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NOLTR 64-69

THE PRESENTATION OF THE PROPELLANT
FLOW IN A CONSTANT ACCELERATION GUN
BY THE METHOD OF CHARACTERISTICS

NOL

15 JULY 1964



UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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NOLTR 64-69

Ballistics Research Report 128

THE PRESENTATION OF THE PROPELLANT FLOW
IN A CONSTANT ACCELERATION GUN
BY THE METHOD OF CHARACTERISTICS

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ABSTRACT: The propellant flow in a constant acceleration gas gun has been presented by the method of characteristics. The presentation has been done in Eulerian as well as Lagrangian coordinates. To illustrate the properties of constant acceleration flows and to facilitate the investigation by the method of characteristics a simple means of generating the flow has been examined in more detail. The equations describing constant acceleration flows have been reviewed and extended.

PUBLISHED SEPTEMBER 1965

U. S. NAVAL ORDNANCE LABORATORY
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**THE PRESENTATION OF THE PROPELLANT FLOW IN A CONSTANT
ACCELERATION GUN BY THE METHOD OF CHARACTERISTICS**

The investigation presented in this report is part
of the continuing effort to improve the performance
of hypervelocity laboratory guns.

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List of Symbols

a	speed of sound
c_p	specific heat at constant pressure
g	gravitational acceleration
h	enthalpy
p	pressure
s	Lagrangian space coordinate
t	time
T	temperature
x	Eulerian space coordinate
α	constant acceleration
γ	ratio of specific heats
ρ	density

Subscripts

o	condition at time $t = 0$
x	conditions along x - axis
s	conditions along s - axis

Superscripts

$*$	see Eq. 8
$-$	average value and initial condition

INTRODUCTION

It was first reported by Stanyukovich (Ref. 1) that the launching of a projectile from a gas gun can be accomplished with constant pressure at the base of the projectile if a particular kind of propellant flow is utilized in the gun barrel. Stanyukovich derived the equation of the required flow from the assumption that the flow velocity is a function of time only. Independently of Stanyukovich, Curtis (Ref. 2) and the author (Ref. 3) arrived at the same analytical solution, but used different approaches. Curtis derived the solution from an analogy with the earth's atmosphere, the author from the necessary and sufficient condition to maintain constant pressure of each element of a fluid in motion. Various investigators have elaborated on the solution and suggested modifications of two-stage gas guns to approximate the conditions for a constant base pressure launching of a projectile (Refs. 2 through 6).

In the present report the particular flow required in the gun barrel will be studied by the method of characteristics. The investigation will not be restricted to the propellant flow in the gun barrel, but will be extended to the more general flow phenomenon, which has been termed "constant acceleration flow" by the author. To facilitate the presentation by the method of characteristics the production of a constant acceleration flow by a simple means will be investigated in more detail. The constant acceleration flow equations will be reviewed briefly, for reason of reference, and some relations will be derived which have not been given elsewhere.

CONSTANT ACCELERATION FLOW EQUATIONS

Assuming a one dimensional flow in the absence of viscosity and heat conduction, the author has shown, in reference 3, that the pressure of each fluid element can only remain constant in time if no relative velocity variation exists throughout the flow. This is obviously the case for a fluid at rest or in constant velocity motion. But the fluid also exhibits no relative velocity variation if the same force is acting on the unit mass throughout the fluid at each instant of time. In the absence of body forces, pressure times area forces are active only and it follows that the force is constant and thus produces a flow of constant acceleration.

The proof, that the "no relative velocity variation" is a necessary and sufficient condition for the pressure of each fluid element to remain constant in time, follows from the continuity equation

$$\frac{dp}{dt} + \rho \frac{\partial u}{\partial x} = 0 \quad (1)$$

with $\partial u / \partial x = 0$ one obtains $dp/dt = 0$ and it can be shown (see Ref. 3) that $dp/dt = 0$ as a consequence of $dp/dt = 0$ independent of both the process by which the flow has been generated and the equation of state of the fluid used. For the strictly one-dimensional flow, Euler's equation of motion becomes with $\partial u / \partial x = 0$

$$\frac{\partial u}{\partial t} = \alpha = -1/\rho \frac{\partial p}{\partial x} \quad (2)$$

where α is a constant.

