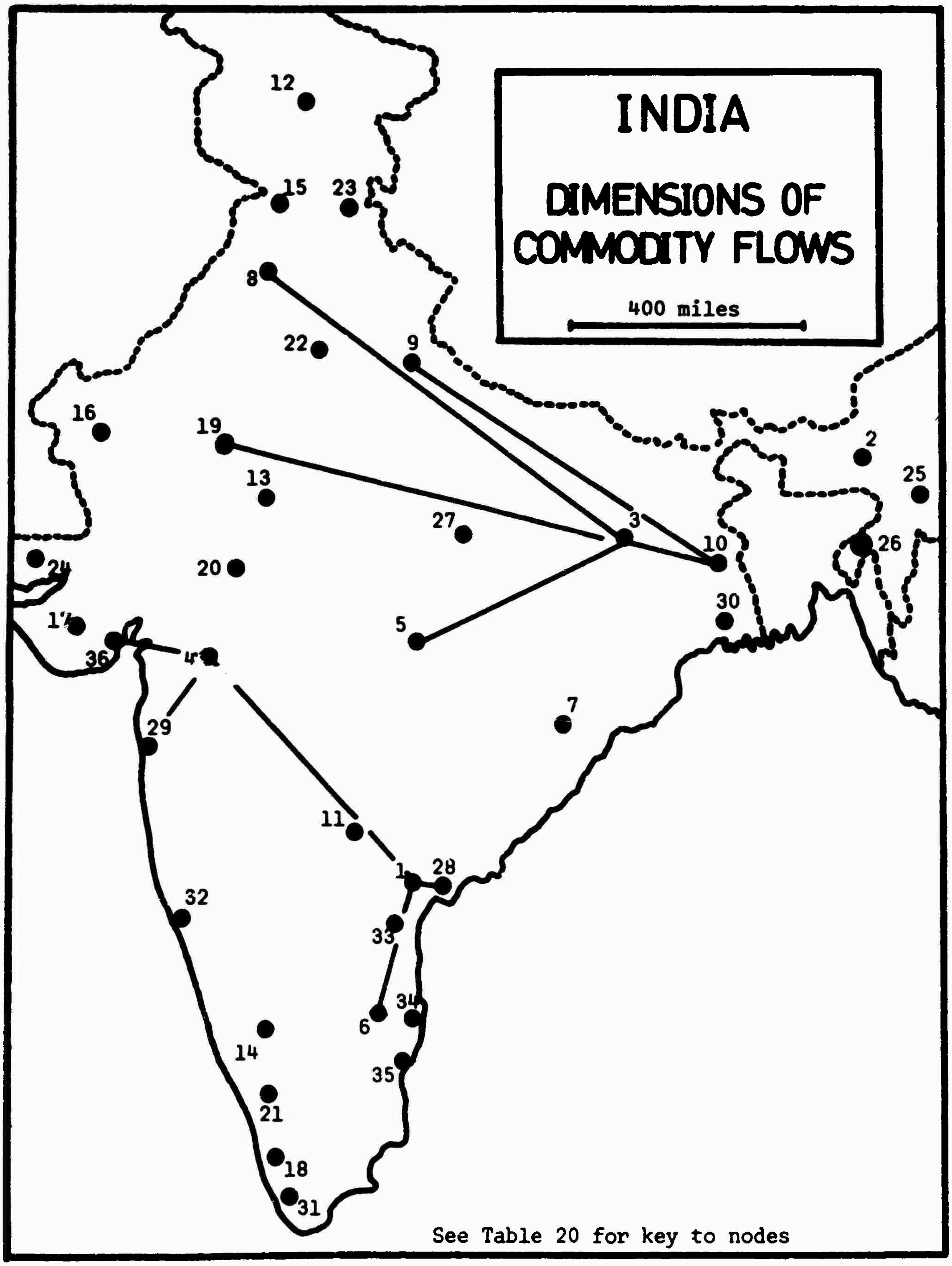


Figure 15: Gram, 1959



See Table 20 for key to nodes

Figure 16: Baled Raw Jute, 1959

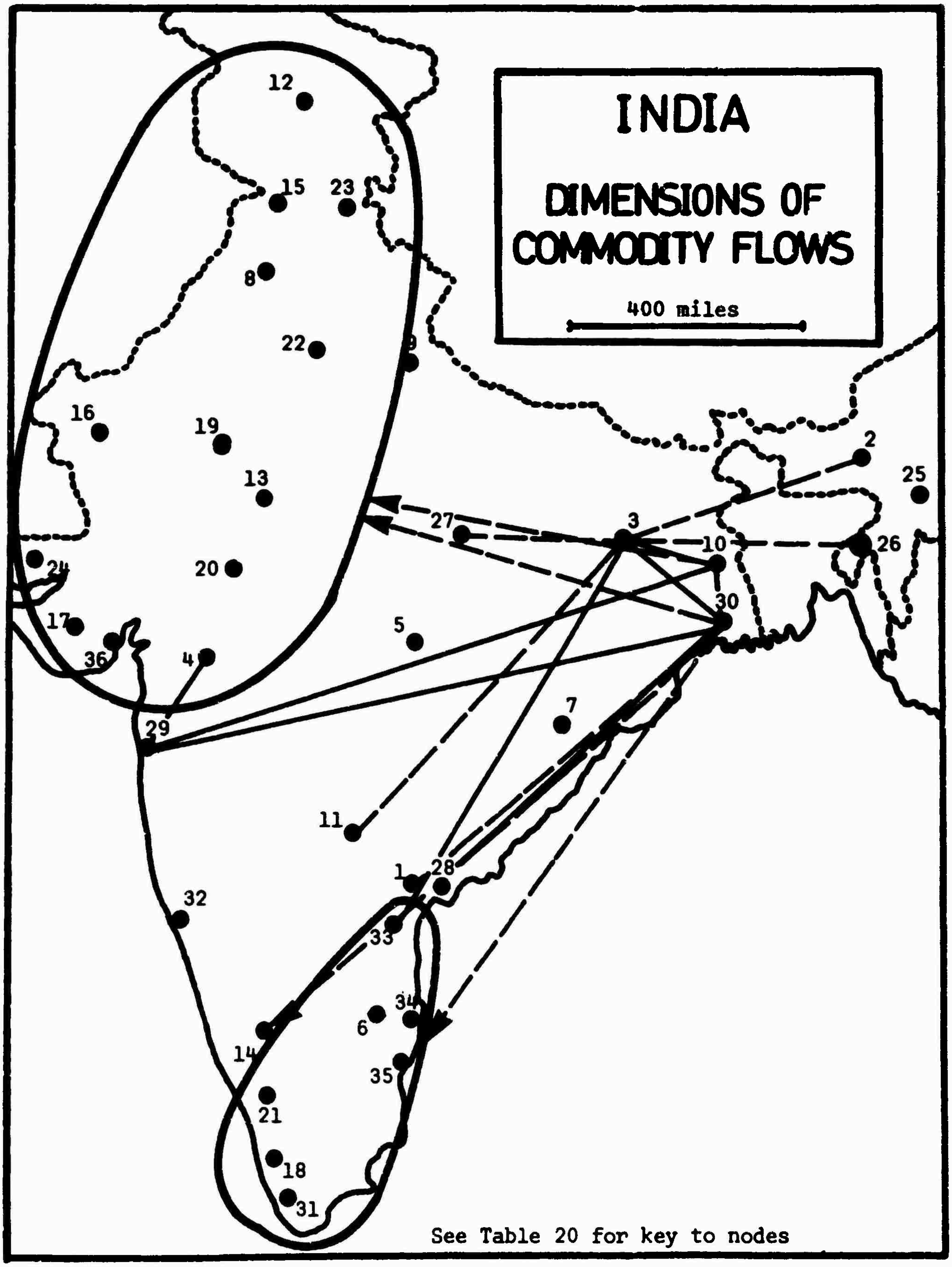


Figure 17: Iron and Steel, 1959

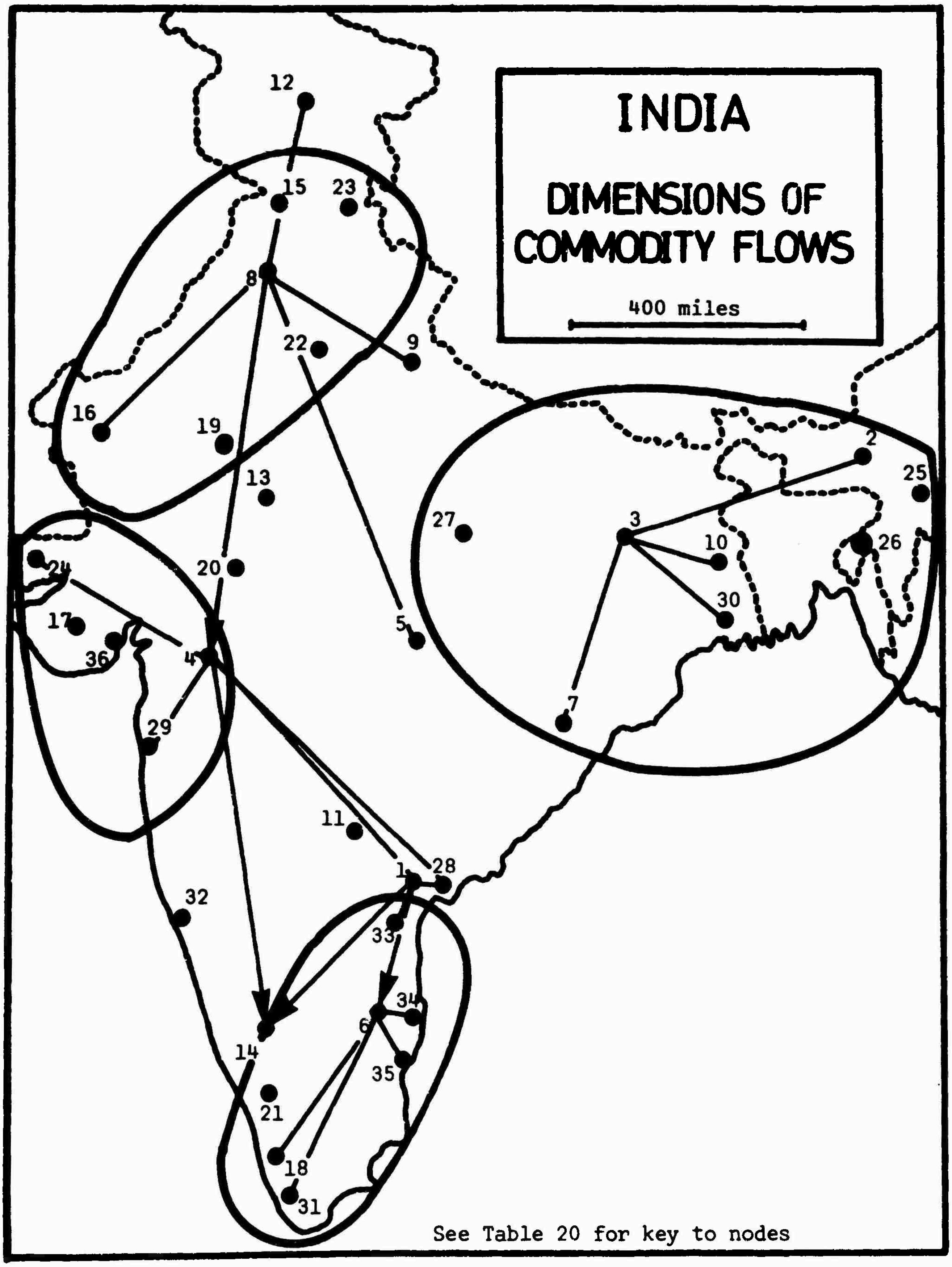


Figure 18: Cattle, 1961

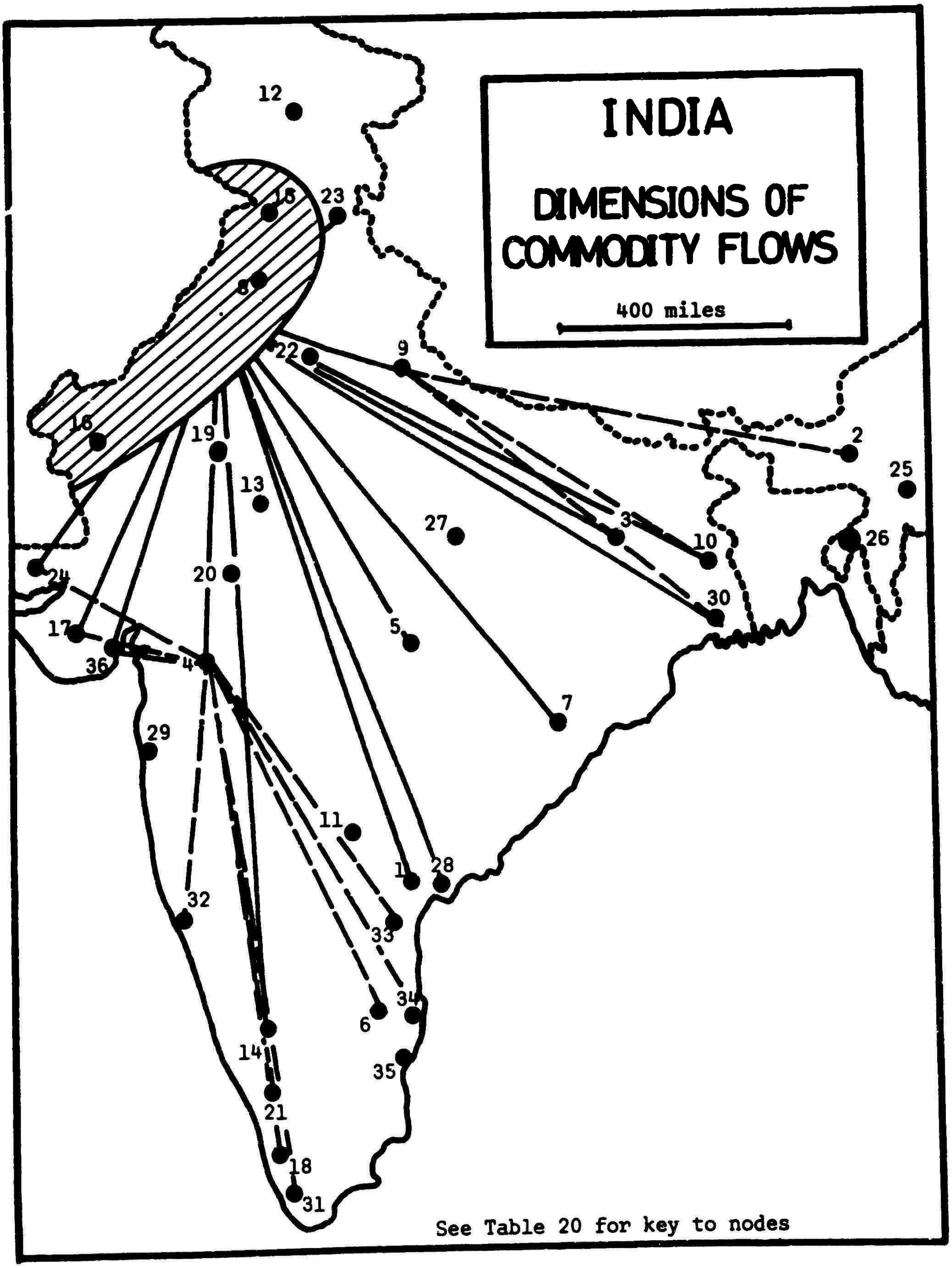


Figure 19: Gram, 1961

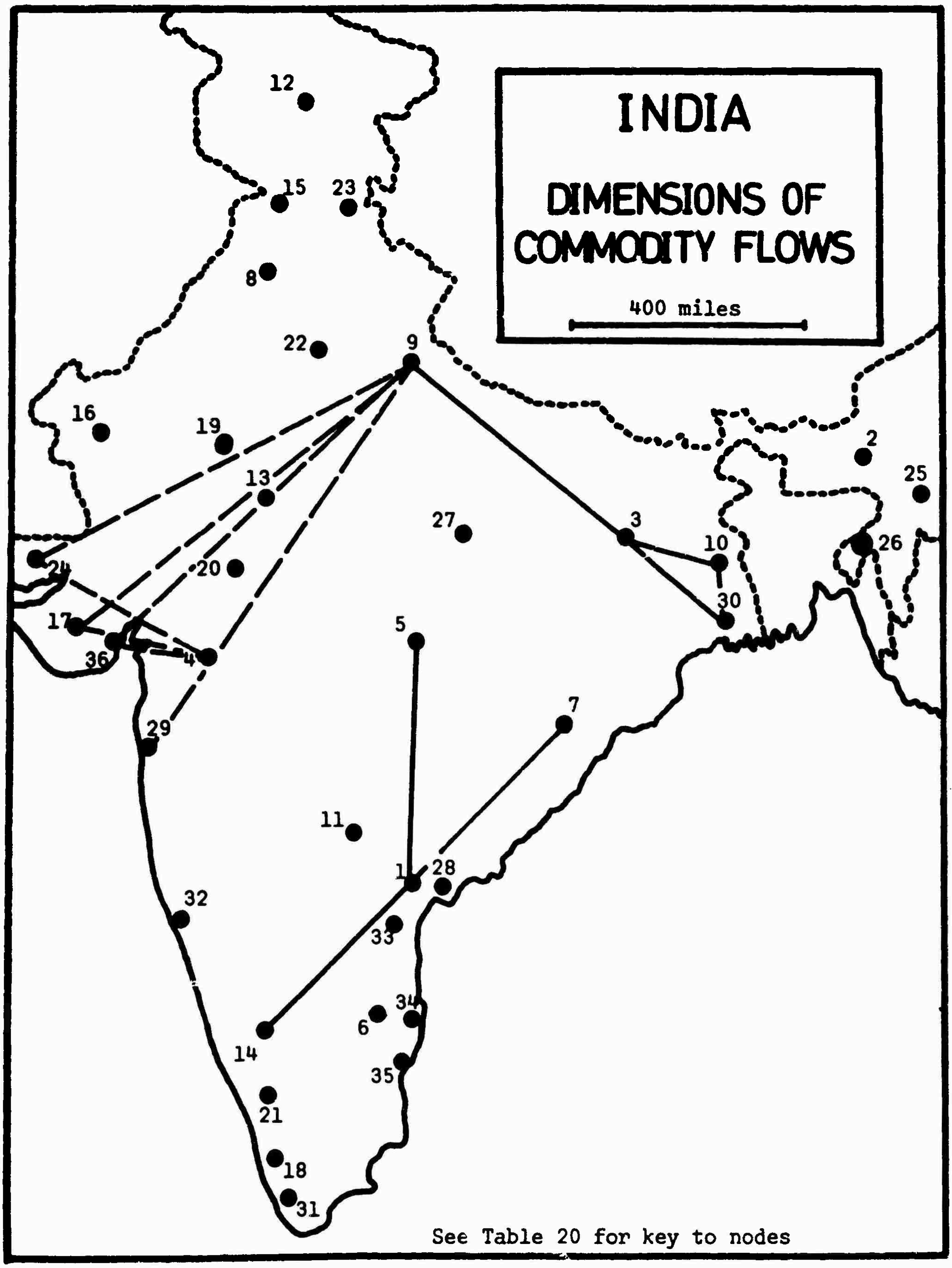


Figure 20: Baled Raw Jute, 1961

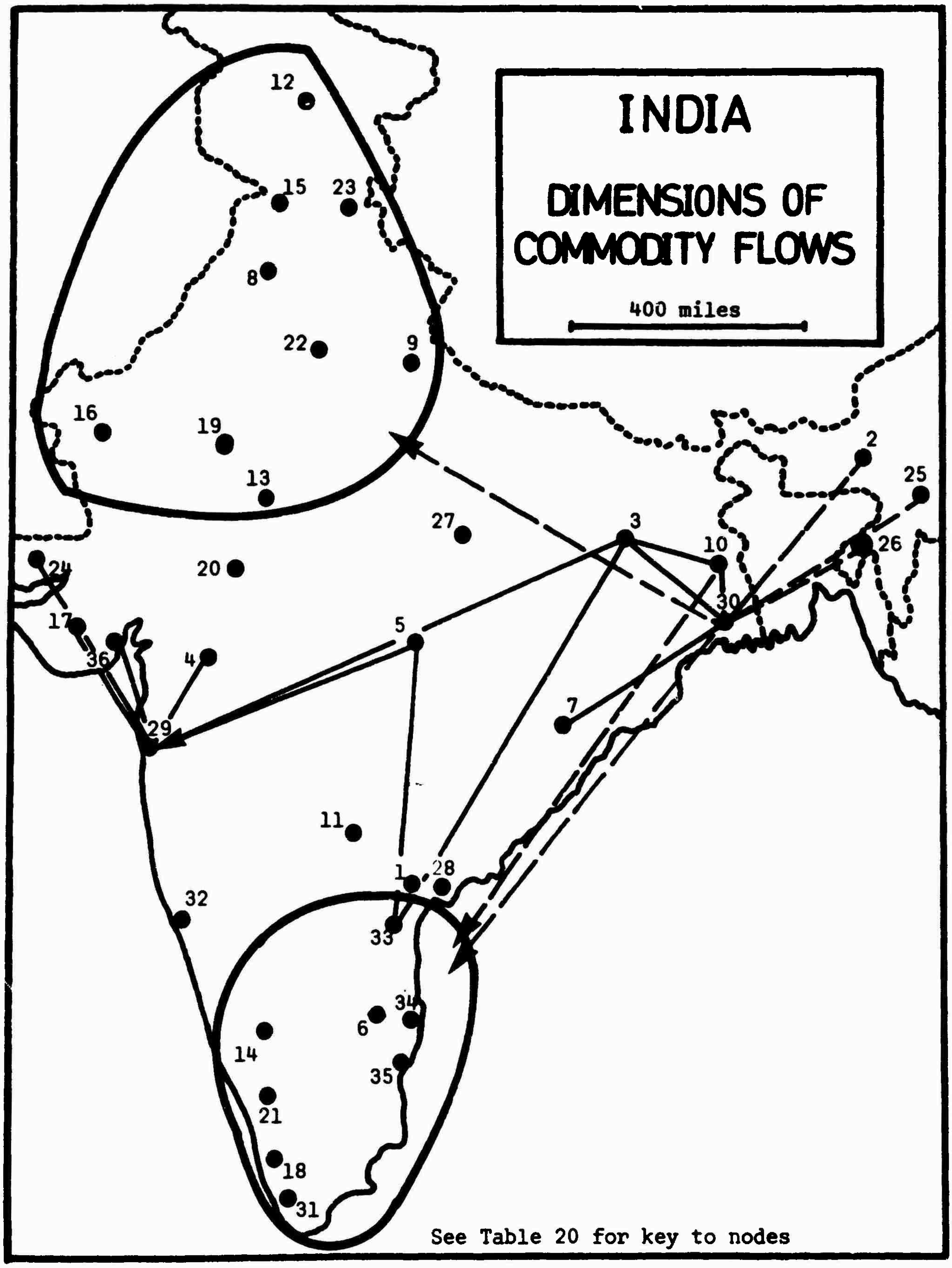


Figure 21: Iron and Steel, 1961

Figure 22: Generated Network One

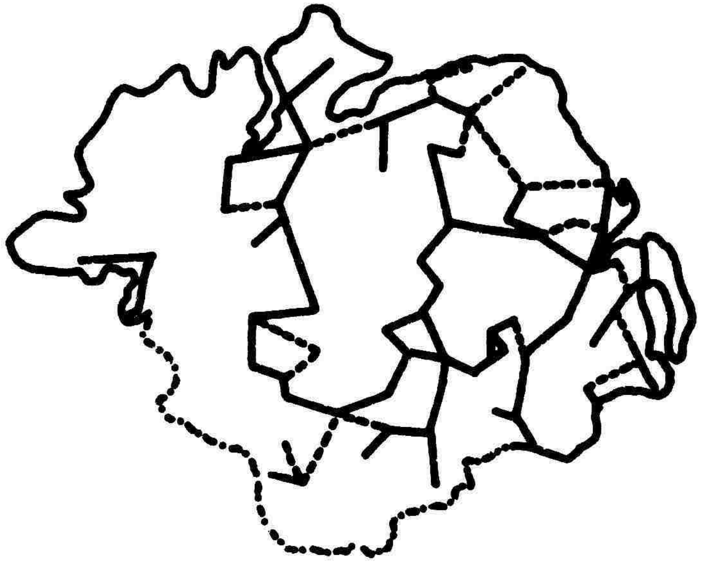


Figure 23: Generated Network Two

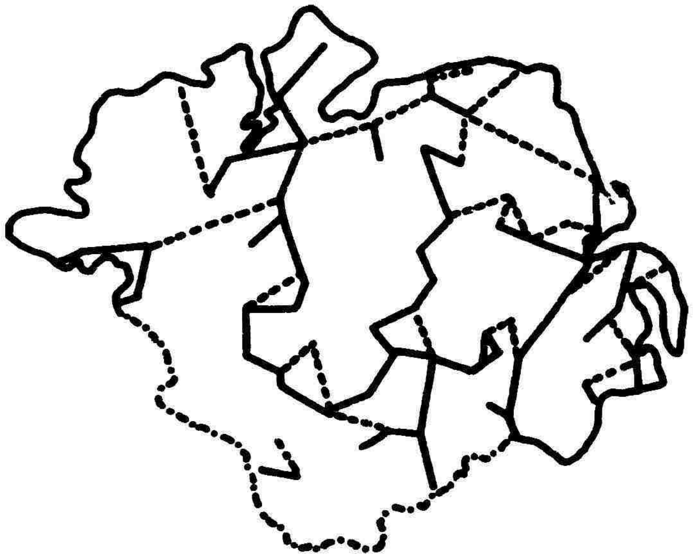


Figure 24: Generated Network Three

Figure 25: Ulster Railroad Network

fewer sources of grain and grain products, and these serve wider areas of the country. Finally, in Figure 12, Calcutta is seen to be the source from which the entire country draws baled jute. Actually this latter pattern is illusory, however, and illustrates the need for complete interpretation of flow systems, for almost 95 percent of all movements of baled jute are from rural Bihar (3) and West Bengal (10) to the Calcutta (30) metropolitan area. These movements were masked in the factor analysis for 1956 because the rest of the country drew baled jute from Calcutta, and this receipt-profile therefore dominated the intercorrelations of the states. As is to be seen in Figures 16 and 20, however, with the breakdown of national dependence upon Calcutta for baled jute, the volumetrically more significant patterns assume a role of increasing importance.

Whereas, in 1956, various types of nodal organization characterized the production and receipts of agricultural goods (these goods therefore occupying various positions with respect to field effects), a more complex form of regional interdependency characterized the shipment and receipt of iron and steel. Each of the major metropolitan centers ships to the other -- Bombay (29), Madras (33), and Calcutta (30). In addition, the Calcutta, West Bengal (10), and Bihar (3) production areas of 1956 link to each other. Finally, Calcutta ships to its own hinterland and to that of Bombay; Bihar also ships to the Calcutta hinterland; and Madras serves its own hinterland in the south. Complex interconnections therefore exist between the metropolitan centers and the manufacturing centers, and the metropolitan centers act as nodes for their non-producing hinterlands.

