

THEORY OF INITIATION OF EXPLOSION IN SOLIDS  
BY AN INTENSE LIGHT FLASH

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The evolution of the temperature profile within a solid capable of exothermic decomposition is derived from an approximate analytic solution of the equation describing heat conduction in the presence of continuously distributed heat sources due to chemical decomposition and to the absorption of an intense, short duration pulse of light. The critical light intensity capable of giving rise to a thermal explosion is deduced and a minimum, critical intensity is shown to exist. The model predicts all the major experimental features of ignition by light.

Several sensitive explosives having high absorption coefficients for light in or near the visible range have been ignited by intense light flashes of short duration <sup>1-4</sup>. The future development of these may lead to light sources capable of igniting a wider range of explosives.

It is known that many methods of igniting explosives depend on the initial formation of a localized region of high temperature - hot spot<sup>1</sup>. The use of an intense light source of short duration

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offers the possibility of introducing controlled hot spots into an explosive and thus of gaining some knowledge of the high temperature decomposition kinetics of the explosive. Blanchard<sup>5,6</sup> has described the initial results of a digital computer study of the relevant thermal regime and the present author<sup>7</sup> has used an approximate analytic model to explain Barchtold's results<sup>4</sup> for silver nitride. Since no generally applicable model of the ignition process has been described, and in view of the usual limitations of numerical solutions even an approximate analytic solution would be useful. In general the thermal regime is extremely complex. Accordingly several simplifying approximations are made in this paper for the sake of mathematical tractability but they have reasonable physical validity and result in little error in the critical ignition intensities.

#### BASIC EQUATION

For simplicity the following one-dimensional regime is considered. Light is incident normally and uniformly on the free surface of a homogeneous, isotropic explosive mass filling the half-space  $z > 0$ . The free surface is in contact with a vacuum and the entire explosive mass is initially at the uniform temperature  $T_0$ . The equation expressing local conservation of energy has the form

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} + q_0 \nu \exp(-E/RT) + \int \alpha E_0^\lambda e^{-\alpha z} dz \quad (1)$$

where the symbols have the following meanings:

$T$  - absolute temperature,  $\sigma$  - density,  $c$  - heat capacity,  
 $t$  - time,  $k$  - thermal conductivity,  $q$  - exothermicity of the  
 explosive decomposition (energy/unit mass),  $\nu$  - frequency factor,  
 $E$  - activation energy,  $\alpha$  - absorption coefficient,  $\lambda$  - wavelength  
 of light,  $E_0^\lambda$  - light energy flux density in the interval  $(\lambda, \lambda + d\lambda)$   
 at  $z = +0$ , differing from the incident value by a reflexion  
 component.

In general both  $E_0^\lambda$  and  $\alpha$  are complicated functions of  
 wavelength, so that an analytic solution of (1) is impossible in  
 the case of polychromatic radiation. If monochromatic radiation  
 of integrated intensity  $E_0$  (energy/unit area/unit time) at  $z = +0$   
 is used, the relevant equation

$$\underbrace{\sigma c}_{\text{I}} \underbrace{\frac{\partial T}{\partial t}}_{\text{II}} = \underbrace{k}_{\text{III}} \underbrace{\frac{\partial T}{\partial z^2}}_{\text{IV}} + q \sigma \nu \exp(-E/RT) + \alpha E_0 e^{-\alpha z} \quad (2)$$

is more tractable. The use of (2) as the basic equation  
 implies that Arrhenius kinetics and exponential absorption  
 adequately describe the regime. It is further assumed that all  
 the quantities involved, with the exception of  $T$  and  $E_0$  are  
 independent of  $z$  and  $t$ . Latent heats of phase transformation,  
 reactant consumption and hydrodynamic effects are ignored.

The boundary condition at  $z = 0$  which satisfies Stefan's  
 radiation law makes a solution difficult. The Newtonian boundary  
 condition.

$$\frac{1}{T - T_0} \frac{\partial T}{\partial z} = \text{const.}, \quad z = 0 \quad (3)$$

will be used below.

If the light is switched on at the time  $t = 0$  the initial conditions are

$$T = T_0, \quad E_0 = 0 \quad \text{when } t < 0 \quad (4)$$

#### INERT PHASE

Equation (2) with conditions (3) and (4) remains extremely intractable but since term III of (2) is small compared with the other terms until explosion is imminent it may be neglected in the first stage of the solution. (2) can now be written in the form

$$\frac{\partial U}{\partial \tau} = \frac{\partial^2 U}{\partial \rho^2} + A e^{-\rho} \quad (5)$$

where  $\tau = \kappa t \alpha^2 / \sigma c$ ,  $\rho = z \alpha$  are a dimensionless time and distance respectively;  $A = f(\tau) = E_0 / \kappa \alpha$  is a temperature proportional to the energy flux and  $U = T - T_0$ . Equations (3) and (4) become

$$\left(\frac{\partial U}{\partial \rho}\right)_{\rho=0} = h U_{\rho=0} \quad ; \quad h = \text{const.} \quad , \quad (6)$$

$$U = 0, \quad f(\tau) = 0 \quad \text{when } \tau < 0 \quad . \quad (7)$$

The Laplace transform of (5) is

$$\frac{d^2 u}{d\rho^2} - pu = - \int_0^\infty e^{-\rho} f(\tau) e^{-p\tau} d\tau = e^{-\rho} \bar{f}(p) \quad (8)$$

say, because of (7), where  $u = \int_0^\infty U e^{-p\tau} d\tau$ .

The Laplace transform of (6) is

$$\frac{du}{d\rho} = hu \quad \text{at } \rho = 0 \quad (9)$$

The general solution of (8) is

$$-u = \frac{e^{-\rho} \bar{f}(\rho)}{1-\rho} + B e^{s\rho} + C e^{-s\rho}; \quad B, C = \text{const.}, \quad s^2 = \rho.$$

$B$  must be zero for admissible (finite) solutions and (9) gives

$$C = -(1+h) \frac{\bar{f}(\rho)}{(1-\rho)(s+h)}$$

so, finally, the solution of (5) is given by

$$U = e^{-\rho} \mathcal{L}^{-1} \left\{ \frac{\bar{f}(\rho)}{\rho-1} \right\} - (1+h) \mathcal{L}^{-1} \left\{ \frac{\bar{f}(\rho) e^{-s\rho}}{(\rho-1)(s+h)} \right\} \quad (10)$$

where the operator  $\mathcal{L}^{-1}$  corresponds to the Laplace inverse transformation.

If the "emission function",  $f(\tau)$ , of the light source is specified, (10) can be put in explicit form by standard methods<sup>8</sup>.

For brevity only the step function

$$\left. \begin{aligned} f(\tau) &= 0, & \tau < 0 \\ f(\tau) &= A_0, & \tau > 0 \end{aligned} \right\} \bar{f}(\rho) = A_0 \rho^{-1}$$

corresponding to a constant flux source with zero rise time, will

be considered here. (10) now has the form

$$\begin{aligned} \frac{U}{A_0} &= e^{-\rho} (e^{-1}) + \frac{h+1}{h} \left[ \text{erfc}(\rho/2x) - e^{h\rho+h^2x} \text{erfc}(\rho/2x + hx) \right] \\ &\quad - (h+1) \left\{ \frac{1}{2} e^{\rho} \left[ \frac{e^{-\rho}}{h+1} \text{erfc}(\rho/2x - x) + \frac{e^{\rho}}{h-1} \text{erfc}(\rho/2x + x) \right] - \frac{h}{h^2-1} e^{h\rho+h^2x} \text{erfc}(\rho/2x + hx) \right\} \end{aligned} \quad (11)$$

where  $\alpha = \tau^{\frac{1}{2}}$  and  $\operatorname{erfc} z = 1 - \operatorname{erf} z = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-\xi^2} d\xi$ .

It is shown in appendix I that if  $\tau \lesssim 10^2$  then (11) can be written in the approximate form

$$\begin{aligned} \frac{U}{A_0} = & 2\alpha i \operatorname{erfc}(c/2\alpha) - e^{-c} + \frac{1}{2} e^{\tau+c} \operatorname{erfc}(x+c/2\alpha) + \frac{1}{2} e^{\tau-c} \operatorname{erfc}(x-c/2\alpha) \\ & + h \left\{ 2\alpha i \operatorname{erfc}(c/2\alpha) + e^{\tau+c} \operatorname{erfc}(x+c/2\alpha) - \operatorname{erfc}(c/2\alpha) - 4\alpha^2 i^2 \operatorname{erfc}(c/2\alpha) \right\} \\ & + O(h^2) \end{aligned} \quad (12)$$

where  $i^n \operatorname{erfc} z = \int_z^{\infty} i^{n-1} \operatorname{erfc} \xi d\xi$ ;  $i^0 \operatorname{erfc} z = \operatorname{erfc} z$

The form of the repeated integrals of the error function complement and related functions is shown in Fig. 1.

If  $h=0$ , (12) has the form

$$\frac{U}{A_0} = 2\alpha i \operatorname{erfc}(c/2\alpha) - e^{-c} + \frac{1}{2} e^{\tau+c} \operatorname{erfc}(x+c/2\alpha) + \frac{1}{2} e^{\tau-c} \operatorname{erfc}(x-c/2\alpha), \quad (13)$$

a solution previously given by Carslaw and Jaeger<sup>8</sup>.

Now in cases of interest  $h \lesssim 10^{-2}$  (see Appendix E) so (13) can be used as a good approximation to (12). Hence (13) is a good description of the evolution of the temperature profile provided that chemical reaction is insignificant. It will not be valid if high efficiency photochemical processes can occur.

#### INERT DECAY OF HOT SPOT

The total energy dissipated per unit area when the light has been switched on for a time  $t_0$  is given by

$$\mathcal{E} = E_0 t_0 = \int_0^{\infty} \sigma_c U dz \quad \text{and thus}$$

$$\int_0^{\infty} U dz = \frac{\mathcal{E}}{\sigma_c} = A_0 \alpha^2 = \bar{T} \quad , \quad \text{say}$$

where  $\alpha^2 = \tau_0 = k t_0 \alpha^2 / \sigma_c$ .

The temperature profile given by a light pulse of fixed integrated energy density,  $\mathcal{E}$  and of dimensionless duration  $\alpha_0^2$  is given by

$$U/\tilde{T} = U/A_0 \alpha_0^2 \quad (14)$$

where  $U/A_0$  is given by (13) and  $\tilde{T}$  is constant.

The profiles given by (14) are plotted in Fig. 2.

If the light source is switched off at a time  $t_0$ , the appropriate emission function is

$$\begin{aligned} f(\tau) &= A_0 [1 - H(\tau - \tau_0)] \quad , \quad \tau > 0 \\ f(\tau) &= 0 \quad , \quad \tau < 0 \\ \bar{f}(p) &= A_0 p^{-1} (1 - e^{-p\tau_0}) \quad , \end{aligned} \quad (15)$$

where  $H$  is the Heaviside function.

The combination of (10) and (15) with  $h = 0$  gives

$$U = U_*(\tau) - U_*(\tau - \tau_0) \quad (16)$$

where  $U_*(\tau)$  is given by equations (7) and (13). (16) describes the entire inert regime corresponding to a rectangular light flash.

An interesting limiting case occurs when  $A_0$  tends to infinity while  $\tau_0$  tends to zero in such a way that the total energy (which is proportional to  $A_0 \tau_0$ ) remains constant.

The appropriate emission function is

$$f(\tau) = \frac{\mathcal{E} \alpha}{\sigma c} \delta(\tau, 0) ; \quad \bar{f}(p) = \tilde{T}$$

and gives the profile



$$\frac{U}{\hat{T}} = \frac{1}{2} e^{-\rho^2/4\tau} \left[ Z_0(x - \rho/2x) + Z_0(x + \rho/2x) \right]$$

from (10) where  $Z_0(z) = e^{z^2} \operatorname{erfc} z$ . (17) is the limiting form of (16) as  $\tau_0 \rightarrow 0$ ,  $\mathcal{E}_0 = \text{const.}$ , and with  $\tau = 0$  yields the limiting case of (13):  $U = \hat{T} e^{-\rho}$ , a result entirely to be expected for the initial temperature distribution produced by a light pulse of infinitesimal duration.

#### REACTIVE PHASE

In the above treatment heat evolution due to chemical reaction has been entirely ignored. In this section the effect of chemical reaction is introduced as a perturbation of the inert evolution of the temperature profile, in the following way. Firstly, chemical reaction during the time for which the light is switched on is ignored and, secondly, the rate of decay of the hot spot due to heat conduction after the light source is switched off is considered to be identical to that of the inert case. The validity of this procedure is examined in appendix II. Although not entirely justifiable this procedure is of the utmost importance in simplifying the subsequent analysis.

If chemical reaction is ignored prior to the time  $\tau_0$  then the inert temperature decay after the time  $\tau_0$  is given by (16).

In particular

$$(U/A_0)_{\rho=0} = \frac{2}{\sqrt{\pi}}(x-x') + Z_0(x) - Z_0(x')$$

where

$$\tau' = \tau - \tau_0, \quad x' = \sqrt{\tau - \tau_0} = \sqrt{\tau'}$$

Hence  $\frac{1}{A_0} \left( \frac{\partial U}{\partial \tau} \right)_{\rho=0} = Z_0(\sqrt{\tau'+\tau_0}) - Z_0(\sqrt{\tau'})$  (18)

in the inert case.

