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FAMILIES OF MODELS

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INTRODUCTION

The greatest drawback to simulation as a technique for analyzing military conflict is that it is likely to be a slow, cumbersome, and expensive way of conducting a study. Most simulations proceed by playing through a detailed case history of a single engagement, and the number of such cases that can be examined is extremely limited. Thus, for studies which require extensive sensitivity analysis or require some form of optimization, simulation is inappropriate. Military events are determined by a large number of parameters most of which are stochastic; that is they consist of probabilities. In addition, for a large number of these, and in particular for those which refer to the future, their values are uncertain. The traditional notion of the fog of war should be supplemented by the notion of the fog of the analysis of war. The outcome of

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a conflict is determined as much by the forces and the strategies of the enemy as they are by the forces and strategies of the friendly side. Unless the analyst can take into account the wide range of options open to the enemy, the analysis is likely to furnish a completely inadequate picture of the worth of new weapons or their modes of employment.

This limitation can be ameliorated, at least in part, through the use of a set or family of models rather than a single model. Using a cooperative group of models at different levels of generality, small, rapidly computable models can be used to survey wide ranges of the parameters, to conduct sensitivity analyses, or perform optimizations. More detailed models can be used to perform feasibility checks and to spell out the details of either force structures or force employments. In this way the advantages of simulation can be retained and at the same time the possibility of dealing with a rich space of parameters and strategies is feasible.

This scheme is actually not too different from the way in which an analyst normally proceeds. It is usually the case that a great deal of thinking-through or back-of-the-envelope calculation is required in order to determine which cases are significant and worth the application of simulation. There are, however, a number of advantages to formalizing this process. Perhaps the biggest advantage

is that in many instances the thinking through at the aggregate level of the problem is the most valuable part of the exercise. In all events, in a family of models, the results of the highest level computations will be of value in themselves; and this is especially true if optimization or the analysis of a fully two-sided war is involved. By formalizing the process, feedback is easier to arrange. That is, if the results of the detailed analyses do not accord with the outcome of the more general computations, modification and rerun of the general model is usually rather easy. Another advantage is that suboptimization in the more detailed models is usually much easier to devise if general guidance from the less detailed models is available; there is greater assurance that suboptimizing procedures will not distort the outcome prohibitively. Intuitive criteria such as plausibility, accordance with more general policy goals or restrictions, and similar contextual considerations are usually much easier to interpret in terms of the aggregated high-level models than they are in terms of the very detailed models. Finally, by formalizing the scheme, much of the burdensome creation of inputs for the more detailed models—and that holds especially for plans or lists of potential events—can be turned over to the machine. This results from the fact that detailed plans or prescriptions of the course of events can be considered

as more or less straight forward unpacking of the more abstract allocations or strategies of the higher level models. This point is clearly related to the one I made above about suboptimization, since one of the elements of the unpacking process is suboptimization over those aspects of a plan which were aggregated together in the general model.

At RAND, we have constructed an experimental family of models dealing with the planning problem for strategic nuclear war and with the problem of designing strategic nuclear forces. The first problem was chosen because the role of detailed operational constraints is crucial. Hence, a very extensive model is required for final evaluation. But on the other hand, the role of the enemy's operational plan is equally significant and this requires the evaluation of each potential plan over all the options open to the enemy. These two requirements in the present state of the art are incompatible with simulation as the only tool. However, they precisely match the structure of a hierarchy of models.

