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DRIVING POINT IMPEDANCE OF LINEAR ANTENNAS IN THE PRESENCE OF A STRATIFIED DIELECTRIC

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ABSTRACT

A linear flat strip antenna lies between several dielectric layers in a direction parallel to the interfaces. Its impedance is formulated variationally as an infinite double integral. For thin half-wave antennas in a homogeneous medium the double integral is evaluated analytically and it gives standard impedance expressions. For antennas of finite width the integrals are evaluated numerically and for a homogeneous medium the impedance has been correlated with the theories of King and Middleton, Wu, with the impedance computed from a complementary slot antenna and with available measurements. The impedance is also computed for antennas in a dielectric layer, and the decrease of the radiation efficiency observed with increasing electrical thickness of the layer is explained with the increased amounts of surface wave power and in part by local dielectric losses. For insulated antennas located in a dissipative medium the theory presented here yields results in better agreement with measurements than the conventional transmission line theory. The transmission line theory is shown to give too large a resistance and reactance at the **full-wave** resonant peaks of the impedance and too low a resistance for short antennas.

1. INTRODUCTION

Numerous investigations have been reported of the input impedance of linear antennas in homogeneous lossy media [King, 1956; King and Harrison, 1960; King and Iizuka, 1963]. Treatments of insulated antennas in a highly dissipative medium have been based on transmission line theory [Moore, 1951, Ghose, 1960, Guy and Hasserjian, 1963]. The theory of insulated antennas in an infinite dissipative medium has been further refined by King [1964] but there are at present no impedance calculations for linear antennas in the presence of stratified media of arbitrary parameters. This may be due in part to the difficulty of treating the cylindrical boundary of the antenna and its insulation simultaneously with the planar boundary of the dielectric layers.

Slot antenna impedance for plasma and dielectric layers have been considered by Galejs [1964, 1965a,b] and by Villeneuve [1965] and the impedance of an insulated loop in a dissipative medium has been calculated by Galejs [1965c]. A similar variational impedance formulation is derived herein for linear antennas surrounded by a stratified dielectric. The linear antenna is assumed to be in the shape of a rectangular strip of zero thickness that is excited at its center and is sandwiched between parallel dielectric layers as shown in Fig. 1. The dielectric regions are surrounded by free space, but the layers may be unsymmetrical with respect to the antenna. For semi-infinite media one of the boundaries between the dielectric and the free space can be made to recede to infinity.

For dielectric layers that may be lossy or that can support surface waves the power radiated into the two lossless half-space regions will generally differ from the power supplied to the antenna. Hence, separate expressions are derived for the input impedance of the antenna and for its radiation resistance. The integrals of the resulting expressions are evaluated in closed form for antennas in a homogeneous medium. The well known results for a thin half wave antenna in free space or in a homogeneous dielectric provides a simple check of the development, as is shown in section 3.1. In section 3.2 the present formulation is compared with available theories for homogeneous media. Calculations for an insulated strip antenna in a dielectric layer are reported in Section 3.3. Section 3.4 considers the insulated antenna in a dissipative medium. Comparisons are made with the conventional transmission line theory and several measurements.

2.0 ANTENNA DRIVING POINT IMPEDANCE AND RADIATION RESISTANCE

The driving point impedance of a flat strip antenna may be computed from the expression

$$Z = -\frac{\iint \underline{E} \cdot \underline{J}_s \, dx \, dy}{[I(x=0)]^2} \tag{1}$$

which is stationary with respect to small changes of the surface current density $\underline{J}_s$ or of the current $I(x,y)$ about its correct value (Harrington [1961], Section 7-9). The surface current $\underline{J}_s$ is assumed to have only an x-component J_x, which in addition, is an even function about x=0, y=0. The antenna fields can be expressed as a superposition of TE and TM modes and it follows from the Appendix that

$$Z = \frac{i\omega\mu_o}{[2\pi\, I(x=0)]^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\left[\iint J_x(x,y)\cos ux \cos vy \, dx\, dy\right]^2 du\, dv \tag{2}$$

where

$$F(u,v) = \frac{v^2}{u^2+v^2}\left[\gamma_{op}\frac{1-R_{aop}}{1+R_{aop}} + \gamma_{iq}\frac{1-R_{aiq}}{1+R_{aiq}}\right]^{-1} - \frac{u^2}{u^2+v^2}\left[\frac{k_{op}^2}{\gamma_{op}}\frac{1+R_{bop}}{1-R_{bop}} + \frac{k_{iq}^2}{\gamma_{iq}}\frac{1+R_{biq}}{1-R_{biq}}\right]^{-1}. \tag{3}$$

The designation of the dielectric layers by subscripts is seen from Fig. 1 and in particular the subscripts op and iq designate dielectric layers next to z=0 for z ≥ 0 and z ≤ 0 respectively. The other symbols are defined as $k_{j\ell} = \sqrt{\omega^2\mu_o\epsilon_{j\ell}+i\omega\mu_o\sigma_{j\ell}}$, $\gamma_{j\ell} = i\sqrt{k_{j\ell}^2-u^2-v^2}$, where j=0 or i and ℓ = 1,2...p or 1,2...q. The reflection coefficients of the TE modes $R_{aj\ell}$ and of the TM modes $R_{bj\ell}$ depend on the dielectric structure for |z| > 0. For stratified dielectric layers as shown in Figure 1 the reflection coefficient $R_{dj\ell}$(d=a or b) in the region of $|z_{\ell+1}| \le |z| \le |z_\ell|$ is related to the reflection coefficient $R_{d(\ell-1)}$ of the region $|z_\ell| \le |z| \le |z_{\ell-1}|$ by the expressions

$$R_{aj\ell} = e^{2\gamma_\ell|z_\ell|}\frac{\left[e^{2\gamma_{\ell-1}|z_\ell|}+R_{a(\ell-1)}\right] - \frac{\gamma_{\ell-1}}{\gamma_\ell}\left[e^{2\gamma_{\ell-1}|z_\ell|}-R_{a(\ell-1)}\right]}{\left[e^{2\gamma_{\ell-1}|z_\ell|}+R_{a(\ell-1)}\right] + \frac{\gamma_{\ell-1}}{\gamma_\ell}\left[e^{2\gamma_{\ell-1}|z_\ell|}-R_{a(\ell-1)}\right]} \tag{4}$$

$$R_{bj\ell} = e^{2\gamma_\ell|z_\ell|}\frac{\left(\frac{k_{\ell-1}}{k_\ell}\right)^2\left[e^{2\gamma_{\ell-1}|z_\ell|}+R_{b(\ell-1)}\right] - \frac{\gamma_{\ell-1}}{\gamma_\ell}\left[e^{2\gamma_{\ell-1}|z_\ell|}-R_{b(\ell-1)}\right]}{\left(\frac{k_{\ell-1}}{k_\ell}\right)^2\left[e^{2\gamma_{\ell-1}|z_\ell|}+R_{b(\ell-1)}\right] + \frac{\gamma_{\ell-1}}{\gamma_\ell}\left[e^{2\gamma_{\ell-1}|z_\ell|}-R_{b(\ell-1)}\right]} \tag{5}$$

where the subscript "j" has been omitted in the right hand sides of (4) and (5). $R_{dj\ell}$ is derived by considering the scalar functions (40) to (43) in two adjacent layers and by requiring the tangential field components (44), (45), (47) and (48) to be continuous across the interface. The computations should start with $\ell=1$ where $R_{dj(\ell-1)} = R_{djo} = 0$ in (4) and (5). A series of computations gives then R_{dj1}, R_{dj2}, R_{dj3} and finally R_{djp} or R_{djq} in (3).

The antenna current density is assumed to be representable by [Galejs 1963, 1965] the trial function

$$J_x(x,y) = [A \sin[k(\ell-|x|)] + B[1-\cos[k(\ell-|x|)]]]f(y) \tag{6}$$

with $f(y) = 1/(2\epsilon)$ = const. Substituting (6) in (2) and utilizing the stationary character of the impedance expression for determining the complex amplitudes A and B, it follows that [Galejs 1963, 1965]

$$\frac{A}{B} = \frac{\gamma_{BB} F_A - \gamma_{AB} F_B}{\gamma_{AA} F_B - \gamma_{AB} F_A} \tag{7}$$

$$Z = \frac{\gamma_{AA}\gamma_{BB} - \gamma_{AB}^2}{\Delta} \tag{8}$$

with

$$\Delta = F_B^2 \gamma_{AA} - 2F_A F_B \gamma_{AB} + F_A^2 \gamma_{BB} \tag{9}$$

$$F_A = \sin k\ell \tag{10}$$

$$F_B = 1 - \cos k\ell \tag{11}$$

$$\gamma_{NM} = \frac{4k^2 i\omega\mu_o}{\pi^2} \int_0^\infty dv \left(\frac{\sin \epsilon v}{\epsilon v}\right)^2 \int_0^\infty du\, F(u,v) g_N(u) g_M(u) \tag{12}$$

$$g_A(u) = \frac{1}{k^2-u^2} (\cos u\ell - \cos k\ell) \tag{13}$$

$$g_B(u) = \frac{1}{k^2-u^2} \left(\frac{k}{u} \sin u\ell - \sin k\ell\right) . \tag{14}$$

The impedance can be also computed for the sinusoidal field distribution term by setting B = 0 in (6). This gives

$$Z_s = \frac{\gamma_{AA}}{F_A^2} . \tag{15}$$

The power radiated into the region outside of the dielectric slabs can be computed as

$$P_r = \mathrm{Re} \iint \underline{E} \times \underline{H}^* \cdot \underline{ds} \tag{16}$$

where the integration is carried out over the outermost surfaces of the dielectric ($z = z_{o1}$ and z_{i1}) and where $d\underline{s}$ is in the direction of the outward normal. After substituting the field expressions of the Appendix, (16) becomes a six fold integral. The x and y integrals are carried out first and they give delta functions. The two subsequent integrations are trivial and the remaining double integral becomes

$$P_r = \frac{16\pi^2}{\omega\mu_o} \int_0^{k_o} du \int_0^{\sqrt{k_o^2-u^2}} dv \; \sqrt{k_o^2-u^2-v^2}\,(u^2+v^2) \tag{17}$$

$$\cdot \left[|A_{oo}|^2 + |A_{io}|^2 + k_o^2|B_{oo}|^2 + k_o^2|B_{io}|^2 \right]$$

where A_{jo} and B_{jo} designate the complex amplitudes of TE and TM modes for $z > z_{o1}$ (when $j=0$) and for $z < z_{i1}$ (when $j=i$). The amplitudes A_{jo} and B_{jo} are related to the corresponding amplitudes in the dielectric region near the antenna A_{js} and B_{js} ($s=p$ for $j=0$, $s=q$ for $j=i$) by matching the tangential field components across the dielectric interfaces. This gives

$$A_{jo} = \left[\prod_{\ell=1}^{s} \frac{e^{\gamma_{j\ell}|z_{j\ell}|} + R_{aj\ell}\, e^{-\gamma_{j\ell}|z_{j\ell}|}}{e^{\gamma_{j(\ell-1)}|z_{j\ell}|} + R_{aj(\ell-1)}\, e^{-\gamma_{j(\ell-1)}|z_{j\ell}|}} \right] A_{js} \tag{18}$$

$$B_{jo} = \left[\prod_{\ell=1}^{s} \frac{e^{\gamma_{j\ell}|z_{j\ell}|} - R_{bj\ell}\, e^{-\gamma_{j\ell}|z_{j\ell}|}}{e^{\gamma_{j(\ell-1)}|z_{j\ell}|} - R_{bj(\ell-1)}\, e^{-\gamma_{j(\ell-1)}|z_{j\ell}|}} \right] \frac{\gamma_{js}}{\gamma_{jo}} B_{js} \tag{19}$$

where $R_{ajo} = R_{bjo} = 0$ and where $\Pi = 1$ in absence of dielectric layers ($s=0$). A_{js} and B_{js} are related to the amplitudes A and B of the trial function for the antenna current density (6) by

$$A_{js} = \frac{v}{1+R_{ajs}} \left[\gamma_{op} \frac{1-R_{aop}}{1+R_{aop}} + \gamma_{iq} \frac{1-R_{aiq}}{1+R_{aiq}} \right]^{-1} H(u,v) \tag{20}$$

$$\gamma_{js} B_{js} = \mp \frac{u}{1-R_{bjs}} \left[\frac{k_{op}^2}{\gamma_{op}} \frac{1+R_{bop}}{1-R_{bop}} + \frac{k_{iq}^2}{\gamma_{iq}} \frac{1+R_{biq}}{1-R_{biq}} \right]^{-1} H(u,v) \tag{21}$$

with

$$H(u,v) = \frac{\omega\mu_o}{u^2+v^2} \left(\frac{1}{2\pi}\right)^2 \frac{\sin v\epsilon}{v\epsilon}\, 2k \left[A g_A(u) + B g_B(u) \right] . \tag{22}$$

The minus sign of (21) should be used with j=o and s=p, the plus sign with j=i and s=q. With P_r of (16) related to A and B by (17) to (22), the radiation resistance R_r may be defined as

$$R_r = \frac{P_r}{|I(x=0)|^2} = \frac{P_r}{|AF_A + BF_B|^2} \tag{23}$$

where the complex ratio A/B is given by (7).