Now, by definition, term IV of (2) is zero for  $\tau > \tau_0$ ,  $\tau' > 0$  so Frank-Kamenetskii's exponential approximation<sup>9,24</sup> can be used to write (2) in the dimensionless form

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \rho^2} + \delta e^{\theta}$$

or approximately

$$\frac{\partial \theta}{\partial \tau} \approx \frac{\partial^2 \theta}{\partial \rho^2} \Big|_{\text{inert}} + \delta e^{\theta}$$
 (19)

where  $\theta = \frac{E}{RT_s^2}(\tau - T_s)$ ,  $\delta = \frac{q\sigma_0 E}{\alpha^2 k RT_s^2} \exp(-E/RT_s)$

and  $T_s = T_0 + U(c=0, \tau=\tau_0)$  (20)

In the approximations the Arrhenius rate expression is expanded about the highest temperature produced in the inert phase of regime, where the effect of chemical reaction is most important. The necessary condition for a thermal explosion to occur in an explosive with zero order kinetics is that the temperature somewhere tend to infinity. In the present regime the temperature is greatest at the origin,  $\rho=0$ . Only the evolution of the temperature at the free surface will be further considered.

Since

$$\left. \frac{\partial \theta}{\partial \rho^2} \right|_{\text{inert}} = \left. \frac{\partial \theta}{\partial \tau} \right|_{\text{inert}} = \frac{E}{RT_s^2} \left. \frac{\partial U}{\partial \tau} \right|_{\text{inert}} \quad \text{for } \tau > \tau_0 .$$

(18) and (19) may be combined to give

$$\frac{d\theta}{d\tau} = Bg(\tau, \tau_0) + \delta e^\theta \quad \text{for } \epsilon = 0 \quad (21)$$

where

$$B = \frac{EA_0}{RT_s^2}, \quad g(\tau, \tau_0) = Z_0(\sqrt{\tau + \tau_0}) - Z_0(\sqrt{\tau})$$

and, for convenience, the prime on  $\tau'$  has been dropped. The ordinary differential equation (21) can be integrated to give

$$\eta_{\text{exp}} \left[ B \int g d\tau \right] = - \int \int \exp \left[ B \int g d\tau \right] d\tau + \text{const} \quad (22)$$

where  $\eta = e^{-\theta}$ .

It is clear that explicit explosion conditions cannot be derived from (22) unless  $g(\tau, \tau_0)$  has a simple form. Fortunately  $g(\tau, \tau_0)$  has simple asymptotic forms at both small and large values of  $\tau_0$ . These two extremes will be considered separately.

#### SHORT DURATION FLASH

If  $\tau_0 \ll 1$  then

$$\begin{aligned} g(\tau, \tau_0) &= Z_0(\sqrt{\tau + \tau_0}) - Z_0(\sqrt{\tau}) \approx \tau_0 \frac{dZ_0(x)}{dx} \\ &= \left[ Z_0(x) - \frac{x^{-1}}{\sqrt{\pi}} \right] \tau_0 \approx - \frac{\tau_0}{\sqrt{\pi}} x^{-1}, \end{aligned}$$

if  $x \ll 1$ .

Hence  $B \int g d\tau \approx \frac{2}{\sqrt{\pi}} B \tau_0^2 \tau^{\frac{1}{2}}$  and (22) with the initial

condition,  $\theta = 0$  when  $\tau = 0$ , gives

$$\eta = \mu(1+y) + (1-\mu)e^y \quad (23)$$

where

$$y = \left\{ \frac{2}{\sqrt{\pi}} \cdot \frac{EA_0 x_0}{RT_s^2} \right\} \tau^{\frac{1}{2}}, \quad \mu = \frac{\pi}{2} \cdot \frac{q\sigma\gamma R}{\kappa\alpha^2 E} \left( \frac{T_s}{T} \right)^2 \exp(-E/RT_s)$$

Fig. 3 shows the nature of (23) for various values of the parameter  $\mu$ . Clearly the condition for explosion to occur is  $\mu > 1$ .

#### LONG DURATION FLASH

If  $\tau_0 \gg 1$ ,  $\tau \ll 1$  then

$$\begin{aligned} g(\tau, \tau_0) &= Z_0(\sqrt{\tau_0 + \tau}) - Z_0(\sqrt{\tau}) \\ &\approx Z_0(\sqrt{\tau_0}) + \tau \left[ Z_0'(\sqrt{\tau_0}) - \frac{1}{\sqrt{\pi\tau_0}} \right] - \left[ \frac{2}{\sqrt{\pi}} \tau + \tau^2 \right] \\ &\approx Z_0(x_0) - 1 \end{aligned}$$

Hence  $B \int_0^\tau g^k \tau \approx - \frac{E\tilde{T}}{RT_s^2} \frac{1 - Z_0(x_0)}{\tau_0} \tau$  and (22) with the appropriate initial condition gives

$$\eta = \varepsilon + (1-\varepsilon)e^{C\tau} \quad (24)$$

where  $C = \frac{E\tilde{T}}{RT_s^2} \frac{1 - Z_0(x_0)}{\tau_0}$

and  $\varepsilon = \frac{\delta}{C} = \frac{q\sigma\gamma\tau_0 \exp(-E/RT_s)}{\alpha^2 \kappa \tilde{T} [1 - Z_0(x_0)]}$

Fig. 4 shows the nature of (24) for various values of the parameter  $\varepsilon$ . Clearly the condition for explosion to occur is  $\varepsilon > 1$ .

### CONDITION FOR EXPLOSION

The conditions for explosion are given by

$$\mu > 1 \quad (\tau_0 \ll 1)$$

$$\varepsilon > 1 \quad (\tau_0 \gtrsim 1)$$

Although critical energy densities,  $\mathcal{E}_{cr}$ , could be computed from these criteria, it is preferable to find an explicit formula for  $\mathcal{E}_{cr}(\tau_0)$  even at the expense of some accuracy. Consequently a simple approximation to  $T_S$  is required. By definition

$$T_S - T_0 = U(\rho=0, \tau'=0) = A_0 \left\{ \frac{2x_0}{\sqrt{\pi}} + Z_0(x_0) - 1 \right\}$$

from (13). Or since  $A_0 \tau_0 = \tilde{\tau}$ ,

$$T_S - T_0 = \tilde{\tau} \Phi^{-1}(x_0)$$

where 
$$\Phi(x) = x^2 \left\{ \frac{2x}{\sqrt{\pi}} + Z_0(x) - 1 \right\}^{-1}$$

It can be shown that if  $x$  is small

$$\Phi(x) = 1 + \frac{4x}{3\sqrt{\pi}} + \frac{1}{3} \left[ \frac{16}{3\pi} - 1 \right] x^2 + \dots$$

$$= 1 + 0.752x + 0.233x^2 + O(x^3)$$

and if  $x$  is large

$$\Phi(x) = \frac{\sqrt{\pi}x}{2} \left[ 1 + \frac{\sqrt{\pi}}{2x} + \frac{1}{2} \left( \frac{\pi}{2} - 1 \right) \frac{1}{x^2} + \dots \right]$$

$$= 0.786 + 0.885x \left[ 1 + \frac{0.286}{x^2} - \frac{0.190}{x^3} + O(x^{-4}) \right]$$

Fig. 5 shows a plot of  $\Phi(x)$  and its linear asymptotes. It is

clear that  $1 + \frac{\sqrt{\pi}}{2}x = 1 + 0.885x$  estimates  $\Phi(x)$

for all  $x \geq 0$  with an error of less than 5%.

The critical condition for a short duration flash,  $\mu = 1$ , may be written in the form

$$\frac{1}{a_0^2 x^2} \exp(-E/RT_0) \left[ \Phi^{-1}(x_0) + \frac{T_0}{\tilde{T}} \right] = 1$$

where  $a_0^2 = \frac{2}{\pi} \left( \frac{T_R}{T_A} \right) \left( \frac{D}{\gamma} \right)$  is constant for a given explosive

and  $T_R = E/R$ ,  $T_R = q/c$ ;  $D = k/\rho c$  is the thermal diffusivity. Since  $\Phi^{-1}(x) \approx -\frac{4x}{3\sqrt{\pi}}$  for small  $x$  and  $T_0/\tilde{T} \leq 0.3$  in cases of interest, the critical condition is given approximately

by

$$\frac{\tilde{T}}{T_A} = K_0 \Phi(x_0)$$

where  $K_0 = \left[ \ln \left( \frac{1}{a_0^2 x^2} \right) \right]^{-1} - \frac{T_0}{T_A}$ .

The critical condition for the long duration flash,  $\xi = 1$ , has the form

$$\frac{\tilde{T}}{T_A} = \Phi(x_0) \left\{ \left[ \ln \frac{2x_0 T_R / \pi (1 - Z_0(x_0))}{\tilde{T} a_0^2 x^2} \right]^{-1} - \frac{T_0}{T_A} \right\} = K \Phi(x_0), \text{ say.}$$

This expression does not yield  $\tilde{T}$  explicitly, but since  $T_0/T_A$  is small ( $\sim 10^{-2}$ ) and  $a_0^2 x^2 \sim 10^{-12}$  for the conventional explosives,  $K$  is insensitive to large variations in the argument of the logarithm.

Thus  $\tilde{T}/T_A \approx K_0 \Phi(x_0)$  and

$$K \approx \left[ \ln \frac{2x_0 / \sqrt{\pi} (1 - Z_0(x_0))}{a_0^2 x^2} \right]^{-1} - \frac{T_0}{T_A} \quad (25)$$

Since  $Z_0(x) \approx -\frac{2}{\sqrt{\pi}} x$  when  $x$  is small, (25) gives the correct limiting value  $K_0$  as  $x_0$  tends to zero.

Thus the critical condition for explosion has the approximate form

$\tilde{T}/A = K \underline{\Phi}$  where  $K$  is given by (25) and is a feebly decreasing function of  $\alpha_0$ , while  $\underline{\Phi}$  increases strongly with  $\alpha_0$  according to the approximate formula

$$\underline{\Phi} = 1 + 0.885 \alpha_0$$

Ignoring the variation in  $K$  the critical condition is

$$\tilde{T} = K_0 \underline{\Phi} T_A \quad (26)$$

This formula should predict critical energy densities with a maximum error of 10% if  $\alpha_0 \ll 1$  or  $1 \lesssim \alpha_0 \lesssim 10$ . It will further be assumed that (26) remains a good approximation for the intermediate range of  $\alpha_0$  where a solution of (21) is difficult. According to (26) the critical energy density is given by

$$E_c = K_0 \left( \frac{\sigma c T_A}{\alpha} \right) \left[ 1 + \frac{\sqrt{\pi}}{2} \alpha_0 \right] = K_0 \left( \frac{\sigma c T_A}{\alpha} \right) \left[ 1 + \sqrt{\frac{\pi D}{4}} \cdot \alpha t_0^{\frac{1}{2}} \right] \quad (27)$$

#### DISCUSSION

In the derivation of critical energy densities for ignition numerous assumptions and approximations have been introduced, the validity of which requires examination before a comparison between (27) and experimental measurements is attempted.

##### 1. CONSTANCY OF PARAMETERS.

For real explosives there are two sources of variation of the relevant thermochemical parameters. Temperatures occurring in

the ignition regime range between 300°K and ca 2000°K so that significant variations, particularly in the activation energy<sup>10</sup>, are to be expected. The error introduced in this way can be minimized by selecting averaged values for the density and heat capacity and the high temperature values for the activation energy and  $K_0$ .

It is shown below that variable composition due to reactant consumption has only a small effect on the parameters.

## 2. LATENT HEAT EFFECTS

If a zone of thickness  $\rho_m \alpha^{-1}$  must be melted before the critical temperature profile becomes established then an extra amount of energy  $\Delta \tilde{E}$  must be expended, where

$$\Delta \tilde{E} \approx \rho_m \alpha^{-1} L \quad (\text{or } \Delta \tilde{T} = \frac{L}{c} \rho_m)$$

and  $L$  is the latent heat. Thus

$$\frac{\Delta \tilde{E}}{E_0} = \frac{\Delta \tilde{T}}{T_0} = \frac{\rho_m}{K_0 \alpha} \cdot \frac{L}{c T_0} \approx \frac{L}{c T_0} \frac{\ln[U_0 / (T_m - T_0)]}{K_0 \alpha}$$

where  $T_m$  is the melting point of the explosive. Since  $(L/c)$  is about 100°K for both organic and inorganic explosives, the latter ratio may be as small as 10% for a sensitive explosive with a high melting point (e.g.  $\text{AgH}_3$ ) or as large as unity for an insensitive explosive with a low melting point (eg. TNT). The value of  $E_0$  given by (27) is of the correct order of magnitude but for a more accurate estimate the amount  $\sigma L \alpha^{-1} \ln[U_0 / (T_m - T_0)]$  should be added.



