3ND-PBPO-2491

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AD619872

TECHNICAL REPORT

U. S. NAVAL APPLIED SCIENCE LABORATORY

NAVAL BASE

BROOKLYN 1, NEW YORK

3ND USNASL-9800-P-1A (REV. 7-63)

ANALYTICAL STUDIES OF HOLLOW SPHERES
FOR
LOWER DENSITY HIGHER STRENGTH BUOYANCY MATERIALS

SF 020-04-05, Task 1013

Lab. Project 9300-57, Progress Report #1

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16 AUG 1965

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ADMINISTRATIVE INFORMATION

Ref: (a) NAVAPLSCIENLAB Program Summary SF 020-04-05, Task 1013 of 1 May 1965.
(b) Timoshenko, S., "Theory of Elastic Stability," McGraw Hill Book Co., New York (1936).

1. As described in reference (a), a Research and Development Program to develop and investigate low density, high strength buoyancy materials for use in deep submergence vehicles is being conducted at the U. S. Naval Applied Science Laboratory. This project is being performed under the direction of C. K. Chatten, Branch Head.

INTRODUCTION

2. The U. S. Naval Applied Science Laboratory has developed a candidate syntactic foam buoyancy material for deep submersible vehicles. This buoyancy material, identified as NASL ML-B3, consists of small hollow glass microspheres on the order of 10 to 100 microns (.0004 to .004 in. diameter) in a rigid epoxy resin matrix. This material has a density of .6-.7 gms/cm^3 (37-44 $\#/ft^3$) and is capable of withstanding a uniform hydrostatic pressure of 10,000 psi.

3. In order to obtain a higher strength, lower density material several approaches can be taken together or separately; these are:

a. Develop and use a lower density, higher strength matrix consisting of other resin systems.

b. Incorporate lower density hollow microspheres in the existing or improved new matrix, or

c. Incorporate higher strength, larger hollow spheres in a syntactic foam matrix (1-4 in. diameter).

4. Based on the existing state of the art of glass and resin technology, the third approach offers the greatest immediate promise. Spheres have the best geometric shapes for resisting hydrostatic compression, and hollow shapes have the added property of very low density. Therefore, it is mainly via larger hollow thin spheres that the overall density of the system can be reduced with the proper strength requirements.

OBJECT

5. The object of this program is to develop a low density, high strength buoyancy system of materials with a target density of 0.3 to 0.4 gm^3/cm^3 (19-25 $\#/ft^3$) which will be able to withstand 13,500 psi hydrostatic pressure. It is the intent of this phase of the program to perform analytical studies of significant variables for glass, ceramics, and metal hollow spheres in order to determine promising optimum materials and sphere sizes.

APPROACH

6. To determine the significant variables the following assumptions are made regarding a thin sphere (see Figure 1):

a. It is under uniform external pressure

b. Has no constraint

c. Has uniform outer radius

d. Has uniform inner radius

e. Will fail by elastic buckling.

7. Some development effort will be required to meet or closely approach the conditions assumed above, since the limitations of present production methods may introduce some variations from these ideal conditions. Variations between actual and theoretical test results will be taken into account during the evaluation phase of the program. The intent is to approach the "ideal" conditions as closely as possible.

8. The formula for critical unit external pressure, P', at which elastic buckling of a thin sphere occurs under uniform external pressure, was first investigated by Zoelly as contained in reference (b) and is as follows:

$$P' = \frac{2Et^2}{r_o^2 \sqrt{3(1 - \nu^2)}} \qquad (1)$$

where

E = Modulus of Elasticity

ν = Poissons Ratio

r_o = Outer Radius of Sphere

r_i = Inner Radius of Sphere

t = $r_o - r_i$ = Thickness of Shell

P' = Hydrostatic Pressure At Which Elastic Buckling Occurs.

This equation can be used to obtain optimum densities with the required strength considerations when applying the proper boundary conditions.

For any one material spherical shell, equation (1) can be rearranged:

(1a) $$\frac{t^2}{r_o^2} = \frac{P' \sqrt{3(1 - \nu^2)}}{2E}$$

where

E is constant for any one material

ν is constant for any one material

P' is constant and equal to 13,500 psi for this particular design application.

$\therefore$ By Inspection

(1b) $$\frac{t^2}{r_o^2} = K_1^2$$

$$\frac{t}{r_o} = K_1$$

Where K_1 is a constant for a particular material at a specific design pressure.

