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The Economics of Uncertainty VI

by

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Chapter VI

The Bernoulli Principle - Economic Observations and Experiments.

6.1 In Chapter III we indicated how the Bernoulli Principle could be derived as a theorem from three simple axioms. This was a purely mathematical result. The economic significance and the practical usefulness of the result depend on the answers to the following two questions:

(i) Do the axioms hold in practice, i.e. are they observed by people who make decisions under uncertainty?

(ii) If the axioms hold, what is the shape of the utility function which represents the preference ordering of a typical decision maker in the different situations we want to study?

6.2 To illustrate the first point, we shall discuss an example, due to Allais [2].

Allais considers the following two situations:

Situation 1.

We have to choose between the prospects A and B

A will give a certain gain of $1 million

B will give:

Either $5 million with probability 0.10

or $1 million with probability 0.89

or Nothing with probability 0.01

Most people seem to prefer prospect A i.e. to take the million rather than the risk of getting nothing

If this decision is based on a preference ordering, which satisfies our three axioms, there must exist a function u(x), such that

$$u(1) > 0.1\ u(5) + 0.89\ u(1) + 0.01\ u(0)$$

We then consider:

Situation 2.

Here we have to choose between the prospects C and D

C will give:

Either \$1 million with probability 0.11

or nothing with probability 0.89

D will give:

Either \$5 millions with probability 0.1

or nothing with probability 0.9

In this situation most people seem to prefer D .

If this decision is based on the same preference ordering, we must have

$$0.1\ u(5) + 0.9\ u(0) > 0.11\ u(1) + 0.89\ u(0)$$

If we add the two inequalities, we obtain

$$0.1\ u(5) + u(1) + 0.9\ u(0) > 0.1\ u(5) + u(1) + 0.9\ u(0)$$

This is obviously a contradiction, since we have assumed strict preferences and strict inequalities.

If in Situation 2 we had chosen C , a similar argument will give

$$0.11\ u(1) > 0.1\ u(5) + 0.01\ u(0)$$

It is obviously possible to find a function $u(x)$ which satisfies this condition, i.e. which can be interpreted as the utility function representing the preference ordering of the decision maker.

6.3 The example of Allais illustrates two points:

(i) If we have observed that a person chose A in Situation 1, we can predict that he will choose C in Situation 2 - under the assumption that he makes his decision in a rational manner.

(ii) If a person in Situation 1 has chosen A , he must choose C in Situation 2, if he wants to be consistent, i.e. the first choice commits him.

In 1952 Allais prepared a number of examples of this kind, and asked several prominent economists how they would choose in such situations. The questionnaire circulated by Allais has been published [1], but not the systematic analysis of the replies. It is however generally known that leading economists made choices which implied an inconsistent preference ordering.

Savage admits that he was trapped by the example we have quoted, and that he chose A and D . He adds however that when the contradiction was pointed out to him, he reconsidered the problem and reversed his choice in Situation 2 from D to C . Savage states that when he did this, he felt that he corrected an error. ([10] p. 103).

Some people don't seem to feel this way. If a person insists that in Situation 1 he prefers A and in Situation 2 D , there is nothing we can do. We can only note that if he has some general rule for making decisions which leads to these choices in the two situations, this rule cannot satisfy our three axioms, i.e. there are good reasons for calling the rule inconsistent.

Samuelson discussing these axioms recalls a story about an old farmer who considered the whole world as crazy, except himself and his wife. This farmer used to add "some times I am not quite certain about her". Samuelson is tempted to assume that only he himself and Professor Savage are so rational that they will always observe the axioms, but he adds "some times I am not quite certain about myself".

6.4 So far we have taken it for almost self-evident that in Situation 1 "ordinary" rational people would prefer A to B . However is this - on second thought - really so obvious? What would after all an ordinary person do with a million dollars? He would not, and probably could not just spend it. It is likely that he would invest most of the money, and that means really that he would exchange the prospect A for a prospect more like B .