### 3. REACTANT CONSUMPTION

It can be shown that if  $\alpha_0$  is small the fraction of undecomposed explosive  $\xi$  satisfies the condition

$$-\ln \xi < \frac{\alpha_0^2}{\frac{E}{RT_s} + \frac{\sqrt{\pi}}{2} \alpha_0} \left( \frac{-t_0}{\theta_R} \right) \exp \left\{ -t_0 \frac{\sqrt{\pi} \alpha_0}{2} \right\} \quad \text{if } t < 1$$

in the neighbourhood of critical conditions, where  $\theta_0 = E(T_0 - T_s)/RT_s^2$

and  $\theta_R = ET_R/RT_s^2$ . In cases of interest  $-\theta_0/\theta_R \lesssim 0.5$  and

$\theta_0 \approx -10$  so that  $\xi > 0.8$  throughout the explosive "induction period"<sup>12</sup>,  $t < 1$ , provided that  $\alpha_0 \lesssim 0.2$ . Exact numerical solutions<sup>6</sup> show that reactant consumption remains of little importance for times rather larger than  $\alpha_0 = 0.2$ . The small amount of reactant consumption during the "induction period" justifies the neglect of hydrodynamic effects.

### 4. DELAY TIME TO EXPLOSION

If  $\tau_0$  is small the reduced time,  $\tau_*$ , required for the surface temperature to become infinite is given by

$$\mu(1+y_*) + (1-\mu)e^{y_*} = 0$$

from (23). If the regime is critical,  $\mu = 1$  and  $\tau_*$  becomes infinite.

If, however, the regime is slightly supercritical,  $\mu \geq 1.1$ , this

author<sup>11</sup> has shown that  $1 < \tau_*/\tau_0 < 9$ , where  $\tau_0 = \delta^{-1}$  is

the reduced adiabatic explosion time at the temperature  $T_s$ ,

so that for temperatures which are only slightly supercritical the

delay time to explosion is of the same order of magnitude as the

adiabatic explosion time at the surface temperature.

Thus in the immediate neighbourhood of critical<sup>a</sup> conditions

$$\tau_* \approx 10\tau_{\infty} \approx 10 (\theta_0)_{cr}^{-2} \approx 10 K_0^2$$

and since  $K$  is typically  $\sim 10^{-1}$  for the conventional explosives

$$\tau_* \approx 10^{-1}.$$

Hence the assumption that  $\alpha \ll 1$  made in the derivation of (23) is reasonably valid.

Similarly if  $\tau_0$  is large (24) gives  $\tau_*/\tau_{\infty} = \epsilon \ln \frac{\epsilon}{\epsilon-1}$ , so that if  $\epsilon = 1.1$ ,  $\tau_*/\tau_{\infty} = 2.6$ . The delay time near critical conditions is given by

$$\tau_* \approx 3\tau_{\infty} \approx 3 (\beta)_{cr}^{-1} \approx 3 K^2 \left( \frac{T_A}{T_R} \right) \tau_0$$

and since  $\frac{T_A}{T_R} \sim 5$  for the conventional explosives,  $\tau_* \approx \tau_0 / 6$

Hence the assumption that  $\alpha \ll 1$  does not remain valid throughout the explosive induction period unless  $\tau_0 \lesssim 1$ .

##### 5. DEPARTURE FROM EXPERIMENTAL CONFIGURATION

Actual experiments on ignition by light are carried out using a light flash with a non-rectangular emission function focussed on a finite area of an explosive slab of finite thickness. The conditions under which the results for the present idealized configuration remain valid in practice must be considered.

It can be seen from Fig.2 that the light flash causes significant heating only in a thin surface layer of thickness  $\text{ca. } \alpha^{-1}$  (corresponding to  $\rho = 1$ ). In fact, it can be shown from (13)

that the reduced distance to the inflexion in the temperature profile,  $\rho_i$ , is given by

$$\rho_i \approx \ln \sqrt{\pi} = 0.572 \quad \text{if} \quad x_0 \ll 1$$

$$\rho_i \approx \ln(\sqrt{\pi} x_c) \quad \text{if} \quad x_c \gg 1$$

and that if  $\rho/x$  is large the temperature excess  $U$  decays as  $e^{-\rho}$ .

If the rear surface of the explosive slab corresponds to  $\rho > 4$  the light is attenuated by a factor greater than  $e^4 \approx 50$ , so a slab thicker than  $4\alpha^{-1}$  reflects negligible radiation from its rear surface. In addition heat losses from the rear surface into a vacuum or a solid substrate are small provided that  $\tau_0 \lesssim 10$ , i.e.  $\rho_i < 3$ .

Consequently the idealized analysis remains applicable if the thickness of the explosive slab exceeds  $10\alpha^{-1}$ , say. These considerations are in good agreement with Borchtold's observations<sup>4</sup> that the critical ignition energy of an  $\text{Ag}_3\text{N}$  film deposited on glass is sensibly constant if the film thickness exceeds  $0.5\mu$  when  $\alpha^{-1} \approx 10^{-5}$  cm.

In most cases of interest  $\alpha^{-1} < 10^{-2}$  cm and thus the idealized treatment is applicable to any explosive mass thicker than 1 mm.

If a parallel beam of light is incident normally upon a semi-infinite cylinder of finite diameter  $d$ , the lateral heat

is negligible compared with the heat loss from the surface  $\gamma = 0$  provided that  $d \gg \alpha^{-1}$ . Since this free surface heat loss has been shown to be negligible the idealized results remain good approximations as long as  $d \alpha \gg 1$ .

If a condenser system is used, so that some radiation is no longer incident normally on the free surface, the radiation is degraded into thermal energy closer to the surface  $\rho = 0$  than in the case of normal incidence. The effect on the maximum surface temperature attained is small and the effective width of the heated zone is reduced by less than 30% even when the semi-angle of the light cone,  $\omega$ , is as great as  $45^\circ$ . Ignoring reflexion effects, it can be shown that  $E_{cr}$  is equal to the value given by (27) multiplied by a factor not less than  $\cos \omega$ . In<sup>fact</sup> the reflexion effects, which have been avoided in defining  $E_0$  and related quantities, should make the experimental quantity (critical source energy reaching explosive) / (surface area illuminated) insensitive to the type of condenser system used.

The remarks of this section apply only if no lateral heat sinks in the form of inert confinement or unilluminated explosive are present strongly to modify the predicted temperature evolution.

## 6. REAL LIGHT SOURCES

The output of a real light source is in general so complicated a function of time that the present approach becomes unproductive. If, however, the flash duration,  $t_0$ , is considerably less than  $(D \times 2)^{-1}$ , so that  $\alpha_0 \ll 1$ , then the shape of the emission function is unimportant. To demonstrate this fact,  $h$  is

equated to zero in (10) which can then be written in the form

$$U = \frac{1}{2} [f(\tau)] * \left[ e^{-\rho^2/4x^2} \left\{ Z_0(x + \rho/2x) + Z_0(x - \rho/2x) \right\} \right], \quad (28)$$

where  $[f_1(\tau)] * [f_2(\tau)] = \int_0^\tau f_1(\tau') f_2(\tau - \tau') d\tau'$

is the convolution of  $f_1$  and  $f_2$ .

If  $x_0$  is small it can be shown from (28) that the temperature distribution for  $x = x_0$  satisfies the inequalities

$$e^{-\rho^2} \int_0^{\tau_0} f(\tau) d\tau < U < e^{-\rho^2(1+\tau_0)} \int_0^{\tau_0} f(\tau) d\tau \quad \text{for} \quad \rho \gtrsim x$$

$$\int_0^{\tau_0} f(\tau) d\tau > U > \left(1 - \frac{2x_0}{\sqrt{\pi}}\right) \int_0^{\tau_0} f(\tau) d\tau \quad \text{for} \quad \rho = 0$$

These inequalities show that if  $\tau_0 \ll 1$  the initial temperature distribution and, therefore, the critical energy density for explosion are practically independent of  $\tau_0$  and depend only upon the quantity  $\int_0^{\tau_0} f(\tau) d\tau$  which is proportional to the integrated energy output of the light source. Although the present treatment requires that  $\alpha$  is constant, it should remain valid for polychromatic light such that  $\frac{\Delta\alpha}{\alpha} \ll 1$ . This condition is satisfied by many explosives<sup>1</sup> in the visible region but unfortunately  $\alpha$  is, in general, so small for visible light that existing sources are several orders of magnitude too feeble to give rise to explosion.

Clearly the general case of ignition by polychromatic

light could only be described by numerical methods.

It is concluded that the present idealized model, subject to the reservations of the above discussion, corresponds closely to reported experimental configurations<sup>1-4</sup>,

It is difficult to estimate the effect of the mathematical approximations used in the above analysis upon the critical values of the parameters  $\mu$  and  $\varepsilon$  but it is unlikely that they are in error by a factor as large as 5. Since  $E_{cr}$  varies as the logarithm of  $\mu_{cr}$  or  $\varepsilon_{cr}$  the consequent error in  $E_{cr}$  is not greater than 10% for explosives with activation energies greater than 20 Kcal.mole<sup>-1</sup>. A further error, not greater than 10% provided that  $\tau_0 \lesssim 10$ , is incurred by using the simplified formula (27) instead of the exact explosion criteria. Certainly, (27) should estimate critical energy densities to within one order of magnitude.

#### COMPARISON WITH EXPERIMENT

Equation (27) gives a good approximation to the critical energy density for ignition provided that the duration of the light flash does not exceed a standard conduction time  $t_c = (D\alpha)^{-1}$  by more than about one order of magnitude. The criterion becomes inaccurate for long duration flashes. The thermal diffusivity of most explosives lies in the range  $10^{-3}$ - $10^{-4}$  cm<sup>2</sup>sec<sup>-1</sup>, while the absorption coefficient in cases of interest ranges from  $10^2$  to  $10^5$  cm<sup>-1</sup>. Thus the standard conduction time  $t_c$  may range from 0.1  $\mu$  sec (at large  $\alpha$ ) to 1 sec (small  $\alpha$ ). The value of

$t_c$  is most important in determining the way in which the critical energy varies with flash duration.

#### EFFECT OF FLASH DURATION

If the conditions of an experimental determination of  $\mathcal{E}_c$  as a function of  $t_c$  are such that  $t_0 \gg t_c$  then (27) predicts

$$\mathcal{E}_c = K_0 \sigma c T_A \sqrt{\frac{\pi}{4} D} \left[ t_0^{\frac{1}{2}} + \sqrt{\frac{t_c}{\pi}} \cdot t_c^{\frac{1}{2}} \right] \approx K_0 \sigma c T_A \sqrt{\frac{\pi}{4} D} \cdot t_0^{\frac{1}{2}} \quad (28)$$

In this case the critical energy varies linearly with (and is almost proportional to) the square root of the flash duration. It is interesting to note that if the feeble variation of  $K_0$  with  $\alpha$  is ignored (28) predicts that the critical energy density

$$\mathcal{E}_c \approx \sqrt{\frac{\pi}{4} K_0 \sigma c} \cdot T_A K_0 t_0^{\frac{1}{2}} \quad (29)$$

is independent of the absorption coefficient. This fact suggests that (29) may remain valid for polychromatic light satisfying the condition,  $t_0 \gg t_c$  or  $\alpha^2 \gg (Dt_0)^{-1}$ . Berthold<sup>1,4</sup> has ignited Berthollot's "explosive silver",  $Ag_3N$ , with polychromatic light for which  $\alpha \gtrsim 10^4 \text{ cm}^{-1}$  using flashes of millisecond duration. Thus in this case the condition  $t_0 \gg t_c$  can be satisfied if  $D \gg 10^{-5} \text{ cm}^2 \text{ sec}^{-1}$ . Although  $D(Ag_3N)$

not been measured, it is reasonable to assume that the latter condition is satisfied. Fig.6 shows the variation of  $E_{cr}$  with the square root of the flash duration calculated from Berchtold's data<sup>4</sup>. It can be seen that the variation is almost linear. Moreover Berchtold reports that, allowing for experimental error, the critical energy is independent of the range of wavelengths of the light used. This fact supports the tentative suggestion made above concerning "long" duration flashes of polychromatic radiation (see appendix III).

With the opposite extreme of a "short" duration flash the predicted behaviour is quite different. If  $\alpha^2 t_0 D \ll 1$ , (27) can be written in the form

$$E_{cr} = K_0 \frac{\sigma_c T_A}{\alpha} \left[ 1 + \sqrt{\frac{\pi t_0}{4 t_c}} \right] \approx K_0 \sigma_c T_A / \alpha$$

Thus the critical energy of a short flash should be almost independent of the flash duration and inversely proportional to the absorption coefficient (ignoring the variation of  $K_0$  with  $\alpha$ ). Unfortunately suitable "short" flash duration experiments have not yet been performed since the presently available light sources cannot deliver adequate power within a bandwidth for which  $\alpha$  is both sensibly constant and sufficiently large. (A giant-pulse "laser" system operating in the presently available power range but with a resonant wavelength of 2-3000 Å would constitute a suitable radiation source). However it has been observed experimentally<sup>1</sup> that the critical ignition energy of a short



duration flash increases strongly as the effective mean absorption coefficient of the spectral range employed is increased.

An extremely important conclusion to be drawn from (27) is that as the flash duration tends to zero the critical energy density tends to the finite value

$$E_{min} = \frac{\sigma c T_R}{\alpha} \left\{ \left[ \ln \left( \frac{1}{\alpha_0^2 \alpha^2} \right) \right]^{-1} - \frac{T_0}{T_R} \right\} \quad (30)$$

which is the minimum energy density, at the wavelength corresponding to  $\alpha$ , that is capable of giving rise to explosion."

