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THERMAL TRANSIENT RESPONSE OF UNDERGROUND SHELTERS

BY

LT D. F. Sampsell, CEC, USNR

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ABSTRACT

The sound design of environmental control systems for underground shelters is based on knowledge of the thermal transient response under certain climatic conditions. To gain insight into the transient response of underground shelters, a series of tests using a scale model shelter was conducted. Model-prototype considerations resulted in several model temperature distortions which could be accounted for by analytical techniques. The corrected model results agreed well with results from an analog computer study which considered the same prototype shelter. The model results also indicated that shelter shape does not significantly affect its transient response.

Another phase of this study was the development of a non-computer design procedure for determining the environmental control system capacity required for a given set of climatic and soil conditions. The design procedure was applied to a number of shelter locations and climatic conditions to test its performance. As expected, the solutions indicated that ventilation rate and air-conditioning capacity depend heavily on climate, initial soil temperature, and shelter area per person.

This report is intended as a record of the analytical and experimental methods evolved under the task to date. All analytical and experimental techniques are described in detail, and numerical examples for a typical model run and design procedure solution are given. Further refinement and application of the model study techniques and non-computer design procedure are presently underway, and it is hoped that this work will lead to recommended design data.

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NOMENCLATURE

Primary symbols (presented in order of appearance in text).

x, y, z = distance into the thermal medium (soil); ft.

q = heat rate; Btu/hr, Btu/hr-person.

t = temperatures; $^{\circ}\text{F}$

k = thermal conductivity; Btu/hr-ft- $^{\circ}\text{F}$.

ρ = density; lb/ft³.

c = specific heat; Btu/lb- $^{\circ}\text{F}$.

θ = time; hours, days.

A = area; ft², ft²/person.

V = volume; ft³.

L = any length corresponding on the model and prototype; ft.

α = thermal diffusivity; ft²/hr.

N = primary non-dimensional parameter.

h = average convective heat transfer coefficient along the boundary layer; Btu/hr-ft²- $^{\circ}\text{F}$.

Bi = Biot number; hL/k_w , (non-dimensional).

Q = ventilation rate; ft³/hr-person.

R = air-conditioning capacity; tons/person, Btu/hr-person.

F = sensible heat factor; non-dimensional.

w = absolute humidity; lb moisture/lb dry air.

H = enthalpy of condensation; Btu/lb.

n = number of shelter occupants.

Superscripts

* denotes a non-dimensional ratio; $()_m / ()_p$.

Subscripts

x,y,z = quantities with respect to distance in the thermal medium.

s = a stored quantity.

w = quantities with respect to the shelter wall (soil).

m = quantities with respect to the model.

p = quantities with respect to the prototype.

a = quantities with respect to air inside the shelter.

A = quantities with respect to air outside the shelter.

o = an initial condition; $\theta = 0$.

θ = a condition at time $\theta \geq 0$.

S = quantities with respect to sensible heat.

L = quantities with respect to latent heat.

T = a total quantity.

g = quantities with respect to heat generated.

r = quantities with respect to air conditioning.

v = quantities with respect to ventilation.

NOTE: All other symbols are defined as they appear in the text.

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INTRODUCTION

Underground blast and fallout shelters are usually designed to provide protection and habitable living conditions at the lowest reasonable cost per person. The requirement for economical underground shelter facilities has led to recognizing the soil surrounding the shelter as a heat sink which can allow a significant reduction in the ventilation and air-conditioning requirements. The determination of the required environmental control system capacity, including heat absorption by the soil, is a difficult problem, but digital and analog computer solutions have been devised to solve it.^{1,2} These solutions give the thermal transient response of shelters under certain operating and climatic conditions. Environmental control studies have also been conducted with actual underground shelters.^{3,4} Unfortunately tests with actual shelters have been limited. All of these studies (including similar work not referenced) has given considerable insight into the environmental control of underground shelters; however it was felt that additional insight and evidence was needed in view of the limited experience with actual shelters. It was noted that this information, coupled with the results of prior work, might provide the basis for a non-computer procedure for the design of underground shelter environmental control systems. Since the majority of prior work was basically analytical, it was decided that additional analytical work would be duplication of effort. On the other hand, tests with actual shelters are expensive and unwieldy experimentally and logistically. Thus it was decided to approach the shelter problem by means of model studies. The model would provide a degree of realism which cannot be easily achieved with analysis. For example, the shape of the shelter could be duplicated, and heat flow through the soil would actually occur in three dimensions (most of the prior analytical work assumed an idealized shelter shape and heat flow in one dimension).

To simplify construction of the model and reduce experimental complexities, it was decided to model a steel arch structure. This type of shelter has been considered in previous work, ^{2,3,4} and it does not involve consideration of large concrete masses relative to soil masses. The model shelter was buried in a suitable soil medium and heated appropriately to simulate ventilation, air-conditioning, and the physiological thermal response to dry-bulb temperature of people engaged in moderate activity. Temperatures were measured by thermocouples placed inside the shelter, on the shelter walls, and in the soil surrounding the shelter. The model exhibited temperature distortions because of the method of heat input, and because not all of the non-dimensional parameters

could be adjusted to satisfy the laws of similitude. However, all of these distortions were accounted for analytically. The results of the model experiments, when distortions were corrected, showed excellent agreement with the analog computer solutions.² A quantitative comparison of the model behavior with results from other work could not be performed because of insufficient data reported; nevertheless the results were similar in general terms. In general, the model showed that: (1) after occupation the air temperature rises very sharply during the first day after which there is a marked decrease in temperature rise and the response is quite flat -- approximating a step function -- however a steady-state condition is not usually reached during the expected period of occupancy (10 to 14 days); (2) the shape of the shelter does not greatly affect its transient response, thus prior assumptions of one-dimensional heat flow appear to be reasonable. An unanticipated result of the model study is that the distortion analysis can be applied to full-size shelters, which raises the possibility that a program of environmental control studies covering a range of locations and climatic conditions might be undertaken on a single prototype shelter installation.

After the model studies indicated that the geometrical shape of the shelter does not significantly affect its transient response, and that the response might be approximated by a step function, it was found that a non-computer procedure for the design of underground shelter environmental control systems could be developed. The procedure involves a certain degree of trial and error and a working knowledge of psychrometric charts; otherwise it is conceptually straightforward, flexible, and can be used without difficulty by trained personnel. During the early portion of the transient response, the design procedure gives only fairly accurate results because the assumption of a step function response is somewhat in error. However, the time of interest is really at the end of the expected occupancy period, and at this point the procedure usually gives results within 1°F of the analog computer and model study results.

This report considers the experimental and analytical methods used to develop the model study and non-computer design procedure, and it is divided into three parts. The first part deals with the model study, the second part is devoted to the non-computer design procedure, and the third part gives general recommendations. The design procedure is applied to a number of shelter locations and climatic conditions; however the results are to be used only for purposes of comparison. Further refinement and application of the design procedure must be undertaken before it can be used with confidence. This work is receiving continued attention, and it will be reported at a later date.

PART I. THE MODEL STUDY

Development of the Non-Dimensional Parameters

The non-dimensional parameters used for the shelter model study can be derived conveniently by considering the heat transfer phenomena involved. The nomenclature used is listed at the end of the text, and the assumptions taken are given in Appendix A.

To begin the analysis, consider a differential element in cartesian coordinates with heat flow in all three dimensions (see Figure 1). The energy balance between heat conducted and heat stored is:

$$(dq_x + dq_y + dq_z) = dq_s + (dq_{x+dx} + dq_{y+dy} + dq_{z+dz}) \quad (1)$$

At the surfaces of the differential volume, the Fourier conduction equation is expressed in terms of partial differential equations:

$$dq_x = -k_w(dy)(dz) \frac{\partial t}{\partial x} \quad (2)$$

$$dq_{x+dx} = -k_w(dy)(dz) \left[\frac{\partial t}{\partial x} + \frac{\partial^2 t}{\partial x^2} dx \right] \quad (3)$$

(Similar relations can be developed for dq_y , dq_{y+dy} , dq_z , and dq_{z+dz})
The heat stored within the volume element, dq_s , is expressed as

$$dq_s = \rho_w c_w(dx)(dy)(dz) \frac{\partial t}{\partial \theta} \quad (4)$$

To expedite the analysis, let $dA = (dx)(dy) = (dy)(dz) = (dx)(dz)$ and $dV = (dx)(dy)(dz)$.

Now, by substitution, Equation 1 can be expressed as

$$\begin{aligned} -k_w(dA) \left(\frac{\partial t}{\partial x} + \frac{\partial t}{\partial y} + \frac{\partial t}{\partial z} \right) &= \rho_w c_w(dV) \frac{\partial t}{\partial \theta} - k_w(dA) \left[\frac{\partial t}{\partial x} + \frac{\partial^2 t}{\partial x^2} dx \right. \\ &\quad \left. + \frac{\partial t}{\partial y} + \frac{\partial^2 t}{\partial y^2} dy + \frac{\partial t}{\partial z} + \frac{\partial^2 t}{\partial z^2} dz \right] \end{aligned} \quad (5)$$

The first order terms containing $\frac{\partial t}{\partial x}$, $\frac{\partial t}{\partial y}$, and $\frac{\partial t}{\partial z}$ can be eliminated immediately so that

$$k_w (dA) \left[\frac{\partial^2 t}{\partial x^2} dx + \frac{\partial^2 t}{\partial y^2} dy + \frac{\partial^2 t}{\partial z^2} dz \right] = \rho_w c_w (dV) \frac{\partial t}{\partial \theta} \quad (6)$$

Equation 6 is dimensionally consistent; therefore the quantity, L , can be substituted for all quantities involving length, and Equation 6 can be expressed dimensionally as

$$k_w L^2 \left(\frac{t}{L^2} L \right) = \rho_w c_w L^3 \frac{t}{\theta} \quad (7)$$

which can be simplified to

$$k_w = \frac{\rho_w c_w L^2}{\theta} \quad (7a)$$

It should be noted that the quantities k_w , ρ_w , and c_w have not been expressed in dimensional form because these quantities are recognized soil properties. Equation 7a can be arranged to yield a nondimensional parameter.

$$N_1 = \frac{\alpha_w \theta}{L^2} \quad (8)$$

$$\text{where } \alpha_w = \left(\frac{k}{\rho c} \right)_w$$

A basic principle of similitude is that

$$\frac{\alpha_w \theta}{L^2} \text{ model} = \frac{\alpha_w \theta}{L^2} \text{ prototype} \quad (9)$$

The quantity, L , is a convenient length for proportionately scaled models. It does not matter which length is chosen, but the length chosen on the prototype must correspond to the length taken on the model.

The parameter, N_1 , has been derived from the interaction between heat conducted and heat stored. It is this interaction which affects the transient temperature response in the thermal medium. N_1 can be used to determine prototype time values for corresponding model time values.

$$\text{Let } L^* = L_m / L_p$$

$$\text{and } \alpha_w^* = \alpha_{wm} / \alpha_{wp}$$

Equation 9 can be solved for θ_p .

$$\theta_p = \frac{\alpha_w^*}{L^{*2}} \theta_m \quad (10)$$

A non-dimensional parameter for temperatures in the soil medium at any point can be derived from the Fourier conduction equation.

$$dq = -k_w (dA) \left[\frac{\partial t_w}{\partial x} + \frac{\partial t_w}{\partial y} + \frac{\partial t_w}{\partial z} \right] \quad (11)$$

The quantities q , k_w , and t_w will be retained, and upon substitution of L^2 for A , and L for x , y , and z , Equation 11 can be expressed dimensionally as

$$q = k_w L^2 \frac{t_w}{L} \quad (12)$$

The resulting non-dimensional parameter is

$$N_2 = \frac{k_w L t_w}{q} \quad (13)$$

The temperature, t_w , varies as a function of time, θ , from some datum temperature, t_{w0} , at $\theta = 0$. Therefore, N_2 can be expressed more conveniently as

$$N_2 = \frac{k_w L (t_w - t_{w0})}{q} \quad (13a)$$

Applying the basic principles of similitude,

$$\frac{k_w L (t_w - t_{w0})}{q} \text{ model} = \frac{k_w L (t_w - t_{w0})}{q} \text{ prototype} \quad (14)$$

If we let $q^* = q_m/q_p$ and $k_w^* = k_w/k_{w_p}$, the prototype shelter wall temperature rise, $(t_w - t_{w_o})$, can be expressed as

$$(t_w - t_{w_o})_p = \frac{k_w^* L^*}{q^*} (t_w - t_{w_o})_m \quad (15)$$

Heat flow from the shelter air to the shelter wall is expressed by the familiar convection equation written below in differential form.

$$dq = h(dA) (t_a - t_w) \quad (16)$$

where: h is the average convective heat transfer coefficient along the boundary layer.

t_a is the air temperature beyond the boundary layer.

This equation immediately suggests a non-dimensional parameter when L^2 is substituted for A .

$$N_3 = \frac{hL^2}{q} (t_a - t_w) \quad (17)$$

Letting $h^* = h_m/h_p$ and imposing the principle of similitude, we have an expression for the prototype air-wall temperature difference.

$$(t_a - t_w)_p = \frac{h^* L^{*2}}{q^*} (t_a - t_w)_m \quad (18)$$

Model-Prototype Temperature Distortions

Equations 15 and 18 are basic temperature relations between the prototype and model. When considered together, these equations yield an important result. Recall Equation 15 and compare it to Equation 18. Note that a one-to-one correspondence between prototype and model temperature differences in each equation will exist only when

$$\frac{h^* L^{*2}}{q^*} = \frac{k_w^* L^*}{q^*} = 1 \quad (19)$$

from which

$$\frac{h^*L^*}{k_w^*} = Bi^* = 1 \quad (19a)$$

where: Bi is the Biot number*

$$Bi^* = Bi_m / Bi_p$$

The requirement that $Bi^* = 1$ severely restricts the flexibility of the model to simulate prototype values of h and k_w ; therefore, this requirement cannot always be met and temperature distortions will result.

There are two methods of accounting for these distortions: (1) imposing a one-to-one correspondence between model and prototype shelter wall temperatures, and (2) imposing a one-to-one correspondence in heat flux between the model and prototype

Case 1 Condition

Considering the Case 1 condition, it is noted from Equation 15 that $K_w^*L^* = q^*$. Substituting this requirement into Equation 18,

$$(t_a - t_w)_p = \frac{h^*L^{*2}}{k_w^*L^*} (t_a - t_w)_m \quad (18a)$$

provided $t_{w_o p} = t_{w_o m}$

Since $Bi = hL/k_w$

$$(t_a - t_w)_p = Bi^* (t_a - t_w)_m \quad (18b)$$

When $Bi^* \neq 1$, an air temperature distortion between prototype and model will occur.

