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UNSTEADY AERODYNAMICS FOR ADVANCED CONFIGURATIONS

PART I—APPLICATION OF THE SUBSONIC KERNEL FUNCTION TO NONPLANAR LIFTING SURFACES

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FOREWORD

This report covers the research conducted by the Space and Information Systems Division of North American Aviation, Inc., Downey, California, for the Aerospace Dynamics Branch, Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract No. AF 33(657)-10399.

The work was performed to advance the state of the art of flutter prevention for flight vehicles as part of the Research and Technology Division, Air Force Systems Command's exploratory development program. This research was conducted under Project No. 1370 "Dynamic Problems in Flight Vehicles," and Task No. 137003, "Prediction and Prevention of Dynamic Aerothermoelastic Instabilities." Mr. James Olsen of the Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory was the Project Engineer.

Mr. L. V. Andrew was the Program Manager for North American Aviation. Mr. H. T. Vivian, under the guidance of Dr. H. Ashley, laid out the form of the solution and wrote the computer program. Dr. E. R. Rodemich made many significant contributions to the numerical analysis.

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This technical documentary report has been reviewed and is approved.

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ABSTRACT

In this part, equations for pressure distributions and generalized aerodynamic forces are derived for a thin nonplanar lifting surface in simple harmonic motion at subsonic speeds. A digital computer program, written in Fortran IV, is also presented herein. The computer program will generate up to a ten by ten matrix of generalized aerodynamic forces when given data for the geometry of a planar lifting surface with a folded planar tip, the flight Mach number, the reduced frequency of motion, and some control constants. Control surface deflections are not accounted for in this study.

The kernel function method given by Watkins, Runyan, and Woolston (Reference 1), which relates the pressure distribution to the downwash on a planar lifting surface, has been extended and applied to a nonplanar lifting surface. Hsu's technique (Reference 4) of employing Gaussian quadrature formulas is used when integrating the product of the kernal function and the lift function over the planform area.

Recommendations are made to extend the method to account for blunted leading edges and the accompanying airfoil thickness and to account for control surface deflections.

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SYMBOLS

a	Local speed of sound (constant in linear theory)
b_o	Root semichord
b (s)	Local semichord
C_p	Pressure coefficient
$\overline{C}_p$	Time independent factor of oscillatory part of C_p
i, j, k	Unit vectors parallel to the coordinate axes
i	$\sqrt{-1}$
k	Reduced frequency, $\omega b_o / U_\infty$
k_1	kr_1
K	Kernel function
K_0, K_1, K_2	Modified Bessel functions of the second kind
M	Local Mach number (same as free stream Mach number, M_∞, in linear theory)
n	Coordinate measured normal to wing surface
n(x, s, t)	Contribution to n of elastic deformation of the wing
n_t (x, s)	Contribution to n of wing thickness
$\overline{n}$ (x, s)	Time independent factor of n(x, s, t)
p	Pressure
$\vec{q}$	Fluid velocity
R	Gas constant
R	$\sqrt{(x-\xi)^2 + \beta^2 r_1^2}$

List of Symbols continued on next page.

r_1	$\sqrt{(y-\eta)^2 + (z-\zeta)^2}$
s	Curvilinear coordinate on the wing (page 10)
T	Absolute temperature
t	Time
u, ℓ	Subscripts indicating upper and lower wing surfaces
u, v, w	Components of perturbation velocity
U, W	x- and z-components of $\vec{V}$
U_∞	$\lvert\vec{V}\rvert$ (speed at infinity)
$\vec{V}$	Uniform fluid velocity at infinity
$\overline{W}$	Time independent factor of normal velocity
x, y, z	Cartesian coordinates
x_o	$x-\xi$
y_o	$y-\eta$
β	$\sqrt{1-M^2}$
γ	Ratio of specific heats
$\gamma(s)$	Local angle between wing surface and xy-plane
$\Delta\bar{p}$	Time independent factor of pressure difference between wing surfaces, $\bar{p}_\ell - \bar{p}_u$
ξ, η, ζ	Cartesian coordinates
$\bar{\xi}$	Coordinate on the wing (page 22)
ϕ	Velocity potential
φ	Perturbation velocity potential
$\bar{\varphi}$	Time independent factor of φ
$\bar{\psi}$	Acceleration potential
ρ	Fluid density
ω	Angular frequency (radians per unit time)