#### EFFECT OF AMBIENT TEMPERATURE

If  $E_{cr}$  is measured for a series of values of  $T_0$  using a light source of constant duration then according to (27)

$$\frac{\partial E_{cr}}{\partial T_0} = - \frac{\sigma c}{\alpha} \bar{\Phi}(\alpha_0) = \text{const.} \quad (31)$$

provided that the variation of the thermochemical parameters over the relevant temperature range is negligible. Moreover  $\partial E_{cr} / \partial T_0$  will remain sensibly constant even if large variations in  $\alpha_0$  (such that  $\alpha_0 \ll 1$ ) are permitted. The linear variation of  $E_{cr}$  with  $T_0$  predicted by (31) will only be valid as long as  $T_0$  is considerably less than the "explosion temperature" of the system explosive plus environment. The applicability of the foregoing analysis depends not only upon the fact that  $E/RT_0$  is large

but also upon the assumption that the system is inherently stable in the neighbourhood of the temperature  $T_0$ . However, a truly adiabatic explosive mass necessarily explodes if it is initially at any temperature greater than absolute zero. A real explosive mass is stabilized below a certain temperature  $T_{cr}$  by its finite rate of heat loss to the environment and above that temperature either decomposes quietly<sup>12,13</sup> or explosively<sup>9,14,15</sup> according to the nature of its reaction kinetics. The range of ambient temperature for which the present analysis is applicable cannot be uniquely stated since  $T_{cr}$  depends upon the environment and is not, therefore, a characteristic parameter of the explosive. The explosion criteria derived here are valid if the reaction rate of the system is negligible at the ambient temperature.

If  $\tilde{C}_{cr}$  is plotted against  $T_0$ , the low temperature linear asymptote of the resultant curve intercepts the temperature axis at

$$T_i = T_A \left[ \ln \left( \frac{1}{a_0^2 \alpha^2} \right) \right]^{-1},$$

a value independent of flash duration.

A number of explosives have been shown to obey the linear law predicted by (31). Fig.7 illustrates the results obtained by McAuslan<sup>16</sup> for lead styphnate. A linear law is clearly a satisfactory description of the results over a wide temperature range. The value of  $T_i$  is approximately 750°K. Similar results have

been obtained with the following explosives for which the values of  $T_i$  are given in brackets :  $\text{AgNO}_3$  (555°K)<sup>17</sup>,  $\text{PbN}_6$  (655°K)<sup>4</sup>,  $\text{NH}_3\text{NI}_3$ , "nitrogen iodide" (355°K)<sup>18</sup>,  $\text{AgNO}_3 \cdot \text{Ag}_2\text{C}_2$  (ca 800°K)<sup>4</sup>. It is important to note that, for a given wavelength,  $T_i(\lambda)$ , unlike  $T_{\text{cr}}$ , is an eigen-property of the explosive. The reported approximate equality of  $T_i$  and  $T_{\text{cr}}$ <sup>1,4</sup> is to be regarded as fortuitous. From the definitions of  $T_i$  and  $T_s$

$$T_s - T_0 = (K/K_0) (T_i - T_0)$$

under critical conditions. Hence if  $\kappa_0$  is small ( $K/K_0 \approx 1$ ),  $T_s \approx T_i$  and  $T_i$  is approximately the minimum surface temperature that must be produced by a short duration flash in order to cause an explosion. As a further illustration of the significance of  $T_i$  it is noted that (26) may be written in the form

$$\frac{E_{\alpha} \alpha}{\sigma c} = \tilde{T}_{\alpha} = \Phi(T_i - T_0) \quad (32)$$

For a short duration flash  $E_{\alpha}$  is the amount of energy required to raise unit area of a surface "skin" of thickness  $\alpha^{-1}$  to the temperature  $T_i$ .

#### ORDERS OF MAGNITUDE

The above treatment has been concerned with the quantity  $\epsilon^{+0}$  whereas the experimentally important quantity is  $\epsilon^{-0}$ , the incident energy density outside the explosive. These two variables are related by the formulas<sup>19</sup>

$$\frac{c+c_0}{c} = \frac{1}{t_\lambda} \epsilon^{-\alpha}, \quad \frac{1}{t_\lambda} = \frac{4\pi n}{(n+1)^2 + K_0^2}, \quad K_0 = \frac{\alpha \lambda}{4\pi}$$

where  $n$  is the refractive index and  $K_0$  the absorption index.

Now for the organic explosives,  $n \sim 1.5 - 1.6$ , and for the inorganic explosives<sup>1</sup>,  $n \sim 2 - 5$ , and  $\alpha \lesssim 10^5 \text{ cm}^{-1}$  so that

$\frac{1}{t_\lambda}$  is greater than 90% except in the neighbourhood of a strong absorption edge ( $\alpha \sim 10^5 \text{ cm}^{-1}$ ) where it may fall to ca 50%.

Thus in discussing orders of magnitude it is reasonable to equate  $\frac{1}{t_\lambda}$  to unity.

Table 1 gives the values for some common explosives of the thermochemical parameters needed to calculate  $E_{cr}$  from (27).

It is noteworthy that both  $c_0 T_0$  and  $D$  are almost constant at the values  $10^4 \text{ cal. cm}^{-3}$  and  $10^{-3} \text{ cm}^2 \text{ sec}^{-1}$ , respectively, in spite of the large variation of explosive character among the substances considered. Since  $a_0 \lesssim 10^{-9} \text{ cm}$ ,  $\alpha \lesssim 10^5 \text{ cm}^{-1}$  and

$$T_0/T_n \simeq 1.5 \times 10^{-2}$$

$$K_0 = \left[ \ln \left( \frac{1}{a_0^2 \alpha^2} \right) \right]^{-1} \frac{T_0}{T_n} \lesssim 4 \cdot 10^{-2}$$

Hence the critical energy densities for short duration flashes

are of the order of

$$\frac{4 \cdot 10^2}{\alpha} \text{ cal. cm}^{-2} \simeq \frac{10^3}{\alpha} \text{ j. cm}^{-2},$$

(where  $\alpha$  is expressed in  $\text{cm}^{-1}$ ) whether the explosive be

primary or secondary. Most of the experimental workers have

not reported the absolute magnitude of  $E_{cr}$  but some quantity

considered to be proportional to it (such as the square of the voltage on a storage condenser). Consequently a detailed numerical comparison of the present theory and experimental results cannot be made. However, Courtney-Pratt and Rogers<sup>1,22</sup> have reported absolute data for  $\text{AgN}_3$  using radiation in the near ultraviolet for which  $\alpha = 10^3 - 10^4 \text{ cm}^{-1}$ ,  $\tau_0 \approx 0.15$ . They find  $E_{cr} \approx 8 \times 10^{-2} \text{ cal cm}^{-2}$ . This value is yielded by (27) and Table 1 if  $\alpha = 1.39 \times 10^3 \text{ cm}^{-1}$ , and by (32) with the experimental result  $T_1 = 555^\circ\text{K}$  if  $\alpha = 1.63 \times 10^3 \text{ cm}^{-1}$ .

Both these values of the absorption coefficient lie within the range corresponding to the wavelength range employed. Berchtold<sup>4,1</sup> has also estimated that  $E_{cr}$  is of the order of 1 joule  $\text{cm}^{-2}$  for a large number of substances. The relevant values of  $\alpha$  are not reported, but the present theory requires them to have the reasonable magnitude of ca  $10^3 \text{ cm}^{-1}$ . Clearly the present theory predicts the correct magnitude of the critical energy density.

The value of  $E_{cr}$  given by (27) could be extremely low for a well designed light pulse. For instance, in the neighbourhood of the fundamental absorption edge the absorption coefficient may be as great as  $10^5 \text{ cm}^{-1}$  so that the critical energy density for the corresponding radiation is about  $10^{-2} \text{ joule cm}^{-2}$ , if the flash duration is less than  $0.1 \mu\text{s}$ . Clearly a high absorption coefficient makes the most efficient use of the incident radiation. If  $\alpha$  is large an incident intensity of about  $10^3 \alpha^{-1} \text{ joule cm}^{-2}$  delivered

in less than  $10^2 \lambda^{-2}$  seconds will suffice to initiate the conventional explosives ( $\lambda$  is in units of  $\text{cm}^{-1}$ ). This requires a source power in excess of  $10 \alpha \text{ watt. cm.}^{-2}$ , say one Megawatt  $\text{cm.}^{-2}$  near an absorption edge. (The ruby laser system of McClung and Hellworth<sup>23</sup> can operate at this power for ca  $0.2 \mu\text{s}$  at a wavelength of  $6943 \text{\AA}$ ). Because most explosives have very low absorption coefficients for radiation in the visible range ( $10^{-1}$  to  $10 \text{ cm.}^{-1}$ ), existing sources cannot deliver the adequate critical intensities in this part of the spectrum alone. Consequently, with the exception of some coloured "exotic" substances, e.g.  $\text{Ag}_3\text{N}^4$ , initiation is caused by the ultraviolet radiation accompanying the ineffective "light" flash. The apparently dissimilar "sensitivities" of primary and secondary explosives must be ascribed in the main to the fortuitous positioning of their absorption edges with respect to the energy distribution of existing high intensity, short duration flashes. For example a xenon flash tube has a number of sharp lines between 5000 and 12,000  $\text{\AA}$  and, more importantly, a fairly flat continuous background with a 20% power range of 2,400 - 22,000  $\text{\AA}$ . Moreover, cut-off is rapid beyond these limits, being complete on the short wavelength side at ca 2000  $\text{\AA}$ . The secondary class consists mostly of organic substances whose crystals possess molecular lattices. PETN, a typical member of this class, transmits well down to 3000 $\text{\AA}$ . Its absorption coefficient rises to  $10^3$ - $10^4 \text{ cm.}^{-1}$  at 2000  $\text{\AA}$ . The primary class consists of such ionic materials as  $\text{AgN}_3$  for which absorption sets in at 3850  $\text{\AA}$ .  $\alpha$  becomes ca  $10^3 \text{ cm.}^{-1}$  at 3000  $\text{\AA}$  rising

approximately to ca  $10^5 \text{ cm}^{-2}$ . Consequently a x-ray flash which is critical for  $\text{AgH}_2$  would be subcritical for PETN by nearly two orders of magnitude even if the two explosives had identical thermochemistry.

#### NUMERICAL SOLUTIONS

Blanchard <sup>5,6</sup> and Bouchon <sup>6</sup> have described computer solutions of the finite difference equations corresponding to the flash initiation of an explosive with first order kinetics. The relevant values of  $\alpha_0$  lie in the range from  $10^{-3}$  to 30 and the general form of the solutions agrees well with the present theory. One important difference emerges. Critical conditions can be found numerically only by trial and error - a process costly in computer time. The majority of the computed results are presented in terms of "useful energy",  $\mathcal{E}_u$ , computed as the product of flux density,  $\mathcal{E}_0$ , and the time taken for explosion to occur. This procedure corresponds to a situation in which light continues to illuminate the explosive for some time after a critical temperature profile has been established owing to the existence of an apparent explosive induction time. It is shown in appendix II that for a flash of constant intensity and zero rise

$$\mathcal{E}_u / \mathcal{E}_c \approx 1 + \frac{\tau_{\infty}}{\tau_0} \ln(1 + \tau^{-1})$$

where  $\tau_{\infty}$  is defined in section 4 and  $\tau$  is given by

$$\Gamma = \tau_0 \left( 1 + \frac{\sqrt{\pi}}{2} z_0 \right)^{-1}.$$

When  $\tau_0$  is large,  $\Gamma \approx 1$  and  $E_u \approx E_{cr}$ , but when  $\tau_0$  is small  $E_u/E_{cr} \approx 1 + \frac{\tau_0}{K} \ln(1 + K\tau_0^{-1})$ . Since  $E_{cr}$  is finite at  $\tau_0 = 0$ ,  $E_u$  tends to infinity as  $\tau_0$  tends to zero.

$E_u$  is a poor approximation to the critical energy density for very short flashes.

This result is in agreement with Moerkhamp's experiments<sup>18</sup> on nitrogen iodide in which  $E_u t_0$  was found to be roughly constant. This particular form  $E_u \propto t_0^{-1}$  was conditional by the shape of the light flash employed and is of no special significance.

#### CONCLUSIONS

The present model of flash initiation, which takes only thermal effects into account, satisfactorily describes all the major experimental features of the process. This supports experimental evidence<sup>1, 2, 4, 16, 22</sup> that initiation by light is essentially a thermal regime. In the case of the conventional explosives the sensitivity towards initiation by light is affected very little by thermochemistry but strongly by the magnitudes of the absorption coefficient and flash duration.

The agreement between the present model and experiment suggests that the conditions necessary for the occurrence of a thermal explosion are also sufficient to engender a macroscopic explosion or a transition to a stable detonation regime, although further experimental work on secondary explosives may show that



and is not universally true.

The model should assist the design of flash ignition experiments and enable useful combinations of thermochemical parameters to be extracted from their results.

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APPENDIX I : EFFECT OF HEAT TRANSFER AT FREE SURFACE

The boundary condition at  $z=0$  which satisfies Stefan's law for black body radiation has the form

$$\kappa \frac{\partial T}{\partial z} = \sigma_s \left[ (T_c + U_c)^4 - T_c^4 \right],$$

where  $\sigma_s = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ sec}^{-1} \text{ K}^{-4}$  is Stefan's constant

This may be written in the form

$$\left( \frac{\partial v}{\partial z} \right)_{z=0} = \frac{\sigma_s}{\kappa \alpha} \left[ (T_c + U_c)^4 - T_c^4 \right] = h U_c$$

according to (6). Hence the maximum value of  $h$  is

$$h_{\max} \approx \frac{\sigma_s U_c^3}{\kappa \alpha}, \quad \text{since } U_c \lesssim 2000^\circ \text{C.}$$

Adopting the representative value  $\kappa = 5 \times 10^4 \text{ erg cm}^{-1} \text{ sec}^{-1} \text{ K}^{-1}$ ,

it is found that  $h_{\max} < 10 \alpha^{-1}$  where  $\alpha$  is in units of  $\text{cm}^{-1}$ .

