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COMPUTING PROGRAMS FOR SURFACE WAVE TRAINS OF POINT SOURCES

by

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ABSTRACT

Descriptions are given for four programs which compute the components of velocity in the wave train of a unit source. The programs give the velocity at an arbitrary position in any direction and at any distance from the source. The programs have been used to compute a table of components of velocity for 27000 positions.

ZUSAMMENFASSUNG

Es werden vier Programme beschrieben, die die Geschwindigkeitskomponenten im Wellenzug einer Einheitsquelle berechnen. Die Programme liefern die Geschwindigkeit für einen beliebigen Punkt in jeder Richtung und in jedem Abstand von der Quelle. Die Programme sind benutzt worden, um eine Tabelle für die Geschwindigkeitskomponenten an 27000 Punkten zu berechnen.

RÉSUMÉ

On présente des descriptions pour quatre programmes qui calculent les composants de vitesse dans le train d'ondes d'une source unitaire. Les programmes indiquent, dans une position arbitraire, la vitesse dans n'importe quelle direction et à n'importe quelle distance de la source. Les programmes ont été utilisés afin de constituer un tableau des composants de vitesse pour 27.000 positions.

FOREWORD

The objective of this report is the release of enough information so that a programmer can utilize the existing programs for particular applications without waiting for a full documentation of the symbolic and numerical analysis. The work was supported initially by the Office of Naval Research under Project No. NR 062-203, but is supported currently as part of the Foundational Research Program of the Naval Weapons Laboratory, Project No. R360FR103/2101/R0110101. Assistance for programming and checkout was contributed by W. H. Langdon and E. J. Hershey. The date of completion of this report was 30 June 1965.

APPROVED FOR RELEASE:

/s/BERNARD SMITH
Technical Director

INTRODUCTION

The objective of work on ship waves at the Naval Weapons Laboratory is the computation of waves around ships of finite breadth in water of finite depth. Thus the scope of the work is not limited to the far field directly astern of the ship. A computation of the flow over the hull of a ship is basic to the computation of both the friction drag and the wave drag on the ship. The computation would help to improve the efficiency of ships. Unfortunately, it has not been possible heretofore to bring the calculator into competition with the towing tank. It costs more money to set up a model on the calculator than it costs to make a model and test it in a towing tank. A major effort has been directed therefore toward improvement of the method of computation.

The time consuming step in the method is the calculation of velocity in the wave train which trails behind a unit source. The components of velocity are derived from Havelock's double Fourier integral in wave number space. Following a suggestion from the David Taylor Model Basin, A. R. DiDonato[15] has developed a method of integration in which the path of integration in the complex plane is so displaced as to avoid a singularity in the integrand. On the other hand, radial integration through the singularity introduces the complex exponential integral and simplifies the double integration to single integration. Azimuthal integration still must be applied to an integrand which is partly oscillatory and partly monotonic.

The computation of special functions for random arguments is achieved on a digital calculator by reference to subroutines. For the ship wave application the subroutines may require several seconds for computation. The development of subroutines has been continued through many generations to what is considered to be the ultimate from the standpoint of efficiency. Four systems of subroutines and programs are the subject of the present report. The programs are coded in language for the Naval Ordnance Research Calculator but are not available in FORTRAN. The NORC does thirteen decimal digit arithmetic at the rate of 15000 operations per second in response to three address instructions. The possibility of translating the programs to other languages is under consideration.

Between the four programs it is possible to obtain output with better than six decimal digit accuracy, whereas the mathematical model for ship waves is in error insofar as it does not meet exactly the physical boundary conditions. The components of velocity from the computing programs are to be used in the formation of a matrix which may be nearly singular, and unless a relatively high level of accuracy is maintained, the inversion of the matrix may introduce more error than the mathematical model.

In the first computing program, the azimuthal integration is completed by the application of an integration rule to computed values of the integrand. Inasmuch as the angular variable of integration is cyclical, the high accuracy rule of integration is the trapezoidal rule for equally spaced arguments. This method has been used for submerged bodies of revolution, but it is impractical for surface ships. Thousands of intervals of integration are required, and it may take an hour to evaluate one velocity on NORC.

A breakthrough has been achieved through an integration by parts which makes it possible to integrate through many oscillations at a time. It may be recognized that the integrand contains two factors one of which is monotonic while the other contains all of the fluctuation of the integrand. If the first factor is approximated in terms of the powers of a parameter common to both factors, then the approximation can be integrated term by term through the use of recurrence equations.

