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TECHNICAL REPORT

U. S. NAVAL APPLIED SCIENCE LABORATORY

NAVAL BASE
BROOKLYN 1, NEW YORK

FATIGUE OF STRUCTURAL ELEMENTS
DEVELOPMENT OF THEORY AND MEASUREMENT
OF RESIDUAL STRESSES AT TEE FILLET WELDS IN
1-1/2 in. HY-80 STEEL

Lab. Project 9300-23, Progress Report 1

SF 013-013-02, Task 2025

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Lab. Project 9300-23
Progress Report 1

SUMMARY

A theory has been developed for the calculation of relaxation strains when a hole is drilled into a plate with a linearly varying stress field. With this theory a technique was developed for the measurement of residual stresses at the toe of tee fillet welds. The above technique was employed for the measurement of residual stresses at the toe of tee fillets in 1 1/2 in. HY-80 steel with the fillet in the as welded, ground, shot-peened, ground and shot-peened, and mechanically-peened condition. It was found that experimental data conform to the assumed theory, and that residual stresses in as welded tee fillet welds in both the transverse and longitudinal directions approach the yield strength of the steel. It was also found that residual stresses are reduced approximately 25 percent by grinding, 30 percent by shot peening and 50 percent by grinding and shot peening. Mechanical peening drastically affected residual stresses by converting high tension at the toe of the fillet weld to high compression of approximately the same magnitude.

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ADMINISTRATIVE INFORMATION

Ref: (a) BuShips ltr F013-03-02 Ser 442-135 of 26 July 1963
(b) NAVAPLSCIENLAB Program Summary of 1 Nov 1964 pgs 151-156
(c) DTMB Report 1742 of Aug 1963 - An Evaluation of the Hole-Relaxation Method of Determining Surface Residual Stresses by Peter M. Palermo.
(d) NAVAPLSCIENLAB Lab. Proj. 9300-23, Technical Memorandum No. 2 of 19 Oct 1964.
(e) S.E.S.A. Proceedings, Vol XLIV, No. 1, 1956 Measuring Non-Uniform Residual Stresses by the Hole Drilling Method by R.A. Kelsey.

1. In compliance with references (a) and (b), the U.S. Naval Applied Science Laboratory is pursuing a research program on the Fatigue of Structural Elements. One phase of this program covers the determination of residual stresses at the toe of tee fillet welds in HY-80 steel for the as welded, ground, shot-peened, ground and shot-peened, and mechanically-peened-to-contour condition. This is a progress report on that phase of the program specifically related to the analysis and measurement of the above residual stresses.

ACKNOWLEDGMENT

2. Consultation and guidance, provided by Dr. V.L. Salerno, Laboratory Consultant in Applied Mechanics, in the development of theory given in Appendix A is hereby acknowledged. The Bureau of Ships Program Manager for this work is G. Sorkin (Code 341A) and the Bureau of Ships Project Engineer is C.H. Pohler (Code 442)

BACKGROUND

3. It has been well established that residual stresses developed during welding have, in most instances, a deleterious effect on fatigue life. To alleviate this condition, a considerable amount of effort has been exerted to develop improved fabrication procedures by the introduction of grinding, shot peening, mechanical peening, and combinations of these operations. A knowledge of the effects that these operations have on the magnitude and distribution of residual stresses would be of great value in the further development of improved techniques for the fabrication of structures which are imparted with favorable, or made free of, residual stresses.

4. At present there is a great need for suitable methods of measuring residual stresses in fabricated structures. In this connection the David Taylor Model Basin has investigated the problem and has found that the hole-relaxation method is presently the most suitable for application to Naval structures. An evaluation of this method was reported in ref. (c). The method entails the judicious placement of strain gages around a point at which residual stresses are to be determined and measuring relaxation strains when a small hole is drilled at the point. From a knowledge of strain distribution before and after the hole is

is traversed parallel to the weld. These residual stresses may be represented by

$$\sigma_x = cy + A \qquad (1)$$

$$\sigma_y = ky + B$$

where A, B, c and k are constants to be evaluated. If a hole of given radius is drilled at some point o which is taken as the origin of coordinates, as shown in Fig. 1, and the change in radial strain near the edge of the hole is measured by means of a strain gage, this change in strain, the relaxation strain, is given by the following equation:

$$
\begin{aligned}
E\epsilon_r = & - \left(\frac{A+B}{2}\right)(1+\nu)\frac{a^2}{r^2} \\
& + \left(\frac{A-B}{2}\right)\left[-4\frac{a^2}{r^2} + 3(1+\nu)\frac{a^4}{r^4}\right]\cos 2\theta \\
& + \frac{kr}{4}\left[(\nu-3)(\nu+1)\frac{a^2}{r^2} + \nu(\nu+1)\frac{a^4}{r^4}\right]\sin\theta \qquad (2) \\
& - \frac{cr}{4}\left[(1+\nu)\frac{a^4}{r^4}\right]\sin\theta \\
& + \left(\frac{k-c}{4}\right) r\left[(5+\nu)\frac{a^4}{r^4} - 4(1+\nu)\frac{a^6}{r^6}\right]\sin 3\theta
\end{aligned}
$$

where:

ϵ_r is the relaxation strain in radial directions

E is Young's modulus of elasticity

ν is Poisson's ratio

drilled, the magnitude of residual stresses at the point may be calculated. The above hole-relaxation method has been found to give satisfactory results in uniform stress fields and also in non-uniform stress fields provided relaxation strains are measured in a manner to give average values for the point.

5. Preliminary studies, ref. (d), made by the NAVAPLSCIENLAB on the magnitude and distribution of residual stresses at the toe of tee fillet welds indicated that the residual stress field is not uniform and that the above hole-relaxation technique would give low results if strain gages were not placed all around the holes as required for a non-uniform residual stress field. Placement of strain gages all around the hole was found to be possible only when the center of the hole was located at a distance of 0.3 in. or more from the base of the fillet. This procedure yielded average residual stresses at the point represented by the hole center and not at the toe of the fillet as desired. However, the results did indicate that the residual stresses normal and parallel to the weld varied linearly with distance from the weld over the area for which data were obtained.

6. The above problem was pursued further by this Laboratory and steps were taken to develop the theory for the calculation of relaxation strains when a hole is drilled in a linearly varying stress field and thereby permitting the determination of residual stresses at the toe of tee fillet welds. This report covers the above development of theory and related laboratory experiments.

OBJECTIVE

7. The object of this report is to:

a. present the development of the theory for the determination of relaxation strains around a hole drilled in a linearly varying stress field.

b. develop a technique for the application of this theory to the determination of residual stresses at the toe of tee fillet welds.

c. present results obtained by use of this technique in the measurement of residual stresses at the toe of tee fillet welds in the as welded, ground, shot-peened, ground and shot-peened and mechanically-peened condition.

THEORETICAL ANALYSIS

8. Referring to sketches (a) and (b) of Fig. 1, it is assumed that residual stresses at the surface of a plate near a fillet weld are in a state of plane stress; that because of geometry the principal stresses σ_1, and σ_2 are in the x and y directions, parallel and normal to the weld, respectively; that these residual stresses are zero at some distance from the weld and rise linearly as they approach the weld, and that the stress field does not vary as the plate

is traversed parallel to the weld. These residual stresses may be represented by

$$\sigma_x = cy + A \qquad (1)$$
$$\sigma_y = ky + B$$

where A, B, c and k are constants to be evaluated. If a hole of given radius is drilled at some point o which is taken as the origin of coordinates, as shown in Fig. 1, and the change in radial strain near the edge of the hole is measured by means of a strain gage, this change in strain, the relaxation strain, is given by the following equation:

$$\begin{aligned} E\epsilon_r = & -\frac{A+B}{2}(1+\nu)\frac{a^2}{r^2} \\ & + \frac{A-B}{2}\left[-4\frac{a^2}{r^2} + 3(1+\nu)\frac{a^4}{r^4}\right]\cos 2\theta \\ & + \frac{kr}{4}\left[(\nu-3)(1)\frac{a^2}{r^2} + (\nu+1)\frac{a^4}{r^4}\right]\sin\theta \qquad (2) \\ & - \frac{cr}{4}(1\ \frac{a^4}{r^4})\sin\theta \\ & + \frac{k-c}{4}r\left[5(\ )\frac{a^4}{r^4} - 4(1\ )\frac{a^2}{r^2}\right]\sin 3\theta \end{aligned}$$

where:

ϵ_r is the relaxation strain in radial directions

E is Young's modulus of elasticity

ν is Poisson's ratio

θ is the angle that direction of radial strain makes with the axis, Fig. 1

a is the radius of the hole

r is the radius of the gage circle

A,B,c and k are the constants of equ. (1)

Since there are four constants A,B,c and k to be determined, at least four independent measurements of ϵ_r are to be made in four radial directions θ. It should be noted that the first two terms of equation (2) represent the known expression for the relaxation strain in a biaxial stress field where A and B are the principal stresses. The remaining terms of equation (2) represent the linear variation of the principal stresses with y where the constants c and k are proportionality constants.

9. Equation (2) is developed in detail in Appendix A.

SPECIMENS

10. Ten tee fillet welded, NAVAPLSCIENLAB plate type fatigue specimens fabricated from 1 1/2 in. thick HY-80 steel were used in this work. A typical specimen is shown in Fig. 2. Two specimens were in each of the as welded, ground, shot-peened, ground and shot-peened, and mechanically-peened condition. These specimens were identical to the corresponding types subjected to fatigue at this Laboratory under the High Strength Steel Program.

STRAIN GAGE ARRANGEMENTS

11. Two strain gage arrangements as shown in Fig. 3 were employed in the measurement of radial relaxation strains by the hole drilling technique as follows:

(a) Arrangement 1- Satisfactory for a biaxial stress field in which the principal stresses are uniform, and normal and parallel to the weld. The indications from gages c and d are averaged to give the relaxation radial strain parallel to the weld and the indication from gage b is taken as the relaxation radial strain normal to the weld. These two strains are used to calculate the average residual stresses existing at the hole center prior to drilling. The first two terms on the right hand side of equation 2, are applicable for this calculation.

(b) Arrangement 2- Intended primarily for a non-uniform biaxial stress field. Eight strain gages spaced 45 degrees apart are shown in Fig. 3, although only four independent measurements are required for the case where principal stresses are normal and parallel to the weld. Three pairs of gages e and f, c and d, and g and h are symmetrically located and actually provide only three relaxation strains when properly averaged. Thus, five independent relaxation strains are obtained with this arrangement which tends to give a uniformly weighted representation of the stress field around the hole. Substitution of these relaxation strains into equation 2 and by application of the method of least squares the resulting five equations may be reduced to four which in turn may be solved simultaneously for the unknown parameters A,B,c and k.

METHOD

12. The following techniques and procedures which are essentially in accordance with recommendations of DATMOBAS report, ref. (c), were adhered to in the drilling of holes and measurement of residual strains:

a. The hole centers were located on opposite sides of the stiffeners of the plate type specimens as shown on Fig. 4.

b. Foil strain gages, type C6-1X1M50A, with a gage length of 0.050 in., carefully cemented at appropriate locations and protected with a coat of wax, were used for the measurement of relaxation strains. Fig. 5 is a photo of strain gage arrangement 2 applied at a ground fillet.

c. Strain gages were located on a 5/8 in. dia. circle concentric with the drilled hole.

d. Each set of gages was connected to a Baldwin switching and balancing unit and type N or Model 120 Baldwin strain indicator.

e. A portable Bux magnetic base drill, carefully aligned by means of a centering pin in the chuck directly over the center of the hole, was used for drilling.

f. Drilling was accomplished in increments using 3/16, 5/16, 3/8 and 7/16 in. dia. drills in succession while a stream of compressed air was directed towards the drill to blow the chips away, thereby avoiding damage to the gages.

g. Each hole was drilled to a depth of 7/16 in.

h. Measurements of relaxation strain were taken for all gage locations after ea increment of drilling.

i. Prior to drilling the 3/16 in. dia. hole when using strain gage arrangement 1, it was necessary to remove the fillet weld in the way of the drills flush with the plate surface, using a 1/2 inch end mill mounted in the Bux drill. The effects of this operation were recorded as relaxation strains and were considered in the calculation of residual stresses.

13. Relaxation strains were measured on each of the 10 available plate type specimens using the hole-drilling technique described above. Measurements were made on each plate with each of the two strain gage arrangements shown in Fig. 3. The reference gage line for each of the fabrication conditions was located as shown in Fig. 6. For the as welded and shot-peened fillets the reference gage line was located 1/32 in. away from the toe of the weld. For the ground, ground and shot-peened and mechanically-peened conditions, the reference gage line was located along a line representing the toe of the weld prior to finishing by the respective fabrication procedures.

RESULTS

14. Measured radial relaxation strains obtained with strain gage arrangement 1 are shown plotted against hole diameter on Figs. 7 to 16 for fillet welds in the as welded, ground, shot-peened, ground-and-shot-peened, and mechanically-peened condition. Similar data obtained with strain gage arrangement 2 on the same series of plates are shown on Figs. 17 to 26. It will be observed that for the as welded, ground, and mechanically-peened specimens all gages showed an increase in the magnitude of the relaxation strain as the hole diameter was increased. Both positive and negative relaxation strains were obtained. In general, the positive values indicate release of compressive residual stresses and the negative values indicate release of tensile residual stresses. In the case of shot-peened and shot-peened after grinding specimens the measured tensile residual strains rose to a maximum and then dropped off as the hole diameter was increased.

15. Using data taken from the curves of Figs. 7 to 16 at a hole diameter of 0.438 in., residual stresses were calculated as described in paragraph 11a for strain gage arrangement 1. Similarly, using data taken from the curves of Figs. 17 to 26 at a hole diameter of 0.438 in., constants A,B,c and k were calculated as described in paragraph 11b for arrangement 2. By substituting these constants into equations 1, the residual stress field in the plate at the fillet weld was determined. A tabulation of residual stresses calculated in the above manner for the two strain gage arrangements is given in Table I.

DISCUSSION

16. A discussion of the results given in Table 1 is made with full cognizance of the following limitations:

(a) The theory is based on a plane stress field which is uniform through the thickness of the plate whereas the actual residual stress field varies through the thickness.

(b) The theory is developed for stresses within the elastic range whereas the magnitude of radial stresses measured indicates that the tangential stresses at the very edge of the hole are in the plastic range. In this connection, since the stress concentration factor at the edge of a hole is 2 for equal and uniform biaxial stresses, it is required that the initial biaxial stresses be no greater than one-half the elastic limit stress if plastic flow at the edge of the hole is to be precluded.

(c) The results have not been corrected to nullify the above limitations.

17. Notwithstanding the above limitations, work reported in ref. (c) indicates that for the conditions of measurement used errors of the order of 5 percent may be expected in measuring residual stresses up to 90 percent of the yield strength of the material. Errors for those residual stresses in Table I reported as being equal to or greater than yield strength, may be greater than 5 percent because tangential strains at the edge of the hole would be well into the plastic range.

18. The assumption of a linearly varying stress field for residual stresses near a fillet weld appears to be substantiated by a comparison of measured and calculated radial relaxation strains. This comparison is given on Figs.27 to 36. The curves marked theoretical are a plot of equation 2, paragraph 8, which represents a linearly varying stress field. The values of A,B,k and c used in equation 2 were determined from the experimental data using the procedure of paragraph 11b. The experimental data shown as open circles in Figs. 27 to 34 fall very close to the corresponding theoretical curves. The experimental data for the mechanically-peened fillets Figs. 35 and 36 do not show the same close agreement because the surface residual stresses were changed from tensile to compressive by the peening operation. In addition, mechanical peening which is a manual process would tend to produce non-uniform compressive residual stresses. The approximate linearly varying compressive residual stress field arises from a combination of high residual compressive stresses in the peened area and the lower residual compressive stresses already existing in the adjacent base plate surface prior to welding.

19. The close agreement between the experimental data and theoretical curves, Figs. 27 to 36, tends to confirm the assumption that residual stresses at the base of tee fillet welds vary linearly with distance from the weld. In addition, this close agreement and symmetry of experimental data with respect to $\theta = 90$ degrees confirms the assumption that the surface principal residual stresses are normal and parallel to the fillet weld.

20. Referring again to Table I, it is seen that residual stresses determined on the basis of a linearly varying stress field are significantly higher than those determined on the basis of a uniform stress field. This is particularly true for residual stresses normal to the weld, because gage b in arrangement 1 is located at a point where relaxation strain is low. This low value, which is even lower than the average, is used to represent the average strain normal to the weld in the uniform stress field.

21. The results of Table I for arrangement 2 indicate that surface residual stresses at the toe of as welded tee fillet welds in 1 1/2 inch thick HY-80 steel plate are at least equal to the yield strength of the steel. The lowest residual stress shown is 88,800 psi parallel to the weld and the highest shown is 124,000 psi normal to the weld. Obviously, since the ultimate strength of the HY-80 steel is 105,000 psi these high values of residual stress indicate large strains associated with plastic behavior were developed during the drilling process.

22. The results of Table I also show that:

a. Grinding the fillet weld to contour reduced residual stresses moderately. The values of 85,900 and 37,200 psi for σ_1 appear far apart. This difference may be due to difficulty in locating the critical section after grinding for accurate placement of strain gages.

b. Shot peening reduced residual stresses significantly. Although this process is intended to put surface metal in compression, the results indicate tensile residual stresses. It appears that high tensile residual stresses immediately below the surface were effective in controlling the final state of stress at the surface.

c. Shot peening after grinding appears to have had about the same net effect on the magnitude of residual stresses as shot peening alone. It had been expected that the combination would result in a greater reduction. However, various factors, such as, actual initial residual stresses, location of ground contour, selection of critical section for strain gages, severity of shot peening, etc, affect the final result.

d. Mechanical peening drastically affected residual stresses by converting high tension at the toe of the fillet weld to high compression of approximately the same magnitude. The extensive plastic deformation associated with mechanical peening is the sole cause for the large reversal in residual stresses.

CONCLUSIONS

23. An approximate theory has been developed for the determination of surface residual stresses in a linearly varying stress field by the hole-drilling method. This theory is given in Appendix A

24. The above approximate theory is particularly adaptable to the determination of residual stresses at the toe of tee fillet welds and has been found practicable by actual application to tee fillet welds in the as welded, ground, shot-peened, ground and shot-peened, and mechanically-peened conditions.

25. The theory is also applicable to uniform stress fields and therefore may be universally applied. At least four strain gages are required if the principal stress directions are known. Five gages are required if they are not known.

26. Experimental data tend to confirm the approximate theory and the assumption of a linearly varying stress field

27. The above results and conclusions are applicable to determination made with a 7/16 in. diameter drilled hole and a 5/8 in. diameter gage circle. Smaller holes with the same gage circle may require calibration

28. Residual stresses at the toe of as welded tee fillet welds in 1 1/2 in. thick HY-80 plate are tensile in both the transverse and longitudinal directions and approach the yield strength of the steel. Results reported in Table I are higher than the yield strength because of the large strains associated with stress concentration in the tangential direction at the edge of the hole and the resulting plastic deformation.

29. Grinding reduces the surface tensile residual stresses in tee fillet welds by roughly 25 percent.

30. Shot peening reduces the surface tensile residual stresses in tee fillet welds by roughly 50 percent.

31. Shot peening after grinding reduces surface tensile residual stresses by roughly 50 percent.

32. Mechanical peening reverses the surface residual stresses in tee fillet welds from tensile to compressive residual stresses of approximately the same magnitude.

FUTURE WORK.

33. Additional methods for measuring residual stresses under shipboard conditions will be studied. These include the use of photo stress methods and portable x-ray diffraction equipment. In addition, the theory for stress distribution around a slit is being studied for possible application to measuring residual stresses at a point.

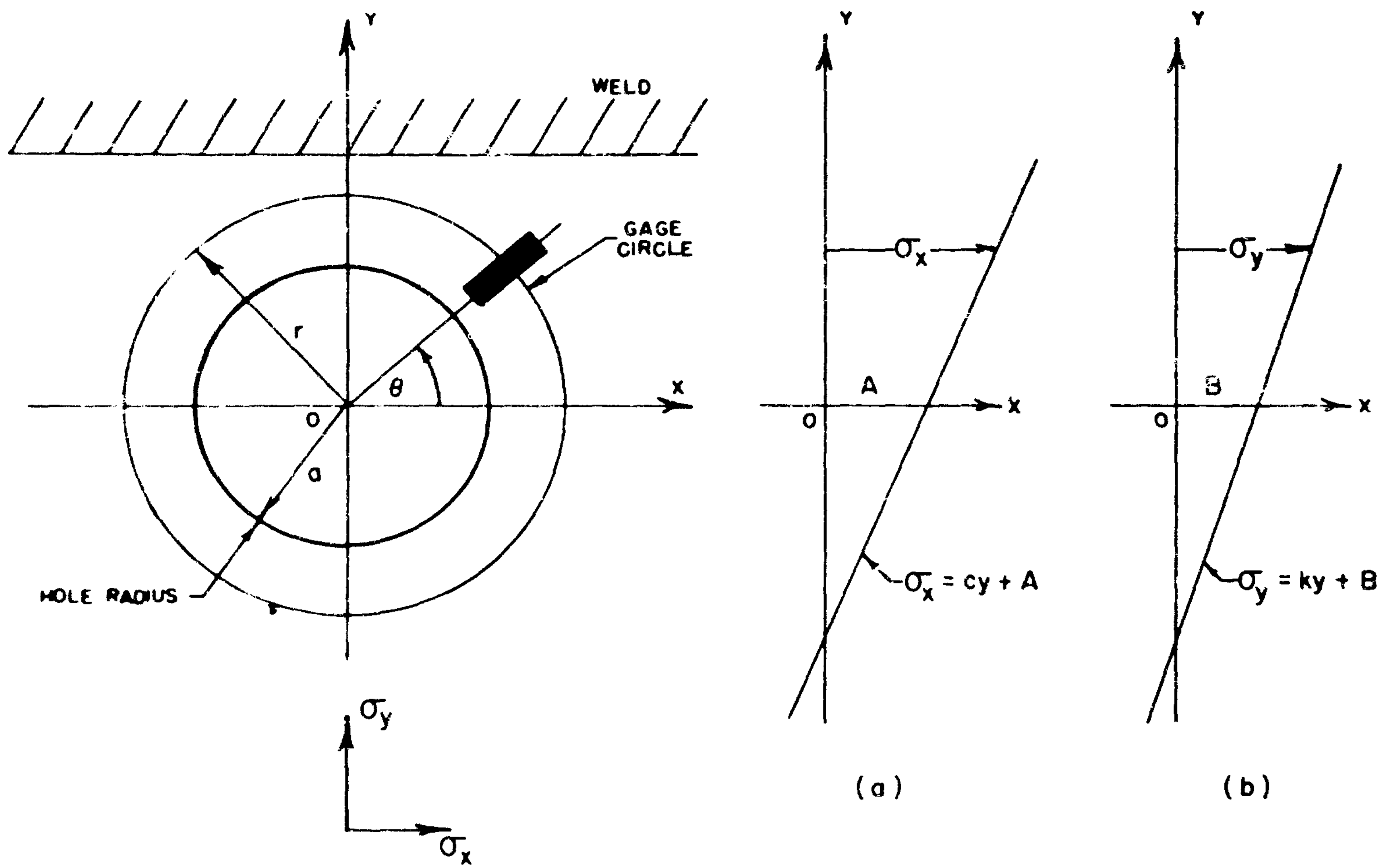

FIGURE I - COORDINATE SYSTEM AND ASSUMED RESIDUAL STRESS DISTRIBUTION

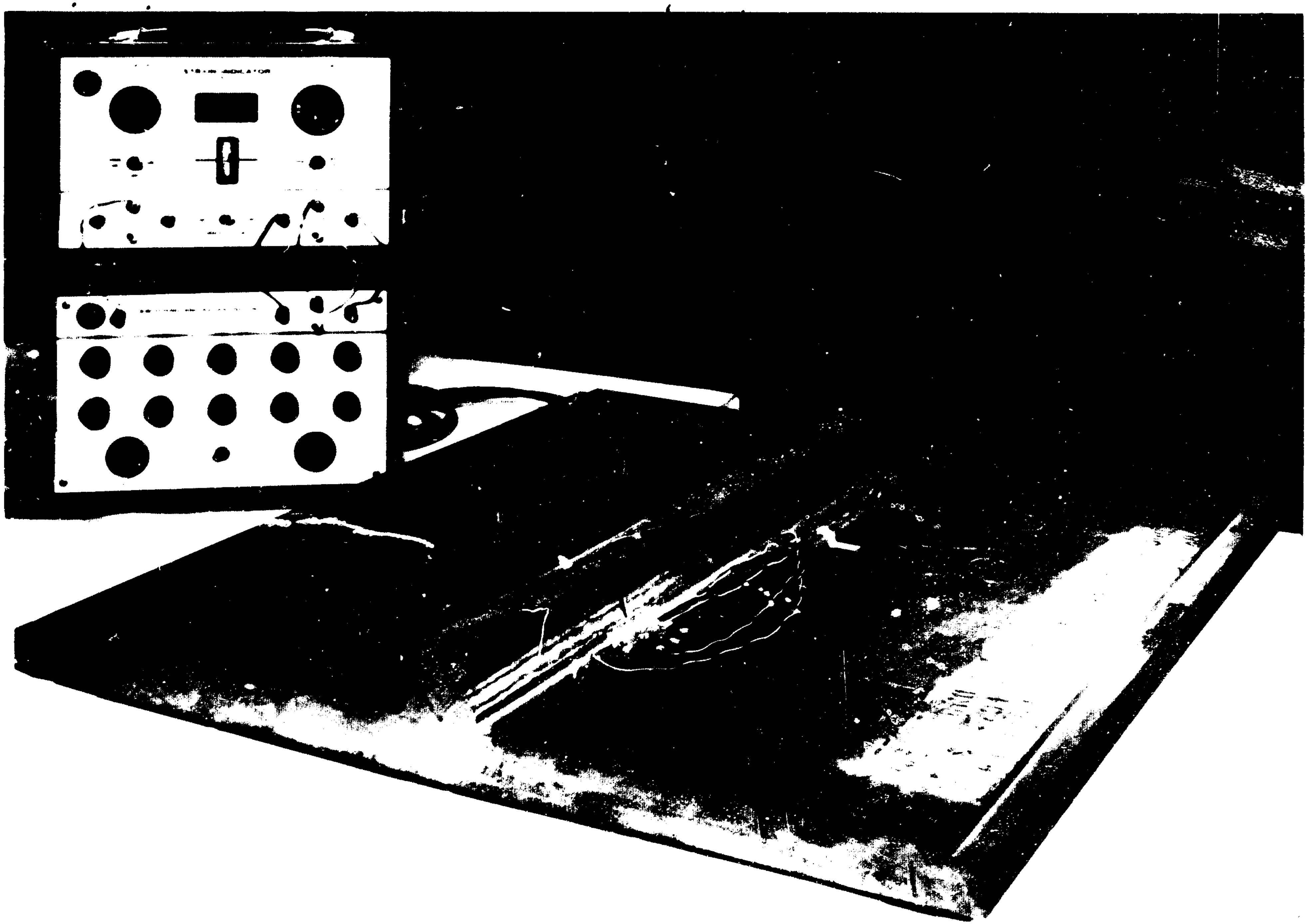

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FIGURE 2 - PLATE TYPE SPECIMEN OF HY-80 STEEL WITH GROUND FILLET WELD AND INSTRUMENTATION FOR MEASURING RESIDUAL STRESSES

