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TRANSLATION

DISTRIBUTION OF DROPS ACCORDING TO SIZE IN ATOMIZING LIQUIDS BY MEANS OF CENTRIFUGAL ATOMIZERS

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L. G. Golovkov

The article presents further support of the function of distribution of drops according to size proposed in the work (5), explains the method of paramater derivation for this function and establishes the numerical value of its parameters for centrifugal atomizers.

The most fruitful concept for the establishing of analytic dependences between parameters which determine the process of atomization and sizes of the spectrum, in our view proves to be the concept expressed for the first time by Mugele and Evans (4) and developed afterwards by Tresh (5). Its basis is composed by two propositions: a) in the spectra of the atomized liquid there are always contained drops of the greatest size (D_{max}); b) drops of all the remaining sizes occur as a result of statistical conformity to rule.

Tresh explains the statistical character of the functions of distribution of drops according to size by the fact that at the original moment of scattering of the jet there occurs a disordered exchange of energies among the separate particles of liquid through the merging of individual drops, their secondary atomization, repeated merging, etc. This chaotic process also results in the formation in the spectrum of the atomized liquid of only a small number of very big and very small drops.

Here the author proposes the following dependences for computing the spectra of the atomized liquid: the equation for determining D_{max}

$$\left(\frac{\sigma}{\rho_{ж} v^2 D_{max}}\right)\left(1+\frac{10^6 \mu_{ж}^2}{\rho_{ж}\sigma D_{max}}\right)^{1/11}\left(1-0,5\ \frac{\rho_г}{\rho_{ж}}\right)=4,8\cdot 10^{-6} \qquad (1)$$

and the function of the density of the distribution of the drops sizes

$$\psi(\delta)=\frac{dn}{n_0 d\delta}=\frac{1}{\delta^3}\exp\left(-\frac{\beta}{\delta}\right)\Big/\int_0^1 \exp\left(-\frac{\beta}{\delta}\right)\frac{d\delta}{\delta^3}. \qquad (2)$$

The expression (2) is a well-founded semiempirical function of the distribution of the drops by sizes and consequently, it is favorably distinguished from the other purely empirical equations (for example (1-3, 6) and of other authors). However, the equation (1) and the numerical value of the parameter β (equal to 0.35) were established by Trësh for developing the experiments in pneumatic atomization of liquids and therefore proved to be not suitable for centrifugal atomizers.

In this article there is analyzed a supplementary basis of the function of distribution proposed by Trësh (2), methods are given for the determining of the parameter β of this function, there is established its numerical value for centrifugal atomizers, and there is also refined the generalized dependence for determining D_{max}.

The function (2) was established by Trësh by the methods of statistical thermodynamics with the postulation: 1) of constancy of the overall volume of the drops; 2) of constancy of the surface of the drop mixture in the period of chaotic exchange of energies among the particles of the liquid. If the first assumption causes no particular uncertainties, then the second does not prove to be manifest. Below there is given the proof of Trësh's second postulate.

The known peculiarities of the conditions of the existence of molecules on the boundary separating liquid and gaseous phases are the reason that the increase of the surface of the separation requires the expenditure of work. The amount of this work referred to unit of area of surface represents the surface tension or the specific isobar potential Z (7).

In the formation of the surface by the area Δ f the change in the isobaric potential amounts to

$$\Delta Z = \sigma \Delta f. \tag{3}$$

On the other hand it is known that for any process $(\partial Z/\partial T)_p = -S$. Consequently, the change in the entropy in this process is equal to $\Delta S = -(\partial(\Delta Z)/\partial T)_p$, or with the taking into consideration of the relationship (3)

$$\Delta S = -\left[\frac{\partial(\sigma\Delta f)}{\partial T}\right]_p = -\Delta f\left(\frac{\partial\sigma}{\partial T}\right)_p - \sigma\left[\frac{\partial(\Delta f)}{\partial T}\right]_p.$$

By disregarding the change in the surface connected with the change in the temperature (in view of the insignificant coefficient of the volumetric expansion of liquids) we shall get

$$\Delta S = -\Delta f\left(\frac{\partial\sigma}{\partial T}\right)_p. \qquad (4)$$

Now let us assume that in the process under consideration of the chaotic exchange of energies between the particles of the liquid their total surface increases by ($\Delta f > 0$). Then according to the relationship (3) and (4) $\Delta Z > 0$ and $\Delta S > 0$ inasmuch as $(\partial\sigma / \partial T)_p$ always is negative (with increased temperature the Surface tension always becomes less).