If for instance the person who selected A would spend \$10.000. - and invest the rest in growth stock, he would in reality change A for a prospect of the type:

B':

\$ 3 millions with probability 0.10

\$ 1 million with probability 0.89

\$10.000 with probability 0.01

The point we want to make, is that many people may say they prefer A to B , and defend their choice. Having done this, they may well make decisions, which imply that they prefer B' to A .

It is doubtful if examples of this kind can contribute much to our knowledge about economic behavior under uncertainty. Most people are not used to toss coins or throw dice for millions of dollars, and one should probably not attach too much significance to their statements as to how they would make decisions in such situations. One should at least admit that rational people may well make "mistakes" when they state how they would decide in situations which they never had to consider seriously.

6.5 In order to gain significant knowledge about economic decisions under uncertainty, we can either conduct controlled experiments in laboratory conditions, or we can try to analyse observable economic behavior. In the following we shall explore both these approaches. We shall first quote the conclusions of some of the most shrewd observers of economic activity:

(i) Adam Smith states flatly: "The chance of gain is by every man more or less over-valued, and the chance of loss is by most men under-valued, and by scarce any man who is in tolerable health and spirits, valued more than it is worth." ([12] Book I, Chapter X.) In our terms this means that Adam Smith had observed that most people had a "risk preference" - i.e. that their "attitude to risk" had to be represented by a convex utility function (increasing marginal utility of money). It is worth noting that Adam Smith did not arrive at this conclusion by introspection. He thought that he proved his statement by observing that one can only make a modest profit in insurance, but that one can make a fortune by organizing lotteries.

(ii) One hundred years later, Alfred Marshall comes to exactly the opposite conclusion. He discusses the "evils of uncertainty" and observes that most people are willing to pay quite handsomely to get rid of these evils ([7] Book V, Chapter VII, and Book VI, Chapter VIII). To prove this statement he refers to insurance companies which have "great expenses in advertising and administration", and still make a good profit.

Marshall wrote at the height of the Victorian Age, when lotteries no longer were a part of respectable economics. Marshall's conclusion was that people generally had a "risk aversion", i.e. that the utility of money must be represented by a concave function.

3.6 Most modern economists seem to have accepted Marshall's view. They are quite willing to admit that some people like to gamble, so that risk preference undoubtedly exists, but they do not consider this a serious factor in the economy. The current school of thought is that most respectable people - the people whose opinion matters - have a risk aversion. The evidence one can quote to support this view is quite overwhelming. Casinos may exist, but they are of no real importance in economic life. The economy is essentially made up of "responsible" people who buy insurance and who diversify their investments.

As a counter example against this prevailing view we can consider the development of <u>Premium Bonds</u> in some European countries (England, Norway, Sweden, etc.)

To illustrate this let us assume that a government sells one million bonds, each at $100.-. If the interest rate is 4%, the total annual interest payment will be $4 million - or $4.- to each bond holder. The idea behind the premium bond is that instead of paying $4.- on each bond, a number of bonds, say 400, are drawn at random each year and each of these receive $10,000. All bonds are eventually reimbursed, so the bondholder cannot lose anything except the interest.

This means that if a person holds one orthodox bond, maturing in one year, he will receive $104 with certainty, after one year.

If he holds one Premium Bond, he has a prospect which wil give

$10,100. - with probability 0.0004

$ 100. - with probability 0.9996

A person with risk aversion will prefer the orthodox bond. There is, however, good evidence that many people prefer the premium bond. By issuing bonds of this kind, several European governments have been able to borrow at interest rates considerably lower than the current market rate for orthodox loans.

6.7 We shall not continue these speculations. It is, however, worth noting that the development of the modern welfare states can only be explained by some general risk aversion.

