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1965-31

K. J. Harte

Bottom
of the Spin-Wave Spectrum
in a Magnetic Film

28 July 1965

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

BOTTOM OF THE SPIN-WAVE SPECTRUM
IN A MAGNETIC FILM

K. J. HARTE

Group 24

TECHNICAL NOTE 1965-31

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ABSTRACT

The bottom of the spin-wave spectrum is examined for the case of a thin film magnetized in its plane. The bias field H_b and wave number k_b for the onset of first-order spin-wave instability at high microwave power is calculated. In addition, the critical thickness d_c , at which the uniform mode saturation process changes from first to second order, is obtained. These results are in fairly good agreement with experiment.

Accepted for the Air Force
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TABLE OF CONTENTS

	ABSTRACT	ii
I.	INTRODUCTION	1
II.	FIELD AND WAVE NUMBER AT THE BOTTOM OF THE SPECTRUM	1
	A. Thin-Film Limit	3
	B. Thick-Film Limit	7
	C. Intermediate Case	9
III.	CRITICAL THICKNESS	10
IV.	COMPARISON WITH EXPERIMENT	12
	REFERENCES	13

I. INTRODUCTION

Suhl¹ has shown that saturation of the uniform mode in ferromagnetic resonance will occur, at a critical microwave field, via the nonlinear coupling to the uniform mode of pairs of spin waves of energy $\hbar\omega_0/2$, and their ensuing unstable growth. ($\omega_0 = 2\pi \times$ drive frequency.) However, if such spin waves do not exist, these "first-order" processes cannot occur, and the uniform mode does not saturate until some higher field at which "second-order" processes appear: growth of spin waves of energy $\hbar\omega_0$. In the latter case at some value of the bias field H , less than its resonance value H_0 , spin waves of energy $\hbar\omega_0/2$ again become available and result in a subsidiary absorption. Thus the spin-wave energy at the bottom of the spectrum determines the nature of possible instabilities. As a measure of this bottom, we may take the field H_b , defined as the value of H for which the energy $\hbar\omega_{\vec{k}_b}$ of the lowest-lying spin wave is equal to $\hbar\omega_0/2$.

Calculation of H_b in a bulk ferromagnet is completely straightforward. In a thin film magnetized in its plane, on the other hand, the spin-wave spectrum is badly distorted at low wave numbers, essentially because of the creation of non-cancelling surface poles. It is the purpose of this note to calculate the resultant bottom field and wave number. As a by-product, we obtain the critical thickness at which the transition from first-order to second-order processes takes place. Finally, we compare our theory with experimental results.

II. FIELD AND WAVE NUMBER AT THE BOTTOM OF THE SPECTRUM

The dispersion relation for a film of thickness $d = 2L$, with a field \vec{H} in the film plane, is given by²

$$\omega_{\vec{k}}^2 = [\omega_h + \omega_e k^2 L^2 + \omega_m \chi(kL)] [\omega_h + \omega_e k^2 L^2 + \omega_m \tilde{\chi}(kL) \sin^2 \Phi] \quad (1)$$

where

$$\omega_h = HY \quad (2a)$$

$$\chi(\kappa) = \kappa^{-1} e^{-\kappa} \sinh \kappa \quad (2d)$$

$$\omega_e = \frac{2A}{M_o L^2} \quad (2b)$$

$$\tilde{\chi}(\kappa) = 1 - \chi(\kappa) \quad (2e)$$

$$\omega_m = 4\pi M_o \gamma \quad (2c)$$

and the wave vector \vec{k} lies in the film plane at an angle ϕ to \vec{H} . (A = exchange constant, M_o = saturation magnetization, γ = |gyromagnetic ratio|.) This dispersion law is based on a "thin-film approximation," which assumes excitations that are constant across the film thickness. Equation (1) reduces to the bulk spectrum as $kL \rightarrow \infty$ ($\chi \rightarrow 0$, $\tilde{\chi} \rightarrow 1$); in the thin-film limit $kL \rightarrow 0$, $\chi \rightarrow 1$ and $\tilde{\chi} \rightarrow kL$. The term $\omega_m \chi$ results from surface poles, which tend to cancel as $kL \rightarrow \infty$. The term $\omega_m \tilde{\chi} \sin^2 \phi$ results from volume poles, whose density vanishes as $kL \rightarrow 0$.