*The Biot number has the same grouping of variables as the Nusselt number. The fundamental difference between these two numbers is that the Biot number involves the thermal conductivity of a solid, and the Nusselt number the thermal conductivity of a fluid. Thus the Biot number is an index of the relative resistance to heat flow in a solid and fluid, whereas the Nusselt number expresses a comparison of the convective capability to the conductive capability of the fluid only.

$$\text{Let } \Delta t_a = t_{a_m} - t_{a_p} \quad (20)$$

Since $t_{w_m} = t_{w_p}$ (by definition for Case 1)

$$\Delta t_a = (t_a - t_w)_m - (t_a - t_w)_p \quad (21)$$

$$(t_a - t_w)_p = \left(\frac{q}{hA}\right)_p \quad (22)$$

$$(t_a - t_w)_m = \frac{1}{Bi^*} (t_a - t_w)_p \quad (18b)$$

$$\text{Thus } (t_a - t_w)_m = \frac{1}{Bi^*} \left(\frac{q}{hA}\right)_p \quad (23)$$

Substituting from Equations 22 and 23 into Equation 21:

$$\Delta t_a = \frac{1}{Bi^*} \left(\frac{q}{hA}\right)_p - \left(\frac{q}{hA}\right)_p \quad (21a)$$

$$\Delta t_a = \left(\frac{q}{hA}\right)_p \left(\frac{1}{Bi^*} - 1\right) \quad (21b)$$

Or, the air temperature distortion may be expressed as

$$\Delta t_a = \left(\frac{q}{hA}\right)_p \left(\frac{1 - Bi^*}{Bi^*}\right) \quad (21c)$$

(Note that the distortion equals zero when $Bi^* = 1$)

The model-prototype air temperature relation is found from Equations 20 and 21c, where:

$$t_{a_p} = t_{a_m} - \left(\frac{q}{hA}\right)_p \left(\frac{1 - Bi^*}{Bi^*}\right) \quad (24)$$

Since the model may not have the same initial temperature as the chosen prototype, all air temperatures should be replaced by temperature differences between temperatures at time θ and initial temperatures at time $\theta = 0$.

$$(t_{a_\theta} - t_{a_0})_p = (t_{a_\theta} - t_{a_0})_m - \left(\frac{q}{hA}\right)_p \left(\frac{1 - Bi^*}{Bi^*}\right) \quad (24a)$$

Case 2 Condition

If the Case 2 condition is imposed (heat flux in model and prototype are equal), the model-prototype air temperature relation is found to be

$$(t_{a_{\theta}} - t_{a_o})_p = (t_{a_{\theta}} - t_{a_o})_m - q' \left(\frac{h_p - h_m}{h_p h_m} \right) + \left(\frac{k_w^*}{L^*} - 1 \right) (t_{w_{\theta}} - t_{w_o})_m \quad (B-15a)$$

(See Appendix B for derivation)

Note that the Case 2 condition involves model distortion of both the air and wall temperatures, whereas the Case 1 condition results in model distortion of the air only. Also note that Case 1 and Case 2 are separate experimental conditions. Thus Equations 24a and B-15a are not strictly interchangeable. Although both Cases 1 and 2 were attempted experimentally, only Case 1 will be developed in detail within this report.

Description of the Model

The underground shelter model was a 1/7 scale steel arch structure similar to that used for several actual shelter tests which have been conducted.^{3,4} See Figure 2. The model was buried in dry beach sand to assure a homogenous soil medium and reasonably consistent degree of compaction, and also to eliminate the problem of moisture migration. The moisture content of soil affects its bulk thermal properties; thus moisture migration during a test would result in a requirement to account for varying thermal properties. (Effects of moisture migration would be considerably more significant for the model than the prototype because the unscaled model temperature gradients in the soil are steeper.) The thermal properties of the sand (conductivity and specific heat) were measured by a specially constructed conductivity tester and standard calorimetric techniques respectively. (The conductivity tester was designed to measure thermal conductivity by means of steady-state conduction between two concentric cylindrical surfaces.) Model temperatures were measured by copper-constantan thermocouples placed inside the shelter, on the shelter walls and floor, and in the sand surrounding the shelter. Thermocouples for measuring soil temperatures were soldered to brass rings which were mounted on wooden dowels protruding radially from the shelter walls and floor (see Figure 2). Heating pad units stripped of padding were used in conjunction with a wattmeter and variac to provide controlled sensible heat to the model. The pads were operated far below their rated output to minimize radiant heat exchange. Latent heat input (part of human heat output) was not provided for the model; nor were a ventilation system and an air-conditioning system provided. Instead, the sensible heat output was modified in accordance with a heat balance which is described below. Although this procedure resulted in an air temperature distortion (which will be discussed), it greatly simplified the model system and allowed maximum experimental control.

Heat Input to the Model

The heat balance, which determined the heat input to the model, required consideration of human sensible and latent heat outputs, climatic conditions for the geographical location of the prototype shelter in question, and ventilation rate and air-conditioning capacity for the prototype operating conditions being modeled. Sensible heat was added to the model according to the net heat load which would be imposed on the prototype shelter walls. The capacitance effect of the shelter air could be neglected because the thermal capacitance of air compared to that of the shelter walls is very small.

The sensible heat load imposed on the prototype shelter wall is:

$$q_{S_p(\text{per person})} = (835 - 8t_a)_p - R(F) - \rho_a c_a Q(t_a - t_A)_p \quad (25)$$

Human heat Air- Ventilation
output* conditioning

The latent heat load imposed on the prototype shelter wall is:

$$q_{L_p(\text{per person})} = (8t_a - 435)_p - R(1-F) - \rho_a QH(w_a - w_A)_p \quad (26)$$

Human heat Air- Ventilation
output* conditioning

The total heat load imposed on the model shelter wall is:

$$q_{T_p} = q_{S_p} + q_{L_p} \quad \text{when } q_{L_p} > 0 \quad (27)$$

$$q_{T_p} = q_{S_p} \quad \text{when } q_{L_p} \leq 0 \quad (28)$$

Notice that q_{L_p} must be positive to be considered. A latent heat load is manifested by moisture condensation on the prototype shelter walls. Evaporation from the shelter wall requires the presence of moisture. This mass transfer phenomena could not be controlled analytically for the model, thus a blanket restriction that evaporation from the shelter walls not be considered was necessary.

The relative humidity inside prototype shelters is usually very high, thus it can be assumed that the shelter air is at or near saturation for purposes of computing q_{L_p} . q_{S_p} and q_{L_p} can be evaluated from Equations 25

*Mathematical expression suggested by Drucker² to approximate metabolic heat output of average persons engaged in moderate activity. (See ASHRAE Guide and Data Book, Fundamentals and Equipment, 1961, pp. 106-107.)

and 26 as a function of dry-bulb temperature. The basic problem is to assure that the heat input to the model is varied properly according to the dry-bulb temperature inside the model. At this point it should be recalled that the latent heat input is added as sensible heat to provide greater control over the model. Since sensible heat transfer between the shelter air and walls requires a temperature difference, $q_s = hA(t_a - t_w)$, the addition of latent heat by means of sensible heat results in another air temperature distortion. This distortion can be corrected as follows:

$$\text{Let } (t'_a - t_w) = \left(\frac{q_T}{hA}\right)_p = \left(\frac{q_s + q_L}{hA}\right)_p \quad (29)$$

$$(t_a - t_w) = \left(\frac{q_s}{hA}\right)_p \quad (30)$$

$$(t''_a - t_w) = \left(\frac{q_L}{hA}\right)_p \quad (31)$$

$$\text{Since } q_{s_p} + q_{L_p} = q_{T_p} \quad (27)$$

$$(t'_a - t_w) + (t''_a - t_w) = (t'_a - t_w) \quad (32)$$

$$t_a + t''_a - 2t_w = t'_a - t_w \quad (32a)$$

$$t'_a = t_a + (t''_a - t_w) \quad (32b)$$

$$\text{Let } \Delta t'_a = (t''_a - t_w)$$

$$\text{Then } \Delta t'_a = \left(\frac{q_L}{hA}\right)_p \quad (\text{the temperature distortion}) \quad (31a)$$

$$\text{And } t'_a = t_a + \Delta t'_a \quad (34)$$

Incorporating this correction into Equations 24a and B-15a we have:

$$(t_{a\theta} - t_{a_o})_p = (t_{a\theta} - t_{a_o})_m - \left(\frac{q_L}{hA}\right)_p - \left(\frac{q_T}{hA}\right)_p \left(\frac{1 - Bi^*}{Bi^*}\right) \quad (24b)$$

$$(t_{a\theta} - t_{a_o})_p = (t_{a\theta} - t_{a_o})_m - \left(\frac{q_L}{hA}\right)_p - q_{T_p} \left(\frac{h_p - h_m}{h_p h_m}\right) + \left(\frac{k^*}{L^*} - 1\right)(t_{w\theta} - t_{w_o})_m \quad (B-15b)$$

Preparations for a Model Run

Model runs were made using both the Case 1 and Case 2 conditions. However, the Case 1 condition (which imposes equal prototype and model wall temperature rises) is considerably less complex; thus this condition will be used exclusively to describe the procedure for setting up a model run.

As mentioned earlier, the basic problem of setting up a model run is properly relating heat input to the air temperatures inside the shelter. Once the climatic conditions at the chosen geographical location have been determined from weather data, and prototype values for the many physical and operating conditions have been assigned, the heat balances (Equations 25 and 26) are solved for the range of air temperatures expected to occur in the prototype shelter. The family of solutions for q_{T_p} as a function

of t_{a_p} are the basic heat inputs to the prototype shelter wall. These heat inputs must be modified for the model as follows: Recalling that $(t_w - t_{w_o})_p = (t_w - t_{w_o})_m$ for the Case 1 condition, Equation 15 requires that

$$q^* = k_w * L^* \quad (35)$$

Since $q^* = q_m / q_p$

$$q_m = k_w * L^* q_p \text{ on a per person basis.} \quad (36)$$

The total heat input to the model is

$$q_{T_m} = n k_w * L^* q_{T_p}$$

where: n is the number of shelter occupants.

Now, the model air temperatures will be distorted because Bi^* may not equal unity, and the latent heat portion, q_{L_p} , of the total heat input,

is supplied to the model as sensible heat. The distortions for the Case 1 condition are part of Equation 24b. These distortions are found using the solutions of the heat balances so that for each prototype shelter air temperatures, t_{a_p} , there is a distorted model shelter air

temperature rise, $(t_{a_\theta} - t_{a_o})_m$; and the model heat input, q_{T_m} , is plotted

as a function of model air temperature rise (see Figure 3). This curve is used during the model run. After the run has been completed, model

air temperature distortions are applied (with a change in algebraic sign) to the experimental air temperature data to yield corrected air temperatures for the prototype. For this purpose it is convenient to plot prototype air temperatures as a function of model air temperature rise (see Figure 4). Using the Case 1 condition, the model gives the prototype shelter wall temperature rise directly -- no correction is necessary.

Since experimental data is taken in real time, the time of each model temperature observation must be converted to prototype time after the run has been completed -- the equivalent number of prototype hours for each hour of model (or real) time must be found. This conversion is made from Equation 10.

$$\theta_p = \frac{\alpha_{w*}}{L_*^2} \theta_m \quad (10)$$

The conversion factor, α_{w*}/L_*^2 , is found before the model run is begun so that the duration of the run can be anticipated.

A Typical Model Run

To examine the behavior of the model let us consider a model run for Washington, D. C., at summer climatic conditions. The performance of the model can be compared to analog computer results for the same location by using the input data reported by Drucker.² The basic climatic and shelter operating conditions were evaluated as follows:

Average air temperature outside shelter:	$t_A = 77^\circ\text{F}$
Average air absolute humidity outside shelter:	$w_A = 0.0162 \text{ lb/lb}$
Initial soil temperature:	$t_{w_o} = 68^\circ\text{F}$
Ventilation rate per person:	$Q = 6 \text{ cfm}$
Air-conditioning rate per person:	$R = 0$

All other input data and detailed calculations for setting up the model run are given in Appendix C. The heat input curve is given in Figure 3, and the temperature conversion curve is given in Figure 4.

During the model run, temperatures were recorded as a function of real time. Model shelter air and wall temperatures were determined by averaging the response of several thermocouples in each instance. As the air temperature changed, the heat input, q_{T_m} , was varied according to the

curve shown in Figure 3. After the experimental data had been taken, prototype air temperatures were found from the distorted model air temperatures by means of Figure 4. As mentioned earlier, it was not necessary to correct the shelter wall temperature data (other than to account for the difference between the initial soil temperatures of the prototype and model). Real time was converted to prototype time by means of the time conversion factor, α_w^* / L^2 . This conversion factor was 13.6; thus 1 hour of model time was equivalent to 13.6 hours of prototype time (see Appendix C).

Results of the Model Run

The results of the model run for Washington, D. C. are shown in Figures 5 and 6. Figure 5 shows the results when using the Case 1 condition, and Figure 6 shows the results for the Case 2 condition. The solid line curve for the air temperatures were developed from the analog computer results obtained by Drucker.² Reference 2 only shows the analog results for the shelter conditions at the tenth day, however, Professor Drucker was kind enough to provide the complete analog transient solution for purposes of comparison. The solid line shelter wall temperature curve was obtained from the convection equation using values of q_s calculated

from Equation 25. It should be noted that the model results using both Cases 1 and 2 agree closely with the analog computer results. It should also be noted that the transient response closely resembles a step function. The air temperature rise is very steep initially, and then it levels off so that by the fourth or fifth day of occupancy the response is quite flat, although still rising. This step-like response was observed in all model runs, but it was less pronounced in situations where the total air temperature rise exceeded 15°F.

The results shown in Figures 5 and 6 are the average shelter temperature conditions. Actually a periodic air temperature variation inside the shelter occurs because the outside air temperature fluctuates periodically over a 24-hour interval. The maximum daily air temperature is found by adding a temperature increment to the data shown. This increment is a function of the daily outside air temperature range and the ventilation rate. Temperature increments for various daily outside air temperature ranges and shelter ventilation rates have been determined from Drucker's data, and can be found from the family of curves presented in Figure 7.*

*The curves shown in Figure 7 are valid for the shelter area/person and soil thermal property values used by Drucker (see Appendix C). However, it is believed that the curves have general applicability because the effect of shelter area per person and soil thermal properties on the basic transient response is much more significant than the effect of these parameters on inside air fluctuations.

The average daily temperature range in Washington, D. C. during the summer is 18°F (according to Weather Bureau data). From Figure 7, a ventilation rate of 6 cfm/person under these conditions will result in a temperature increment of 1.7°F. Thus the maximum shelter air temperature occurring during the tenth day of occupancy is $84.5 + 1.7 = 86.2^\circ\text{F}$. Now, dry-bulb temperature alone is not a complete index of comfort; but dry-bulb temperature considered with respect to absolute humidity (or wet-bulb temperature) leads to an index of comfort, called effective temperature.

Effective temperature is an empirically determined index of the degree of warmth perceived by the human body on exposure to different combinations of temperature, humidity, and air movement. The temperature was determined by trained subjects who compared the relative warmth of various air conditions in two adjoining rooms by passing back and forth from one room to another. The results of these tests have led to an effective temperature chart (see ASHRAE Guide and Data Book, 1961, p. 109). For the sake of convenience, a portion of this chart has been transposed to a modified psychrometric chart (Figure 8). The effective temperature lines have been drawn for still-air conditions.

To determine the maximum effective temperature which would occur in the shelter being studied, the maximum absolute humidity must be found. For this purpose, recall Equation 26 -- the heat balance used to calculate the latent heat load imposed on the shelter walls.

$$q_{L_p} = (8t_a - 435)_p - R(1-F) - \rho_a QH(w_a - w_A)_p \quad (26)$$

If q_{L_p} is positive, condensation is occurring on the shelter walls and the air is saturated at the wall temperature. If negative, q_{L_p} is considered equal to zero and the absolute humidity, w_{a_p} , can be found by rearranging Equation 26.

$$w_{a_p} = w_{A_p} + \frac{(8t_a - 435)_p - R(1-F)}{\rho_a QH} \quad (38)$$

To find $w_{a_{\max p}}$, evaluate Equation 38 using $t_{a_{\max p}}$.

$$t_{a_{\max p}} = 86.2 \text{ F}$$

$$R = 0$$

$$\begin{aligned}
w_{a_{\max p}} &= 0.0162 + \frac{255}{2.84 \times 10^4} \\
&= 0.0162 + 0.009 \\
&= 0.0252 \text{ lb/lb}
\end{aligned}$$

Turning to the psychrometric chart, Figure 8, it can be seen that $t_{a_{\max p}} = 86.2^\circ\text{F}$, and $w_{a_{\max p}} = 0.0252 \text{ lb/lb}$ correspond to an effective temperature of very nearly 85°F . The results reported by Drucker for the tenth day of shelter occupancy are $t_{a_{\max p}} = 86.5^\circ\text{F}$ and $ET = 85^\circ\text{F}$.