In the second computing program, the monotonic factor is approximated in terms of positive integral powers of the common parameter. This has the advantage that only single valued complex functions are required for integration, and no distinction must be made between different branches of the Riemann surface. It has the disadvantage that irrational functions must be approximated, and the range of approximation is limited. Hundreds of intervals of approximation are required, but it takes only 32 seconds to evaluate one velocity on NORC.

The classical approach to a surface disturbance by Kelvin[1,2,3] was an application of the principle of stationary phase to the determination of the first terms of an asymptotic expansion. Considerations of the possibility of development into full expansions have been made for the line of wave crests by Peters[12] and by Ursell[13].

In the present approach to submerged sources, the integrand is expressed as the sum of a monotonic term and an oscillatory term. In terms of a common parameter the phase of the oscillatory term is expressed as a polynomial of at most the third degree, while the amplitude of the oscillatory term is expressed as a power series of unlimited degree. Symbolic expressions for the terms are very complicated.

In the third computing program, symbolic analysis is combined with numerical computation to generate the series expansions. The asymptotic expansion is valid at large distances astern of the source or ahead of the source but the Stokes phenomenon makes trouble abeam of the source. A representative computing time is 6 seconds to evaluate one velocity on NORC.

In view of the relatively high efficiency of the asymptotic expansion, a further effort was made to improve the integration by parts. The convolution theorem offers the best promise of transformation of integrals without loss of accuracy, but it leads to integrals of multiple valued functions for which new subroutines have had to be developed. The invention of algorithms for keeping the path of integration on contiguous branches of the Riemann surface required the guidance of graphical analysis, and the invention of formulae for determining ranges of approximation for balanced accuracy required the guidance of experimental computations.

In the fourth computing program, the monotonic factor is approximated in terms of positive and negative half integral powers of a parameter common to both factors. When either of two branch points in the complex plane is selected to be the origin of the parameter, that branch point is replaced by a branch point elsewhere on the Riemann surface, and the range of approximation can be expanded. Only four intervals of approximation are required, but it still takes 9 seconds to evaluate one velocity on NORC.

The computation of special functions at greater speed can be achieved through interpolation in a table of sufficient size. The time for random access in the table increases while the time for interpolation decreases with increase in the size of the table. For the ship wave application the table could contain more than a hundred thousand numbers. A table of this size could not all be stored in the core memory of many calculators, and the access time from magnetic tape would determine the time of operation.

For studies in interpolation a table has been prepared with 27000 sets of components of velocity. The table is accessible to the IBM 7090 series of calculators. The table has been recorded in the binary coded decimal mode on magnetic tape with one 80 column card image per record. This table still is not large enough to provide a universal basis for interpolation, and a further enlargement of the table is under consideration.

Interpolation is especially difficult where both transverse and divergent wave systems are present in the wave train. Computation determines the sum of the wave systems, but otherwise their amplitudes and phases are arbitrary. Separation of the wave systems by analysis leads to asymptotic series which are accurate only at large distances from the source. Convergence of the asymptotic series can be accelerated through the use of converging factors, but experience has indicated that converging factors themselves can be asymptotic. Development of rational functions in the range of difficulty has been unsuccessful because zeros appeared in the denominators of the rational functions.

Documentation of the symbolic and numerical analysis is under preparation but will require a long time. In the meantime, it is easy to use any of the programs for computations. Only the first two programs provide for water of finite depth, although all four programs provide for water of infinite depth. The present report is restricted to the case of infinite depth, and a later report will take up the case of finite depth.

ANALYSIS

Formulae for the velocity in the wave train of a point source have been derived for infinite depth by Havelock[10] and for finite depth by Lunde[11]. A brief derivation is reproduced herein as part of the documentation of notation.

Let the point source be moving at a depth h below the surface with a speed U. A point in the fluid is located at Cartesian coordinates x, y, z in a righthanded coordinate system which is moving with the source. The origin of coordinates is at the surface directly over the source. The x-coordinate is the distance forward, the y-coordinate is the distance to the right, and the z-coordinate is the distance downward.