The density of a hollow sphere system:

(2) $$\rho \text{ sphere} = \frac{\text{Weight of Material Sphere}}{\text{Volume of Sphere}}$$

(2a) $$\text{Volume of Sphere} = \frac{4}{3}\pi r_o^3$$

(2b) $$\text{Weight of Sphere} = \rho \text{ material } \frac{4}{3}\pi (r_o^3 - r_i^3)$$

(2c) $$\rho \text{ sphere} = \frac{\rho \text{ material } \frac{4}{3} (r_o^3 - r_i^3)}{\frac{4}{3}\pi r_o^3}$$

Simplifying we have:

$$\rho \text{ sphere} = \frac{\rho \text{ material } (r_o^3 - r_i^3)}{r_o^3} \tag{2d}$$

since $\quad r_i^3 = (r_o - t)^3$

Expanding

$$r_i^3 = (r_o^3 - 3tr_o^2 + 3r_ot^2 - t^3)$$

Substituting into the density equation (eq. 2d)

$$\rho \text{ sphere} = \rho \text{ material } \frac{r_o^3 - (r_o^3 - 3tr_o^2 + 3r_ot^2 - t^3)}{r_o^3} \tag{2e}$$

$$\rho \text{ sphere} = \rho \text{ material } \frac{3tr_o^2 - 3r_ot^2 + t^3}{r_o^3} \tag{2f}$$

$$\rho \text{ sphere} = \rho \text{ material } \frac{3t}{r_o} - \frac{3t^2}{r_o^2} + \frac{t^3}{r_o^3} \tag{2g}$$

From equation (1b)

$$K_1 = \frac{t}{r_o}$$

Substituting eq. (1b) into equation (2g)

$$\rho \text{ sphere} = \rho \text{ material } [3K_1 - 3K_1^2 + K_1^3] \tag{2h}$$

It can be seen from equation (2h) that for any one particular material at the same applied critical compression pressure, the optimum sphere density is a constant for any r_o chosen. (See Appendix A, and Table I).

PROCEDURE

9. An extensive group of materials were reviewed. The candidate material selected and listed in Table I (Material and Corresponding Hollow Sphere Properties) were based on the following requirements:

a. High Modulus of Elasticity

b. Low Material Density

c. Possible ability to be fabricated into desired shapes.

RESULTS

10. Figures 2 to 6 show the important variables influencing the system density, resulting buoyancy and their interrelationships; they are:

Figure 2 - Outer Diameter to Thickness Ratios of Hollow Spheres

Figure 3 - Outer Diameter to Volume Ratios of Hollow Spheres

Figure 4 - Net Buoyancy vs. Outer Sphere Diameter

Figure 5 - Net Buoyancy vs. Sphere Weight

Figure 6 - Comparison of Material and Corresponding Sphere Densities.

11. These relationships have been calculated from equations of the preceding section and Appendix A. The basic guidelines for selection will be made from these figures and Table I for the candidate materials showing optimum (low) density and therefore greatest buoyant conditions to withstand 13,500 psi hydrostatic pressure (see Discussion of Results and Conclusion).

DISCUSSION OF RESULTS

12. Figure 2 illustrates Outer Diameter to thickness Ratios of Hollow Spheres. As shown in the analysis, the ratio of $t/D_o = 1/2K_1 = K_2$ (where D_o is the outer diameter) and is a constant for any one material at a particular design pressure. It can be seen in Figure 2 and in Table I that the ceramics have the lowest K_1; next in line are the metals and finally glass. K_1 is an indication of the sphere thickness per unit outer radius.

13. Figure 3 depicts diameter as a function of volume. The top curve shows the total volume of the sphere which includes both the shell and the hollow center. The lower curves show the corresponding volume of material for each diameter, therefore relative percentages by volume can be quickly determined. Note that glass has the greatest volume of material per unit diameter followed by the metals. Beryllia, silicon carbide and alumina have negligible differences in their corresponding volumes.

14. Figure 4 illustrates the net buoyancy as a function of outer sphere diameter. The materials with the greatest buoyancy are the ceramics, then glass, followed by the metals. It is interesting to note that glass, which has the highest t/r_0 ratio, has a higher buoyancy than the metals. Since the spheres will eventually be contained in a matrix, the largest diameter sphere is not necessarily the optimum choice. A combination of smaller diameter spheres may give an overall more buoyant system (See Appendix A and Table I), depending on size, packing and geometric shape of the final system.

15. Figure 5 illustrates the net buoyancy as a function of the sphere weight. By arbitrarily selecting a weight and crossing the graph horizontally, it can be seen that there is a significant variation in buoyancy for the spectrum of different candidate material hollow spheres.

16. Figure 6 illustrates the comparisons of material and hollow sphere density. While glass has the lowest material density, its sphere density lies in between the ceramics group and the metals group. The ceramics (alumina, beryllia and silicon carbide) have higher material densities and yet much lower sphere densities. This apparent inconsistency is due to the thickness of material necessary in the design to withstand the critical hydrostatic pressure of 13,500 psi. Table I indicates that glass has the highest K_1 value. It follows, then, from the preceding discussion that the final sphere density is a function of (1) K_1 (the ratio of thickness to outer radius), (2) the material density. This relationship is given in paragraph 8.

17. Table I, which lists all candidate materials with their corresponding material and hollow sphere properties, shows that the ceramics have the lowest sphere densities (with beryllia as the lowest and alumina the highest of the ceramics group), followed by glass and then the metals.