Model Studies on Shelter Shape

The excellent agreement between the model study results and the analog computer solutions, as illustrated by the run for Washington, D. C., suggests that the shape of the shelter may not significantly affect its transient response (the Drucker study assumed one-dimensional heat flow). If the assumption of one-dimensional heat flow is sound, the shelter surfaces can be laid out as the plane face of a semi-infinite slab. Under these conditions, the diffusion equation (6) can be simplified to

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha_w} \left(\frac{\partial t}{\partial \theta} \right) \quad (39)$$

For the semi-infinite slab, Carslaw and Jaeger⁶ suggest a solution of the form:

$$t = t_0 + \frac{2q}{k_w A} \left[\frac{\alpha_w \theta}{\pi} e^{-\frac{x^2}{4\alpha_w \theta}} - \frac{x}{2} \operatorname{erfc} \left(\frac{x}{2\sqrt{\alpha_w \theta}} \right) \right] \quad (40)$$

which, upon substitution into Equation 39, leads to an expression for the temperature distribution in the slab (soil) as a function of time following a step change in heat flux.

$$t = t_0 + \frac{2q}{k_w A} \left[\frac{\alpha_w \theta}{\pi} e^{-\frac{x^2}{4\alpha_w \theta}} - \frac{x}{2} \operatorname{erfc} \left(\frac{x}{2\sqrt{\alpha_w \theta}} \right) \right] \quad (41)$$

At $x = 0$, Equation 41 yields an expression for wall temperatures as a function of time.

$$t_{w\theta} = t_{w_0} + \frac{2q}{k_w A} \left(\frac{\alpha_w \theta}{\pi} \right)^{\frac{1}{2}} \quad (42)$$

The model was tested against the one-dimensional heat flow relations as follows: at time $\theta = 0^-$, $q = 0$, and the model was in thermal equilibrium; at time $\theta = 0^+$, the heat flux was raised to a constant value equal to the maximum heat flux which would occur in a prototype shelter with normal occupancy. During the test, wall and soil temperatures were measured, but the heat rate was not changed. The experimental data was then compared to theoretical curves obtained from Equations 41 and 42 evaluated in terms of the model (see Appendix C for model parameter values). Since the model was not being run with respect to a prototype, there were no temperature distortions or time conversions. However, the total heat input to the soil was roughly equivalent to that experienced with a model run for a typical prototype.

The theoretical curves and experimental data for the transient response at the shelter wall are shown in Figure 9; and the theoretical curve and experimental data for the temperature distribution in the soil after 8 hours are given in Figure 10. The experimental results agree very satisfactorily with the theoretical curves. This evidence indicates that shelter shape effects are probably not serious. Although polydimensional diffusion from the shelter edges and arch was expected (this was seen by plotting isotherms in the surrounding soil), the exponential nature of the temperature gradient (Figure 10) shows that the major portion of the thermal energy absorbed by the soil occurred close to the shelter wall thereby lessening the effects of polydimensional diffusion. For this reason, it is believed that the polydimensional diffusion effect would have been minor even if the shelter had been a parallelepiped.

Use of Model Study Relationships for Prototype Shelter Runs

It is conceivable that the underground shelter transient response could be studied using a prototype shelter and model study relationships. For example, having run a prototype shelter buried in a given soil, it may be desirable to find what the transient response would have been if the shelter had been buried in a different soil. Assuming that the soil thermal properties can be represented by single values, parameters k_w^* , ρ_w^* , and c_w^* can be found. Other parameters are:

$$q^* = 1$$

$$\text{where: } ()^* = ()_1 / ()_2$$

$$h^* = 1$$

Subscript 1 refers to the actual prototype.

$$L^* = 1$$

Subscript 2 refers to the prototype with modified soil properties.

Since the ratio of heat fluxes is equal to unity, a Case 2 condition exists, and Equation B-15b is applicable. This equation can be greatly simplified because $h^* = 1$, $L^* = 1$, and latent heat is not added as sensible heat; however, a wall temperature distortion occurs because $k_w^* \neq 1$.

$$(t_{a_\theta} - t_{a_o})_2 = (t_{a_\theta} - t_{a_o})_1 + (k_w^* - 1) (t_{w_\theta} - t_{w_o})_1 \quad (B-15c)$$

Since the initial air temperature, t_{a_o} , is synonymous for both (1) and (2), Equation B-15c can be further simplified to

$$t_{a_\theta 2} = t_{a_\theta 1} + (k_w^* - 1) (t_{w_\theta} - t_{w_o}) \quad (B-15d)$$

For $k_w^* \geq 1$, the modified soil thermal conductivity is less than that for the actual prototype, and one would expect that higher air temperatures would occur in the shelter if it were buried in such a soil (assuming $\rho_w^* = 1$, $c_w^* = 1$). Checking Equation B-15d, it is seen that, for $k_w^* \geq 1$, a temperature increment is added to the actual prototype air temperature data to obtain air temperatures for the modified prototype soil. The converse is true for $k_w^* < 1$.

The time conversion factor for the prototype buried in soil having modified thermal properties is found from Equation 10, which can be simplified because $L^* = 1$.

$$\theta_2 = \alpha_w^* \theta_1 \quad (10a)$$

where: $\alpha_w^* = (k/\rho c)_w^*$

It should be recalled that for the model-prototype case the time conversion factor involved L^{*2} which allowed many hours of prototype time to be run per hour of model (or real) time. For the prototype-prototype case, the time conversion factor can be less than or greater than unity. Practically speaking, the prototype-prototype time conversion factor will not vary far from unity -- rarely exceeding the range from 1/2 to 2. Moreover, this conversion factor will have little effect on the shelter transient response because the response closely resembles a step function (see Figures 5 and 6).

Since the prototype shelter heat input to the wall varies naturally with climatic conditions and conditions of the air inside the shelter, there is no need to consider the heat balances (Equations 25 and 26) and the problems associated with heat control. On the other hand, the

functional character of the heat input appears to preclude a non-dimensional treatment of climatic conditions; thus a given prototype installation cannot be tested for a range of climatic conditions greater than that which occurs naturally unless the temperature and humidity of the ventilation air is controlled, or controlled heat inputs are employed as in the model.

CONCLUSIONS TO PART I

As a tool for obtaining engineering data, the model shelter is not completely satisfactory because considerable effort is required to set up, conduct, and analyze each run. However, the model study proved to be of considerable value in terms of developing a method for the analysis of complex heat transfer systems and generally substantiating the computer programs which have been developed by others. More specifically the results of model experimentation have shown that:

1. The thermal transient response of an underground shelter can be examined by model studies.
2. The analog computer studies conducted by Drucker, et al,² for the fully buried underground steel arch shelter are probably sound.
3. Shelter shape does not seriously affect the thermal transient response. Shape effects include temperature distortions due to poly-dimensional heat diffusion at the edges and over the arch. These effects are minor because the temperature gradient does not extend appreciably into the soil surrounding the shelter during the expected period of occupancy.
4. One-dimensional conduction in the soil can be assumed with confidence.

PART II. NON-COMPUTER DESIGN PROCEDURE FOR DETERMINING THE CAPACITY OF UNDERGROUND SHELTER ENVIRONMENT CONTROL SYSTEMS

Introduction

The non-computer design procedure for determining the capacity of underground shelter environment control systems depends on analytical tools developed in Part I. Briefly the procedure is as follows: The shelter soil system is conceived as a semi-infinite slab having a plane face equal in area to the total shelter area (per person basis). The maximum dry-bulb temperature and effective temperature tolerable (or desired) inside the shelter at the end of the expected occupancy period are determined, and trial values for the ventilation rate and air-conditioning capacity are assigned. Next, the maximum dry-bulb temperature is converted to an average value to account for daily air temperature fluctuations inside the shelter. It is assumed that the air temperature rises to the average value upon occupancy and remains at this value during the entire occupancy period. From the heat balance equations (25 and 26) evaluated for the average inside air temperature and climatic conditions at the intended shelter location, the sensible and latent heat rate inputs to the shelter wall are determined. Now, responding to the rise in air temperature, the wall temperature rises with time in accordance with the expression developed for a semi-infinite slab subject to a step increase in heat flux (Equation 42). Eventually a maximum wall temperature is reached above which the sensible heat to the shelter wall cannot be supported at the average air temperature, and the occupancy period must be terminated. Once the maximum wall temperature has been ascertained (from the convection equation), the allowable occupancy period is determined from Equation 42. If the allowable occupancy period is less than that required, the ventilation rate and/or air-conditioning rate must be adjusted to lower the heat input to the shelter walls. If the allowable occupancy period is considerably greater than that required, the environmental control system capacity should be reduced.

It should be noted that two fundamental assumptions are necessary for the design procedure: (1) the air temperature response in an actual shelter can be thought of as a step function, and (2) one-dimensional heat flow from the walls and floor of an actual shelter into the surrounding soil prevails. The bases for these assumptions are covered in Part I. Other assumptions are given in Appendix A.

The design procedure requires a certain degree of trial and error; however, it is straightforward, flexible, and can be used without difficulty by trained personnel. The use of the design techniques will become apparent from the step-by-step procedural outline and numerical example below.

Outline of the Design Procedure

I. Determine size of the shelter and number of occupants (usually governed by operational requirements). Ascertain 24-hour average climatic conditions (dry-bulb temperature and wet-bulb temperature) and the daily temperature range at the intended location from weather data.^{5,7} Entering the psychrometric chart with dry-bulb and wet-bulb temperature (or relative humidity) data, the absolute humidity can be found. Determine (or assume) soil thermal properties at the intended location.

II. Determine maximum dry-bulb temperature and effective temperature tolerable (or desired) inside the shelter at the end of the expected occupancy period (usually 10 - 14 days). The following table of acceptable and tolerable thermal limits for healthy persons at rest and properly clothed is suggested by Yaglou:⁸

	Effective Temperature (minimum air movement)
Lowest acceptable for continuous exposure— manual dexterity may be affected.	50°F
Optimum for comfort, with 60% relative humidity.	68°F - 72°F
Perspiration threshold. Acceptable for continuous exposure.	78°F
Endurable in emergencies for at least two weeks— possible heat rash in prolonged exposures.	85°F
Possible heat exhaustion in unacclimatized persons.	88°F
Possible heat exhaustion in acclimatized persons.	92°F

III. Assume a trial ventilation rate and air-conditioning capacity.

IV. Test the ventilation rate and air-conditioning capacity for the maximum air temperature by ascertaining effective temperature through consideration of the latent heat balance (evaluate Equation 26 at $t_{a \max}$).

A. Latent heat output per person.

$$q_{L_g} = (8t_{a_{\max}} - 435) \quad (43)$$

B. Latent heat removed by air-conditioning, per person.

$$q_{L_r} = (1-F)R \quad (44)$$

C. Latent heat removed by ventilation air, per person.

$$q_{L_v} = \rho_a QH(w_{a_{\text{sat}}} - w_A) \quad (45)$$

where: $w_{a_{\text{sat}}}$ is the absolute humidity of air saturated at the maximum shelter air dry-bulb temperature, $t_{a_{\max}}$.

D. Net latent heat to the shelter wall.

$$q_{L_w} = q_{L_g} - q_{L_r} - q_{L_v} \quad (46)$$

A positive value of q_{L_w} indicates a saturated air condition at $t_{a_{\max}}$ and the effective temperature will equal $t_{a_{\max}}$. If this effective temperature exceeds the effective temperature allowed in Step II, the ventilation rate and/or air-conditioning capacity should be increased. If q_{L_w} is negative, it is considered equal to zero (see Section I, Heat Input to the Model), and w_a must be calculated from Equation 38a.

$$w_a = w_A + \frac{q_{L_g} - q_{L_r}}{\rho_a QH} \quad (38a)$$

Now enter the modified psychrometric chart with $t_{a_{\max}}$ and w_a to find the effective temperature. If this value exceeds the maximum effective temperature allowed in Step II, the ventilation rate and/or air-conditioning capacity should be increased.

When the environment control system capacity is adequately sized for $t_{a_{\max}}$ and ET_{\max} , the allowable duration of shelter occupancy under these conditions can be found from the steps following.

V. Having completed Step IV so as to satisfy the effective temperature requirements determined in Step II, find the average dry-bulb temperature, t_a , tolerable (or desired) at the end of the expected shelter occupancy period by subtracting a temperature increment from $t_{a_{max}}$. This temperature increment, which accounts for daily temperature fluctuations inside the shelter, is obtained from Figure 7. Note that the increment is a function of ventilation rate and daily temperature range at the intended location.

VI. Now calculate the net heat input to the shelter walls by evaluating the heat balance equations using t_a .

A. Sensible heat balance

1. Heat generated per person

$$q_{S_g} = (835 - 8t_a) \quad (47)$$

2. Heat removed by air-conditioning, per person

$$q_{S_r} = RF \quad (48)$$

3. Heat removed by ventilation air, per person

$$q_{S_v} = \rho_a c_a Q (t_a - t_A) \quad (49)$$

4. Net sensible heat to the shelter wall.

$$q_{S_w} = q_{S_g} - q_{S_r} - q_{S_v} \quad (50)$$

B. Latent heat balance (follows format of Step IV)

$$q_{L_w} = (8t_a - 435) - (1-F)R - \rho_a QH(w_{a_{sat}} - w_A) \quad (26)$$

where: $w_{a_{sat}}$ is the absolute humidity of air saturated at t_a .

C. Total net heat input to the shelter wall.

$$q_{T_w} = q_{S_w} + q_{L_w} \quad \text{when } q_{L_w} > 0 \quad (27)$$

$$q_{T_w} = q_{S_w} \quad \text{when } q_{L_w} \leq 0 \quad (28)$$

From Section I, q_{L_w} is restricted to positive values because evaporation from the shelter walls is difficult to manage analytically.

VII. Having determined the heat inputs to the shelter wall for a given air temperature, t_a , find the wall temperature required to satisfy the convection equation for sensible heat transfer. This wall temperature is the maximum value allowed at the end of shelter occupancy.

$$t_{w_{\max}} = t_a - \frac{q_{S_w}}{hA} \quad (51)$$

where: A is the total shelter area per person.