Now  $\alpha^{-1} \lesssim 10^{-3} \text{ cm}$  in cases of interest so that  $h_{\max} < 10^{-2}$  and

$h \ll h_{\max}$  for most of the regime.

If  $h \neq 0$  the surface temperature is slightly lower than that given by (13). It can be shown that the fractional reduction in the excess surface temperature ( $U$ ) is  $ca hx$ , which is always small provided that  $\tau_c < 10^2$ .

It can further be shown that the reduced distance between the surface and temperature maximum given by (12) is approximately  $hx$  at all times. Thus in cases of interest the maximum temperature occurs at a point extremely close to the free surface and (13) is a good approximation to (11).

APPENDIX II EFFECT OF CHEMICAL REACTION

Heat evolution due to chemical reaction will remain small during the illumination if the flash duration is much less than the isothermal half life of the explosive at the maximum temperature produced by the light flash i.e. if

$$\gamma t_0 \exp(-E/RT_0) \ll 1$$

Conventional explosives satisfy this condition if  $t_0 \lesssim 10^{-6}$  sec.

A reasonable estimate of the effect of chemical reaction can be obtained by finding the extent of reaction in an explosive constrained to follow the inert temperature evolution of the free surface as given by (13). The rate equation for a first order reaction is

$$\frac{d\xi}{dt} = -\xi \gamma \exp(-E/RT)$$

where  $\xi$  is the fraction of explosive undecomposed. Since  $\xi = 1$  when  $t = 0$ , this can be written in the form

$$-\ln \xi = \gamma \int_0^{t_0} \exp(-E/RT) dt$$

where  $T(t)$  is to be derived from (13).

The integral can be evaluated approximately by using the Frank-Kamenetski expansion<sup>9</sup> about the maximum inert temperature,  $T_0$ , where the contribution to the integral is most important<sup>24</sup>.

This procedure yields

$$-\ln \xi = (\theta_0 \tau_0)^{-1} \int_0^{\tau_0} e^{\theta} d\tau$$

$$< \frac{2\tau_0^2}{\theta_0^2 \theta_0^2 \tau_0} [e^{\theta_0} - \theta_0 - 1]$$

since  $-\theta \leq -\theta_0 \left(1 - \frac{x}{x_0}\right)$ .

Hence  $-\ln \xi \lesssim \frac{-2x_0^2}{\theta_0 \theta_R \tau_\infty}$ ,

since  $\theta_0 \lesssim -10$  in cases of interest.

The ratio of the temperature increase due to reaction in the absence of conduction and the increase due to the absorption of radiation is given by

$$R = \theta_R(1-\xi)/-\theta_0 \lesssim -\frac{\theta_R}{\theta_0} \left[1 - \exp\left\{-\frac{2\tau_0}{\theta_0 \theta_R \tau_\infty}\right\}\right] \quad (\text{II.1})$$

If  $\tau_0 \ll 1$  then  $R \lesssim \frac{S}{\theta_0^2} \cdot 2\tau_0$ . For a critical heating regime

$$\frac{S}{\theta_0^2} = \frac{2}{\pi} \quad \text{since } \mu = 1. \quad \text{Hence}$$

$$R_{cr} \lesssim \frac{4}{\pi} \tau_0$$

and heating due to chemical reaction is negligible while the light is switched on if  $\tau_0 \lesssim 0.1$  say.

If  $1 < \tau_0 \ll \theta_R^2 \approx 10^2$  the condition  $\xi = 1$  gives  $R_{cr} = \frac{2x_0}{-\theta_0}$

so  $R$  remains small if  $x_0 \lesssim 1$ . The neglect of chemical reaction prior to the time  $t_0$  is therefore justified for a critical flash if  $t_0 \lesssim t_0$ .

During the reactive phase of the regime the rate at which the surface temperature falls owing to conduction differs from that of the inert case but an argument similar to that of

Rideal and Robertson<sup>25</sup> suggests that the effect is small. Throughout the induction period<sup>9,24</sup>,  $\theta < 1$ , with which this treatment is exclusively concerned, the temperature increase due to reaction remains less than  $T_g^2/T_A$  while the total temperature drop across the hot spot is initially  $T_g - T_o$ . In all cases of interest the ratio of these two quantities,  $|\theta_o|$ , is greater than 10 because of the high activation temperatures  $T_A$  involved. Consequently until the end of the induction period is reached the perturbation of the temperature profile caused by chemical reaction is small and the use of (19) is reasonable.

Blanchard<sup>5,6</sup> and Bouchon<sup>6</sup> have shown that if an extremely intense flash is used the effect of conduction is negligible and if the light source remains on after a critical profile has been established no abrupt change of  $\partial v/\partial t$  is observed. The development of the free surface temperature of such a regime obeys the equation

$$\frac{dv}{d\tau} = B + \delta e^\theta, \quad (II.2)$$

$$\theta = \theta_o \text{ when } \tau = 0,$$

where  $B = \frac{A_o E}{RT_g^2}$  represents the heating due to the light alone.

The solution of (II.2) is given by

$$\eta = -\frac{\delta}{B} + \left(\eta_o + \frac{\delta}{B}\right) e^{-B\tau}, \quad (II.3)$$

where  $\eta_o = e^{-\theta_o}$ .

The variation of  $\theta - \theta_0$  with  $\tau$  given by (II.3) is shown in Fig. 8. The reduced time required to produce a critical temperature profile ( $\dot{t} = 0$ ) is given by

$\exp(\bar{B}\tau|_{\theta=0}) = \eta_0 (1 + \delta/B\eta_0) / (1 + \delta/B)$  and the reduced time taken to reach infinite temperature is given by

$$\exp(\bar{B}\tau|_{\theta=\infty}) = (1 + \delta/B\eta_0) / (\delta/B\eta_0)$$

Hence the "useful energy" of Blanchard and Bouchon is related to the critical energy by the expression

$$\begin{aligned} E_u/E_{cr} &= 1 + (\tau|_{\theta=\infty} - \tau|_{\theta=0}) / \tau|_{\theta=0} \\ &= 1 + \frac{\tau_{\infty}}{\tau_0} \frac{\delta}{B} \ln(1 + B/\delta) \end{aligned}$$

where  $\delta$  corresponds to critical conditions.

Since the value of  $(\delta/B)$  or  $\tau$  from (26) is approximately  $\tau_0/K\Phi(x_0)$ ,  $E_u$  becomes considerably greater than  $E_{cr}$  if  $\tau_0 \ll K_0^{-1}$ .

Appendix III.

Polychromatic radiation.

It is possible to obtain good approximate solutions of the present problem provided that certain restrictions are imposed on the relevant range of  $\alpha$ . The reasoning closely follows that of the foregoing analysis and the expressions involved are cumbersome. Consequently only an outline of the procedure and the major results will be given below.

The quasi-inert initial phase of the monochromatic regime is governed by the equation

$$\sigma c \frac{\partial U}{\partial t} = \kappa \frac{\partial^2 U}{\partial z^2} + \alpha E_0 e^{-\alpha z} \quad (\text{III,1})$$

with the conditions

$$\frac{\partial U}{\partial z} = 0, \quad z = 0, \quad t > 0 \quad (\text{III,2})$$

$$U = 0, \quad E_0 = 0, \quad t < 0 \quad (\text{III,3})$$

and since this system is linear its solutions,  $U$ , are additive. Although the general solution is awkward the solutions of interest for the case of a rectangular light pulse are relatively simple. In particular, from (13)

$$U_s = \frac{t_0}{\sigma c} \int \alpha E_0^\lambda \bar{\Phi}^{-1}(x_0) d\lambda, \quad (\text{III,4})$$

where the integral is to be taken over the entire emission range of the light source. The cases of "long" and "short" duration flashes will be considered separately.

Short flash. If  $\alpha^2 \ll (Dt_0)^{-1}$  then

$$\bar{Q}^{-1}(x_0) \approx 1 - \frac{4x_0}{3\sqrt{\pi}} \quad \text{and (III,4) gives}$$

$$U_s = \frac{t_0}{\sigma c} \int \alpha E_0^\lambda d\lambda \cdot \left[ 1 - \frac{4}{3\sqrt{\pi}} \sqrt{\frac{kt_0 \alpha^2}{\sigma c}} \right]$$

where  $\hat{\alpha} = \int \alpha^2 E_0^\lambda d\lambda / \int \alpha E_0^\lambda d\lambda$  is an average

absorption coefficient. (It is assumed that the light has a constant colour,  $E_0^\lambda = E_1(\lambda) \cdot E_2(t)$ , so that  $\hat{\alpha}$  is constant). The inert decay given by com-

bining solutions like (18) is

$$\sigma c \frac{dU}{dt} = -t_0 \sqrt{\frac{k}{\sigma c}} \int \alpha^2 E_0^\lambda d\lambda \cdot t^{-\frac{1}{2}}, \quad \beta = 0,$$

where time is measured from  $t_0$ . Hence the analogue of (21) is

$$\frac{d\theta}{dt} = - \frac{Et_0}{\sigma c RT_s^2} \sqrt{\frac{k}{\sigma c}} \int \alpha^2 E_0^\lambda d\lambda \cdot t^{-\frac{1}{2}} + \frac{q^2 E}{c RT_s^2} \exp(-E/RT_s) e^\theta$$

which with the initial condition  $\theta = 0$  when  $t = 0$  has the solution (cf.(25))



$$\eta = M(1+Y) + (1-M)e^Y \quad (\text{III,6})$$

where

$$Y = \frac{2Et_0}{\sigma c RT_s^2} \sqrt{\frac{k}{\pi \sigma c}} \int \alpha^2 E_0^\lambda d\lambda \cdot t^{\frac{1}{2}}$$

and

$$M = \frac{\pi}{2} \cdot \frac{q RT_s^2 \exp(-E/RT_s) \cdot \nu (\sigma c)^3}{k t_0^2 c E \left\{ \int \alpha^2 E_0^\lambda d\lambda \right\}^2}$$

Clearly the condition for explosion is  $M > 1$  or, re-arranging and using (III,5) the critical condition is

$$\begin{aligned} \frac{t_0}{\sigma c T_A} \int \alpha E_0^\lambda d\lambda &= \frac{1}{\sigma c T_A} \int \alpha E^\lambda d\lambda = \frac{\bar{\alpha}}{\sigma c T_A} \int E^\lambda d\lambda \\ &\approx \left[ 1 + \frac{4}{3\sqrt{\pi}} \sqrt{\frac{k t_0 \alpha^2}{\sigma c}} \right] \left[ \left\{ \ln \left( \frac{1}{\alpha_0^2 \alpha^2} \right) \right\}^{-1} - \frac{T_0}{T_A} \right], \end{aligned}$$

where  $\bar{\alpha} = \int \alpha E_0^\lambda d\lambda / \int E_0^\lambda d\lambda$ .

The critical energy is given by

$$E_c = \int E^\lambda d\lambda = \frac{\sigma c T_A}{\bar{\alpha}} \left[ 1 + \frac{4}{3} \sqrt{\frac{k t_0 \alpha^2}{\pi \sigma c}} \right] \left[ \left\{ \ln \left( \frac{1}{\alpha_0^2 \alpha^2} \right) \right\}^{-1} - \frac{T_0}{T_A} \right]$$

which should be compared with (26).

Long flash. If  $\alpha^2 \gg (Dt_0)^{-1}$  then

$$\begin{aligned} \bar{\Phi}^{-1}(\alpha_0) &\approx \left[ 1 + \frac{\sqrt{\pi}}{2} \alpha_0 \right]^{-1} \\ &\approx \left( \frac{\sqrt{\pi}}{2} \alpha_0 \right)^{-1} - \left( \frac{\sqrt{\pi}}{2} \alpha_0 \right)^{-2} \end{aligned}$$

and (III,4) gives

c

$$U_s = \sqrt{\frac{4t_0}{\pi\kappa\sigma c}} \int E_0^\lambda d\lambda - \frac{4}{\pi\kappa} \int \alpha^{-1} E_0^\lambda d\lambda$$

The inert decay is given by

$$\begin{aligned} \sigma c \frac{dU}{dt} &= - \int \alpha E_0^\lambda (1 - Z_0(x_0)) d\lambda \\ &\approx - \int \alpha E_0^\lambda \left[ 1 - \frac{1}{\sqrt{\pi} x_0} \right] d\lambda \\ &= - \int \alpha E_0^\lambda d\lambda + \frac{\int E_0^\lambda d\lambda}{\sqrt{\pi\kappa t_0 \sigma c}} \\ &= - \int \alpha E_0^\lambda d\lambda \cdot \left[ 1 - \sqrt{\frac{\sigma c}{\pi\kappa t_0 \alpha^2}} \right] \\ &\approx - \int \alpha E_0^\lambda d\lambda \end{aligned}$$

since  $\alpha^2 \gg (Dt_0)^{-1}$  . Hence the analogue

of (21) is

$$\frac{d\theta}{dt} = - \frac{E}{RT_s^2} \frac{\int \alpha E_0^\lambda d\lambda}{\sigma c} + \frac{q\nu E}{cRT_s^2} \exp(-E/RT_s) e^\theta$$

which with the initial condition  $\theta = 0$  when  $t = 0$  gives

$$\eta = \omega + (1 - \omega) e^\chi$$

where

$$\chi = \frac{E}{RT_s^2} \frac{\int \alpha E_0^\lambda d\lambda}{\sigma c} \cdot t$$

and

$$\omega = \frac{q\sigma\nu \exp(-E/RT_s)}{\int \alpha E_0^\lambda d\lambda}$$

0

Explosion occurs if  $\omega > 1$  (cf. (24)).