In accordance with the usual assumption of incompressible, irrotational flow, the velocity in the fluid is expressed as the negative gradient of a velocity potential. Under steady state conditions the potential in the moving reference frame is given by the expression

$$Ux + \varphi(x, y, z) \tag{1}$$

where Ux is the potential for uniform flow in a direction opposite to the motion of the source and $\varphi(x, y, z)$ is the potential for the local disturbance in the wave train. The computing routines give the Cartesian components u, v, w of velocity in accordance with the equations

$$u = -\frac{\partial\varphi}{\partial x} \qquad v = -\frac{\partial\varphi}{\partial y} \qquad w = -\frac{\partial\varphi}{\partial z} \tag{2}$$

Let the configuration of the free surface be expressed by the equation

$$z - f(x, y) = 0 \tag{3}$$

For steady state conditions the velocity at the free surface is tangent to the surface and the potential obeys the boundary equation

$$\left(U + \frac{\partial\varphi}{\partial x} \right) \frac{\partial f}{\partial x} + \frac{\partial\varphi}{\partial y}\frac{\partial f}{\partial y} - \frac{\partial\varphi}{\partial z} = 0 \tag{4}$$

The equation is linearized, through neglect of the term

$$\frac{\partial\varphi}{\partial x}\frac{\partial f}{\partial x} + \frac{\partial\varphi}{\partial y}\frac{\partial f}{\partial y} \sim 0 \tag{5}$$

to give the boundary equation

$$U\frac{\partial f}{\partial x} - \frac{\partial\varphi}{\partial z} = 0 \tag{6}$$

Let the motion of the fluid be determined by the Bernoulli equation,

$$\frac{1}{2}\left\{\left(U+\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right\} + \frac{p}{\rho} - gz = \text{constant} \tag{7}$$

where ρ is the density, p is the pressure, and g is the acceleration of gravity. At the free surface the pressure is constant, and the equation is linearized, through neglect of the term

$$\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2 \sim 0 \tag{8}$$

to give the boundary equation

$$U\frac{\partial\varphi}{\partial x} - gf = 0 \tag{9}$$

The free surface is eliminated from Equations (6) and (9) by differentiation to give the equation

$$\frac{\partial^2\varphi}{\partial x^2} - \varkappa_0\frac{\partial\varphi}{\partial z} = 0 \tag{10}$$

where the critical wave number $\varkappa_0$ is defined by the equation

$$\varkappa_0 = \frac{g}{U^2} \tag{11}$$

Along the line behind the source the wave length λ of the transverse waves is given by the equation

$$\lambda = \frac{2\pi}{\varkappa_0} \tag{12}$$

and a Froude number based upon a length l is given by the equation

$$\frac{U}{\sqrt{gl}} = \frac{1}{\sqrt{\varkappa_0 l}} \tag{13}$$

Analysis shows that the potential φ may be expressed as the sum of three potentials in accordance with the equation

$$\varphi(x, y, z) = \varphi_1(x, y, z) + \varphi_2(x, y, z) + \varphi_3(x, y, z) \tag{14}$$

where φ_1 is the potential of the source in an unbounded fluid, φ_2 is the potential of an image source over the free surface, and φ_3 is the potential of the wave train.

The potential φ_1 is given by the equation

$$\varphi_1 = \frac{1}{\{x^2 + y^2 + (z-h)^2\}^{\frac{1}{2}}} \tag{15}$$

and its derivatives are given by the equations

$$-\frac{\partial \varphi_1}{\partial x} = \frac{x}{\{x^2 + y^2 + (z-h)^2\}^{\frac{3}{2}}} \tag{16}$$

$$-\frac{\partial \varphi_1}{\partial y} = \frac{y}{\{x^2 + y^2 + (z-h)^2\}^{\frac{3}{2}}} \tag{17}$$

$$-\frac{\partial \varphi_1}{\partial z} = \frac{z-h}{\{x^2 + y^2 + (z-h)^2\}^{\frac{3}{2}}} \tag{18}$$

The potential φ_2 is given by the equation

$$\varphi_2 = -\frac{1}{\{x^2 + y^2 + (z+h)^2\}^{\frac{1}{2}}} \tag{19}$$

and its derivatives are given by the equations

$$-\frac{\partial \varphi_2}{\partial x} = -\frac{x}{\{x^2 + y^2 + (z+h)^2\}^{\frac{3}{2}}} \tag{20}$$

$$-\frac{\partial \varphi_2}{\partial y} = -\frac{y}{\{x^2 + y^2 + (z+h)^2\}^{\frac{3}{2}}} \tag{21}$$

$$-\frac{\partial \varphi_2}{\partial z} = -\frac{z+h}{\{x^2 + y^2 + (z+h)^2\}^{\frac{3}{2}}} \tag{22}$$

The potential φ_3 is expressed as a Fourier integral.