In Figure 14 the pattern of Figure 10 is repeated, although the hinterland of Bombay is more clearly defined with Bombay State as its specialized source. Increasing specialization in production of grain in the northwest had led to an increasing focussing of the country upon that area, however, as is evidenced by Figure 15. The opposite has happened in the case of jute; small alternative sources are being tapped, and long-distance movements have been eliminated -- no longer does the country rely upon Calcutta, and the importance of the Bihar-West Bengal production area is brought out. Figure 17 shows the effects of increasing domestic production of iron and steel in Calcutta, West Bengal, and Bihar, and the elimination of many of the imports through Madras. The complex intermetropolitan cross-flows have been removed. Calcutta serves the Madras hinterland; Calcutta-West Bengal serve the hinterlands of Bombay and Delhi. Bihar ships throughout the Calcutta region, and there are complex cross-flows between Calcutta, West Bengal, and Bihar.

Few further changes are evidenced in Figures 18-21. Some penetration of the Madras hinterland by Bombay is seen in Figures 18 and 19. In Figure 20 the Bihar-West Bengal-Calcutta triangle stands out more clearly. In Figure 21 the effects of new steel production in Madhya Pradesh are in evidence. The Bihar-West Bengal-Calcutta industrial belt still has complex cross-flows, and ships to the

Calcutta region. Calcutta serves the Madras hinterland, and West Bengal that of Delhi. But the new steel mills of Bihar and Madhya Pradesh also ship to the port metropoli of Bombay and Madras, and Bombay in turn is reestablished as the node for its own hinterland.

Several conclusions may be drawn from these examples:

(1) As production becomes less ubiquitous, local "field effects" are replaced by those which are broader in scale, and ultimately embrace the nation.

This generalization applies to the flow patterns of different commodities at any point in time and to any particular commodity through time.

(2) Later stages of the productive process (for example, manufacturing) are more likely to be characterized by complex regional interdependencies than are earlier stages (for example, primary production of agricultural commodities).