The set we have designed consists of a three-level pyramid of war games. See Fig. 1. The first, which we call STROP—a contraction of the somewhat pretentious name Strategic Optimizing routine—is a highly aggregated two-sided war game. It is designed to run on a high speed computer, in this case the IBM 7044, and will evaluate the outcome of a pair of Red and Blue war plans in about

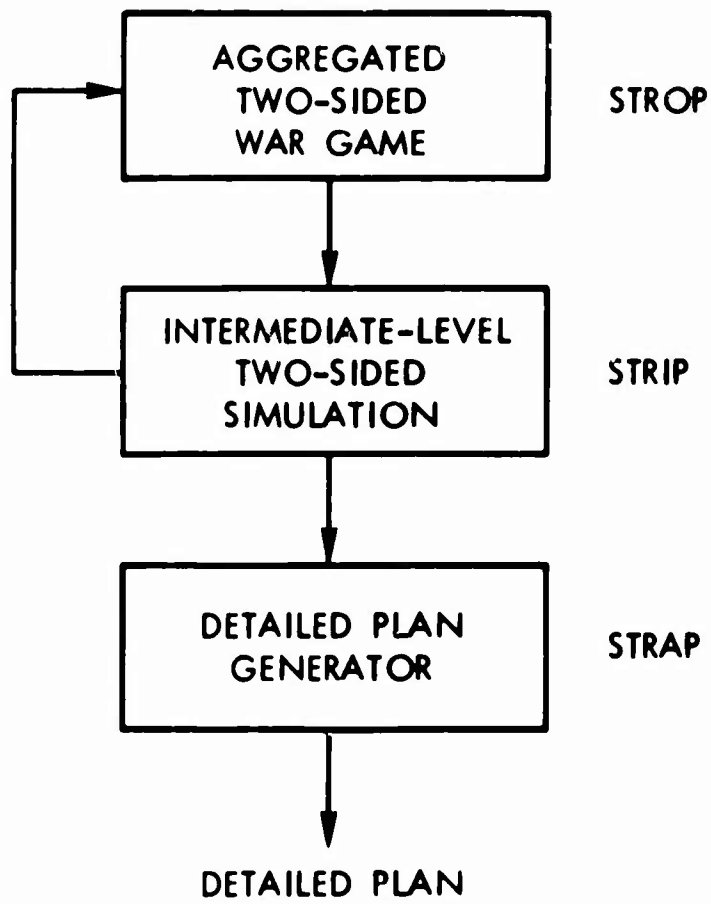


Fig.1—Planning Model Family

one one hundredth of a second. By war plan, at this level of aggregation is meant an allocation of strategic forces to potential targets. With this running time, it is feasible to survey a large number of possible allocations on each side. In our test program, we have been using samples of 168 allocations for each side. This means we have been running a total of 168 by 168 or about 28,000 potential wars. This sample along with the analysis to be described below takes about ten minutes to complete.

The second model we call STRIP, short for Strategic Intermediate Planner. STRIP is also a two-sided war game, in this case of a simulation variety, which is intermediate in level of detail. It contains some geography—launch bases and targets are aggregated in geographic regions. Time is divided roughly into half-hour periods. Forces and targets are less drastically aggregated. There may be a number of kinds of ICBM's, bombers, and targets. STRIP takes a pair of allocations from STROP, unpacks these into plans at the appropriate level of detail, and carries out a simulation of the two-sided engagement. STRIP requires about 1/10 of a minute to run. Hence, if the outcome of STROP analysis is not a unique pair of preferred strategies for each side, but say a small matrix like a ten by ten, it is quite feasible to run all of the cases through STRIP.

The final model in the set is a detailed plan generator which we call STRAP, short for Strategic Actions Planner. STRAP takes a plan from STRIP and develops it into a specific launch schedule for each missile and bomber, taking into account operational constraints such as bomber-tanker mating, fuel consumption, time of flight, number of bombs on target, as well as taking into account the effect of specific timed damage to enemy defenses on bomber and missile requirements. STRAP requires about four hours to generate a complete strategic war plan.

It would, of course, be feasible to take a pair of plans generated by STRAP and feed them into a highly detailed battle model such as the model STAGE employed by Headquarters USAF. We have no plans at RAND at the moment to carry out such an extension since our interest is primarily concentrated on the other end of the scale. Because time is short, I am going to spend most of my time today discussing STROP. And in any case, the other models are more or less of a standard simulation type.