With no losses present in the dielectric layers the power carried by surface waves that are guided along the dielectric interfaces can be computed as

$$P_s = \mathrm{Re} \iint \underline{E} \times \underline{H}^* \cdot \underline{ds} \tag{24}$$

where the integration is carried out over the surfaces $x = \pm \infty$ and $y = \pm \infty$ and where is in the direction of the outward normal. Considering an antenna located at the center of a single lossless dielectric layer (p=q=1) of thickness $2a = z_{o1} - z_{i1}$, the integrals representing the fields in (24) are singular for those values $\lambda^2 = u^2 + v^2$, where $R_{ao1} = R_{ai1} = 1$ or $R_{bo1} = R_{bi1} = -1$. This gives

$$\tan\left(ak_o \sqrt{\epsilon_r - w^2}\right) = \frac{\sqrt{w^2 - 1}}{\sqrt{\epsilon_r - w^2}} \tag{25}$$

and

$$\tan\left(ak_o \sqrt{\epsilon_r - w^2}\right) = -\frac{\sqrt{\epsilon_r - w^2}}{\epsilon_r \sqrt{w^2 - 1}} \tag{26}$$

with $1 < (w = \lambda/k_o) < \sqrt{\epsilon_r}$ and $\epsilon_r = \epsilon_{o1}/\epsilon_o = \epsilon_{i1}/\epsilon_o$ as the equations for determining the characteristic values λ for TE and TM modes respectively. Equation (25) has real solutions for the TE modes for any thickness of the dielectric layer. Equation (26) has real solutions for the TM modes only for $ak_o \sqrt{\epsilon_r - 1} > \pi/2$. For lossless dielectric there are no complex solutions which satisfy the radiation condition following Whitmer [1948].

Substituting the field expressions of the Appendix, (24) becomes a six-fold integral. In the integrals over the $x = \pm \infty$ ($y = \pm \infty$) surfaces the y (x) integrals give delta functions, which makes one of the v (u) integrals trivial. The two u (v) integrals are computed by evaluating the residues at the poles given by (25) and (26). The z integrations are trivial and (24) is reduced to

$$P_s = P_s \Big|_{TE} + P_s \Big|_{TM} \tag{27}$$

where

$$P_s\Big|_{TE} = \frac{\omega\mu_o}{8\pi}\left[\frac{\gamma_{oo}}{\lambda(1-a\gamma_{oo})}\right]^2 \left\{a\left[1 + \frac{\sin(-2i\gamma_{o1}a)}{(-2i\gamma_{o1}a)}\right] - \frac{\cos^2(-i\gamma_{o1}a)}{\gamma_{oo}}\right\} \cdot (I_{x1} + I_{y1}) \tag{28}$$

$$P_s\Big|_{TM} = \frac{\omega\mu_o}{8\pi}\left[\frac{\gamma_{o1}^2}{k_{o1}\lambda}\right]^2 \left[\epsilon_r \frac{\gamma_{oo}^2 - \gamma_{o1}^2}{(\epsilon_r\gamma_{oo})^2 - \gamma_{o1}^2} + a\,\gamma_{oo}\right]^{-2} \cdot \left\{a\left[1 - \frac{\sin(-2i\gamma_{o1}a)}{(-2i\gamma_{o1}a)}\right] + \frac{\gamma_{o1}^2}{\gamma_{oo}^3}\sin^2(-i\gamma_{o1}a)\right\}(I_{x2}+I_{y2}) \tag{29}$$

$$I_{x1} = \int_{v=o}^{\lambda} \frac{dv\, v^2}{\sqrt{\lambda^2-v^2}} \left|g(\sqrt{\lambda^2-v^2})\; h(v)\right|^2 \tag{30}$$

$$I_{y1} = \int_{u=o}^{\lambda} du\,\sqrt{\lambda^2-u^2} \left|g(u)\; h(\sqrt{\lambda^2-u^2})\right|^2 \tag{31}$$

$$I_{x2} = \int_{v=o}^{\lambda} dv\,\sqrt{\lambda^2-v^2} \left|g(\sqrt{\lambda^2-v^2})\; h(v)\right|^2 \tag{32}$$

$$I_{y2} = \int_{u=o}^{\lambda} \frac{du\, u^2}{\sqrt{\lambda^2-u^2}} \left|g(u)\; h\sqrt{\lambda^2-u^2}\right|^2 \tag{33}$$

$$g(u) = 2k\left[A\; g_A(u) + B\; g_B(u)\right] \tag{34}$$

$h(v) = \sin \epsilon v/\epsilon v$, $g_A(u)$ and $g_B(u)$ are defined by (13) and (14), γ_{oo} is negative and γ_{o1} is purely imaginary. The antenna input resistance R_s associated with the surface wave power is computed from (27) by analogy with (23).

3.0 NUMERICAL CALCULATIONS

3.1 The Free Space Impedance of a Thin Antenna

The antenna impedance (8) or (15) is expressed in terms of double integrals (12), which can be evaluated analytically for the antenna in a homogeneous medium. The free space impedance (15) of a thin half-wavelength antenna will be considered as a simple example. For $\ell = \lambda/4 = \pi/(2k_o)$, k of the trial functions (6) set equal to $k_o = \omega\sqrt{\mu_o \epsilon_o}$, and with $R_{dj\ell} = 0$ γ_{AA} of (12) simplifies to

$$\gamma_{AA} = \frac{2\omega\mu_o}{\pi^2} \int_0^\infty dv \left(\frac{\sin \epsilon v}{\epsilon v}\right)^2 \int_0^\infty \frac{du \cos^2(\frac{\pi}{2}\frac{u}{k})}{\sqrt{k^2-u^2-v^2}(k^2-u^2)} . \tag{35}$$

With $F_A = 1$, substitution of (35) in (15) gives the real part of the impedance as

$$\mathrm{Re}\, Z_s = \frac{2\omega\mu_o}{\pi^2} \int_0^k \frac{du \cos^2(\frac{\pi}{2}\frac{u}{k})}{k^2-u^2} \int_0^{\sqrt{k^2-u^2}} \frac{dv}{\sqrt{k^2-u^2-v^2}} \tag{36}$$

$$= \frac{\omega\mu_o}{\pi} \int_0^k \frac{du \cos^2(\frac{\pi}{2}\frac{u}{k})}{(k-u)(k+u)} .$$

After expanding the denominator in partial fractions and changing the variables of integration to (k-u) and (k+u)

$$\mathrm{Re}\, Z_s = 60 \int_0^2 \frac{\sin^2(\frac{\pi}{2}x)}{x} dx = 30\,[\log(2\pi) + 0.5772 - \mathrm{Ci}(2\pi)] \tag{37}$$

$$= 73.1$$

where Ci(x) is the cosine integral. The imaginary part of the impedance is obtained as

$$\mathrm{Im}\, Z_s = -\frac{2\omega\mu_o}{\pi^2} \left[\int_0^k du \int_{\sqrt{k^2-u^2}}^\infty dv + \int_k^\infty du \int_0^\infty dv\right] \tag{38}$$

$$\cdot \left[\frac{\cos^2(\frac{\pi}{2}\frac{u}{k})}{k^2-u^2}\left(\frac{\sin \epsilon v}{\epsilon v}\right)^2 \frac{1}{\sqrt{v^2+u^2-k^2}}\right] . \tag{38}$$

The v-integrations become elementary by noting that $(\sin \epsilon v/\epsilon v) \approx 1$ over the range of v where $\sqrt{v^2+u^2-k^2} \neq v$. (This applies strictly if $\epsilon \to 0$ and $u \leq u_o$ = finite.) It follows that

$$\mathrm{Im}\, Z_s = \frac{\omega\mu_o}{\pi^2 k^2} \int_0^\infty \frac{du \cos^2(\frac{\pi}{2}\frac{u}{k})}{1-(\frac{u}{k})^2}\left[\log\left|1-\frac{u}{k}\right| + \log(1+\frac{u}{k})\right] \quad (39)$$

$$= \frac{240}{\pi} \int_0^\infty \frac{dy \log y \sin^2 \pi y}{4-y^2} = -30\, \mathrm{Si}(2\pi) = -42.5$$

where Si(x) is the sine integral. The numerical values (37) and (39) are recognized as standard results [Kraus, 1950, Sec. 10.3].

Considering the previous example of free space and a thin half-wavelength antenna, the radiation resistance R_r (23) simplifies under similar assumptions to Re Z_s of (36) and (37).

3.2 Antennas in a Homogeneous Dielectric

Impedance calculations for an antenna of a finite width are first carried out for homogeneous media where other solutions are already available.

The free space impedance of a $\lambda/2$ antenna is shown in Figure 2 for various values of the width ϵ or of the parameter $\Omega = 2\ln(4\ell/\epsilon)$. Also shown in Figure 2 are the impedance of a cylindrical antenna of radius $a = \epsilon/2$ (Table II.30.1 of King [1956]), and the impedance Z computed from the admittance Y of a complementary slot antenna [Galejs, 1964] using the formula $Z = \mu_o Y/[2\epsilon(1+i \tan\delta)]$. The assumption of a sinusoidal current distribution is shown to give an incorrect resistance for antennas of finite width. For $\Omega < 10$ the reactance computed by King [1956], is nearly the same as for the assumed sinusoidal current distribution and it differs from the other computations. There is good agreement between the various computations for antennas of $\Omega > 11$.

The admittance of the strip antenna in a lossy medium is compared in Figure 3 with the admittance of a complementary slot antenna [Galejs, 1964], with measurements of Iizuka and King [Fig. 10, 1962] and with the theory of Wu [1961, Gooch, et al 1963]. The antenna admittance calculated from King [1956] for a lossless dielectric is also indicated in Figure 3. The antenna length is equal to one-half wave length for low dielectric losses and $\epsilon_r = \epsilon_d/\epsilon_o$ changes from 78 to 69 with increasing losses as indicated by Iizuka and King [1962]. There is good agreement between the various theoretical curves for $\tan\delta_d < 2$. The measured conductance appears too low for a low loss dielectric. The theory of Wu gives lower susceptance figures for high loss tangents. The admittance of the complementary slot has been computed using a complex value of $k=k_o \sqrt{\epsilon_d/\epsilon_o} \sqrt{1+i \tan \delta_d}$ in the trial functions (6), while the admittance of the strip antenna was computed using real values of $k=k_o \sqrt{\epsilon_d/\epsilon_o}$. This may account for some of the differences between the two sets of calculations for larger values of $\tan \delta_d$.