VIII. Finally, calculate the allowable duration of shelter occupancy under the set conditions. The transient response of a semi-infinite slab subject to a step increase in heat flux has been considered in Part I, Model Studies on Shelter Shape. At the wall, the temperature response is expressed by Equation 42. This equation can be rearranged to give the allowable duration of shelter occupancy (when $t_{w_{\max}}$ is reached).

$$\theta = \frac{\pi}{\alpha_w} \left[\frac{k_w A}{2q_{T_w}} (t_{w_{\max}} - t_{w_o}) \right]^2 \quad (52)$$

Simplifying

$$\theta = \left[\frac{(t_{w_{\max}} - t_{w_o})}{Cq_{T_w}} \right]^2 \quad \text{hours} \quad (53)$$

$$\theta = \left[\frac{(t_{w_{\max}} - t_{w_o})}{4.9Cq_{T_w}} \right]^2 \quad \text{days} \quad (54)$$

where: t_{w_o} is the initial soil temperature

$$C = \frac{1.13}{A\sqrt{(k\rho c)_w}}$$

If the result of Equation (54) is equal to or greater than the expected duration of shelter occupancy, the ventilation rate and air-conditioning capacity assumed in Step III are sufficient. If the calculated duration, θ , greatly exceeds the expected occupancy term, the ventilation rate and/or the air-conditioning rate should be reduced to provide a more economical environment control system. Naturally, if the calculated duration is less than the expected occupancy period, the capacity of the environment control system should be increased.

Numerical Example

As a numerical example of the hand calculation method, consider the shelter and conditions used to illustrate the model study. Let $t_{a_{\max}} = 86^{\circ}\text{F}$ and $ET_{\max} = 85^{\circ}\text{F}$ at the end of shelter occupancy (results of the model study). Recall that $Q = 6$ cfm/person and $R = 0$. Furthermore, $t_A = 77^{\circ}\text{F}$, $t_{w_0} = 68^{\circ}\text{F}$, $w_A = 0.0162$ lb/lb, and the daily temperature range is 18°F . See Appendix C for the soil properties and other parameter values. Having completed Steps I - III, the trial ventilation rate and air-conditioning capacity are tested.

Step IV. Determine ET_{\max} from latent heat balance evaluated at $T_{a_{\max}}$.

$$\begin{aligned} q_{L_w} &= (8 \times 86 - 435) - 2.84 \times 10^4 (0.0272 - 0.0162) \\ &= -59 \text{ Btu/hr-person} \end{aligned}$$

Since q_{L_w} is negative, it is considered equal to zero, and $w_{a_{\max}}$ must be found from Equation 38a.

$$\begin{aligned} w_{a_{\max}} &= 0.0162 + \frac{(8 \times 86 - 435)}{2.84 \times 10^4} \\ &= 0.0251 \text{ lb/lb} \end{aligned}$$

Entering the modified psychrometric chart (Figure 8) with $t_{a_{\max}} = 86^{\circ}\text{F}$ and $w_{a_{\max}} = 0.0251$ lb/lb, ET_{\max} is found to be just slightly less than 85°F ; thus the ventilation rate and air-conditioning capacity appear to be adequate for $t_{a_{\max}}$. However, the duration of allowable shelter occupancy for these conditions must still be found.

Step V. The average shelter air temperature, t_a , at the end of 10 days is found by subtracting an increment, Δt_a , from $t_{a_{\max}}$. Entering Figure 7 with a daily temperature range of 18°F and ventilation rate at 6 cfm/person, $\Delta t_a = 1.7^\circ\text{F}$, and thus $t_a = (86 - 1.7) = 84.3^\circ\text{F}$.

Step VI. Net heat input to shelter walls.

A. Sensible heat

$$\begin{aligned} q_{S_w} &= (835 - 8 \times 84.3) - 6.5(84.3 - 77) \\ &= 113.6 \text{ Btu/hr-person} \end{aligned}$$

B. Latent heat

$$\begin{aligned} q_{L_w} &= (8 \times 84.3 - 435) - 2.84 \times 10^4 (0.0258 - 0.0162) \\ &= -33 \text{ Btu/hr-person} \end{aligned}$$

C. Total heat

$$\begin{aligned} q_{T_w} &= q_{S_w} \quad \text{since } q_{L_w} < 0. \\ &= 113.6 \text{ Btu/hr-person} \end{aligned}$$

Step VII. Maximum wall temperature

$$\begin{aligned} t_{w_{\max}} &= 84.3 - \frac{113.6}{42.4} \\ &= 81.6 \text{ F} \end{aligned}$$

Step VIII. Allowable duration of shelter occupancy.

$$\begin{aligned} \theta &= \left[\frac{(81.6 - 68)}{4.9 \times 7.85 \times 10^{-3} \times 113.6} \right]^2 \\ &= 10.1 \text{ days} \end{aligned}$$

Since the expected shelter occupancy period for this case is 10 days, the ventilation and air-conditioning rates appear to be sufficient. It should be noted that the non-computer design procedure has given results which agree very closely with the model study and analog computer results reported in Part I.

Discussion

When the hand calculation method is used for a number of assumed air temperature (t_a) conditions, the family of solutions generated takes on the appearance of a transient response even though the solutions do not actually constitute a transient. The family of solutions for the particular shelter considered in this report is shown in Figure 11. It should be noted that significant temperature discrepancies between the hand calculation solutions and the Drucker analog solution (true transient) occur at duration times less than five days, but that the two solutions appear to converge at approximately 10 days. Since we are really only interested in the temperature condition of the shelter at the end of the expected occupancy period, the aforementioned temperature discrepancies are of small consequence.

As a matter of interest, let us consider the nature of the hand calculation method to appreciate the cause of the temperature discrepancies seen in Figure 10. To begin with, the hand calculation method is similar to the model study analysis in that the net heat transfer to the shelter walls is found by a heat balance. However, when using the hand calculation method, the shelter air temperature response is idealized to a step function when the final air temperature, $t_{a \max}$, is assumed in

Step II. (Remember that this temperature is reduced to an average temperature which is held constant over the entire shelter occupancy period.) The use of a constant air temperature is required to fix a constant heat input, q_{T_w} , to the shelter walls, and thereby greatly

simplify the calculation procedure. If q_{T_w} were allowed to vary with

time, a finite difference solution would be necessary; such a solution is more appropriate for a computer than hand calculation. Once the constant heat flow to the shelter wall has been established, the wall temperature rises according to Equation 42 to a temperature, $t_{w \max}$, determined by

Equation 51 which was derived from the convection equation. At this point, the equations governing the heat transfer phenomena are satisfied and a solution exists. The time at which a solution exists for a given heat transfer rate to the shelter wall is the upper bound for shelter occupancy (duration) at the assumed $t_{a \max}$. Now, the hand calculation

method works because a prototype shelter exhibits an extremely sharp initial air temperature rise which approximates a step function response. However, remember that heat flow to the shelter wall is a function of shelter air temperature. Thus, the extent to which a prototype shelter deviates from the idealized step function response will determine the degree of discrepancy between the hand calculation solution and prototype

behavior. At time $\theta < 5$ days, the prototype temperature response over the total time interval shows a significant deviation from a step function response; but at time $\theta = 10$ days, the temperature response is flat, and over the total time interval the initial deviation from a step response is less significant (see Figure 11).

The hand calculation solution can be applied to the buttoned-up shelter situation without theoretical temperature discrepancies from prototype behavior because heat flow to the shelter wall is not a function of shelter air temperature. The "ventilation" terms of Equations 46 and 50 which cause the functional relationship between air temperature and heat flow are no longer considered. In this case:

$$q_{T_w} = q_{S_w} + q_{L_w} \quad (64)$$

$$= (835 - 8t_a) - RF + (8t_a - 435) - R(1-F) \quad (66)$$

$$= 400 - R \text{ Btu/hr-person}$$

When there is no air-conditioning, the heat flow to the shelter wall is simply equal to the total metabolic heat output of the occupants.

Additional Considerations for the Design Procedure

Although the environment control system for the shelter located in Washington, D. C. appears to be sufficient for the chosen design conditions, the conditions should be examined to determine limitations of shelter habitability.

First of all, the chosen average outdoor temperature of 77°F for Washington, D. C. represents a 10% design condition. A temperature expressed in terms of % design condition represents a climatic condition which prevails or is exceeded during the summer months (June through September). For example, the 10% design dry-bulb average temperature corresponds roughly with the 24-hour average temperatures during the summer months. Raising the 10% design dry-bulb temperature for a given location by 5°F usually leads to design conditions in the 1 to 5% range, depending on location. For Washington, D. C., the 4% design dry-bulb temperature is 82°F, and the 1% design dry-bulb temperature is 86°F. The point is that the 24-hour average temperature in Washington, D. C. could exceed 77°F for several consecutive days, and a shelter having a marginal environmental control system designed for 10% conditions could become dangerously uncomfortable. This argument also extends to the use of wet-bulb temperature design data.

Another point which deserves consideration is the metabolic heat output expressions used in this study (part of Equations 25 and 26) which are based on a total heat output of 400 Btu/hr-person. Total

metabolic heat output depends on the general activity level of shelter personnel. Although the total value of 400 Btu/hr-person is widely accepted, Strobe, et al,³ recommend an average total heat output of approximately 500 Btu/hr-person. If this recommended value had been used for the shelters considered in this report, the required capacity of the environment control system would have been somewhat greater. The determination of sensible and latent heat outputs as a function of air temperature for total heat outputs exceeding 400 Btu/hr-person can be found from the ASHRAE Guide and Data Book.⁵

Additional Solutions Using the Design Procedure

Following the work of Drucker,² the non-computer design procedure was applied to various shelter conditions at a number of locations where summer climatic conditions might be severe enough to cause shelter habitability problems. The solutions are summarized in tabular form below. These solutions should not be used for design purposes because the non-computer design procedure has not been fully substantiated; however, they are useful for comparison purposes to determine the effect of climate, shelter population level, and initial soil temperature. The shelter physical parameters and soil properties remain unchanged from those values given in Appendix C. Maximum effective temperature allowed was 85°F.

Q is ventilation rate (cfm/person)	Normal 24-hour average dry-bulb temperature:	78.2°F
	Daily temperature range:	18°F
R is air-conditioning capacity (tons/person)	10% Design average dry-bulb temperature:	77°F
	4% Design average dry-bulb temperature:	82°F
	1% Design average dry-bulb temperature:	86°F
	Absolute Humidity:	0.0162 lb/lb
	Initial Soil Temperature:	68°F

Q is ventilation rate (cfm/person)

R is air-conditioning capacity (tons/person)

$t_A = 77^\circ\text{F}$ (10%)
Normal occupancy

$$\begin{array}{r} R \mid 0 \\ Q \mid 6 \end{array}$$

30

$t_A = 82^\circ\text{F} (4\%)$
Normal occupancy

$$\frac{Q}{R} = \frac{6}{0.005}$$

$t_A = 86^\circ\text{F} (17^\circ\text{C})$
Normal occupancy

$$\frac{Q}{6} \quad \frac{R}{7.01}$$

$t_A = 77^\circ\text{F}$ (10%)
50% Overpopulated

$$\frac{Q}{8.5} \quad \frac{R}{0}$$

Buttoned-up Conditions (Q = 0, Normal Occupancy)

<u>R</u>	<u>Time to Reach 85°ET (hours)</u>
0	18
0.01	47.8

Q is ventilation rate (cfm/person) 72.1°F

R is air-conditioning capacity (tons/person)

Normal 24-hour average dry bulb temperature: 18°F

Daily temperature range: 71°F

10% Design average dry-bulb temperature: 76°F

5% Design average dry-bulb temperature: 83°F

1% Design average dry-bulb temperature: 0.0150 lb/lb

Absolute Humidity: 65°F

Initial Soil Temperature:

$t_A = 71^\circ\text{F} (10\%)$	$t_A = 76^\circ\text{F} (5\%)$	$t_A = 83^\circ\text{F} (1\%)$	$t_A = 71^\circ\text{F} (10\%)$
Normal occupancy	Normal occupancy	Normal occupancy	50% Overpopulated
$\frac{Q}{R}$	$\frac{Q}{R}$	$\frac{Q}{R}$	$\frac{Q}{R}$
4.5	5	9	6
0	0	0	0

Button-up Conditions ($Q = 0$, Normal Occupancy)

<u>R</u>	<u>Time to Reach 85°F (hours)</u>
0	27
0.01	68.6

CHICAGO, ILLINOIS
(Great Lakes)

Q is ventilation rate (cfm/person) Normal 24-hour average dry-bulb temperature: 75.70°F (JUL)
 R is air-conditioning capacity (tons/person) Daily temperature range: 21°F

10% Design average dry-bulb temperature: 73.50°F
 5% Design average dry-bulb temperature: 78.50°F
 1% Design average dry-bulb temperature: 86.50°F
 Absolute Humidity: 0.0150 lb/lb
 Initial Soil Temperature: 66°F

$t_A = 73.50^\circ\text{F}$ (10%)	$t_A = 78.50^\circ\text{F}$ (5%)	$t_A = 86.50^\circ\text{F}$ (1%)	$t_A = 73.50^\circ\text{F}$ (10%)
Normal occupancy	Normal occupancy	Normal occupancy	50% Overpopulated
$\frac{Q}{4.5}$ $\frac{R}{0}$	$\frac{Q}{6}$ $\frac{R}{0}$	$\frac{Q}{4.5}$ $\frac{R}{0.01}$	$\frac{Q}{7}$ $\frac{R}{0}$

Buttoned-up Conditions (Q = 0, Normal Occupancy)

$\frac{R}{\text{Time to Reach } 85^\circ\text{FET (hours)}}$
0 23.8
0.01 61.2

Q is ventilation rate (cfm/person)	Normal 24-hour average dry-bulb temperature:	79.6°F
	Daily temperature range:	18°F
R is air-conditioning capacity (tons/person)	10% Design average dry-bulb temperature:	81°F
	4% Design average dry-bulb temperature:	86°F
	1% Design average dry-bulb temperature:	91°F
	Absolute Humidity:	0.0153 lb/lb
	Initial Soil Temperature:	70°F

$t_A = 81^{\circ}\text{F}$ (10%)	$t_A = 86^{\circ}\text{F}$ (4%)	$t_A = 91^{\circ}\text{F}$ (1%)	$t_A = 81^{\circ}\text{F}$ (10%)
Normal occupancy	Normal occupancy	Normal occupancy	50% Overpopulated
$\frac{Q}{6}$	$\frac{Q}{7}$	$\frac{Q}{4}$	$\frac{Q}{7.5}$
$\frac{R}{0}$	$\frac{R}{0}$	$\frac{R}{0.015}$	$\frac{R}{0}$

Buttoned-up Conditions ($Q = 0$, Normal Occupancy)

<u>R</u>	<u>Time to Reach 85°ET (hours)</u>
0	13
0.01	36.1

Q is ventilation rate (cfm/person)	Normal 24-hour average dry-bulb temperature: 84.1°F
	Daily temperature range: 17°F
R is air-conditioning capacity (tons/person)	10% Design average dry-bulb temperature: 82.5°F
	1% Design average dry-bulb temperature: 87.5°F
	Absolute Humidity: 0.0183 lb/lb
	Initial Soil Temperature: 75°F

$\frac{Q}{6}$	$\frac{R}{0.0125}$
---------------	--------------------

$$\frac{Q}{3} \quad \frac{R}{0.02}$$
$$\frac{Q}{4.5} = \frac{R}{0.015}$$

<u>R</u>	<u>Time to Reach 85°Ft (hours)</u>
0	4
0.01	13.9
0.02	71.3

Q is ventilation rate (cfm/person)

R is air-conditioning capacity (tons/person)

Normal 24-hour average dry-bulb temperature: 91.3°F

Daily temperature range: 30°F

10% Design average dry-bulb temperature: 89°F

1% Design average dry-bulb temperature: 94°F

Absolute Humidity: 0.0111 lb/lb

Initial Soil Temperature: 73°F

$t_A = 94^{\circ}\text{F} (17^{\circ}\text{C})$

Normal occupancy

$\frac{Q}{4.5}$	$\frac{R}{0.01}$
-----------------	------------------

$t_A = 89^{\circ}\text{F}$ (10%)
50% Overpopulation

Q	R
4.5	0.0125

Buttoned-up Conditions (Q = 0, Normal Occupancy)

<u>R</u>	<u>Time to Reach 85°ET (hours)</u>
0	7
0.01	21.5
0.02	100

Conclusions

1. Based on a review of the non-computer design procedure, the most influential parameters affecting the capacity requirements for environment control systems appear to be:

- a. Total metabolic heat output per person and proportion of sensible to latent heat.
- b. Outdoor climatic conditions (including daily dry-bulb temperature variations).
- c. Initial soil temperature.
- d. Soil thermal conductivity.
- e. Total shelter surface area per person.