Consequently, with $\Delta f > 0$ the indicated process should be accompanied simultaneously both by increase in the isobaric potential Z and by increase in the antropy S. According to the second law of thermodynamics the simultaneous increase of these two values in a spontaneous process is not possible. Therefore the value Δf cannot be positive.

The proposition of the reduction in the total surface of the drops ($\Delta f < 0$) also leads to a contradiction with the second law of thermodynamics ($\Delta Z < 0$ and simultaneously $\Delta S < 0$). The contradiction disappears when $\Delta f = 0$, i. e., when the surface of the drop mixture in the period of its "disordered" state is unchanged, which was what was to be proved.

In the solution of each statistical problem after establishing the matheform of the function of the distribution, it is necessary to develop methods of determining the numerical values of the parameters of this function on the basis of the existing results of observations, i. e., to derive its parameters. The parameters of the function (2) are D_{max} and β.

In the processing of the experimental data on the atomization of liquids by centrifugal atomizers with the aid of the theory of dimensionalities there was obtained the following generalized equation for determining D_{max}:

$$\frac{D_{max}}{\varepsilon}=\left(\frac{\sigma}{\Delta\rho_{\Phi}\varepsilon}\right)^{1/6}\left(1+\frac{10^{6}\mu_{ж}^{2}}{\rho_{ж}{}^{\sigma\varepsilon}}\right)^{1/10}\left(1-0{,}5\,\frac{\rho_{г}}{\rho_{ж}}\right). \qquad (5)$$

With regard to the parameter β as depends on two basic methods of primary processing of experimental data on th atomizing of liquids, there are proposed below two operations for its determination.

For the case where to measurement in the sample there is subjected each of its elements (every drop) the determination of β is done by the method of the maximum plausibility (8) which gives the most nearly "precise" value for the parameter β.

In determining in the sample the number of drops which fall into different intervals as to size for finding the numerical values of β one uses the method of moments.

Method of Maximum Similitude. The relative frequency of the drops dn/n_0 in the formula (2) can be treated as the probability of the occurrence of drops of dimensions of from δ to $\delta + d\delta$, and therefore the function (2) is the probability density of the random value δ. By accomplishing on the equation (2) the functional conversion $D = D_{max}\delta$, we shall get the function of the density of the distribution of directly in accordance with the diameter

$$\varphi(D)=\frac{dn}{n_0 dD}=\frac{1}{D^3}\exp\left(-\frac{\beta D_{max}}{D}\right)\Big/\int_0^{D_{max}}\exp\left(-\frac{\beta D_{max}}{D}\right)\frac{dD}{D^3} \qquad (6)$$

or after integration

$$\varphi(D)=\frac{\beta^2 D_{max}^2\exp(-\beta D_{max}/D)}{D^3(1+\beta)\exp(-\beta)}. \qquad (7)$$

We propose now that we have a sample of the volume O_n consisting of n independent values for the dimensions of the drops

$$D_1;\ D_2;\ D_3\ \dots\ ;\ D_n.$$

The probability of the occurrence of a given sample

$$P(O_n;\ \beta)=\varphi(D_1;\ \beta)\,\varphi(D_2;\ \beta)\dots\varphi(D_n;\ \beta), \qquad (8)$$

representing a function of the fixed values of the dimensions of the drops D_1, D_2, D_3, ... , D_n and the variable parameter β is also called the function of similitude.

It is apparent that if the selected value of β is such that $P(O_0;\beta)$ is very small, then this event "practically" should not occur in a single experiment. Consequently, such an hypotheses about the value of β should be rejected as not probable.