3.3 Antenna in Dielectric Layers

The impedance of a linear antenna located either in the center or on the surface of dielectric layers of various thicknesses 2d is shown in Figures 4-9 along with the corresponding efficiency figures. All the calculations of these figures are made for antennas of $\Omega = 8$ and the heavy horizontal lines indicated on the sides of the figures denote the impedance for d=0 or ∞ computed from a complementary slot antenna. The radiation efficiency is computed as a ratio between the power radiated normally through the dielectric surface as indicated in (16) and the power supplied to the antenna. The surface wave efficiency denotes the ratio between the total power carried by the surface wave for a lossless dielectric in the direction parallel to the dielectric surface and the power supplied to the antenna. For a lossy dielectric that part of surface wave power which flows outside of the dielectric layer will be eventually dissipated inside the dielectric and this power will not appear as part of the power radiated outside the lossy dielectric in (16). The sum of the surface wave efficiency and the radiating efficiency are a little less than unity for a low loss dielectric ($\tan \delta_d = 0.03$) in figures 4 and 5, and appreciably less than unity for higher dielectric losses ($\tan \delta_d = 0.1$ and 1) in figure 6, when significant dissipation occurs in immediate vicinity antenna. For antennas located in the center of the dielectric layer the radiation efficiency is decreased by increasing the dielectric constant of the layer (figure 5), but it is unchanged for the several antenna lengths shown in figure 4.

For antennas located on the surface of the dielectric layer (figures 7 and 8) the radiation efficiency is practically unaffected by changes of the antenna length shown in figure 7, and it is also decreased for higher values of ϵ_d. The calculations of surface wave efficiency have not been carried out in the case of antennas located on the surface of the dielectric layer.

For dielectric layers of $\epsilon_d/\epsilon_o < 1$ (approximation of a plasma layer by an isotropic dielectric) the impedance curves of figure 9 are much smoother than for $\epsilon_d/\epsilon_o > 1$, when the layers could support surface waves. Also the radiation efficiency is decreased more gradually with increasing layer thickness.

Variational approximations to the current distributions of the antenna configurations shown in figures 4-9 are computed from equations (6) and (7) and are shown in figures 10 and 11. There are significant deviations from the usually assumed half wave sinusoidal current distribution and these deviations become more significant with increasing thickness of the dielectric layer d, the antenna length ℓ or the dielectric constant ϵ_d. Thick layers of a lossy dielectric make the current distribution approach a triangular one. A similar change is observed with increasing layer thickness d if $\epsilon_d < \epsilon_o$ and with the decreasing ϵ_d in figure 11 when the antenna tends to become electrically shorter.

The above approximate current distributions are the best possible fits to the actual current distribution using the trial function (6) and should be indicative of the trends observed in the actual current distributions with changing antenna parameters.

3.4 Insulated Antenna in Dissipative Medium

The calculated admittance of an insulated strip antenna in a dissipative medium is compared with admittance measurements by Iizuka [1963] in Figure 12. A cylindrical wire of radius a insulated by a cylindrical dielectric shell of outer radius b from the surrounding dissipative medium is compared with a strip line of width 4a laying in the center of a dielectric layer of thickness 2(b-a). In the limit of a highly conducting outer medium the characteristic impedance of the line Z_o in the cylindrical geometry of b/a = 1.4 to 10 approximates Z_o of the corresponding planar geometry with a 10 percent accuracy [Reference data, 1956] which may indicate an approximate equivalence of the two geometries for tan $\delta_d \gg 1$. The comparisons of the calculated and measured admittance data indicates a qualitative agreement.

In Figure 13 the impedance of an insulated cylindrical wire measured by Guy [Figures 3.24 and 3.25, 1962] is compared with transmission line calculations following Guy and Hasserjian [1963] and with calculations for the approximately equivalent model of the strip antenna. The calculated resistance data agree very well with measurements but there is only fair agreement with the measured reactance. The transmission line theory gives too low resistance figures for small antenna lengths and it also produces too large resonance peaks in resistance and reactance. These observations are particularly pronounced in the present example where tan δ_g varies from 2.4 to 0.3 in the frequency range indicated in Figure 13. The same basic differences between transmission line theory and the measured data can be seen from other measurements with higher loss tangents (Figures 3.12 to 3.29 of Gay [1962] or Figures 3 and 5, Fenwick and Weeks [1962]).

The impedance of an insulated strip antenna located near the interface between free space and a lossy dielectric half space (ground) is shown in Figure 14 for several values of the loss tan δ_g. The transmission line theory is applied to an approximately equal geometry with a cylindrical conductor in Figure 15. For tan $\delta_g = 100$ there is fair agreement between the two sets of calculations, but for tan $\delta_g = 10$ the transmission line theory gives an increase in the size of the resonant peaks, while they are definitely lower for the calculations shown in Figure 14. Also there is a significant increase of the resistance for short antenna lengths and there is a relatively small variation of the resistance component with the changing antenna length for tan $\delta_g = 1$. The high values of resistance for a small antenna lengths indicate significant power dissipation in the vicinity of the antenna. For a dipole in direct contact with the ground, the resistance should approach infinity, but insulation and the finite antenna length have the effect of keeping this resistance finite. The radiation efficiency should be improved by decreasing values of tan δ_g, but for short antenna lengths and for tan $\delta_g = 10$ the efficiency is nearly the same as for tan $\delta_g = 1$, due to the very high resistance components shown in Figure 14 for tan $\delta_g = 1$. The resistance components shown for a short antenna in Figure 14 with $\epsilon_g/\epsilon_o = 6$ are higher than those of Figure 13 with $\epsilon_g/\epsilon_o = 80$ for the lower frequencies, because in the latter case there is a large difference in dielectric constant between the dielectric and ground, and as a consequence, less of the near fields is stored in the ground with smaller attendant ohmic losses.

3.5 Conclusions

In the present variational formulation the antenna impedance is expressed as a double integral that is evaluated numerically, except for the trivial case of a thin half-wave antenna. For an antenna in a homogeneous medium the present formulation gives reasonable agreement with other theories [King, 1956; Wu, 1961] and it has been correlated also with impedances computed from complementary slot antennas [Galejs, 1964]. The antenna impedances and radiation efficiencies have been also computed for antennas on or within dielectric layers and radiation efficiencies of 60 percent or more can be achieved with dielectric layers of moderately dielectric constants ($\epsilon_d/\epsilon_o < 3$) and with layer thickness of less than 0.01 λ.

For insulated antenna within an infinite dissipative medium the calculations have been correlated with measurements of Iizuka [1963] and Guy [1962] and also with transmission line theory. Further comparisons have been made between the present formulation and the transmission line theory for insulated antenna near the interface between free space and a semi-infinite lossy dielectric. The transmission line theory is shown to give too large resonance peaks of the input resistance and reactance. Also it gives too low resistance for shorter antennas.

4.0 ACKNOWLEDGEMENT

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APPENDIX

FIELD COMPONENTS

The fields excited in the geometry of Fig. 1 will be expressed as a superposition of TE and TM modes that are derived from scalar functions Ψ_j and Φ_j respectively. The region of $z \geq o$ next to the antenna is designated by a subscript o, the region of $z \leq o$ - by a subscript i. For an $\exp(-i\omega t)$ time dependence of the fields the scalar functions are of the form

$$\Psi_o = \iint A_o(u,v)\, e^{-iux}\, e^{-ivy}\left(e^{\gamma_o z} + R_{ao} e^{-\gamma_o z}\right) du\, dv \qquad (40)$$

$$\Phi_o = \iint B_o(u,v)\, e^{-iux}\, e^{-ivy}\left(e^{\gamma_o z} + R_{bo} e^{-\gamma_o z}\right) du\, dv \qquad (41)$$

$$\Psi_i = \iint A_i(u,v)\, e^{-iux}\, e^{-ivy}\left(e^{-\gamma_i z} + R_{ai} e^{\gamma_i z}\right) du\, dv \qquad (42)$$

$$\Phi_i = \iint B_i(u,v)\, e^{-iux}\, e^{-ivy}\left(e^{-\gamma_i z} + R_{bi} e^{\gamma_i z}\right) du\, dv \qquad (43)$$

where R_{aj} and R_{bj} are reflection coefficients. The field components are related to Ψ_j and Φ_j by

$$E_{xj} = \frac{\partial}{\partial y}\Psi_j + \frac{\partial^2}{\partial z\, \partial x}\Phi_j \qquad (44)$$

$$E_{yj} = -\frac{\partial}{\partial x}\Psi_j + \frac{\partial^2}{\partial z\, \partial y}\Phi_j \qquad (45)$$

$$E_{zj} = -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Phi_j \qquad (46)$$

$$H_{xj} = \frac{1}{i\omega\mu_o}\left[\frac{\partial^2}{\partial z\, \partial x}\Psi_j + k_j^2 \frac{\partial}{\partial y}\Phi_j\right] \qquad (47)$$

$$H_{yj} = \frac{1}{i\omega\mu_o}\left[\frac{\partial^2}{\partial z\, \partial y}\Psi_j - k_j^2 \frac{\partial}{\partial x}\Phi_j\right] \qquad (48)$$

$$H_{zj} = -\frac{1}{i\omega\mu_o}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \qquad (49)$$

The current of a linear density J_x is assumed to flow only in the x direction along the antenna and the field components satisfy the boundary conditions $E_{xi} = E_{xo}$, $E_{yi} = E_{yo}$, $H_{xi} = H_{xo}$, $H_{yi} = H_{yo} + J_x$, for $z = o$. After substituting (40) to (43) in (44) and (45) and applying the E_x and E_y boundary conditions it follows that

$$A_o(1+R_{ao}) = A_i(1+R_{ai}) \tag{50}$$

$$\gamma_o B_o(1-R_{bo}) = \gamma_i B_i(-1+R_{bi}) \tag{51}$$

The H_x and H_y boundary conditions lead to

$$- \gamma_o B_o (1-R_{bo}) \left[\frac{k_o^2(1+R_{bo})}{\gamma_o(1-R_{bo})} + \frac{k_i^2(1+R_{bi})}{\gamma_i(1-R_{bi})} \right] = \frac{\omega\mu_o u}{u^2+v^2} \left(\frac{1}{2\pi} \right)^2 \iint J_x e^{iux} e^{ivy} dx\, dy \tag{52}$$

$$A_o(1+R_{ao}) \left[\gamma_o \frac{1-R_{ao}}{1+R_{ao}} + \gamma_i \frac{1-R_{ai}}{1+R_{ai}} \right] = \frac{\omega\mu_o v}{u^2+v^2} \left(\frac{1}{2\pi} \right)^2 \iint J_x e^{iux} e^{ivy} dx\, dy \tag{53}$$

The amplitudes A_j and B_j are related to the linear current density of the antenna J_x by (50) to (53). The scalar functions Ψ_j and Φ_j and the field components (44) to (49) are thus uniquely related to J_x.