The constant acceleration flow exhibits a pressure gradient which does not vary with time and a constant temporal

velocity gradient. A temperature gradient is present in the flow unless the flow has been produced by an isothermal process. Using Lagrangian coordinates one can easily show that none of the properties of each fluid element changes with time. The integration of the equation of motion leads to

$$\alpha x + \int_{p_0}^p \frac{dp}{\rho} = f(t)$$

where p_0 is a reference pressure and $f(t)$ is the time dependent constant of integration. Since neither p nor ρ depend on time, differentiation with respect to time gives

$$f'(t) = \alpha \frac{dx}{dt} = \alpha u = \alpha^2 t$$

if $u = 0$ for $t = 0$. The general equation of a constant acceleration flow then becomes

$$\int_{p_0}^p \frac{dp}{\rho} + \alpha x - \frac{1}{2} \alpha t^2 = 0 \quad (3)$$

For an isentropic p - ρ relation of an ideal gas one obtains from Eq. 3

$$\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{\gamma-1}{\gamma} \frac{p_0}{\rho_0} \alpha x + \frac{\gamma-1}{2\gamma} \frac{p_0}{\rho_0} \alpha^2 t^2 \quad (4)$$

where ρ_0 and p_0 denote their values at $x = 0$ and $t = 0$. Introducing the speed of sound in Eq. 4 we have

$$a^2 = a_0^2 - (\gamma-1) \alpha x + \frac{\gamma-1}{2} \alpha^2 t^2 \quad (5)$$

where $a = a_0$ for $x = 0$ and $t = 0$. This equation has first been derived by Stanyukovich (Ref. 1)

CONSTANT ACCELERATION FLOW PROPERTIES

The simplest and most illustrative way to produce a constant acceleration flow is to set a horizontal pipe enclosing a certain amount of fluid into motion of constant acceleration. Depending on the degree of acceleration and the length of the pipe, the transition of the enclosed gas from rest into the accelerated motion may or may not lead to the formation of a shock wave. In order to avoid this it is assumed here that the acceleration α is approached sufficiently slowly. The adjustment of the enclosed gas to constant acceleration motion then has taken place isentropically. To describe the conditions within the gas in Eulerian coordinates, the time will be counted from the instant the acceleration α is reached. The origin of the coordinate system is placed at the location of the reference pressure p_0 (Eq. 4) at time $t = 0$ and held stationary. The positive x -axis points into the direction of motion of the fluid. The conditions in each cross section of fluid are then given as functions of position and time by Eqs. 4 and 5. Simpler expressions of the constant acceleration flow are obtained by means of Lagrangian coordinates. The coordinate system then moves with the fluid and the origin is attached to the cross section which exhibits the reference pressure p_0 after the acceleration α is reached. Since none of the gas properties change with time the conditions throughout the fluid are, according to Eqs. 4 and 5, given by

$$\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{\gamma-1}{\gamma} \frac{p_0}{p_0} \alpha s = \left(\frac{p}{p_0}\right)^{\gamma-1} \quad (6)$$

and

$$a^2 = a_0^2 - (\gamma-1) \alpha s \quad (7)$$

The coordinate s , used here for reason of distinction, represents the distance of a given fluid particle from the reference cross section at the time α is reached. The values of s increase in the direction of the accelerated motion. In describing the flow phenomenon further, we will use Lagrangian presentation for reason of convenience.

The above equations indicate that the pressure p , the density ρ , and the sound speed become zero for a certain coordinate s^* . This cross section is given by

$$s^* = \frac{\gamma}{\gamma-1} \frac{p_0}{\rho_0 \alpha} = \frac{a_0^2}{(\gamma-1)\alpha} \quad (8)$$

A vacuum thus exists in that part of the pipe for which $s = s^*$.

The temperature distribution is obtained from Eq. 7 as

$$T = T_0 - \frac{\alpha}{c_p} s \quad (9)$$

where c_p is the specific heat at constant pressure. The temperature thus decreases linearly with s and becomes zero for $s^* = c_p T_0 / \alpha$. The enthalpy also decreases linearly with s according to

$$h = h_0 - \alpha s \quad (10)$$

with zero enthalpy for the particle at $s^* = h_0 / \alpha$.

The variation of pressure, density temperature, and sound speed as a function of s are shown in Fig. 1. The pressure and density approach gradually with horizontal tangent the value zero. The speed of sound according to its parabolic form of variation