I. INTRODUCTION

The first published numerical method for solving the subsonic pressure distribution problem for planar lifting surfaces undergoing simple harmonic motion was developed at NASA's Langley Research Center by Watkins, Runyan, and Woolston (Reference 1). Watkins, et al., presented two methods of handling the numerical integration of the kernel function in the region where high-order singularities exist. Both methods involved a dense concentration of integration points in the neighborhood of the singularity. Using these methods, it is possible to obtain downwash integrals, in terms of the pressure-loading coefficients, at any arbitrary set of points on the surface (e.g., at all the kinematic downwash points known from previously determined vibration mode data). However, in order to reduce the running time on the computer, the downwash integrals were obtained at a selected set of collocation points, such as those at intersections of quarter, half, and three-quarter chord stations and like half-span stations. When downwashes were matched exactly (and thus, boundary conditions) at these collocation points, responsibility was placed upon the user to evaluate the kinematic downwashes there. A least-square error surface fitted to the mode data was commonly used to evaluate them. Furthermore, if the user desired that the boundary conditions be satisfied at a greater number of points, it was necessary that he use a correspondingly greater number of loading functions.

Procedures were then described by Rodden and Revell (Reference 2) and the correct form of the equations were presented by Fromme (Reference 3) for calculating pressure-loading coefficients which match a greater number of kinematic downwashes than coefficients, in the sense that the sum of squares of amplitudes of differences of complex numbers are minimized. Since it was still the responsibility of the user to evaluate the kinematic downwashes at the collocation point, least-square error procedures were used twice: once implicitly and once explicitly.

Hsu (Reference 4) significantly advanced the logical development of the kernel function approach when he established an optimum set of collocation and integration points. He started with the previously established chordwise pressure functions based on steady-state, two-dimensional, incompressible aerodynamics, and with spanwise loading functions, based on steady-state lifting-line theory. He concluded that there is sufficient reason to believe that these functions display the proper characteristics near the edges of lifting surfaces oscillating in a compressible fluid.

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Returning to the two-dimensional case, Hsu established that if the chordwise distribution of modal deflections (and thus downwashes) is accurately represented by a polynomial of degree 2N-1 and is approximated by a polynomial of degree N-1, then the integral for the sectional load is evaluated with zero error by a N-point Gaussian quadrature if the difference between the accurate and the approximate representation of the downwashes is made equal to zero at each of the N points (i.e., the chordwise collocation stations). Conversely, still for the two-dimensional case, Hsu established that if the product of the pressure function and the kernel, divided by the Jacobi-Gauss weight factor (which produces the square root singularity at the leading edge), is accurately represented by a polynomial of degree 2N-1, then the integral for the downwash at any one of the collocation stations is evaluated with zero error by a N-point Gaussian quadrature. These N points are then made the chordwise integration stations.

For the spanwise direction, using lifting-line theory, Hsu similarly established M-spanwise collocation stations and M + 1 interdigitated spanwise integration stations plus the conditions under which the Gaussian quadrature can be used with zero error.

It is important to note that the kernel of the integral equation for the downwashes in unsteady, three-dimensional, compressible flow cannot be accurately represented by a polynomial of finite degree. It is equally important to note, however, that, because of the edge characteristics of the pressure and loading functions, the Gaussian quadratures employed at Hsu's optimum point set evaluate the integrals with the least squared error for a given number of integration points. We have yet to match the boundary conditions using Hsu's method.