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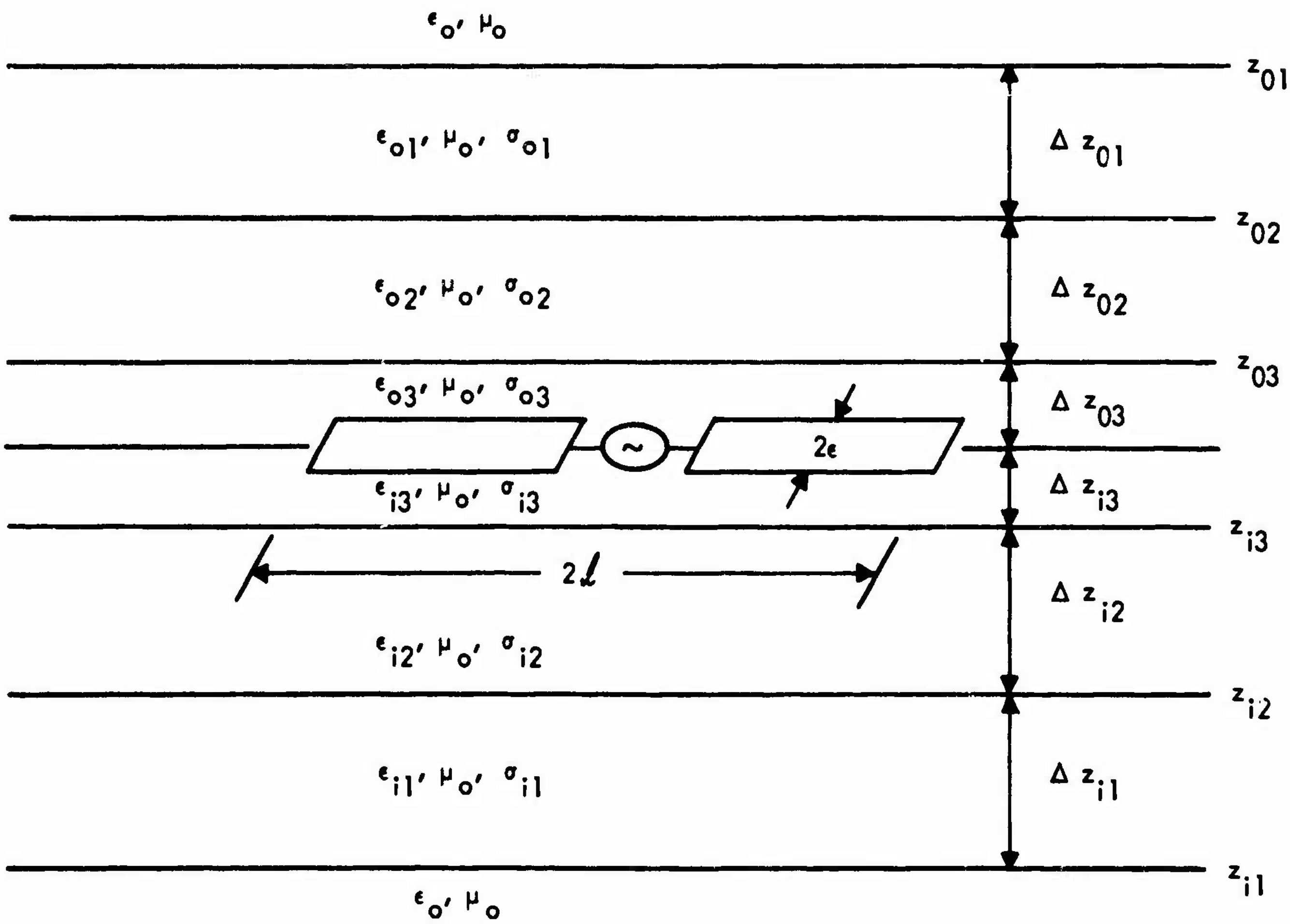

Figure 1. Antenna Geometery.

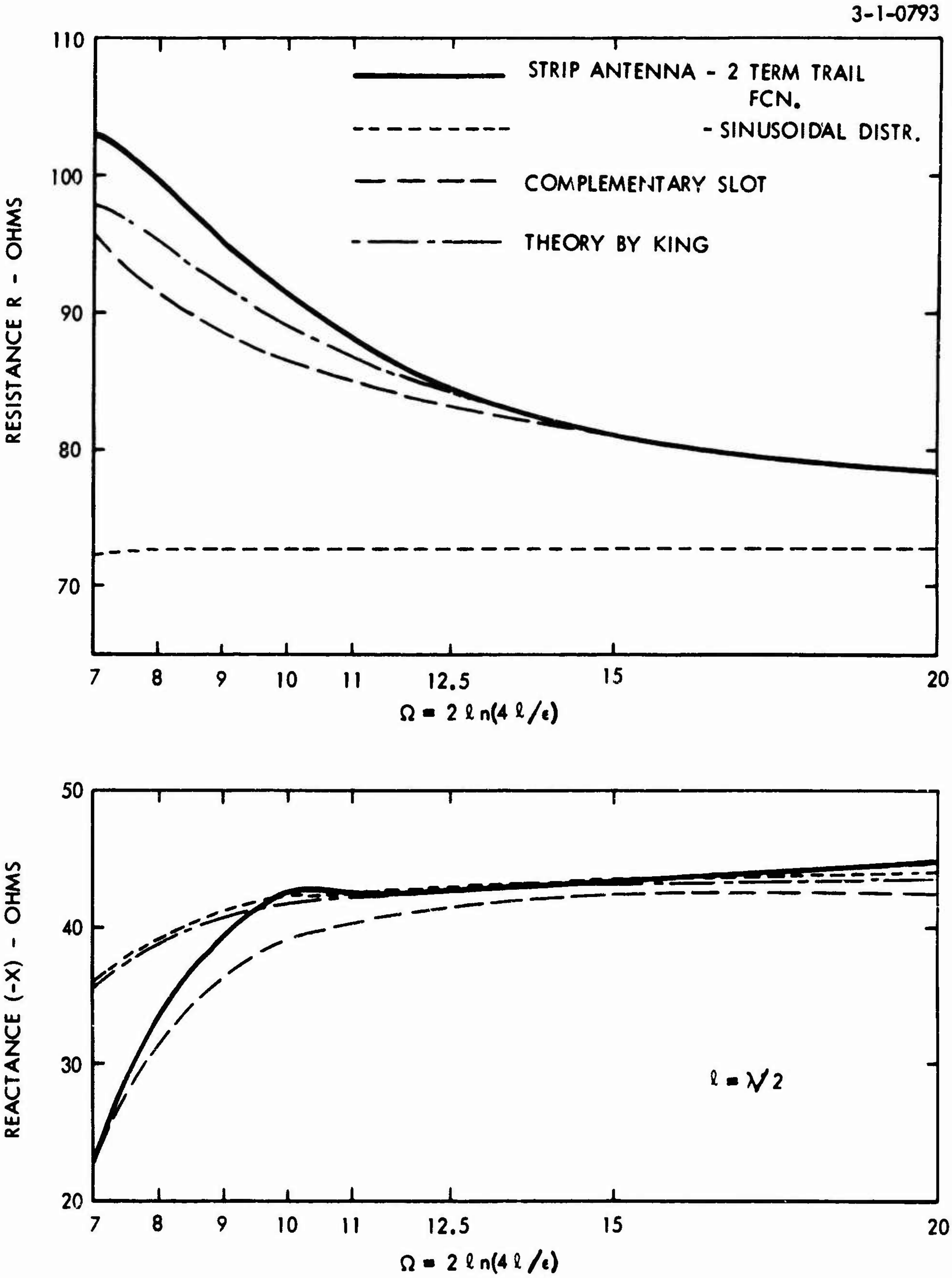

Figure 2. Free Space Antenna Impedance Z = R + iX.

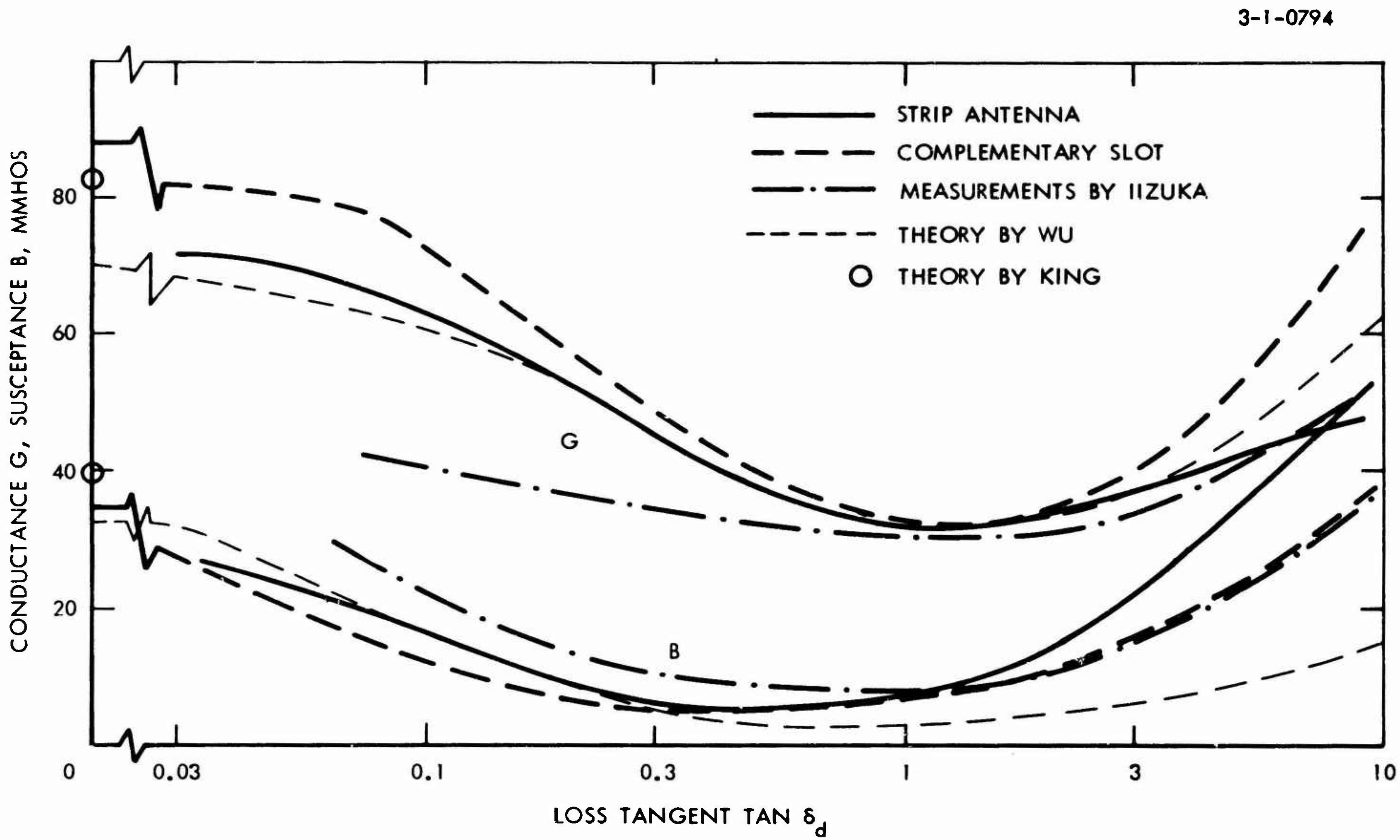

Figure 3. Admittance of a Thin Antenna in a Dissipative Medium.