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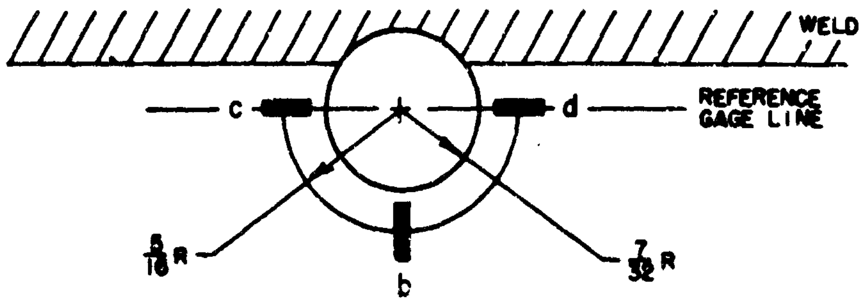
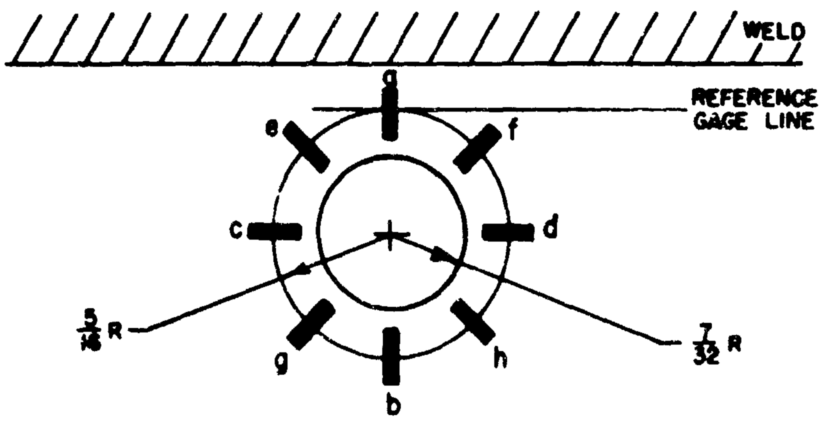
LAB PROJECT 9300-23
PROGRESS REPORT

STRAIN GAGE ARRANGEMENT I
FOR UNIFORM BIAXIAL STRESS FIELD



STRAIN GAGE ARRANGEMENT 2
FOR NON-UNIFORM BIAXIAL STRESS FIELD



GAGES SPACED AT 45° INTERVALS

FIGURE 3-STRAIN GAGE ARRANGEMENTS FOR MEASURING RELAXATION STRAINS AT TEE-FILLET WELDS BY HOLE DRILLING TECHNIQUE

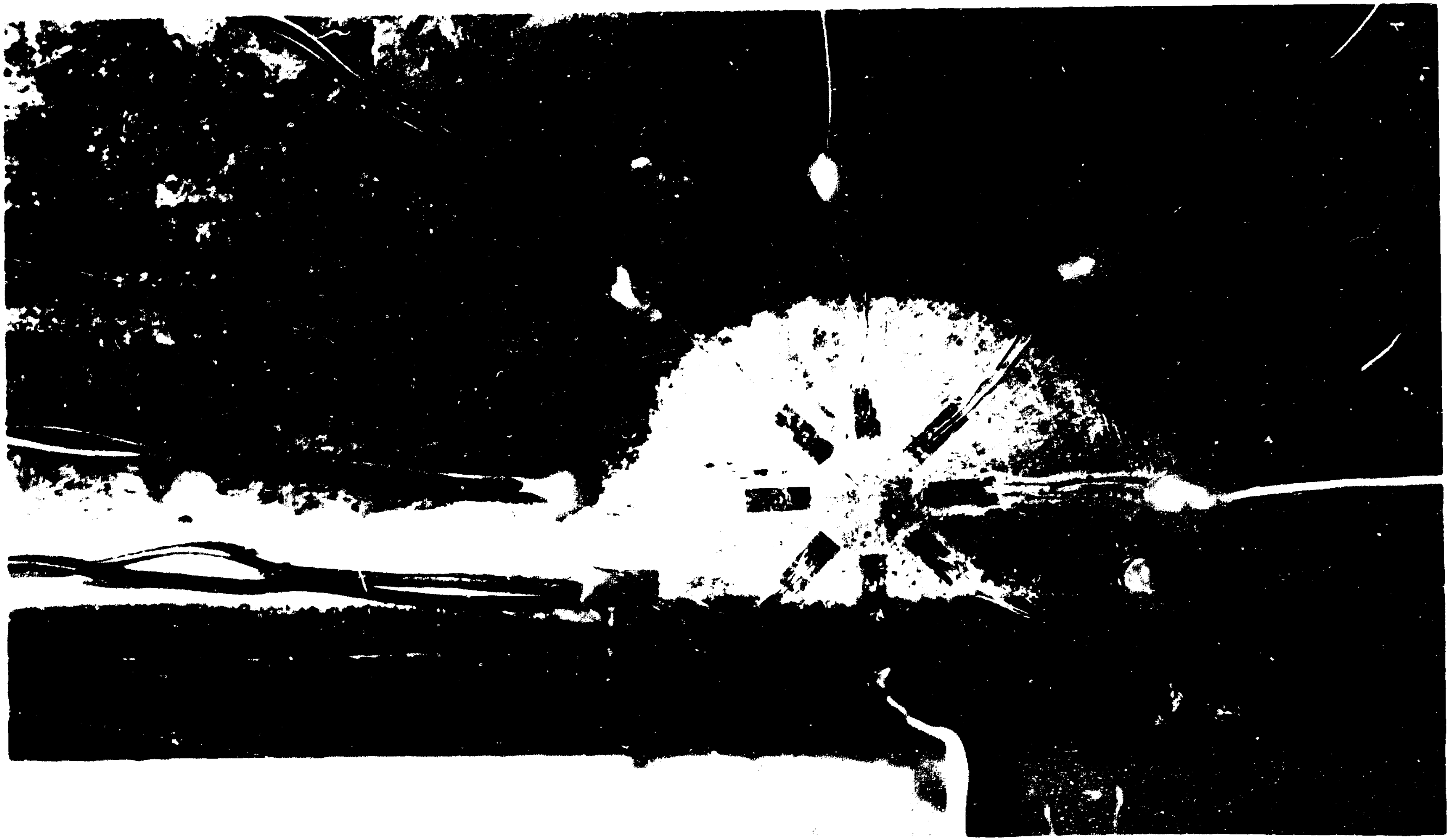


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FIGURE 5 - STRAIN GAGE ARRANGEMENT 2 APPLIED AT A GROUND FILLET WELD

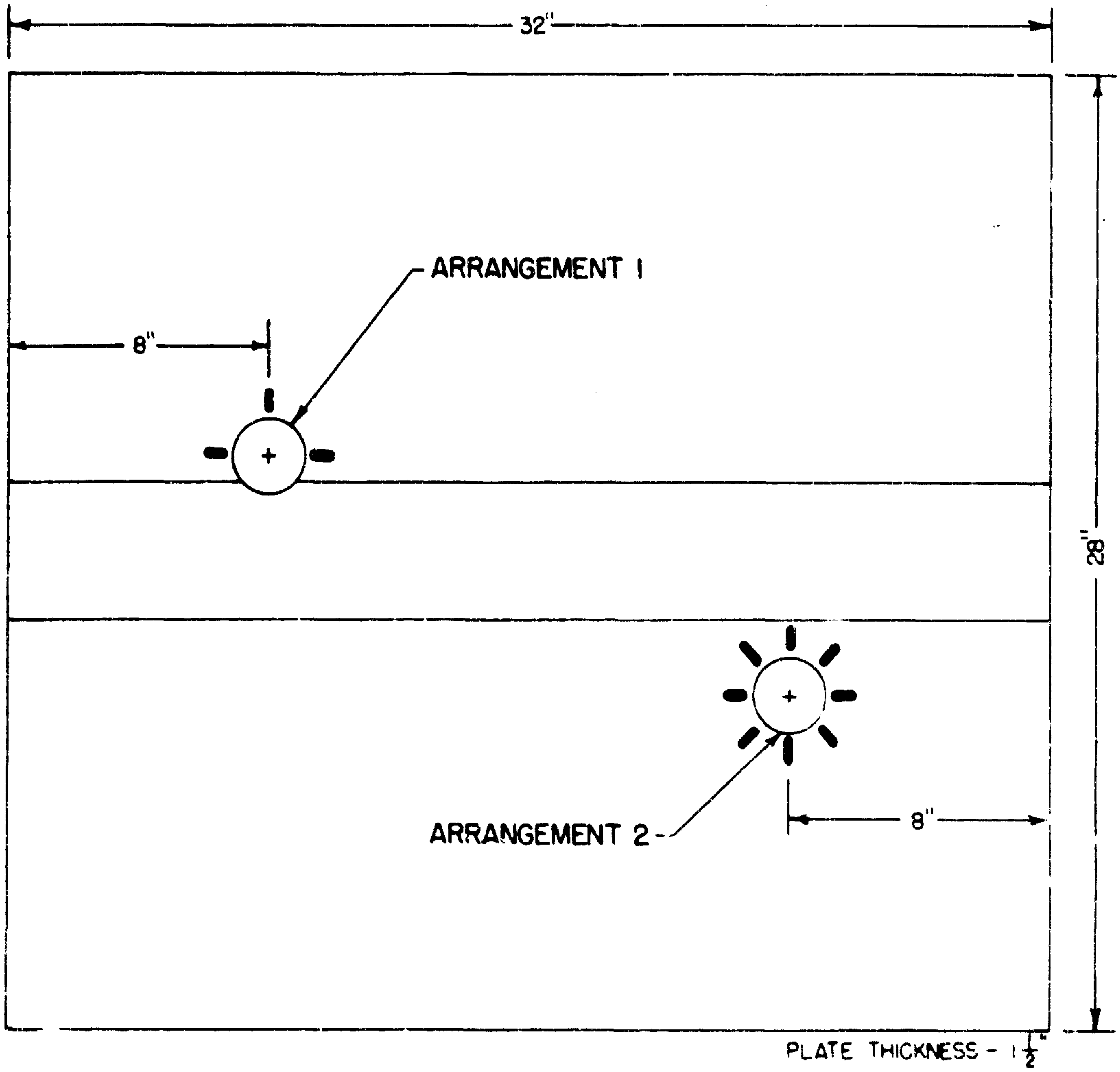

FIGURE 4-STRAIN GAGE ARRANGEMENT AND HOLE LOCATIONS ON HY-80 STEEL TEE-FILLET WELDED PLATE TYPE SPECIMEN

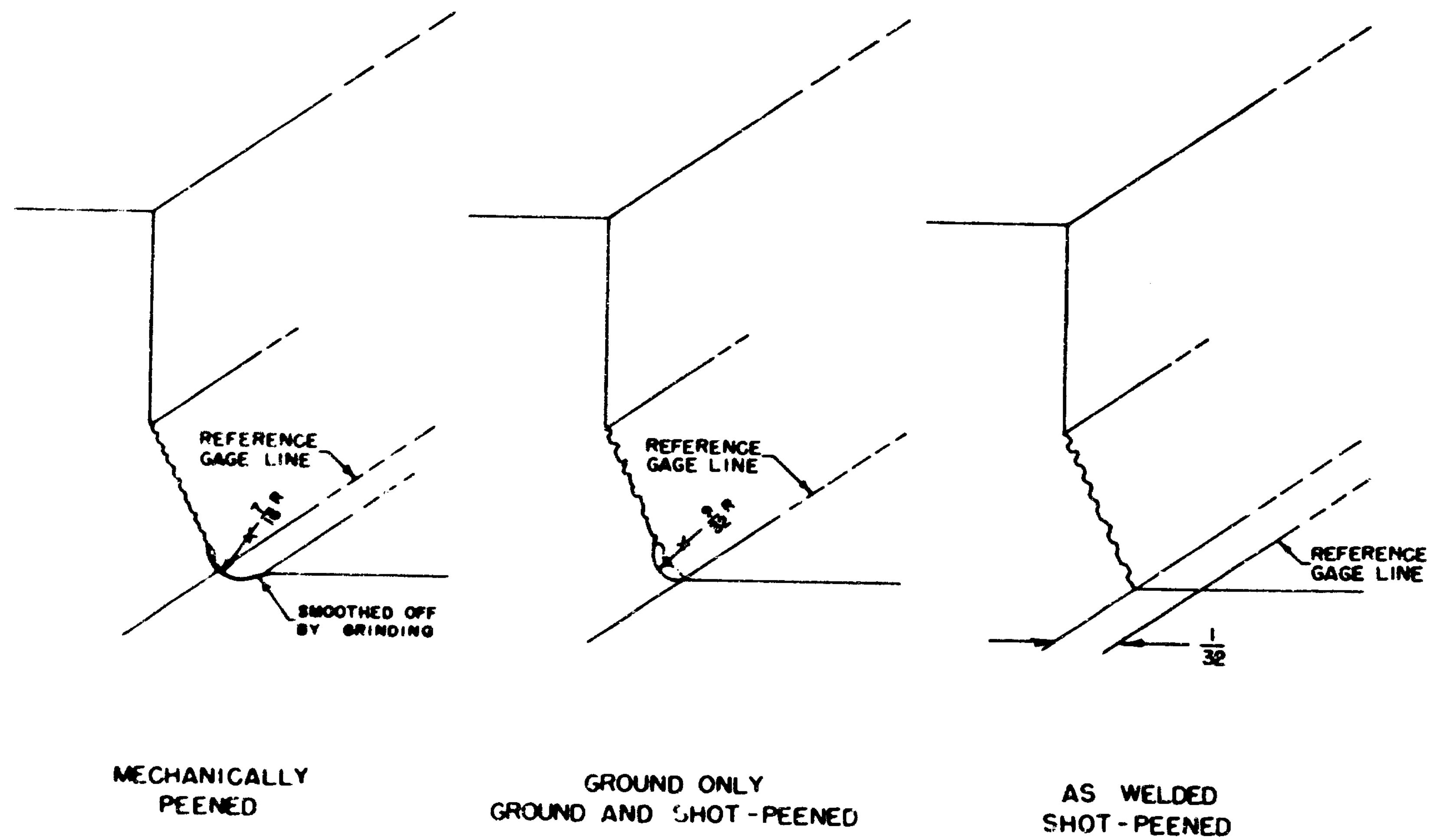

FIGURE 6 - LOCATION OF REFERENCE GAGE LINE FOR GAGES c AND d IN ARRANGEMENT 1 AND GAGE a IN ARRANGEMENT 2

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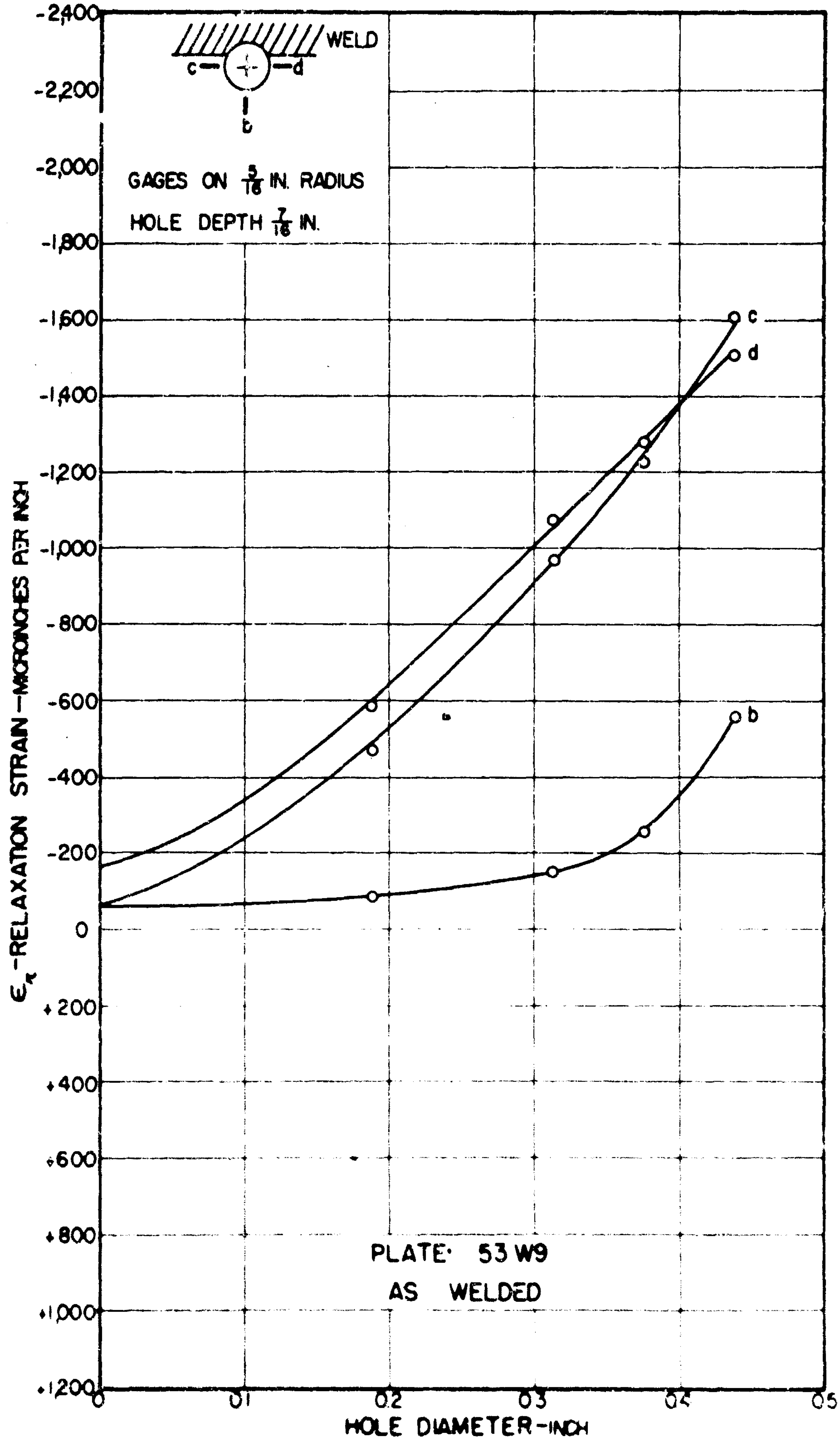