The welfare state seeks to guarantee a decent standard of living to everybody, even if he through bad luck or handicaps is unable to contribute much to the National Income. This means of course that those who by their luck or skill make a large contribution to the National wealth must share with the less fortunate members of the society. If a society, by free choice, and democratic procedure introduces a welfare state, risk aversion must in some way dominate the decision process. There may, however, be a minority of risk lovers who would prefer a society with greater chances and greater risks, and this may explain some of the dissatifaction with the welfare state which often finds eloquent expression in some of the advanced European countries. Complaints about lack of opportunity, are probably inevitable in a society which seeks to provide security for all.

It is generally accepted that insurance will increase "social welfare" - a concept which we shall not define - if people have aversion to risk. It is obvious, as has been suggested by some writers that welfare can be further increased by organizing lotteries - to accomodate people with risk preference.

It is easy to express such vague feelings in general terms, which may contain at least a grain of truth. It is, however, difficult to spell out the assumptions with precision and derive valid conclusions. In the following chapters we shall discuss such problems in more detail, and try to come to grips with the real issues involved.

3.8 Observation of actual economic behavior may not always provide reliable information about what the decision maker really wants. We must admit that people can make the wrong decision "by mistake". This may happen if the choice situation is very complex, or if they do not take enough time and care to analyse the situation.

This leads us to distinguish between the two traditional approaches to our problem.

(i) In business administration one tends to take a normative attitude, i.e. to look for the best possible decision - the decision which intelligent persons like ourselves would make. We may however soon become aware, that finding the best decision may involve more work than it really is worth. We may then settle for something less, and make the decision, which is just good enough, which satisfies certain minimum requirements. This means that instead of being optimizers, we become satisfizers, a term due to Simon [11].

(ii) In general economics we tend to take a more descriptive attitude. We want to find out what rules - if any - business men follow when they make decisions under uncertainty. If we know these rules, we may be able to predict what will happen in the economy as a whole - as the collective outcome of the decisions made by a number of individuals, who together make up the economy.

It may be possible to construct a general theory, based on the assumption that business men follow crazy decision rules - for instance that they cut prices only when the moon is full. It is, however, likely that such a theory would not fit facts, i.e. that the observations we can make in the economy, contradict some conclusion we can derive from the theory.

When we are building an economic theory, it is simplest to assume that people behave rationally, i.e. that they know their own interests and that the actions we observe are precisely the actions which will advance these interests in the best possible manner.

This may not be a very realistic assumption, but it is not easy to replace it by a better one. To say that people do not always act rationally in business, is not a very useful statement. If the statement shall be useful, it must specify when and how, in what circumstances and how often the rules of rationality are broken.

The best - if not the only - way of obtaining such information, seems to be by controlled experiments.

6.9 Before we discuss experimental work, it is necessary to say a few words about <u>subjective probabilities</u>, a concept which we shall study in more detail in Chapter XIII.

Assume that we have to choose between the prospects A and B.

A will give a gain x_1 if the event E_1 occurs.

B will give a gain x_2 if the event E_2 occurs.

Let us further assume that preferences can be represented by a utility function $u(x)$ of unknown form, and let

$$\Pr(E_1) = p_1 \quad \text{and} \quad \Pr(E_2) = p_2$$

If we for some reason prefer A to B , the Bernoulli Principle implies:

$$p_1 \, u(x_1) > p_2 \, u(x_2)$$

If we consider the probabilities as known, this choice gives us some information about the utility function which represents the underlying preference ordering.

If on the other hand, we take the utility function u (x) as known, the observed choice gives us some information about the subjective probabilities assigned to the events E_1 and E_2 .

If for instance $x_1 = x_2$ we conclude from

A preferred to B

that

$$\Pr(E_1) > \Pr(E_2)$$

at least in the mind of the person who made the choice.

Let now:

E_1 = When I toss this coin it will fall heads.