STROP

For the model at the highest level of the hierarchy it would be preferable to have a war game which was capable of an analytic solution. At the moment, we are frustrated in this desideratum for two reasons. One is that even with the model scraped to the bare bone as it were, it is still too complex to be solved by known game-theoretic techniques. Secondly, nuclear war is a highly nonzero sum game. The outcome can be utter disaster for both sides. Unfortunately at the present time, there does not exist a satisfactory solution criterion for nonzero sum games—especially in such highly noncooperative situations as central nuclear war. As an alternative to an analytic solution it is possible, by using the high speed computer, to sample an extensive set of allocations on each side and analyze the resulting matrix of outcomes with a criterion that appears acceptable to the military. The simplest criterion is, of course, that of dominance. In STROP the outcome is represented by damage to value targets on each side. The value of withheld forces, for example, can be measured in terms of the potential damage to value targets of the other side. An allocation for Blue, call it X, is said to dominate another allocation Y if, no matter which allocation is chosen by Red, the result in damage to Red's value targets if Blue employs X, is greater than the damage to Red's value targets given Y, and

conversely the damage to Blue's value targets, if he employs X is less than the damage to Blue's value targets, if he employs Y. Interchanging the two damages gives the corresponding criterion for Red.

This double dominance is a very strong condition and normally would not be expected to apply to many allocations on either side. As it happens, however, in the cases we have looked at—and there doesn't seem to be much reason for thinking they are unique—the dominance criterion has resulted in a very large reduction in the size of the outcome matrix. Typically, the vast majority of allocations are dominated out. The usual order of reduction is from a 168 by 168 to about a seven by seven which is a reduction by a factor of 24 by 24 or greater than 500. Although we have not been able to account for all the effects that produce this large reduction, there appear to be three major considerations: In the first place, factors such as vulnerability of the target and weapon size and CEP strongly determine whether a weapon is efficiently used against a particular target system. Secondly, rapidly decreasing returns for additional weapons on the same target system appears to be the normal situation. And finally, complex missions which require long chains of probabilistic interactions, for example, the counterforce employment of missiles against missiles, are inherently inefficient. Admittedly, analyzing a sample out of the

game matrix is risky.⁽¹⁾ However, it is not as risky as analyzing a sample consisting of a single case. We have performed the experiment of increasing the density of our sample and have found that the resulting undominated strategies lie well within the submatrix defined by the undominated strategies of the coarser sample. In short, the payoff appears to be a reasonably well-behaved function of the strategies.

The elements of STROP are firstly offensive forces— one kind of bomber, one kind of missile; secondly, active defenses—bomber area defenses, missile defenses, and bomber local defenses; thirdly, offensive force targets—bomber fields and missile sites; and finally value targets. A strategy, as mentioned before, consists of a distribution of bombers and missiles across the military and value targets. Missile defenses, however, are not targeted. The routine generates a sample of strategies by selecting allocations to the various target systems and varying these by fixed increments so that the total allocation to all of the target systems for each offensive weapon adds up to one. The routine then, methodically takes each pair of allocations in turn and computes the results of the mutual interaction taking into account the generation and flight times of the weapons.

As remarked above, the payoff criterion is damage to value targets on both sides. But damage to the other kinds of targets is, of course, computed and is also

recorded. Parameters are also included which allow the determination of which side initiates the engagement and the amount of warning time available to the other side. It is also possible to specify whether a given side is acting in a purely preplanned manner or is capable of changing his allocation during the conflict to take account of losses due to enemy attack.

EXAMPLE

Figures 2-4 indicate the initial conditions for one of the test runs which we made with STROP. It should be pointed out that any resemblance of these numbers to real numbers is purely coincidental. They were cooked up to create a test case that had some air of verisimilitude. Several comments are in order. The bomber defense kill probability is the original kill probability before damage to the defenses. The routine does evaluate the effect of an attack on the enemy's bomber defenses. In this case, Red initiates the exchange as indicated by the zero execute time. Blue is responding as indicated by the execute time of 30 minutes for missiles and 15 minutes for bombers. The bomber execute time represents the capability of launching on positive control. Blue is following a pre-planned mode of attack. Red on the contrary is using a retargeting capability.