FIGURE 7 - VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

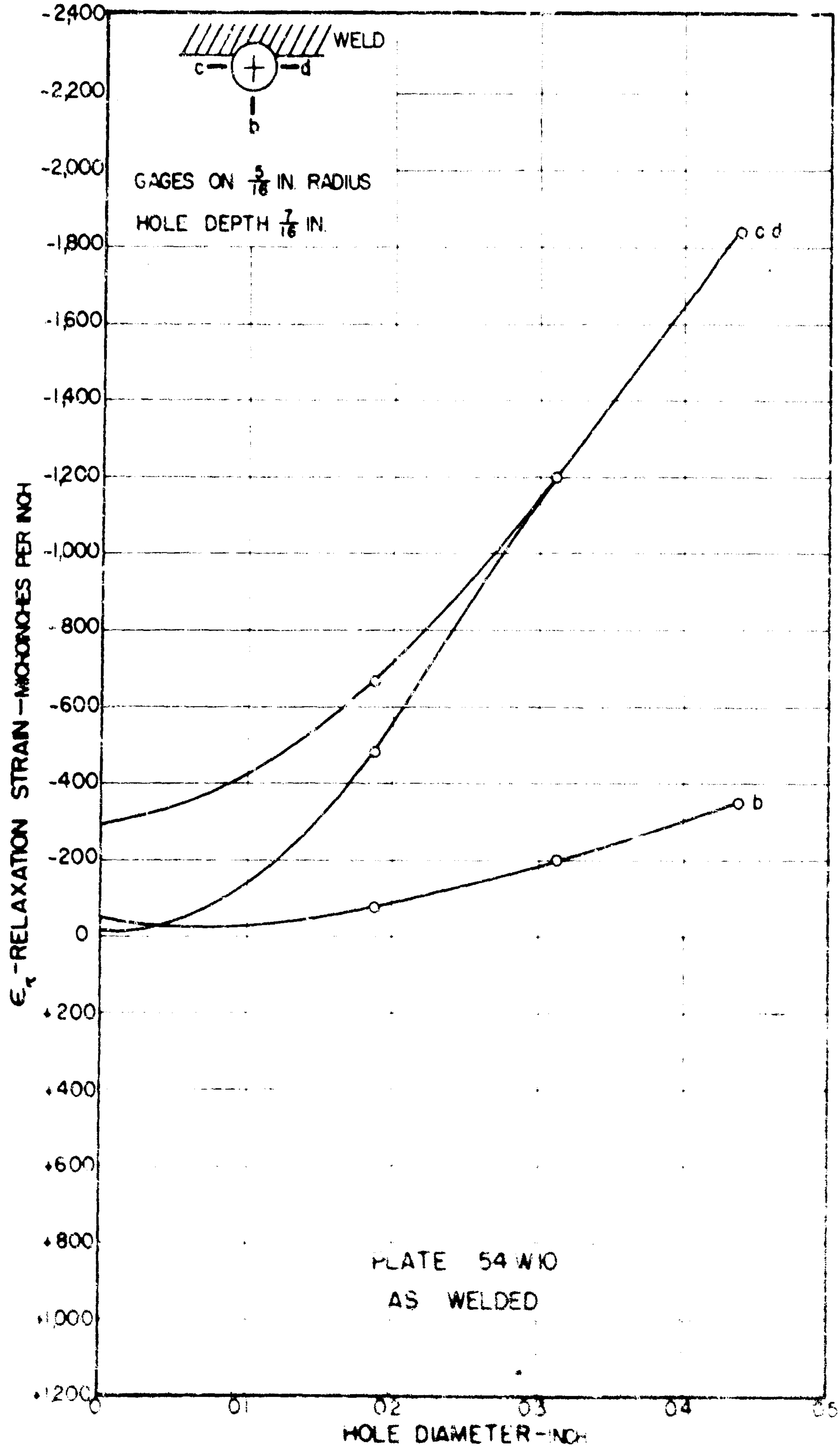

FIGURE 8 - VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

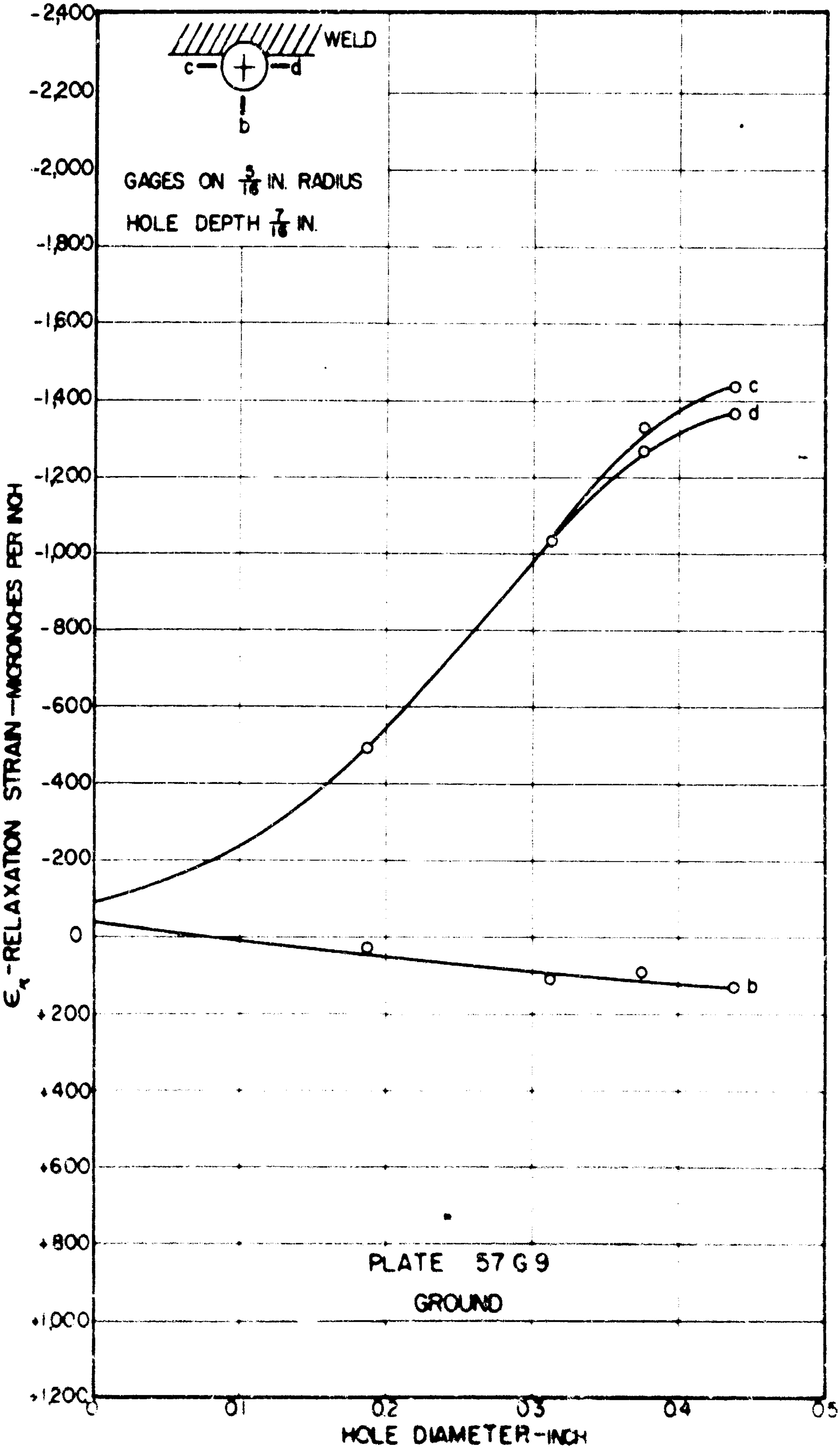

FIGURE 9 - VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

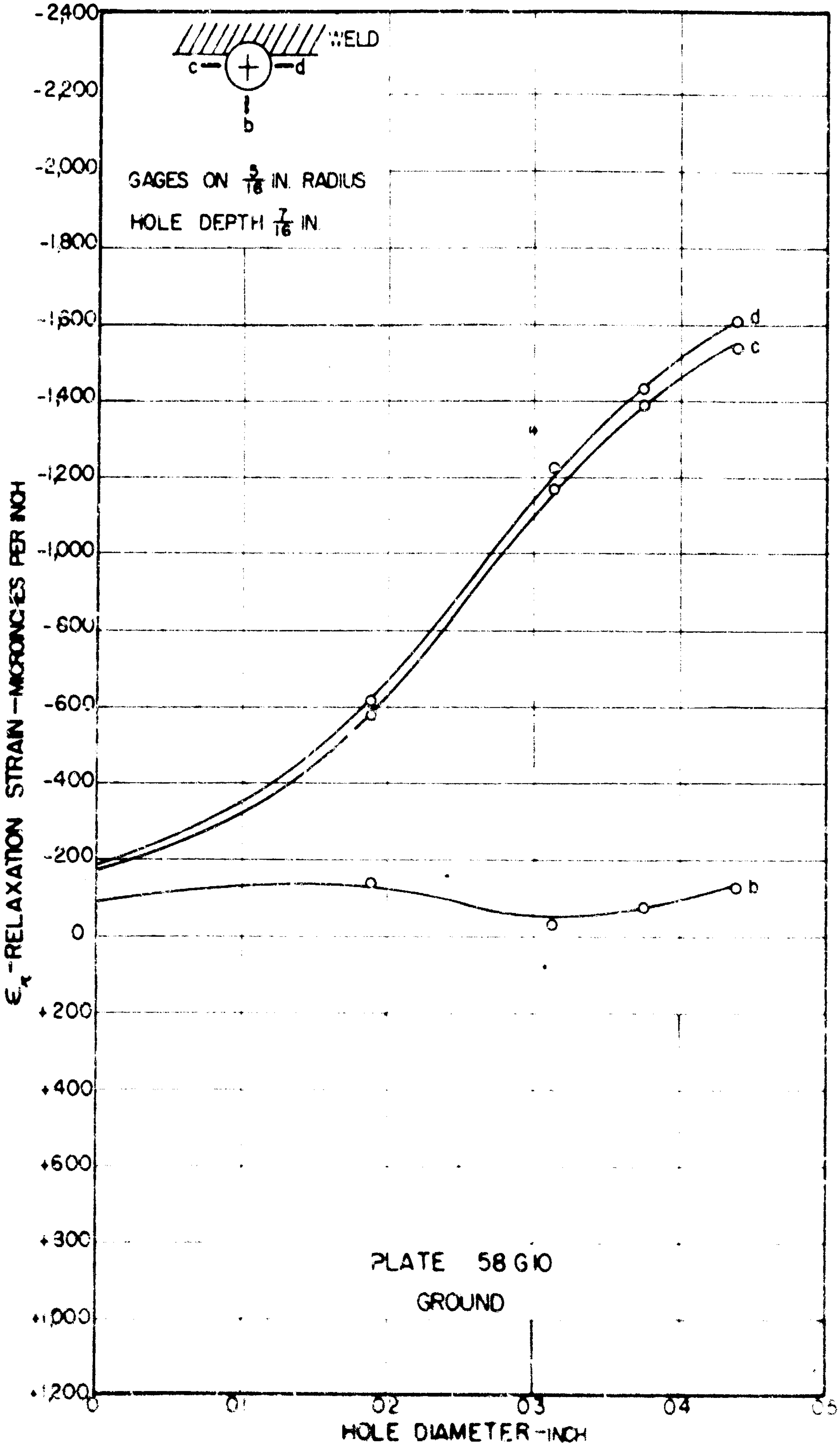

FIGURE 10- VARIATION OF RELAXATION STRAIN
WITH INCREASING HOLE DIAMETER

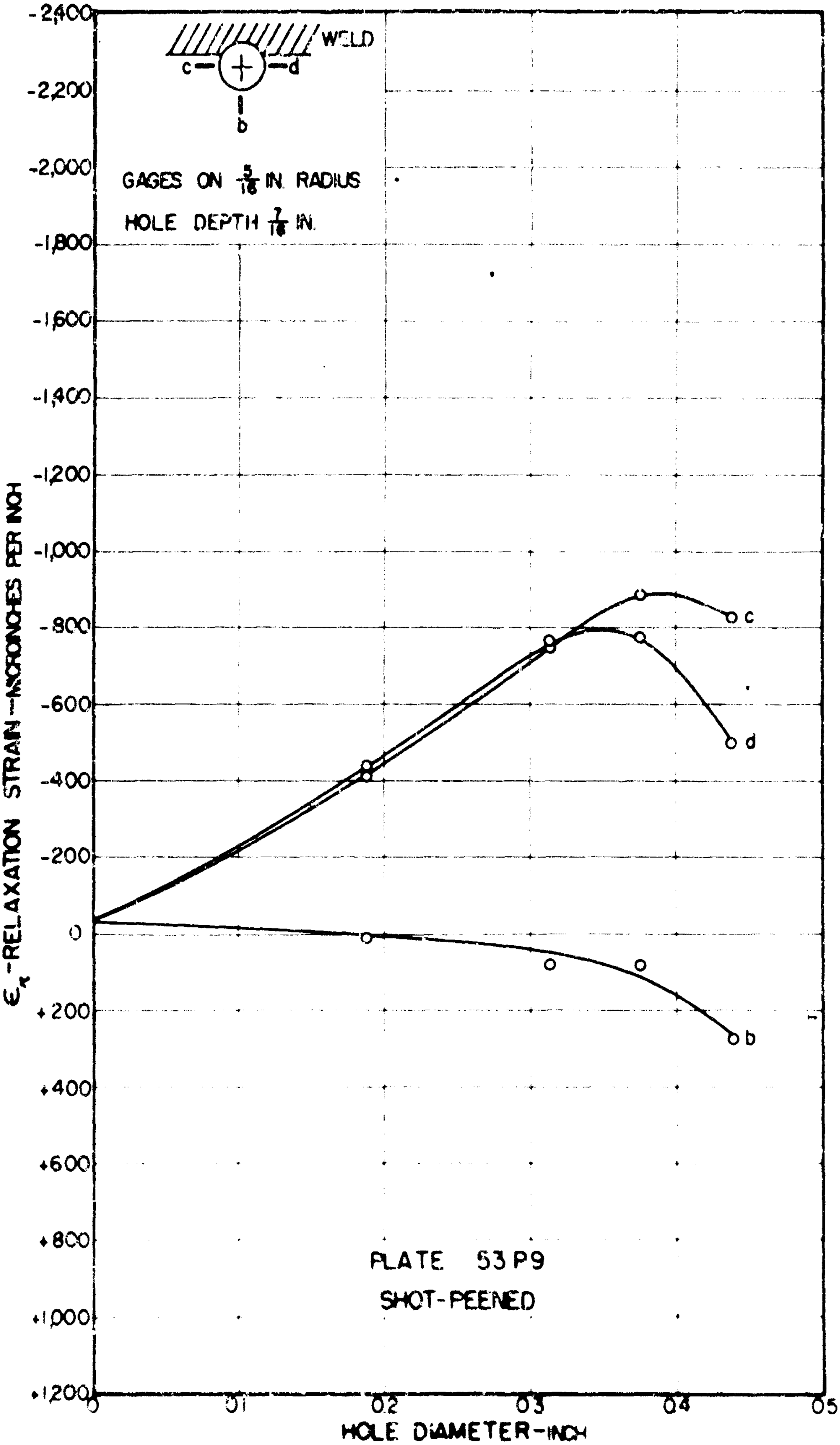

FIGURE 11 - VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

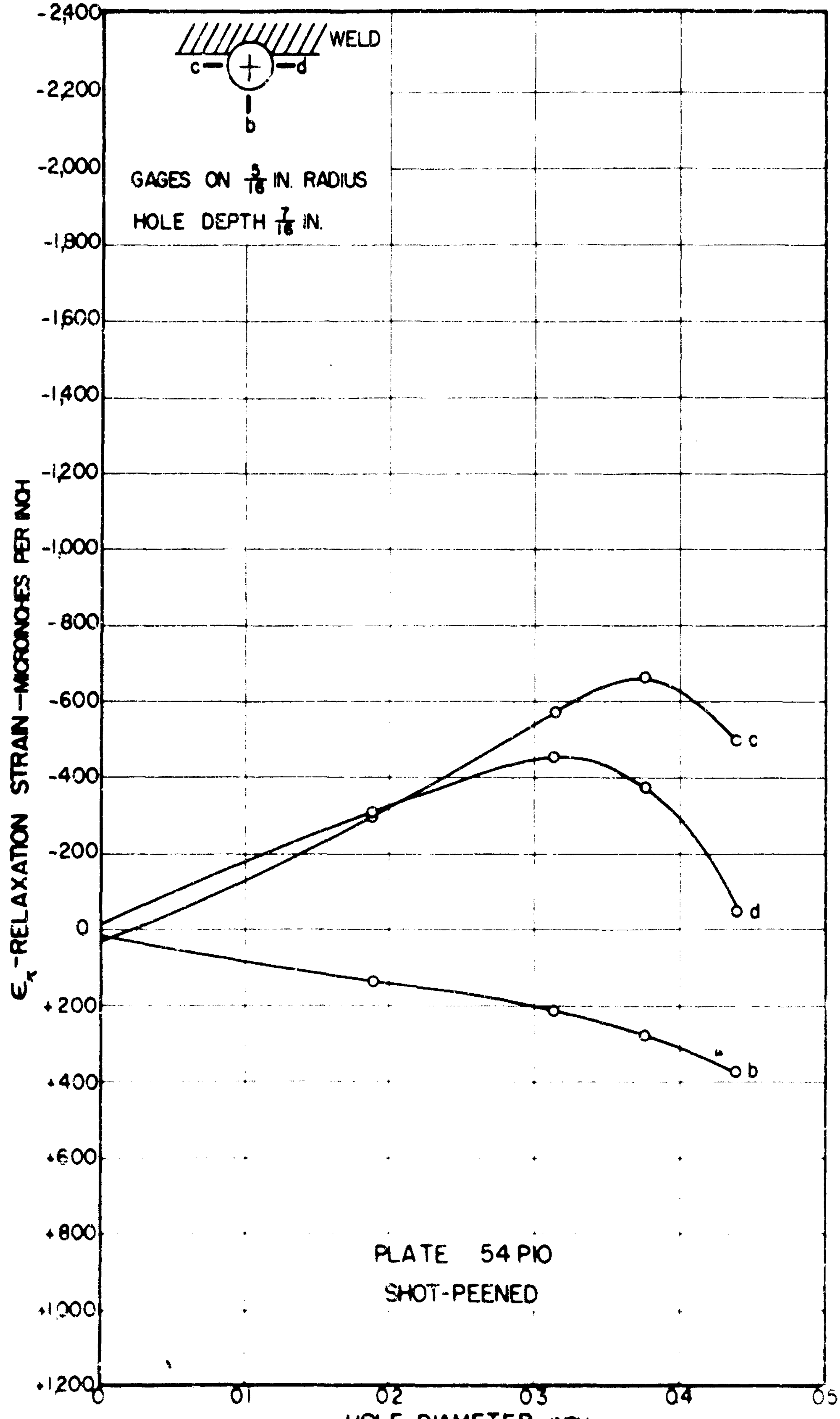

FIGURE 12—VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

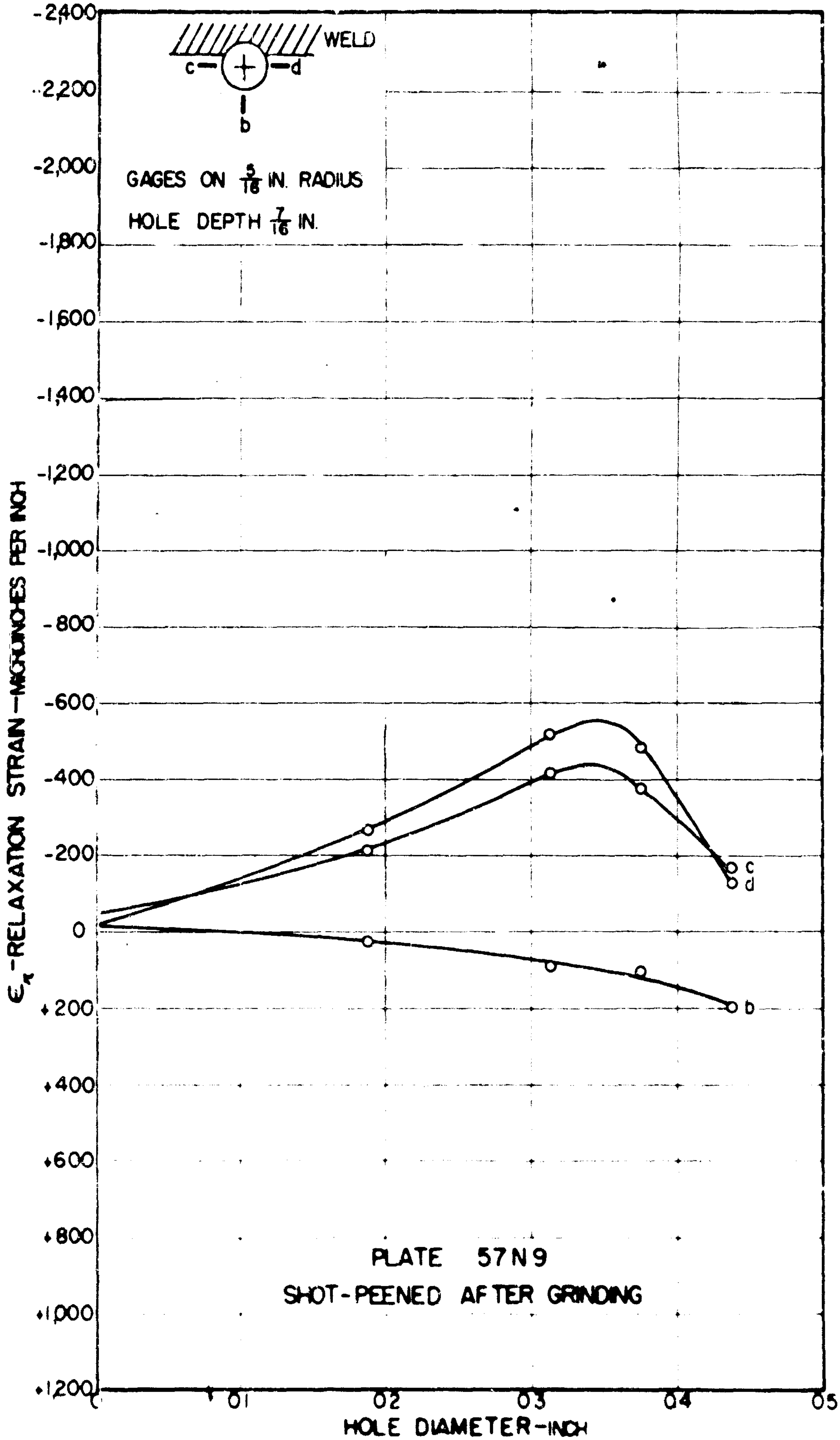

FIGURE 13 - VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

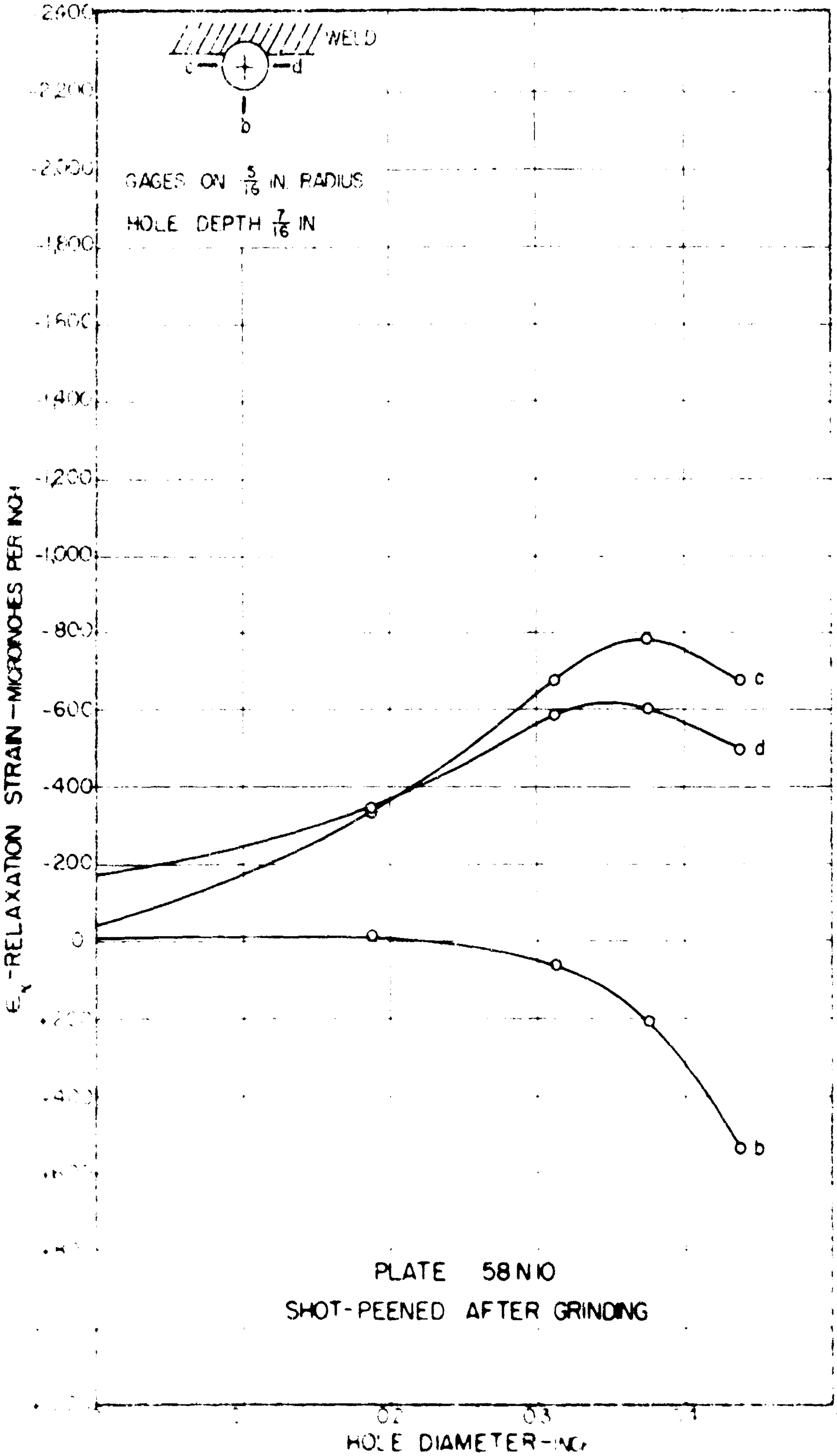