A function $F(x, y)$ is expressed by the Fourier equation

$$F(x, y) = \iint A(\varkappa, \theta)\, e^{i\varkappa(x\cos\theta + y\sin\theta)}\, \varkappa d\varkappa d\theta \tag{23}$$

where $\varkappa$, θ are polar coordinates in wave number space while the amplitude $A(\varkappa, \theta)$ is given by the inverse equation

$$A(\varkappa, \theta) = \frac{1}{4\pi^2} \iint F(x, y)\, e^{-i\varkappa(x\cos\theta + y\sin\theta)}\, dxdy \tag{24}$$

In the particular case where the function is given by the equation

$$F(x,\ y) = \frac{1}{\sqrt{x^2 + y^2}} = \frac{1}{r} \tag{25}$$

the amplitude is given by the equation

$$A(\varkappa,\ \theta) = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{\infty} e^{-i\varkappa r\cos\phi}\ dr d\phi \tag{26}$$

where r, ϕ are polar coordinates in coordinate space.

It may be deduced, with the aid of the equations[14]

$$J_0(z) = \frac{1}{\pi} \int_0^{\pi} \cos(z \cos\theta)\, d\theta \tag{27}$$

and

$$\int_0^{\infty} J_0(z) dz = 1 \tag{28}$$

that the amplitude is given by the equation

$$A(\varkappa,\ \theta) = \frac{1}{2\pi\varkappa} \tag{29}$$

Generalization to a three dimensional solution of Laplace's equation leads to the equation

$$\varphi_1(x,\ y,\ z) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} \int_{0}^{\infty} e^{-\varkappa|z - h| + i\varkappa(x\cos\theta + y\sin\theta)} d\varkappa d\theta \tag{30}$$

for the potential of the point source in an unbounded fluid. To this must be added a term such that the sum of terms satisfies the boundary condition. Differentiation and substitution shows that the amplitude of the additional term is given by the equation

$$A(\varkappa,\ \theta) = \frac{1}{2\pi\varkappa} \frac{(\varkappa_0 + \varkappa \cos^2\theta)}{(\varkappa_0 - \varkappa \cos^2\theta)} \tag{31}$$

or after rearrangement, by the equivalent equation

$$A(\varkappa,\ \theta) = -\frac{1}{2\pi\varkappa} + \frac{\varkappa_0}{\pi\varkappa(\varkappa_0 - \varkappa \cos^2\theta)} \tag{32}$$

The first term in this expression for amplitude leads to the equation

$$\varphi_2(x,\ y,\ z) = -\frac{1}{2\pi} \int_{-\pi}^{+\pi} \int_{0}^{\infty} e^{-\varkappa(z + h) + i\varkappa(x\cos\theta + y\sin\theta)} d\varkappa d\theta \tag{33}$$

for the potential of the image source over the free surface. The second term in the expression for amplitude leads to an equation which can be simplified through the substitutions

$$\delta = \frac{\varkappa_0}{\cos^2\theta}\left\{(z + h) - i(x \cos\theta + y \sin\theta)\right\} \tag{34}$$

and

$$\varkappa = \frac{\varkappa_0}{\cos^2\theta} - \frac{u}{(z + h) - i(x \cos\theta + y \sin\theta)} \tag{35}$$

The function $e^{-\delta}$ satisfies both Laplace's equation and the boundary equation for the free surface. It is added in just the right amount to make the wave train trail behind the source if the path of integration with respect to u proceeds along a straight line in the complex plane from $u = -\infty$ toward the origin, bypasses the origin on a half circle of small radius below the origin, and continues on a straight line to $u = \delta$. The integrand is analytic and the path of integration may be deformed in accordance with the Cauchy theorem on analytic functions. The potential of the wave train is given by the equation

$$\varphi_s(x,\ y,\ z) = \frac{\varkappa_0}{\pi}\int_{-\pi}^{+\pi}\frac{e^{-\delta}}{\cos^2\theta}\int_{-\infty}^{\delta}\frac{e^u}{u}\,du\,d\theta \tag{36}$$