The interconnections in evidence will be those linking metropolitan and manufacturing centers together by multiple cross hauling. This finding specifies some of the conditions under which complex regional organization emerges at higher levels of economic development.

(3) Certain "field effects" (nodal regions) repeat themselves in a variety of commodities, and appear to be stable through time even though the part of the region serving as the node differs from one community to another.

The most notable of these stable fields or regions are the hinterlands of the major metropolitan centers.

(4) Whereas there are, on the one hand, stable elements of structure, on the other hand individual commodities may display great change in flow patterns in relatively brief periods of time. Increasing specialization, the emergence of new production centers, governmental activity (for example, India's import restrictions on most commodities, including iron and steel), and the like, have immediate and often drastic effects upon flow structures.

Conversely, it appears that the results of these changes may be quite predictable, calling for wider nodal regions or more pronounced regional interdependencies in the form of complex interconnections of metropolitan centers and manufacturing areas. Thus, just as networks display progressive changes in character as the degree of economic development, and hence specialization, of the areas they serve increases, so do the flows over these networks. Since increasing proportions of the flows come, at higher levels of development, from more specialized producing areas and involve output from manufacturing industry, the aggregate flow patterns take on wider and more complex forms. These forms constitute the demands for transportation connections which the transport networks seek to supply, and knowledge of them should certainly serve to improve models which seek to predict the processes of network growth and spread.

The Structure of Local Service Airline Routes

Figures 1 and 2 displayed the route structure of local service airlines in Guatemala and Honduras. It was pointed out that these systems appeared to be in some way basically different, since the system in Guatemala seemed to focus upon Guatemala City, whereas the Honduran network appeared to have no such overall focal point. Table 21 displays various indices pertaining to network structure for these two nations.