The next four figures illustrate the outcome of the interaction of a typical pair of Red and Blue strategies. Figure 5 shows the strategy allocations themselves. Blue has allocated no missiles to bomber defenses, has allocated 40 per cent of his missiles to bomber air fields, none to missiles, and 60 per cent to cities. The city category is included as a spill-over feature. None of the games that we have conducted in this test series indicate an advantage to either side to withhold forces. Blue has allocated

ORIGINAL NUMBER OF BOMBER SQUADS	MISSILE KILL PROBABILITY AGAINST BOMBER SQUADS	BOMBER KILL PROBABILITY AGAINST MISSILE SQUADS	BOMBER KILL PROBABILITY		MISSILE KILL PROBABILITY			
			W	AP	W	AP		
700	.46	.43	.54	.97	.89	.56	1	4
300	.43	.43	.54	.97	.89	.56	1	4
109	.43	.43	.54	.97	.89	.56	1	4
64	.43	.43	.54	.97	.89	.56	1	4
54	.43	.43	.54	.97	.89	.56	1	4
400	.46	.43	.54	.96	.89	.55	1	3
245	.45	.45	.64	.96	.89	.55	1	3
264	.45	.45	.64	.96	.89	.55	1	3
435	.45	.45	.64	.96	.89	.55	1	3

Fig. 2—Initial Conditions, I

	DEFENDED CITIES		KILL PROBABILITY	
	S A M	A B M	S A M	A B M
B L U E	102	50	.65	.75
R E D	100	60	.70	.75

Fig. 3—Initial Conditions, II

EXECUTE TIME IN MINUTES	WARRING IN TIME	MISSILE GENERATION RATE	BOMBER GENERATION RATE	TARGETTING MODE	PAYOFF FUNCTION PARAMETERS	ALLOCATION INCREMENT 0/0
30	0	0	0	0	4000	20
30	0	60	10	2	400	20
15	0	15	50			
0	0	100	100			
0	0	0	0			
0	0	80	0	1	3500	20
0	0	100	50	0	350	20
0	0	100	100			
0	0	0	240			
0	0	0	0			

Fig. 4—Initial Conditions, I

B U R D	MISSILE ALLOCATION TO					BOMBER ALLOCATION TO				
	STIRN ADTE EXG Y	BOD ME BF EER NS ES	JOF MI BE EL RDS	MIS SIL ES	CIT IES	SAV E	BOD ME BF EER NS ES	TO CIT IFS	SAV E	
27	0.0	0.40	0.0	0.60	0.0	0.0	1.0	0.0	0.0	
105	0.20	0.40	0.0	0.40	0.0	0.20	0.80	0.0	0.0	

Fig. 5—Typical Allocation

his bombers entirely to cities. Red has allocated 20 per cent of his missiles to bomber defenses, 40 per cent to air fields, again none to missiles, and 40 per cent to cities. He has allocated 20 per cent of his bombers to bomber defenses, and 80 per cent to cities. Figure 6 shows the outcome of the interaction of these two allocations. Of course, the original number of missiles survived because there was no attack on missiles. Bombers have been reduced considerably and bomber defenses on the Blue side have been reduced considerably. Both sides have lost most of their bomber fields and both sides have lost a large number of their cities. The effective kill probabilities are indicated because the routine takes into account the fact that there may be multiple warheads on target, and the effectiveness per warhead of multiple warheads is less than the effectiveness would be if only one warhead were allocated per target. I, perhaps, should remark parenthetically that the notion of city that we are using here is more strictly interpreted as city unit. That is, the targets are more like DGZ's and, a large urban complex might be represented by a number of city target units. Figure 7 indicates some internal accounting numbers which are nevertheless of interest to the analyst. The occupancy number indicates the average number of missiles or bombers that will still be on the sites or air fields when enemy missiles arrive. The double entries for missile and bomber warheads per target in the case of cities indicates the different