FIGURE 14-- VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

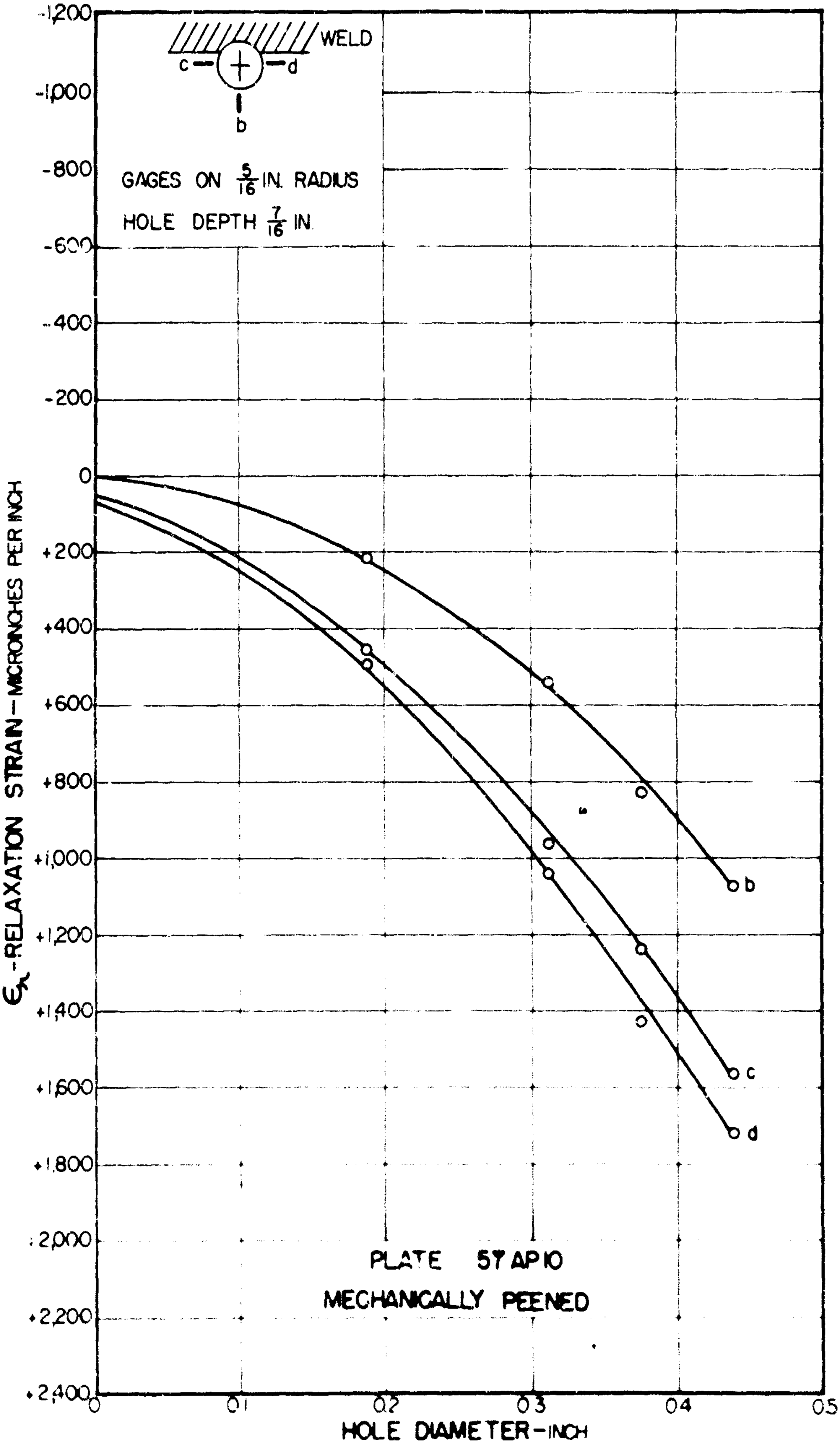

FIGURE 15 - VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

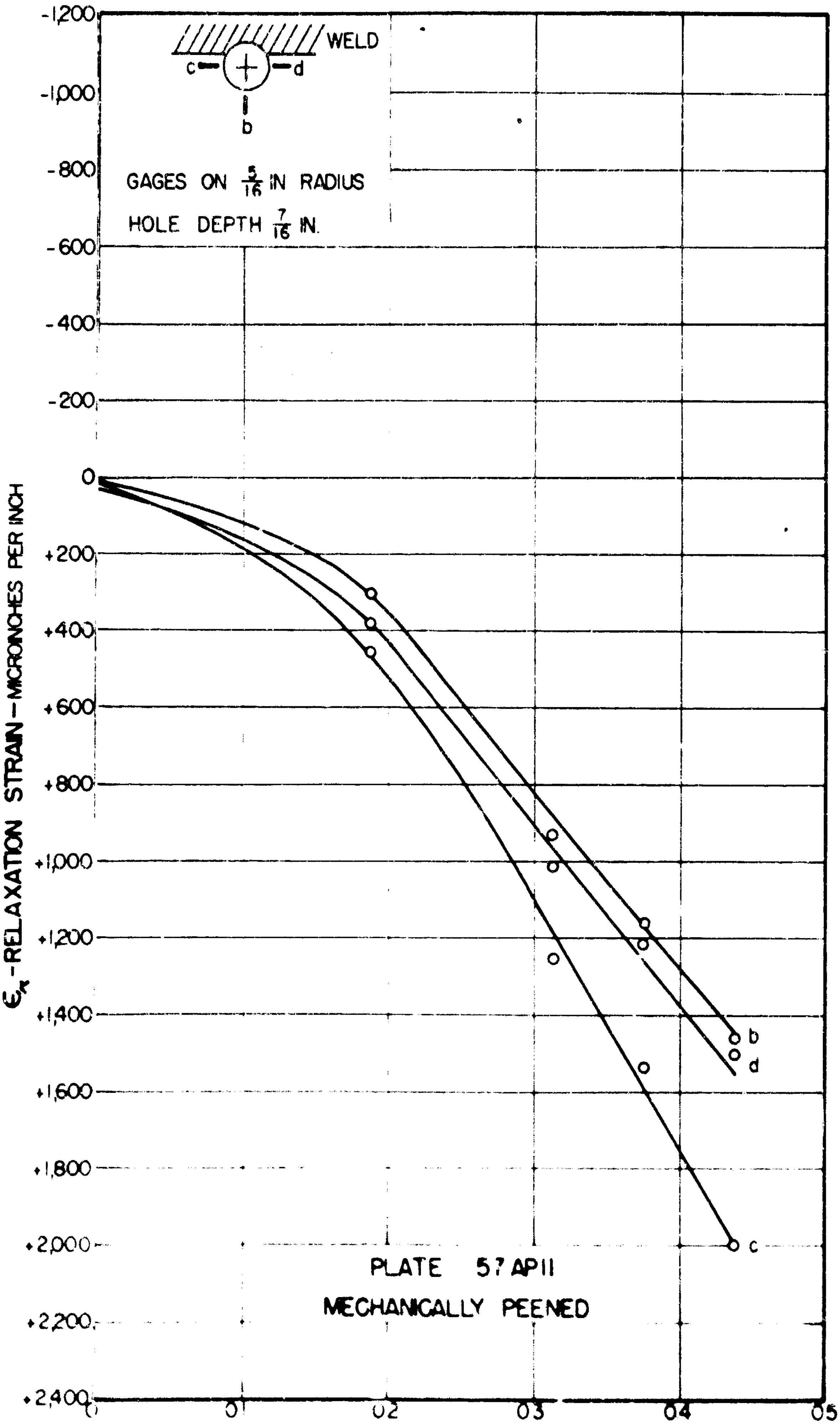

FIGURE 16–VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

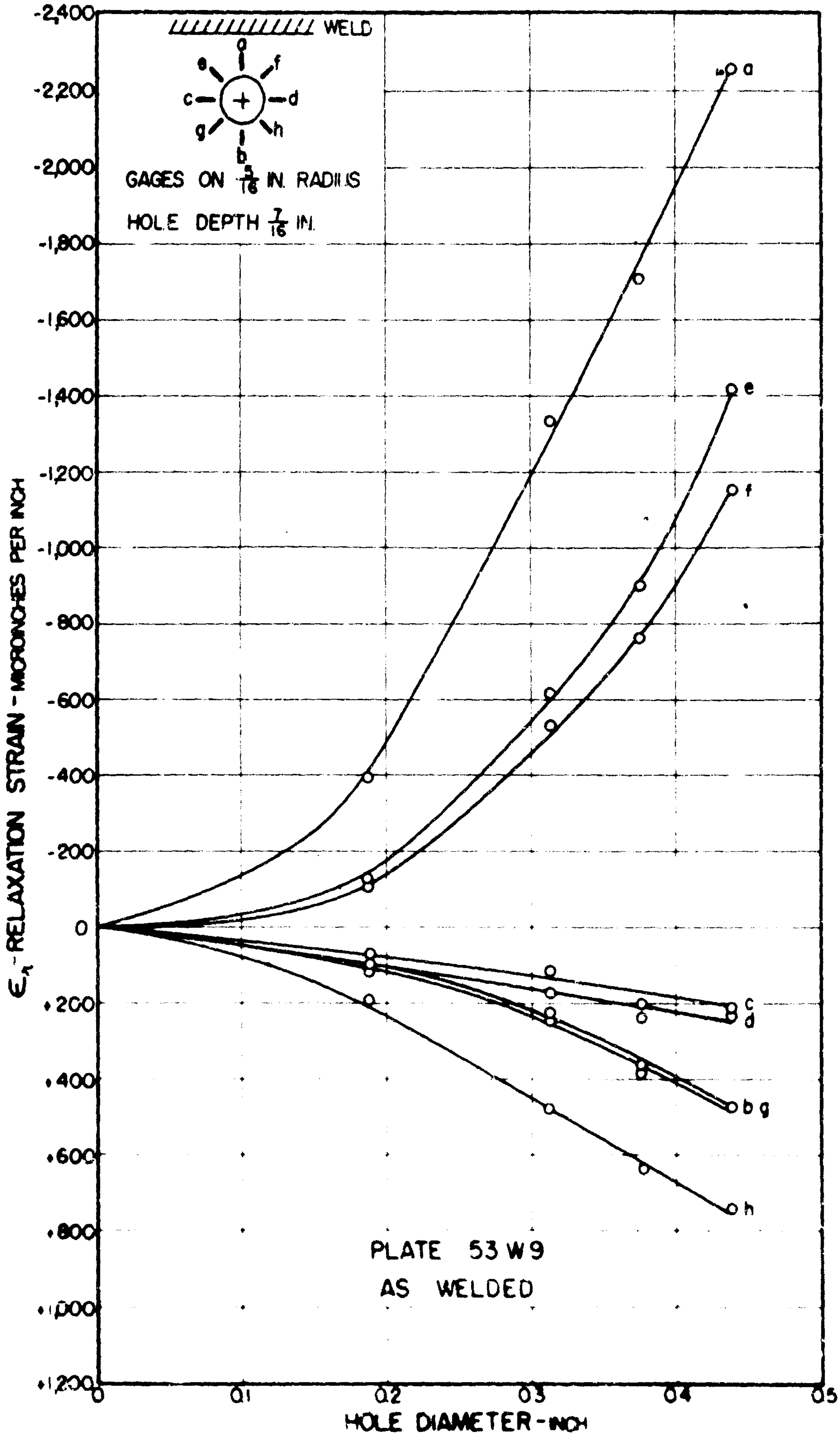

FIGURE 17 - VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

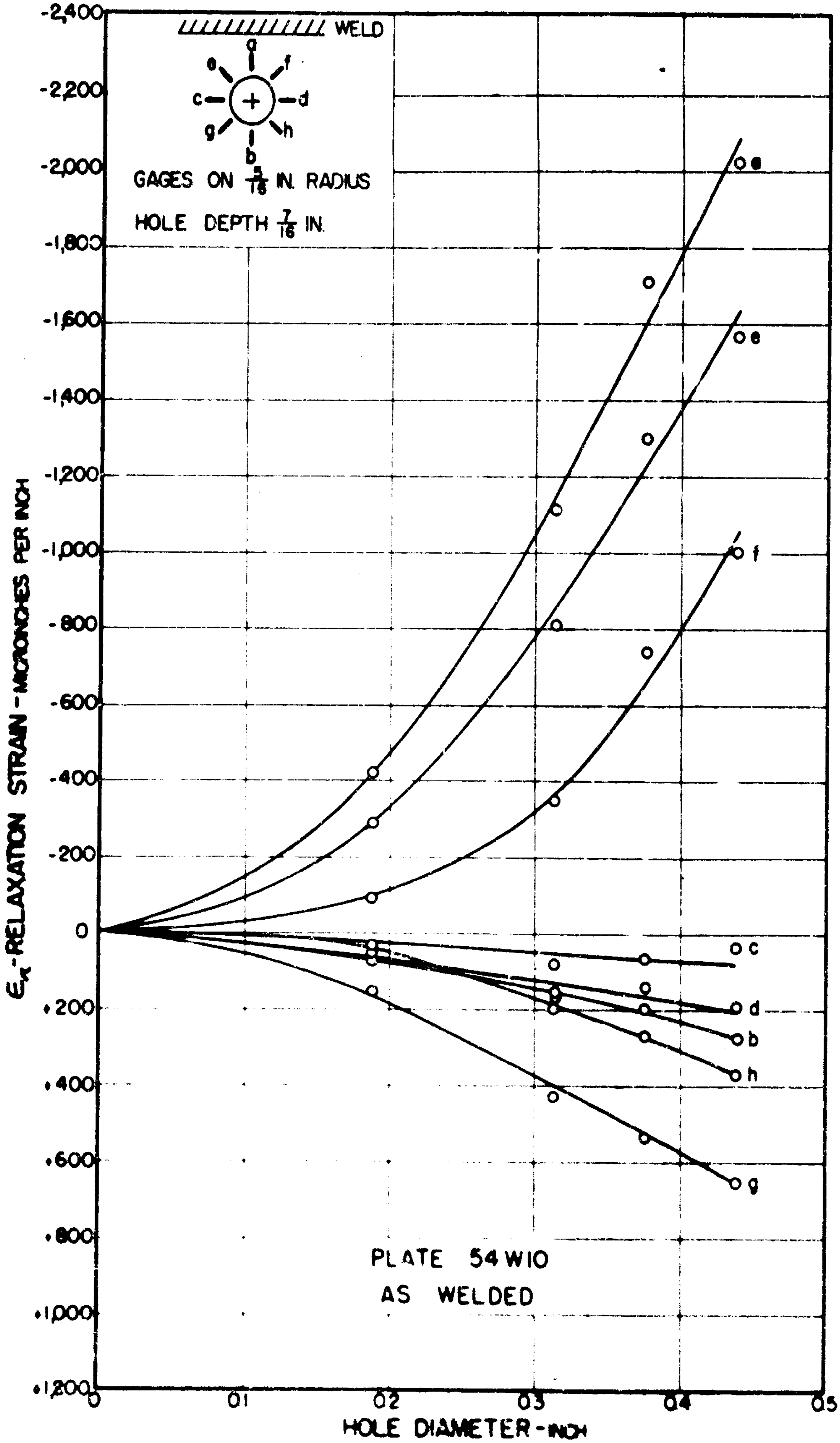

FIGURE 18 - VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

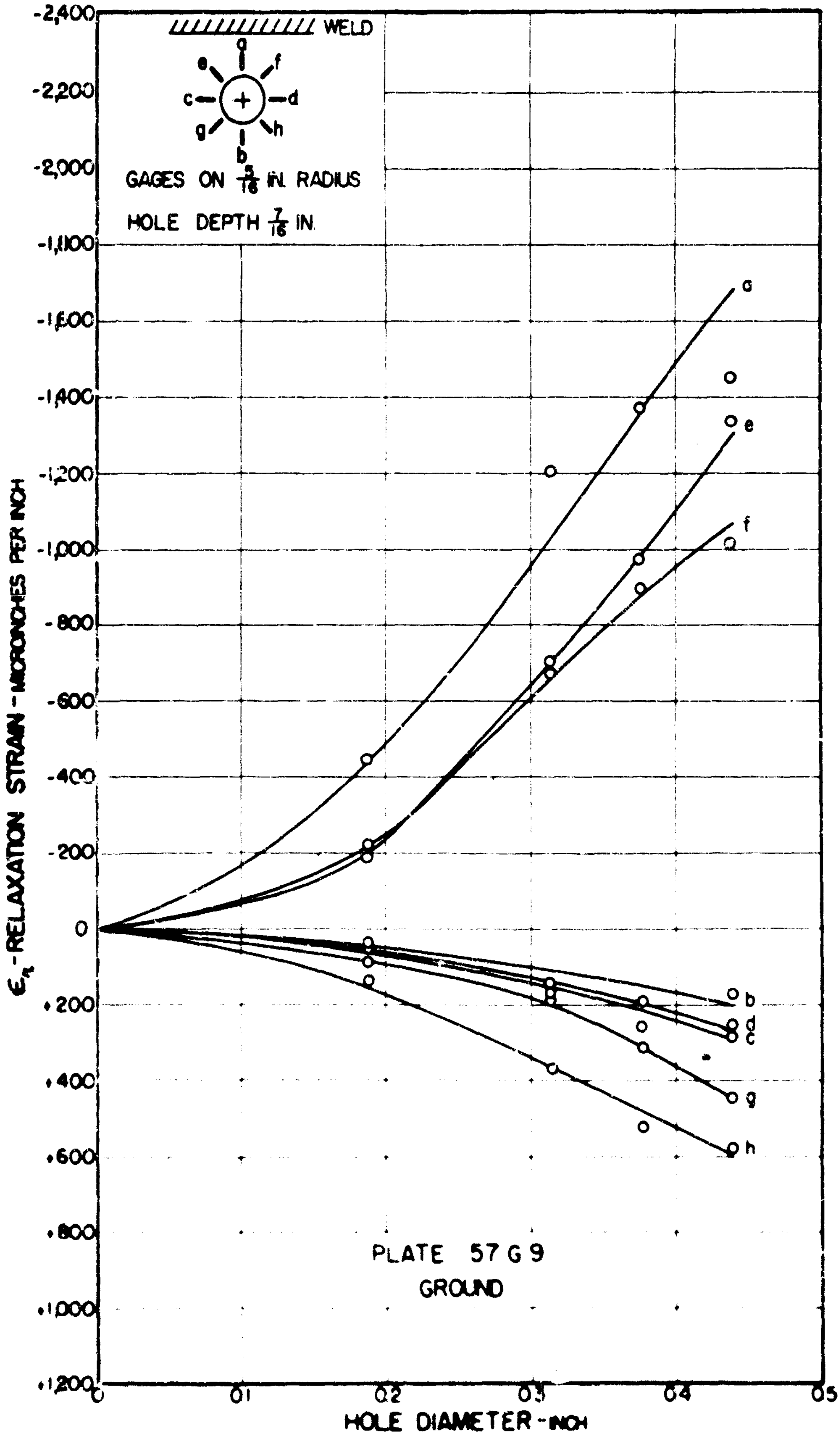

FIGURE 19 - VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

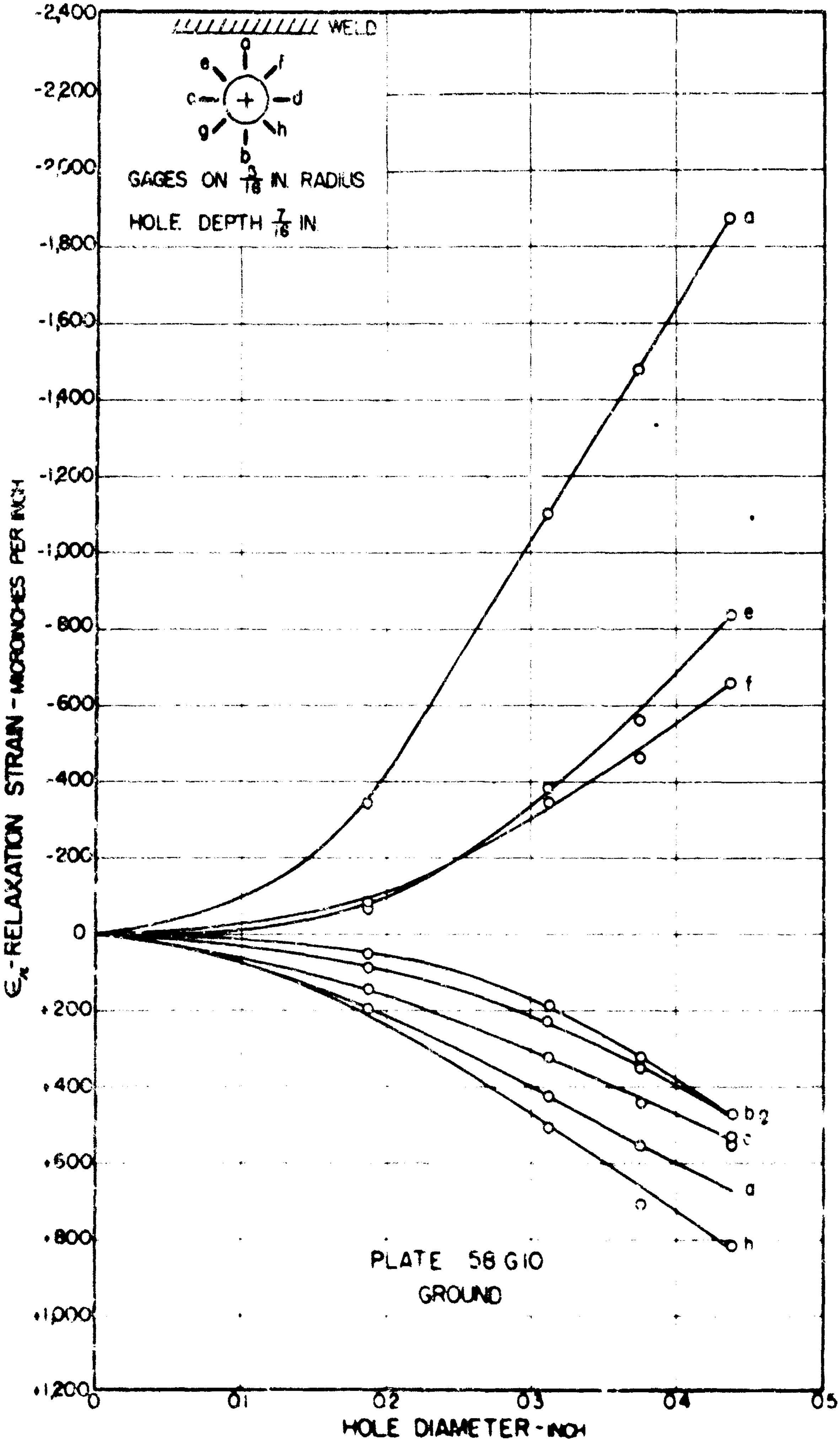

FIGURE 20 VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

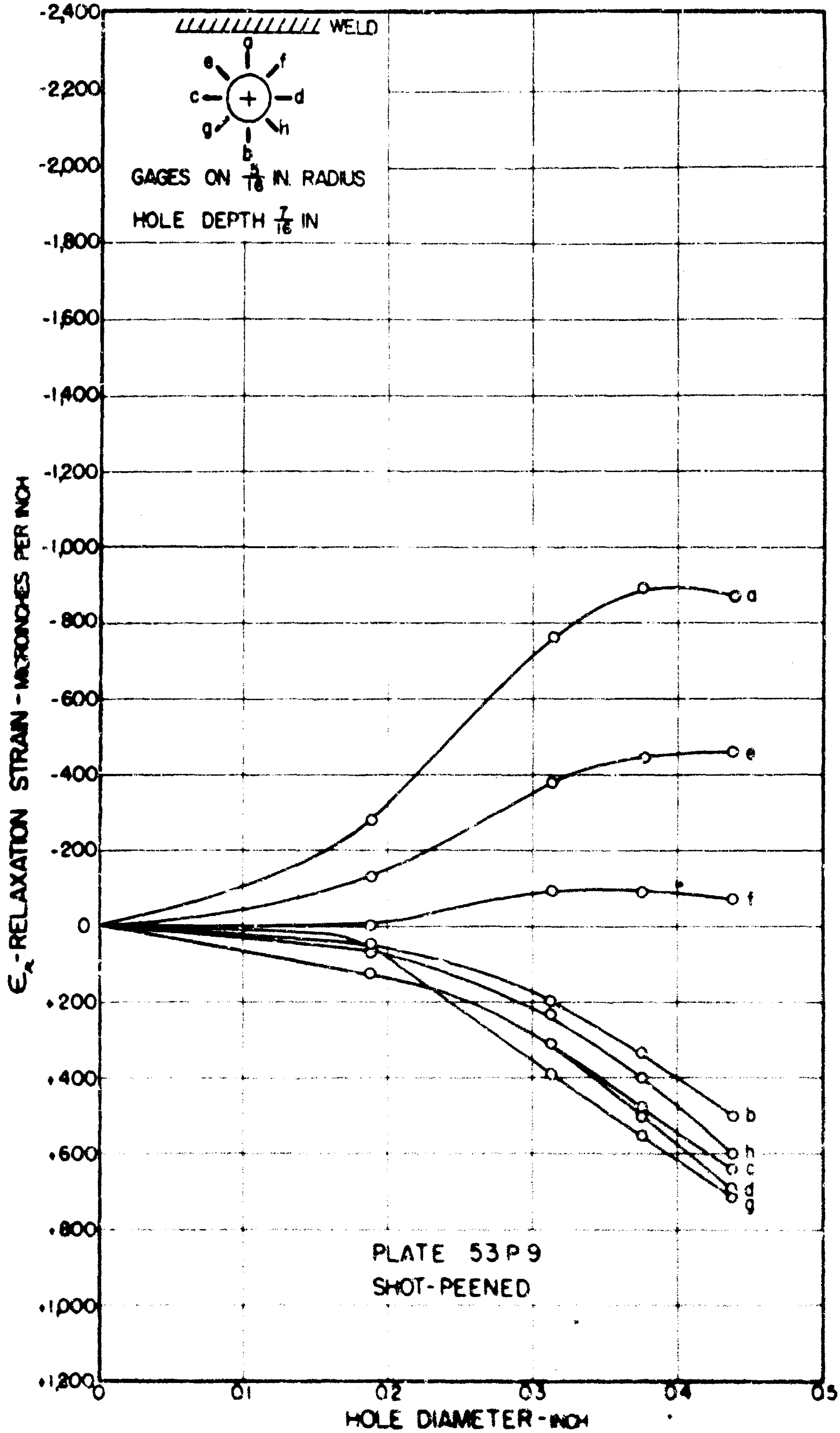