The field H_b and wave vector \vec{k}_b at the bottom of the spectrum are given by Eq. (1) and

$$\frac{\partial \omega_{\vec{k}}}{\partial k} = 0 \quad (3a)$$

$$\phi = 0, \quad (3b)$$

which become

$$\omega^2 = [\omega_b + \omega_e \kappa^2 + \omega_m \chi(\kappa)][\omega_b + \omega_e \kappa^2] \quad (4a)$$

$$2\omega_e \kappa [\omega_b + \omega_e \kappa^2 + \omega_m \chi(\kappa)] + [2\omega_e \kappa + \omega_m \chi'(\kappa)][\omega_b + \omega_e \kappa^2] = 0 \quad (4b)$$

where

$$\omega = \omega_{\vec{k}_b} \quad (5a)$$

$$\omega_b = H_b \gamma \quad (5b)$$

$$\kappa = k_b L \quad (5c)$$

(Note that ω/π is the drive frequency.)

An exact analytical solution to Eqs. (4) for κ and ω_b is impossible to obtain. However, good approximations may be found in the thin-film ($\kappa \ll 1$) and thick-film ($\kappa \gg 1$) limits; a numerical solution may be used for intermediate thicknesses.

A. Thin-Film Limit

With the definitions

$$\mu = \frac{\omega}{\omega_e} \quad (6a)$$

$$\beta = \frac{\omega}{\omega_m} \quad (6b)$$

$$b = \frac{\omega_b}{\omega_m} \quad (6c)$$

$$\eta = \kappa \mu^{-1} \quad (6d)$$

Eqs. (4) become

$$b^2 + b\chi - \beta^2 + \mu\eta^2(2b + \chi) + \mu^2\eta^4 = 0 \quad (7a)$$

$$b\chi' + 2\eta(2b + \chi) + \mu\eta^2(4\eta + \chi') = 0 \quad (7b)$$

where [from the $\kappa \ll 1$ expansion of Eq. (2d)]

$$\chi = 1 - \mu\eta + \frac{2}{3} \mu^2 \eta^2 + \dots \quad (8a)$$

$$\chi' = -1 + \frac{4}{3} \mu\eta - \mu^2 \eta^2 + \dots \quad (8b)$$

We solve Eqs. (7) for b and η by a perturbation expansion in the parameter μ by assuming

$$b = \sum_{n=0}^{\infty} b_n \mu^n \quad (9a)$$

$$\eta = \sum_{n=0}^{\infty} \eta_n \mu^n \quad (9b)$$

Then, with the help of Eqs. (6d) and (8), we equate to zero coefficients of each power of μ in Eqs. (7) to obtain

$$\begin{aligned} b_0 &= \frac{1}{2} (\sqrt{1 + 4\beta^2} - 1) & \eta_0 &= \frac{1}{4} \left(1 - \frac{1}{\sqrt{1 + 4\beta^2}}\right) \\ b_1 &= \eta_0^2 & \eta_1 &= \frac{2}{3} \eta_0^2 (1 - 18\eta_0 + 24\eta_0^2) \\ b_2 &= \eta_0 \eta_1 & & \text{etc. ,} \end{aligned} \quad (10)$$

to any desired order.

In general, the condition for validity of this solution is

$$\mu \ll 1 \quad (11a)$$

i. e. ,

$$L \ll \sqrt{\frac{A}{2\pi M_0^2}} \equiv \lambda_0 = 50 \text{ \AA}^* \quad (11b)$$

*For Permalloy ($A = 10^{-6}$ erg/cm; $4\pi M_0 = 10^4$ oe).