TABLE 21

GUATEMALA AND HONDURAS:

INDICES OF LOCAL SERVICE AIRLINE ROUTE STRUCTURE

	Guatemala	Honduras
Number of nodes	20	32
Number of routes	20	45
Mean number of connections/node	2.0	2.8
Cyclomatic number	1	14
Gamma index*	10.5	9.1
Alpha index*	0.6	3.0

*Nonplanar basis

It seems that while the Honduran network is larger in size and displays a higher average number of connections per node, it is less directly connected than the Guatemalan network, but on the other hand displays a higher percentage of redundant routes. These indices provide some information about the basic network structure in the two countries; but they do not provide information on the marked visual differences in structure.

In a previous technical report (Reference 5), this line of analysis was pursued further and two nations were selected (Argentina and Venezuela) and the connection matrices corresponding to their local service airline networks were subjected to principal components analysis. The results of this analysis appeared to indicate the existence of the following structural characteristics:

(1) An overall field effect centered upon the primate city (Caracas or Buenos Aires).
(2) Major regionalization effects which broke out large distinctive portions of the network.
(3) Minor regionalization effects which isolated specific structural details within the major regions.

The factors which were isolated accounted for some 38 percent of the observed variation, and it was noted that in no case were "neighborhood effects" (the notions that points are most likely to be linked to their near neighbors) or the existence of long strings or chains pointed out.

Subsequent analysis has thrown considerable doubt upon these earlier conclusions, and it now appears that principal components analysis, when combined with varimax rotation to simple structure, will only isolate small clusters of nodes which display similar pattern of direct linkages. This can be demonstrated through a reexamination of the Venezuelan local air service network. The network examined here (circa 1963) differs from the one originally examined (circa 1961) but not in a significant fashion. Some of the results of the principal components analysis are shown in Table 22. Twelve significant factors, accounting for 87.2 percent of the variance, were extracted through the application of principal components analysis with varimax rotation to simple structure. The first factor, accounting for 13.6 percent of the variance, isolated a group of five nodes apparently centering on Maraciabo; the second factor (11.4 percent), a group of four nodes lying between Caracas and Maraciabo; the third factor (10.9 percent), an isolated group of four nodes south of Caracas, etc. The first seven of these minor regions are disjoint, but factors 8 and 9 identify clusters of nodes which overlap slightly with previously identified groups.

Factor 3 provides a fairly clear-cut case in point. Nodes 17 and 18 have identical patterns of direct connection to and from the

TABLE 22

ROTATED FACTOR LOADINGS - CONNECTION MATRIX

VENEZUELAN AIR ROUTES - 1963

	Variable	Factor*				
		1	2	3	4	5
1.	Caracas	-0.177	0.086	0.005	0.146	0.240
2.	Las Piedras	0.132	0.015	0.013	0.019	0.078
3.	Maracaibo	0.743	-0.053	0.069	0.070	0.003
4.	Coro	-0.176	0.327	0.166	0.191	-0.165
5.	Porto Cabello	-0.073	0.803	0.044	0.047	0.049
6.	Barquismeto	0.023	0.341	0.035	0.030	0.049
7.	Valera	0.094	-0.170	0.112	0.104	-0.130
8.	Sta. Barbera	0.909	-0.052	0.096	0.098	-0.080
9.	Casigua	0.894	0.031	0.077	0.082	-0.062
10.	Merida	0.047	-0.163	0.117	0.113	-0.127
11.	Barinas	0.087	0.243	-0.119	-0.538	0.227
12.	San Antonio	0.726	-0.144	0.111	0.109	-0.109
13.	St. Domingo	-0.151	-0.193	0.145	-0.416	-0.133
14.	Guasdualito	-0.147	-0.153	0.150	-0.836	-0.134
15.	Palmarito	-0.078	-0.001	0.074	-0.909	-0.061
16.	San Fernando	-0.019	0.062	-0.955	0.001	0.085
17.	Caicara	-0.093	-0.065	-0.810	0.097	-0.086
18.	Puerto Paez	-0.093	-0.065	-0.810	0.097	-0.086
19.	Puerto Ayacucho	-0.103	-0.075	-0.950	0.111	-0.103
20.	Valle de la Pascua	-0.026	0.053	-0.002	0.009	0.040
21.	Anaco	-0.080	-0.037	0.074	0.095	0.048
22.	Barcelona	-0.108	-0.053	0.098	0.113	0.827
23.	Cumana	-0.030	0.050	0.008	0.012	0.967
24.	Porlamar	-0.141	-0.113	0.141	0.154	0.765
25.	Guiria	-0.096	-0.062	0.094	0.095	-0.033

TABLE 22 - Continued

Variable	Factor* 1	2	3	4	5
26. Pedernales	-0.084	-0.062	0.086	0.090	-0.068
27. Maturin	-0.070	0.018	0.058	0.057	0.250
28. Tucupita	-0.088	-0.057	0.090	0.093	-0.075
29. San Tome	-0.186	-0.207	0.201	0.233	0.236
30. Cuidad Bolivar	-0.151	-0.122	0.158	0.168	-0.056
31. Puerto Ordaz	-0.097	-0.047	0.093	0.093	-0.073
32. El Dorado	-0.086	-0.045	0.085	0.084	-0.067
33. Canaima	-0.056	-0.011	0.054	0.051	-0.059
34. Icabaru	-0.106	-0.068	0.105	0.109	-0.087
35. Santa Elena	-0.084	-0.052	0.085	0.087	-0.064
36. San Felipe	-0.149	0.875	0.140	0.149	-0.133
37. Carupano	-0.149	-0.071	0.140	0.108	0.214
38. Elorza	-0.094	-0.073	0.105	-0.867	-0.089
Percent of Communality over all Variables	13.6	11.4	10.9	9.9	7.7

*Only the first five out of twelve significant factors are shown here.

network; nodes 16 and 19 are nearly identical, and each is quite similar to 17 and 18 (see columns in Table 23). No other nodes in the system have a similar pattern of direct connections. The four nodes in question all load heavily upon factor 3, and no other node or group of nodes does so. The analysis is evidently heavily biased by the (relatively) large number of zero elements in the connection matrix.

It would appear that this problem might be avoided, at least in part, if a denser matrix were the subject for analysis. For a given network a matrix, P, with zeros only along the main diagonal may be generated by setting P_{ij} equal to the minimum number of edges passed over in going from node i to node j. P is commonly referred to as the "shortest path matrix".

TABLE 23

SUBNET CONNECTIONS AND LOADINGS

Node	16	17	18	19	Loading on Factor Three*
16	-	x	x	x	-0.955
17	x	-		x	-0.810
18	x		-	x	-0.810
19	x	x	x	-	-0.950

*Percent of communality over all factors, 10.9 percent.

Note: All other connections are identical except that node 16 is also linked to Caracas.

The shortest path matrix for the Venezuelan network was developed (Reference 32) and was subjected to the same form of analysis as the connection matrix. The rotated factor loadings are displayed in Table 24. Here, five significant factors explaining some 92 percent of the variance were isolated (versus 12 factors and 87 percent in the previous case). This analysis classifies nodes

TABLE 24

ROTATED FACTOR LOADINGS - SHORTEST PATH MATRIX

VENEZUELAN AIR ROUTES - 1963

		Factor				
	Variable	1	2	3	4	5
1.	Caracas	0.692	-0.157	0.294	-0.511	-0.374
2.	Las Piedras	0.859	-0.027	0.155	-0.333	-0.221
3.	Maracaibo	0.940	0.032	0.086	-0.224	-0.161
4.	Coro	0.809	0.027	0.107	-0.313	-0.146
5.	Porto Cabello	0.676	-0.075	0.220	-0.468	-0.267
6.	Barquismeto	0.859	-0.031	0.165	-0.349	-0.216
7.	Valera	0.931	0.047	0.070	-0.198	-0.111
8.	Sta. Barbera	0.941	0.093	0.015	-0.127	-0.099
9.	Casigua	0.943	0.136	-0.038	-0.049	-0.046
10.	Merida	0.944	0.072	0.036	-0.146	-0.103
11.	Barinas	0.452	0.047	0.776	-0.340	-0.191
12.	San Antonio	0.955	0.090	0.017	-0.125	-0.094
13.	St. Domingo	-0.010	0.330	0.923	0.007	0.084
14.	Guasdualito	0.046	0.292	0.946	-0.035	0.055
15.	Palmarito	0.187	0.208	0.938	-0.142	-0.025
16.	San Fernando	0.455	0.001	0.127	-0.841	-0.218
17.	Caicara	0.352	0.059	0.061	-0.888	-0.157
18.	Puerto Paez	0.352	0.059	0.061	-0.888	-0.157
19.	Puerto Ayacucho	0.326	0.077	0.043	-0.909	-0.140
20.	Valle de la Pascua	0.628	-0.188	0.292	-0.497	-0.345
21.	Anaco	0.606	-0.220	0.288	-0.486	-0.343
22.	Barcelona	0.384	-0.338	0.090	-0.302	-0.766
23.	Cumana	0.507	-0.004	0.179	-0.387	-0.687
24.	Porlamar	0.229	-0.151	-0.009	-0.182	-0.918
25.	Guiria	0.119	-0.477	-0.100	-0.083	-0.800

TABLE 24 - Continued

Variable	Factor 1	 2	 3	 4	 5
26. Pedernales	0.193	-0.635	-0.021	-0.146	-0.618
27. Maturin	0.250	-0.669	-0.018	-0.191	-0.642
28. Tucupita	0.160	-0.679	-0.030	-0.121	-0.588
29. San Tome	0.338	-0.366	0.110	-0.302	-0.672
30. Cuidad Bolivar	-0.023	-0.900	-0.193	-0.015	-0.354
31. Puerto Ordaz	0.051	-0.830	-0.124	-0.051	-0.466
32. El Dorado	-0.163	-0.893	-0.314	0.085	-0.161
33. Canaima	-0.063	-0.885	-0.189	0.019	-0.316
34. Icabaru	-0.244	-0.835	-0.384	0.144	-0.040
35. Santa Elena	-0.281	-0.787	-0.417	0.170	0.017
36. San Felipe	0.623	-0.037	0.182	-0.436	-0.225
37. Carupano	0.086	-0.381	-0.128	-0.062	-0.883
38. Elorza	0.077	0.268	0.945	-0.059	0.038
Percent of Communality over all Factors	46.6	27.3	9.3	5.4	3.4

upon the basis of similarity in their profiles of accessibility from the system. Again the factors break out groups of nodes, but this time the groups are larger and they tend to correspond somewhat more closely to the visual impression we receive of network structure. Factor 1 isolates a major group of nodes centered on the Caracas-Maraciabo axis and identifies this as a major structural component of the network. Factor 2 points out a cluster of nodes in the SE which are tributary to Maturin; factors 3 and 4 break out smaller, isolated clusters lying south and southwest of Caracas; while factor 5 isolates a small region in the northeast lying between Maturin and Caracas; this latter region shows a small degree of overlap with the group of nodes isolated by factor 2.