The critical condition may be written in the form

$$\frac{T_0}{T_A} + \frac{U_3}{T_A} = \left[ \ln \left( \frac{q_{0\nu}}{\int \alpha E_0^\lambda d\lambda} \right) \right]^{-1}$$

or using (III,7),

$$E_{cr} \approx T_A K_1 \sqrt{\frac{\pi \kappa \sigma c t_0}{4}} \left[ 1 + \frac{4\sigma c}{\sqrt{\pi \kappa t_0} \langle \alpha \rangle^2} \right] \quad (\text{III,8})$$

where

$$K_1 = \left[ \ln \left( \frac{q_{0\nu}}{\int \alpha E_0^\lambda d\lambda} \right) \right]^{-1} - \frac{T_0}{T_A}$$

and  $\langle \alpha \rangle = \left\{ \overline{\alpha^{-1}} \right\}^{-1}$  is the harmonic mean

value of the absorption coefficient.

The use of this approximation

$$K_1 \approx \left[ \ln \left( \frac{1}{\bar{\alpha}^2 \hat{\alpha}^2} \right) \right]^{-1} - \frac{T_0}{T_A}$$

introduces only a small error into (III,8).

Thus (26) retains its validity provided the relevant average values  $\hat{\alpha}$ ,  $\bar{\alpha}$ ,  $\langle \alpha \rangle$  of  $\alpha$  are selected where

$$\bar{\alpha} = \int \alpha E_0^\lambda d\lambda / \int E_0^\lambda d\lambda,$$

$$\hat{\alpha} = \int \alpha^2 E_0^\lambda d\lambda / \int \alpha E_0^\lambda d\lambda,$$

$$\langle \alpha \rangle = \int E_0^\lambda d\lambda / \int \alpha^{-1} E_0^\lambda d\lambda.$$

If a more accurate approximation is required then the criteria  $M_{\alpha} = 1$ ,  $\omega_{cr} = 1$  should be used. In conclusion two important deductions can be made from this appendix.

- a) As the flash duration tends to zero the critical energy density for polychromatic light tends to the finite value

$$E_{min} = \frac{\sigma c T_A}{\alpha} \left[ \left\{ \ln \left( \frac{1}{\alpha_0^2 \alpha^2} \right) \right\}^{-1} - \frac{T_0}{T_A} \right]$$

- b) The critical energy of a polychromatic flash of "long" duration (i.e. for which  $D t_0 \alpha^2 \gg 1$ ) is virtually independent of the absorption coefficient and has the approximate value

$$E_{cr} = T_A \left( \frac{\pi k \sigma c}{4} \right)^{\frac{1}{2}} \left[ \left\{ \ln \left( \frac{1}{\alpha_0^2 \alpha^2} \right) \right\}^{-1} - \frac{T_0}{T_A} \right] t_0^{\frac{1}{2}}$$

i.e. is proportional to the square root of the flash duration.

R E F E R E N C E S

1. Bowden and Yoffe, Fast Reactions in Solids, Butterworths London, 1958
2. Berchtold and Eggert, Naturwiss., 1953, 40, 55
3. Berchtold, Proc. Roy. Photographic Soc. of Great Britain, London 1953, 250, Published 1955.
4. Berchtold, Ph.D. Thesis ETH No. 2376, Zurich, 1953
5. Blanchard, Comp. Rend. 1963, 256, 2550
6. Blanchard and Bouckon, Trans. Far. Soc. this issue?
7. Boddington, Ph.D. Thesis, Cambridge University, 1962.
8. Cardaw and Jaeger, Conduction of Heat in Solids, Oxford, 1959
9. Frank-Kamenetskii, 1947, Diffusion and Heat Exchange in Chemical Kinetics (Moscow) - English translation, Princeton University Press, 1955
10. Wenograd, Trans. Far. Soc., 1961, 57, 1612
11. Boddington, Proceedings of IX Symposium on Combustion, Cornell, 1962, p 287, Academic Press, New York, 1963.
12. Todes and Melentiev, Acta Physiocoohim, URSS, 1941, 14, 27
13. Thomas, Proc. Roy. Soc. 1961, 262A, 192
14. Semenov, Z. Phys.Chem. 1928, 48, 571
15. Thomas, Trans, Far. Soc. 1960, 56, 833
16. McAuslan, Ph.D. Thesis, Cambridge, 1957
17. Yoffe, private communication.
18. Moerkemper, Z. Electrochem. 1954, 58, 387
19. Moss, Optical properties of semi-conductors, Butterworths, London, 1959
20. Taylor, Detonation in solid explosives, Clarendon Press Oxford, 1952

Carlaw

21. Tomlinson, Technical Report No. 1740, 1958, Picatinny Arsenal, Dover, N.J.
22. Courtney-Pratt and Rogers, Nature, 1955, 175, 632
23. McClung and Hellworth, Journ. Appl. Phys. 1962, 33, 828
24. Gray and Harper, Trans. Far. Soc., 1959, 55, 581
25. Rideal and Robertson, Proc. Roy. Soc., 1948, 195A, 135

TABLE 1. THERMOCHEMICAL PARAMETERS OF SOME EXPLOSIVES

Substance	$a_0$ (Å)	$\sigma cT_A$ ( $10^4 \text{ cal cm}^{-3}$ )	$D$ ( $10^{-3} \text{ cm}^2 \text{ sec}^{-1}$ )
PETN	$1.9 \times 10^{-3}$	1.41	1.02
RDX	$3.6 \times 10^{-3}$	1.31	1.23
HMX	$9.6 \times 10^{-4}$	1.46	1.27
Tetryl	$2.0 \times 10^{-1}$	0.74	1.95
$\text{AC}_2\text{N}_3$	$7.1 \times 10^{-4}$	1.25	1.49
$\text{PbN}_6$	$2.5 \times 10^{-1}$	1.11	1.73

( Values calculated from data given in references<sup>1,20,21</sup> )

Fig. 1.

The error function complement and related functions.

The  $n^{\text{th}}$  repeated integral  $i^n \operatorname{erfc} z$  is  
labeled  $i^n$ , where  $i^0 = \operatorname{erfc} z$ .  $Z_0(z)$  is the function  
 $e^{z^2} \operatorname{erfc} z$ . The function  $\frac{2}{\sqrt{\pi}} e^{-z^2}$  is  
equivalent to  $i^{-1} \operatorname{erfc} z$ .

Ordinate: (not named)

Abscissa:  $z$



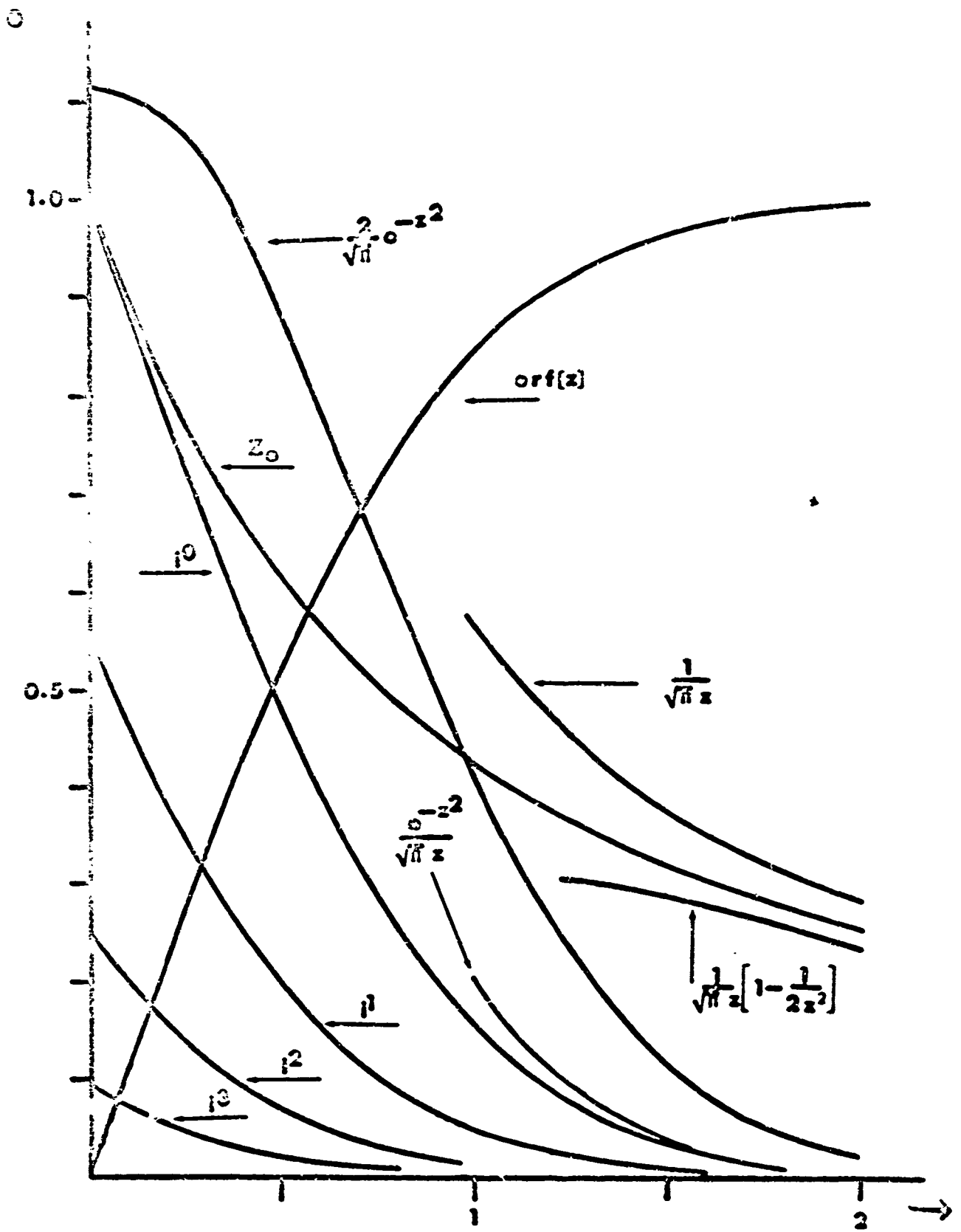


Fig 1

Fig. 2.

The temperature profiles produced by a rectangular flash of constant integrated intensity,  $E$ , and of varying duration  $\alpha^2 t_c$  according to equations (13) and (14). The area under each of the curves equals that enclosed by the dashed rectangle.

Ordinate:

$$U/\bar{T}$$

Abscissa:

$$\rho$$

$$(\rho_0)$$

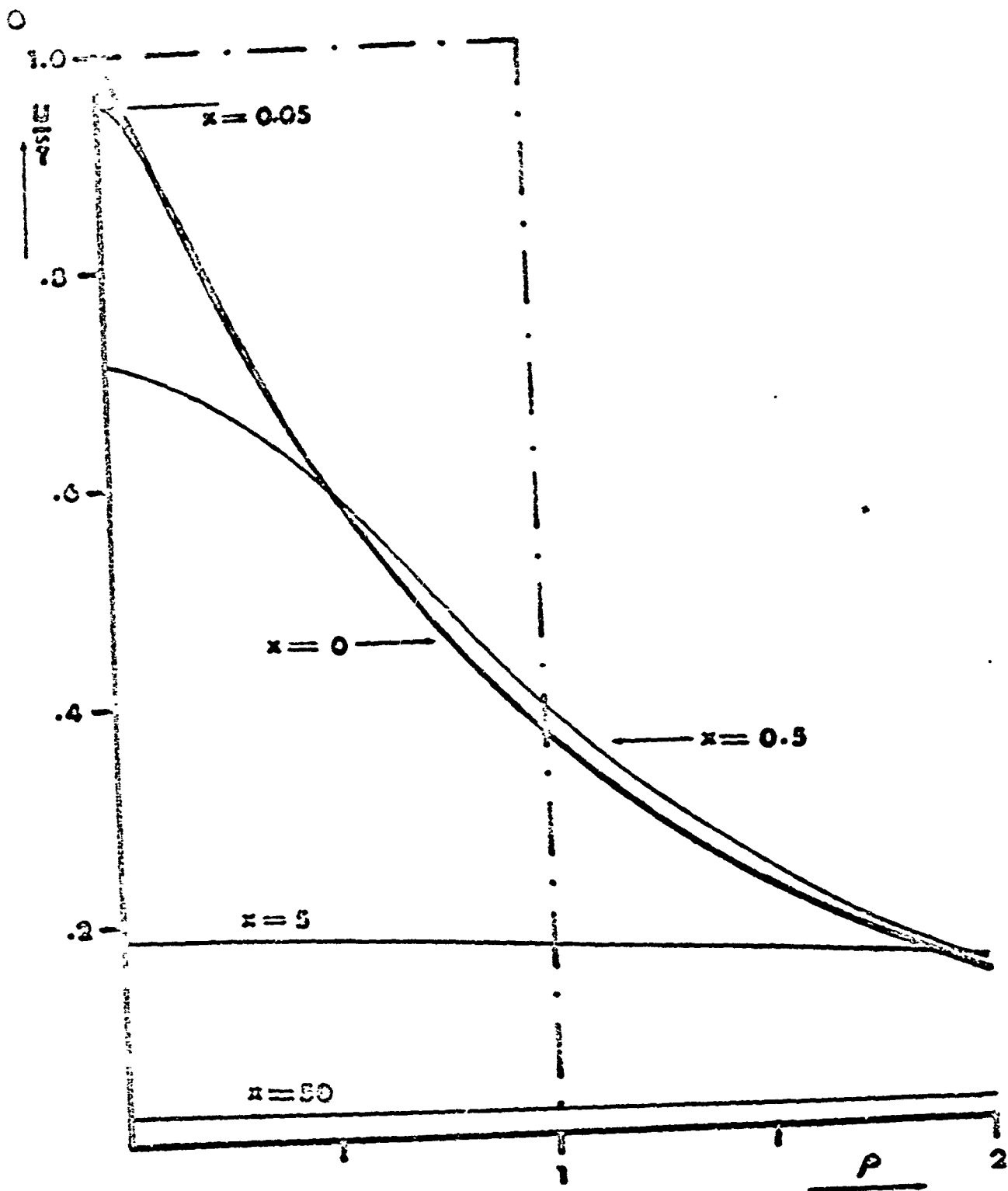


Fig 2

Fig. 3.