However, if

$$4\beta^2 \ll 1 \quad (12a)$$

i. e. ,

$$\frac{\omega}{\pi} \ll \frac{\omega_m}{2\pi} \approx 30 \text{ kMcps}^* \quad (12b)$$

then from Eqs. (6) and (10) we obtain

$$\omega_b = \frac{\omega^2}{\omega_m} [1 - \beta^2 + 2\beta^4 + \dots + \frac{1}{4}\sigma(1 - 6\beta^2 + \dots) + \frac{1}{12}\sigma^2 + \dots] \quad (13a)$$

$$\kappa = \frac{1}{2}\sigma[1 - 3\beta^2 + \dots + \frac{1}{6}\sigma + \dots] \quad (13b)$$

where

$$\sigma = \mu\beta^2 = \frac{\omega^2}{\omega_e \omega_m} \quad (14)$$

The conditions for validity are now (12b) and

$$\sigma \ll 2 \quad (15b)$$

i. e. ,

$$L \ll \frac{\gamma}{\omega} 4\sqrt{\pi A} \approx 500 \text{ \AA}^\dagger \quad (15b)$$

The line at the left in Fig. 1 shows H_b , as given by Eq. (13a), as a function of thickness for Permalloy with $\omega/\gamma = 1530$ oe. The corresponding values of $k_b = \kappa/L$ are plotted in Fig. 2.

B. Thick-Film Limit

For this case, we let

$$\delta = \mu^{-\frac{1}{3}} = \left(\frac{\omega_e}{\omega_m}\right)^{\frac{1}{3}} \quad (16a)$$

$$\zeta = (\kappa\delta)^{-1} \quad (16b)$$

[†] For Permalloy at $\omega/\pi = 8.5$ kMcps.