The downwash matching problem in Hsu's approach is basically the same as in Watkin's approach; we merely have a more logical choice of points at which to match them. In the examples Hsu used to demonstrate his approach, he chose to use the same number of pressure-loading functions as collocation points. However, the approach is not dependent upon that choice. If a smaller number of pressure-loading functions are used, then the procedures described by Rodden, Revell, and Fromme may be used to compute pressure-loading coefficients which yield a minimum sum of squares of amplitudes of differences in downwashes.

A need has arisen for application of the kernel function method to non-planar lifting surfaces on future aerospace vehicles. Application is also required to more conventional non-planar surfaces such as T-tail, V-tail, and wing-vertical tail combinations.

Professor H. Ashley outlined the application to the folded tip configuration. A computer program based on Ashley's work was developed for steady-state flow by L. Johnson, et al., of the Los Angeles Division of North American Aviation, Inc.

The work reported herein is based on Professor Ashley's outline. However, the expression for the kernel has been greatly simplified by Dr. E. R. Rodemich of North American Aviation, Inc., Space and Information Systems Division.

II. FUNDAMENTAL EQUATIONS OF FLUID MOTION

Consider a body immersed in a compressible, nonviscous, perfect fluid and assume the fluid flow to be isentropic and irrotational. Under these conditions, a velocity potential ϕ, exists:

$$\vec{q} = \nabla\phi \tag{1}$$

where $\vec{q}$ is the velocity vector of a fluid element and ∇ is the gradient operator (See Reference 4.) Also under these conditions, the isentropic (constant entropy) pressure-density relationship is valid. Thus,

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s = \gamma RT \tag{2}$$

Other equations which govern the flow are the continuity equation for conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\vec{q}) = 0 \tag{3}$$

and Euler's equations for conservation of momentum

$$\frac{D\vec{q}}{Dt} = \frac{-1}{\rho} \nabla p \tag{4}$$

where, $\frac{D}{Dt}$, the substantial derivative, is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{q} \cdot \nabla \tag{5}$$

These equations may be combined, as described in Reference 5, to yield the nonlinear, unsteady flow equation

$$\nabla^2 \phi - \frac{1}{a^2} \left[\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial q^2}{\partial t} + (\vec{q} \cdot \nabla) \frac{q^2}{2} \right] = 0 \tag{6}$$

Consider, then, that the fluid motion consists of a perturbation superimposed on a uniform stream velocity $\vec{V} = Ui + Wk$ parallel to the xz-plane of a rectangular Cartesian coordinate system. Then the velocity potential may be expressed as the sum of a uniform part and a perturbation part

$$\phi = Ux + Wz + \varphi \tag{7}$$

and, similarly, the velocity vector becomes

$$\vec{q} = \vec{V} + \nabla\varphi = \nabla\phi \tag{8}$$

The pressure coefficient at any point in an isentropic flow field is

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} \tag{9}$$

where $p - p_\infty$ is the difference between local pressure and free-stream pressure, $U_\infty = |\vec{V}|$, and $1/2\, \rho_\infty U_\infty^2$ is the free-stream dynamic pressure. From Kelvin's equation (Reference 5) for isentropic flow

$$C_p = \frac{2}{\gamma M_\infty^2} \left\{ \left[1 + \frac{\gamma - 1}{2} M_\infty^2 \left(1 - \frac{\vec{q}\cdot\vec{q} + 2\frac{\partial\varphi}{\partial t}}{U_\infty^2} \right) \right]^{\frac{\gamma}{\gamma-1}} - 1 \right\} \tag{10}$$

A complete statement of the fundamental problem requires specification of the boundary conditions. The boundary conditions at infinity depend upon the free-stream velocity. When it is less than the speed of sound in the fluid, the disturbances to the flow die out and are not felt at infinity. When it is greater than the sonic speed, then in the region where disturbances are felt, even at infinity, the component of flow due to the disturbance is directed away from the source of disturbance and otherwise the free-stream flow is undisturbed. The boundary conditions at the surface of the body require that the flow be tangent to the surface everywhere on the body. This condition is satisfied by the equation

$$\frac{D}{Dt} B(x, y, z, t) = 0 \tag{11}$$

where

$$B(x, y, z, t) = 0 \tag{12}$$

is the equation for the position of the surface at any time t, and the substantial derivative D/Dt is defined by Equation 5.