CONCLUSIONS

18. From the theoretical analysis described herein the following conclusions can be drawn:

a. For a hollow sphere of a particular material at 13,500 psi external pressure the optimum sphere density is a constant for any desired outer diameter.

b. The following candidate materials are potentially suitable and are listed in order of ascending optimum sphere densities at 13,500 psi external pressure:

MATERIAL	OPTIMUM SPHERE DENSITY
Beryllia	7.8#/ft^3
Silicon Carbide	8.1#/ft^3
Alumina	10.8#/ft^3
Glass	14.8#/ft^3
Aluminum	16.2#/ft^3
Titanium	21.5#/ft^3
Steel	27.8#/ft^3

19. The preceding conclusions are based on an analytical solution of a hollow sphere and will be used as the basic guideline for choice. The final selection of the optimum suitable candidate will be based on the following factors and their relative importance:

a. Analytical solution

b. Cost and availability of materials

c. Manufacturing methods

d. Uniformity of inner and outer diameters

e. Parameters dealing with environmental structural integrity, such as the effects of sympathetic implosion

f. Integration into the matrix

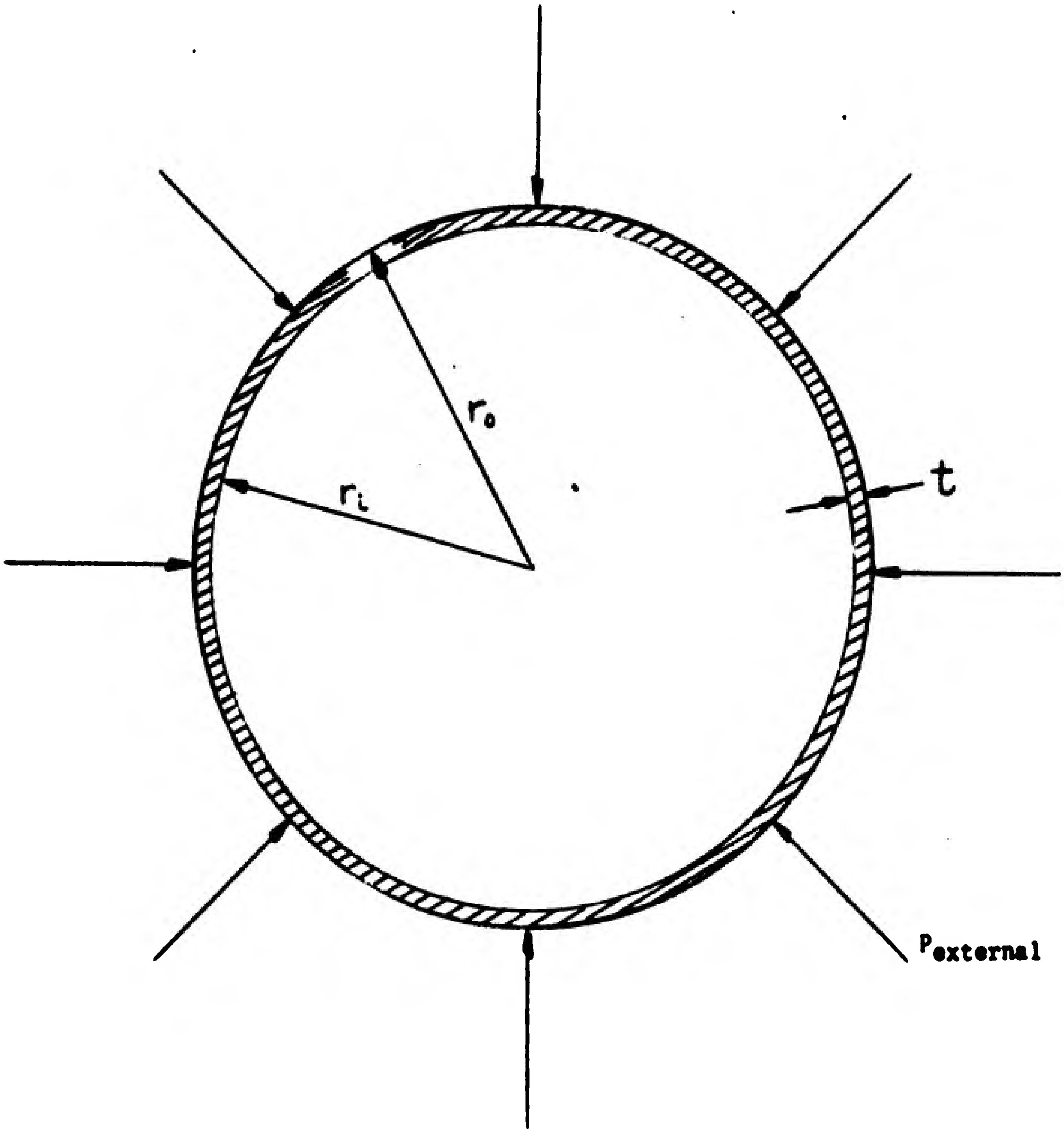
g. Test results.