2. Comparing the results of the additional solutions in the previous section, a number of trends become evident: (valid only for the locations considered).

- a. Ventilation alone at 3 cfm/person, considered adequate for control of oxygen and carbon-dioxide levels, is not sufficient to maintain effective temperatures below 85°F during the assumed shelter occupancy period.
- b. For the 10% design condition, ventilation alone in excess of 3 cfm/person is sufficient for the locations studied except Houston and Phoenix. For these two locations, note that the initial soil temperatures and average air temperatures are relatively high.
- c. An increase of 5°F in the outside air temperatures (over the daily range) from the 10% design conditions will require the use of air conditioning in Washington, Houston, and Phoenix. However, in Providence, Chicago, and St. Louis, where initial soil temperatures are relatively low, no air-conditioning is required if ventilation rates are increased over that required for 10% conditions. This latter situation occurs because the soil acts as a heat sink, and also because ventilation air is capable of carrying away considerable amounts of latent heat in the form of water vapor.

- d. Under 1% design conditions, the use of air conditioning is required in all locations except Providence where an increase in ventilation rate over the 5% condition is sufficient. In Chicago, St. Louis, and Houston, the use of air conditioning is accompanied by a reduction in ventilation rate from the 4 to 5% design condition. This reduction in ventilation rate occurs because high outdoor dry-bulb temperatures (greater than t_a allowed) result in a sensible heat gain to the shelter. In Phoenix, the ventilation adds sensible heat to the shelter even at the 10% conditions, thus ventilation is maintained at low levels for all design conditions.
- e. For all locations except Houston and Phoenix, increasing shelter population by 50% can be taken care of under the 10% design weather conditions, by increasing the ventilation rate per person over that required for normal occupancy; the ventilation rate at Houston and Phoenix is more than 150% that which is normally required. Due to the high outdoor dry-bulb temperature at Houston and Phoenix the ventilation rate must be held to low levels and the air conditioning capacity be increased accordingly. Shelter overpopulation has the effect of reducing the shelter surface area per person.
- f. If a 24-hour buttoned-up period is required, air conditioning is necessary for those locations where the initial soil temperature is above 68°F (Washington, St. Louis, Houston, Phoenix).

These conclusions are not intended as design rules; indeed, they are not specific enough, and they are based on calculations for a particular set of shelter and soil parameter values (Appendix C). The intent is to demonstrate that a shelter environment control system can, and should, be tailored to the specific location of the shelter. Continuing environment control studies are devoted to giving the shelter designer effective analytical methods to meet this goal.

Suggestions for Future Work

1. Investigate simplifying the non-computer design procedure by the use of charts, graphs, or nomograms.
2. If sufficient data from an actual shelter test can be found, compare the results of the test with a solution of the design procedure evaluated with the actual test parameter values. Two major problems will be to determine representative soil thermal properties and 24-hour average climatic conditions.

3. Expand the non-computer design procedure to include concrete structures. A suggested solution method is to use an equivalent coefficient of heat penetration (kpc), in Equation 54.

$$(kpc)_{equiv.} = A(kpc)_{soil} + B(kpc)_{concrete}$$

where: A and B are coefficients which account for the relative thermal energy absorption of concrete and soil in the composite wall. Both A and B are positive, and $A + B = 1$

4. Expand the non-computer design procedure to include partially buried structures. In this case only part of the shelter surface area would be considered with respect to the semi-infinite slab (heat sink). The balance of the area would be considered as the plane face of a thick wall subject to a 24-hour average sol-air temperature. The net heat into or out of this thick wall would be considered in determining the heat load to the semi-infinite slab.

5. Expand the design procedure to include "basement" shelters in which all walls contact the soil (semi-infinite slab). In this case the net heat load through the ceiling of the shelter can be determined by standard air conditioning techniques for the thermal evaluation of buildings. This net heat load would be considered in determining the heat load to the semi-infinite slab.

PART III. GENERAL RECOMMENDATIONS

1. The use of marginal design criteria with respect to controlling the thermal environment of shelters should be carefully studied. An inadequately designed shelter may offer protection from fallout, but it may also offer extreme discomfort, disease, and even death from intolerably high thermal levels.
2. As a step toward better shelter design, the metabolic heat output of people in various modes of activity associated with normal shelter life should be better understood.
3. The design of the shelter thermal environment control system should be based on a careful analysis of the climatic conditions at the intended location. Whenever possible, conservative design data (less than 10%) should be used.
4. The possibility of a low-cost packaged air conditioning unit suitable for shelter use should be investigated.
5. The proper management of shelter ventilation as a function of outside climatic conditions might reduce or eliminate the air conditioning requirements in some locations where excessive dry-bulb temperatures occur during the day.
6. An economic study of achieving lowest cost environment control systems by trade-offs between ventilation and air conditioning should be undertaken. Considering the problem of collective protection, it might be more economical to provide air conditioning with reduced ventilation rates. Moreover, in many cases, this approach will adequately control the shelter environment up to 1% design conditions.

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Appendix A

ASSUMPTIONS

The Prototype Shelter

1. The shelter is sufficiently buried so that temperature gradients in the soil extending from the surface will not affect the temperature response of the shelter.
2. The initial soil temperature is uniform.
3. The soil is homogenous, and its thermal properties are uniform.
4. The soil thermal properties are independent of temperature.
5. The shelter metal skin and concrete deck can be treated as part of the soil system without introducing serious error.
6. The thermal properties of air are uniform and independent of temperature.
7. The temperature of the inlet air varies sinusoidally between the daytime maximum and nighttime minimum.
8. The absolute humidity of the inlet air is constant.
9. Perfect air mixing occurs inside the shelter.
10. The convective heat transfer coefficient inside the shelter is constant.
11. Sensible and latent heat generated by the shelter occupants can be expressed by the relations: (There are no other heat sources).

$$q_s = (835 - 8t_a) \text{ Btu/hr-person}$$

$$q_L = (8t_a - 435) \text{ Btu/hr-person}$$

12. Condensation which occurs on shelter walls does not evaporate even if the surface temperature subsequently rises.
13. The effect of solar radiation upon the surface of the soil can be neglected.

The Model Shelter

1. All assumptions for the prototype shelter extend to the model shelter.
2. The heat response of the heating pads is instantaneous.
3. Net radiation from the heating pads to the shelter wall can be neglected.

The Non-Computer Design Procedure

1. All assumptions for the prototype shelter extend to the design procedure.
2. The air temperature transient response can be idealized to a step function.
3. The shape of the shelter does not seriously affect its transient response.

Appendix B

CASE 2 METHOD OF ACCOUNTING FOR MODEL-PROTOTYPE TEMPERATURE DISTORTIONS

The Case 2 method imposes the condition that there is a one-to-one correspondence in heat flux between the model and prototype. In pursuing this approach, both the model shelter wall temperature and air temperature will be distorted. To find the distortion of the model air temperature when $h^* \neq 1$, consider an imaginary model for which $h^* = 1$, and for which the imaginary wall temperature equals the actual model wall temperature. The imaginary model serves as an intermediate between the actual model and prototype, and it yields correct air-wall temperature differences.

$$q'_p = q'_{m_i} = q'_m \quad (B-1)$$

$$h_p = h_{m_i} \quad (B-2)$$

$$t_{w_{m_i}} = t_{w_m} \quad (B-3)$$

And

$$(t_a - t_w)_p = (t_a - t_w)_{m_i} = h^*(t_a - t_w)_m \quad (B-4)$$

where: $q' = q/A = h(t_a - t_w)$

subscript m_i refers to corrected model temperatures (imaginary model)

subscript m refers to distorted model temperatures (actual model)

$$h^* = h_m/h_p = h_m/h_{m_i}$$

From Equation B-4

$$t_{a_m} = \frac{(t_a - t_w)_{m_i}}{h^*} + t_{w_m} \quad (B-5)$$

$$= \frac{t_{a_{m_i}} - t_{w_{m_i}} + h^* t_{w_m}}{h^*} \quad (B-5a)$$

$$t_{w_m} = t_{w_{m_i}} \text{ by definition}$$

$$t_{a_m} = \frac{t_{a_{m_i}} + t_{w_m} (h^* - 1)}{h^*} \quad (\text{B-5b})$$

$$\text{Now } q' = h_{m_i} (t_{a_{m_i}} - t_{w_m}) \quad (\text{B-6})$$

$$\text{Thus } t_{w_m} = t_{a_{m_i}} - \frac{q'}{h_i} \quad (\text{B-7})$$

Substituting Equation B-7 into B-5b and rearranging, we have

$$t_{a_m} = \frac{t_{a_{m_i}}}{h^*} + (t_{a_{m_i}} - \frac{q'}{h_{m_i}}) (1 - \frac{1}{h^*}) \quad (\text{B-8})$$

$$t_{a_m} = \frac{t_{a_{m_i}}}{h^*} + t_{a_{m_i}} - \frac{t_{a_{m_i}}}{h^*} - \frac{q'}{h_{m_i}} + \frac{q'}{h_{m_i} h^*} \quad (\text{B-8a})$$

$$t_{a_m} = t_{a_{m_i}} + q' \left(\frac{1}{h_{m_i} h^*} - \frac{1}{h_{m_i}} \right) \quad (\text{B-8b})$$

$$\text{Now } h_{m_i} h^* = h_{m_i} \left(\frac{h_m}{h_p} \right) = h_p \left(\frac{h_m}{h_p} \right) = h_m \quad (\text{B-9})$$

because $h_{m_i} = h_p$ by definition

Substituting the result of Equation B-9 into B-8b and rearranging, we have

$$t_{a_m} = t_{a_{m_i}} + q' \left(\frac{h_p - h_m}{h_p h_m} \right) \quad (\text{B-8c})$$

Note that the last term of Equation B-8c is the air temperature distortion, and that this distortion is equal to zero when $h_p = h_m$.

Remember that the determination of the prototype air temperature from the corrected model temperature requires that the distortion of the model shelter wall temperature be found. Here again, the imaginary model concept is useful.

$$t_{a_p} = t_{w_p} + (t_a - t_w)_p \quad (B-10)$$

Now, by definition

$$(t_a - t_w)_{m_i} = (t_a - t_w)_p \quad (B-11)$$

Thus

$$t_{a_p} = t_{w_p} + (t_a - t_w)_{m_i} \quad (B-12)$$

Since $q'^* = 1$ for the Case 2 condition

$$t_{w_p} = \frac{k^*}{L^*} t_{w_m} \quad (B-13)$$

$$(\text{recall that } t_{w_m} = t_{w_{m_i}})$$

Substituting from Equation (B-13) into (B-12) and rearranging

$$t_{a_p} = t_{a_{m_i}} + \left(\frac{k^*}{L^*} - 1\right) t_{w_m} \quad (B-14)$$

Now the imaginary model air temperature, $t_{a_{m_i}}$, can be eliminated from

Equation (B-14) by substitution from (Equation B-8c)

$$t_{a_p} = t_{a_m} - q' \left(\frac{h_p - h_m}{h_p h_m} \right) + \left(\frac{k^*}{L^*} - 1 \right) t_{w_m} \quad (B-15)$$

Since the model may not have the same initial temperature as the chosen prototype, all temperatures should be replaced by temperature differences between temperatures at time θ and the initial temperature at time $\theta = 0$.

$$(t_{a_\theta} - t_{a_o})_p = (t_{a_\theta} - t_{a_o})_m - q' \left(\frac{h_p - h_m}{h_p h_m} \right) + \left(\frac{k^*}{L^*} - 1 \right) (t_{w_\theta} - t_{w_o}) \quad (B-15a)$$

Appendix C

BASIC DATA AND CALCULATIONS FOR SETTING UP THE MODEL RUN -- WASHINGTON, D. C.

The basic input data for the model run, Washington, D. C. -- summer conditions -- corresponds to the values used by Drucker for his analog computer solution for the same location. This procedure allows a comparison of the model behavior to the analog computer solution.

Temperatures and Absolute Humidity

Average air temperature outside shelter:	$t_A = 77^{\circ}\text{F}$
Average air absolute humidity outside shelter:	$w_A = 0.0162 \text{ lb/lb}$
Initial soil temperature(initial air temperature inside shelter equals initial soil temperature):	$t_{w_o} = 68^{\circ}\text{F}$

Air Properties

Specific heat:	$c_a = 0.241 \text{ Btu/lb-}^{\circ}\text{F}$
Density:	$\rho_a = 0.075 \text{ lb/ft}^3$

Soil Properties

Thermal conductivity:	$k_{w_p} = 0.83 \text{ Btu/hr-ft-}^{\circ}\text{F}$
Specific heat:	$c_{w_p} = 0.30 \text{ Btu/lb-}^{\circ}\text{F}$
Density:	$\rho_{w_p} = 140.0 \text{ lb/ft}^3$

Other Parameters

Convective heat transfer coefficient:	$h_p = 1.47 \text{ Btu/hr-ft}^2\text{-}^{\circ}\text{F}$
Enthalpy of condensation:	$H = 1054 \text{ Btu/lb}$
Total shelter area:	$A_p = 2884 \text{ ft}^2$

Length of shelter occupancy:	$\theta_{\max} = 10 \text{ days}$
Total heat output/person:	$q_{Tg} = 400 \text{ Btu/hr}$
Ventilation rate/person	$Q = 6 \text{ cfm}$
Air conditioning rate/person	$R = 0$
Sensible heat factor:	$F = 0.75$
Number of persons in shelter:	$n = 100 \text{ people}$

Model parameters were equal to the prototype values except as follows:

Temperatures

Initial soil temperature (initial air temperature was same as initial soil temperature):	$t_{w_o} = 72^{\circ}\text{F}$
--	--------------------------------

Soil Properties

Thermal conductivity:	$k_{wm} = 0.167 \text{ Btu/hr-ft-}^{\circ}\text{F}$
Specific heat:	$c_{wm} = 0.189 \text{ Btu/lb-}^{\circ}\text{F}$
Density:	$\rho_{wm} = 92.3 \text{ lb/ft}^3$
Convective heat transfer coefficient (determined by tests with constant heat input):	$h_m = 1.15 \text{ Btu/hr-ft}^2\text{-}^{\circ}\text{F}$
Total shelter air:	$A_m = 69.8 \text{ ft}^2$

At this point, a number of parameters required by Equation 24b can be computed. (All computations on a per person basis where applicable.)

$$(hA)_p = 1.47 \times 2884/100 = 42.4 \text{ Btu/hr-}^{\circ}\text{F/person}$$

$$L^* = (A_m/A_p)^{\frac{1}{2}} = (69.8/2884)^{\frac{1}{2}} = 0.155$$

$$h^* = h_m/h_p = 1.15/1.47 = 0.782$$

$$k_w^* = k_{w_m} / k_{w_p} = 0.167 / 0.83 = 0.201$$

$$Bi^* = \frac{h^* L^*}{k_w^*} = \frac{0.782 \times 0.155}{0.201} = 0.605$$

$$\left(\frac{1 - Bi^*}{Bi^*} \right) = \left(\frac{1 - 0.605}{0.605} \right) = 0.654$$