The restructuring of the basic data matrix appears to have eliminated some of the problems encountered in the earlier analysis, but the results noted here should not be classed as more than a tentative structure until the problem has been more fully investigated.

Concluding Remarks

The material discussed in this chapter has dealt with theoretical and empirical investigations of what has been termed the internal structure, or layout, of transport networks. The problem was one that had received only cursory attention in the past and the present investigations, while illuminating certain points, have failed to provide an entirely satisfactory set of results. Part of the problem appears to lie in the line of attack which was chosen; the graph-theoretic model of network structure has proved to be strong in some areas, but disappointingly weak in others. The loss of information about the angular characteristics of the network has prevented us from attaining certain of our research goals -- the analogy drawn between the indices used in the present study and the moments of a frequency distribution still holds, we have about as much information as might be provided by, say, only the first central moment.

While much of the investigative effort devoted to this topic has not been as productive as was originally hoped, the work has nevertheless provided a number of invaluable guidelines for future explorations of this topic.

IV. FORECASTING DETAILED STRUCTURE

Previous chapters have identified the types of information about the structure of transportation systems that may be derived from a knowledge of general areal characteristics. This information includes notions regarding stock levels, numbers of modes and edges in the system etc., as well as certain generalizations relating to regionalization patterns within the overall network. One of the earlier studies (Garrison and Marble, Reference 4) also noted that it was apparently possible to "explain" many observed direct links in networks on the basis that only nearby towns were directly connected. The present chapter outlines some further studies (Boyce, Reference 2) which were conducted to examine further the basic details of network structure.

The research undertaken addressed itself to an examination of the pattern of direct connections between nodes on the system. The basic approach was to ask first to what extent the direct connections could be described by what was termed a regional nearest neighbor model, and secondly to determine to what extent a specific network could be generated from a predefined set of nodes using these same concepts.

Theoretical Basis

An Introduction to Nearest Neighbor Methods

Consider the space or plane surrounding each point i, where i is any point located on that plane. One means of describing the relationship between point i, and other nearby points j, is to arrange the j points in order of their distance as measured from point i. The closest point to i is then designated the first nearest neighbor of point i. Similarly, the second, third, ...n-th nearest points are the second, third, ...n-th nearest neighbors. The technique of point pattern description which operates in this manner is quite similar to the measurement techniques utilized in the order method nearest neighbor statistics.

A variation on the order method measurements may be defined by dividing the space or plane surrounding point i into a number of regions. In this context, the region has an area equal to every other region, and each region borders on the point i. The method of sectoring which provides this unique set of n regions is accomplished by extending radials outward from i such that the included angle between each pair of adjacent radials is equal. In order to define the regions in a general manner, and so as not to bias the resulting regional distributions, the direction of the initial radial is determined in a random manner. The points falling into the regions surrounding the point i are called regional near neighbors of i.

Near neighbor measurements are a means of describing spatial patterns by evaluating distances between nearest and other near neighbors in a point distribution. These methods are a part of a larger literature on statistical pattern analysis first developed by statistical ecologists. In the context of the present study, the consideration of nearest neighbor methods is confined to a two-dimensional unbounded space, possessing the metrical properties postulated in Euclidean geometry. The location of each point, i, (i = 1,2,3...n) in this space is given by the co-ordinates, x_i, y_i. The distance between two points, i and j, is:

$$r_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{1/2},$$

and

$$r_{ij} = r_{ji},\ r_{ii} = 0 \ .$$

The space surrounding each point, i, is partitioned into K equal size regions or sectors, and the point, i, is known as the centroid of the set of regions. For K equal to 1, the region is the entire space. For K equal to 2, each region, k_1 and k_2, is a half-space, and so forth. For any centroid, the j^{th} nearest neighbor in the k^{th} region (k = 1,2,3,...K) of a plane divided into K regions is designated $j(k/K)$. The labels are assigned in a specified manner to conform to the following rules:

(1) Within any region, k, the j labels satisfy the rule,

$$r_{i1}(k/K) = r_{i2}(k/K) = \dots = r_{in}(k/K) : (k = 1,2,\dots,K) .$$

(2) Among the k neighbors of order, j, the labels satisfy the rule,

$$r_{ij}(1/K) = r_{ij}(2/K) = \dots = r_{ij}(K/K) : (j = 1,2,\dots,n) .$$

The labels can only be assigned after the space has been partitioned.

Transportation Networks in a Nearest Neighbor Context

A transportation network, in the simplest sense, consists of a set of nodes and links. A node (intersection, center, or junction) represents the end of one link or the coincident ends of two or more links, and has a specified location in space. A link (route or connection) is a line segment which joins a pair of nodes. The combination of these two primitives, nodes and links, may be taken to represent a simple model of a transportation network.

The analysis to be developed here attempts to indicate which pairs of nodes are connected in terms of two variables, the number of links per node, a, and the number of regions per node, K. The number of links per node, known as the *degree* of a node, is a simple measure of network structure and connectivity, and is simply the number of links which end at one point. The number of regions per node, K, refers to the nearest neighbor notion of partitioning the area around a node into K equal sized regions. Together, these two variables provide a simple means of describing or classifying which pairs of nodes are linked.

Consider a node of degree a which acts as the centroid of a two-dimensional space which has been sectored into K regions. Providing K is sufficiently large, the following statement will be true. There exist a regions in which the first regional nearest neighbor, $1(k/K)$, is linked to the centroid, and $(K-a)$ regions in which no link occurs. As the number of regions is reduced, the probability that a region will contain more than one link increases, this probability being a function of the included angle between the links. But, the descriptive ability of regional nearest neighbor measures tends to increase as the number of regions is reduced, in that the same link relationships are described by a fewer number of regions; however, to the extent that more than one link falls in a region, the description is less meaningful. As the number of regions, K, approaches the number of links or degree of the node, a, the link relationships are more and more precisely described.

The links or connections may be classified according to whether they connect the centroid to the first, second, third,...n^{th} regional nearest neighbor. Further, if more than one link occurs in a given region, then the shortest link (the link to the nearest neighbor) is designated the first link, and the second shortest link is designated the second link, etc. If for a given node, the number of regions is equal to the degree of that node, and each region contains one link connecting the centroid to the first regional nearest neighbor, then in the present context the centroid is termed *completely nearest neighbor*. For certain types of transportation networks such as the common rectangular street system, all nodes are specified by the description *completely nearest neighbor* since, in the case mentioned for example, the degree is equal to the number of regions, $a = K = 4$, and each link connects the centroid to its first regional nearest neighbors.

One means of describing the link-node characteristics of transportation networks is to examine the extent to which the links connect first regional nearest neighbors, when the number of regions is carried from one to K, where K is large compared to the highest degree of node on the network.

If the number of regions is less than the degree of the node, then some special considerations must be introduced. For this purpose, the possible number of links to the first regional nearest neighbor is defined as equal to the number of regions used, or the degree of the node, whichever is smaller. For example, with a node of degree four and only two regions, it is apparent that only two of the four links can connect the centroid to its first regional nearest neighbors; whereas if four regions were used, all four links would connect the centroid to its first regional nearest neighbors. The possible number of links, then, constitutes a ceiling on the number of links which may connect first regional nearest neighbors to the centroid. By varying the number of regions, <u>k</u>, between one and <u>K</u>, a general description of the link-nodes relationships on the network may be obtained.

A Hypothesis on the Structure of Transportation Networks

The use of regional nearest neighbor measures, as described above, provides a basis for making some general assertions regarding the structure of transportation networks. For certain types of networks, such as the rectangular grid system cited above, the statement that the network is completely nearest neighbor is sufficient to describe all the existing link-node relationships. Generalizing from this specific case to that of any transportation network, the hypothesis is set forth that the link-node relationships on any network <u>tend</u> to be completely nearest neighbor.

This hypothesis provides a basis for studying existing transportation networks. The completely nearest neighbor hypothesis also provides a basis for the generation of synthetic transportation networks in the following manner. Given an estimate of the degree of each of the nodes on the network in question, the basic generation procedure requires that each node be connected to its first regional nearest neighbors in such a fashion that the resulting number of connections is equal to the degree of the node. A number of detailed procedures for this synthetic network generation are developed in Boyce, Reference 2.

The tendency of a transportation network to be completely nearest neighbor may well be due in part to the spatial distribution of the nodes. As noted above, a uniform node distribution such as the rectangular grid street system produces a completely nearest neighbor network. However, nodes which are distributed randomly, or in clustered patterns, may deviate significantly from this characteristic.

Empirical Analysis

The Northern Ireland Rail Network

The network chosen for examination was the railroad network of Northern Ireland in the year 1900. This network was chosen primarily because it had been used in previous research on a related problem, and for this reason data were readily available. (Garrison and Marble, Reference 4.) The information available from the previous research consisted of network maps for the six counties making up Northern Ireland, for 10-year intervals, beginning in 1850 and continuing until 1900, with an additional map for the 1960 network. Also, the populations of all cities and towns in Northern Ireland were available on a 10 year basis for those towns whose populations exceeded 1500 at any census between 1851 and 1951. The railroad network for the year 1900 was chosen from this body of available information since this period represented the most extensive development of the Ulster Railway System. Since 1900 the network has diminished considerably in extent. Inasmuch as the population of most Northern Ireland towns has declined since 1900, in nearly every case the population of the towns for which data are available exceeded 1500 persons in 1900. In a later phase of the study, population data for towns exceeding 500 population in 1891 became available.

In accordance with the population data available at the time these analyses were made, the threshold population required for a town to be entered as a node on the railroad network was set at 1500 persons, attained between 1851 and 1951. This value resulted in 46 urban areas being defined as nodes on the rail net. An additional 21 nodes were also placed on the network due to their function as a junction or a terminal of a route. Thus, a total of 67 nodes was defined for the network. Railroad routes passing not more than 1 mile from a town of over 1500 population were taken as direct links to that town; otherwise, the town was considered not on that route. In the case of junctions occurring within 2 miles of a designated node, the node was taken to be the junction and the corresponding links drawn directly to that node.