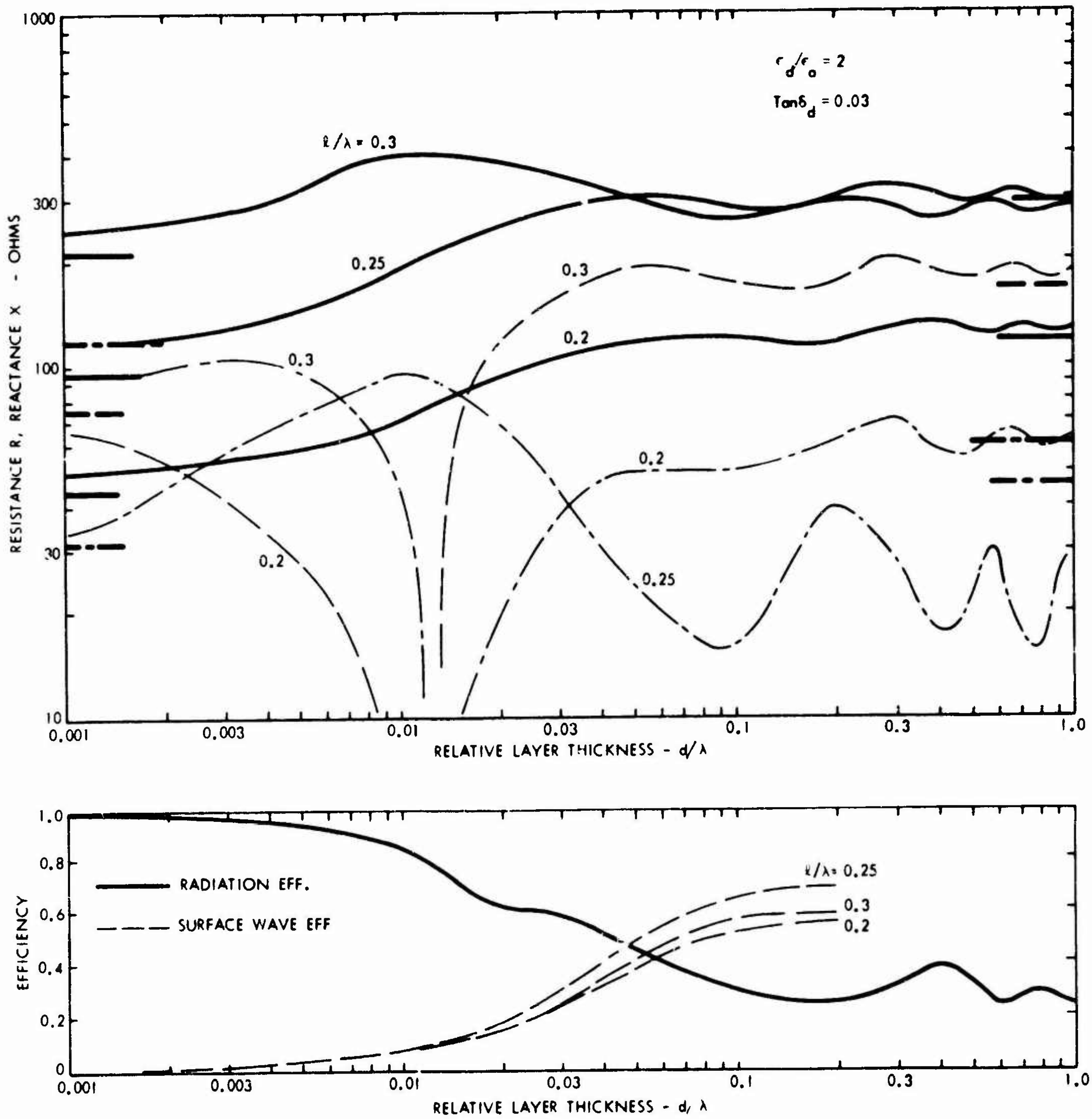

Figure 4. Impedance and Efficiency of Antennas in the Center of a Dielectric Layer. Effects of Antennas Length.

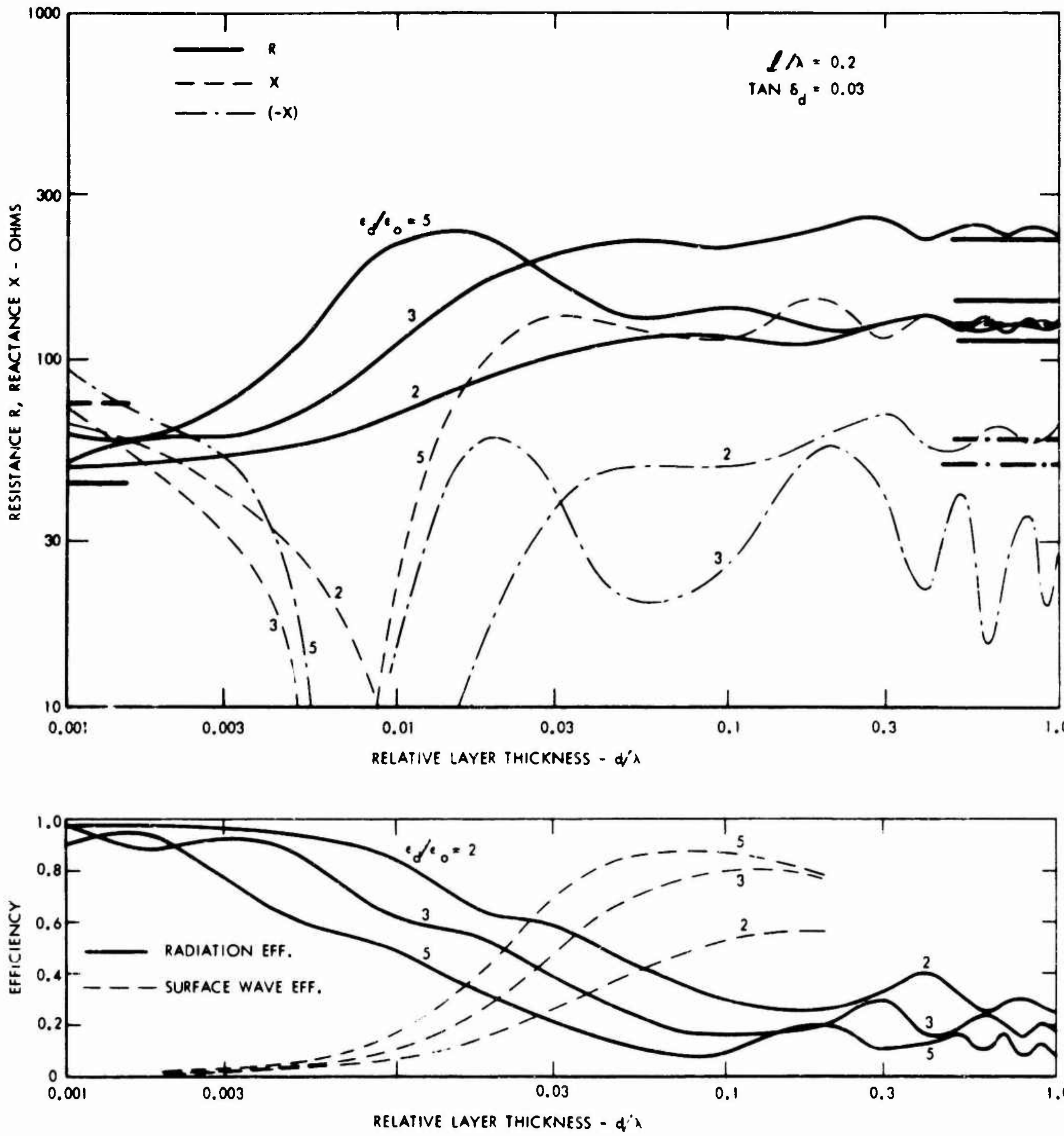

Figure 5. Impedance and Efficiency of Antennas in the Center of a Dielectric Layer. Effects of the Dielectric Constant.

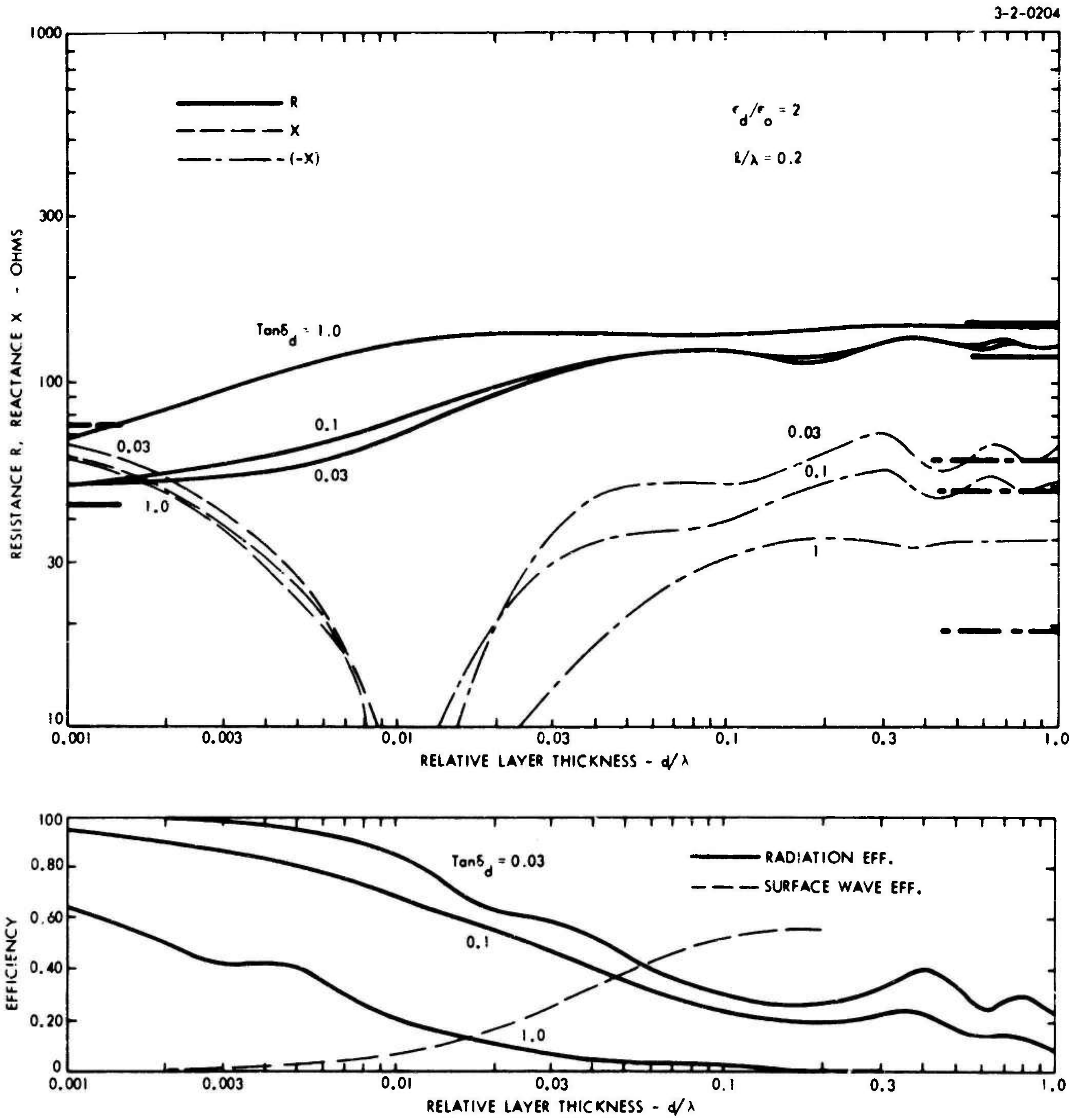

Figure 6. Impedunce and Efficiency of Antenna in the Center of a Dielectric Layer. Effects of Dielectric Losses.