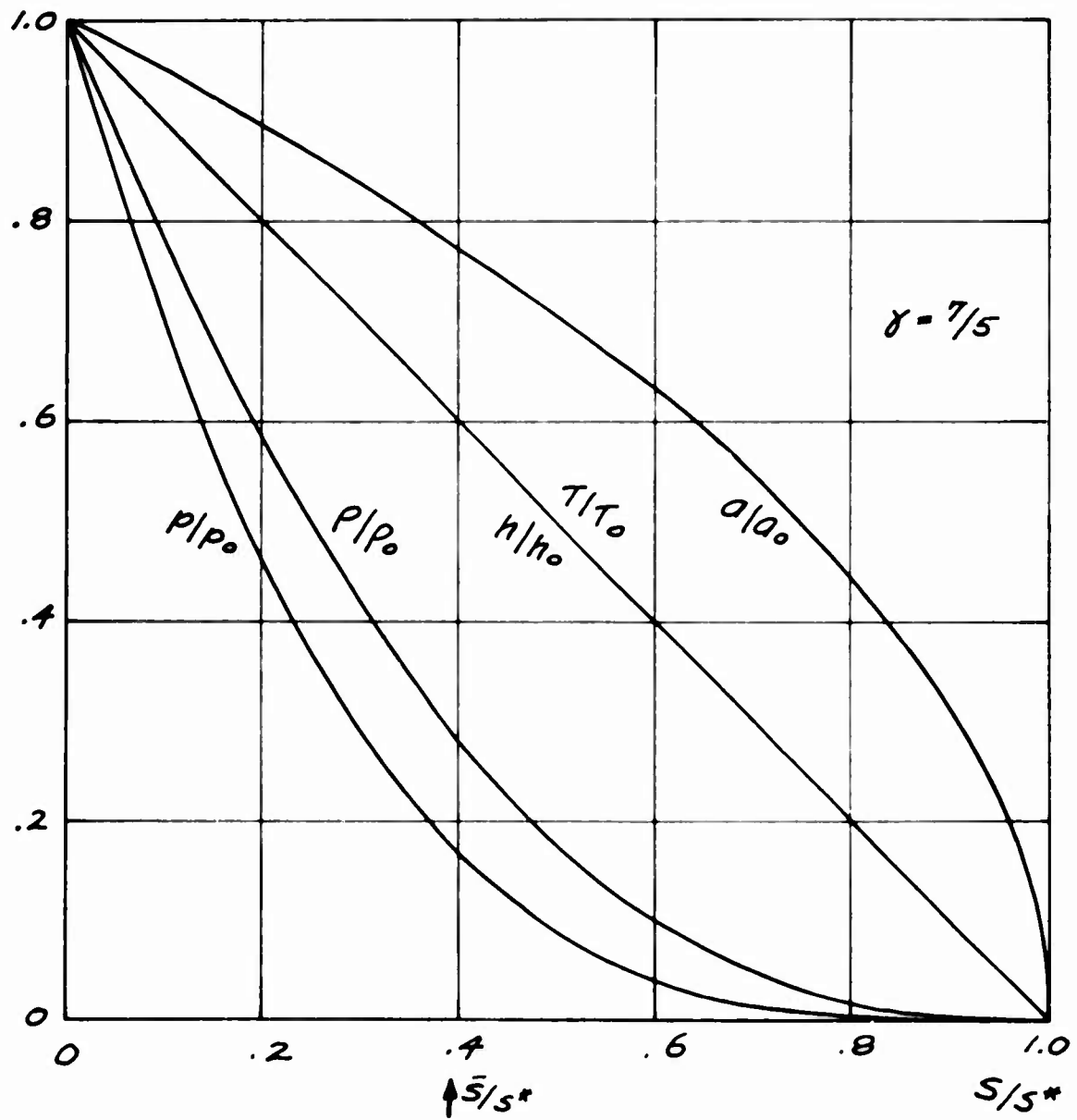


Fig. 1

$$a^2 = (\gamma - 1) \alpha (s^* - s) \quad (11)$$

decreases abruptly with vertical tangent to zero.

For values of $s > s^*$ the enthalpy and temperature becomes negative, the pressure, density, and sound speed become imaginary. A constant acceleration flow cannot physically be realized beyond s^* . It will be shown later that the zero density particle path plays the particular role of an envelope of characteristics.

The relation will now be derived between the conditions in the pipe before and after the acceleration is reached. This could be done by evaluating the average density $\bar{\rho}$ from an integration of Eq. 6. The average density can, however, be derived with less effort by direct integration of Euler's equation at constant time. It follows then for a pipe of length s that

$$p_0 - p = \alpha \int_0^s \rho ds = \alpha m_g = \alpha \bar{\rho} s \quad (12)$$

where m_g is the mass of the enclosed gas. Eq. 12 is valid independent of the p - ρ relation. The above equation becomes particularly simple if the pipe is of length s^* . It follows then that

$$p_0 = \alpha \bar{\rho} s^* \quad (13)$$

and one obtains by replacing s^* by its value

$$p_0 = \frac{\gamma}{\gamma - 1} \bar{p} \quad (14)$$

It is now interesting to determine at what distance \bar{s} the average density $\bar{\rho}$ occurs. At this point all state variables will be the same as they had been before the pipe was accelerated. Eq. 14 and Eq. 6 give