Analysis of the Northern Ireland Rail Net

The regional nearest neighbor measures were used to examine the link-node relationships on the Northern Ireland rail net. A computer program was written to perform the required computations. Briefly, the operations performed by this program are as follows:

(1) A random angle between 1 and 360 degrees is generated for the purpose of orienting the regional boundaries.
(2) The area around each node (centroid) is sectored into K regions, where the sectors are randomly oriented. For the purpose of the network analyzed here, K was varied from one to six regions.
(3) The distance and the direction from the centroid to all other nodes in the system is computed.

(4) Every node is entered in a table in which the rows correspond to the K regions, and the nodes are arranged in order of their distance from the centroid.

With this program it was possible to generate rapidly for each node six sets of regions, one set for each value of $\underline{K}$ ($k = 1,\ldots,6$). Each table of nodes produced by the above steps specifies the first, second, ... sixth regional nearest neighbors for each region. Using these tables, the regional order of those nodes linked to the centroid was then identified. In those cases where two nodes in the same region were linked to the centroid, the links were designated as first or second links according to their lengths. In the case of $k = 1$, that is, only one region, first, second, ... $\underline{a}$ such links were identified, where $\underline{a}$ is equal to the degree of the node. The regional order of those nodes linked to the centroid was tabulated for nodes of each degree and for each set of regions.

Since the sectors were randomly oriented on each centroid, the tables of regional nearest neighbors produced by the analysis may vary slightly each time the sectors are laid down. Experience in using a six-sector system indicates that the distribution of the regional order of the linked nodes fluctuates only very slightly for different orientations of the region boundaries. While it might have been somewhat more desirable to conduct a large number of trials and then average the results in order to determine the regional order of each linked node, it was felt that modest variations noted in the six-sector case did not justify the additional research effort.

Results of the Regional Nearest Neighbor Measures

A table is presented in this section in order to display the results of the analysis of the Northern Ireland railroad network, using the regional nearest neighbor measures. In evaluating these results, the reader should recall the discussion of the completely nearest neighbor hypothesis for the study of link-node relations in a transportation network. This hypothesis states that when the number of regions is equal to the degree of the node, the links of a transportation network tend to connect nodes which are first regional nearest neighbors.

Table 25 is the significant portion of a larger table summarizing the link-node relationships for nodes of degree 1 through 4 and for the six sets of regions containing one through six sectors. Listed down the left edge of the table are six major rows representing the six sets of regions with one, two, three, four, five, and six sectors. Each of these six regional sets is further differentiated in terms of the number of nodes of degree 1, 2, 3, 4 and the total number of nodes. The first main column of the table shows the number of link ends for each sector and degree classification. This is simply the number of links per node, or the degree of the node, multiplied by the number of nodes of that degree. Completing the headings at the top of the table are two major divisions based on linkage relationships. The first major column represents the number of nodes which are linked to their first, second, third or higher regional nearest neighbor by the first (that is, shortest) link in that region. The second major column indicated the number of nodes which are linked to the various regional nearest neighbors by the second shortest link in that region. Both major columns have a single column headed "Beyond" in which are summarized all linked neighbors beyond the last order listed. Two additional major columns, the number of nodes with the third link to each regional nearest neighbor and the number of nodes with the fourth link to each regional nearest neighbor are omitted from this table, but the information they contain is summarized in the column headed "All Other Links".

As an example of how to use the table, take the third major row where K = 3 (that is, the three-region system). Now examine the line dealing with nodes of degree 4. There are six nodes of degree 4 or 24 links associated with these nodes. Looking across this row, note that 16 of the 24 link ends are connected to a first nearest neighbor in one of the three regions. One link bypasses the first nearest neighbor. In seven regions, two links per region are observed. These seven second-order links are entered in the second major column with three links to the second nearest neighbor in a region and one link each to a third and fourth regional nearest neighbor. Finally, two links are entered in the summary "Beyond" column. Inasmuch as only three regions were used in this analysis of nodes of degree 4, it is only possible for eighteen links to be classified as first links. In other words, the sectoring system requires that at least six links share a region with six of the first eighteen links. The table entries indicate, however, that only seventeen of the eighteen regions were occupied by a first link, which in turn indicates that one region had no link at all. In a similar manner, each row entry in the table indicates the total number of links of first and second order, and the regional order of the nodes to which they are connected.

Generation of Synthetic Networks

The analysis of the Northern Ireland rail net, and subsequent work reported in Boyce (Reference 14) suggests certain procedures which might be adopted for the generation of synthetic networks. These procedures rely chiefly upon the observation that nodes in a transport system tend to be linked to their first regional nearest neighbors. A number of strategies based upon this regularity may be set forth, but only one approach will be detailed here. Early in the course of the research study on the generation of networks, it was necessary to pick out one combination of the various degree assignment and link allocation methods discussed above for development and use on the networks being examined. After some additional experimentation, a decision was made to develop a method of estimating the final degree

TABLE 25

SUMMARY OF LINK-NODE CHARACTERISTICS

Number of Regions	Degree of Node	Number of Link-ends	Number of regions with first link to _ RNN*				Number of regions with second link to _ RNN				All Other Links
			1st	2nd	3rd	Beyond	1st	2nd	3rd	Beyond	
K = 1	1	15	11	2	1	1					
	2	40	18	1		1	8	6	1	5	
	3	63	19	2			8	8	4	2	20
	4	24	6				5			1	12
	Total	142	54	5	1	2	21	14	5	8	32
K = 2	1	15	12	1	2						
	2	40	29	3	2	2	4				
	3	63	32	8	2	1	11	2	3	4	
	4	24	12				5	1	1	1	4
	Total	142	85	12	6	3	20	3	4	5	4
K = 3	1	15	12	3							
	2	40	32	4	2	1	1				
	3	63	42	7	3	1	5	2	1	2	
	4	24	16	1			3	1	1	2	
	Total	142	102	15	5	2	9	3	2	4	
K = 4	1	15	14	1							
	2	40	37	2	1						
	3	63	53	5	2		2		1		
	4	24	17	2	2		3				
	Total	142	121	10	5		5		1		
K = 5	1	15	14	1							
	2	40	36	3	1						
	3	63	53	7	3						
	4	24	18	2	3		1				
	Total	142	121	13	7		1				
K = 6	1	15	15								
	2	40	36	4							
	3	63	55	4	3	1					
	4	24	19	3	1		1				
	Total	142	125	11	4	1	1				

*Regional Nearest Neighbor

of the node as a function of two variables -- the local characteristics of the node, in this case, and the position of the node with respect to the other nodes on the network. This decision in turn implied that the link allocation procedure would be of a type which required that the degree of the node remain fixed during the allocation process. Also, since the degree of the node would be given as an input to the link allocation procedure, it was decided to set a number of regions equal to the degree of each node. During the course of the study, the efforts to develop a method of degree estimation and a method of link allocation were carried on concurrently. This procedure was possible since the degree of the nodes on the Ulster rail net, which was being used as an experimental network in the development of these methods, could be taken as a direct input to the link allocation procedure. The results of the study to develop a method of degree estimation are reported in Boyce (Reference 14).

Link Allocation Procedures

The question of link allocation in the generation of synthetic transportation networks may be treated as a question of which pairs of nodes should be connected when the degree or number of links per node is given. The link allocation procedure is based upon the first regional nearest neighbor regularity observed in actual transportation networks. Although when the number of regions equals the degree of the node, 67-73 percent of all links do connect nodes to their first regional nearest neighbors, there remains some 25-30 percent of the links which are observed as connecting second, third,...regional nearest neighbors. Thus, the link allocation procedure, while based on the first regional nearest neighbor regularity, must also account for certain deviations from this tendency. The link allocation procedures investigated in this chapter are designed to simulate these deviations from the basic first regional nearest neighbor rule by varying the order in which the nodes are taken up as centroids.

A trial and error process of network generation, in which the order of the nodes and the number of links allocated in each iteration varied from one trial to the next, was used to develop the link allocation methods proposed here. In the development of these methods many trials were run, several of which resulted in comparatively poor networks, but which provided ideas for new trials and experiments. Three of the more successful examples are presented in this chapter, together with an outline of the generating procedure used to produce each synthetic network.

The basic link allocation procedure used in all of the trials discussed here is as follows. Given the degree of each node as actually found on the network (or as estimated by a procedure of the type outlined in Boyce, Reference 14),

(1) Proceed to the node to be used as the current centroid (the order in which the nodes are taken up is discussed in each case);

(2) Sector the area around the centroid, setting the number of regions, K, equal to the degree of that centroid;

(3) Identify the first nearest neighbor in each region. In the event that the first regional nearest neighbor is:

    (a) a node of degree zero,
    (b) a node of degree 1, if the centroid is also a node of degree 1,
    (c) separated from the centroid by an open body of water such as a gulf or large lake,
    (d) separated from the centroid by a previously drawn link,

    then pass over the first nearest neighbor and take the next nearest neighbor that meets the above qualifications. Then ascertain whether the nearest qualified node in each region has an unallocated link.

(4) Link the nodes identified in part (3), in a manner to be specified in the description of each trial.

Sections (2) and (3) of the above allocation procedure are the same for each trial link allocation. Sections (1) and (4) vary according to the specific method of allocation used and are described in detail in the descriptions of the trials which follow. The restrictions based on the identification of the nearest available neighbor in Section (3) require some explanation. In the entire link allocation process the degree of a node is not allowed to change. Thus, if a node of degree zero appears as the first regional nearest neighbor, this node may be disregarded and the second nearest neighbor can then be considered. If the first regional nearest neighbor happens to be a node of degree 1, and the centroid is also degree 1, there is a reasonable basis for not allowing these two nodes to be connected. If such a connection were allowed, these two nodes would be isolated from the remainder of the system, being connected only with each other. Inasmuch as the networks considered in this study are assumed to be of the connected type (there are no isolated links in the system), a simple restriction on the occurence of an isolated link is certainly reasonable. A further restriction on the allocation of links across large open bodies of water is imposed. A final restriction prohibits links from intersecting and thereby requires that no new nodes be formed, which is a reasonable requirement in this procedure.