FIGURE 21 - VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

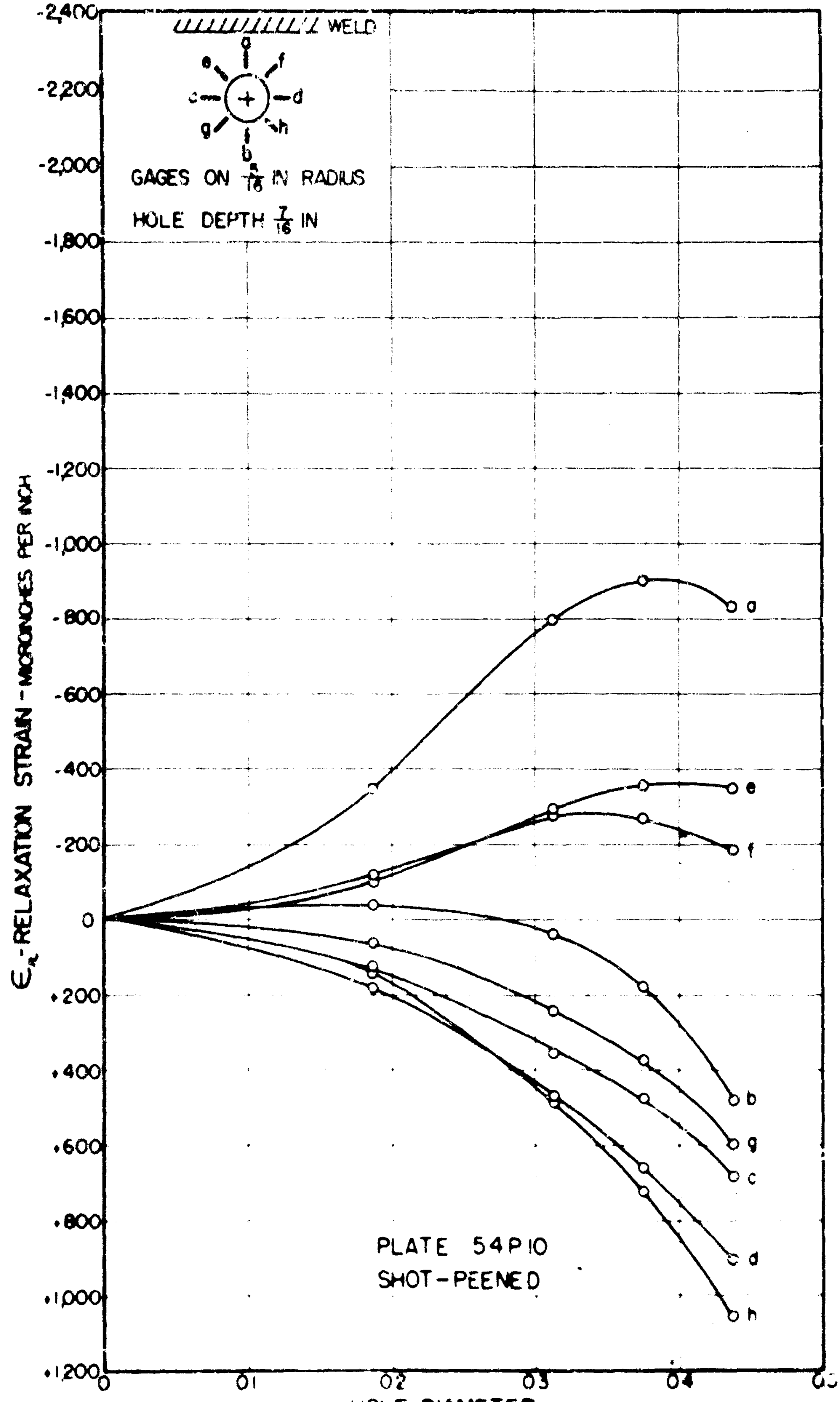

FIGURE 22- VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

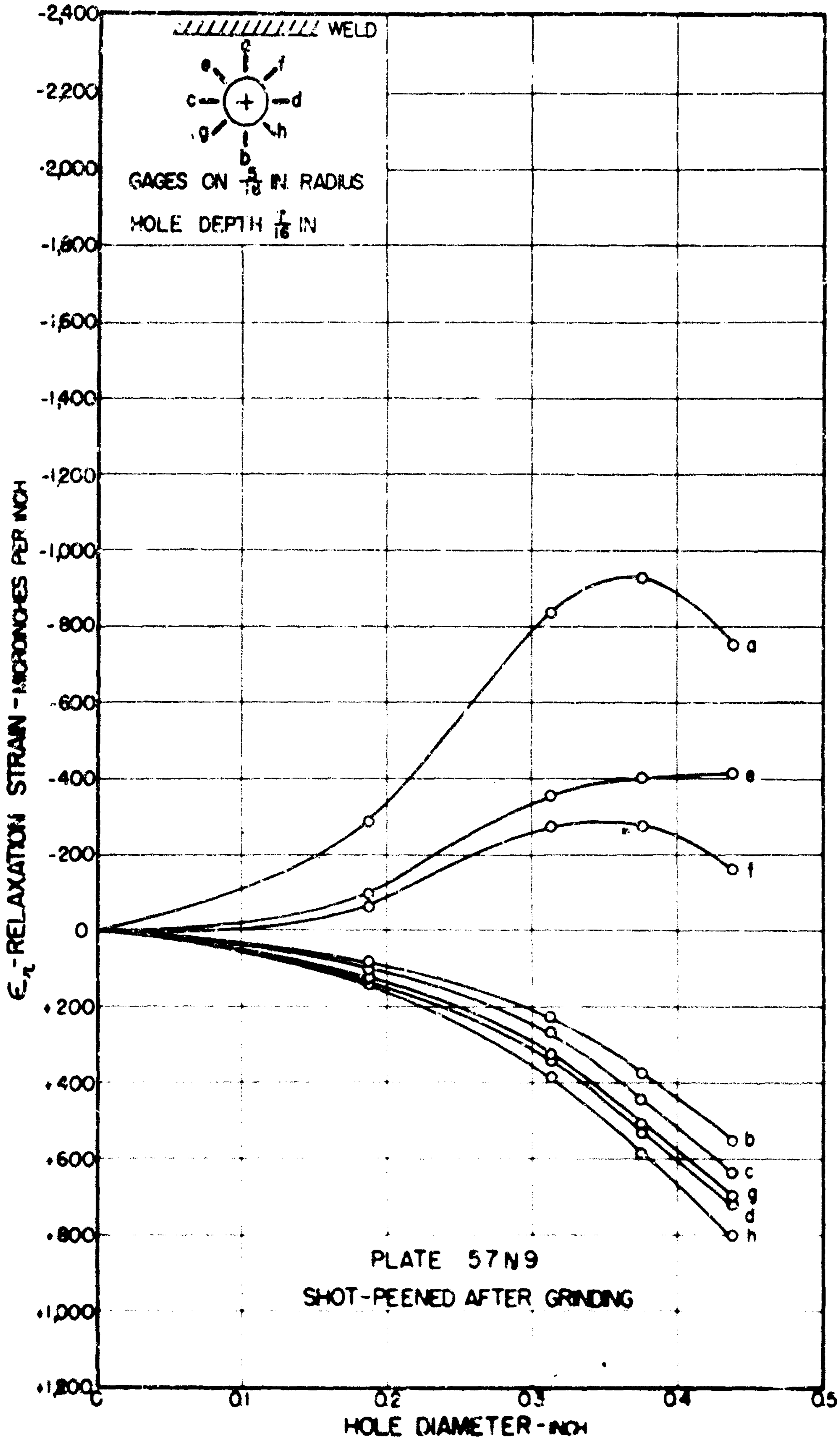

FIGURE 23 - VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

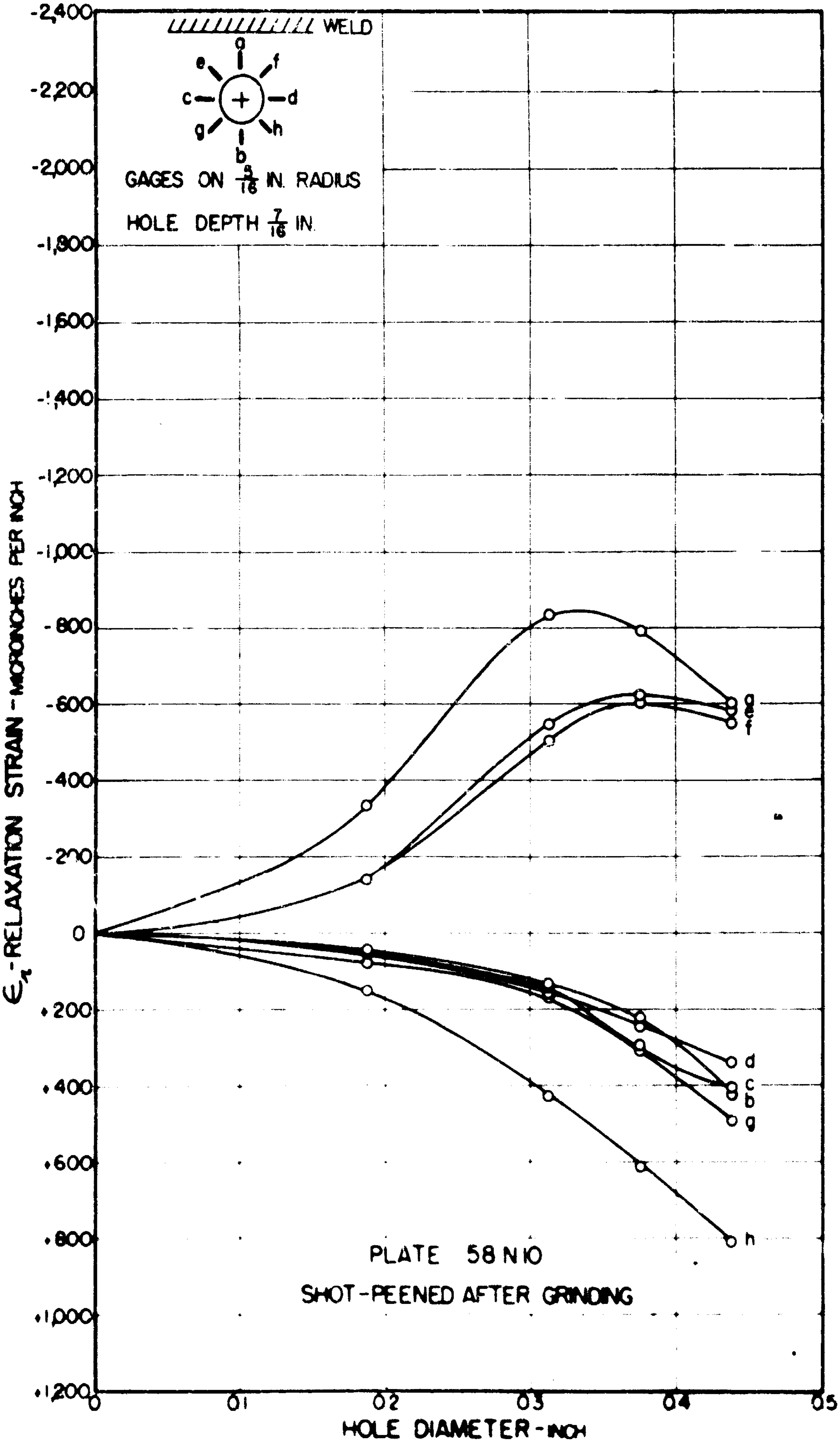

FIGURE 24 - VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

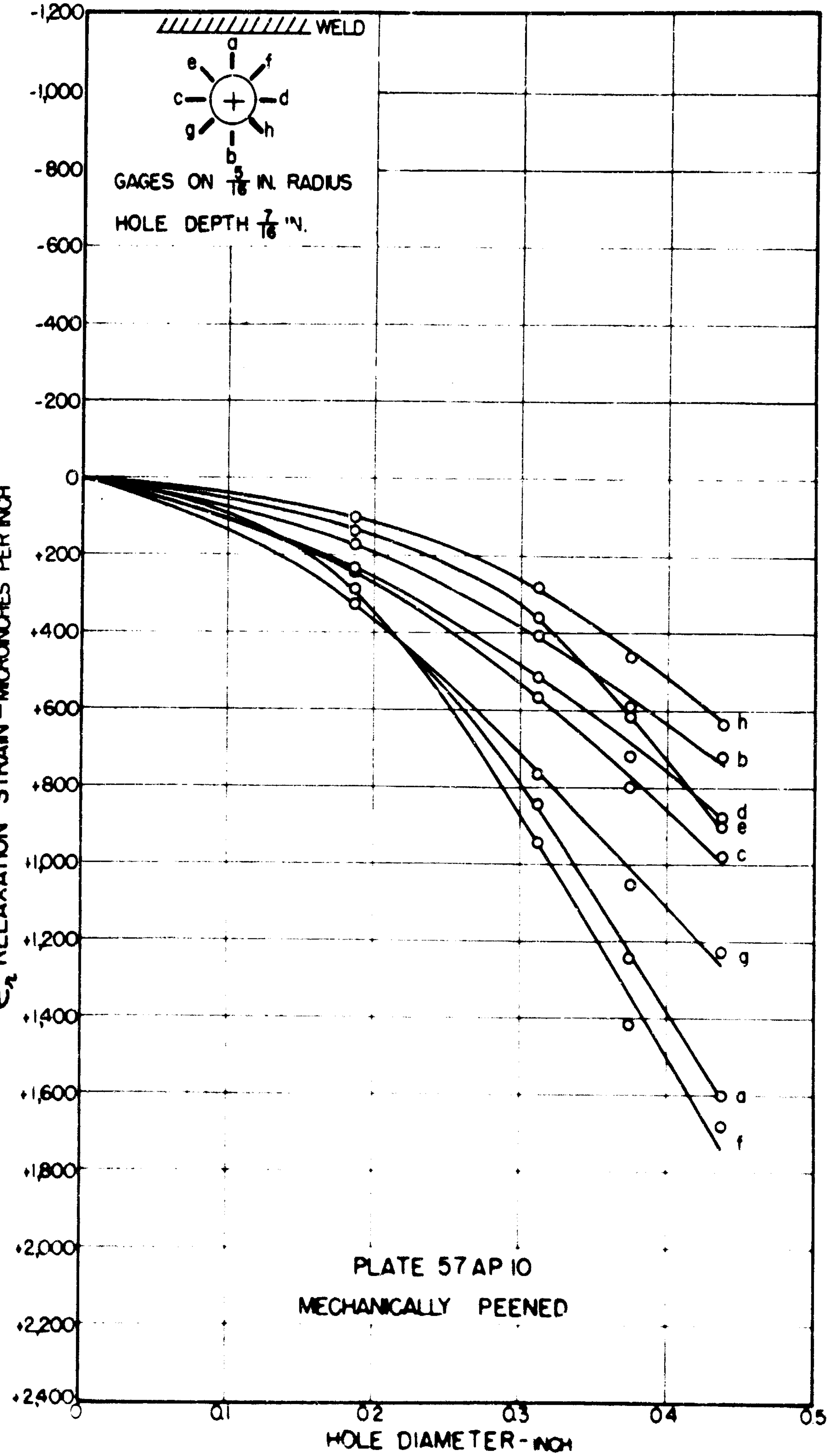

FIGURE 25-VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

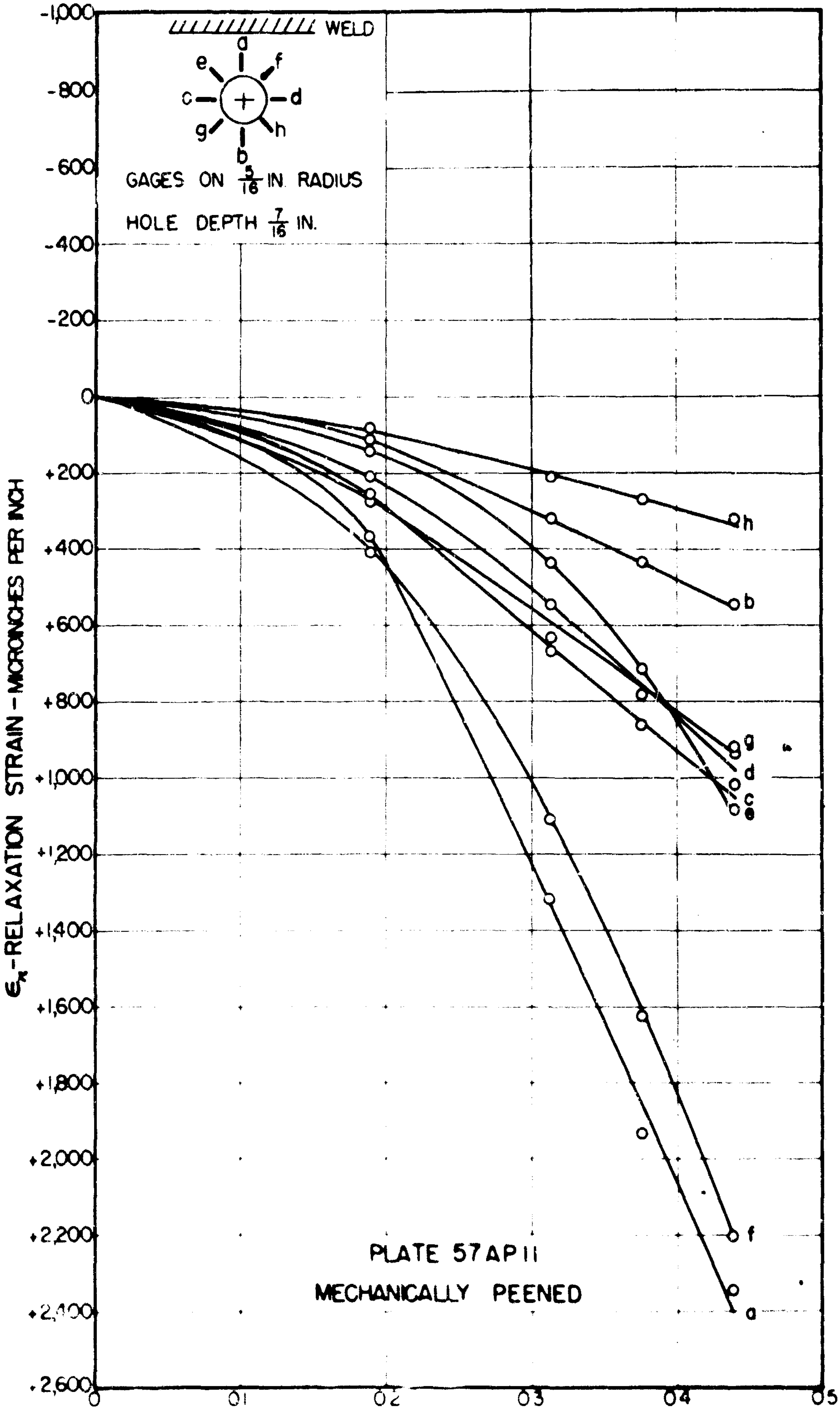

FIGURE 26-VARIATION OF RELAXATION STRAIN WITH INCREASING HOLE DIAMETER

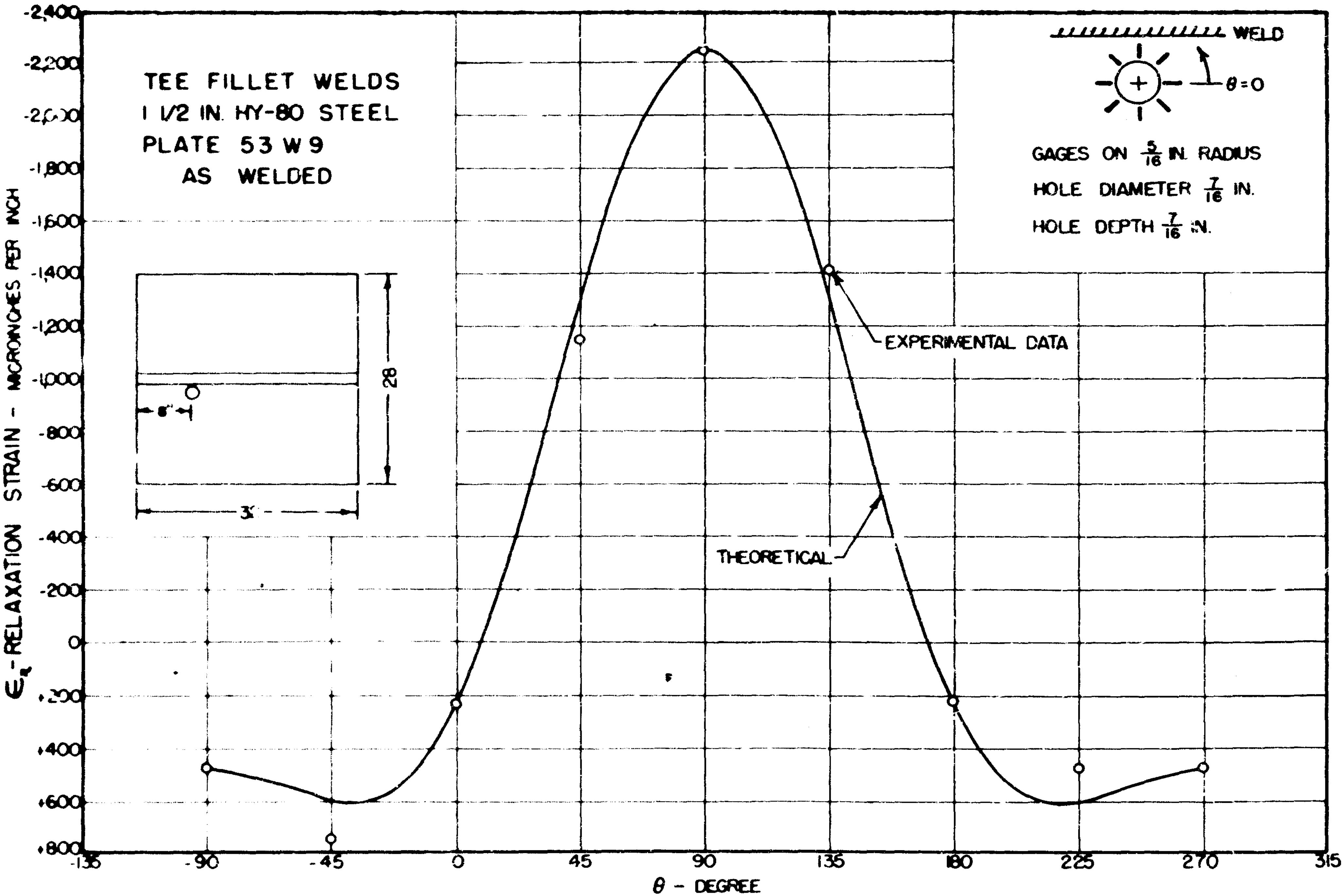

FIGURE 27- VARIATION OF RADIAL RELAXATION STRAIN WITH ANGLE AROUND DRILLED HOLE

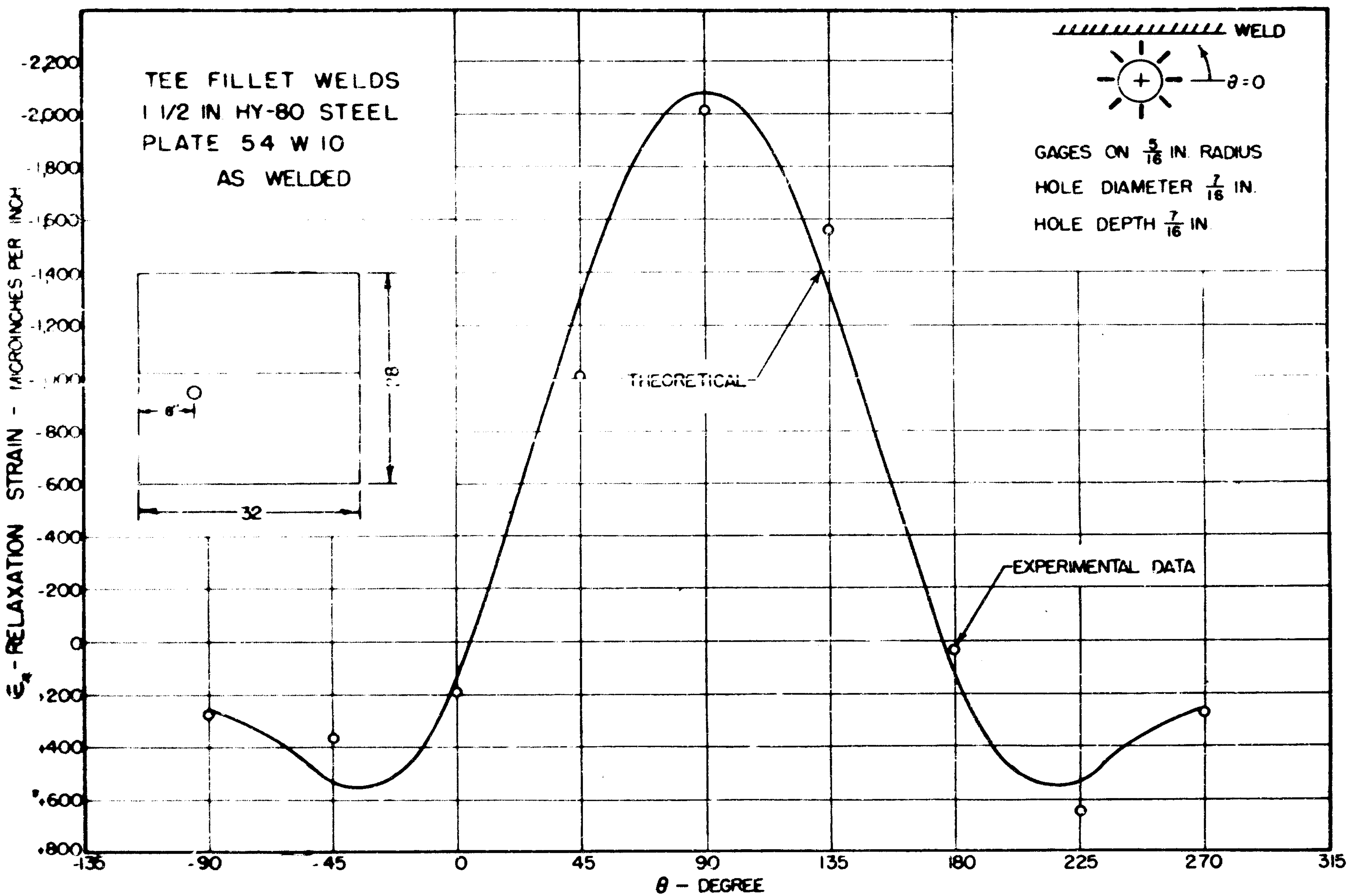