E_2 = Our university team will win its next football game.

x_1 = \$ 10

x_2 = \$ 100

Let us further assume that the person we study knows something about probability, and that he really believes

$$\Pr(E_1) = 1/2$$

If he prefers A to B, we have

$$1/2 \, u(10) > \Pr(E_2) \, u(100)$$

or

$$\Pr(E_2) < \frac{u(10)}{2u(100)}$$

From this we can obviously conclude that the person believes that

$$Pr\ (E_2) < 1/2$$

i.e. that odds are that our team wil lose.

We can further conclude:

(i) Either he thinks there is a very low probability that our team shall win

(ii) or his utility function is very flat.

In general, we can explain observed choices either by assuming that the utility function has a particular shape, or by making assumptions about how a person forms subjective probabilities or "expectations" on the basis of the information available to him.

The Bernoulli Principle makes it possible to separate the two elements in the decision problem.

6.10 The first attempt to measure utility by controlled experiments was made by Mosteller and Nogee [8] in 1950, who studied a group of Harvard undergraduates and some members of the Massachusetts National Guard.

Mosteller and Nogee found that their subjects were not perfectly consistent in their choices, but that the theory in spite of this had a considerable predictive power. They also found that the utility functions of the Harvard students differed significantly from utility functions which represented the preferences of the guardsmen.

We shall not discuss this experiment in further detail. Instead we shall give a brief account of another experiment conducted by Davidson, Suppes and Siegel [5]. This may give some useful indications about the problems we encounter in "experimental economics" - problems which are unfamiliar to most economists, since economics is not usually considered an experimental science.

6.11 The experiment of Davidson, Suppes and Siegel consisted essentially of asking the subjects to bet either "heads" or "tails" in situations like the following:

If you bet heads, you will:

(i) gain 5 cents if right

(ii) lose 5 cents if wrong

If you bet tails, you will:

(i) gain 6 cents if right

(ii) lose 5 cents if wrong

The situation can be represented by the "payoff matrix":

	You bet	
Coin falls	Heads	Tails
Heads	5	- 5
Tails	- 5	6

When the problem is presented in this abstract manner it seems quite obvious that one should bet on tails. This conclusion does, however, rest on the assumption that the test-person believes that the coin is equally likely to fall heads or tails, and that this belief carries through in all his decisions. This is a hypothesis about human behavior, which can - and should - be tested experimently. It is possible that people may have a certain preference for betting "heads", at least in less transparent situations than the one we have considered. Wishful thinking does obviously exist in real life.

Davidson, Suppes and Siegel test this hypothesis, and find that all their subjects behave rationally in this respect, i.e. that they understand when two events are equally probable, and the implications this have.

6.12 To design an experiment in any field requires a considerable amount of care to avoid bias of various kinds. When the results of an experiment are published, it is necessary to give a full description of the design, so that a reader can evaluate how significant these results are. In this respect the book by Davidson Suppes and Siegel is a model of precision, even if their theoretical background can be criticized.

They obtained their subjects through the Stanford University Student Employment Service, among students, who had stated that they were willing to do "relatively unattractive" jobs, such as stapling documents. When a student recruited for such work reported for duty, he was told that the subject for a certain experiment had failed to turn up. He was then asked if he would take part in the experiment instead of stapling documents for two hours at $1 an hour. It was explained to the student that the experiment involved some gambling, and that he might earn less than $2 by two hours participation. He was however assured that the average earning of the participants in the experiment would be more than $2.

The purpose of this cloak and dagger tactics was to obtain a "random sample". If the subjects of the experiment had been chosen from a class in "Decision Theory", or recruited by an "honest" advertisment, one would in all probability have obtained a sample which would not have been representative of the Stanford student population. It is however not obvious that this would have reduced the value of the experiment.

Of 20 students recruited in this way, 19 agreed to take part in the experiment. The 20th preferred to staple documents for two hours, and

and walk away with his $2. He was never heard of again. It is in some sense to be regreted that he was let off so easily. It would have been interesting to have him explain his decision.