Differentiation of this potential requires the equation

$$\frac{d}{d\delta}\,e^{-\delta}\int_{-\infty}^{\delta}\frac{e^u}{u}\,du = \frac{1}{\delta} - e^{-\delta}\int_{-\infty}^{\delta}\frac{e^u}{u}\,du \tag{37}$$

which may be integrated by parts to give the equation

$$\frac{d}{d\delta} e^{-\delta} \int_{-\infty}^{\delta} \frac{e^{u}}{u} du = - e^{-\delta} \int_{-\infty}^{\delta} \frac{e^{u}}{u^2} du \qquad (38)$$

The derivatives of φ_3 are given by the equations

$$- \frac{\partial \varphi_3}{\partial x} = - i \frac{\kappa_0^2}{\pi} \int_{-\pi}^{+\pi} \frac{e^{-\delta}}{\cos^4 \theta} \int_{-\infty}^{\delta} \frac{e^{u}}{u^2} du \cos\theta \, d\theta \qquad (39)$$

$$- \frac{\partial \varphi_3}{\partial y} = - i \frac{\kappa_0^2}{\pi} \int_{-\pi}^{+\pi} \frac{e^{-\delta}}{\cos^4 \theta} \int_{-\infty}^{\delta} \frac{e^{u}}{u^2} du \sin\theta \, d\theta \qquad (40)$$

$$- \frac{\partial \varphi_3}{\partial z} = + \frac{\kappa_0^2}{\pi} \int_{-\pi}^{+\pi} \frac{e^{-\delta}}{\cos^4 \theta} \int_{-\infty}^{\delta} \frac{e^{u}}{u^2} du d\theta \qquad (41)$$

Only the real parts of the complex computations are reported by the computing programs.

PROGRAMMING

The computation on NORC requires four magnetic tapes.

The input is on tape code 01. The input consists of four word blocks which are numbered serially in ascending order of block number.

Each block of input contains a set of values for the coordinates

$$h, x, y, z$$

The blocks of input are terminated with an end of file.

The output is on tape code 02. The first block of output is numbered 9999 and contains the six quantities

$$\varkappa_0, d, n, 0, 0, 0$$

where $\varkappa_0$ is the critical wave number, d is the depth of water, and n is the number of intervals for trapezoidal integration. The remaining output consists of seven word blocks which are numbered like the input. Each block of output contains a set of values of the quantities

$$h, x, y, z, u, v, w$$

The blocks of output are terminated with an end of file.

The library is on tape code 09. The subroutines for the library are in Deck 2500.

The programs are on tape code 12. The blocks of programming are in Deck 2513, which is arranged as shown on the following page.

Block Number	Memory Address	Program
9999	0002 - 0007	Start #1
0001	3861 - 3892	Assembly
0002	0704 - 0928	Computation
0003	3952 - 4000	Input - Output
9998	0002 - 0007	Start #2
0001	3861 - 3892	Assembly
0002	0704 - 1355	Computation
0003	3952 - 4000	Input - Output
9997	0002 - 0007	Start #3
0001	3873 - 3892	Assembly
0002	0420 - 1159	Computation
0003	3937 - 4000	Input - Output
9996	0002 - 0007	Start #4
0001	5001 - 5221	Assembly
0002	2989 - 3991	Computation
0003	4940 - 5000	Input - Output.

End of File.

Each computing program consists of a set of three program blocks which are preceded by a starting block. To select a program it is only necessary to key into location 0001 an instruction which reads in the starting block and then start the calculator with the instruction in location 0001. The starting instruction for each of the four programs is as follows.

Program	Starting Instruction			
#1. Trapezoidal Rule	1294	0002	0007	9999
#2. Integration by Parts	1294	0002	0007	9998
#3. Asymptotic Series	1294	0002	0007	9997
#4. Integration by Convolution	1294	0002	0007	9996

At the start of a run, the assembly routine reads into memory addresses 3894 - 4000 an assembly subroutine from the library. The assembly routine assembles subroutines from the library, reads in the rest of the program, and transfers control to the input-output routine. The space required for subroutines by the four programs is as follows.

Program		Memory Addresses
#1.	Trapezoidal Rule	0008 - 0703
#2.	Integration by Parts	0008 - 0703
#3.	Asymptotic Series	0008 - 0418
#4.	Integration by Convolution	0008 - 2988

During the progress of computation, numbers are accumulated and stored temporarily in common storage in the memory. The space required for common storage by the four programs is as follows.