$$\left(\frac{\gamma-1}{\gamma}\right)^{\gamma-1} = 1 - \bar{s}/s^* \quad (15)$$

From Eq. 14, 15, 9 and 6 one obtains

<p>For $\gamma = 7/5$</p> $\begin{aligned} \bar{s} &= 0.39 s^* \\ \bar{\rho} &= 0.29 \rho_0 \\ \bar{T} &= 0.61 T_0 \\ \bar{p} &= 0.17 p_0 \end{aligned}$	<p>For $\gamma = 5/3$</p> $\begin{aligned} \bar{s} &= 0.46 s^* \\ \bar{\rho} &= 0.40 \rho_0 \\ \bar{T} &= 0.54 T_0 \\ \bar{p} &= 0.22 p_0 \end{aligned}$
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To illustrate the generation of a constant acceleration flow further, it will be assumed that the flow is produced by erecting a pipe enclosing a certain amount of gas from a horizontal to a vertical position. The acceleration then equals the acceleration of gravity g . A Lagrangian coordinate system will be placed with the origin at the bottom of the pipe and the positive s -axis vertically up. Assume the pipe has been filled in a horizontal position with air of one amagat at a temperature of $T = 300^\circ\text{K}$. One finds then that the pressure becomes zero at the upper end of the pipe if its length is $s^* = 31\text{km}$. The density at the bottom enclosure of the pipe has increased to 3.5 amagat, the temperature to 495°K , and the pressure to 5.8 atmospheres. The distance \bar{s} , where the density of one amagat would be found, is at a height of $\bar{s} = 12\text{km}$.

This artificial atmosphere, which was created by an isentropic adjustment from the horizontal to the vertical position of the pipe, differs noticeably from an ordinary atmosphere. If by heat conduction the same temperature could be established throughout the enclosed gas, the conditions then become similar to those of the surrounding atmosphere. The "flow" now present in the pipe represents an isothermal constant acceleration flow. The existence of this flow in the earth's atmosphere was proved by the author (Ref. 3) employing Eulerian coordinates. The properties of an isothermal constant acceleration flow will now be given.

With $p/\rho = \text{const.}$ the solution of the general equation of a constant acceleration flow (Eq. 3) becomes in Eulerian coordinates

$$p_0/\rho_0 \ln p/p_0 + \alpha x - \frac{1}{2} \alpha^2 t^2 = 0 \quad (16)$$

In Lagrangian coordinates one obtains

$$p/p_0 = \rho/\rho_0 = \exp(-p_0/\rho_0 \alpha s) \quad (17)$$

The Lagrangian presentation shows best the similarity to an isothermal atmosphere where the pressure decreases exponentially with increasing height. The significant difference of the constant acceleration flows involving either an isentropic or an isothermal process is that in the isothermal case the pressure becomes zero only at infinity.

CONSTANT ACCELERATION FLOW CHARACTERISTICS

The equations of the characteristics can easily be derived since a and u are known as functions of x and t . We write the characteristics in the form $\frac{x-1}{2} u \pm a = k$. The families of curves which present the net of the characteristics are then generated by using k as a parameter. With $u = 0$ for $t = 0$ it is most convenient to use as a parameter a_x which represents the local speed of sound along the x -axis. The value of a_x is given by Eq. 5. We have

$$a_x = a_0 \left(1 - \frac{\gamma-1}{a_0^2} \alpha x \right)^{1/2} \quad (18)$$

where a_0 equals the sound speed of the fluid element initially at position $x = 0$. The equations expressing the net of characteristics become

$$\frac{x-1}{2} u \pm a = a_x \quad (19)$$

and with

$$a = a_0 \left(1 + \frac{\gamma-1}{2} \frac{\alpha^2}{a_0^2} t^2 - \frac{\gamma-1}{a_0^2} \alpha x \right)^{1/2}$$

$$u = \alpha t$$

one obtains by rearrangement

$$\left(t \mp \frac{2a_x}{(3-\gamma)\alpha} \right)^2 = \frac{4}{(3-\gamma)\alpha} \left[x - \frac{(3-\gamma)a_0^2 - 2a_x^2}{(\gamma-1)(3-\gamma)\alpha} \right] \quad (20)$$

The characteristics are parabolas. The axes of the parabolas are all parallel to the x -axis. With $x' = []$ and $t' = ()$ the parabolas become

$$t'^2 = \frac{4}{(3-\gamma)\alpha} x' \quad (21)$$

All parabolas are of this identical shape.

Eliminating a_x between the expression for x' and t' we derive an equation which describes the locus of the vertices of the parabolas as

$$t^2 = \frac{2(\gamma-1)}{\alpha(3-\gamma)} \left(\frac{a_0^2}{(\gamma-1)\alpha} - x \right) \quad (22)$$

Since the vertices of the parabolas represent for the characteristics the points of zero slope, the equation (22) indicates where sonic conditions occur in the flow. The above parabola will therefore be called the sonic parabola. Its equation could also have been derived from the condition $u + a = 0$.

The sonic parabola intersects the x -axis at a distance of $x = \frac{a_0^2}{(\gamma-1)\alpha}$ where $a_x = 0$. The intersection with the t -axis is at $t = \pm \frac{a_0}{\alpha} \sqrt{2/3-\gamma}$. At this time sonic conditions occur in a constant acceleration gun at the gun barrel entrance. The particle path describes a parabola of the form

$$t^2 = 2/\alpha \left[x - \frac{a_0^2}{(\gamma-1)\alpha} \right] \quad (23)$$

where x_0 indicates the initial position of the particle.