III. LINEARIZED EQUATIONS OF MOTION

Linearization of the equations of motion is not dependent upon an explicit form of the body equation, Equation 12, so long as the normal derivatives of the equation are everywhere nearly perpendicular to the free-stream direction. Thin lifting surfaces at small angle of attack satisfy this condition and are treated herein and in Parts 2 and 4 of this report. The special considerations required for thick bodies and high angles of attack are treated in Parts 3 and 5. The following development is, therefore, restricted to thin airfoils.

We first obtain the specialized form of Equation 7 when the uniform stream velocity lies along the x-axis; i. e., $W = 0$ and, therefore, $\vec{V} = Ui$, $U_\infty = U$. The velocity potential is

$$\phi = Ux + \varphi \tag{13}$$

and the velocity vector of a fluid element becomes

$$\vec{q} = (U + u)\, i + vj + wk \tag{14}$$

where

$$u = \frac{\partial \varphi}{\partial x},\ v = \frac{\partial \varphi}{\partial y},\ \text{and } w = \frac{\partial \varphi}{\partial z}$$

The perturbation velocities u, v, and w are assumed to be much smaller than the free-stream velocity; i. e., $u,\ v,\ w \ll U$.

The linearization procedure when applied to Equation 10 yields the fully linearized pressure coefficient

$$Cp = -\frac{2}{U_\infty^{\,2}}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\varphi \tag{15}$$

and when applied to Equation 6 yields the fully linearized unsteady flow equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} - \frac{1}{a_\infty^2}\left[U^2 \frac{\partial^2 \varphi}{\partial x^2} + 2U\frac{\partial^2 \varphi}{\partial x \partial t} + \frac{\partial^2 \varphi}{\partial t^2}\right] = 0 \tag{16}$$

Next, we write the body equation for a thin, nonplanar lifting surface (Figure 1), in terms of a curvilinear coordinate

$$s = s(y)$$

s represents the integral of distance along the line of the mean position of the airfoil from the centerline to y,

$$s(y) = \int_o^y ds \tag{17}$$

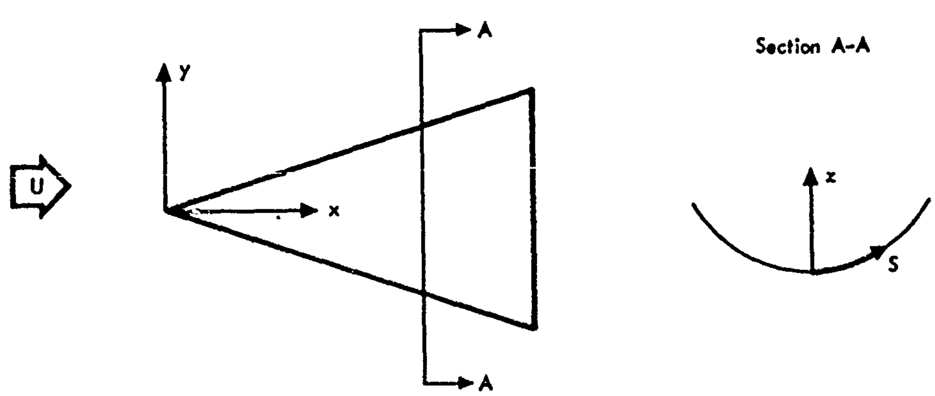


Figure 1. Generalized Curvilinear Planform

The position of the surface in terms of s and n (the normal to S), is separated into two parts; one part for the upper (or inner) surface, and the other for the lower (or outer) surface