6.13 We shall follow Davidson, Suppes and Siegel and use the notation (a,b) for a 50-50 chance of getting either a or b.

As bench-marks the experimenters took a = -4 cents and b = 6 cents. They then determined two numbers c and d, so that

$$(-4, -4) \sim (6, c)$$

and

$$(6, 6) \sim (-4, d)$$

This was done by asking the subjects to select the most preferred from pairs of bets of the type described in paragraph 6.11. The subject had to make a decision, i.e. he was not allowed to state that he was indifferent between two bets. The bets were presented in an order, which did not reveal any systematic pattern.

Since only a finite number of tests could be made, this procedure could only give interval estimates for c and d. If for instance the subject decides that

$$(-4, -4) > (6, -15)$$

$$(-4, -4) < (6, -10)$$

we can only conclude that there is a number c in the interval ($10 \leq c \leq 15$) such that

$$(-4, -4) \sim (6, -c)$$

Similarly we can obtain an interval for the value of d, which satisfies the condition

$$(6,6) \sim (-4, d)$$

In terms of a utility function, this means that

$$\tfrac{1}{2}\ u(6) + \tfrac{1}{2}\ u(6) = \tfrac{1}{2}\ u(-4) + \tfrac{1}{2}\ u(d)$$

or

$$u(d) = 2u(6) - u(-4)$$

A utility function is, as we have seen in Chapter III determined only up to a linear transformation. Hence we can choose two values arbitrarily, for instance our two benchmarks.

This led Davidson, Suppes and Siegel to select the utility function which satisfies the conditions

$$u(-4) = -1 \quad \text{and} \quad u(6) = 1$$

as a convenient representation. From this it follows immediately that

$$u(-c) = -3 \quad \text{and} \quad u(d) = 3$$

When c and d have been determined for a subject, one can determine two other numbers f and g such that

$$(d, f) \sim (6, -c)$$

$$(-c, g) \sim (-4, d)$$

From the first of these relations we obtain

$$u(d) + u(f) = u(6) + u(-c)$$

or

$$u(f) = u(6) + u(-c) - u(d)$$

$$= 1 - 3 - 3 = -5$$

and similarly $u(g) = 5$.

6.14 Of the 19 subjects which took part in the experiment, 4 made decisions which were inconsistent in the sense that the underlying decision rule - if any - could not be represented by a utility function.

The authors ([5] remark page 66) that two of these subjects showed a "considerable disinclination" to gamble, and that it really was a mistake to include them in the experiment. These two students would really have preferred to earn $ 2.- by stapling documents, and should not have been talked into gambling the money which they probably needed.

The other two were very nervous during the experiment. They made a mess of their decisions, and they seemed quite aware of this themselves.

For the remaining 15 persons, the experiment gave four values of the utility function - in addition to the two values which are chosen arbitrarily.

For instance, for Subject 1, the experimentors found:

$$- 18 < f < - 15$$

$$- 11 < c < - 10$$

$$11 < d < 12$$

$$14 < g < 18$$

If we take the middle point of each of the intervals, the utility function of this subject is given by the table to the left.

Subject 1

x	u(x)
- 16.5	- 5
- 10.5	- 3
- 4	- 1
6	1
11.5	3
16	5

Subject 2

x	u(x)
- 32	- 5
- 11.5	- 3
- 4	- 1
6	1
15	3
32.5	5

The table to the right gives the utility function of Subject 2, and it is evident that this function represents a preference ordering quite different from that of Subject 1.

6.15 Our short summary can not do full justice to the well designed experiment of Davidson, Suppes and Siegel. We shall, however, not discuss it any further because this will lead us into a number of problems which are of marginal interest to the subject covered by these lectures. It may, however, be useful to refer the reader to two papers by Becker, De Groot, and Marschak [3] and [4] which provide further refinements of the experimental technique, and to a paper by Marschak [6] which discusses some of the principles involved.