The method maximum probability applicable to the given case consists in this that the "best" value for β is determined in a way as the variable argument of the function (8) at which it reaches the maximum value.

In other words, the "best" value for β we get from the condition

$$P\ (O_n;\beta)=\max$$

or, what is the same, from the condition

$$\ln P\ (O_n;\beta)=L(D_1;\ D_2;\ \dots\ ;\ D_n;\beta)=\max,$$

inasmuch as $P(O_n;\beta)>0$.

The function of probability for the distribution (7) in accordance with the equation (8) has the form

$$P(O_n;\beta)=\frac{\beta^{2n}D_{\max}^{2n}\exp\left(-D_{\max}\sum_{i=1}^{i=n}\frac{\beta}{D_i}\right)}{D_1^3D_2^3\dots D_n^3(1+\beta)^n\exp(-n\beta)},$$

and therefore

$$L=2n\ln\beta+\ln D_{\max}^{2n}-\beta D_{\max}\sum_{i=1}^{i=n}\frac{1}{D_i}-\sum_{i=1}^{i=n}\ln D_i^3+n\beta-n\ln(1+\beta),$$

The value of β sought is determined by the condition of the extremum $\partial L/\partial\beta=0$, hence

$$\frac{2}{\beta}-\frac{1}{1+\beta}=\left[\frac{D_{\max}}{n}\sum_{i=1}^{i=n}\frac{1}{D_i}\right]-1.$$

After designating the right side of the expression obtained by b we shall have

$$\beta=(1-b+\sqrt{(1-b)^2+8b})/2b. \qquad (9)$$

Thus, if one measures in the sample from the drop mixture the dimensions of each individual drop, then the finding of the parameter is reduced to very simple operations, the computation of the value $b=\left[\frac{D_{\max}}{n}\sum_{i=1}^{i=n}\frac{1}{D_i}\right]-1$ and the use of the formula (9).

In this situation $D_{\max}$ should be determined in accordance with the criterional equation (5).

Method of Moments. The essence of this method of deriving parameters consists in this that according to the Khinchin theorem with a great volume of sampling 0_n from the general aggregate of objects any theoretical moment of the order i of random magnitude is approximately equal to the corresponding empirical moment. In this situation the greater the volume of the sampling the more nearly accurate this apporximate equality will be. For our case the most convenient is to use the first initial moments of the random value D, or what is the same, the theoretical $M_T(D)$ and the emperical $M_{\text{э}}(D)$—the mathematical expectancy of the random value D.

Inasmuch as the random value of D (diameter of the drop) changes from 0 to $D_{\max}$ and its probability density is given by the equation (6), therefore

$$M_T(D)=\frac{\int_0^{D_{\max}} D\exp\left(-\frac{\beta D_{\max}}{D}\right)\frac{dD}{D^3}}{\int_0^{D_{\max}} \operatorname{epx}\left(-\frac{\beta D_{\max}}{D}\right)\frac{dD}{D^3}}$$

or after integration

$$M_{\text{т}}(D) = \beta D_{\max}/(1+\beta). \tag{10}$$

With the second method of processing the primary data on the atomization the results of the experiment present themselves thus: in the intervals of grouping 0—D_1; D_1—D_2, ..., D_{m-1}—D_m the numbers of drops respectively, are n_1, n_2, ..., n_m. For the composition of $M_{\text{э}}(D)$ let us have recourse to the following operation. Let us designate by D_i the average of the ith interval of grouping, ΔD_i the width of this interval, and by $\varphi_i(D_i)$ the relative frequency of drops of the size $D_i \pm 0.5 \Delta D_i$. Then $\varphi_i(D_i) = n_i/n_0 \Delta D_i$ and

$$M_{\text{э}}(D) = \sum_{i=1}^{i=m} D_i \varphi_i(D_i) \Delta D_i. \tag{11}$$

On the basis of Khinchin's theorem $M_T(D) \approx M_{\text{э}}(D)$, hence

$$\frac{\beta D_{\max}}{1+\beta} = \sum_{i=1}^{i=m} D_i \varphi_i(D_i) \Delta D_i.$$

by designating

$$a = \frac{\sum_{i=1}^{i=m} D_i \varphi_i(D_i) \Delta D_i}{D_{\max}}, \tag{12}$$

we finally get

$$\beta = a/(1-a). \tag{13}$$

Consequently, the determination of β in this case is reduced to the computation of the value and the application of the formula (13). In this situation $D_{\max}$ also should be computed in accordance with the equation (5).