FIGURE 28 · VARIATION OF RADIAL RELAXATION STRAIN WITH ANGLE AROUND DRILLED HOLE

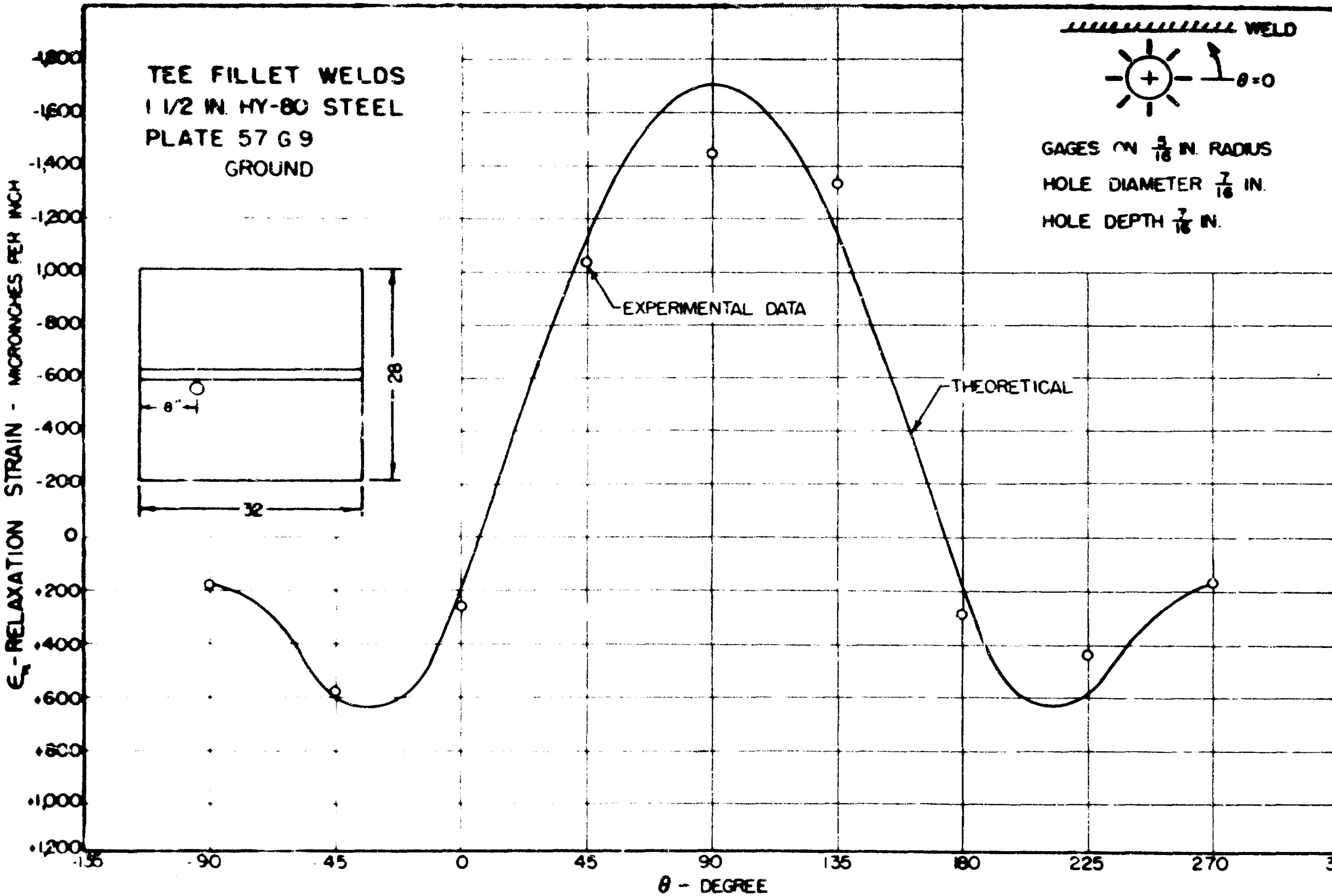

FIGURE 29 - VARIATION OF RADIAL RELAXATION STRAIN WITH ANGLE AROUND DRILLED HOLE

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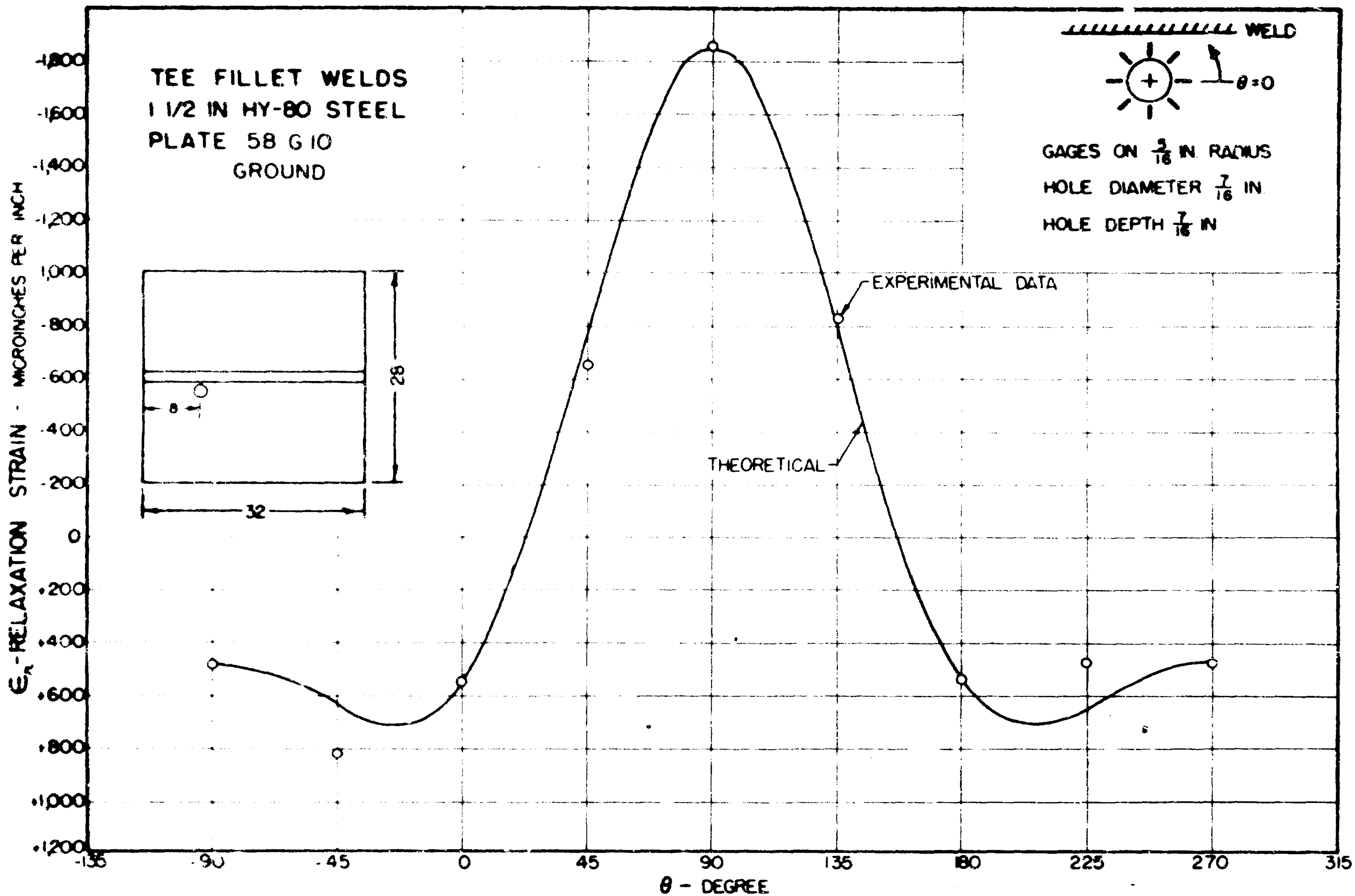

FIGURE 30 - VARIATION OF RADIAL RELAXATION STRAIN WITH ANGLE AROUND DRILLED HOLE

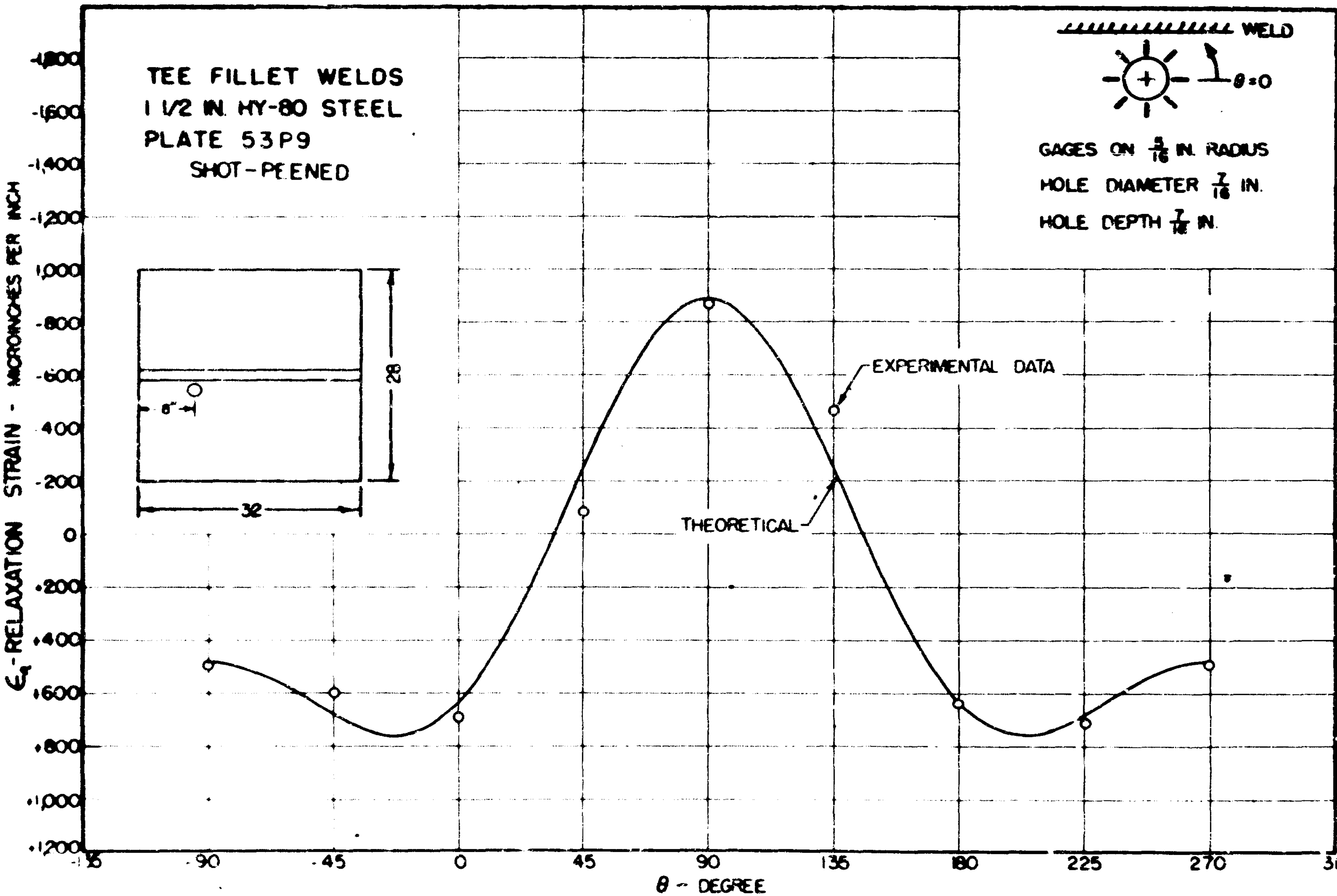

FIGURE 31 - VARIATION OF RADIAL RELAXATION STRAIN WITH ANGLE AROUND DRILLED HOLE

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PROGRESS REPORT 1

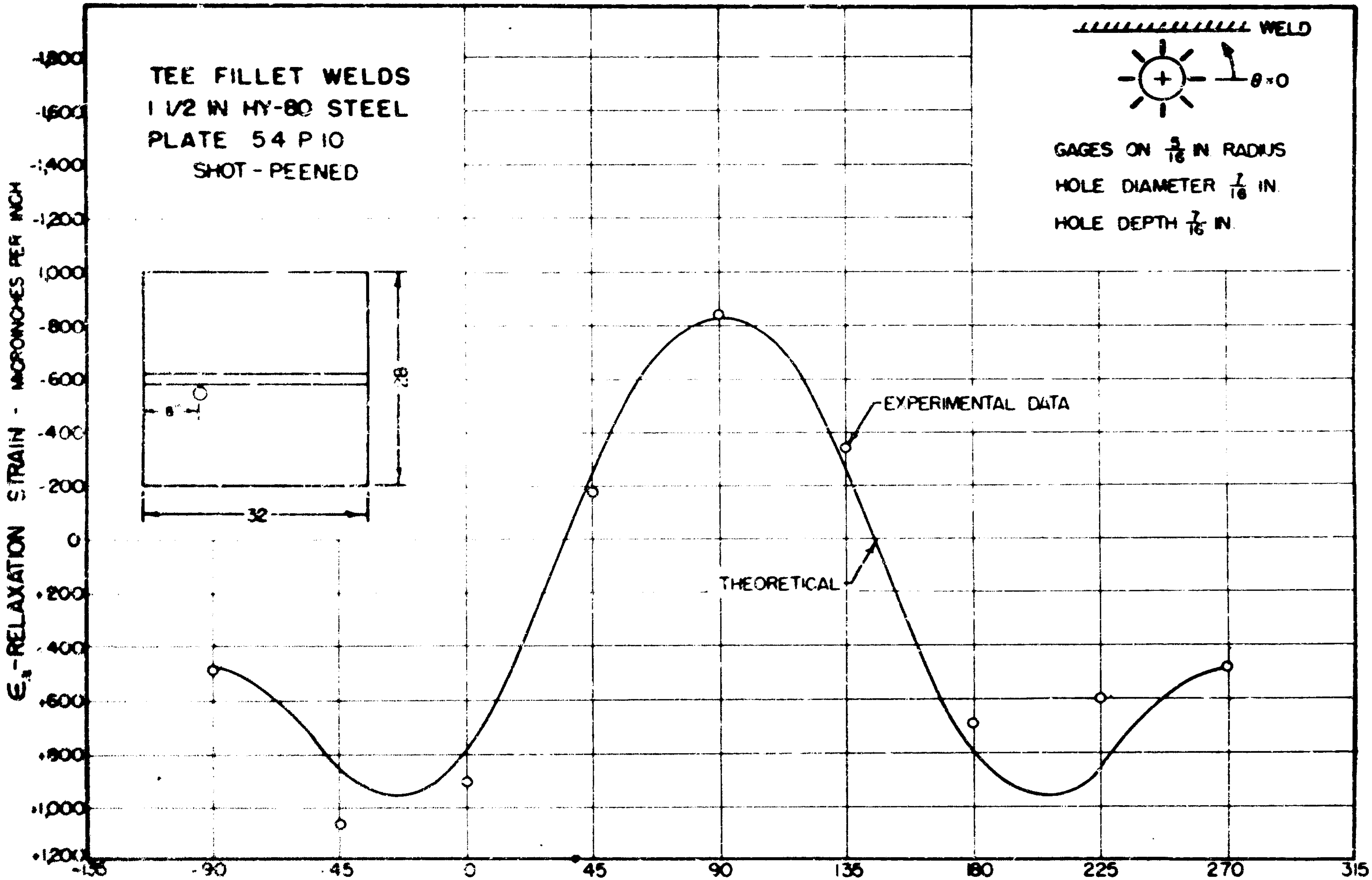

FIGURE 32- VARIATION OF RADIAL RELAXATION STRAIN WITH ANGLE AROUND DRILLED HOLE

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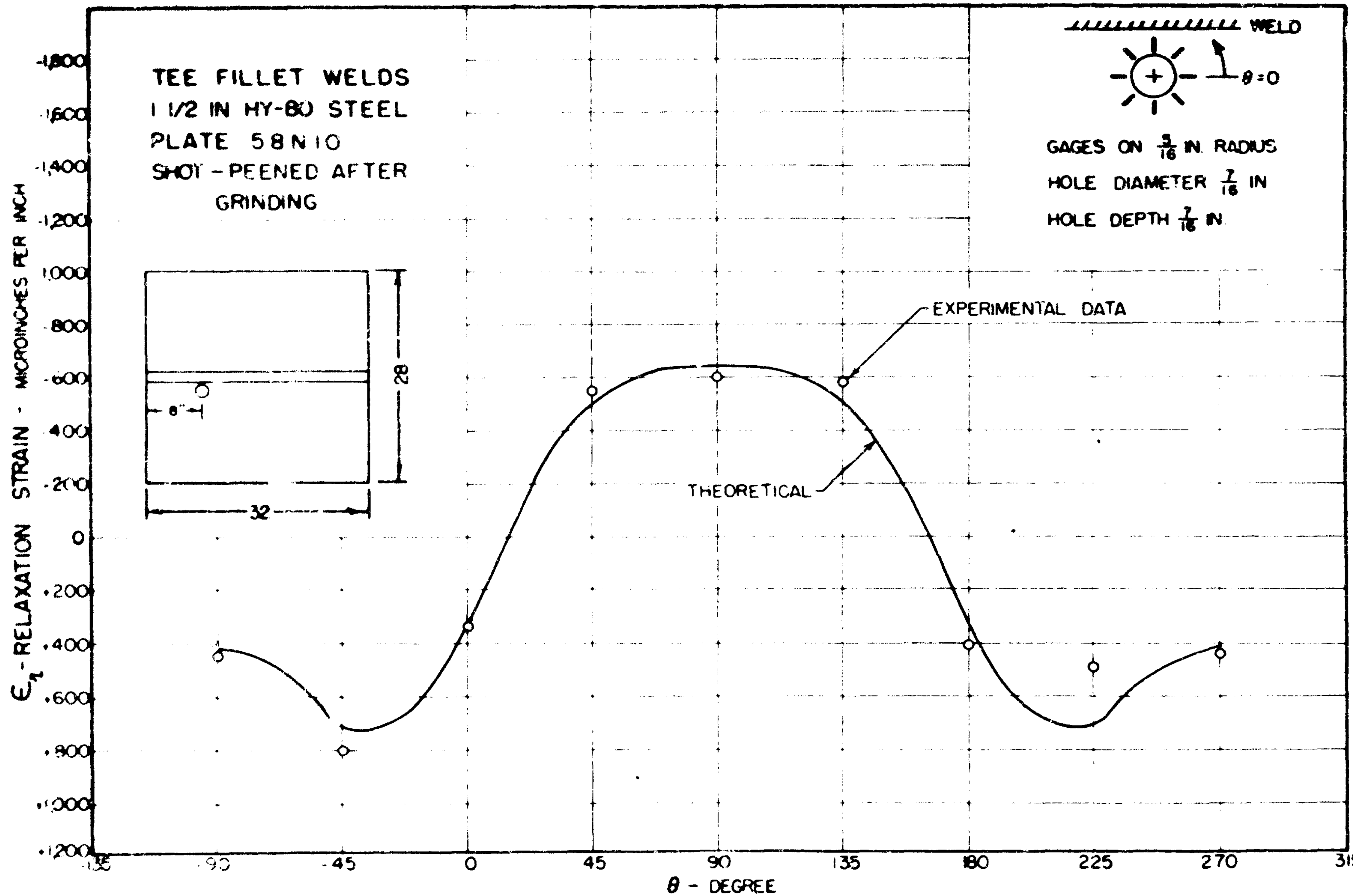