Two particle paths, of initial position $x = 0$ and $x = a_0^2/(\gamma-1)\alpha$, the sonic parabola and two characteristics are shown in Fig. 2. A schematic net of characteristics is illustrated by Fig. 3. Notations have been omitted in this figure so as not to distract from its Op-Art value.

We observe that the system of characteristics is symmetrical to the x -axis and that the separation of neighboring characteristics reduces with increasing x value. The intersection points of infinitely closely spaced characteristics

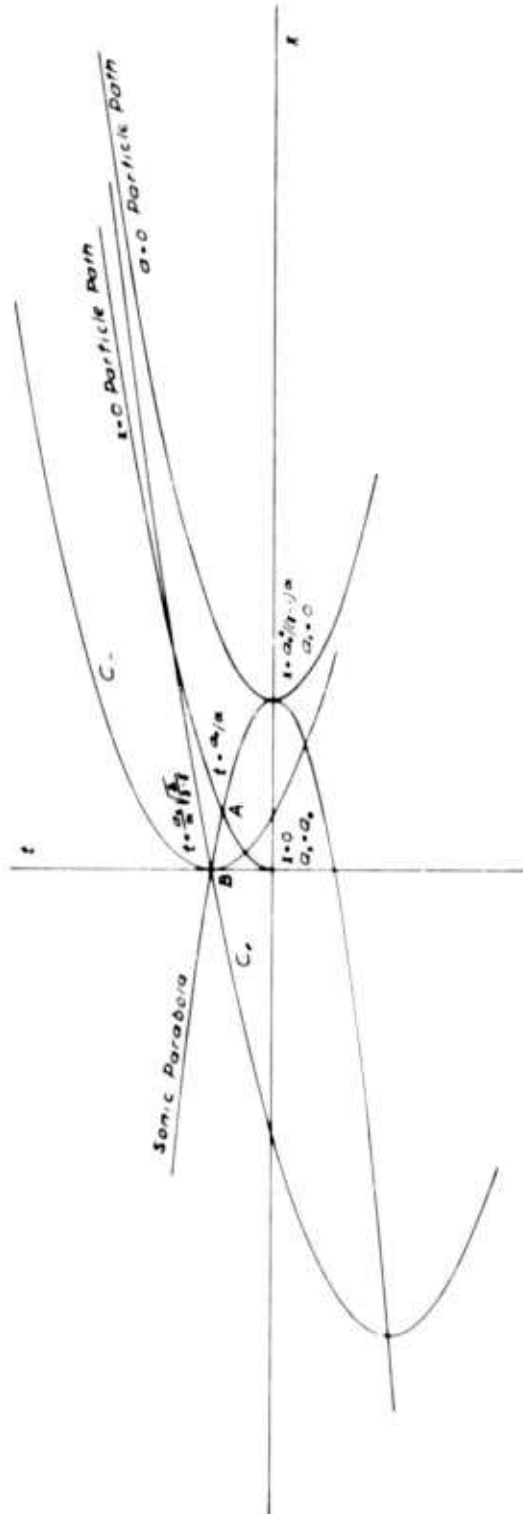


Fig. 2

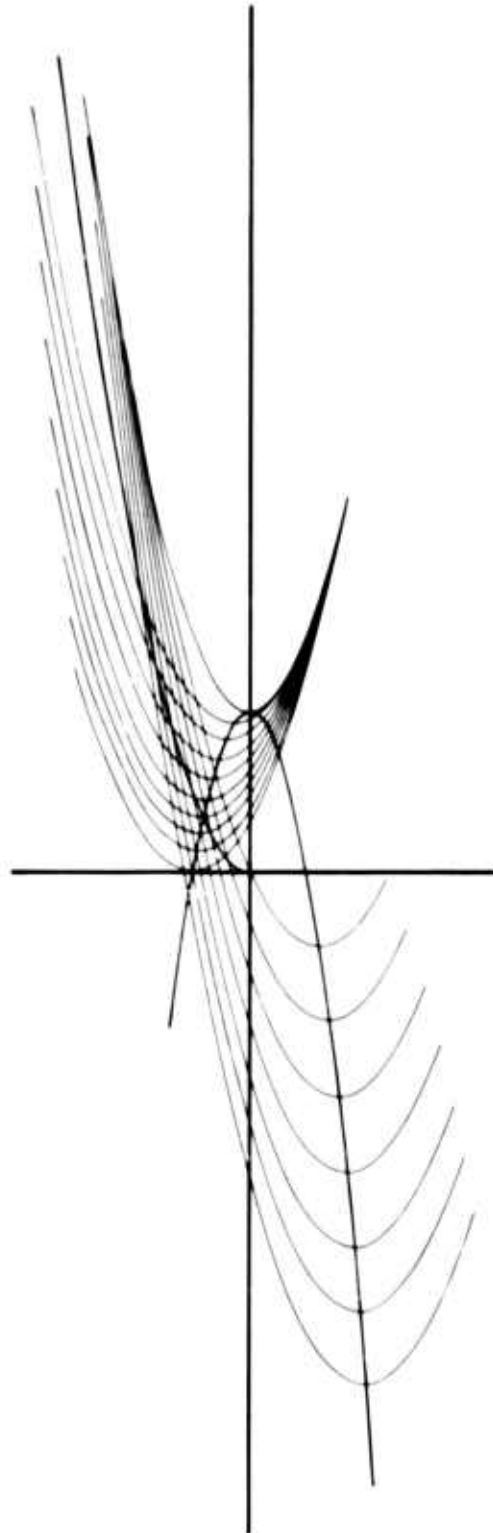


Fig. 3

define an envelope with the equation

$$t^2 = \frac{2}{\alpha} \left[x - \frac{a_0^2}{(\gamma-1)\alpha} \right] \quad (24)$$

This envelope is a parabola whose vertex is at $x = \frac{a_0^2}{(\gamma-1)\alpha}$ where $a_x = 0$. Its equation describes the path of the particle initially at a location where $a_x = 0$. The gas element which moves along this path exhibits, as has been shown, zero pressure, temperature, density, and sound velocity. The physically realizable segment of the x, t plane in which a constant acceleration flow produced by an isentropic process can exist is thus the above zero density particle path and the x -axis. To the right of the zero density particle path the sound velocity assumes imaginary values. The separation of neighboring characteristics increases again in this region.

For the particular case of a constant acceleration gun with projectile position at $x = 0$ for $t = 0$, the constant acceleration flow exists only to the left of the particle path originating at $x = 0$. Sound velocity is reached at the base of the projectile where the path of the particle originally at $x = 0$ intersects the sonic parabola. It occurs at the time