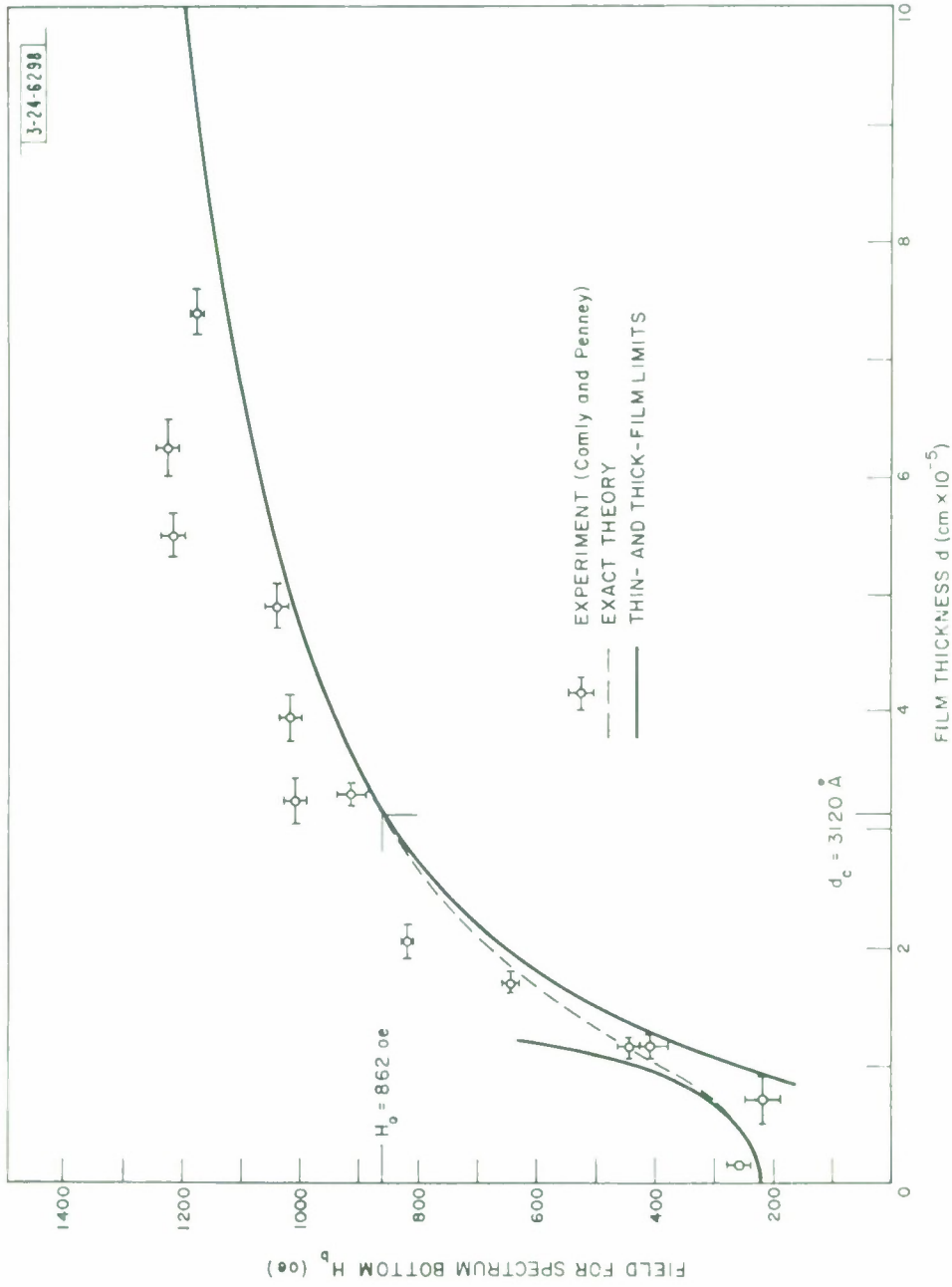


Fig. 1. Field for bottom of the spin-wave spectrum versus film thickness. Theoretical curves are based on the nominal values $A = 10^{-6}$ erg/cm, $4\pi M_0 = 10^4$ oe, $w/\gamma = 1530$ oe. The experiments were performed on 83% Ni-17% Fe films at about 8.56 kMcps.