Evolution of the surface temperature after a short duration flash.

The curves belong to the family  $\eta = \mu(1+y) + (1-\mu)e^y$ .

The regime is critical (and fails to develop an explosion) when  $\mu = 1$ . The dashed curve gives the locus of the maxima in  $\eta$  (temperature minima) and is asymptotic to the line  $\eta = y$  at large  $y$ .

Ordinate:  $\eta = e^{-\delta}$

Abscissa:  $y \quad \left\{ \alpha t^{\frac{1}{2}} \right\}$

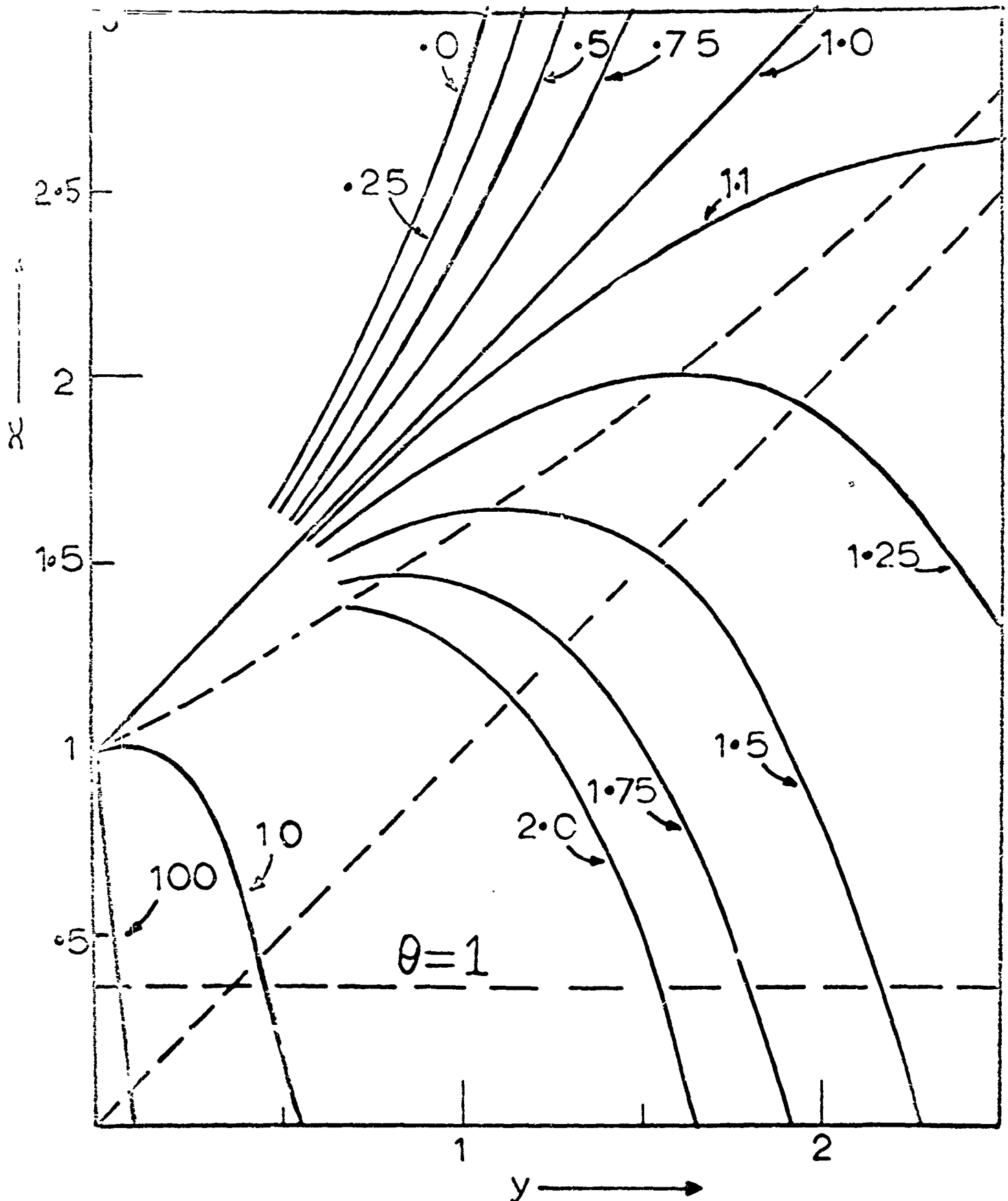


Fig 3

Figure

Evolution of the surface temperature  
after a long duration flash.

The curves belong to the family :

$\eta = \varepsilon + (1 - \varepsilon) e^{C\tau}$  . The regime is critical (and  
fails to develop an explosion) when  $\varepsilon = 1$  .

Ordinate:  $\eta = e^{-\delta}$

Abcissa:  $C\tau \quad \{ \propto t \}$

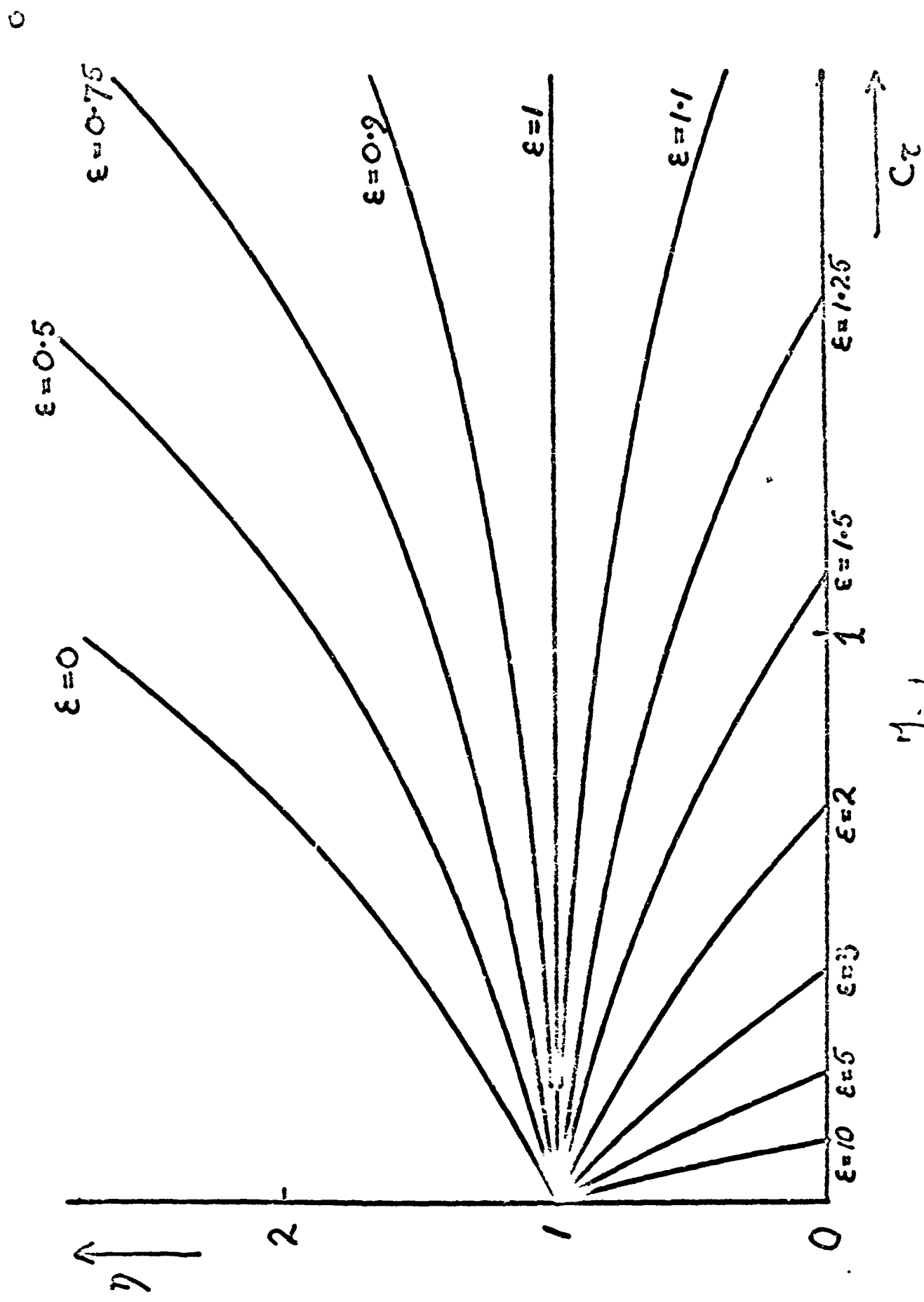


Fig 4

0

Fig. 5.

The function  $\Phi(x) = x^2 \left\{ \frac{2x}{\sqrt{\pi}} + e^{x^2} \operatorname{erfc} x - 1 \right\}^{-1}$ .

The lines A and B are the asymptotes of  $\Phi$  for low and high values of  $x$ , respectively. The line E, which is parallel to B, represents the function used to estimate  $\Phi$  for all values of  $x$ .

$\Phi(x_0)$  is approximately equal to the ratio  $E_a(x_0) / E_{min}$ .

Abscissa :  $x$

Ordinate : (not named)



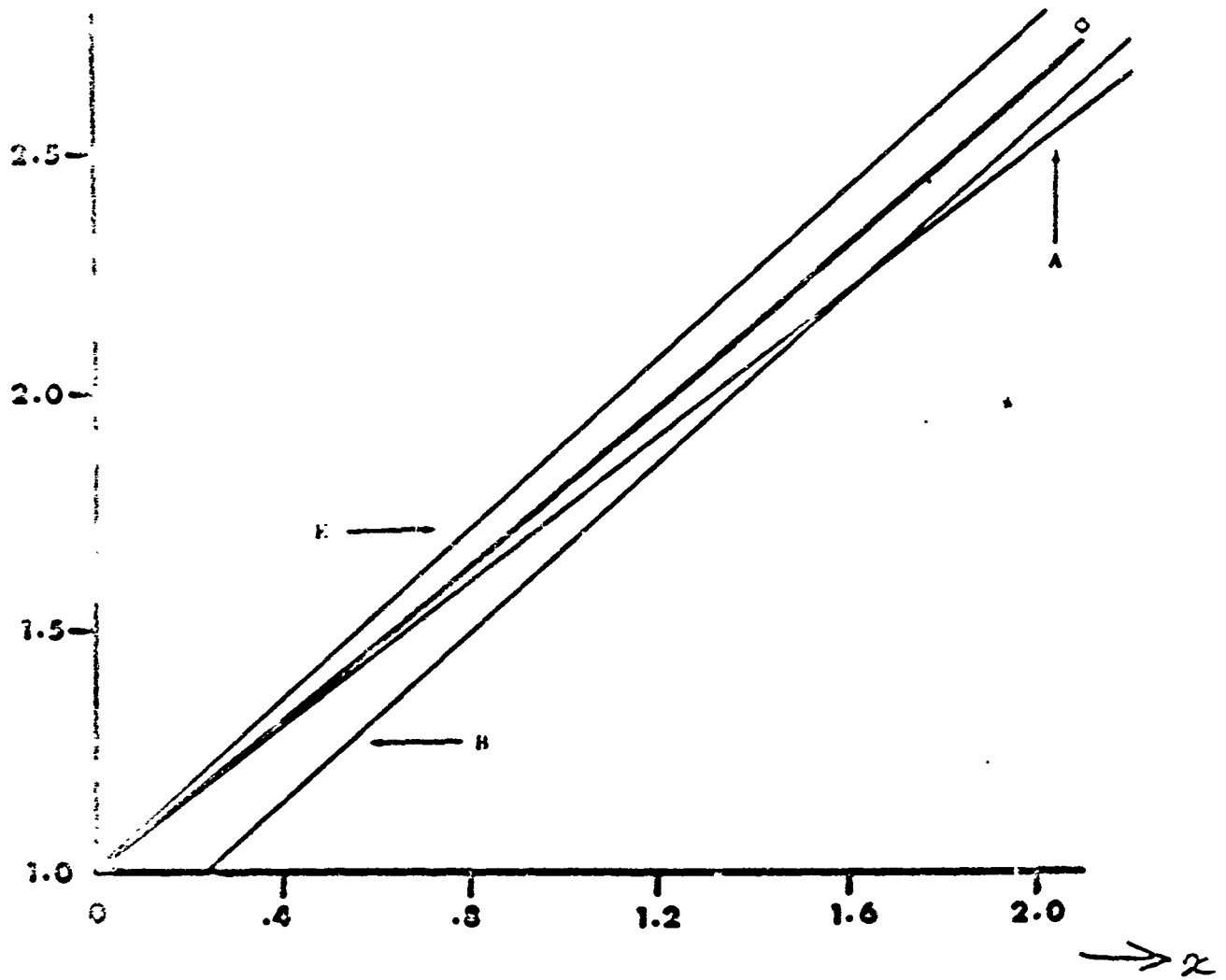


Fig 5

Fig. 6.