$t = a_0 / \alpha$. If $x = 0$ is the position of the barrel entrance, sonic conditions occur there at the time indicated by the intersection of the sonic parabola with the positive t -axis. This time is $t = \frac{a_0}{\alpha} \sqrt{\frac{2}{\gamma-1}}$. The propellant flow will be restricted to $u = a$ from now on for a chambered gun.

The outstanding feature of the constant acceleration flow is especially well illustrated by presenting the characteristics in Lagrangian coordinates. We select again a_x as the parameter. The equations of the characteristics for the

isentropically produced constant acceleration flow follow from the relations which express u and a as functions of the particle coordinate s and time t . We have $u = t$ and $a(s, t) = a_0 \left(1 - \frac{\gamma-1}{a_0^2} \alpha s\right)^{1/2}$ since in Lagrangian coordinates $\partial a / \partial t = 0$. We obtain then

$$\frac{\gamma-1}{2} \alpha t \pm a_0 \left(1 - \frac{\gamma-1}{a_0^2} \alpha s\right)^{1/2} = a_x \quad (25)$$

or after rearrangement

$$\left(t \pm \frac{2 a_s}{(\gamma-1) \alpha}\right)^2 = \frac{4}{(\gamma-1) \alpha} \left[\frac{a_0^2}{(\gamma-1) \alpha} - s\right] = \frac{4}{(\gamma-1) \alpha} [s^* - s] \quad (26)$$

where a_x has been replaced by a_s and $\frac{a_0^2}{(\gamma-1) \alpha}$ by s^* . Setting $() = t'$ and $[] = s'$ the parabolas can be transformed into one parabola with the equation

$$t'^2 = \frac{4}{(\gamma-1) \alpha} s' \quad (27)$$

The vertices form the envelope

$$s = \frac{a_0^2}{(\gamma-1) \alpha} = s^* \quad (28)$$

For this s value we have a_s , pressure, density, and temperature equal to zero. To the right of the envelope the sound velocity assumes imaginary values. The positive sign in equation (25) is satisfied by the lower branch of the parabola of equation (26) and its extension as the upper branch into the region of imaginary sound speeds, the negative sign by the upper branch of the parabola and its extension into the imaginary sound speed region. The net of characteristics is illustrated by Fig. 4. The sonic parabola, which is also