At our disposal there were experimental data on the spectra of the atomization of liquids in the form of the above indicated tables. Therefore the numerical value for the parameter β for the centrifugal atomizers was determined by the method of moments.

In the table there are shown the results of the computations of the value β for 15 centrifugal atomizers, the geometrical characteristics of which changed from 2.09 to 7.50 and the drop in the pressure on them $6 \cdot 10^5$—$30 \cdot 10^5$ n/m^2.

From the table it is seen that the value β practically does not depend on the geometrical characteristics of the atomizers nor on the system of their operation, and it varies with the maximum scattering by not more than 10% around the average value $\beta_{op} = 0.190$.

Table

Original Data and Results of Determining Parameter β

№ опыта 1)	Геометрическая характеристика форсунки A 2)	Перепад давления на форсунке $\Delta p_ф \cdot 10^{-5}$, $н/м^2$ 3)	Число отобранных капель n_0 4)	β
1.	4,72	9	2835	0,198
2	3.00	10	4521	0,171
3	6,31	13,5	3183	0,182
4	5,45	7,5	1543	0,186
5	3,65	20	3713	0,206
6	4,61	30	2603	0,188
7	3,65	10	4008	0,192
8	5,13	12	4996	0,193
9	7 50	14	8438	0,199
10	3,65	28	5404	0,192
11	3,92	10	3448	0,192
12	4,25	6	6244	0,186
13	3,45	10	5747	0,168
14	2,09	12	8246	0,180
15	4,34	9	3384	0,205

1) ordinal number of experiments; 2) geometrical characteristics of the atomizer A; 3) drop in pressure on the atomizer $\Delta p_\phi \cdot 10^{-5}$, n/m^2; 4) number of selected drops n_0.

After determining the numerical value of the parameter it was clear how well the selected function of distribution describes the existing data from observations. In Figure 1 there is shown for a single atomizer the emperical distribution D in the form of a graduated curve (histogram) and the theoretical distribution (6) with the computation for this atomizer of the value β. In Figure 2 there is given for this same atomizer the composition of the integral curves of the distribution: experimental F(D) and theoretical $\Phi(D)$. As is known, $F(D) = n(D)/n_0$ and

$$\Phi(D) = \int_0^D \varphi(D)\,dD = \int_0^D \frac{\beta^2 D_{\max}^2 \exp(-\beta D_{\max}/D)}{D^3 (1+\beta)\exp(-\beta)}\,dD = \frac{\beta D_{\max}\exp(-\beta D_{\max}/D)}{(1+\beta)\exp(-\beta)} \times \left[\frac{1}{D} + \frac{1}{\beta D_{\max}}\right]. \quad (14)$$

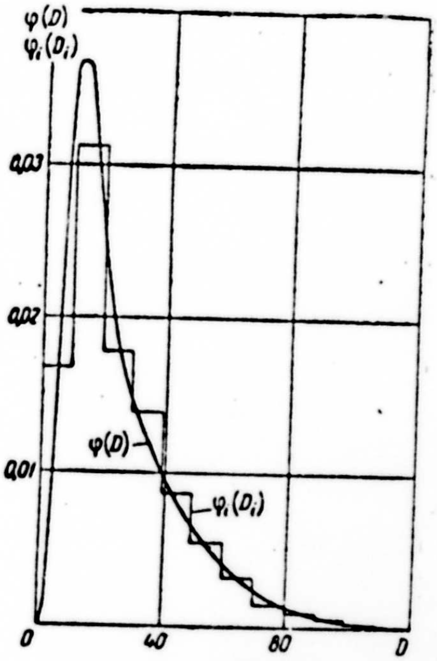

Figure 1. Histogram $\varphi_i(D_i)$ and graph of function corresponding to it of distribution density φ (D) for one centrifugal atomizer.