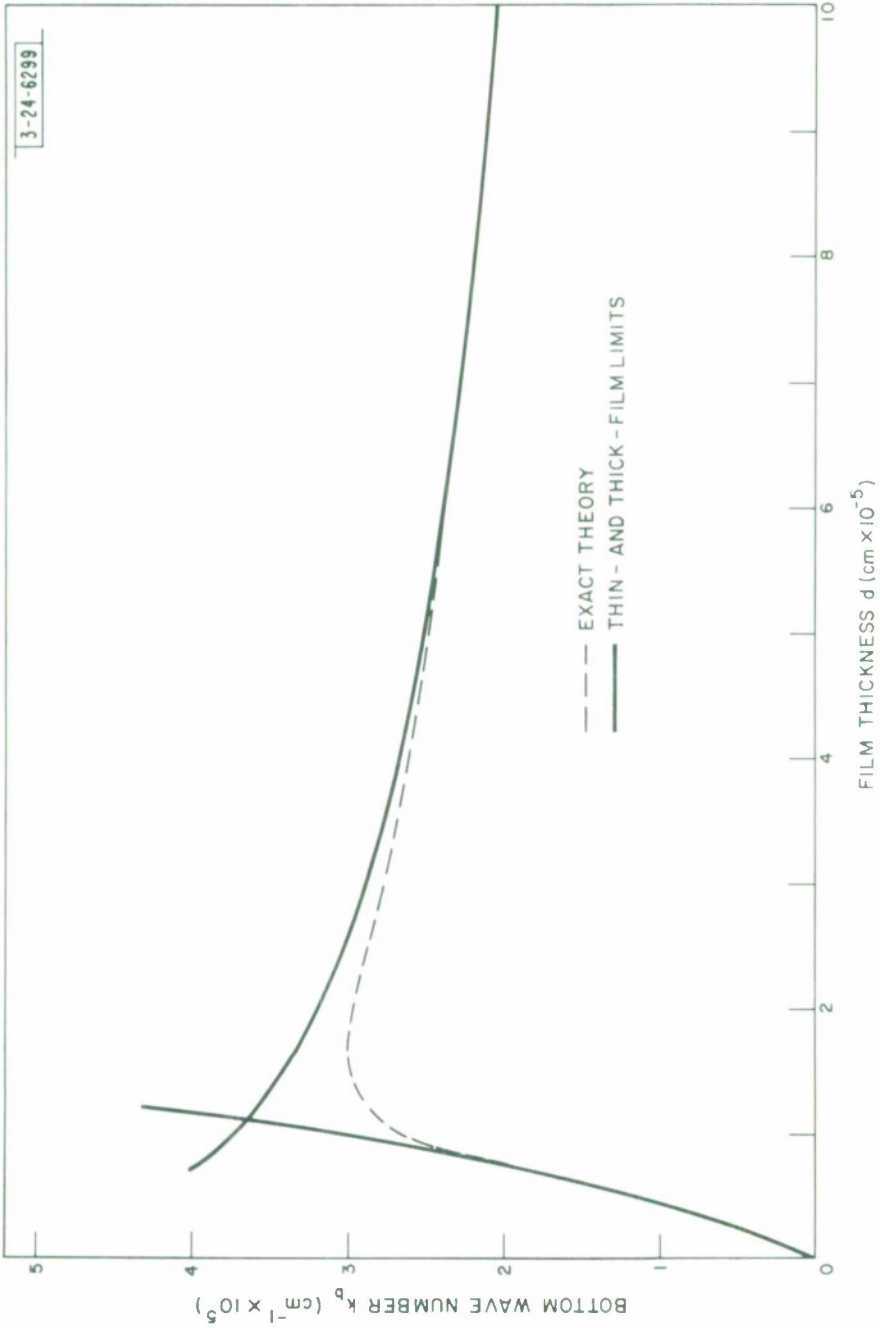


Fig. 2. Wave number of lowest lying spin wave versus film thickness at $H = H_b$. Film parameters are the same as in Fig. 1.

so that Eqs. (4) become

$$\zeta^4 (b^2 + b\chi - \beta^2) + \delta \zeta^2 (2b + \chi) + \delta^2 = 0 \quad (17a)$$

$$b\chi' \zeta^3 + \delta \chi' \zeta + 2\delta^2 \zeta^2 (2b + \chi) + 4\delta^3 = 0 \quad (17b)$$

where [from the $\kappa \gg 1$ expansion of Eq. (2d)]

$$\chi \approx \frac{1}{2} \delta \zeta \quad (18a)$$

$$\chi' \approx -\frac{1}{2} \delta^2 \zeta^2 \quad (18b)$$

Proceeding as before, we assume solutions of the form

$$b = \sum_{n=0}^{\infty} b^{(n)} \delta^n \quad (19a)$$

$$\zeta = \sum_{n=0}^{\infty} \zeta^{(n)} \delta^n \quad (19b)$$

and find

$$\begin{aligned} b^{(0)} &= \beta & \zeta^{(0)} &= 2 \\ b^{(1)} &= -\frac{3}{4} & \zeta^{(1)} &= \frac{1}{3\beta} \\ b^{(2)} &= \frac{1}{8\beta} & \zeta^{(2)} &= \frac{1}{6\beta^2} \\ b^{(3)} &= \frac{1}{48\beta^2} & & \text{etc.} \end{aligned} \quad (20)$$

or

$$\omega_b = \omega \left(1 - \frac{3}{4} \rho + \frac{1}{8} \rho^2 + \frac{1}{48} \rho^3 + \dots \right) \quad (21a)$$

$$\kappa = \frac{1}{2\delta} \left(1 - \frac{1}{6} \rho + \frac{1}{12} \rho^2 + \dots \right) \quad (21b)$$

where

$$\rho = \frac{\delta}{\beta} = \frac{1}{\omega} (\omega_e \omega_m^2)^{\frac{1}{3}} \quad (22)$$