These experiments have a fundamental importance in the sense that they throw light on the basic psychological processes behind the economic decisions made under uncertainty. However, much of this work belongs to experimental psychology rather than to economics. The experiments conducted so far have little direct economic significance, and the authors themselves have never claimed this. During an experiment a student may well behave as if the loss of 20 cents was a minor catastrophe. On the basis of this observed behavior we can construct a utility function which will represent the preference ordering of the student. It is however not very likely that this utility function will make it possible to predict the decisions of the same student when he buys his lunch after the experiment, or when he goes out for a date on the evening.

It is worth noting that one of the papers already referred to [4] observed that behavioral patterns remained substantially the same if subjects were asked to select among hypothetical bets for large amounts of dollars, or if they were offered for cents with promise of real payoff. The shame associated with losing 20 cents in an experiment, which the subject considers a "game of skill", may well be just as great as the loss of utility which the subject assigns to the loss of S|1000.- which he does not have.

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Appendix to Chapter XII.

12.13 In paragraph 12.10 we studied a simple numerical example, and in Table 1 we gave the values of the function V(S,Z) for some integral values of S and S .

In paragraph 12.13 we studied this function for non-integral values of S , and we found the expression:

$$V(S,Z) = V([S],Z) + (S-[S]) \frac{r_1^{[S]+1} - r_2^{[S]+1}}{r_1^{Z+1} - r_2^{Z+1}}$$

which is valid only when Z is an integer. For the sake of completeness we shall derive the corresponding formula for non-integral Z.

12.19 If both S and Z are non-integers, we have to distinguish two cases:

(1) $S - [S] > Z - [Z]$

In this case the first dividend payment will be:

$$(S - [S]) - (Z - [Z])$$

This payment can at the earliest be made after [Z] - [S] periods, and ruin can occur at the earliest after [S] + 1 periods. The generating function for the probabilities that the first dividend shall be paid is then

$$W([S],\ [Z]-1) = \frac{r_1^{[S]+1} - r_2^{[S]+1}}{r_1^{[Z]+1} - r_2^{[Z]+1}}$$

From this it follows that

$$V(S,Z) = (S - [S] - Z + [Z] + V(Z,Z))\ W([S],\ [Z]-1)$$

Noting that V(Z,Z) = V([Z], [Z]), we find that this expression can be written:

$$V(S,Z) = V([S], [Z]) + \{S - [S] - Z + [Z]\} \frac{r_1^{[S]+1} - r_2^{[S]+1}}{r_1^{[Z]+1} - r_2^{[Z]+1}}$$

<u>(ii) $S - [S] < Z - [Z]$</u>

In this case the first dividend payment will be:

$1 + (S - [S]) - (Z - [Z])$

The payment can be made at the earliest after $[Z] - [S] + 1$ periods. By using the same argument as in the preceding case we find.

$$V(S,Z) = V([S], [Z]) + \{S - [S] - Z + [Z]\} \frac{r_1^{[S]+1} - r_2^{[S]+1}}{r_1^{[Z]+2} - r_2^{[Z]+2}}$$

12.20 In paragraph 12.14 we studied $V(S,Z)$ as a function of S for a fixed value of Z. The graph of this function is as indicated by Figure 1.

We found that this function could be interpreted as the utility function governing the company's decisions under uncertainty. We saw that the discontinuities in the function could lead to decisions which might surprise an observer who tried to study the company's attitude to risk, without knowing the long-term prospects of the company.

12.21 Let us now study $V(S,Z)$ as function of Z. Figure 2 shows the graph of this function for $S = 1.4$.

We can now assume that the company holds the capital $S = 1.4$, and that the Board debates whether a dividend should be paid or not.