FUTURE WORK

20. Future work will continue in the areas listed in Section 19. It is planned to evaluate glass and alumina spheres first, because the state of the art of fabrication of these materials in hollow sphere shape appears to be more advanced than for other lower hollow sphere density materials.

Figure 1

THIN SPHERE UNDER UNIFORM EXTERNAL PRESSURE
(NO CONSTRAINT)



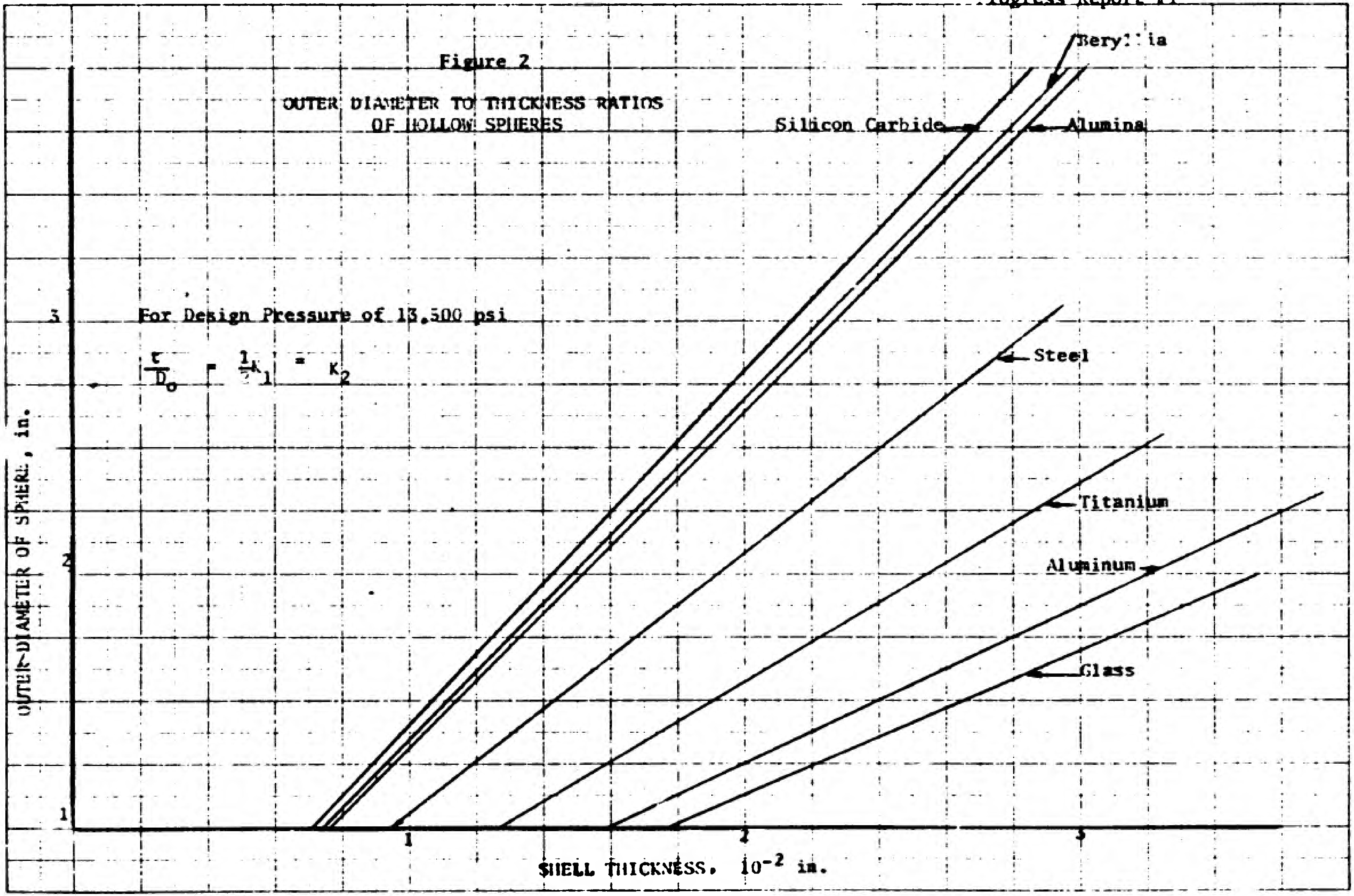


Figure 2
OUTER DIAMETER TO THICKNESS RATIOS
OF HOLLOW SPHERES
For Design Pressure of 13,500 psi
Beryllia
Silicon Carbide
Alumina
Steel
Titanium
Aluminum
Glass
OUTER-DIAMETER OF SPHERE, in.
SHELL THICKNESS, 10^{-2} in.
1
2
3
1
2
3