	SURVIVING NUMBER OF	EFFECTIVE MISSILE KILL PROBABILITY	EFFECTIVE BOMBER KILL PROBABILITY	F ID								
M I S S I L E S				NEP								
B O M B E R S				AFR								
B O D M E E R N S				LEO								
B O F M I B E E L R D S				NB								
C I T I E S				BSA								
B O D M E B F E E R N S				OEB								
B O F M I B E E L R D S				M I								
				BKL								
				EII								
				RLT								
				LY								
B L U E	700	.46	.18	.43	.54	.97	.86	.6				
R E D	435	145	264	3	164	.46	.29	.45	.64	.89	.89	.55
								.45				

Fig. 6—Final Conditions, I

OCCUPANCY	PERCENT SURVIVING GROUND LOSS	MISSILE WARHEAD PER TARGET	BOMBER WARHEAD PER TARGET								
M I S S I L E S	B O M B E R S	M I S S I L E S	B O M B E R S								
0. 0	3.26	1. 0	0.44	1. 0	5.18	1. 0	1. 0	1. 0	1. 0	1.35	7.93
0. 0	1.96	1. 0	0.59	1. 0	2.72	1. 0	1. 0	1. 0	1.35	1. 0	1.86

Fig. 7—Final Conditions, II

allocations that were selected for those cities which were locally defended. The number 14.9 may seem a little high for bomber warheads per target; however, if you reflect that each bomber carries four warheads, in this particular case that's less than four bombers on an average to a defended city.

Figure 8 illustrates the payoff, 278 cities destroyed for Blue, 273 for Red, no missiles or bombers saved on either side. The column headed Value is really a misnomer, it's simply the difference between the damage to the city targets on the two sides. I'll say something about the last two columns in just a moment. STROP conducts a war game of the sort just indicated for the entire sample of 28,000 pairs of Red and Blue allocations, creates a matrix of these outcomes, reduces it by dominance, and prints-out the undominated allocations. In this particular instance the undominated matrix was a little larger than usual. It was a 22 by 9. This represents a very large reduction from the original 168 by 168. But frankly it still isn't a very good representation of what you might call preferred allocations on each side. It also is not particularly comprehensible at a glance and, therefore, I won't bother to show it to you.

	C I D T E I S E T S R O Y E D	S A M V I E S D S I L E S	S A V C E M D B E R S	V A L U E	P A Y O F F	F A L L O U T
	260	0	0	-24	-52	303
	236	0	0	24	-6	84

Fig. 8—Payoff

SENSITIVITY TO PAYOFF

In order to get a further refinement of the analysis—which for all practical purposes means a further reduction in the size of the undominated matrix—several routes can be taken. One route is to define the payoff more completely in terms of the difference in the damage to the two sides. This criterion has several advantages. One, it is very simple; two, it transforms a game into a zero sum game; and three, it includes in a single criterion the desire on the part of a given side to both achieve as large a damage against the other side as possible, and to minimize the damage to himself. It has several drawbacks: First, it implies a very simple minded trade-off between damage to targets on the other side and damage to ones' own cities; secondly, it does not reflect the fact that there are levels of damage to either side beyond which the nation would not be considered viable; and finally it does not reflect the fact that as damage approaches the level, let's call it the critical level, at which one side no longer is viable, no simple trade-off of target for target could be appropriate. The last two drawbacks can be compensated for by introducing into the payoff a factor which we have called the Assumption of Increasing Concern. This assumption is illustrated in Fig. 9 which shows the outcome of the engagement as a point in the two-dimensional space defined by damage to Red and damage to Blue.