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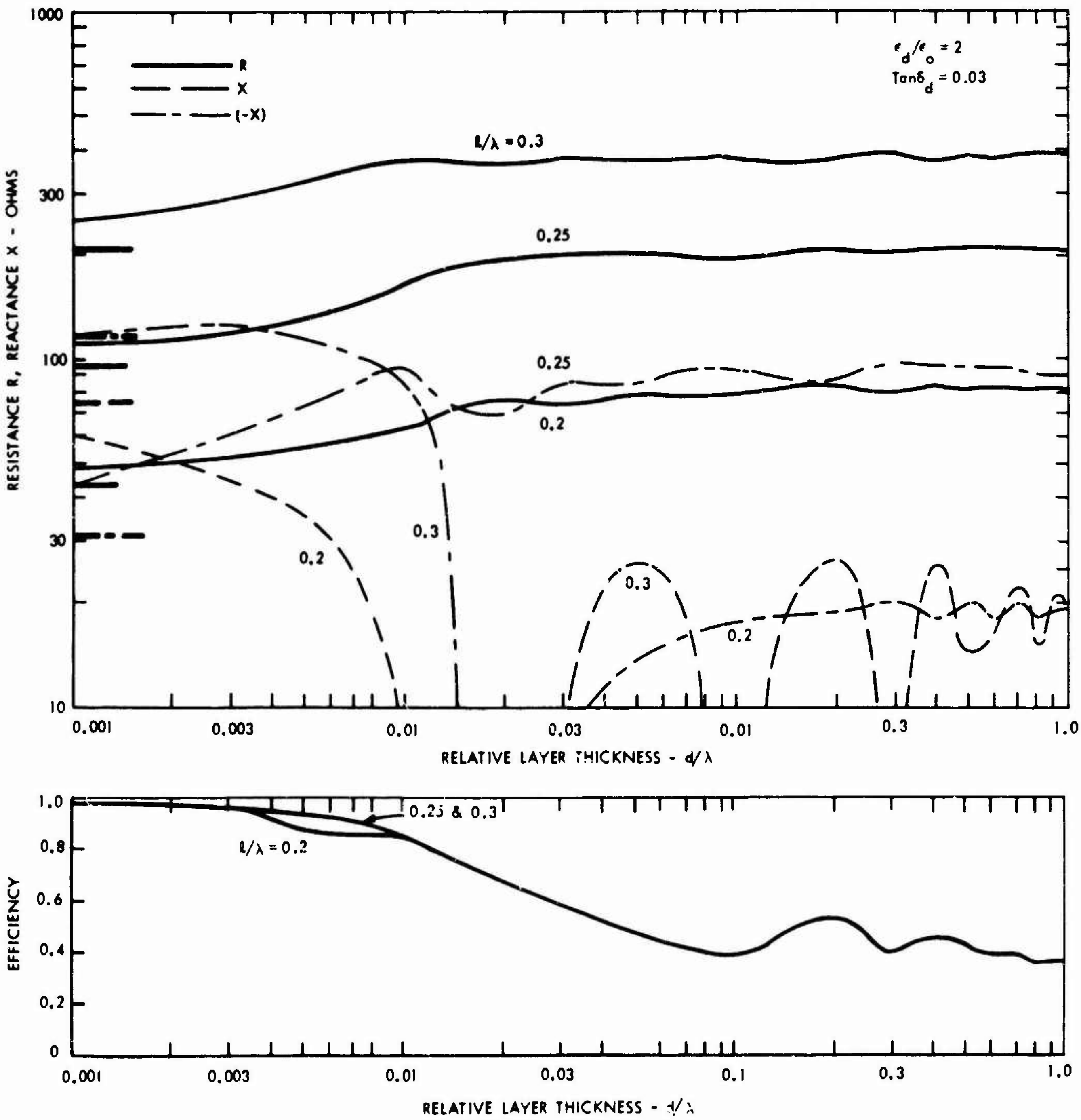

Figure 7. Impedance and Efficiency of an Antenna on the Surface of a Dielectric Layer. Effects of Antenna Length.

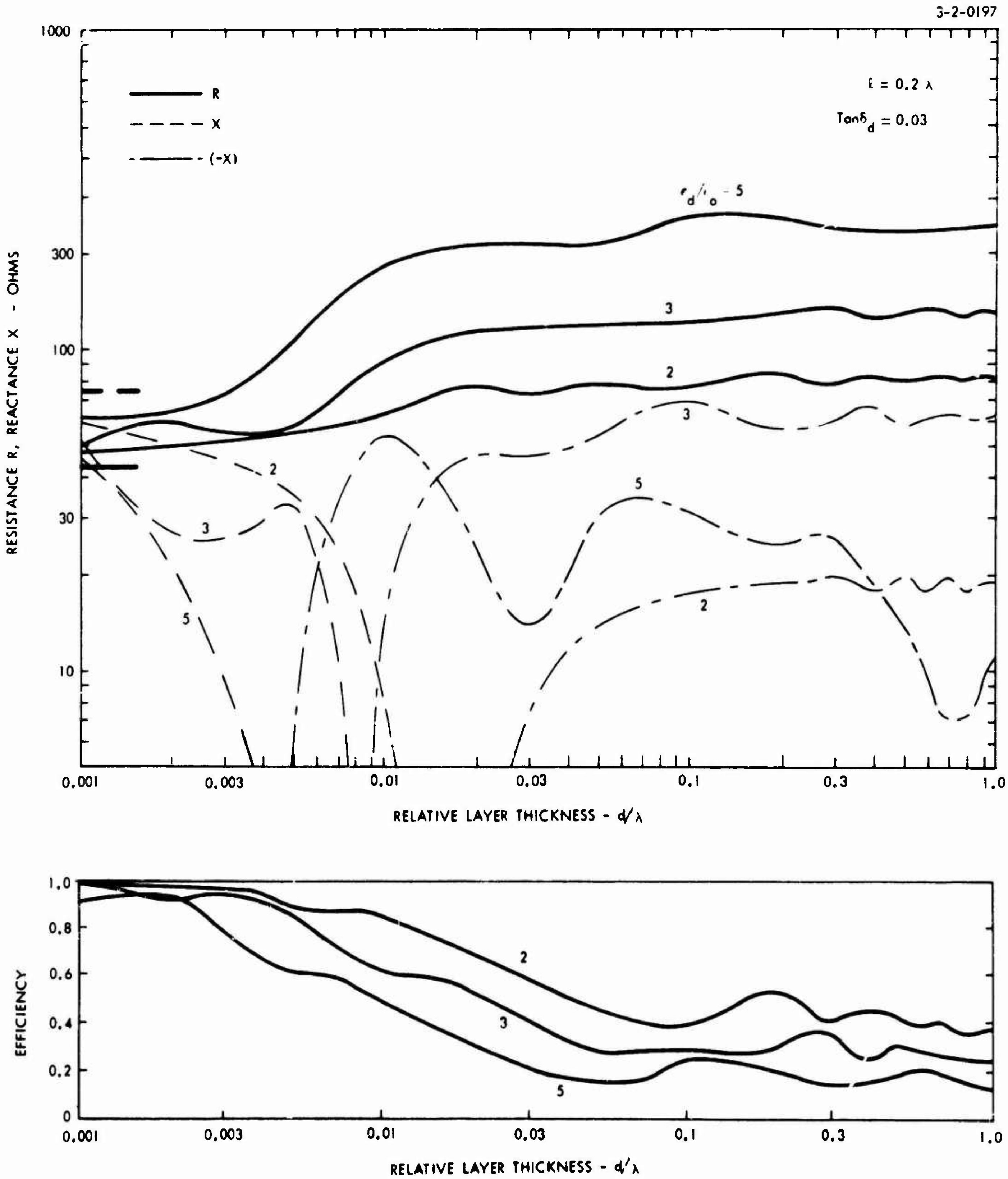

Figure 8. Impedance and Efficiency of Antennas on the Surface of a Dielectric Layer. Effects of the Dielectric Constant.

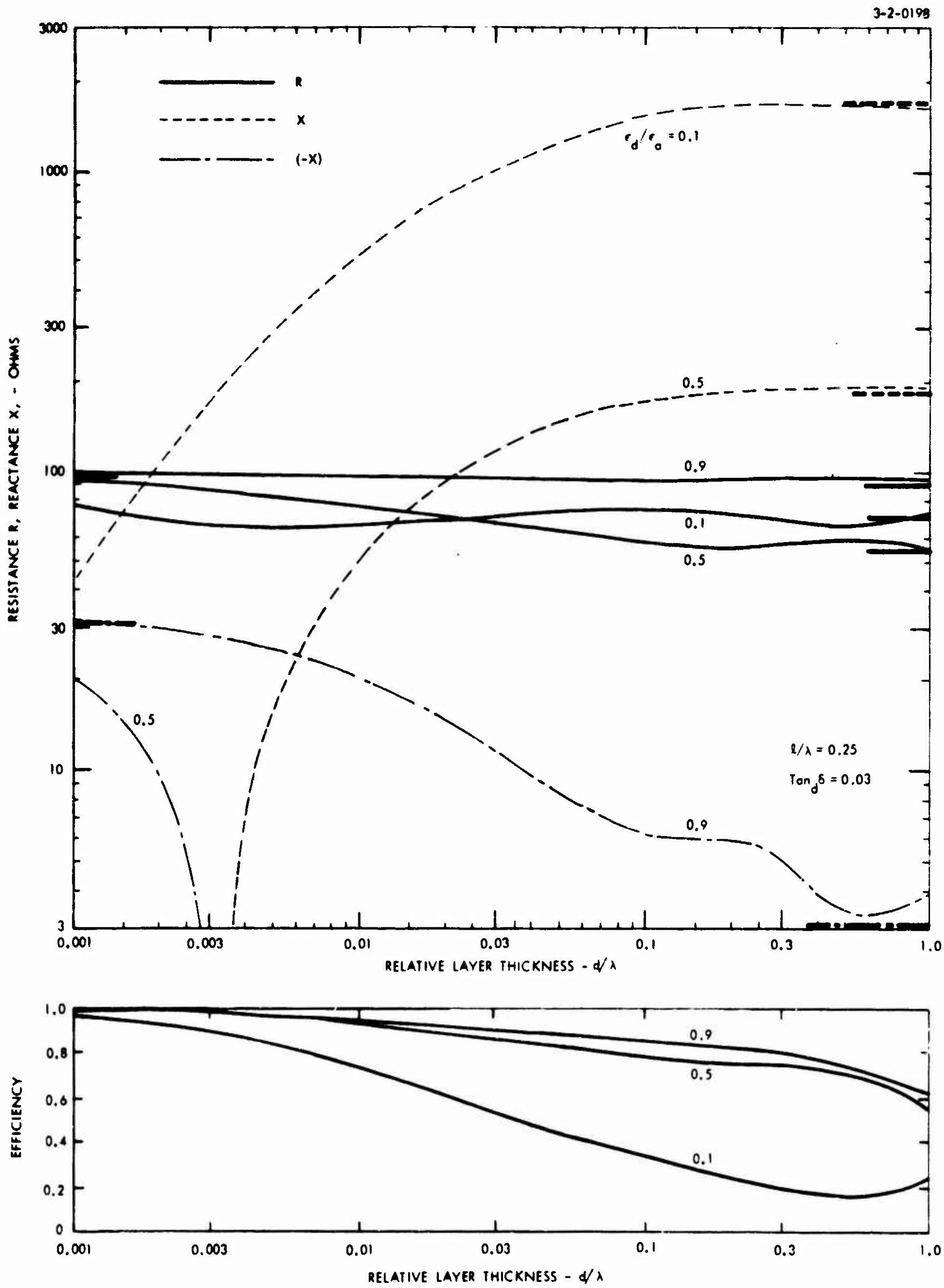

Figure 9. Impedance and Efficiency of Antennas in the Center of a Plasma Layer.