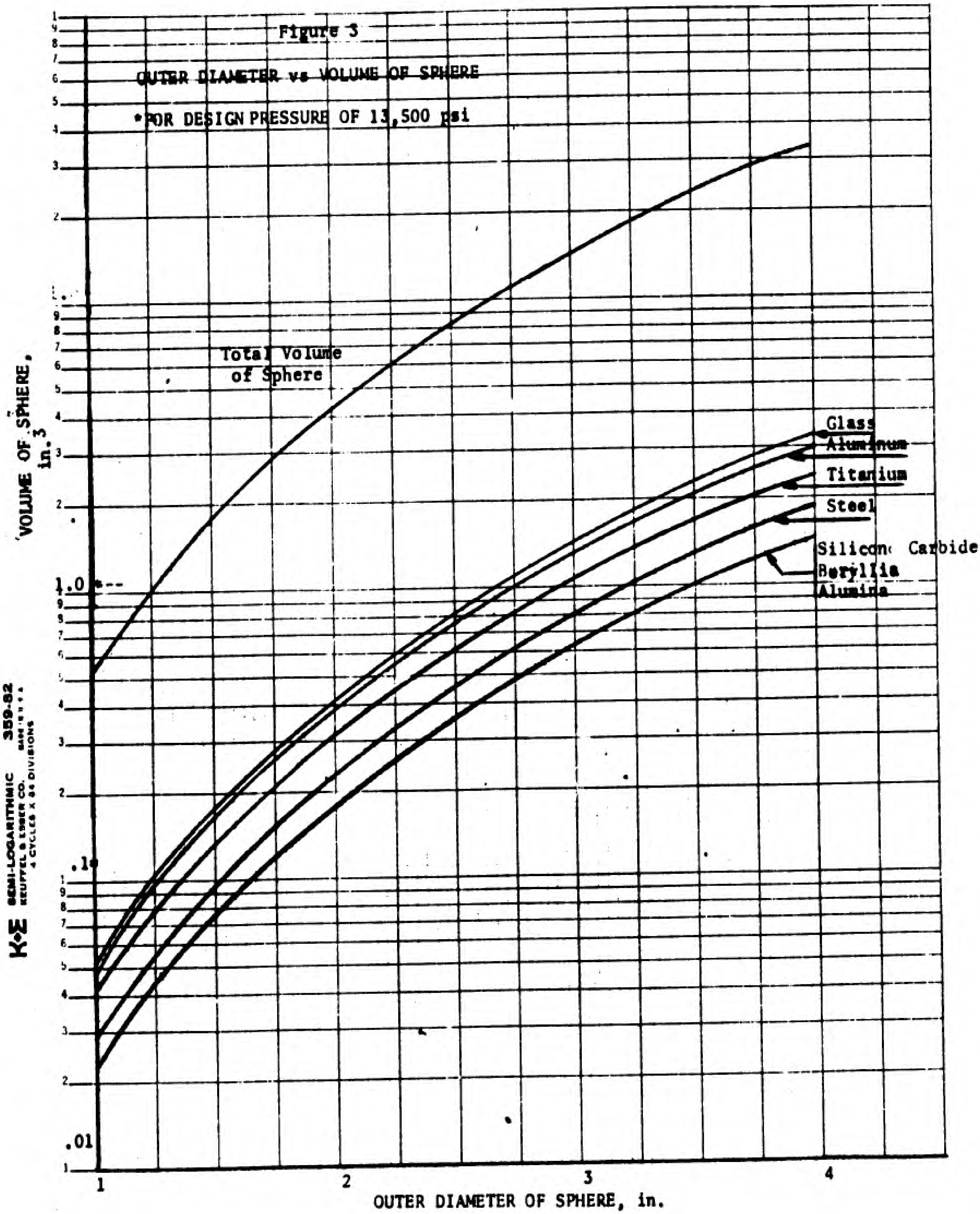


Figure 3
OUTER DIAMETER vs VOLUME OF SPHERE
*FOR DESIGN PRESSURE OF 13,500 psi
Total Volume
of Sphere
Glass
Aluminum
Titanium
Steel
Silicon Carbide
Beryllia
Alumina
VOLUME OF SPHERE, in.3
1.0
.1
.01
1
2
3
4
OUTER DIAMETER OF SPHERE, in.
K-E SEMI-LOGARITHMIC 359-82
KEUFFEL & ESSER CO.
4 CYCLES X 84 DIVISIONS