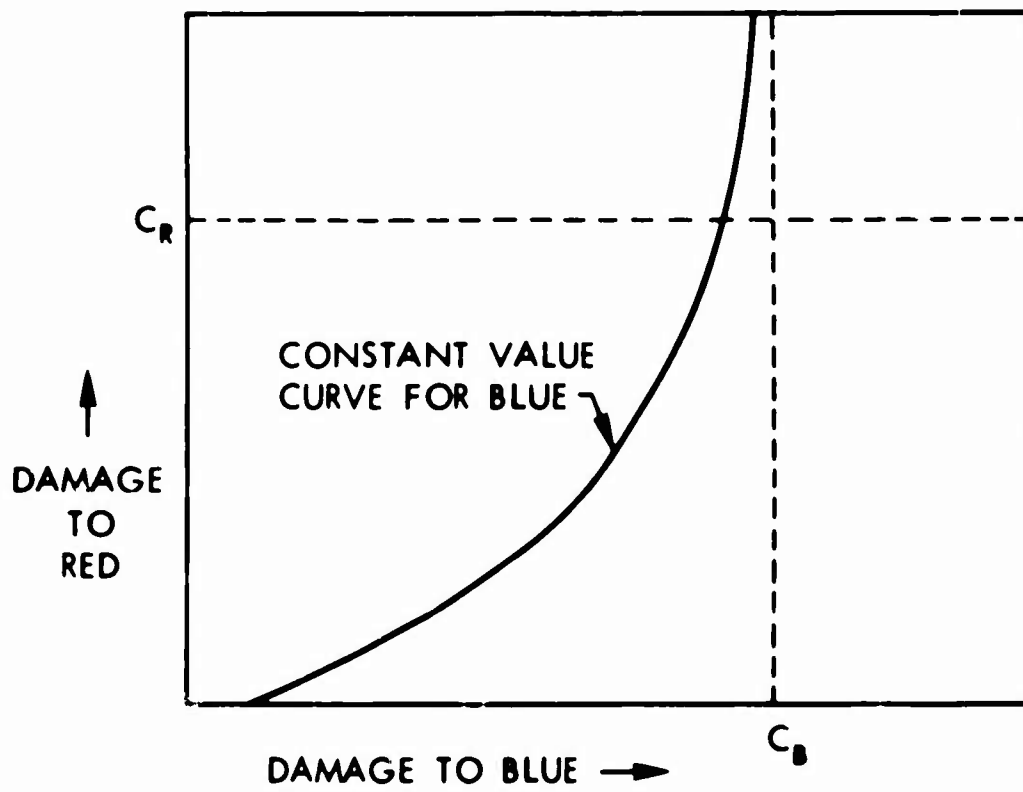


Fig. 9—Assumption of Increasing Concern

Presumably at some level of damage to Blue which we have labeled here the critical level, Blue has lost the war no matter what the status of Red, and similarly there is such a critical level for Red. At levels of low damage to both sides, it may be reasonable to assume that there is some form of trade-off of target for target. But as Blue's level of damage approaches that of the critical the relative value of his own targets over those of enemy targets increases, and an equal-value curve will then presumably become asymptotic to the line represented by the critical level. (2)

A simple expression for such a payoff is indicated in Fig. 10. Here the simple difference in damage to value targets is modified by an expression containing a scaling factor in the numerator and in the denominator, the difference between Blue's damage and his critical damage. As you can see at lower levels of damage the expression is practically equivalent to the difference between the two damages. But at high levels of damage the correction factor takes over and as Blue's damage approaches the critical level it becomes negatively infinite. This is only one out of a very large number of possible formulations which have the properties mentioned above. It does have the advantages of simplicity and of fairly direct interpretation. When this payoff is applied to the undominated matrix remaining and the matrix analyzed again using dominance, but in this case dominance on the new payoff, the matrix receives a

$$P_B = D_R - D_B - \frac{A}{C_B - D_B}$$
$$P_R = D_B - D_R - \frac{B}{C_R - D_R}$$

	<u>RED</u>	<u>BLUE</u>
PAYOFF	P_R	P_B
DAMAGE	D_R	D_B
CRITICAL LEVEL	C_R	C_B
SCALING FACTOR	B	A

Fig. 10—Simple Form of Assumption
of Increasing Concern

further reduction and in most cases it is reduced to a one by one. That is what happens in the illustrative case I showed you a moment ago, and the allocations shown were precisely the unique set of preferred allocations for Red and Blue.