Some Synthetic Networks

Three link allocation procedures were attempted, and the resulting transportation networks are presented in Figures 22 through 25. Figure 25 is the existing railroad network in 1900, Figures 22, 23 and

24 are three synthetic networks which resulted from various link allocation procedures. Each procedure consisted of two parts, and in each figure the links allocated in the first part are indicated by solid lines while the links allocated in the second part are indicated by dashed lines. The success of each link allocation procedure may be determined to some extent by comparing each figure with Figure 24, the existing network. A second measure of the success of each network generation is the number of links on the existing network which were correctly allocated on the generated network, expressed as a percent of the total number of links on the existing network. For network one, 70 of the 98 links on the existing network were correctly allocated, or a total of 71 percent. On network two, 70 percent of the links on the existing network were allocated, and on network three 75 percent of all links were in the correct place. A third gross measure of the success of each generated network is the extent to which the network is connected. A connected network is one in which it is possible to move from any node on the network to any other node on the network via some path. Inasmuch as the networks considered in this study are connected, it is logical to ask if the generated networks are also connected. Of the networks generated, none are connected. Generated network two, then, on the basis of the percent of links which are correctly allocated and the extent to which the network is connected must be considered the most successful of the synthetic network generated in this study.

Evaluation of the Network Generation Procedures

The networks generated in this study, as shown in Figures 22,23, and 24 are regarded as satisfactory evidence that the problem of link allocation in the generation of transportation networks can be handled adequately through the use of techniques of the type outlined here. These three networks, and in particular network two, are considered to be adequate representations of the existing Ulster rail net. Further, now that the basic structure of the network has been established through the use of the regional nearest neighbor regularities, it is possible to improve on this network significantly by applying local optimization techniques such as the Delta-Wye transformation. Without knowing or ever having seen the existing network being generated here, it is felt that one could make modifications in generated network two through the addition and rearrangement of a few selected links. The characteristics of the resulting network should be extremely close to the actual network for this area. No attempt was made in this study to make these alterations.

Little effort within the confines of the current study was made to evolve a means of testing the generated networks for their correspondence of similarity with the existing network. Two rather gross tests of similarity were proposed based upon the extent to which there is a one-to-one correspondence between the links on the generated and existing networks, or the percent of the actual links which appear on the generated network, and the extent to which the network is connected, assuming that the existing network is also connected. A third test might be described as one of reasonableness or localized efficiency based on available techniques for arriving at a so-called efficient transportation network configuration for two, three, and four nodes. These last techniques all are bound to correct or improve on a configuration such as is found in the area around node 33 in generated network two, and thus these techniques permit an answer to be given to whether a network configuration is reasonable for up to four nodes. However, for systems of more than four nodes no analytical techniques are yet available for specifying a so-called optimal or efficient transportation network. For these reasons a criterion of reasonableness in evaluating the generated networks must be confined to clusters of four or less nodes.

V. TRANSPORTATION SYSTEM PLANNING

Previous chapters have contained reviews of work directed chiefly to forecasting properties of transportation systems and to developing incisive definitions of properties to be forecasted. More and more, broad planning considerations are guiding transportation system development. Attention is directed in this chapter to definitions of transportation planning problems and some ways these problems may be treated. Although it has proved quite difficult to identify and treat planning problems other than at very general levels, the approaches developed here may approximate approaches to be used in the future, and they may, thus, approximate some properties of transportation development situations that will be of concern in the future.

Three problems are of primary concern in this discussion. The first is that of the synthesis of either entire networks or parts of networks, such as adding a single link or making an increment in capacity to a link. The second problem is that of the geometric pattern of routes; and the third is that of the longitudinal configuration of particular routes. The two latter questions -- which are defined more precisely later -- pose the problems discussed in final parts of this chapter.

The succeeding discussions are based largely on previous research.

General Remarks

All planning would appear to involve optimization according to some criteria. Methods of optimization include:

- Calculus
- Selective Search
- Linear Programming
- Integer Programming
- Lagrangian multipliers
- Convex Programming
- Dynamic Programming

Optimization criteria involve minimizing or maximizing some sum or vector (the objective function).

Transportation planning also would appear to require a systems point of view. Transportation activities take place within interdependent systems, where changes in one part of the system may affect other parts.

These observations suggest three questions that are central to transportation planning analysis:

(1) What is being optimized?
(2) What method is in use?
(3) To what system is the method applied and to what system is the optimization relevant?

In spite of the ease with which the planning situation may be described in general terms, no completely satisfactory answers can be found to these three questions. Simple answers can be given, of course, but these are unsatisfactory in that they do not seem to correspond to actual transportation situations.

Forecasting Versus Planning

Although planning is the subject of this chapter, the discussion should not suggest that planning and forecasting are activities that differ in kind. A forecast may be defined as an estimate of the state of events at some date in the future. The forecast may be made in a situation where there is much planning (and control) and the forecasting problem is then simplified. A contrasting situation is one with little planning and much variability. The notion that planning will be more important in the future is a notion about the environment of transportation and about changes in environments of transportation developments.

Who Plans?

Planning precedes most economic, military, and political action; certainly, transportation systems have not been "unplanned" in the past. The difference appears to be in the level or degree of centralization of planning and in the objectives. More and more planning is under the direct control of the central government and is explicitly directed toward reaching national goals, in contrast to planning by the individual operator with respect to his goals. This suggests another question:

> In what ways will the centralization of planning affect the character of transportation development?

This is one of the important questions we have posed but have not answered in this chapter. To state the problem again, we observe that central planning is becoming increasingly important, and we would like to know how this will change the characteristics of transportation, if at all.

Network Synthesis

A very general planning problem is that of constructing and expanding networks. Questions include:

How much budget should be allocated?
What mode (modes) should be used?
Where should nodes and routes be located?
What control policy should be developed (for example, prices charged)?
How should route additions be staged?
What criteria should be used to judge effectiveness?

An Example

Consider a system where certain origins and destinations are known and where at a given time the following variables and constants can be identified;

t_{ij}: the transportation cost between i and j, in units of, say, cents per ton.
x_{ij}: number of movements between i and j in, say, tons.
c_{ij}: the cost of an incremental unit of capacity between i and j.
k^*_{ij}: units of capacity added between i and j.
s_i : the amount to be moved from the i^{th} place.
d_j : the amount of traffic that will terminate at j.
k_{ij}: existing capacity between i and j.
B : budget available for expansion of capacity.

An objective might be to minimize the joint cost of operation and capacity addition: namely,

$$\sum_i \sum_j t_{ij}x_{ij} + \sum_i \sum_j k^*_{ij}c_{ij} = M \text{ (minimum)}.$$

This objective might be subjected to constraints that require that all goods be moved from supplying points:

$$\sum_j x_{ij} = s_i . \qquad i = 1,\ldots,n$$

All terminations are to be made:

$$\sum_i x_{ij} = d_j . \qquad j = 1,\ldots,m$$

Movement along a route shall not exceed existing and new capacity:

$$x_{ij} \geq k_{ij} + k^*_{ij}. \qquad \begin{array}{l} i = 1,\dots,n \\ j = 1,\dots,m \end{array}$$

Finally, budgetary constraints shall not be exceeded:

$$\sum_i \sum_j c_{ij} k^*_{ij} \geq B.$$

A model such as this one might appear to answer or to be easily adapted to answering either the questions asked earlier in this section or the questions asked in the previous section, "General Remarks". For the system as it is defined, it is clear what is being optimized and what method is being used for optimization.

Elaboration of the Example

An important fact about this basic model is that it is readily subject to elaboration in order to bring out particular features of network development that may be of interest in special applications. For instance, a homogeneous commodity is assumed and it is assumed that direct movements are made between supplying and receiving places. The model can be expanded to a multi-commodity model simply by separate labeling of different commodities or by introducing inter-industry relationships. Normally shipments do not pass directly from i to j, but pass through many intervening nodes. This phenomenon may be handled as a transshipment problem, by direct labeling, or using Kirchhoff's constraints and incidence matrices, (Reference 33, Vol. II, p. 636).

The latter device requires constructing the incidence matrix for the transportation network and the conservation of flows at nodes (Kirchhoff's node law); for example,

$$\sum_j e_{ij} q_j = E_i.$$

e_{ij} equal 1 if branch j is positively incident on node i; minus 1 if branch j is negatively incident on node i; and zero if brance j is not incident on node i. q_j is the flow on the j^{th} branch and E_i is efflux out of the network at node i. Charnes and Cooper (Reference 33) point out that this particular scheme permits writing constraining equations in extremely simple form.

This model may be applied to an instance where the network is made up of different modes simply by labeling capacity cost separately for different modes, labeling new capacity separately, and labeling flows separately. The possibility of adding new capacity where capacity did not exist before might be handled using an integer programming formulation. This would enable bringing cost of new right-of-way and associated development into the cost function.

The possibility of relating network development to resource development exists in principle because flows required by the constraining equations may be tied directly to levels of resources available at different points on the network. It is possible, for example, to add a new programming variable to the problem (say r_i, which would be the units of resource developed at place i), and to reconstruct the problem so as to optimize the joint cost of (1) movement on the transportation system, (2) the addition of capacity to the transportation system, and (3) the addition of new resources into the economy. The typical constraints of such a problem might be that the network carry out its function, and that demands by the economy be met.

Is the Model Useful?

The model developed above was presented because this or some similar model may be used for planning purposes. Data and computing requirements are very large, but they are within ranges that may be feasible within the next 10 years or so. The goal of minimum cost is one that is readily accepted, especially be engineers.

Two other points of view may be used for the discussion of this matter of the model's utility. One is the question of whether the model as stated meets the requirements it appears to meet. In the case of costs, for instance, Quandt has pointed out that the units in which c_{ij}'s are measured pose some difficulty (Reference 35). The model is applied to some fixed time period, and there is no reason to believe that the entire cost of the capacity should be retired during that time. Consequently, c_{ij} might be the measure of interest cost for providing the unit of capacity. However, development authorities use highly varied methods of costing, and it may be difficult to determine appropriate cost to be used in the model. Also, the right-hand terms in the objective function are not in the same units as those of the left-hand terms. Another point mentioned by Quandt is that results differ depending upon the time span over which the model is applied. For instance, results of applying the model over a two-year period would be different from those achieved if the model were applied twice with the same targets, that is, for two one-year periods. Again, it may be quite difficult to measure cost. This would appear to be especially true for cost of movement of passengers.

Along the same lines, one might question whether or not the model fits the system under development. Transportation investment and reductions in transportation cost may occasion, for instance, uses of new resources and new patterns of distribution. Relevant cost would extend to the cost of new activities, and there should be some way to utilize information of gains from uses of these resources. Put more broadly, the relevant decision situation extends well beyond immediate questions of uses and capacity cost.