$$B_u\ (x,\ n,\ s,\ t) = n - n_T(x,\ s) - n\ (x,\ s,\ t) \tag{18a}$$

$$B_\ell\ (x,\ n,\ s,\ t) = n + n_T(x,\ s) - n\ (x,\ s,\ t) \tag{18b}$$

where $n_T(x,\ s)$ represents the thickness of the airfoil and $n(x,\ s,\ t)$ represents the elastic deflection of the airfoil. In accordance with Equation 11, we use the operator

$$\nabla = i\frac{\partial}{\partial x} + j'\frac{\partial}{\partial s} + k'\frac{\partial}{\partial n} \tag{19}$$

on Equation 18a to get

$$\nabla B_u(x,\ n,\ s,\ t) = -i\left[\frac{\partial}{\partial x}n_T(x,\ s) + \frac{\partial}{\partial x}n(x,\ s,\ t)\right]$$
$$-j'\left[\frac{\partial}{\partial s}n_T(x,\ s) + \frac{\partial}{\partial s}n(x,\ s,\ t)\right] + k'$$

Substitution into

$$\frac{D}{Dt}B_u(x,\ s,\ n,\ t) = \frac{\partial B_u}{\partial t} + (Ui + \nabla\varphi)\cdot\nabla B_u = 0 \tag{20}$$

of the equation for the upper surface, after higher order terms have been discarded, gives

$$\frac{\partial\varphi}{\partial n} = \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)n(x,\ s,\ t) + U\frac{\partial}{\partial x}n_T(x,\ s) \tag{21}$$

The same procedure, for the lower surface, gives

$$\frac{\partial \varphi}{\partial n} = \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) n\ (x,\ s,\ t) - U\frac{\partial}{\partial x}\ n_T(x,\ s) \tag{22}$$

Finally, we restrict the analysis to that class of problems in which the effects of thickness on the time dependent forces can be neglected. By letting

$$n_T(x,\ s) = 0 \tag{23}$$

we get a single expression for the boundary condition

$$\frac{\partial \varphi}{\partial n} = \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) n\ (x,\ s,\ t) \tag{24}$$

It is evident from Equation 24 that, when the motion of the surface is simple harmonic motion,

$$n(x,\ s,\ t) = \bar{n}\ (x,\ s)\ e^{i\omega t} \tag{25}$$

then,

$$\varphi\ (x,\ s,\ n,\ t) = \bar{\varphi}\ (x,\ s,\ n)\ e^{i\omega t} \tag{26}$$

Substitution of Equations 25 and 26 into Equations 15, 16, and 24 gives

$$\bar{C}_p = -\frac{2}{U_\infty^{\ 2}}\ \frac{D\bar{\varphi}}{Dt} \tag{27}$$

$$\nabla^2 \bar{\varphi} = \frac{1}{a_\infty^{\ 2}}\ \frac{D^2\bar{\varphi}}{Dt^2} \tag{28}$$

$$\frac{\partial \bar{\varphi}}{\partial n} = \frac{D\bar{n}}{Dt} \tag{29}$$

where

$$\frac{D}{Dt} \equiv U \frac{\partial}{\partial x} + i\omega \tag{30}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial n^2} \tag{31}$$

To this degree of approximation, $M = M_\infty$ and $a = a_\infty$.

IV. THE ACCELERATION POTENTIAL

PLANAR WINGS

Equation 27 shows that the calculation of pressure requires taking derivatives of the velocity potential, and Equation 29 states the boundary condition which must be satisfied. Watkins, Runyan and Woolston (Reference 1) solved this problem for planar surfaces in terms of a series of pressure functions, or acceleration potential functions ($\bar{\psi}$), where,

$$\bar{\psi}(x, y, z) = \frac{D}{Dt}\,\bar{\phi}(x, y, z) \tag{32}$$

Integration of Equation 32 gives the general expression for the velocity potential in terms of the acceleration potential:

$$\bar{\phi}(x, y, z) = \frac{1}{U}\, e^{-i\omega \frac{x}{U}} \int_{-\infty}^{x} e^{i\omega \frac{\lambda}{U}}\, \bar{\psi}(\lambda, y, z)\, d\lambda \tag{33}$$

if the velocity at infinity is $\vec{V} = Ui$. The velocity potential ϕ due to a pulsating doublet satisfies Equation 28 (in the planar case $s = y$ and $n = z$); and, because the order of operators is interchangeable, the acceleration potential ψ also satisfies Equation 28.