The conditions for validity of this solution are

$$\rho \ll 1 \quad (23a)$$

i. e. ,

$$L \gg \left(\frac{Y}{\omega}\right)^{\frac{3}{2}} 4\pi\sqrt{2M_0}A \approx 800 \text{ \AA}^{\dagger} \quad (23b)$$

and

$$\delta \ll \frac{1}{2} \quad (24a)$$

i. e. ,

$$L \gg 2\sqrt{2} \lambda_0 \approx 150 \text{ \AA}^* \quad (24b)$$

Note in the bulk limit $L \rightarrow \infty$ that $\rho \rightarrow 0$ so that $\omega_b \rightarrow \omega$ and $k_b \rightarrow 0$. H_b , as given by three terms of Eq. (21a), is shown by the line on the right in Fig. 1, and k_b as determined from two terms of Eq. (21b) is shown in Fig. 2.

C. Intermediate Case

To find numerical solutions, it is convenient to solve Eq. (4b) for $\omega_b + \omega_e \kappa^2$ and substitute this into Eq. (4a), to obtain

$$\beta^2 (\mu\chi' + 4\kappa)^2 + 2\kappa\chi^2 (\mu\chi' + 2\kappa) = 0 \quad (25a)$$

$$\omega_b = -\omega_e \kappa \left(\frac{2\mu\chi}{\mu\chi' + 4\kappa} + \kappa \right) . \quad (25b)$$

Eq. (25a) may then be solved graphically for κ . H_b , as determined from Eq. (25b) is shown by the dashed line in Fig. 1. (Note that the interpolation between the two approximate solutions is smooth.) In Fig. 2 the exact numerical result for the wave number k_b is shown by a dashed line.

III. CRITICAL THICKNESS

The transition from second order to first order instability occurs at a critical thickness $d_c = 2L_c$ such that $\omega_{\vec{k}}$ at the bottom of the spin-wave spectrum is equal to half the uniform precession frequency, or

$$\omega^2 = \left(\frac{Y}{2}\right)^2 H_o (H_o + 4\pi M_o) . \quad (26a)$$

$$H_o = H_b(d_c) . \quad (26b)$$

The critical thickness is conveniently found from a plot of H_b vs. d (see Fig. 1). However, an approximate analytic expression for d_c may be obtained from Eqs. (26) and (21a). Using an iteration method, we find

$$d_c = 2\delta_c^{-\frac{3}{2}} \lambda_o \quad (27a)$$

where

$$\delta_c = \frac{4}{3} \frac{\beta(1-3\beta)}{(1+2\beta)} \left\{ 1 + \frac{1}{9} \frac{(1-3\beta)(2+13\beta)}{(1+2\beta)^2} \left[1 + \frac{1}{9} \frac{(1-3\beta)(2+13\beta)}{(1+2\beta)^2} \right]^2 + \frac{4}{81} \frac{(1-3\beta)(1-7\beta)}{(1+2\beta)^3} + \dots \right\} \quad (27b)$$

provided $\beta < 1/3$. If $\beta > 1/3$, first-order processes cannot occur for any thickness, including bulk samples. In Fig. 3 we plot $\delta_c^{3/2}$, as given by three terms of Eq. (26b), vs. β , with scales at top and right for Permalloy. An interesting feature is the existence of an absolute minimum critical thickness given by