Assuming that the prototype shelter in Washington, D. C. will experience an air temperature rise from 68°F to 85°F, the heat balances (Equations 25 and 26) can be solved for various dry-bulb temperatures within this range. The values obtained thereby are the prototype heat inputs/person as a function of prototype air temperature.

$$q_{S_p} = (835 - 8t_a)_p - R(F) - p_a c_a Q(t_a - t_A)_p \quad (25)$$

$$q_{L_p} = (8t_a - 435)_p - R(1-F) - p_a QH(w_{a_{sat}} - w_a)_p \quad (26)$$

$$q_{T_p} = q_{S_p} + q_{L_p} \quad \text{when } q_{L_p} > 0 \quad (27)$$

$$q_{T_p} = q_{S_p} \quad \text{when } q_{L_p} \leq 0 \quad (28)$$

Now q_{T_p} is the total heat load to the prototype shelter wall; for the model, the heat load is

$$\begin{aligned} q_{T_m} &= nk_w^* L^* q_{T_p} \quad (37) \\ &= 100 \times 0.201 \times 0.155 q_{T_p} \\ &= 3.13 q_{T_p} \text{ Btu/hr} \end{aligned}$$

Recalling that $(t_w - t_{w_o})_p = (t_w - t_{w_o})_m$ for the Case 1 condition, Equation 15 requires that:

$$q^* = k_w^* L^* \quad (C-1)$$

Since $q^* = q_m/q_p$

$$q_m = k_w * L * q_p \text{ on a per person basis} \quad (C-2)$$

and the total heat input to the model is:

$$\begin{aligned} q_{T_m} &= n k_w * L * q_{T_p} \quad (C-3) \\ &= 100 \times 0.201 \times 0.155 q_{T_p} \\ &= 3.13 q_{T_p} \text{ Btu/hr} \\ &= 0.919 q_{T_p} \text{ Watts} \end{aligned}$$

Using the parameters calculated previously, the distorted model temperature rise corresponding to each prototype temperature can be found from Equation 24b rearranged as

$$(t_{a_\theta} - t_{a_o})_m = (t_{a_\theta} - t_{a_o})_p + \left(\frac{q_L}{hA}\right)_p + \left(\frac{q_T}{hA}\right)_p \left(\frac{1-Bi^*}{Bi^*}\right) \quad (24b)$$

$$(t_{a_\theta} - t_{a_o})_m = (t_{a_\theta} - 68) + \left(\frac{q_L}{42.4}\right)_p + 0.654 \left(\frac{q_T}{42.4}\right)_p \quad (C-4)$$

where: $68 \leq t_{a_\theta p} \leq 85$

The several solutions of equation group 25, 26, C-3, and C-4 for q_S , q_L , q_{T_m} , and $(t_{a_\theta} - t_{a_o})$ can be conveniently performed on an IBM 1620 digital computer. Once found, the distorted model temperatures are plotted as a function of q_{T_m} expressed in Watts (see Figure 3). This curve is used

during the model run to determine the sensible heat inputs associated with distorted model air temperatures. Following the model run, the prototype air temperatures are found from the distorted model air temperatures by means of Equation 24b (C-4). It is convenient to plot $t_{a_\theta p}$ as a function of $(t_{a_\theta} - t_{a_o})_m$ as shown in Figure 4.

Since experimental data is taken in real time, the time of each temperature observation must be converted to prototype time. This conversion is made from Equation 10.

$$\theta_p = \frac{\alpha_w^*}{L^{*2}} \theta_m \quad (10)$$

$$\alpha_w^* = \frac{k_w^*}{\rho_w^* c_w^*}$$

$$= \frac{0.201}{0.923 \times 0.66}$$

$$= 0.33$$

$$L^{*2} = A^* = (69.8/2884)$$

$$\theta_p = \frac{0.33 \times 2884}{69.8}$$

$$= 13.6 \theta_m \text{ (hours for each hour of model time)}$$

$$= 0.566 \theta_m \text{ (days for each hour of model time)}$$

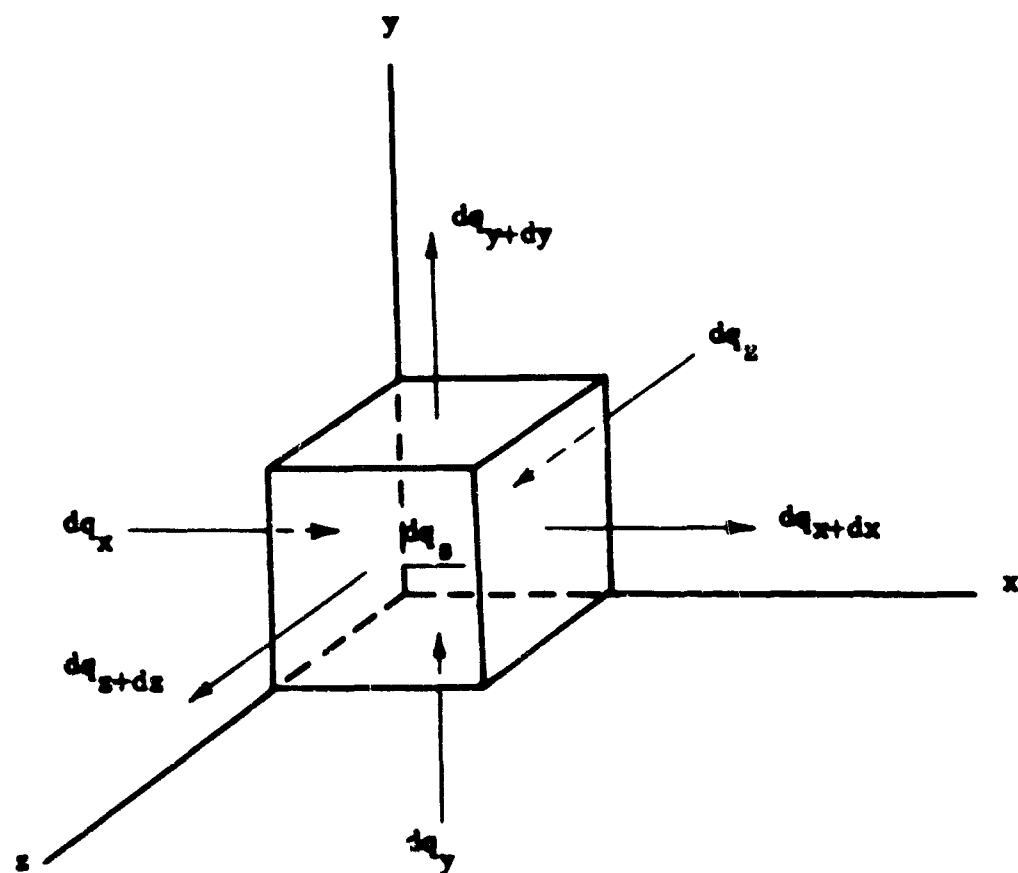


Figure 1. Differential volume element in the thermal medium.

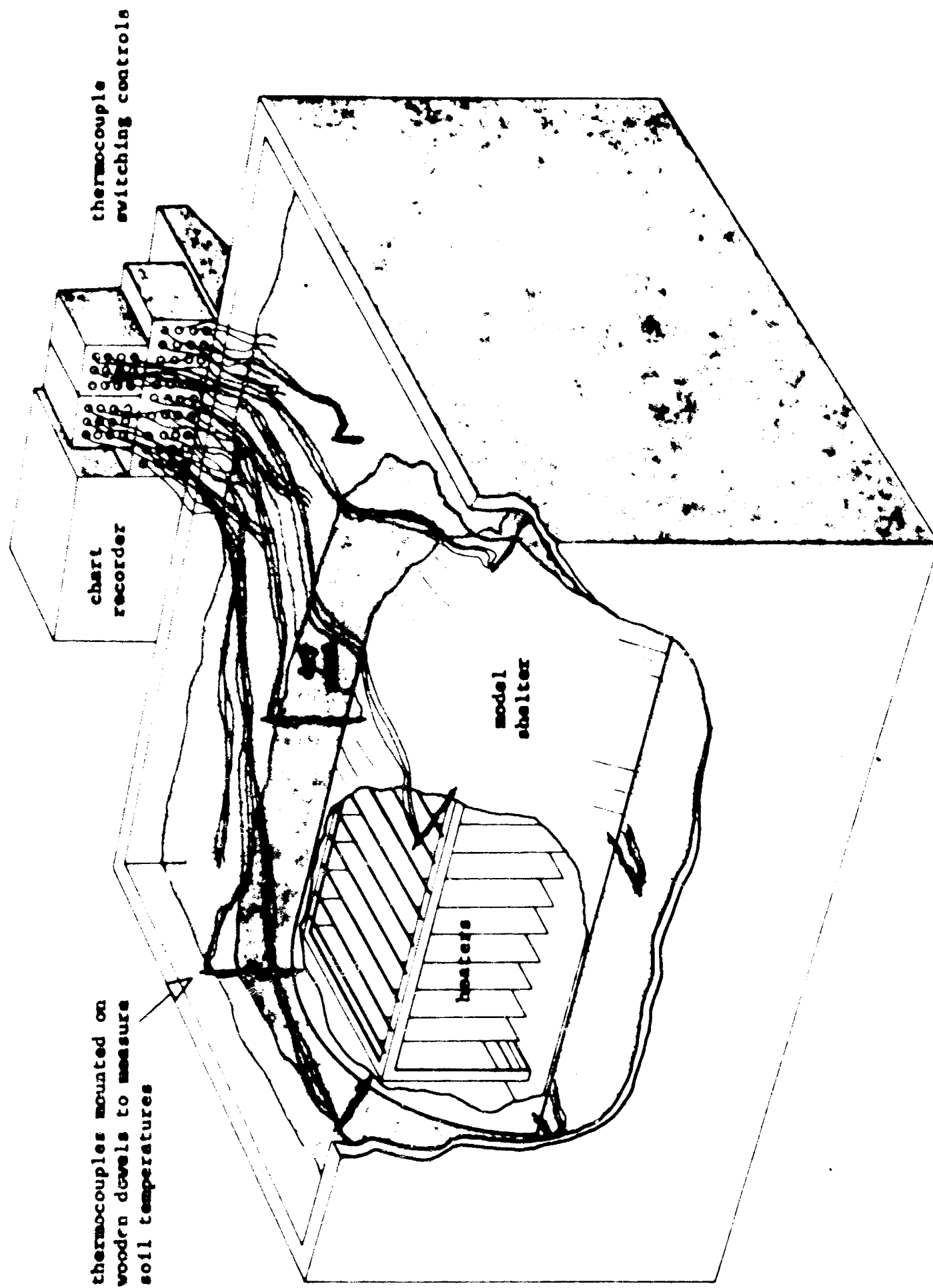


Figure 2. Cut-away view of the model set-up.

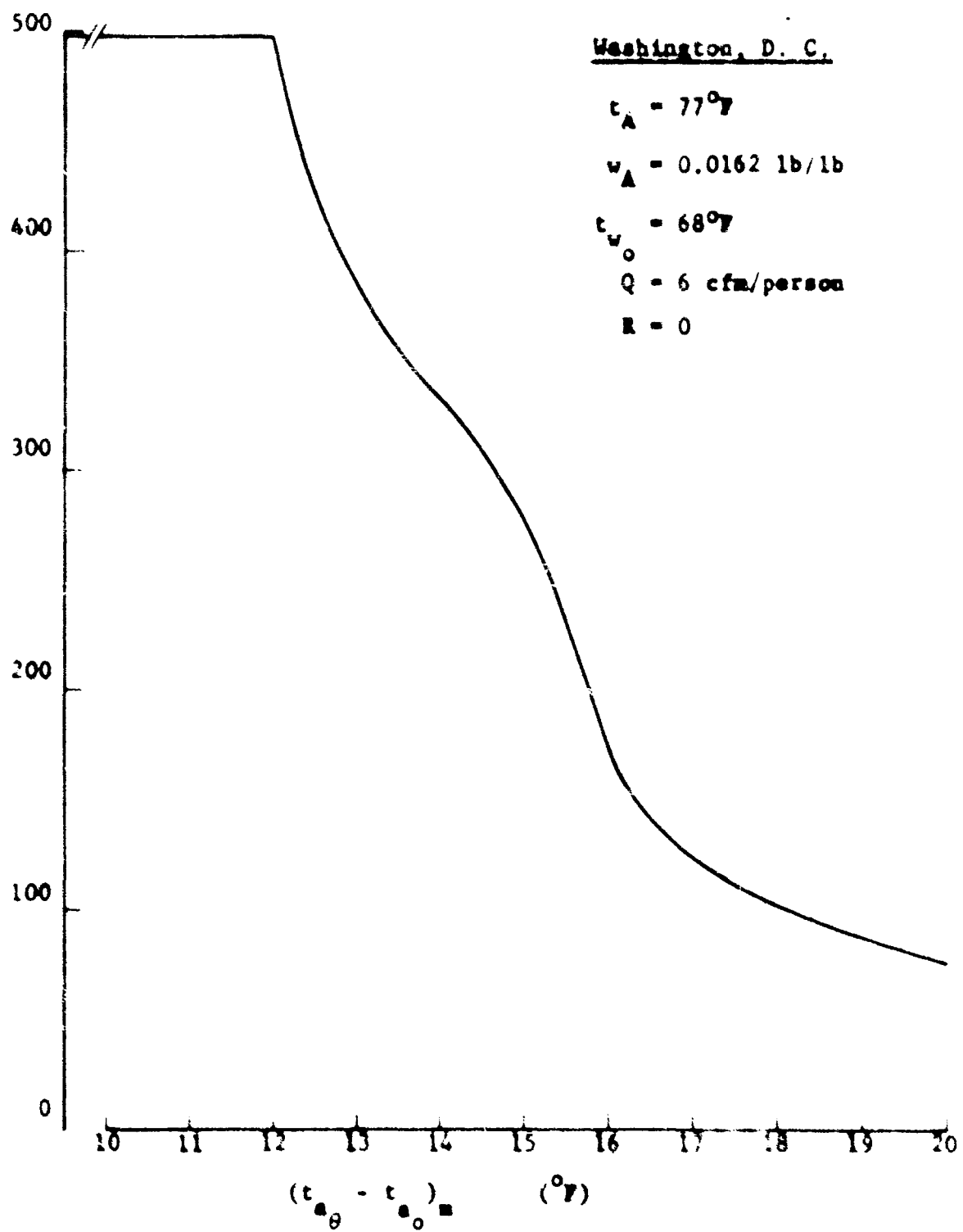


Figure 3. Heat input to the model shelter as a function of distorted model air temperatures -- Case 1 condition.