Program		Memory Addresses
#1.	Trapezoidal Rule	0929 - 3951
#2.	Integration by Parts	1356 - 3951
#3.	Asymptotic Series	1248 - 1999
#4.	Integration by Convolution	3998 - 4936

The calculator starts, restarts, or stops under the control of switches. Computation starts at the beginning of the output tape when Switch 74 is turned off, but restarts after the last block written on the output tape when Switch 74 is on transfer. The computation continues as long as Switch 77 is on transfer, but makes a standard stop after it has completed a block of output whenever Switch 77 is turned off. The programs all operate only in 10K storage.

TABULATION

The results of computation on NORC are recorded card by card on magnetic tape. On each card the first 11 columns are blank, columns 12 to 75 contain four 16 digit words, and the last 5 columns are blank.

Each number word contains 16 digits such that the first two digits give the exponent to base 10 (with negative exponents indicated by 9's complements), the third digit gives the sign (with 0 for + and 1 for -), while the remaining thirteen digits give the decimal coefficient, with the decimal point after the first of the thirteen digits. The data are organized into sets, with values of h, x, y, z, u, v, w in each set.

The sets are packed into blocks such that the first and last columns contain an extra punch in row 11 and the first and last words are beginning of block or end of block words. Each BOB or EOB word contains four fields of 4 digits each such that the PQ field is 0290 or 0291, the R field is the first memory address of information in the block, the S field is the last memory address of information in the block, and the T field is the block number. Each block contains 90 sets of data or 630 number words for 90 distances. There are two groups of distances, for $r \leq 1$ and for $r \geq 1$, within each of which the spacing is similar to the spacing for Chebyshev approximation. Each block gives a different azimuth out of a set of 31 azimuths. There are three groups of azimuth, for $\pi \geq \phi \geq \sin^{-1} \frac{1}{8}$, for $\sin^{-1} \frac{1}{8} \geq \phi \geq \frac{1}{2}\pi$, and for $\frac{1}{2}\pi \geq \phi \geq 0$, within each of which the spacing is the same as the spacing for Lobatto approximation. The blocks of data are organized into files.

Each file starts with a block numbered 9999 which states that $\varkappa_0 = 1$ and $d = \infty$. The blocks of data are numbered 0001 to 0031.

The file ends with an EOF word which gives the file number. Each file gives a different depth out of a set of 10 depths, for $\frac{1}{32} \leq h \leq 16$, where each consecutive depth is twice the previous depth.

The inequality of spacing was intended to provide a more accurate basis for interpolation than is possible with equality of spacing.

DISCUSSION

In the classical analysis of wave resistance by Michell[4] the potential was expressed as the sum of three terms of which two cancelled out in the integration over the surface while the third term alone contributed to the wave resistance. Analogous cancellations arise when the present functions are applied to the limiting case of a thin ship.

Let $d\sigma$ be defined to be the projection on a cross sectional plane of one surface element on the side of the ship. In the limiting case of a thin ship the Bernoulli pressure on this element would contribute

$$-\rho U \frac{\partial\varphi}{\partial x}\, d\sigma \tag{42}$$

to the wave resistance. The total potential is the sum of contributions from other surface elements. Let $d\sigma'$ be defined to be the projection on a cross sectional plane of another surface element. The flux through the projection $d\sigma'$ would be $Ud\sigma'$ for uniform flow. This flux is cancelled at the surface by a local source of strength

$$\frac{U}{4\pi}\, d\sigma' \tag{43}$$

on the median plane. The total wave resistance R_w is given by the double integral

$$R_w = -\frac{\rho U^2}{4\pi} \iint \frac{\partial\varphi}{\partial x}\, d\sigma d\sigma' \tag{44}$$

where the integration with respect to $d\sigma d\sigma'$ is taken over both port and starboard halves of the hull.