Variation of  $E_{cr}$  with the square root of time for  $A_{g_3}N$  calculated from Berchtold's data<sup>4</sup>.  $E_{cr}$  is in arbitrary units.  $a$  - best fit to the 4 points with high  $t_0$ ,  $b$  - best fit to all six points.

Ordinate:  $E_{cr}$  (arbitrary units)

Abcissa:  $10^2 [t_0(\text{sec})]^{1/2}$

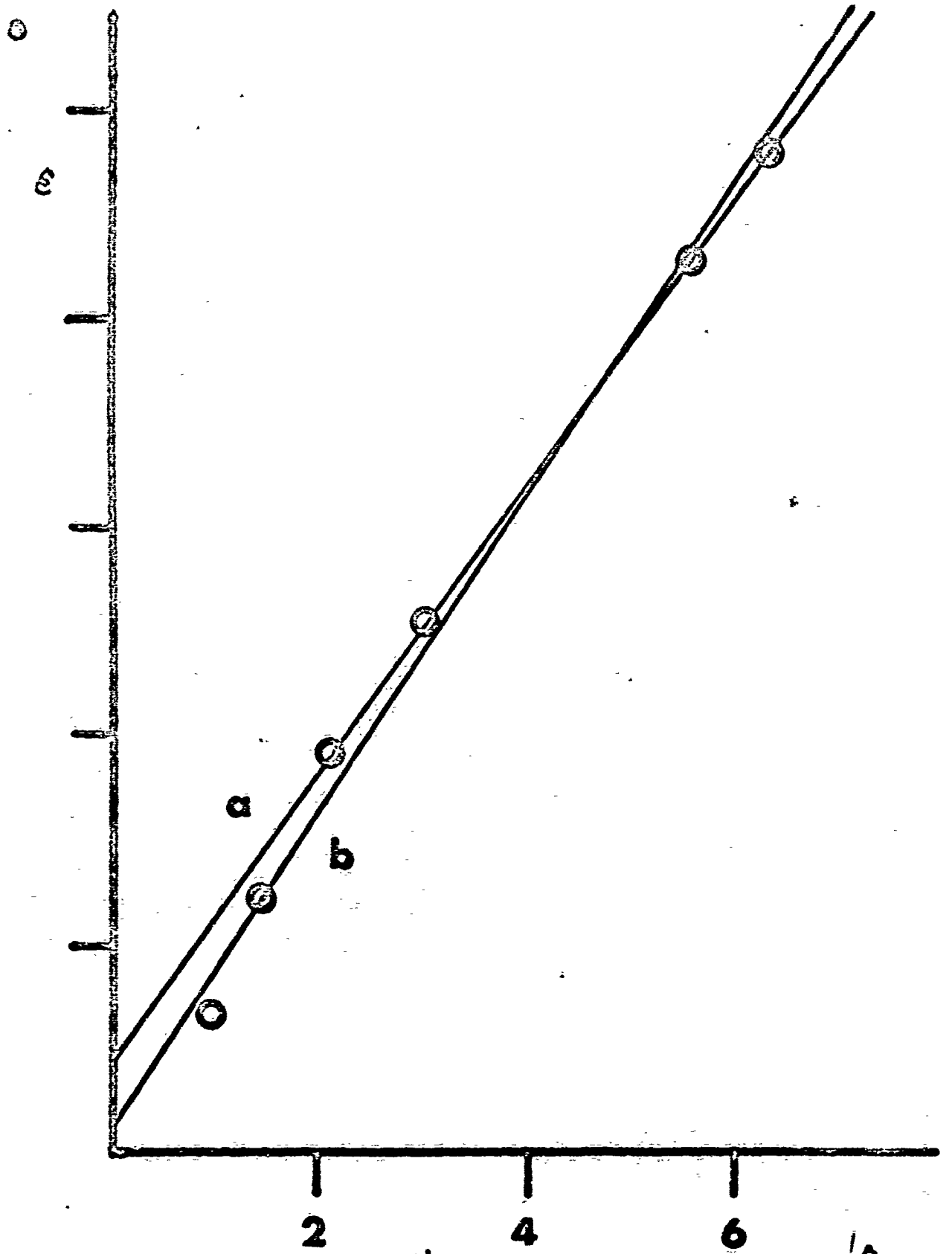


Fig 6

1/80

Fig. 7.

Variation of critical energy with ambient temperature  
in the case of lead styphnate (after McAuslan<sup>1,16</sup>).

- ★ - explosion
- - failure

Ordinate: Critical stored energy ( $\frac{1}{2}CV^2$ ), Joules  
Abscissa: Ambient temperature (°C)

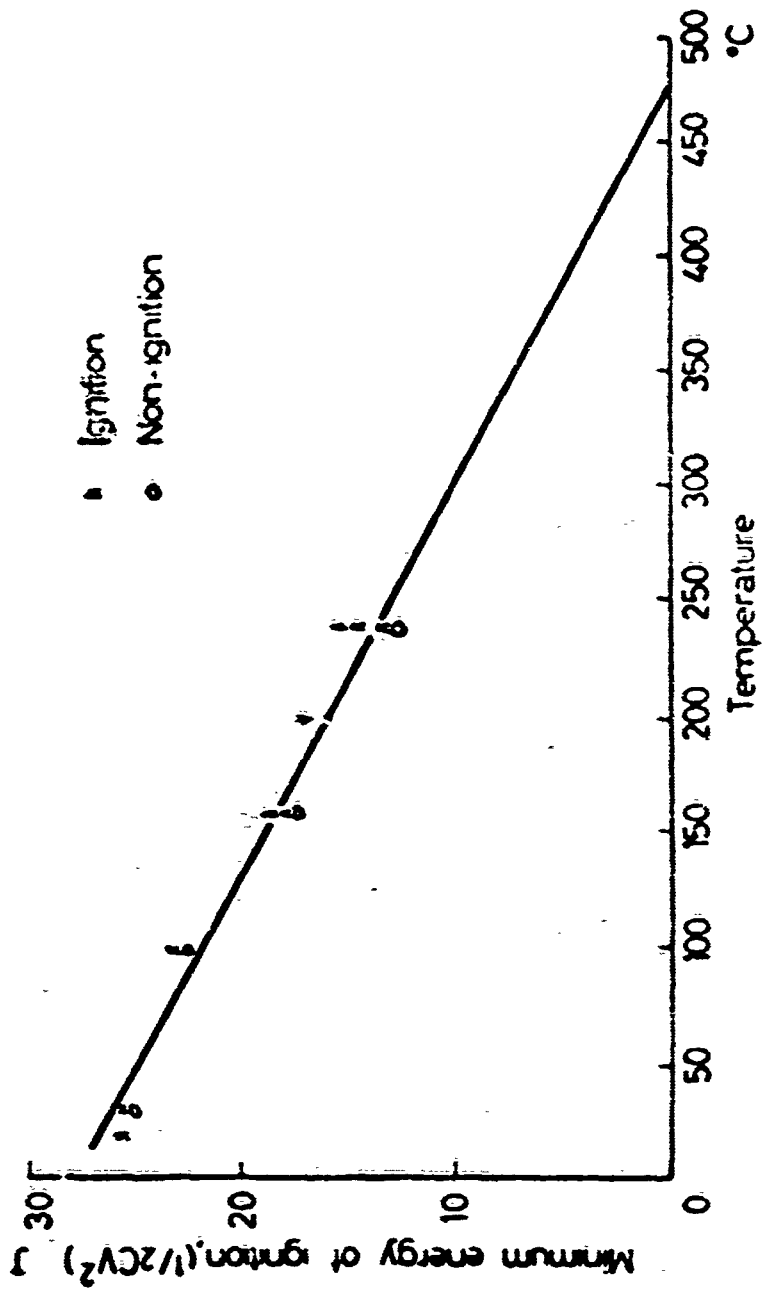


Fig 7

Fig. 8.

Evolution of surface temperature for the case of continuous intense irradiation with zero rise time. The curves are obtained by writing (II.3) in the form

$$\theta - \theta_0 = -\ln \left\{ -\psi + (1 + \psi) e^{-\Delta} \right\}$$

where  $\Delta = B\tau$ ,  $\psi = \frac{I}{3\gamma_0}$ ,  $B = \frac{A_0 E}{RT_0^2}$ .

A critical profile is established in the conventional explosives somewhere between the two dashed lines. The present theory is invalid unless  $\psi \approx 10^{-3}$ .

Ordinate:  $\theta - \theta_0$

Abscissa:  $\Delta$

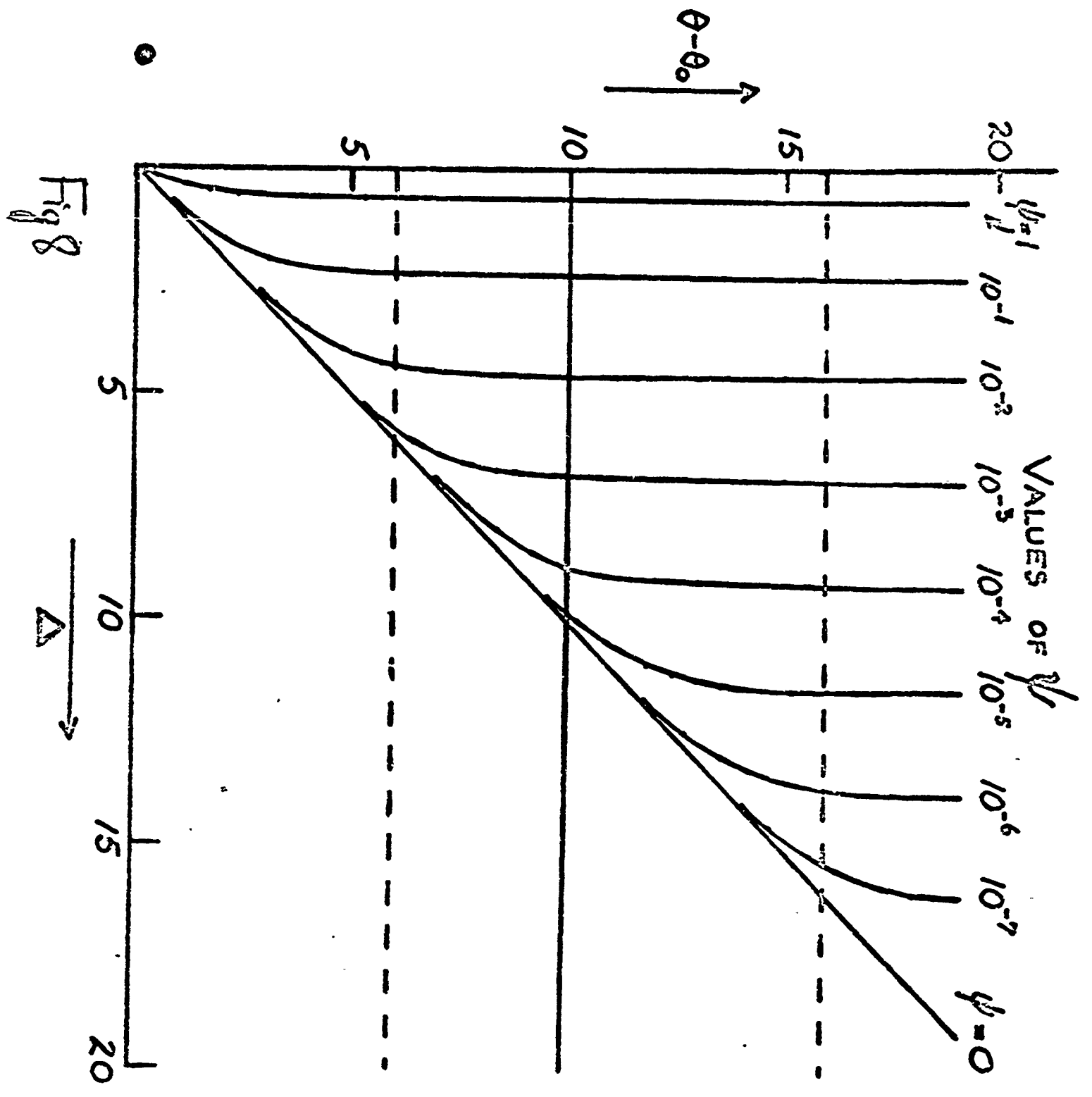


Fig 8