Since the subject of payoff functions for central nuclear war is so controversial, one is inclined to be uneasy in using a payoff function which might be overly sensitive to arbitrary parameters. We examined the result of applying the assumption of increasing concern, not just to the final undominated matrix but to the entire sample matrix. Again the matrix was reduced to a one by one, and as a matter of fact precisely the same matrix as obtained by first applying pure dominance on the damage and then applying the assumption of increasing concern.

If the simple difference value $D_B - D_R$ is used as a payoff, the game becomes zero-sum. If this zero-sum game is solved (for the sample matrix) optimal allocations almost identical to those obtained by the Assumption of Increasing Concern are obtained, and the respective payoffs are identical.

These experiments indicate that the outcome is not very sensitive to the form of the payoff function used, but I must add the caveat that this fact is true only so long as you are not dealing with the most extreme levels of damage. If either side suffers more than its critical

level of damage, or if either side has its total target system destroyed, then it is clear that any of these functions will give a biased result. However, the determination of a preferred strategy for the side that has lost all of its value targets is perhaps a matter of little concern.

FORCE-STRUCTURES MODEL

We are developing an extension of STROP which will apply the analysis described above to the problem of allocating funds among force elements, i.e. to the problem of budget allocation to force structures. The general procedure is about the same as for the analysis of strategic allocations, but the problem is much more complex. In order to evaluate a force structure, it is necessary to assign to it an employment, i.e. a force allocation. Therefore, the strategy space is the combined space of force structures and allocations. This is many orders of magnitude larger than the space of force allocations alone. For example, if we were to take the simple brute force method of playing a STROP game for every sample pair of force structures, with the sampling about as gross for force structures or for strategic allocations, we would need a continuous machine run of about five years' duration in order to play through one force structure survey.

Since such a run would be rather absurd, we have adopted the tactic of exploring the combined force allocation and strategy spaces in a heuristic fashion, relying on the rough continuity of the outcome as a function of forces and allocations. In essence, what we do is pick a representative force structure for each side, and make a full STROP run for that representative pair. Then we explore outward from the representative sample, and for

each new pair of Red and Blue forces, we explore the strategy space only within the vicinity of the preferred strategies for the previous case. In this way we can, as it were, feel our way through the matrix, making only small changes in force structures and only small changes in preferred strategies until we have exhausted the sample. This technique will reduce the amount of machine time required by a factor of about 10,000, leaving us with a necessity for a run of something of the order of five or six hours for one survey examination of the force structures sample. At the conclusion of this survey, we will have a matrix of outcomes for each pair of Red and Blue force structures and also associated with each such pair the preferred force allocations. We can then perform dominance on this matrix in precisely the same fashion as we do for the force allocation matrix.

Preliminary test runs with this more extensive model indicate that in the area of force structures dominance is quite as powerful as it is in the area of force allocation. Again it appears to be the case, although this is a very tentative conclusion, that by and large most force combinations are extremely inefficient. Another tentative conclusion which has come out of our test runs, but one which certainly must be backed up with further investigation, is that the stability of the payoff under variations in force structures is much greater than the stability of the payoff under variations of strategic allocation. The

reason is that the strategic allocation is itself a stabilizing factor. It is possible to make up for quite large variations in force structure by changing the strategic allocation suitably. Strictly speaking this remark refers to the strategic allocation of the opposite side; that is, a change in force structure by one side, can be compensated for by a change in strategy on the part of the other side. This would mean that the relative balance of a set of forces is not very critical providing you assume that the enemy will have a fairly good idea of the nature of your forces.

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