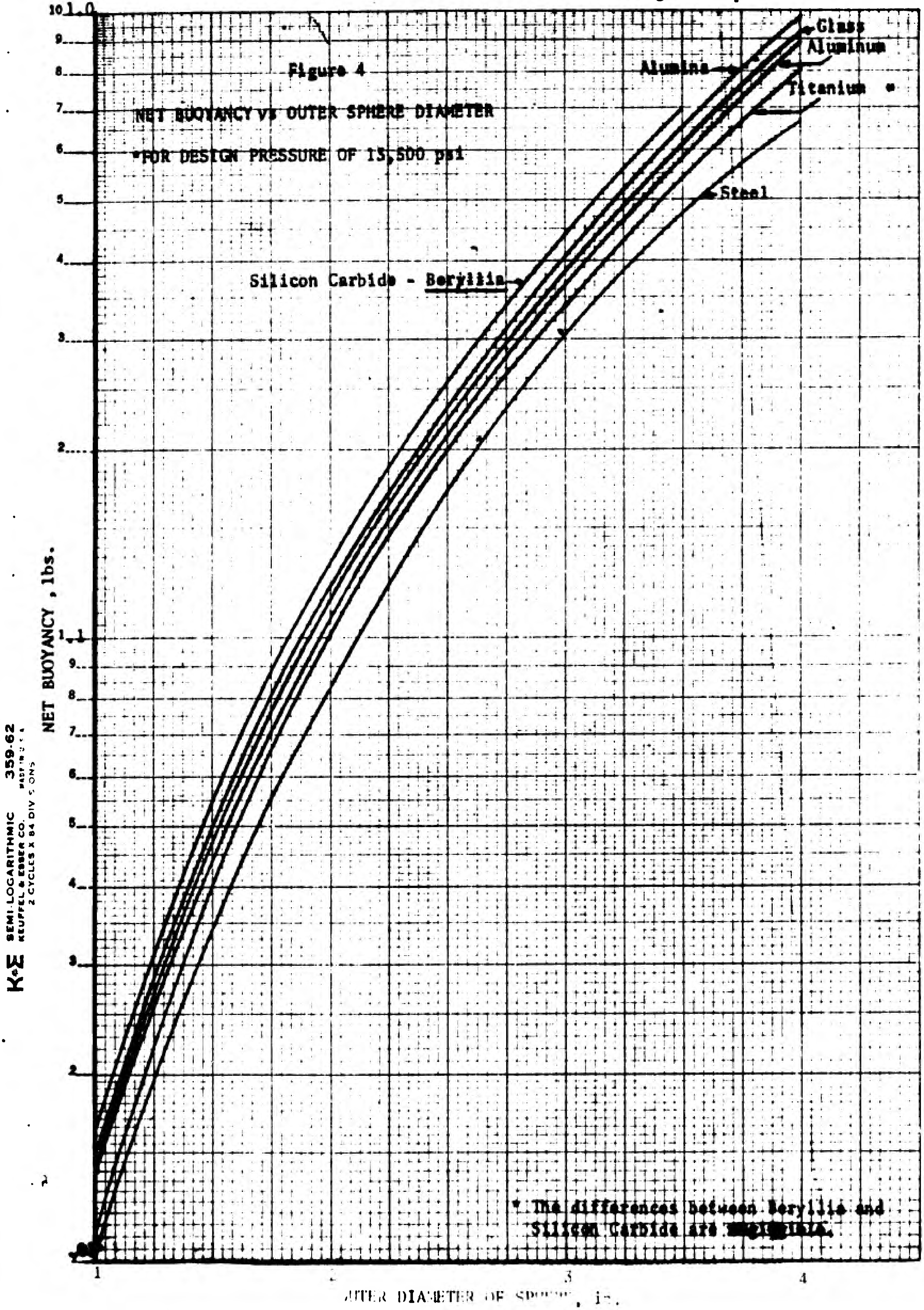


Figure 4
NET BUOYANCY vs OUTER SPHERE DIAMETER
*FOR DESIGN PRESSURE OF 13,500 psi
Glass
Aluminum
Alumina
Titanium
Steel
Silicon Carbide - Beryllia
* The differences between Beryllia and Silicon Carbide are negligible.
NET BUOYANCY, lbs.
OUTER DIAMETER OF SPHERE, in.
K+E SEMI-LOGARITHMIC 359-62 KEUFFEL & ESSER CO. 2 CYCLES X 84 DIVISIONS