FIGURE 33-VARIATION OF RADIAL RELAXATION STRAIN WITH ANGLE AROUND DRILLED HOLE

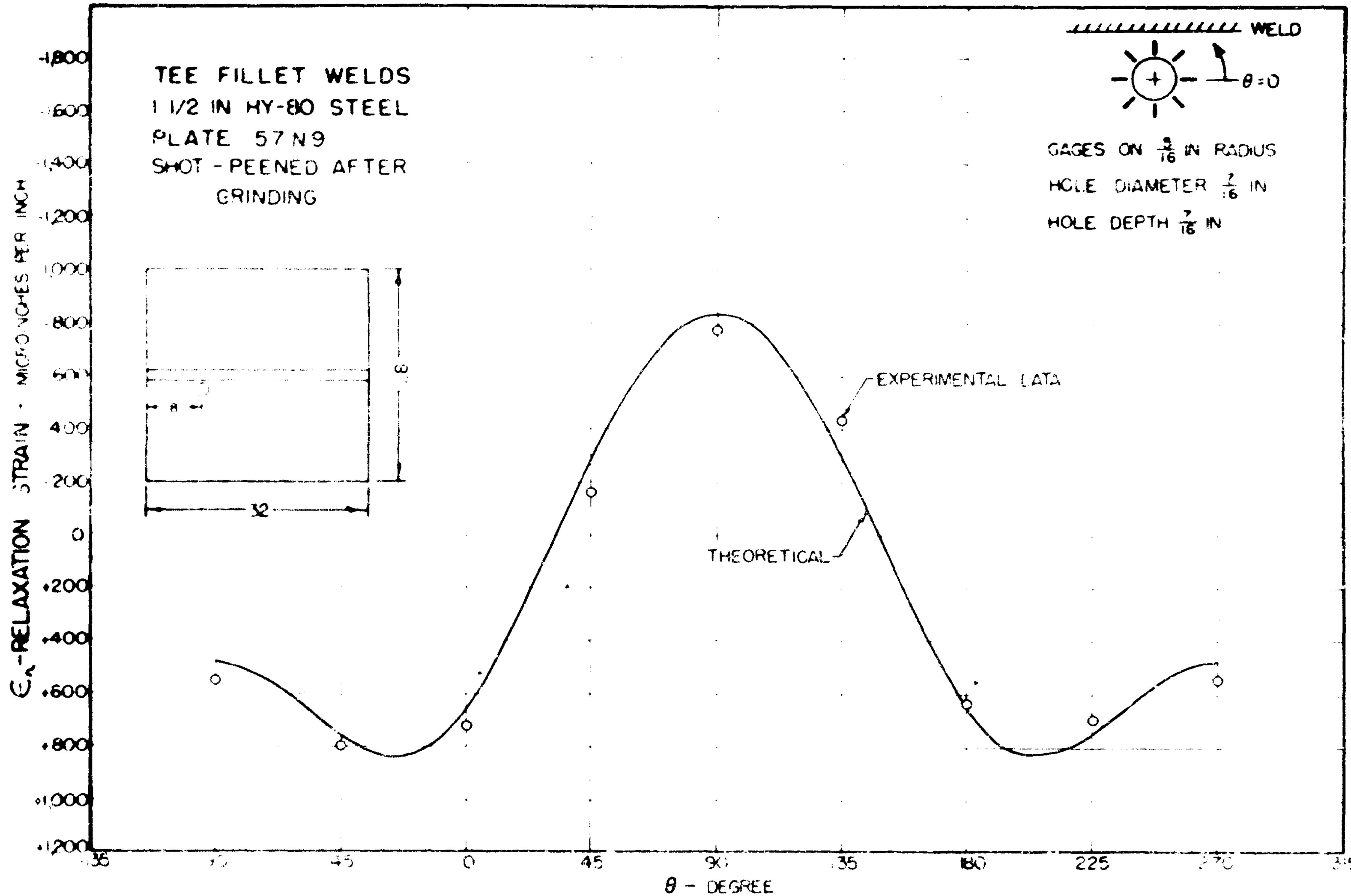

FIGURE 34-VARIATION OF RADIAL RELAXATION STRAIN WITH ANGLE AROUND DRILLED HOLE

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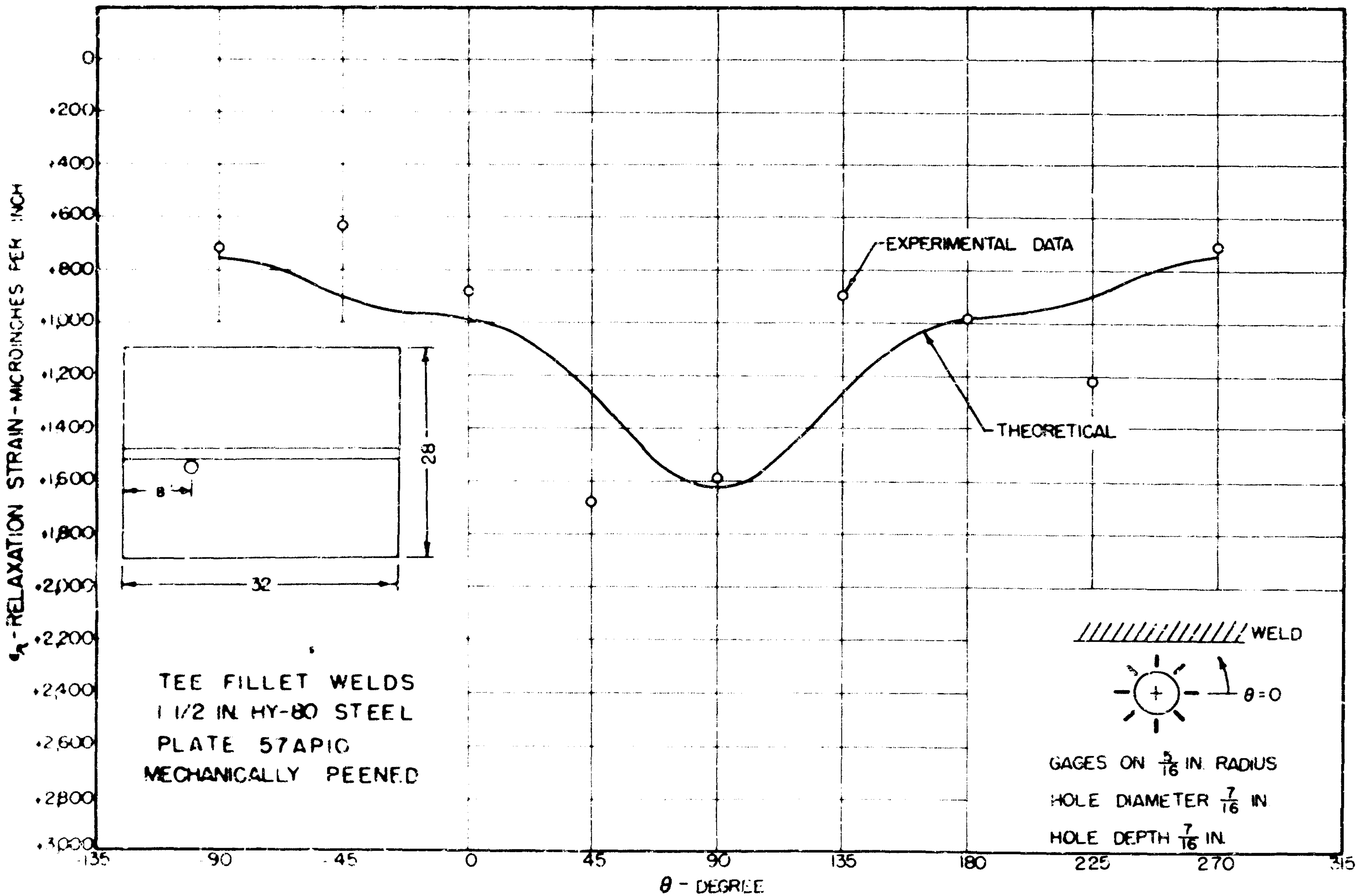


FIGURE 35-VARIATION OF RADIAL RELAXATION STRAIN WITH ANGLE AROUND DRILLED HOLE

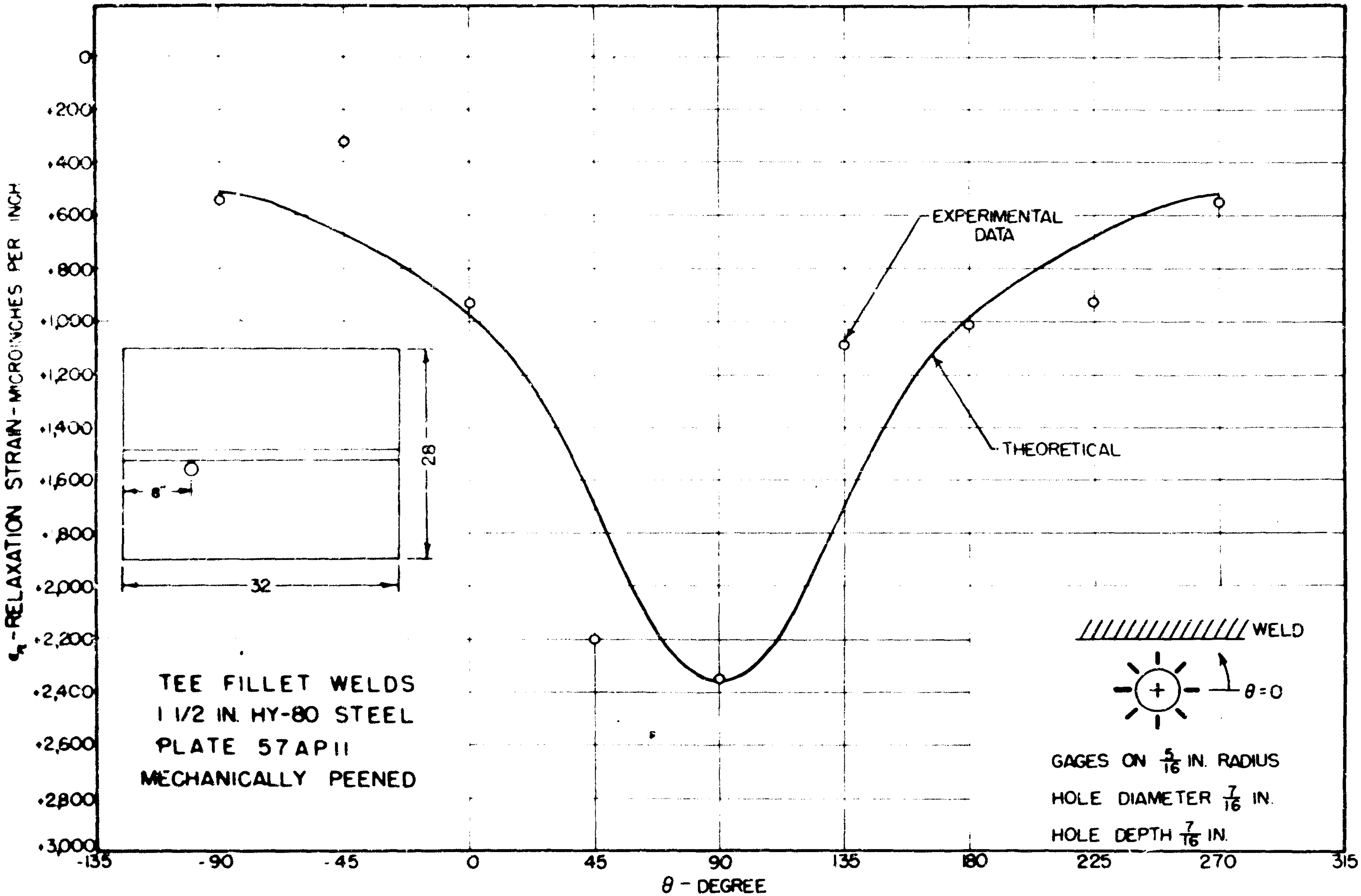


FIGURE 36- VARIATION OF RADIAL RELAXATION STRAIN WITH ANGLE AROUND DRILLED HOLE

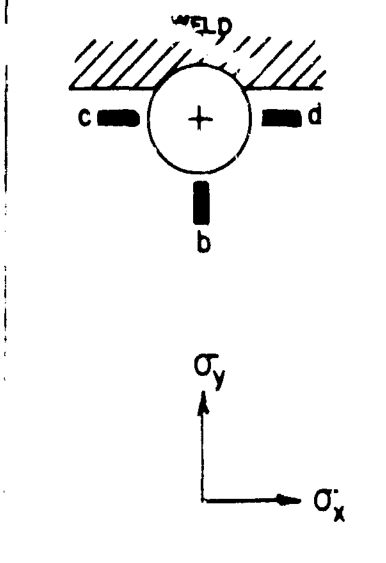
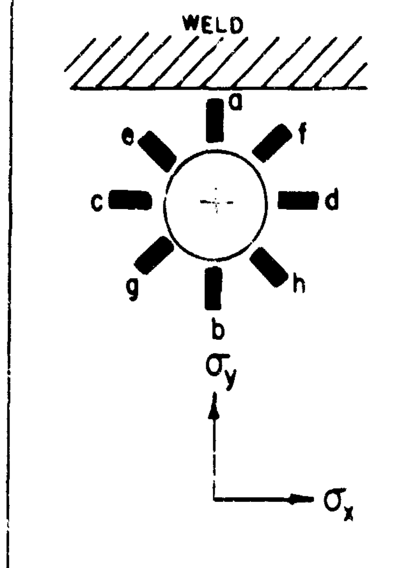
CONDITION OF WELD	SPECIMEN NO.	ARRANGEMENT 1 ASSUMED UNIFORM STRESS FIELD		ARRANGEMENT 2 ASSUMED LINEARLY VARYING STRESS FIELD	
					
		σ_y psi	σ_x psi	σ_y psi	σ_x psi
AS WELDED	53W9	35,300	64,600	124,000	88,800
	54W10	29,700	73,300	116,300	99,200
GROUND	57G9	7,700	52,600	90,100	85,900
	58G10	19,000	61,300	90,600	37,200
SHOT-PEENED	53P9	-5,000	22,800	48,100	21,000
	54P10	-11,700	7,300	46,800	30,100
SHOT-PEENED AFTER GRINDING	57N9	-6,100	4,000	41,100	25,300
	58N10	-15,000	17,800	39,200	58,500
MECHANICALLY PEENED	57AP10	-55,400	-72,100	-78,200	-56,300
	57AP11	-69,000	-79,300	-117,500	-83,600

TABLE I

RESIDUAL STRESSES AT HY-80 TEE-FILLET WELDS DETERMINED FOR TWO STRAIN GAGE ARRANGEMENTS

APPENDIX A

MATHEMATICAL THEORY FOR THE SOLUTION TO THE PROBLEM OF DETERMINING THE STRESS DISTRIBUTION AROUND A HOLE IN AN INFINITE PLATE IN WHICH THE PLANE STRESS FIELD IS SPECIFIED AS:

[illegible] $cy + A$

[illegible] $kx + B$

[illegible] 0

A solution is desired to the problem of determining the stress distribution around a hole in an infinite plate in which the plane stress field, Figure 1, for the unpenetrated plate, is specified as:

$$\sigma_x = cy + A$$
$$\sigma_y = ky + B \qquad (1)$$
$$\tau_{xy} = 0$$

Since this stress field does not satisfy the equilibrium equations the solution will not be exact.

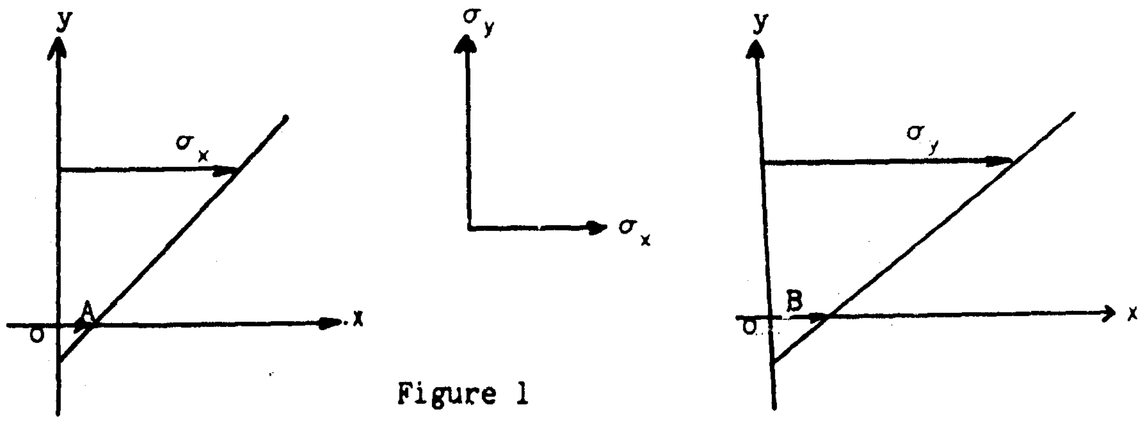


Figure 1

The approximate solution provides a means for determining the strain distribution around the hole. Having this strain distribution a calculation can be made of the relaxation strains that would result from drilling a hole in an unpenetrated plate under the stress field described by equation (1). An experimental determination of the same relaxation strains may also be made using electric strain gages. A knowledge of these calculated and measured relaxation strains permits a determination to be made of the state of residual stress in the unpenetrated plate.

STRAIN EQUATIONS FOR THE UNPENETRATED PLATE

Equations (1) may be expressed in terms of polar coordinates by use of the following transformation equations.

$$\sigma_r = \tfrac{1}{2}(\sigma_x + \sigma_y) + \tfrac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta \qquad (2)$$

$$\sigma_\theta = \tfrac{1}{2}(\sigma_x + \sigma_y) - \tfrac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta - \tau_{xy}\sin 2\theta \qquad (3)$$

$$\tau_{r\theta} = \tfrac{1}{2}(\sigma_y - \sigma_x)\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (4)$$

On substitution we get.

$$\sigma_r = \frac{(A+B)}{2} + \frac{(A-B)}{2}\cos 2\theta \qquad (5)$$

$$+ (c+3k)\frac{r}{4}\sin\theta + (c-k)\frac{r}{4}\sin 3\theta$$

$$\sigma_\theta = \frac{(A+B)}{2} - \frac{(A-B)}{2}\cos 2\theta \qquad (6)$$

$$+ (3c+k)\frac{r}{4}\sin\theta - (c-k)\frac{r}{4}\sin 3\theta$$

$$\tau_{r\theta} = \tfrac{1}{2}[(c-k)\,y + (A-B)]\sin 2\theta \qquad (7)$$

The corresponding strains are

$$\epsilon_r = \frac{1}{E}(\sigma_r - \nu\sigma_\theta) \qquad (8)$$

$$\epsilon_\theta = \frac{1}{E}(\sigma_\theta - \nu\sigma_r) \qquad (9)$$

$$\gamma_{r\theta} = \frac{1}{G}\tau_{r\theta} \qquad (10)$$

On substituting (5), (6) and (7) into (8), (9) and (10) and using subscripts 1 for conditions in the unpenetrated plate, we get:

$$E\epsilon_{r_1} = \frac{(A+B)}{2}(1-\nu) + \frac{(A-B)}{2}(1+\nu)\cos 2\theta$$

$$+ c(1-3\nu)\frac{r}{4}\sin\theta + k(3-\nu)\frac{r}{4}\sin\theta + (c-k)(1+\nu)\frac{r}{4}\sin 3\theta \qquad (11)$$

$$E\epsilon_{\theta_1} = \frac{(A+B)}{2}(1-\nu) - \frac{(A-B)}{2}(1+\nu)\cos 2\theta$$

$$+ c(3-\nu)\frac{r}{4}\sin\theta + k(1-3\nu)\frac{r}{4}\sin\theta - (c-k)(1+\nu)\frac{r}{4}\sin 3\theta \qquad (12)$$

$$E\gamma_{r\theta_1} = (1+\nu)[(c-k)y + (A-B)]\sin 2\theta \qquad (13)$$

STRESS DISTRIBUTION AROUND A CIRCULAR HOLE IN AN INFINITE PLATE WHERE THE STRESS FIELD IN THE UNPENETRATED PLATE IS.

$$\sigma_x = cy; \qquad \sigma_y = 0; \qquad \tau_{xy} = 0$$

In the solution of two dimensional problems, the stresses may be derived from the Airy stress function $U(x,y)$ which satisfies the general biharmonic equation:

$$\frac{\partial^4 U}{\partial x^4} + 2\frac{\partial^4 U}{\partial x^2 \partial y^2} + \frac{\partial^4 U}{\partial y^4} = 0 \tag{14}$$

and the boundary conditions of the problem The stresses derived from the above function are:

$$\sigma_x = \frac{\partial^2 U}{\partial y^2} \; ; \; \sigma_y = \frac{\partial^2 U}{\partial x^2} \; ; \; \tau_{xy} = \frac{-\partial^2 U}{\partial x \, \partial y} \tag{15}$$

It has been shown that the biharmonic function $U(x, y)$ can be expressed as:

$$U(x,y) = \mathrm{Re}\,[\bar{Z}\varphi(Z) + \chi(Z)] \tag{16}$$

Where Re denotes the real part of the term in brackets, and $\varphi(Z)$ and $\chi(Z)$ are analytical functions of the complex variable $Z = x + iy$ and $\bar{Z} = x - iy$. If the functions $\varphi(Z)$ and $\chi(Z)$ are known, the stress components σ_x, σ_y and τ_{xy} can be determined directly from these functions by means of the Kolosov-Mushkelishvili formulae:

$$\sigma_x + \sigma_y = 4[\mathrm{Re}\,\varphi'(Z)] \tag{17}$$

$$\sigma_y - \sigma_x + 2i\tau_{xy} = 2[\bar{Z}\varphi''(Z) + \chi''(Z)] \tag{18}$$

where the prime and double prime denote the first and second derivatives, respectively, with respect to Z.

Substituting $\sigma_x = cy$, $\sigma_y = 0$ and $\tau_{xy} = 0$ into (17) and (18) and making use of the Cauchy-Riemann conditions we get:

$$\Phi_1'(Z) = \frac{-icZ}{4} \tag{19}$$

$$\chi_1''(Z) = \frac{icZ}{4} \tag{20}$$

Subscripts 1 have been used to denote initial conditions in the unpenetrated plate. After a hole is drilled the change in stress state may be represented by the additional stress functions Φ_o' and χ_o'' so that for the final stress state we have:

$$\Phi'(Z) = \Phi_1'(Z) + \Phi_o'(Z) \tag{21}$$

$$\chi''(Z) = \chi_1''(Z) + \chi_o''(Z) \tag{22}$$

Thus the functions Φ_o' and χ_o'' remain to be determined.

Since the effects of the hole on the stress field are localized, the stresses will drop off rapidly with increasing distance from the hole. Thus the corresponding stress functions may be taken in the form

$$\Phi_o'(Z) = \frac{A_2}{Z^2} + \frac{A_3}{Z^3} + \frac{A_4}{Z^4} + ---- + \frac{A_n}{Z^n} \tag{23}$$

$$\chi_o''(Z) = \frac{B_2}{Z^2} + \frac{B_3}{Z^3} + \frac{B^4}{Z^4} + ---- + \frac{B_n}{Z^n} \tag{24}$$

Substituting equations (19), (20), (23) and (24) into (21) and (22) yields

$$\Phi'(Z) = -\frac{icZ}{4} + \frac{A_2}{Z^2} + \frac{A_3}{Z^3} + ---- + \frac{A_n}{Z^n} \tag{25}$$

$$\chi''(Z) = \frac{icZ}{4} + \frac{B_2}{Z^2} + \frac{B_3}{Z^3} + ---- + \frac{B_n}{Z^n} \tag{26}$$

In polar coordinates, the Kolosov-Muskhelishvili Formulae equations (17) and (18) become

$$\sigma_r + \sigma_\theta = 2[\phi'(z) + \bar{\phi}'(\bar{z})] = 4\,\mathrm{Re}\,[\phi'(z)] \tag{17a}$$

$$\sigma_\theta - \sigma_r + 2i\tau_{r\theta} = 2[\bar{z}\phi''(z) + \chi''(z)]e^{2i\theta} \tag{18a}$$

and by subtracting (18a) from (17a)

$$\sigma_r - i\tau_{r\theta} = \phi'(z) + \bar{\phi}'(\bar{z}) - [\bar{z}\phi''(z) + \chi''(z)]e^{2i\theta} \tag{27}$$

The barred terms indicate conjugate functions, i.e.,

$$z = re^{i\theta}\ ;\ \bar{z} = re^{-i\theta}$$

At the edge of the hole $r = a$; $\sigma_r = 0$; $\tau_{r\theta} = 0$

Thus, equation (27) becomes

$$\{\phi'(z) + \bar{\phi}'(\bar{z}) - [\bar{z}\phi''(z) + \chi''(z)]\,e^{2i\theta}\}_{r=a} = 0 \tag{28}$$

From equation (25)

$$\phi'(z)_{r=a} = -\frac{icae^{i\theta}}{4} + \frac{A_2e^{-2i\theta}}{a^2} + \frac{A_3e^{-3i\theta}}{a^3} + \text{-----} + \frac{A_ne^{-ni\theta}}{a^n}$$

$$\bar{\phi}'(\bar{z})_{r=a} = \frac{icae^{-i\theta}}{4} + \frac{\bar{A}_2e^{2i\theta}}{a^2} + \frac{\bar{A}_3e^{3i\theta}}{a^3} + \text{-----} + \frac{\bar{A}_ne^{ni\theta}}{a^n}$$

also

$$-\bar{z}\phi''(z)e^{2i\theta}]_{r=a} = \frac{icae^{i\theta}}{4} + \frac{2A_2e^{-2i\theta}}{a^2} + \frac{3A_3e^{-3i\theta}}{a^3} + \text{-----} + \frac{nA_ne^{-ni\theta}}{a^n}$$

$$-\chi''(z)e^{2i\theta}\}_{r=a} = \frac{-icae^{3i\theta}}{4} - \frac{B_2}{a^2} - \frac{B_3e^{-i\theta}}{a^3} - \frac{B_4e^{-2i\theta}}{a^4} \text{---} \frac{B_ne^{-(n-2)i\theta}}{a^n}$$

Substitution of the above into equation (28) yields

$$\left(-\frac{B_2}{a^2}\right) + \left(-\frac{ica}{4}+\frac{ica}{4}\right)e^{i\theta} + \left(\frac{\bar{A}_2}{a^2}\right)e^{2i\theta} + \left(\frac{\bar{A}_3}{a^3} - \frac{ica}{4}\right)e^{3i\theta}$$

$$+\left(\frac{\bar{A}_4}{a^4}\right)e^{4i\theta} + \text{-----} + \left(\frac{A_n}{a^n}\right)e^{ni\theta} + \left(\frac{ica}{4} - \frac{B_3}{a^3}\right)e^{-i\theta}$$

$$+\left(\frac{A_2}{a^2} + \frac{2A_2}{a^2} - \frac{B_4}{a^4}\right)e^{-2i\theta} + \left(\frac{A_3}{a^3} + \frac{3A_3}{a^3} - \frac{B_5}{a^5}\right)e^{-3i\theta}$$

$$+\left(\frac{A_4}{a^4} + \frac{4A_4}{a^4} - \frac{B_6}{a^6}\right)e^{-4i\theta} + \left(\frac{A_5}{a^5} + \frac{5A_5}{a^5} - \frac{B_7}{a^7}\right)e^{-5i\theta}$$

$$+\text{-------}+\left(\frac{A_n}{a^n} + \frac{nA_n}{a^n} - \frac{B_{n+2}}{a^{n+2}}\right)e^{-ni\theta} = 0$$

To satisfy the above equation all coefficients of $e^{ni\theta}$ must be equal to zero.

$$\therefore B_2 = 0 \qquad \bar{A}_2 = 0; \qquad \bar{A}_3 = \frac{ica^4}{4}; \qquad \bar{A}_4 = 0; \qquad \bar{A}_n = 0$$

$$A_2 = 0; \qquad A_3 = -\frac{ica^4}{4}; \qquad A_4 = 0; \qquad A_n = 0 \quad {}_{n>3}$$

$$B_3 = \frac{ica^4}{4}; \qquad B_4 = 0; \qquad B_5 = -ica^6$$

$$B_6 = 0 \qquad B_n = 0 \qquad n >$$

Substituting these constants back into equations (25) and (26) we get

$$\phi'(z) = -\frac{ica}{4}\left(\frac{z}{a} + \frac{a^3}{z^3}\right) \tag{29}$$

$$\chi''(z) = \frac{ica}{4}\left(\frac{z}{a} + \frac{a^3}{z^3} - \frac{4a^5}{z^5}\right) \tag{30}$$

Putting (29) into equation (17a) yields

$$\sigma_\theta + \sigma_r = 4\mathrm{Re}[\phi'(z)] = 4\mathrm{Re}\left[-\frac{ica}{4}\left(\frac{z}{a} + \frac{a^3}{z^3}\right)\right]$$

$$\sigma_\theta + \sigma_r = cr\left(\sin\theta - \frac{a^4}{r^4}\sin 3\theta\right) \tag{31}$$

Equation (18a) gives upon appropriate substitution for the respective terms

$$\sigma_\theta - \sigma_r + 2i\tau_{r\theta} = \frac{2\bar{z}^2}{r^2}\left[\bar{z}\left\{-\frac{ic}{4}\left(1-\frac{3a^4}{\bar{z}^4}\right)\right\} + \frac{ica}{4}\left(\frac{z}{a} + \frac{a^3}{z^3} - \frac{4a^5}{z^5}\right)\right]$$

$$\sigma_\theta - \sigma_r + 2i\tau_{r\theta} = \frac{cr}{2}\left[\sin\theta + \frac{3a^4}{r^4}\sin 3\theta - \sin 3\theta + \frac{a^4}{r^4}\sin\theta - \frac{4a^6}{r^6}\sin 3\theta\right]$$

$$- \frac{icr}{2}\left[\cos\theta - \frac{3a^4}{r^4}\cos 3\theta - \cos 3\theta - \frac{a^4}{r^4}\cos\theta + \frac{4a^6}{r^6}\cos 3\theta\right]$$

Equating real and imaginary parts

$$\sigma_\theta - \sigma_r = \frac{cr}{2}\left[(\sin\theta - \sin 3\theta) + \frac{a^4}{r^4}(\sin\theta + 3\sin 3\theta) - \frac{4a^6}{r^6}\sin 3\theta\right] \tag{32}$$

$$\tau_{r\theta} = -\frac{cr}{4}\left[(\cos\theta - \cos 3\theta) - \frac{a^4}{r^4}(\cos\theta + 3\cos 3\theta) + \frac{4a^6}{r^6}\cos 3\theta\right] \tag{33}$$

Solving equations (31) and (32) simultaneously we get

$$\sigma_r = \frac{c\,r}{4}\left[\left(1-\frac{a^4}{r^4}\right)\sin\theta + \left(1-\frac{5a^4}{r^4}+\frac{4a^6}{r^6}\right)\sin 3\theta\right] \tag{34}$$

$$\sigma_\theta = \frac{c\,r}{4}\left[\left(3+\frac{a^4}{r^4}\right)\sin\theta - \left(1-\frac{a^4}{r^4}+\frac{4a^6}{r^6}\right)\sin 3\theta\right] \tag{35}$$

Equations (33), (34) and (35) represent the stress distribution around a hole in a plate for which the stress distribution for the unpenetrated plate is

$$\sigma_x = cy; \qquad \sigma_y = 0; \qquad \tau_{xy} = 0$$

It is seen that when $r = a$

$$\tau_{r\theta}\Big]_{r=a} = 0$$

$$\sigma_r\Big]_{r=a} = 0$$

$$\sigma_\theta\Big]_{r=a} = ca(\sin\theta - \sin 3\theta)$$

$$= 0 \text{ for } \theta = 0$$

$$= 2ca \text{ for } \theta = \frac{\pi}{2}$$

or twice the stress in the unpenetrated plate.