In addition to questioning whether or not the model meets the requirements it appears to meet, one can question whether the broad planning model can be articulated to specific decisions. While the broad model may provide guidance, specific decisions must be made about route locations, and the broad model provides little information on the topic. In the model, it was assumed that the locations of the i and j places shipping and receiving goods were known; yet developments of production and consumption may hinge upon transport investment, especially route location. Also, configuration of routes may affect transportation cost as well as capacity cost.

Geographic Details

In the paragraph just preceding, the point is made that a capability is needed to treat details of route location. Cost depends upon the details as do patterns of production and consumption.

The route location problem may be considered at different levels. At one level, we may consider effects of different types of topography on capacity cost and consequent route location. On a somewhat broader level, we may consider alternate geometric arrangements of routes to service several nodes on a system. In the latter case, we may choose from among various patterns of routes that would provide the service. A less specialized case of the second level of attack is that where routes serve continuous areas rather than points (for example, see Reference 27).

The problem of differences in topography from place to place and consequent differences in capacity cost has been attacked only for simple cases (References 12 and 36). In the most simplified case, it has been assumed that a single boundary separates two regions. The problem posed is that of the manner in which a route extends across the two regions. If transportation costs are different in the regions, the route will tend to prolong its course through the region with the lower transportation costs in order to decrease the distance traversed in the region with the higher costs. Several "laws of breaking" have been discovered and rediscovered for this problem.

Route Patterns

Alternate geometric patterns may be used to provide transportation service among places. The grid-iron pattern of routes in the American Middle West is an example of one arrangement. A map of the

Interstate Highway System will provide another example. Routes in the eastern, central, and southern portions of the nation form mainly triangles. Routes in the western portion are arranged more in rectangles. These route patterns illustrate a type of decision situation where an entire pattern of routes is at choice. Another example is provided by requirements to add new routes to a system. Say that a railroad is to be extended to a new city. One could choose to extend a route directly from a city already on the system, from more than one city, and/or a new intersection might be established and a route extended from a non-terminal portion of an existing route.

An Example

In Reference 6 (p.2 ff.) the route patterns question was illustrated using a somewhat forced example from the settlement of the United States. Pre-Colombian transportation consisted of myriad Indian trails and inland water routes. Because little capital was invested in routes and because unit cost of transportation was high, this system may be characterized as one with low fixed cost (roughly, cost of a unit of capacity) and high variable cost (roughly, cost of moving a unit over the route). (For cost definitions see Reference 37.) The development of America by Europeans greatly increased the amount of commerce and, aided by political stability and by innovations of the steamboat, the train, and highway vehicles, it became practicable to invest capital in route capacity in order to reduce the variable or over-the-road cost of transportation. This was the substitution of fixed for variable cost and it was a function of both technological possibilities and magnitudes of the cost involved.

From the standpoint of geographical details, what is interesting about this substitution is the possible realignment of routes. Each and every Indian trail of pre-Colombian America, for instance, was not replaced by a railroad track. The high fixed cost of railroad development constrained the location of railroad routes to a few favorite corridors. The gradual development of other modes of transportation, such as modern highways and air, also, occasioned locational shifts. The present pattern of routes is a result of a series of shifts and relocations that may be described as wye-delta-wye shifts.

To continue with the example, one would expect three Indian villages to be connected directly by trails. The high cost of carrying goods or movement by persons would occasion the seeking out of direct routes from village to village. Geometrically, the trails would form a delta. The delta configuration would not provide the minimum length of network required to link three villages, but trail maintenance with its low cost was certainly less important than the extra distances that would have to be traveled if effort were made to reduce the length of the network of trails.

With gradual economic development and consequent increases in trade and possibilities for taking advantage of new technology, upgrading of routes, say to roads, might be undertaken. In this case, the problem of the fixed cost of routes versus the cost of using them would have to be considered. One possibility might be that of joining the villages with a wye. Roads might be built outward from each village until they intersected at some intermediate point. Travel among villages would be via the intermediate point. This solution would reduce the amount of investment required (that is, the cost of constructing the network), but because routes between particular villages would be longer than those of the delta, the consequent saving in fixed cost would warrant longer routes between particular villages.

Total cost of transportation is a function of the lengths of routes and the magnitudes of flows. With continuing increases in trade, realignment of transportation routes to a delta might be warranted when distance savings were great enough to offset the extra cost of increasing the total length of the network. Scrapping the wye and complete redevelopment of a delta would be very expensive, of course. Change to a delta might occur if a new mode were introduced; with an existing mode, location change might take the form of gradual shifts toward a delta.

A scale effect might be present. In shifting from a delta to a wye (or from a wye to a delta), the new pattern of routes might be selectively restricted to larger urban centers. In the Interstate Highway System, for example, much of the pattern in the eastern part of the United States is delta-like with nodes on the system being larger urban places. This new pattern of routes is superimposed on "finer patterns" of existing road systems.

Several paragraphs have just been used to describe how three points might be joined. This three-point case would seem to be of special interest. Consider the problem of adding a node (say a city) to a network. If we consider that node and the two nearest nodes on the network, the problem is one of three points. This type of consideration would seem to arise more frequently than would four, five, or higher order cases.

Routes Connecting Three Points

The bulk of our work on this topic was presented in an earlier monograph (Reference 6) and, as was noted there, one point of departure was an unpublished paper by M. Beckmann (Reference 38). An isotropic and homogeneous plain was assumed, capacity cost was assumed to be constant, and transport cost was assumed to be proportional to flow. Cost on a distance unit basis was assumed to be a linear function of flow.

$$c(f) = a + b(f)$$

where: a was capacity cost per unit of distance
b was transport cost per unit of flow per unit of distance
f was a measure of flow.

In the wye instance, the total cost for the network was:

$$c = \sum_{i=1}^{3} (a + bf_i) \; |P_o - P_i| ,$$

where the three points to be joined were P_1, P_2, and P_3 and flows on the associated route segments were f_1, f_2, f_3. P_o was the point of intersection. $|P_o - P_i|$ represents the length of the line segment from P_o to P_i. Flows through P_o follow Kirchoff's Law.

If P_o has coordinates (x_o, y_o) then the distance from P_o to P_i is

$$\sqrt{(x_o - x_i)^2 + (y_o + y_i)^2} = |P_o - P_i| .$$

The coordinates (x_o, y_o) that in general make c (total cost) a minimum may be found by taking the partial derivatives of c with respect to x_o and y_o and equating these to zero.

$$\frac{\partial c}{\partial x_o} = 0 \qquad\qquad \frac{\partial c}{\partial y_o} = 0$$

This problem is trivial from a conceptual point of view, but from the point of view of computations it may be quite difficult. Much of the effort in our study was invested in searches for efficient ways to compute the location of P_o. Instances other than the wye, say a simple delta, also presented rather simple conceptual problems, but problems that were difficult to compute.

An end product of our research was a computer program for rapid solution of the three-point problem. The program determined the type of solution to seek as well as the optimal solution for that pattern. Several example problems were processed with the conclusion that over a wide range of flows and cost the wye solution proved optimal.

Variable Terrain

Previous discussions have assumed away those variations in terrain that might occasion variations in the cost of capacity and in the cost of the use of a route. In the planning of details of transportation system development, however, these terrain variations cannot be assumed away. The orientation of a new route may require a compromise between a straight route with high construction cost per unit of distance and a more circuitous route with lower cost per unit of distance. In these terms alone, route location policy might be that of minimizing the total cost of construction for a given inter-nodal link. In rugged terrain the problem may be that of compromise between a very long route with low unit cost and a short route with high unit cost.

Flow Variations

The transportation planning activity must also encompass cost of using the route. If a route is to be used very heavily, then a short route with high construction cost might be warranted, route length may be less important for more lightly utilized routes.

As the economies of nations expand with technological improvements and population increases, transportation movements increase. Consequently, routes are realigned. These statements identify the thinking behind our main interest in the route location problem in variable terrain. Our interest is based upon the simple notion that one property of transportation change in developing areas is route rearrangement. We would like to have some systematic ideas on this subject.

The thoughts in the paragraph above are hardly novel. It is easy to see that much transportation investment takes the form of route realignment and improvement. This has been one aspect, for example, of railroad right-of-way improvement in the United States.

Investigations

Our studies (Reference 12) followed two lines of investigation. In one, studies were made of various approximations of the variable terrain problem using well-known mathematical tools. An approximation found desirable was that of dividing the terrain into "homogeneous" regions. The area to be traversed by the route (in which cost might be infinitely variable) was divided into areas within which cost could be considered constant. This approximation simplified the problem from that of finding a minimum cost curve between points to be connected to that of finding the points of intersection of the optimal route with the boundaries between regions. The number of regions can be made as small as desired.

The infinitely variable terrain problem was investigated using calculus of variations. A Lagrangian multiplier approach was used for the "regional" problem. Various approximations were considered for regional boundaries. Also, a variety of ways to solve the equations representing the problem were investigated. There is no reason why these straight-forward approaches can not be used for the problem.

Our second line of investigation was a search for approximations for the problem that could be calculated to any desired degree of exactness. A simple method was found for finding a first approximation to the optimal route location. This first approximation used a squared measure of distance. Having found this first approximation, a method was developed for improving this approximation to any desired degree.

Evaluation

It proved practicable to develop both explicit mathematical solutions and approximate solutions to the problem of route location over variable terrain. Our approximate solution may have some properties similar to approximations made by engineers and other determining route locations. Our approximations tend toward least cost, and presumably search methods used by engineers also trend in this fashion.

Early in this section it was remarked that our interest in the variable terrain problem stemmed from expected route relocations as nations develop. While we have developed a way to replicate how relocation decisions might be made, this broad relationship has not been investigated at this level. (Work in earlier chapters and earlier in this chapter relates to relocations viewed at a broader level.) Further work needs to be done on this topic.

Summary Remarks

The present chapter is the last of a series of chapters reviewing work on problems of forecasting transportation developments. The first chapter provided an overall view of the research, and summary remarks have been made from time to time in the text of each chapter. Consequently, no overall summary and evaluation would seem to be required.

The present chapter used as a point of the departure the notion that planning decisions concerning transportation development will probably be more highly centralized than has been true in the past. Some planning situations or problems have been stated and investigated. Results have varied depending upon problem definitions and point of view. Some limited problems can be given explicit solutions. Finally, some technical discussion relevant to planning and forecasting problems has been presented.

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