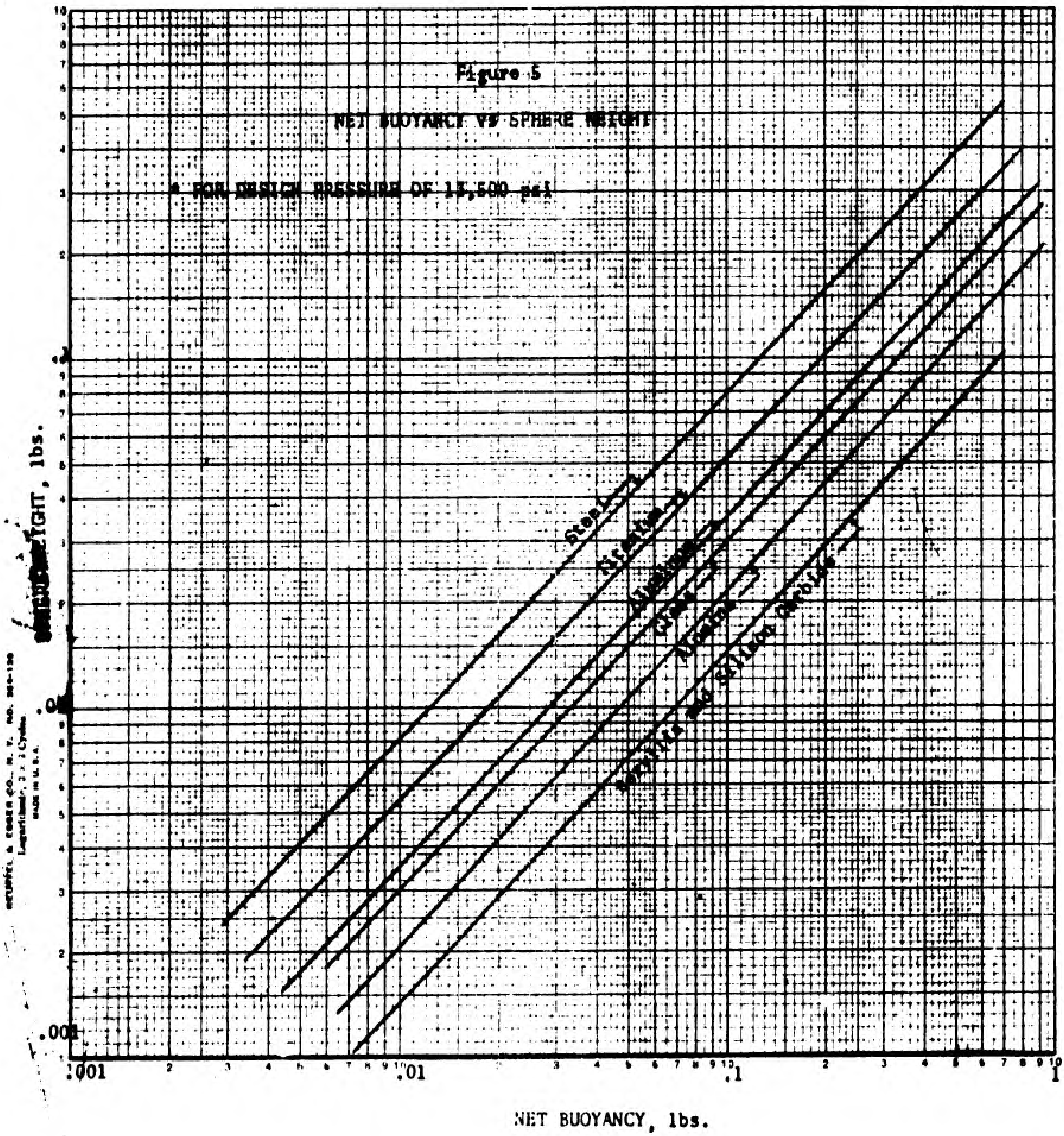


Figure 5
NET BUOYANCY VS SPHERE WEIGHT
* FOR DESIGN PRESSURE OF 13,500 psi
Steel
Titanium
Aluminum
Glass
Alumina
Beryllia and Silicon Carbide
.001
.01
.1
1
NET BUOYANCY, lbs.
WEIGHT, lbs.