The Board may decide that the present capital constitutes a reserve which is just sufficient, i.e. that if the capital should increase above $S = 1.4$, the excess will be paid out as dividend. From Figure 2 we see that this policy decision means that the expected discounted value of the company's dividend payments will be

$$V(1.4,\ 1.4) = 2.69$$

FIGURE 1

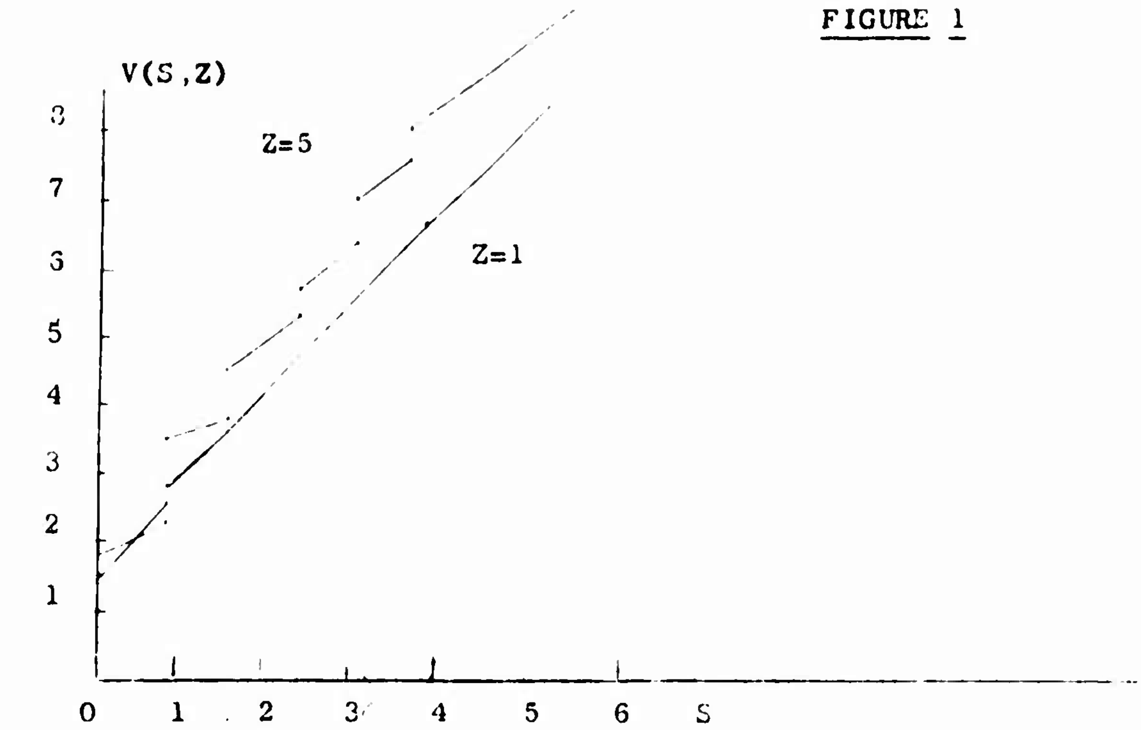
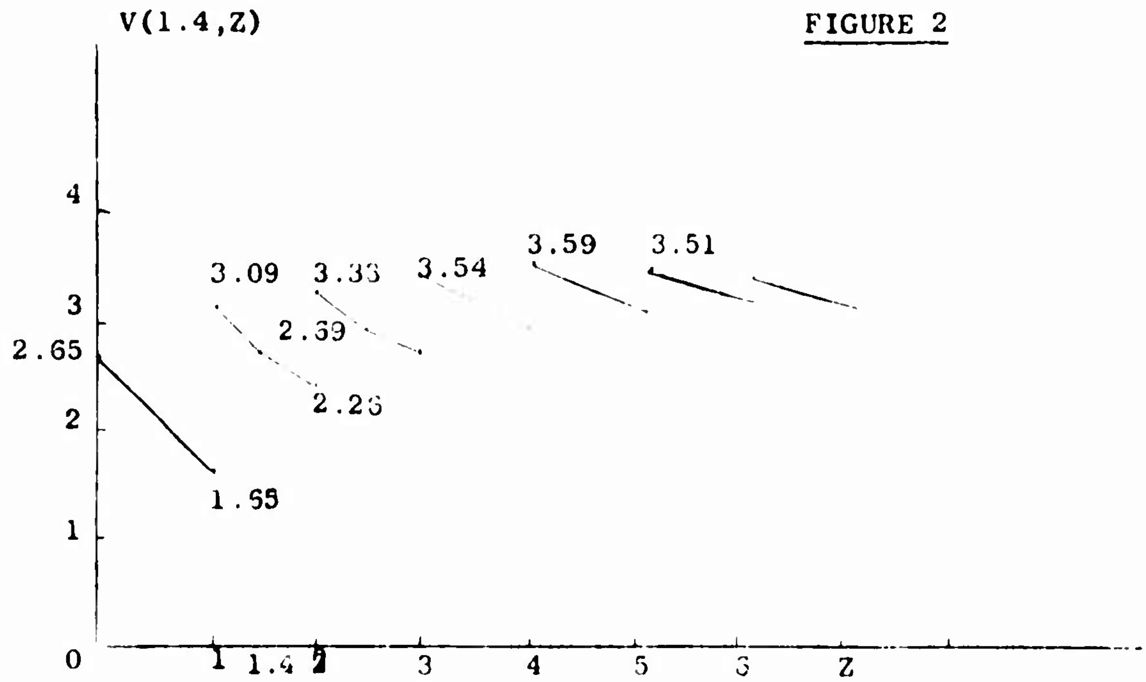