$$(d_c)_{\min} = 61.6 \lambda_o = 3080 \text{ \AA}^* . \quad (28)$$

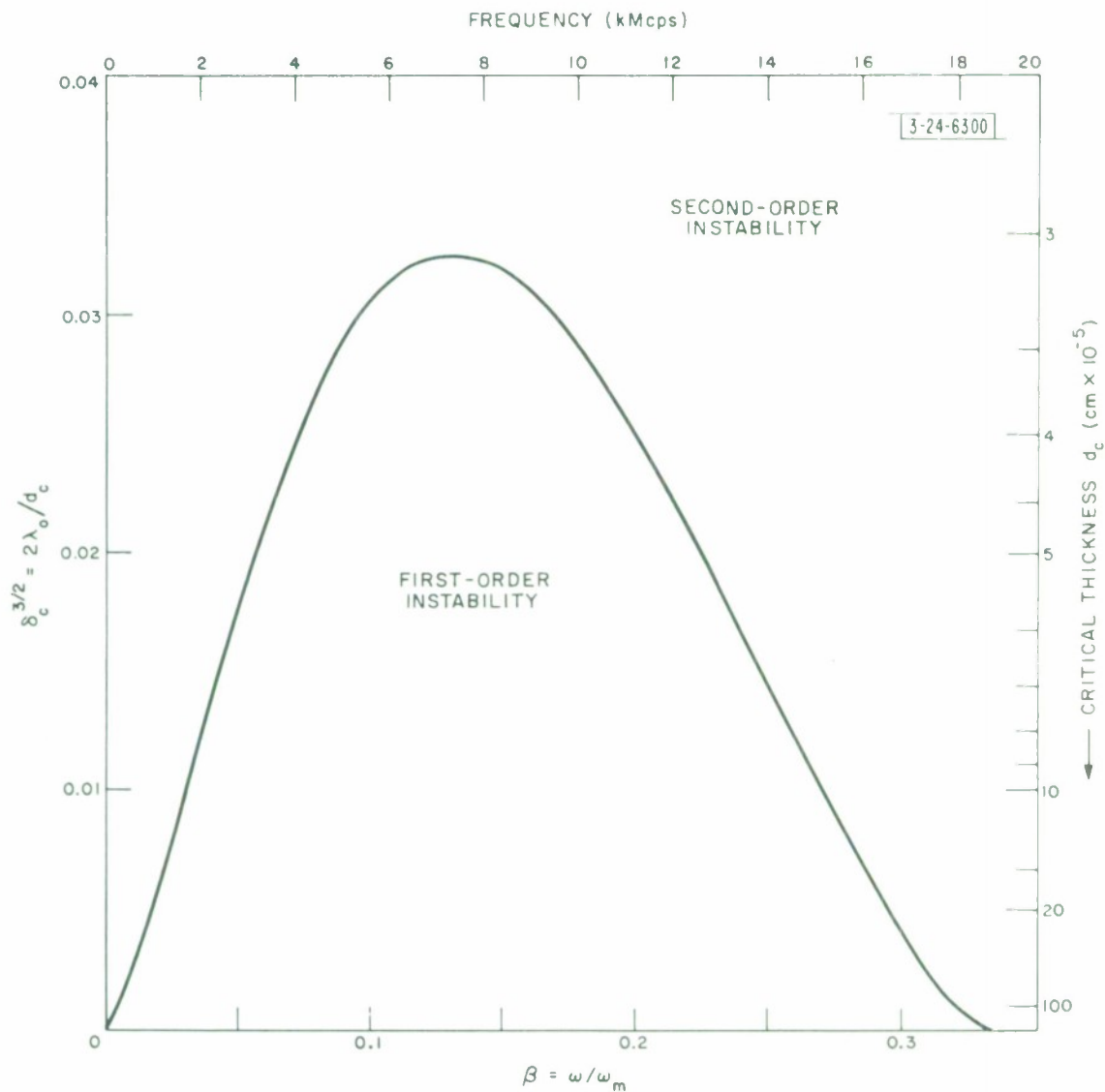


Fig. 3. Normalized inverse critical thickness versus normalized drive frequency. The scales at right and top are computed for $A = 10^{-6}$ erg/cm, $4\pi M_0 = 10^4$ oe.

IV. COMPARISON WITH EXPERIMENT

Comly and Penney³ have measured H_b at 8.56 kMcps in Permalloy films varying in thickness from 150 Å to 7400 Å. Their results are shown in Fig. 1, and are in fairly good accord with theory. The discrepancy probably results from the approximations used to derive Eq. (1): the true normal modes will vary somewhat across the film thickness, lowering the spectrum and therefore raising H_b , particularly for the thicker samples. Comly and Penney also found $2050 < d_c < 3250$ Å; the theoretical value of 3120 Å is consistent with these limits. A more complete account of these experimental results will be published in the near future.⁴

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