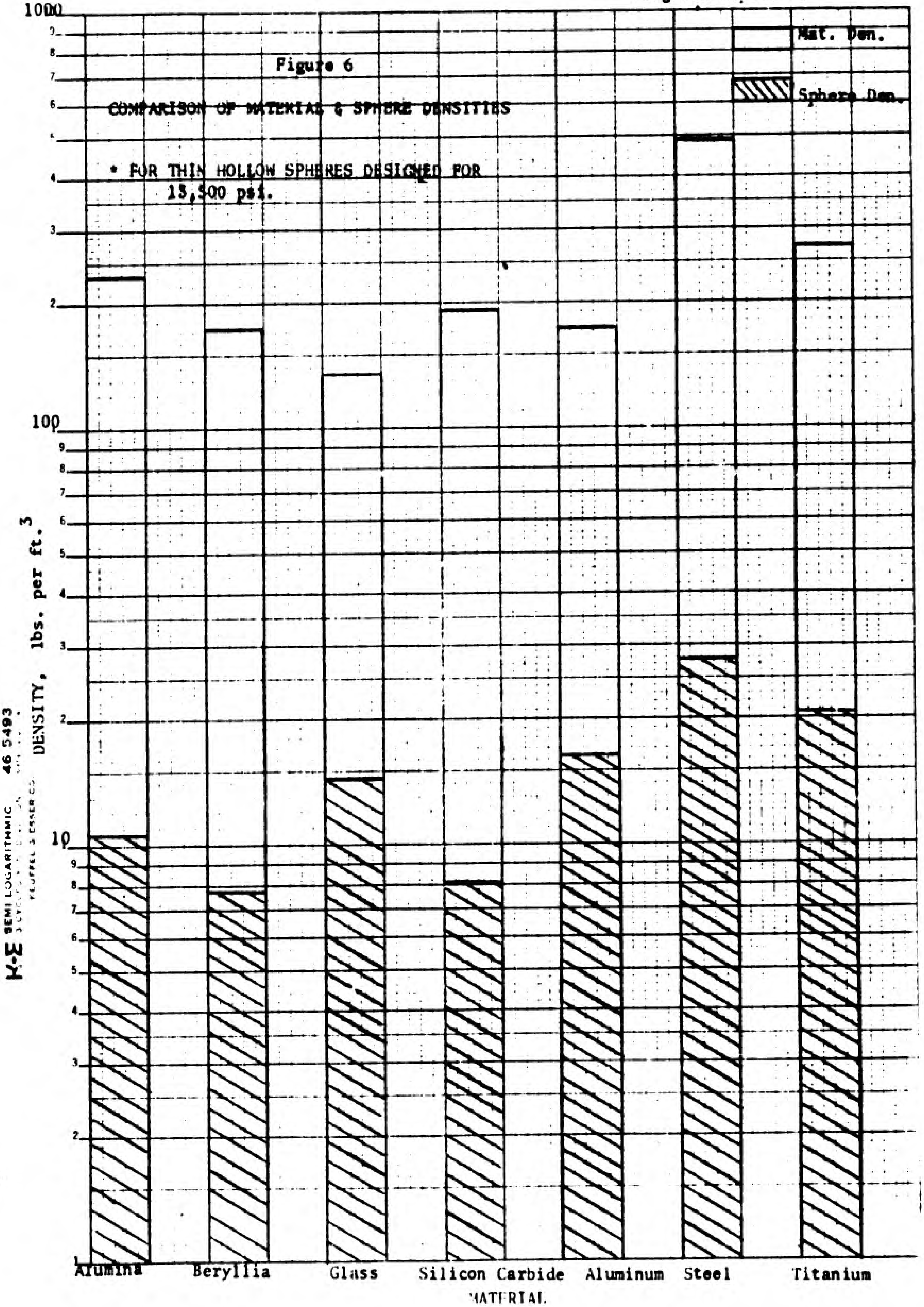


Figure 6
COMPARISON OF MATERIAL & SPHERE DENSITIES
* FOR THIN HOLLOW SPHERES DESIGNED FOR 13,500 psi.
Mat. Den.
Sphere Den.
1000
100
10
1
DENSITY, lbs. per ft.3
K•E SEMI LOGARITHMIC 46 5493
Alumina
Beryllia
Glass
Silicon Carbide
Aluminum
Steel
Titanium
MATERIAL

TABLE I

TABULATION OF MATERIAL AND CORRESPONDING HOLLOW SPHERE PROPERTIES

Material	E	$K \times 10^2$	Material Density	System Density	System Buoyancy Per Unit Volume
	psi $\times 10^{-6}$		#/ft^3	#/ft^3	#/ft^3
Alumina	50.	1.52	238	10.624	51.8
Aluminum	10.8	3.18	173	16.2	46.2
Berrylia	51.	1.48	182	7.821	54.6
Glass	9.1	3.54	139	14.254	48.1
Silicon Carbide	56.	1.40	192	8.148	54.3
Steel	30.	1.94	490	27.8	34.6
Titanium	17.5	2.54	276	21.5	40.9

APPENDIX A

DERIVATION OF CONSTANT BUOYANCY PER UNIT VOLUME IN K TERMS

Buoyancy = Wt. of Water Displaced - Wt. of Hollow Sphere

(3) $$B = \frac{4}{3}\pi r_o^3\ \rho_w - \frac{4}{3}\pi (r_o^3 - r_i^3)\ \rho_{mat}$$

(4) $$\frac{\text{Buoyancy}}{\text{Vol}} = \frac{B}{\frac{4}{3}\pi\ r_o^3} = \frac{\frac{4}{3}\pi r_o^3\ \left\{\rho_w - \left(1 - \frac{r_i^3}{r_o^3}\right) \rho_{mat}\right\}}{\frac{4}{3}\pi\ r_o^3}$$

$$= \rho_w - \left(1 - \frac{r_i^3}{r_o^3}\right) \rho \text{ material}$$

From the previous analysis

$$r_i^3 = (r_o^3 - 3tr_o^2 + 3r_o t^2 - t^3)$$

Substituting into equation (4)

(4a) $$\frac{\text{Buoyancy}}{\text{Vol}} = \rho_w - \frac{r_o^3 - (r_o^3 - 3tr_o^2 + 3r_o t^2 - t^3)}{r_o^3} \rho \text{ material}$$

After eliminating and cancelling terms

(4b) $$\frac{\text{Buoyancy}}{\text{Vol}} = \rho_w - (3K - 3K^2 + K^3)\ \rho \text{ material}$$

It can be seen from equation 4b that for any one particular material at the same applied critical compression pressure the buoyancy per unit volume is a constant for any r_o chosen.