Washington, D. C.

$$t_A = 77^{\circ}\text{F}$$

$$w_A = 0.0162 \text{ lb/lb}$$

$$t_{w_o} = 68^{\circ}\text{F}$$

$$Q = 6 \text{ cfm/person}$$

$$R = 0$$

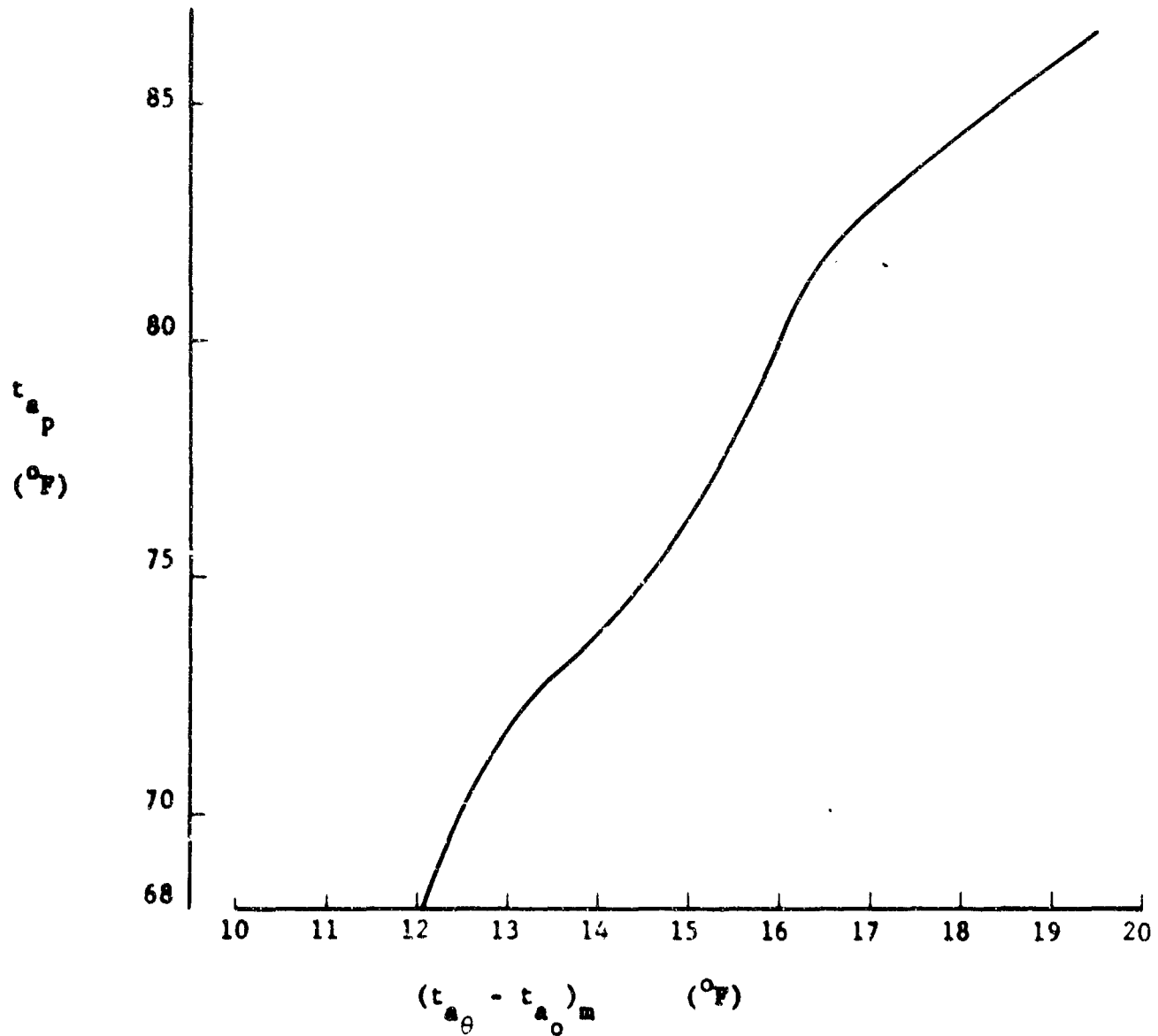


Figure 4. Temperature conversion curve: distorted model air temperature to prototype air temperature -- Case 1 condition.

Washington, D. C.

$$t_A = 77^\circ\text{F}$$

$$w_A = 0.0162 \text{ lb/lb}$$

$$t_{w_o} = 68^\circ\text{F}$$

$$Q = 6 \text{ cfm/person}$$

$$R = 0$$

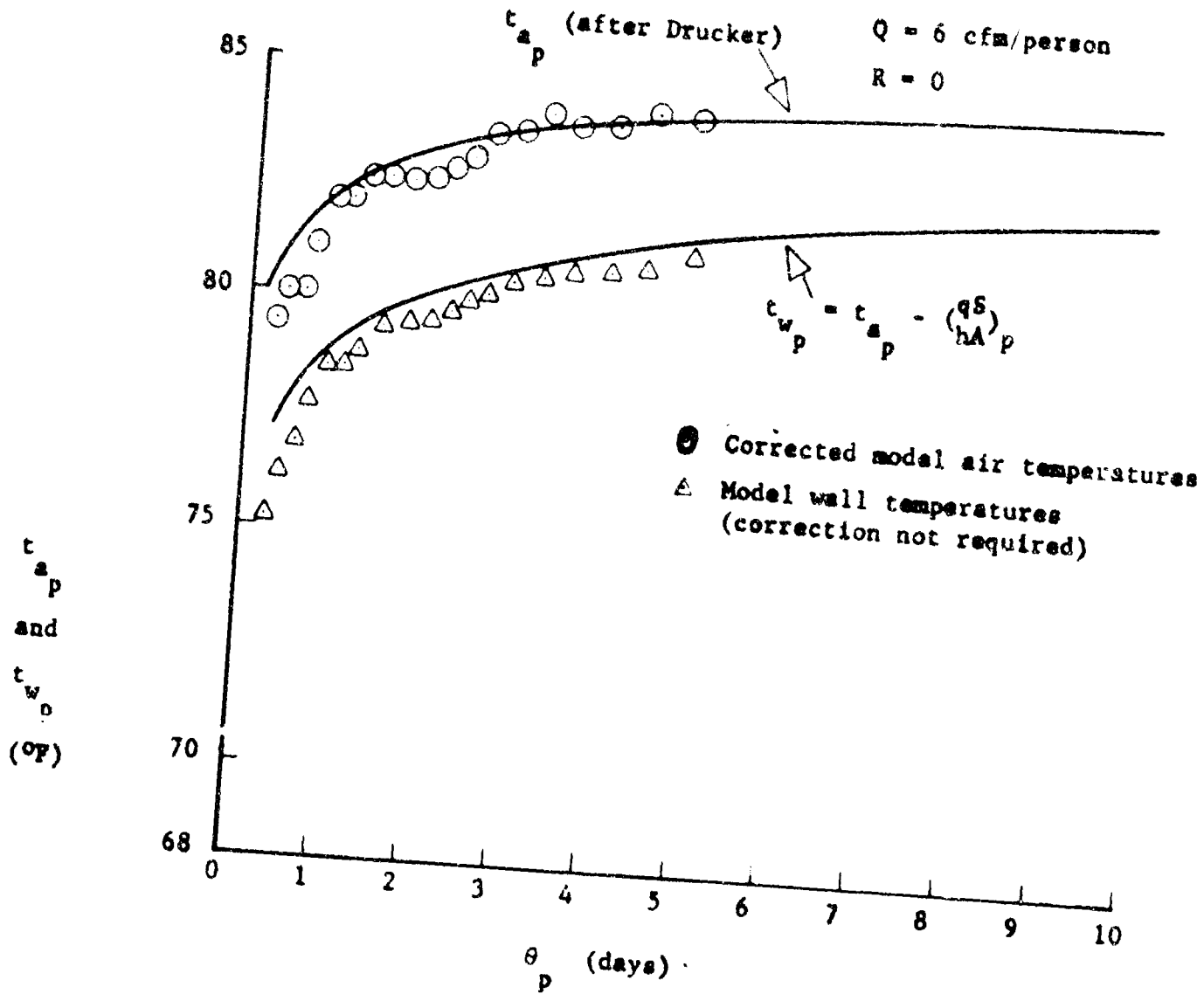


Figure 5. Thermal transient response of the model shelter
Case 1 condition.

Washington, D. C

$$t_A = 77^{\circ}\text{F}$$

$$w_A = 0.0162 \text{ lb/lb}$$

$$t_{w_o} = 68^{\circ}\text{F}$$

$$Q = 6 \text{ cfm/person}$$

$$R = 0$$

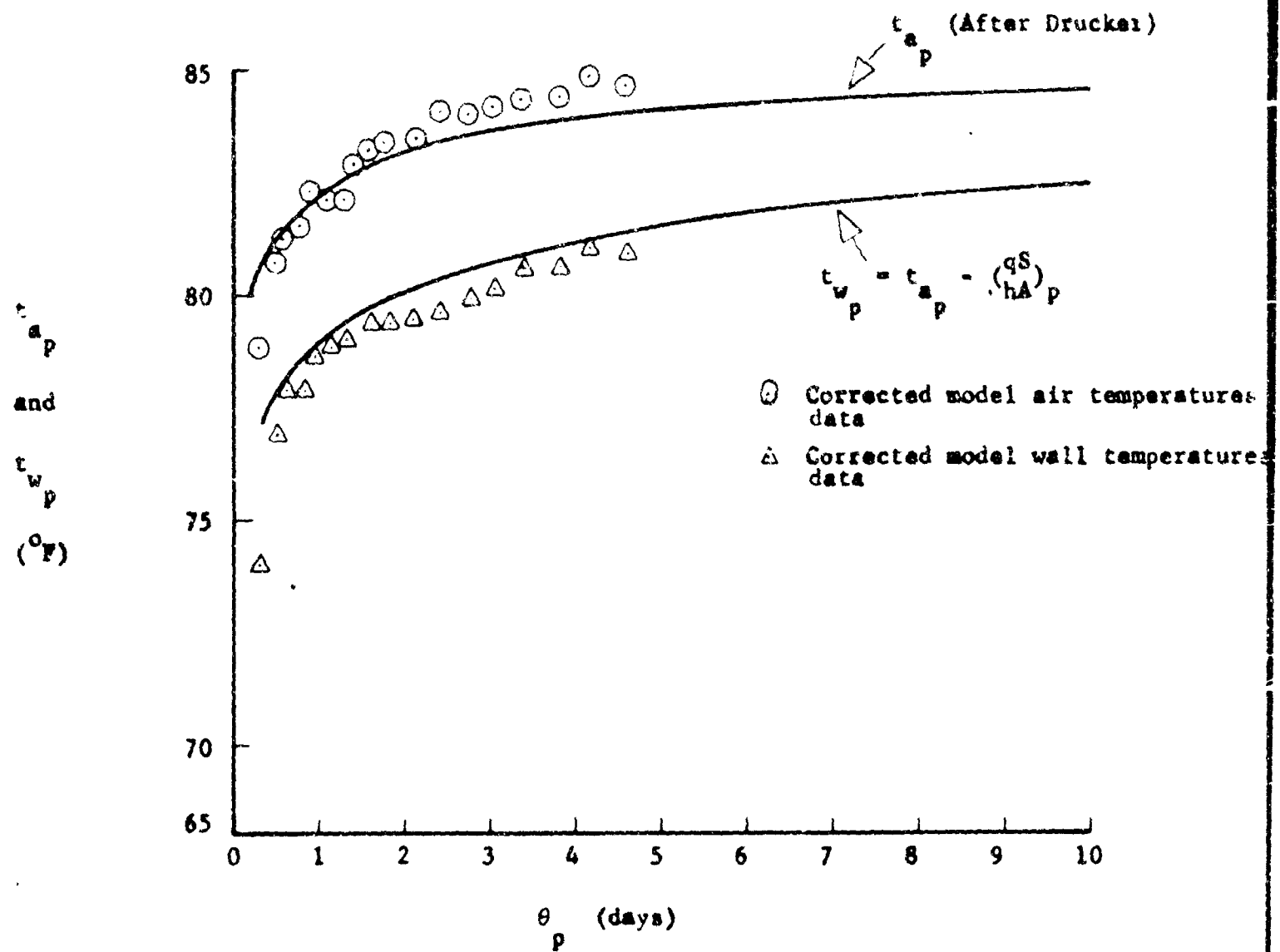


Figure 6. Thermal transient response of the model shelter
Case 2 condition.

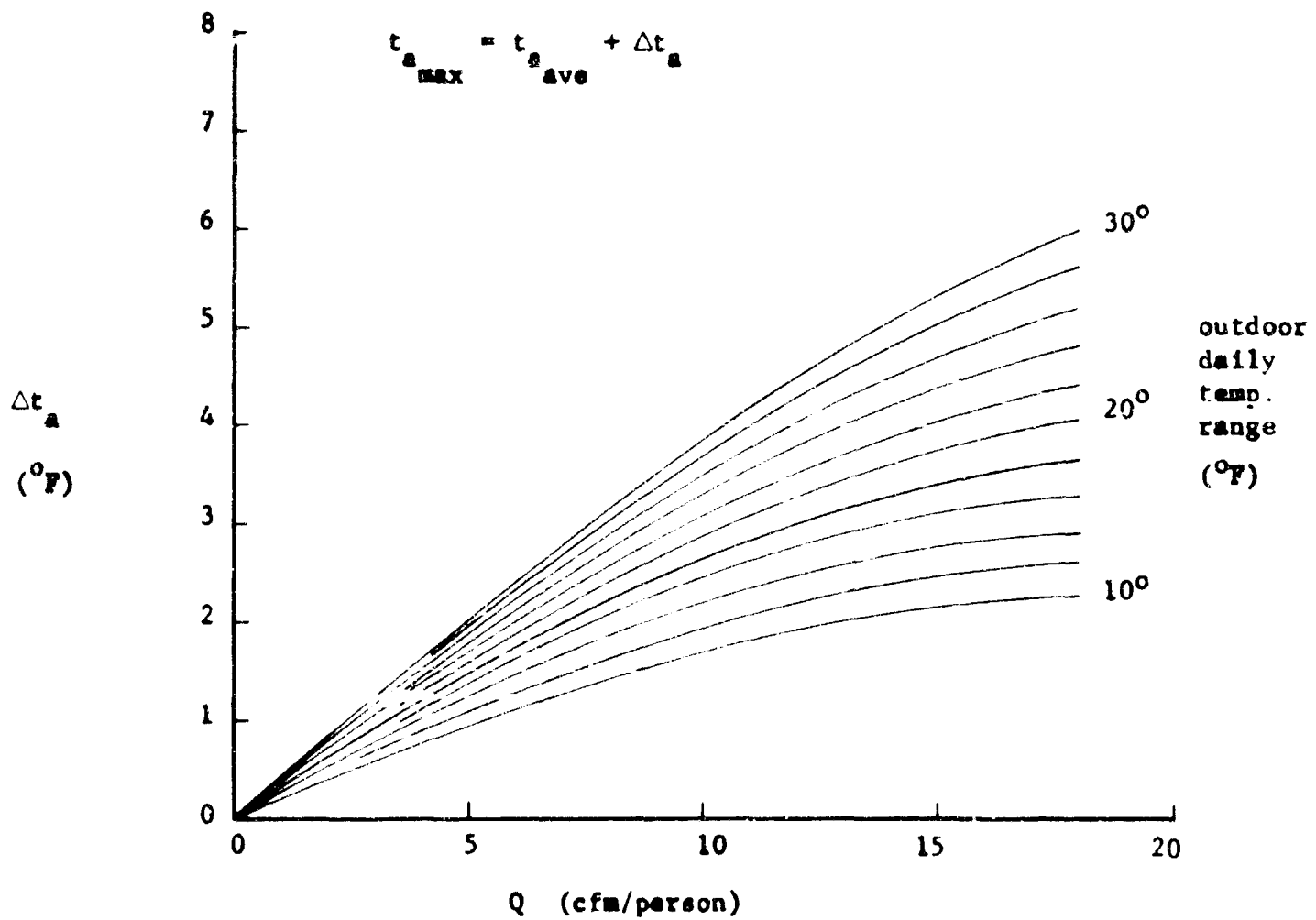


Figure 7. Temperature increments. (1) Increments are added to corrected model temperature data to find maximum dry-bulb temperature attained during the day. (2) For non-computer design procedure, increments are subtracted from $t_{a \text{ max}}$ allowed to find average air temperature attained at the end of shelter occupancy.