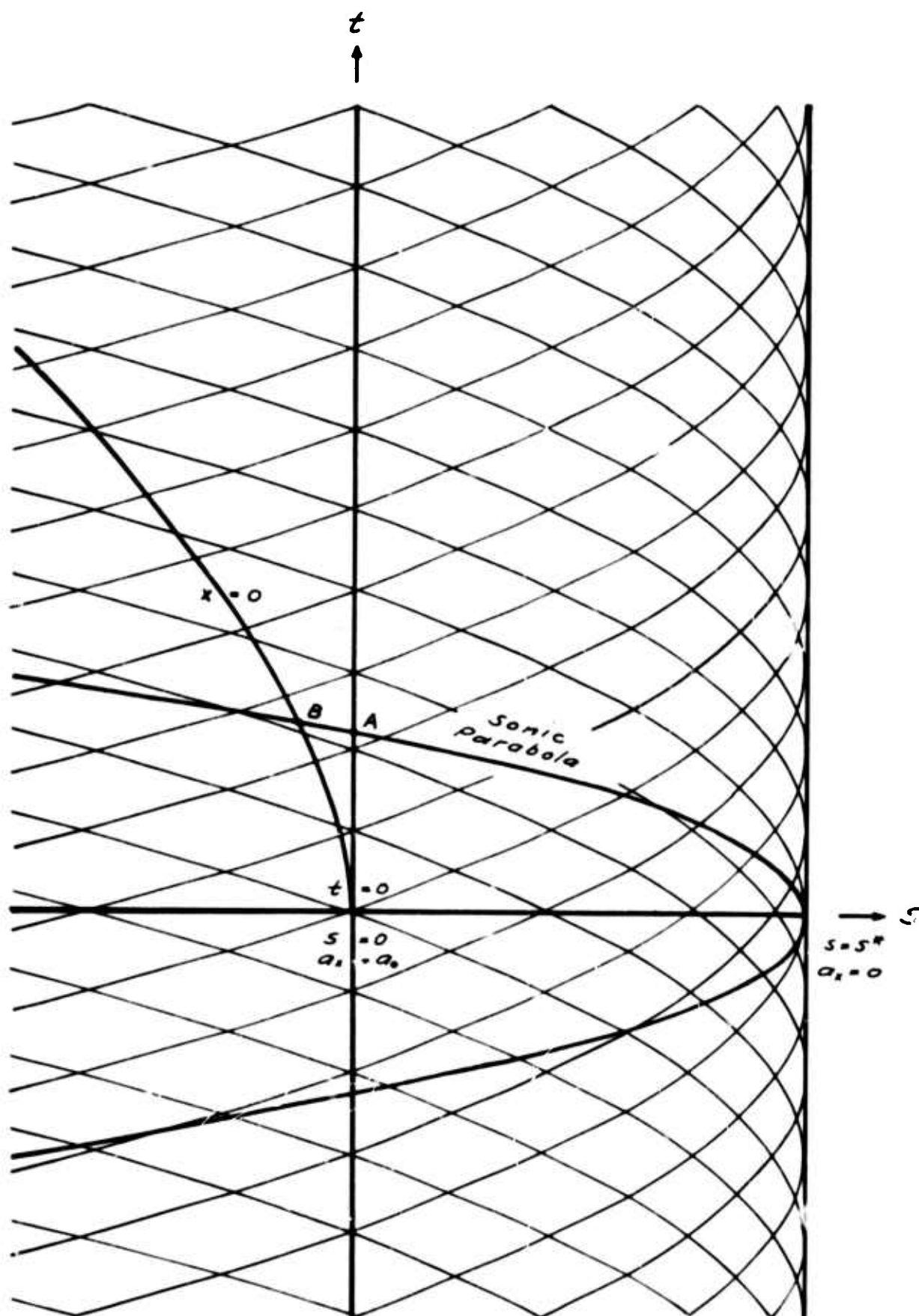


Fig. 4

shown, has in Lagrangian coordinates the equation

$$t^2 = \frac{\gamma-1}{\alpha} \left(\frac{a_0^2}{(\gamma-1)\alpha} - S \right) = \frac{\gamma-1}{\alpha} (S^* - S) \quad (29)$$

For reference the position in time of the $x = 0$ Eulerian coordinate has been included in Fig. 4. Its equation is

$$t^2 = - \frac{2}{\alpha} S \quad (30)$$

For a constant acceleration gun with the projectile at $s = 0$ for $t = 0$ we find from equation 28 that at the time

$$t = a_0 / \alpha \quad (31)$$

sonic conditions are reached at the base of the projectile. Sonic velocity occurs at the barrel entrance $x = 0$ at the intersection point of the parabola represented by Eqs. (28) and (29). We obtain

$$S = - \frac{a_0}{\alpha(3-\gamma)} \quad (32)$$

$$t = \frac{a_0}{\alpha} \sqrt{\frac{2}{3-\gamma}} \quad (33)$$

The Lagrangian presentation illustrates best the fact that the forward and backward running waves completely cancel each other and that, consequently, the properties of the flow cannot change with time.

The effort of drawing the characteristic net in Lagrangian coordinates can be reduced greatly by using an a versus u plot. The characteristics in these coordinates are then two families of straight lines with slopes of $\pm \frac{2}{\gamma-1}$. This is in general

true independent of the particular flow under consideration. The unique difference here is that for a constant acceleration flow in Lagrangian coordinates the sound speed is a function of the initial coordinate alone. We have then $u = u(t)$ and $a = a(s)$ and thus an a, u plot also presents an s, t plot.