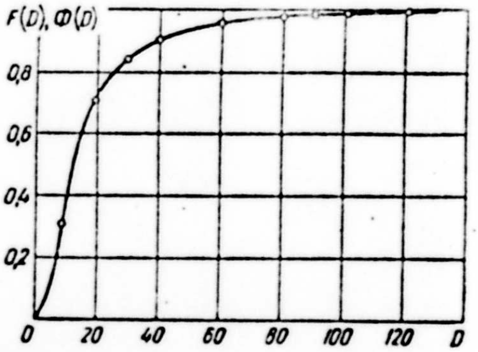

Figure 2. Graph of the experimental F(D) and theoretical Φ (D) of the integral functions of the distribution of the drops of one centrifugal atomizer.

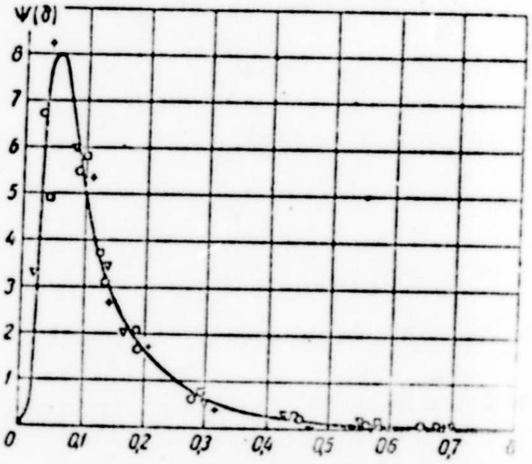

Figure 3. Graph of the generalized function of the distribution of the drops in accordance with sizes ψ (δ) for centrifugal atomizers.

In Figure 3 there is presented the graph the generalized function of the density of the distribution of the drops by the dimensions, i. e., the graph constructed in accordance with the equation (2) with the values $\beta = \beta_{cp} = 0.190$, (subscript cp = average) and the points are plotted according to the experimental data for atomizers which stand out most by their geometrical characteristics in the systems of operation (Experiments 6,9,14,12)

The data of the table and the graphs (Figures 1-3) are evidence of the possibility of using equation (7) for computing the spectra of atomization by centrifugal atomizers with the value of the parameter β equal to 0.190.

For determining the numerical value of β in the case of other qualitatively different methods of atomization, it is necessary to perform the respective experiment.

The relationships (5) and (7) make it possible to compute the spectra of atomization by centrifugal atomizers in accordance with their geometrical dimensions, the given system of operation, and the physical properties of the liquids, and can be used in different fields of technology.

Symbols

ρ = density

μ = dynamic viscosity

σ = surface tension

v = speed of flow of liquid from the atomizer

D = diameter of the drop

δ = D/D_{max}—relative diameter of drop

n_o = overall number

dn = number of drops of the size from o to o + do or from D to D + dD

β experimentally determined parameter of the function of distribution

S = entropy

Δp_o = drop in pressure on the atomizer

ε = thickness of the foam on the liquid at the section of the atomizer nozzle

n_j = number of drops fallen into the ith interval ith interval of grouping

m = number of intervals of grouping

$n(D)$ = number of drops the dimension of which is less than D

The subscripts ж, and г represent respectively, liquid and gas.

Summary

Proceeding from the Second Law of Thermodynamics, the validity of Tresh's assumption (5) for the derivation of the function (2) of droplet size distribution is shown. The mathematical statistic methods are used for obtaining the analytical relations between the parameters of this function. This allowed very simple predicting formula (9) and (13) for the determination of the parameter spectra of the pulverized liquid. From the treatment of the experimental data the value of $\beta = 0.19$ for centrifugal pulverizers was found.

Figures 2 and 3 show how distribution functions (2) and (14) agree with the observation data.

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