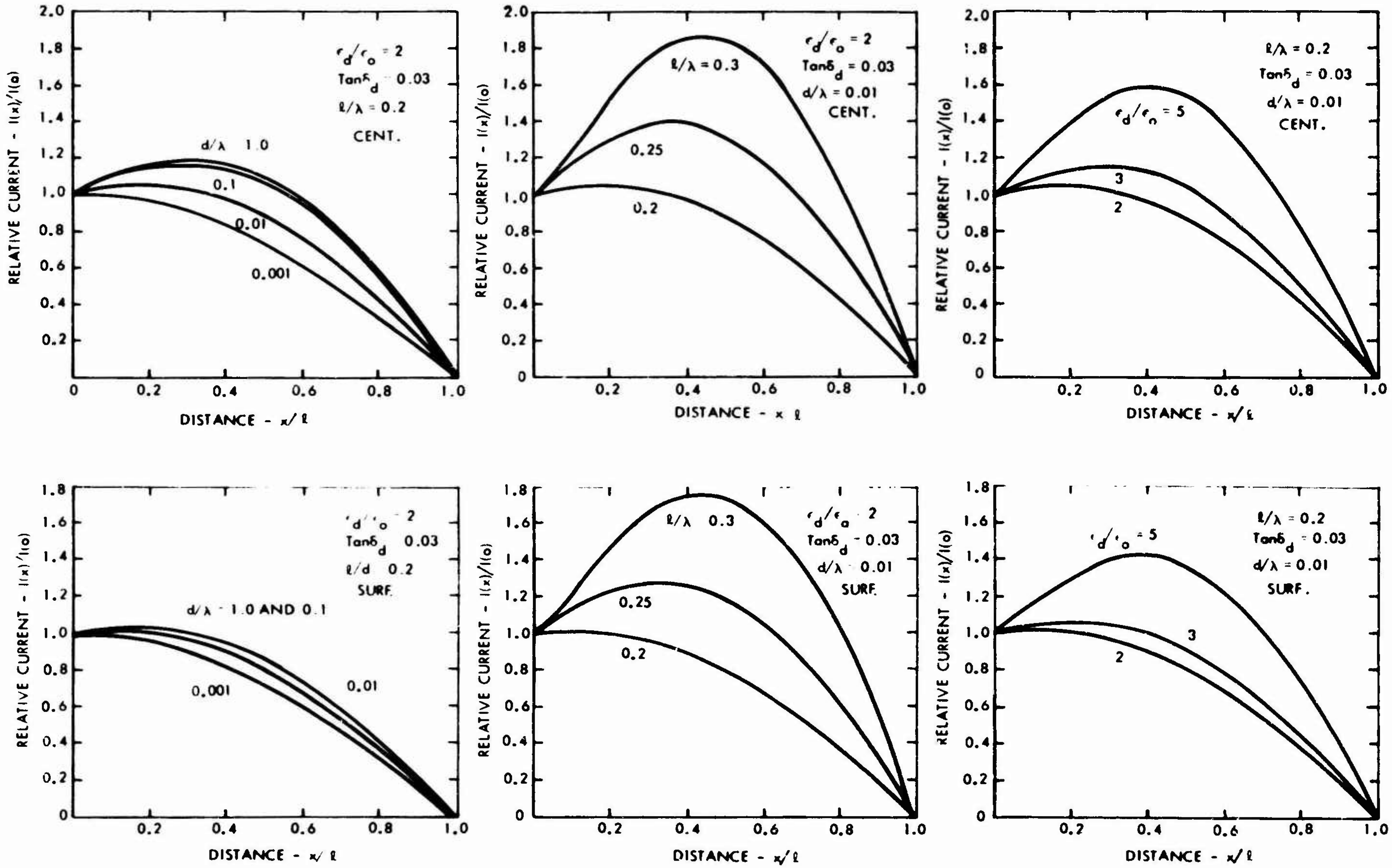

Figure 10. Variational Approximation to the Current Distribution of Antenna in the Center (CENT.) or on the Surface (SURF.) of Dielectric Layers of Thickness 2d.

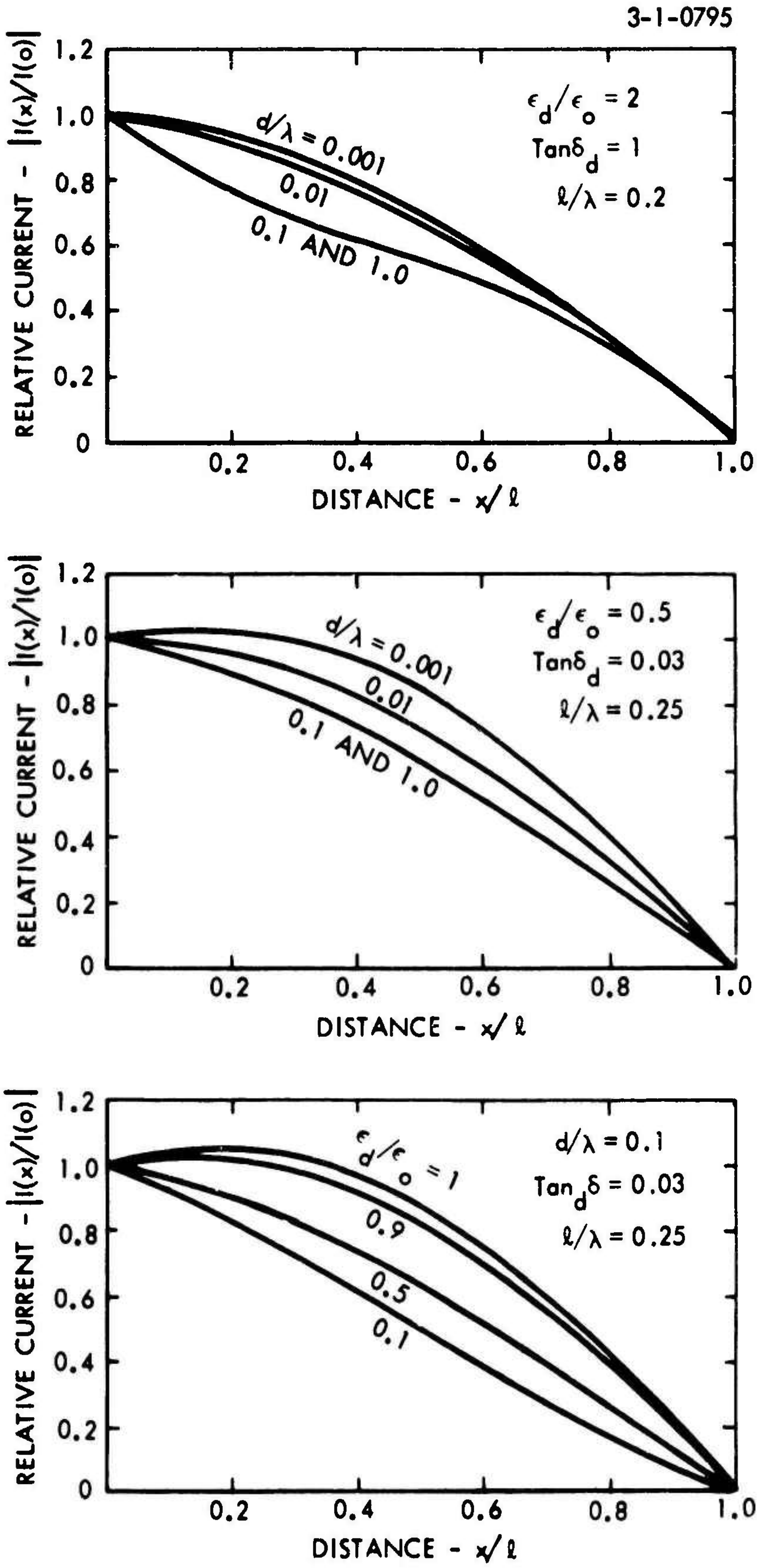

Figure 11. Variational Approximations to the Current Distributions of Antennas in the Center of a Dislectric Layer of Thickness 2d.

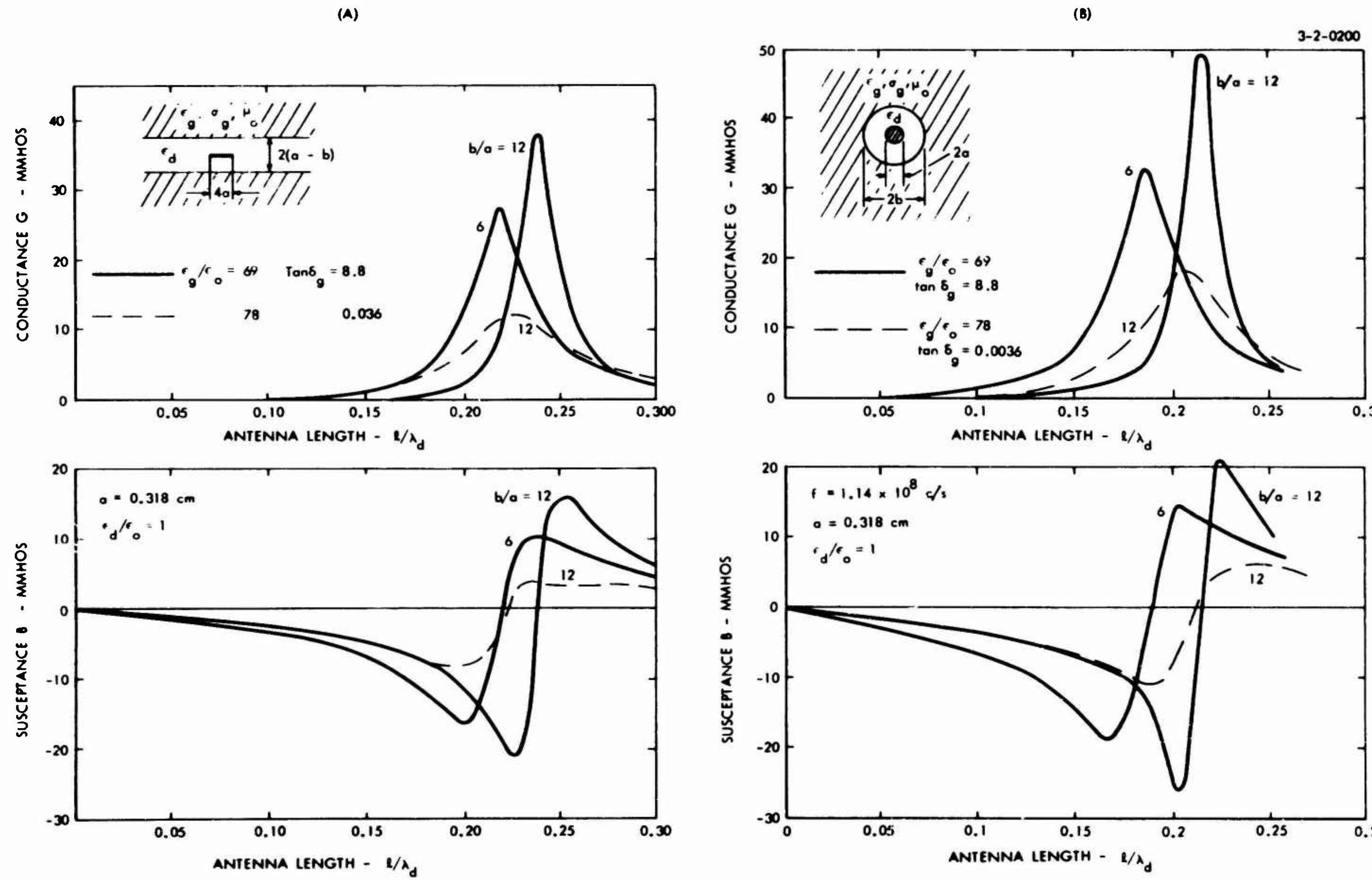

Figure 12. Admittance of an Insulated Antenna in a Dissipative Medium.
A. Calculation. B. Measurements by Iizuka.