In nondimensional coordinates the relations are $u/\alpha = \alpha t/a_0$

and
$$a/a_0 = \left(1 - \frac{\gamma-1}{2} \alpha s\right)^{1/2} = \left(1 - s/s^*\right)^{1/2}$$

The straight line characteristic net is shown in Fig. 5 for the case of $\gamma = 5/3$. The sonic parabola degenerates into two straight lines $u = \pm a$ as shown. The Eulerian coordinates $x = 0$ prescribe a hyperbola of the equation

$$(a/a_0)^2 - \frac{\gamma-1}{2} (u/a_0)^2 = 1 \quad (34)$$

Point A gives the time when at the base of the projectile and point B when at the barrel entrance sonic conditions occur. The C_+ and the C_- characteristics through point B are the same characteristics which are shown in Fig. 2. The C_- characteristic through point B is tangent to the hyperbola.

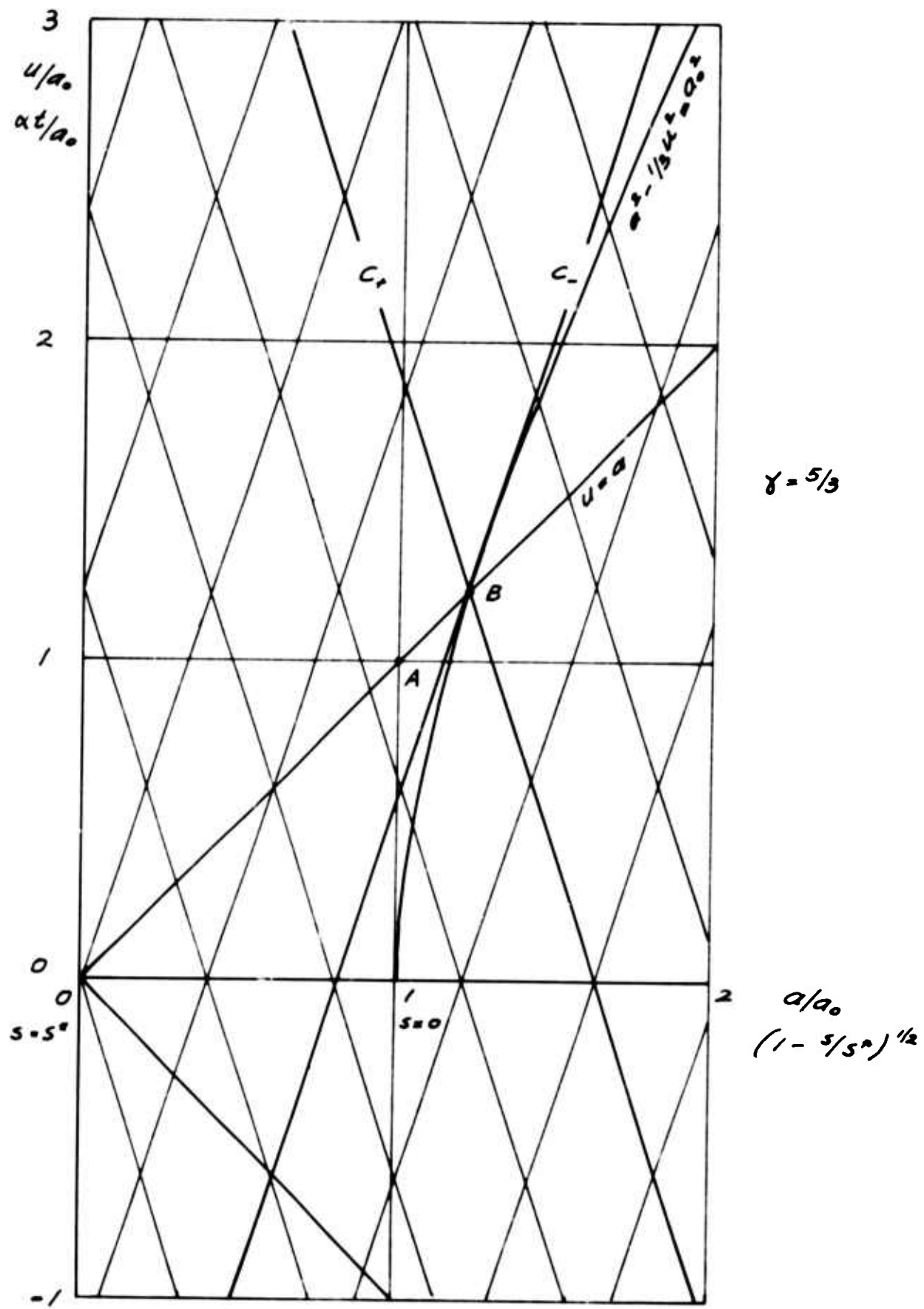


Fig. 5

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