FIGURE 2



The Board may then consider some <u>small</u> changes in this policy. It may for instance propose to pay a dividend of 0.2, and continue operating with a reserve capital $Z = 1.2$. This will increase the expectations to

$$V(1.4,1.2) = 0.2 + V(1.2,1.2) = 2.89$$

The Board may also consider increasing the reserve requirements to $Z = 1.3$, and find that this will reduce dividend expectations to

$$V(1.4,1.6) = 2.47$$

It is easy to see from the Figure that if the Board only considers reserve requirements in the neighborhood of the present capital $S = 1.4$, it may well arrive at the conclusion that the optimal reserve capital is $Z = 1$, giving the dividend expectations

$$V(1.4,1) = 0.4 + V(1,1) = 3.09$$

If however the Board is prepared to consider more ambitious reserve schemes, it should discover that the real optimum may be $Z = 4$, with the corresponding expectations

$$V(1.4,4) = 3.59$$

12.22 This simple example provides an example of how a company can be led to make the "wrong" decision by using techniques of analysis which give the nearest <u>local optimum</u>. Such mistakes are probably made in business, particularly by economists who are fond of calculus and marginal analysis. We shall therefore study how the situation may develop after an initial mistake of this kind.

In our example the company will pay a dividend of 0.4, and continue operating with the declared policy that whenever the capital exceeds 1, the excess shall be paid out as dividend.

If the company is not ruined, its capital will sooner or later increase to S =2. The company may then stick to its declared policy and pay a dividend of 1. The company may however discover that the local optimum technique indicates that it will pay to change the policy.

It is easy to see from Table I in paragraph 12.10 that dividend expectations will be:

(i) By adhering to the established policy

$$V(2,1) = 1 + V(1,1) = 3.69$$

(ii) By changing reserve requirements to Z = 2

$$V(2,2) = 4.19$$

If the company in this situation changes its policy, the same thing may happen again, and bring the company to increase its idea of reserve requirements to 3 and 4. However at this point the process will stop. Dividend cannot be increased by setting reserve requirements higher than 4. This is evident from Table 1.

12.23 When we observe actual business behavior, we may well find processes which - by some stretch of imagination - resemble the one we have outlined. Economists, aided by psychologists, may then think up strange theories to explain these observations. They may for instance suggest that companies tend to become more greedy as they become richer, i.e. the more capital they accumulate, the more capital will they retain as "necessary" reserves. Such exotic theories may contain a grain of truth, but are they really necessary? The simplest explanation is that the companies we observe are not able to handle the mathematics of their decision problem.