STRESS DISTRIBUTION AROUND A CIRCULAR HOLE IN AN INFINITE PLATE WHERE THE STRESS FIELD IN THE UNPENETRATED PLATE IS

$$\sigma_x = 0; \qquad \sigma_y = ky; \qquad \tau_{xy} = 0 \tag{36}$$

Substituting these stresses into equation (17)

$$0 + ky = 4[\mathrm{Re}\,\varphi_1'(z)]$$

where $\varphi_1'(z)$ applies to the unpenetrated plate.

$$\therefore \mathrm{Re}\,\varphi_1'(z) = \frac{ky}{4}$$

$$\varphi_1'(z) = -\frac{ikz}{4}; \quad \varphi_1''(z) = \frac{-ik}{4} \tag{37}$$

Also, by use of equation (18)

$$ky - 0 + 0 = 2[\bar{z}\varphi_1''(z) + \chi_1''(z)]$$

$$\therefore \chi_1''(z) = \frac{ky}{2} + \bar{z}\left(\frac{ik}{4}\right) = \frac{k}{2}\left(y + \frac{i\bar{z}}{2}\right)$$

$$\chi_1''(z) = \frac{ik\bar{z}}{2} - \frac{ikz}{4} \tag{38}$$

After the hole is drilled, the change in stress may be represented by the functions $\varphi_0'(z)$ and $\chi_0(z)$ so that for the final stress state we have

$$\varphi'(z) = \varphi_1'(z) + \varphi_0(z) \tag{21}$$

$$\chi'(z) = \chi_1'(z) + \chi_0(z) \tag{22}$$

Since the effects of the hole in the stress field are localized, the stresses will drop off rapidly with distance from the hole. Thus, $\varphi_0(z)$ and $\chi_0(z)$ may be taken in the form

$$\varphi_o'(z) = \frac{A_1}{z} + \frac{A_2}{z^2} + \frac{A_3}{z^3} + \cdots + \frac{A_n}{z^n}$$

$$\chi_o''(z) = \frac{B_1}{z} + \frac{B_2}{z^2} + \frac{B_3}{z^3} + \cdots + \frac{B_n}{z^n}$$

where A_n and B_n are complex constants. Therefore, on substitution from these and equations (37) and (38) we get

$$\varphi'(z) = -\frac{ikz}{4} + \frac{A_1}{z} + \frac{A_2}{z^2} + \frac{A_3}{z^3} + \cdots + \frac{A_n}{z^n} \tag{39}$$

$$\chi''(z) = \frac{ik\bar{z}}{2} - \frac{ikz}{4} + \frac{B_1}{z} + \frac{B_2}{z^2} + \frac{B_3}{z^3} + \cdots + \frac{B_n}{z^n} \tag{40}$$

At the edge of the hole $\sigma_r = 0$; $\tau_{r\theta} = 0$ and boundary condition, equation (28) applies.

$$\left\{\varphi'(z) + \bar{\varphi}'(\bar{z}) - [\bar{z}\,\varphi''(z) + \chi''(z)]\, e^{2i\theta}\right\}_{r=a} = 0 \tag{28}$$

Substitution into equation (28) with the aid of (39) and (40), yields

$$\begin{aligned}
&\left(-\frac{B_2}{a^2}\right) + \left(-\frac{ika}{4} + \frac{\bar{A}_1}{a} - \frac{ika}{4} - \frac{B_1}{a}\right) e^{i\theta} + \left(\frac{\bar{A}_2}{a^2}\right)e^{2i\theta} + \left(\frac{\bar{A}_3}{a^3} + \frac{ika}{4}\right)e^{3i\theta} \\
&+\left(\frac{\bar{A}_4}{a^4}\right)e^{4i\theta} + \left(\frac{A_n}{a^n}\right)e^{ni\theta} + \left(\frac{2A_1}{a} + \frac{ika}{4} - \frac{B_3}{a^3}\right)e^{-i\theta} \\
&+\left(\frac{A_2}{a^2} + \frac{2A_2}{a^2} - \frac{B_4}{a^4}\right)e^{-2i\theta} + \left(\frac{A_3}{a^3} + \frac{3A_3}{a^3} - \frac{B_5}{a^5}\right)e^{-3i\theta} \\
&+\left(\frac{A_4}{a^4} + \frac{4A_4}{a^4} - \frac{B_6}{a^6}\right)e^{-4i\theta} + \cdots + \left(\frac{A_n}{a^n} + \frac{nA_n}{a^n} - \frac{B_{n+2}}{a^{n+2}}\right)e^{-ni\theta} = 0
\end{aligned} \tag{41}$$

To satisfy equation (41) all coefficients of $e^{ni\theta}$ must be equal to zero. Therefore, setting all quantities in brackets equal to zero, we get

$$B_2 = 0 \qquad\qquad B_1 = \bar{A}_1 - \frac{ika^2}{2}$$

$$\bar{A}_2 = A_2 = 0$$

$$\bar{A}_3 = -\frac{ika^4}{4} \qquad\qquad A_3 = \frac{ika^4}{4}$$

$$A_4 = A_5 = A_n = \bar{A}_n\Big]_{n\geq 4} = 0$$

$$B_3 = 2A_1a^2 + \frac{ika^4}{4}$$

$$B_4 = 0 \qquad B_5 = 4a^2A_3 = ika^6$$

$$B_6 = 0 \qquad B_{n+2}\Big]_{n\geq 4} = 0$$

For the displacements to be single-valued the condition

$$B_1 = -\frac{(3-\nu)}{(1+\nu)}\bar{A}_1$$

must be satisfied. (see Wang "Applied Elasticity" page 192.)

$$\therefore \quad B_1 = \bar{A}_1 - \frac{ika^2}{2} = -\frac{(3-\nu)}{(1+\nu)}\bar{A}_1$$

$$\bar{A}_1 = (1+\nu)\frac{ika^2}{8} \; ; \quad A_1 = -(1+\nu)\frac{ika^2}{8}$$

$$B_1 = \frac{ika^2}{8}(\nu-3)$$

$$B_3 = -\frac{ika^4}{4}$$

Substituting these constants into equations (39) and (40)

$$\Phi'(z) = -\frac{ikz}{4} - \frac{i(1+\nu)ka^2}{8z} + \frac{ika^4}{4z^3} \tag{42}$$

$$\chi''(z) = \frac{ik\bar{z}}{2} - \frac{ikz}{4} + \frac{i(\nu-3)ka^2}{8z} - \frac{i\nu ka^4}{4z^3} + \frac{ika^6}{z^5} \tag{43}$$

By use of equation (17a)

$$\sigma_r + \sigma_\theta = 2[\Phi'(z) + \bar{\Phi}'(\bar{z})]$$

$$\sigma_r + \sigma_\theta = 2[-\frac{ikz}{4} - \frac{i(1+\nu)ka^2}{8z} + \frac{ika^4}{4z^3}$$

$$+\frac{ik\bar{z}}{4} + \frac{i(1+\nu)ka^2}{8\bar{z}} - \frac{ika^4}{4\bar{z}^3}]$$

$$\sigma_r + \sigma_\theta = \frac{ika}{2}[(\frac{r}{a} - \frac{1+\nu}{2}\frac{a}{r})(-2i\sin\theta) + \frac{a^3}{r^3}(-2i\sin 3\theta)$$

$$\sigma_r + \sigma_\theta = \frac{kr}{2}[2(1-\frac{1+\nu}{2}\frac{a^2}{r^2})\sin\theta + 2\frac{a^4}{r^4}\sin 3\theta] \tag{44}$$

From equation (27)

$$\sigma_r - i\tau_{r\theta} = \Phi'(z) + \bar{\Phi}'(\bar{z}) - [\bar{z}\Phi''(z) + \chi''(z)]\, e^{2i\theta}$$

$$\bar{z}\, e^{2i\theta} = re^{-i\theta}\, e^{2i\theta} = z$$

$$-\bar{z}\,\Phi''(z)\, e^{2i\theta} = -z\,\Phi''(z)$$

$$\therefore\ -\bar{z}\,\Phi''(z)\, e^{2i\theta} = \frac{ikz}{4} - \frac{i(1+\nu)ka^2}{8z} + \frac{3ika^4}{4z^3}$$

$$-\bar{z}\,\Phi''(z)\, e^{2i\theta} = \frac{kr}{4}[(-1-\frac{1+\nu}{2}\frac{a^2}{r^2})\sin\theta + 3\frac{a^4}{r^4}\sin 3\theta]$$

$$+\frac{ikr}{4}[(1-\frac{1+\nu}{2}\frac{a^2}{r^2})\cos\theta + 3\frac{a^4}{r^4}\cos 3\theta]$$

Putting $\frac{z^2}{r^2}$ for $e^{2i\theta}$ and

$-\chi''(z)e^{2i\theta} = -\chi''(z)\frac{z^2}{r^2}$ we get from equation (43)

$$-\chi''(z)e^{2i\theta} = -\frac{ikzz^2}{2r^3} + \frac{ikz^3}{4r^2} - \frac{i(\nu-3)ka^2z}{8r^2} + \frac{i\nu ka^4}{4r^2z} - \frac{ika^6}{r^2z^3}$$

$$-\chi''(z)e^{2i\theta} = \frac{ik}{2}[-re^{i\theta} + \frac{r}{2}e^{3i\theta} - \frac{a^2(\nu-3)}{4r}e^{i\theta} + \frac{\nu a^4e^{-i\theta}}{2r^3} - \frac{2a^6}{r^5}e^{-3i\theta}]$$

$$-\chi''(z)e^{2i\theta} = \frac{ik}{2}[-r(\cos\theta + i\sin\theta) + \frac{r}{2}(\cos 3\theta + i\sin 3\theta)$$

$$-\frac{a^2(\nu-3)}{4r}(\cos\theta + i\sin\theta) + \frac{\nu a^4}{2r^3}(\cos\theta - i\sin\theta)$$

$$-\frac{2a^6}{r^5}(\cos 3\theta - i\sin 3\theta)]$$

$$-\chi''(z)e^{2i\theta} = \frac{kr}{2}[(1 + \frac{\nu-3}{4}\frac{a^2}{r^2} + \frac{\nu}{2}\frac{a^4}{r^4})\sin\theta - (\frac{1}{2} + \frac{2a^6}{r^6})\sin 3\theta]$$

$$+\frac{ikr}{2}[(-1 - \frac{\nu-3}{4}\frac{a^2}{r^2} + \frac{\nu}{2}\frac{a^4}{r^4})\cos\theta + (\frac{1}{2} - \frac{2a^6}{r^6})\cos 3\theta]$$

After substitution into equation (27)

$$\sigma_r - i\tau_{r\theta} = \frac{kr}{2}[(1 - \frac{1+\nu}{2}\frac{a^2}{r^2})\sin\theta + \frac{a^4}{r^4}\sin 3\theta]$$

$$+\frac{kr}{4}[(-1 - \frac{1+\nu}{2}\frac{a^2}{r^2})\sin\theta + \frac{3a^4}{r^4}\sin 3\theta]$$

$$+\frac{ikr}{4}[(1 - \frac{1+\nu}{2}\frac{a^2}{r^2})\cos\theta + \frac{3a^4}{r^4}\cos 3\theta]$$

$$+\frac{kr}{2}[(1 + \frac{\nu-3}{4}\frac{a^2}{r^2} + \frac{\nu}{2}\frac{a^4}{r^4})\sin\theta - (\frac{1}{2} + \frac{2a^6}{r^6})\sin 3\theta]$$

$$+\frac{ikr}{2}[(-1 - \frac{\nu-3}{4}\frac{a^2}{r^2} + \frac{\nu}{2}\frac{a^4}{r^4})\cos\theta + (\frac{1}{2} - 2\frac{a^6}{r^6})\cos 3\theta]$$

Equating real and imaginary parts and substituting σ_r into equation (44) to solve for σ_θ we arrive at the three components of stress.

$$\sigma_r = \frac{kr}{4}\left[\left(3 - 3\frac{a^2}{r^2} - \nu\frac{a^2}{r^2} + \nu\frac{a^4}{r^4}\right)\sin\theta + \left(-1 + \frac{5a^4}{r^4} - \frac{4a^6}{r^6}\right)\sin 3\theta\right] \tag{46}$$

$$\tau_{r\theta} = \frac{kr}{4}\left[\left(1 - \frac{a^2}{r^2} + \frac{\nu a^2}{r^2} - \frac{\nu a^4}{r^4}\right)\cos\theta + \left(-1 - 3\frac{a^4}{r^4} + \frac{4a^6}{r^6}\right)\cos 3\theta\right] \tag{47}$$

$$\sigma_\theta = \frac{kr}{4}\left[\left(1 + (1-\nu)\frac{a^2}{r^2} - \nu\frac{a^4}{r^4}\right)\sin\theta + \left(1 - \frac{a^4}{r^4} + 4\frac{a^6}{r^6}\right)\sin 3\theta\right] \tag{48}$$

Equations (46), (47) and (48) represent the stress distribution around a hole in a plate for which the stress distribution for the unpenetrated plate is

$$\sigma_x = 0 \; ; \; \sigma_y = ky \; ; \; \tau_{xy} = 0$$

It is seen that when

$$r = a$$

$$\tau_{r\theta}\Big]_{r=a} = 0$$

$$\sigma_r\Big]_{r=a} = 0$$

$$\sigma_\theta\Big]_{r=a} = \frac{ka}{2}\left[(1-\nu)\sin\theta + 2\sin 3\theta\right]$$

$$= 0 \text{ for } \theta = 0$$

$$= -(1+\nu)\frac{ka}{2} \text{ for } \theta = \frac{\pi}{2}$$

STRESS DISTRIBUTION AROUND A CIRCULAR HOLE IN AN INFINITE PLATE WHERE THE STRESS FIELD IN THE UNPENETRATED PLATE IS

$$\sigma_x = A \quad ; \qquad \sigma_y = B \quad ; \qquad \tau_{xy} = 0$$

The solution to this problem, which is readily found in the technical literature, is

$$\sigma_r = \frac{(A + B)}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{(A - B)}{2}\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\cos 2\theta \tag{49}$$

$$\sigma_\theta = \frac{(A + B)}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{(A - B)}{2}\left(1 + 3\frac{a^4}{r^4}\right)\cos 2\theta \tag{50}$$

$$\tau_{r\theta} = -\frac{(A - B)}{2}\left(1 + 2\frac{a^2}{r^2} - 3\frac{a^4}{r^4}\right)\sin 2\theta \tag{51}$$

STRESS DISTRIBUTION AROUND A CIRCULAR HOLE IN AN INFINITE PLATE IN WHICH THE PLANE STRESS FIELD IN THE UNPENETRATED PLATE IS

$$\sigma_x = cy + A \quad ; \quad \sigma_y = ky + B \quad ; \quad \tau_{xy} = 0$$

The solution to this problem is obtained by applying the principle of superposition to the above detailed solutions. Adding solutions (34), (46) and (49) yields

$$\begin{aligned}\sigma_r = {} & \frac{(A+B)}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{(A-B)}{2}\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\cos 2\theta \\ & + \frac{kr}{4}\left(3 - (3+\nu)\frac{a^2}{r^2} + \nu\frac{a^4}{r^4}\right)\sin\theta \\ & + \frac{cr}{4}\left(1 - \frac{a^4}{r^4}\right)\sin\theta \\ & - \frac{(k-c)}{4}r\left(1 - 5\frac{a^4}{r^4} + 4\frac{a^6}{r^6}\right)\sin 3\theta \end{aligned} \qquad (52)$$

Adding solutions (35), (48) and (50)

$$\begin{aligned}\sigma_\theta = {} & \frac{(A+B)}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{(A-B)}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta \\ & + \frac{kr}{4}\left(1 + (1-\nu)\frac{a^2}{r^2} - \nu\frac{a^4}{r^4}\right)\sin\theta \\ & + \frac{cr}{4}\left(3 + \frac{a^4}{r^4}\right)\sin\theta \\ & + \frac{(k-c)}{4}r\left(1 - \frac{a^4}{r^4} + 4\frac{a^6}{r^6}\right)\sin 3\theta \end{aligned} \qquad (53)$$

Adding solutions (33), (47) and (51)

$$\tau_{r\theta} = -\frac{(A - B)}{2}\left(1 + 2\frac{a^2}{r^2} - 3\frac{a^4}{r^4}\right)\sin 2\theta$$

$$+ \frac{kr}{4}\left[\left(1 - (1-\nu)\frac{a^2}{r^2} - \nu\frac{a^4}{r^4}\right)\cos\theta - \left(1 + 3\frac{a^4}{r^4}\right)\cos 3\theta\right] \qquad (54)$$

$$- \frac{cr}{4}\left[\left(1 - \frac{a^4}{r^4}\right)\cos\theta - \left(1 + 3\frac{a^4}{r^4}\right)\cos 3\theta\right]$$

$$+ (k-c)r\left(\frac{a^6}{r^6}\right)\cos 3\theta$$

STRAIN DISTRIBUTION AROUND A HOLE IN AN INFINITE PLATE IN WHICH THE PLANE STRESS FIELD IN THE UNPENETRATED PLATE IS

$$\sigma_x = cy + A \; ; \quad \sigma_y = \kappa y + B \; ; \quad \tau_{xy} = 0$$

Substituting from equations (52) and (53) into Hooke's Law, equations (8) and (9) and using subscript 2 to denote conditions after the hole is drilled, we get

$$\begin{aligned} E\epsilon_{r_2} = {} & \frac{(A+B)}{2}\left[1-\nu-(1+\nu)\frac{a^2}{r^2}\right] \\ & + \frac{(A-B)}{2}\left[1+\nu-4\frac{a^2}{r^2} + 3(1+\nu)\frac{a^4}{r^4}\right]\cos 2\theta \\ & + \frac{kr}{4}\left[(3-\nu) + (\nu-3)(\nu+1)\frac{a^2}{r^2} + \nu(1+\nu)\frac{a^4}{r^4}\right]\sin\theta \\ & + \frac{cr}{4}\left[1-3\nu-(1+\nu)\frac{a^4}{r^4}\right]\sin\theta \\ & + \frac{(k-c)r}{4}\left[-(1+\nu) + (5+\nu)\frac{a^4}{r^4} - 4(1+\nu)\frac{a^6}{r^6}\right]\sin 3\theta \end{aligned} \qquad (55)$$

$$\begin{aligned} E\epsilon_{\theta_2} = {} & \frac{(A+B)}{2}\left[1-\nu+(1+\nu)\frac{a^2}{r^2}\right] \\ & - \frac{(A-B)}{2}\left[1+\nu-4\nu\frac{a^2}{r^2} + 3(1+\nu)\frac{a^4}{r^4}\right]\cos 2\theta \\ & + \frac{kr}{4}\left[1-3\nu + (\nu+1)^2\frac{a^2}{r^2} - \nu(1+\nu)\frac{a^4}{r^4}\right]\sin\theta \\ & + \frac{cr}{4}\left[3-\nu + (1+\nu)\frac{a^4}{r^4}\right]\sin\theta \\ & + \frac{(k-c)r}{4}\left[-(1+\nu) + (1+5\nu)\frac{a^4}{r^4} - 4(1+\nu)\frac{a^6}{r^6}\right]\sin 3\theta \end{aligned} \qquad (56)$$

RELAXATION STRAINS RESULTING FROM DRILLING A HOLE IN AN INFINITE PLATE IN WHICH THE PLANE STRESS FIELD IN THE UNPENETRATED PLATE IS

$$\sigma_x = cy + A; \quad \sigma_y = ky + B; \quad \tau_{xy} = 0$$

The relaxation strains are

$$\epsilon_r = \epsilon_{r_2} - \epsilon_{r_1} \tag{57}$$

$$\epsilon_\theta = \epsilon_{\theta 2} - \epsilon_{\theta 1} \tag{58}$$

Substituting from equations (11), (12), (55) and (56) into (57) and (58)

$$\begin{aligned}
E\epsilon_r = &- \frac{(A+B)}{2}(1+\nu)\frac{a^2}{r^2} \\
&+ \frac{(A-B)}{2}\left[-4\frac{a^2}{r^2} + 3(1+\nu)\frac{a^4}{r^4}\right]\cos 2\theta \\
&+ \frac{kr}{4}\left[(\nu-3)(\nu+1)\frac{a^2}{r^2} + \nu(\nu+1)\frac{a^4}{r^4}\right]\sin\theta \\
&- \frac{cr}{4}\left[(1+\nu)\frac{a^4}{r^4}\right]\sin\theta \\
&+ \frac{(k-c)}{4}r\left[(5+\nu)\frac{a^4}{r^4} - 4(1+\nu)\frac{a^6}{r^6}\right]\sin 3\theta
\end{aligned} \tag{59}$$

$$\begin{aligned}
E\epsilon_\theta = &\frac{(A+B)}{2}(1+\nu)\frac{a^2}{r^2} \\
&- \frac{(A-B)}{2}\left[-4\nu\frac{a^2}{r^2} + 3(1+\nu)\frac{a^4}{r^4}\right]\cos 2\theta \\
&+ \frac{kr}{4}\left[(1+\nu)^2\frac{a^2}{r^2} - \nu(1+\nu)\frac{a^4}{r^4}\right]\sin\theta \\
&+ \frac{cr}{4}(1+\nu)\frac{a^4}{r^4}\sin\theta \\
&- \frac{(k-c)}{4}r\left[(1+5\nu)\frac{a^4}{r^4} - 4(1+\nu)\frac{a^6}{r^6}\right]\sin 3\theta
\end{aligned} \tag{60}$$

ϵ_r and ϵ_θ are relaxation strains resulting from drilling a hole in a plate for which the plane stress field in the unpenetrated plate is specified by equation (1)