For every pair of elements which constitute a pressure point and a source point there is another pair of elements with the pressure point and the source point interchanged, and for which the coordinate x in the expression for φ is reversed in sign. Any term in φ for which $\partial\varphi/\partial x$ is odd with respect to x will contribute nothing to the wave resistance. This disposes of the terms φ_1 and φ_2. In the case of φ_3, a reversal of x with $y = 0$ replaces the path of integration with respect to u by a complex conjugate path everywhere except on the half circle which bypasses the origin. The only contribution to the wave resistance comes from the integration at the origin. The surviving term is retained if the potentials are replaced in accordance with the substitution

$$-\frac{\partial\varphi_3}{\partial x} \rightarrow \varkappa_0^2 \int_{-\pi}^{+\pi} \frac{e^{-\delta}}{\cos^3\theta}\, d\theta \tag{45}$$

A change in variable of integration in accordance with the equation

$$\cos\theta = \frac{1}{\lambda} \tag{46}$$

leads to the equation

$$\frac{1}{4}\int_{-\pi}^{+\pi} \frac{e^{-\delta}}{\cos^3\theta}\, d\theta = \int_1^{\infty} e^{-\lambda^2 \varkappa_0 (z+h)} \cos \lambda\varkappa_0 x \frac{\lambda^2 d\lambda}{\sqrt{\lambda^2 - 1}} \tag{47}$$

where the integral on the right may be recognized as Michell's function as defined by Birkhoff and Kotik[5]. With due recognition that the Michell integral is expressed by an integration once only over the median plane, it may be verified that the present formulae are consistent with Michell's integral in the limiting case of a thin ship. It might be mentioned that a computing program for generating Michell's function along with other oscillatory parts of the velocity has been constructed incidental to efforts to achieve a successful method for interpolation.

The Michell function is a special case of a set of functions which have been investigated by Wilson[7]. The functions $E_c^{(n)}(\alpha, \beta)$ and $E_s^{(n)}(\alpha, \beta)$ are the real and imaginary parts of a complex function $\mathrm{E}^{(n)}(\alpha, \beta)$ as stated by the equation

$$\mathrm{E}^{(n)}(\alpha, \beta) = E_c^{(n)}(\alpha, \beta) + iE_s^{(n)}(\alpha, \beta) \tag{48}$$

while the complex function is defined by the equation

$$\mathrm{E}^{(n)}(\alpha, \beta) = \int_0^{\frac{\pi}{2}} e^{-\alpha^2 \sec^2\theta + i\beta\sec\theta} \sec^n\theta\, d\theta \tag{49}$$

The functions of different orders have convergent and asymptotic expansions and they are connected by recurrence equations. Values of $\mathrm{E}^{(n)}(\alpha, \beta)$ for $n = -1, 0, 1, 2, 3$ have been tabulated in reference (8).

The Michell function is the real part of the third order Wilson function. A quantitative comparison with functions in the present report is possible only for $\beta = 0$. The substitution

$$\sec\,\theta = \frac{1}{2}\,(1 + t) \tag{50}$$

and evaluation in terms of Macdonald functions gives the equation[6, 14]

$$E_c^{(1)}\,(\alpha,\ 0) = \frac{1}{2}\,e^{-\frac{\alpha^2}{2}} \int_1^\infty e^{-\frac{\alpha^2}{2}t}\,\frac{dt}{\sqrt{t^2 - 1}} = \frac{1}{2}\,e^{-\frac{\alpha^2}{2}}\,K_0(\tfrac{1}{2}\,\alpha^2) \tag{51}$$

whence differentiation with respect to α^2 gives the equation[6, 14]

$$E_c^{(s)}\,(\alpha,\ 0) = \frac{1}{4}\,e^{-\frac{\alpha^2}{2}}\,\{K_0(\tfrac{1}{2}\,\alpha^2) + K_1(\tfrac{1}{2}\,\alpha^2)\} \tag{52}$$

At $x = y = 0$ the path of integration with respect to u lies along the real axis except at the half circle where it bypasses the origin. Integration with respect to u and the substitutions

$$\alpha^2 = \varkappa_0(z + h) \qquad\qquad \beta = \varkappa_0 x \tag{53}$$

lead to the equation

$$-\frac{\partial \varphi_3}{\partial x} = 4\varkappa_0^2\,E_c^{(s)}\,(\alpha,\ 0) \qquad (x = y = 0) \tag{54}$$

This equation is fulfilled to ten digit accuracy by the numerical integrations.

At the ONR seminar on wave resistance, Inui[9] has described tables from which the full value of $\partial\varphi_8/\partial x$ may be computed with both monotonic and oscillatory parts included, but only for the direction astern of the source.

CONCLUSION

Any system which is used to compute velocity for a three dimensional position must respond to a random reference. Subroutines can be utilized for an efficient evaluation, but interpolation may require too bulky a table. The fastest subroutines require 6 to 9 seconds per evaluation on NORC.

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