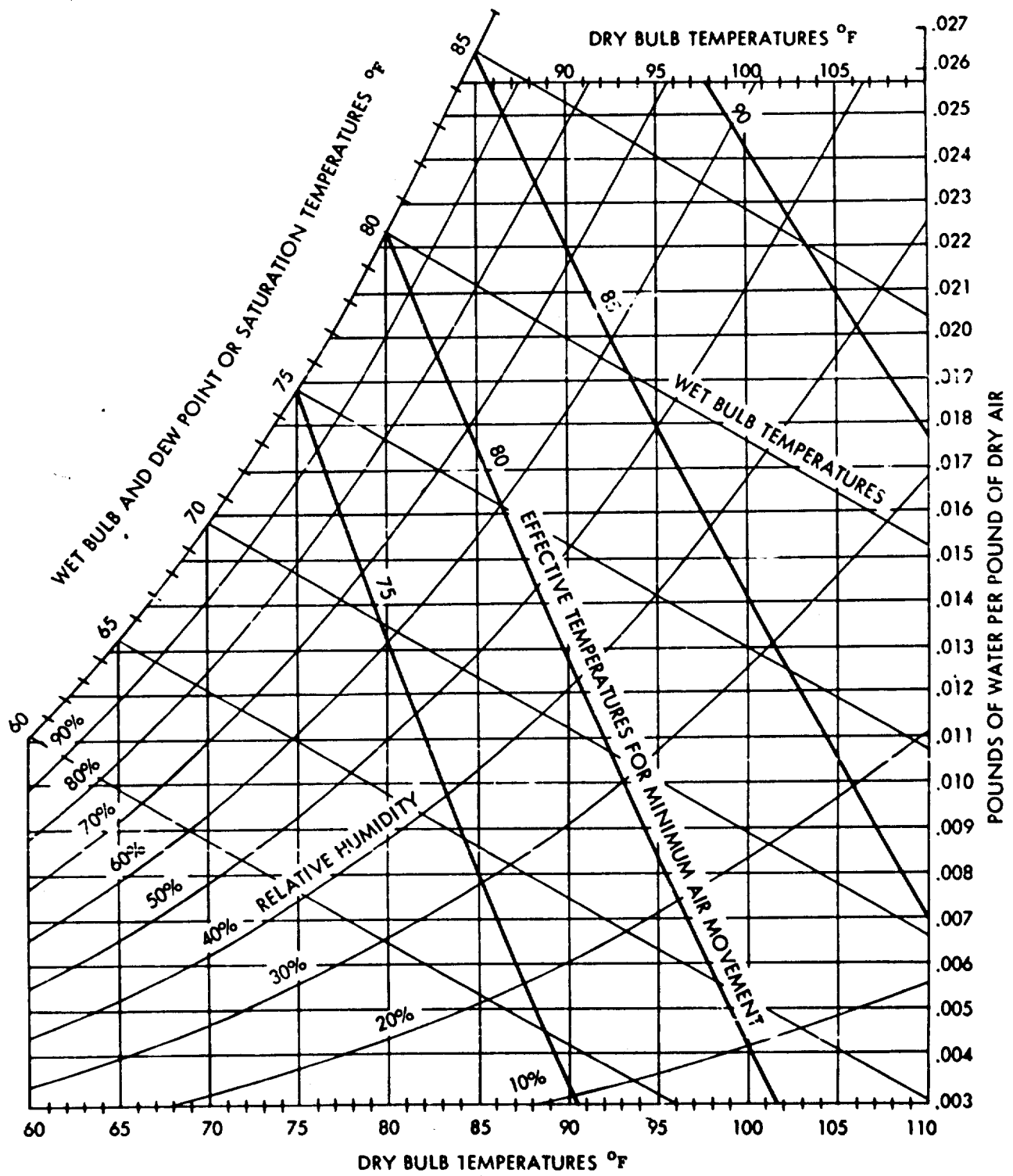


FIG. 8 MODIFIED PSYCHROMETRIC CHART

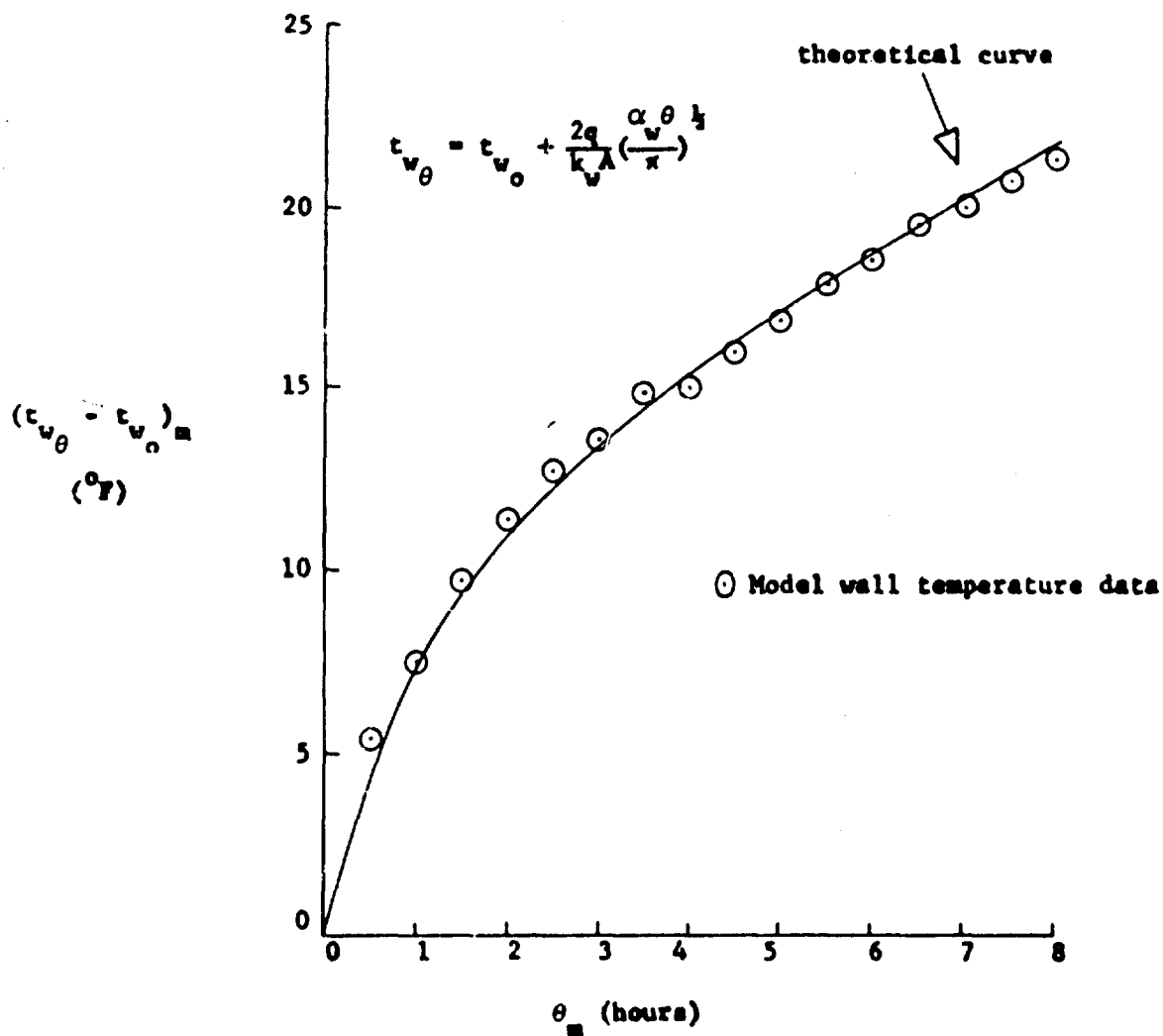


Figure 9. Transient response of the model shelter wall to a constant heat input.

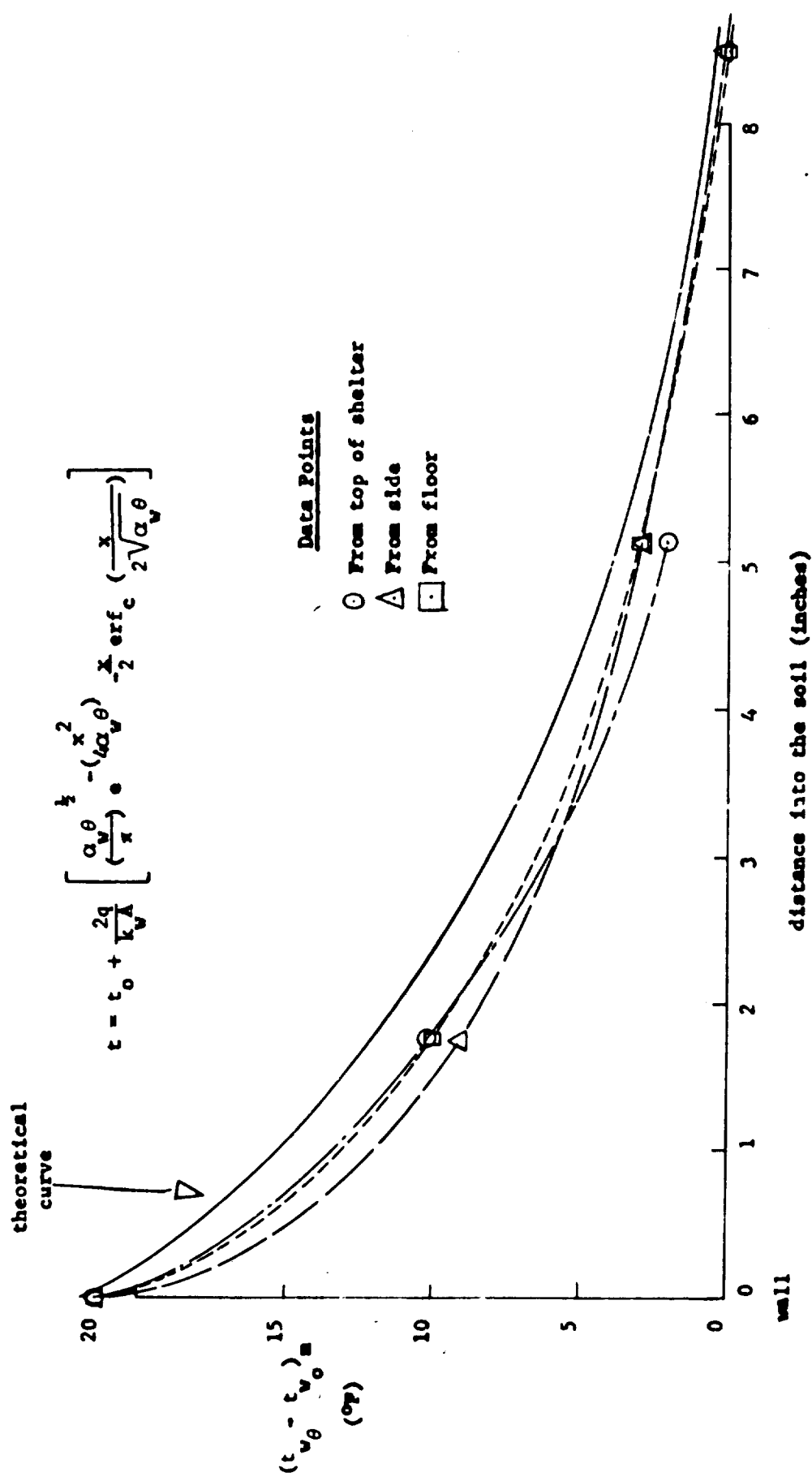


Figure 10. Temperature distributions in the soil surrounding the model shelter after 8 hours (constant heat input).

Washington, D. C.

$$t_A = 77^\circ\text{F}$$

$$v_A = 0.0162 \text{ lb/lt}$$

$$t_{w_o} = 68^\circ\text{F}$$

$$Q = 6 \text{ cfm/person}$$

$$R = 0$$

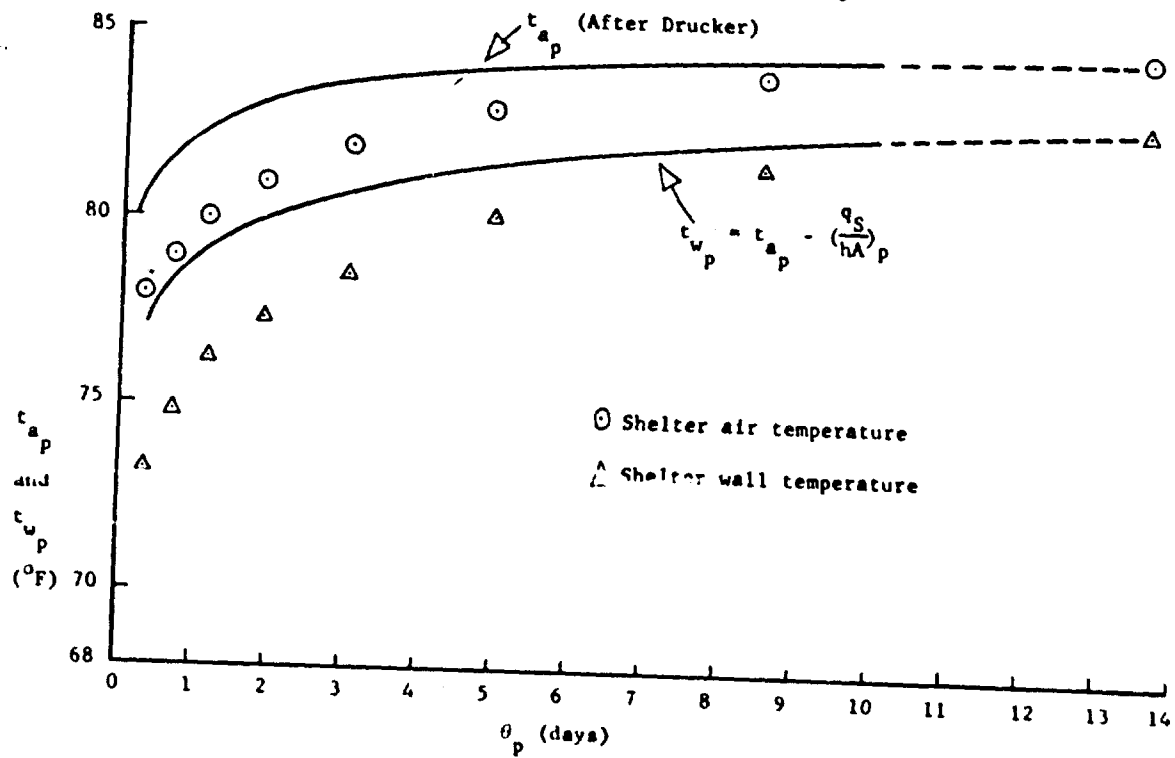


Figure 11. Family of solutions found from the non-computer design procedure.