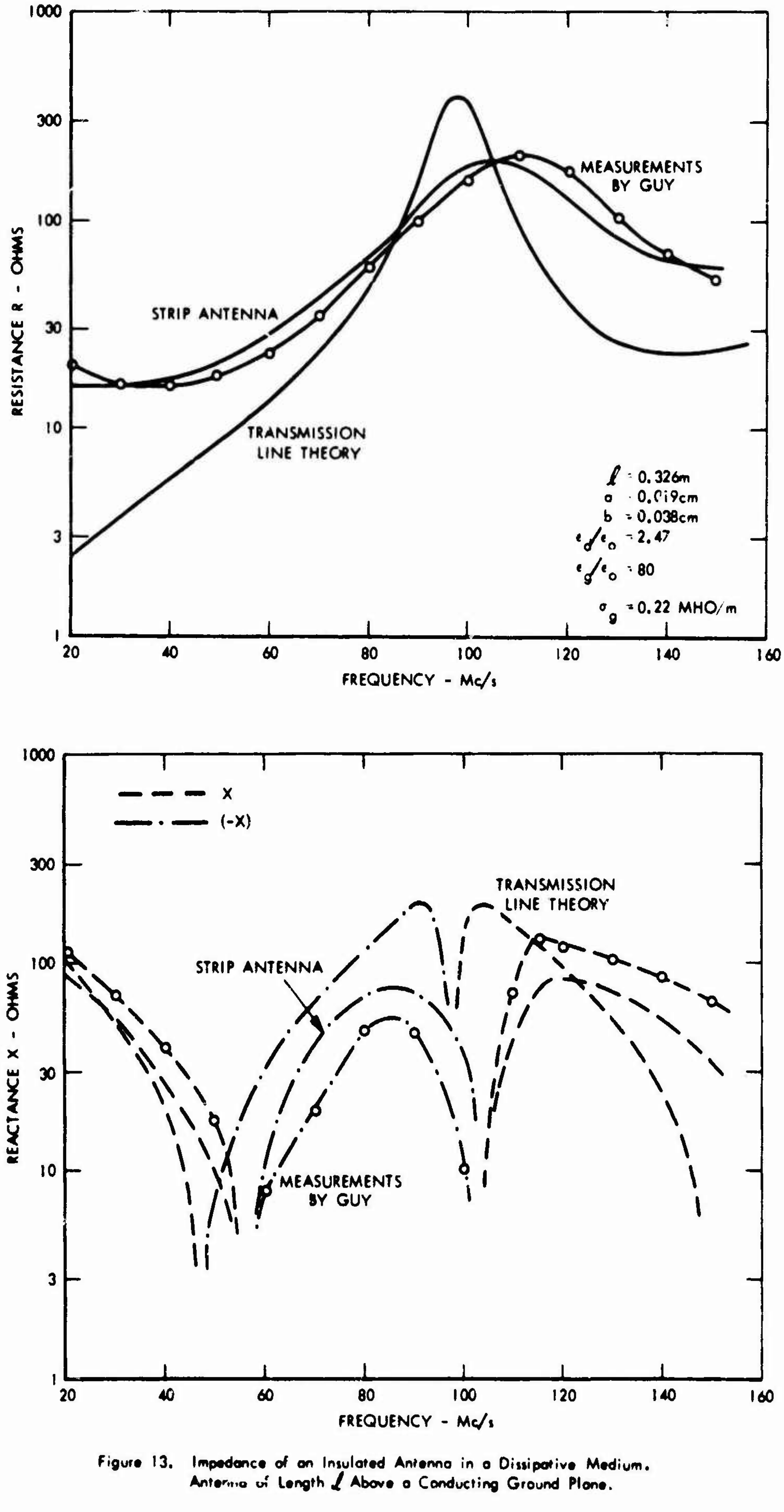

Figure 13. Impedance of an Insulated Antenna in a Dissipative Medium. Antenna of Length ℓ Above a Conducting Ground Plane.

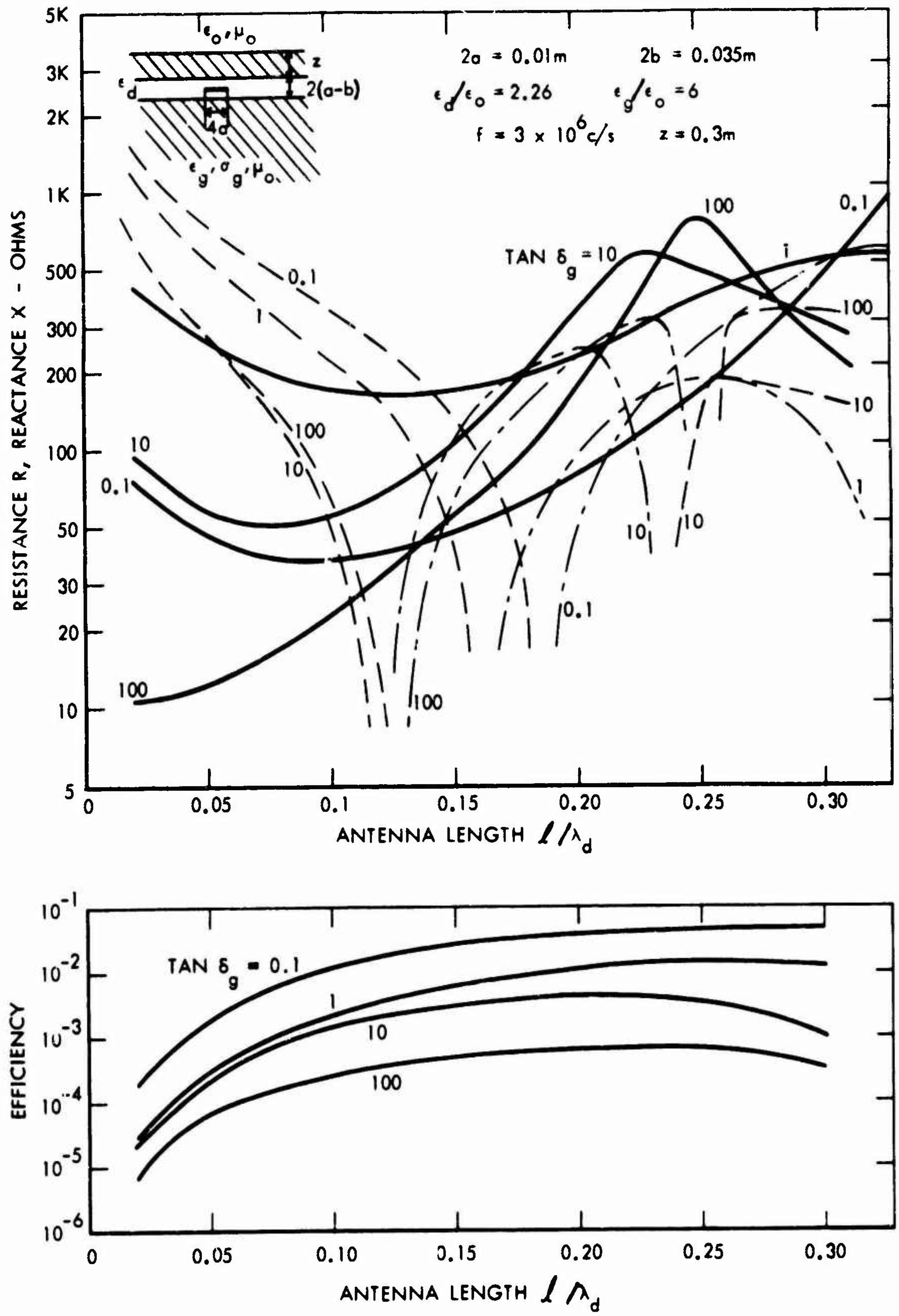

Figure 14. Impedance Z = R + iX of a Buried Strip Antenna and its Radiation Efficiency.

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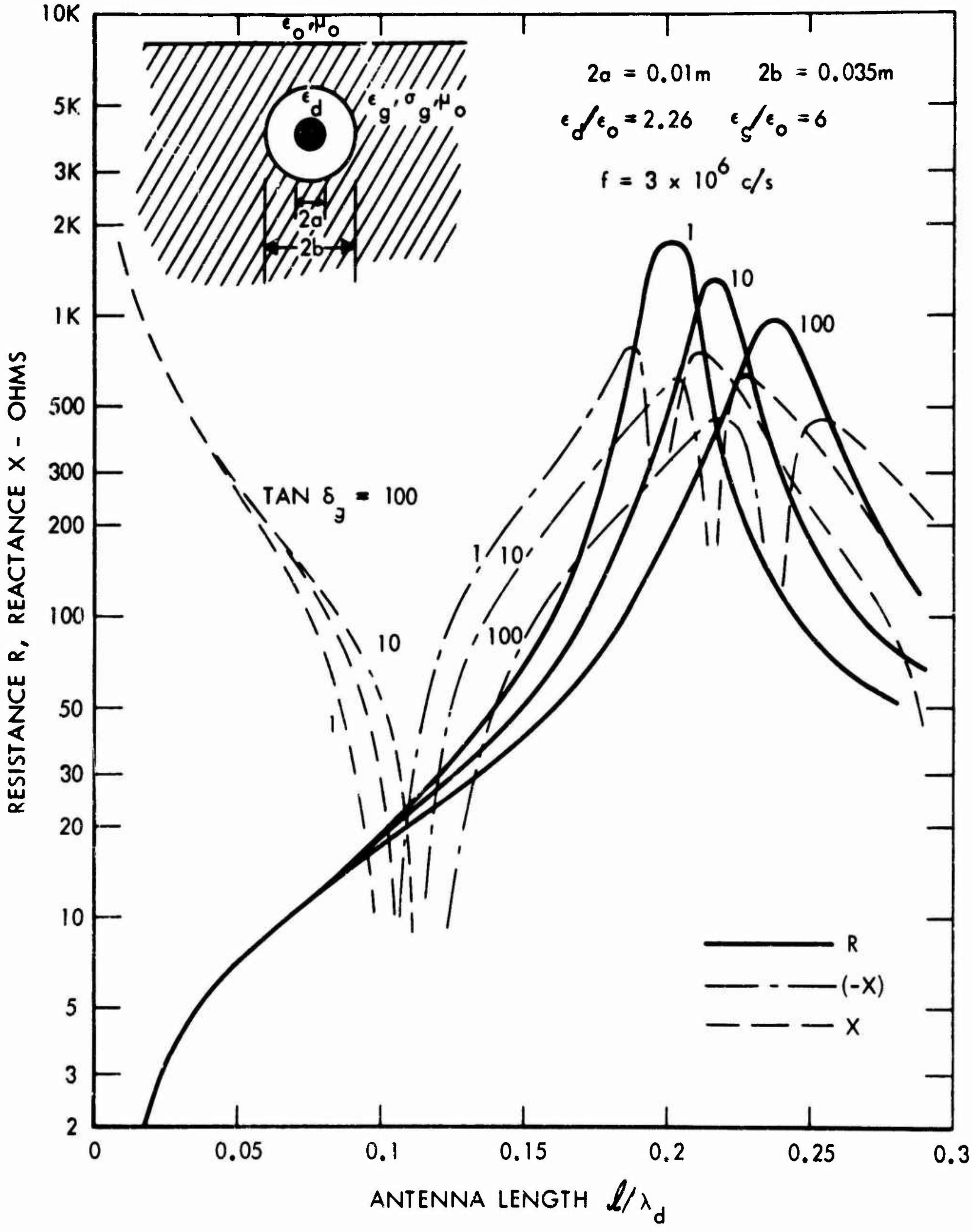

Figure 15. Impedance of a Buried Antennc Z = R + iX Transmission Line Theory.

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13. ABSTRACT A linear flat strip antenna lies between several dielectric layers in a direction parallel to the interfaces. Its impedance is formulated variationally as an infinite double integral. For thin half-wave antennas in a homogeneous medium the double integral is evaluated analytically and it gives standard impedance expressions. For antennas of finite width the integrals are evaluated numerically and for a homogeneous medium the impedance has been correlated with the theories of King and Middleton, Wu, with the impedance computed from a complementary slot antenna and with available measurements. The impedance is also computed for antennas in a dielectric layer, and the decrease of the radiation efficiency observed with increasing electrical thickness of the layer is explained with the increased amounts of surface wave power and in part by local dielectric losses. For insulated antennas located in a dissipative medium the theory presented here yields results

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