A complete discussion of the application of boundary conditions in the planar case is given in Section 6-4 of Reference 5. The application in the nonplanar case is discussed in less detail in the following text.

NONPLANAR WINGS

In the nonplanar case, the acceleration potential at a point (x, s, N) due to a pulsating doublet located at the point (ξ, σ, n), or (ξ, η, ζ) in the direction of n is

$$\bar{\psi}(x, s, N) = -A\,\frac{\partial}{\partial n}\left\{\frac{e^{i\omega\left[\frac{M}{a_\infty \beta^2}(x-\xi) - \frac{R}{a_\infty \beta^2}\right]}}{R}\right\} \tag{34}$$

where

$$\frac{\partial}{\partial n} = \cos\gamma(\eta)\frac{\partial}{\partial \zeta} - \sin\gamma(\eta)\frac{\partial}{\partial \eta} \tag{35}$$

$$R = \sqrt{(x-\xi)^2 + \beta^2\left[(y-\eta)^2 + (z-\zeta)^2\right]} \tag{36}$$

and $\gamma(\eta)$ is angle between the wing and the xy-plane at the point (ξ, η, ζ). The perturbation velocity potential may be built up from a distribution of doublets of acceleration potential over the wing. If A is the infinitesimal doublet strength at (ξ, σ, η), the contribution to $\bar{\varphi}$ from the doublet at this point, from Equations 33 and 34, is

$$\Delta\bar{\varphi}(x,s,N) = \frac{-A}{U}\frac{\partial}{\partial n} e^{-i\omega\frac{x-\xi}{U}} \int_{-\infty}^{x-\xi} \frac{e^{i\omega\left[\frac{\lambda}{U} + \frac{M\lambda}{a_\infty \beta^2} - \frac{R'}{a_\infty \beta^2}\right]}}{R'} d\lambda \tag{37}$$

where

$$R' = \sqrt{\lambda^2 + \beta^2\left[(y-\eta)^2 + (z-\zeta)^2\right]}$$

and the velocity component normal to the surface at (x, s, N) is

$$\Delta\bar{w}(x,s,0) = \lim_{N \to 0} \frac{\partial}{\partial N} \Delta\bar{\varphi}(x,s,N) \tag{38}$$

where

$$\frac{\partial}{\partial N} = \cos\gamma(y)\frac{\partial}{\partial z} - \sin\gamma(y)\frac{\partial}{\partial y} \tag{39}$$

Note that when the operator $\partial/\partial n$ is applied in Equation 37, the partials $\partial/\partial\zeta$ and $\partial/\partial\eta$ may be replaced by $-\partial/\partial z$ and $-\partial/\partial y$, respectively. Substitution of Equation 37 into Equation 38 gives

$$\Delta\bar{w}(x,s,0) = \frac{A}{U} e^{-i\omega\frac{x-\xi}{U}} \lim_{N \to 0} P \int_{-\infty}^{x-\xi} \frac{e^{i\omega(\lambda - MR')/U\beta^2}}{R'} d\lambda \tag{40}$$

where the operator P is

$$P = \left[\cos\gamma(y)\frac{\partial}{\partial z} - \sin\gamma(y)\frac{\partial}{\partial y}\right]\left[\cos\gamma(\eta)\frac{\partial}{\partial z} - \sin\gamma(\eta)\frac{\partial}{\partial y}\right] \quad (41)$$

and finally

$$A = \frac{\Delta\bar{p}\,d\xi\,d\sigma}{4\pi\rho} \quad (42)$$

and $\Delta\bar{p}$ is the complex amplitude of the difference in pressure on the upper and lower sides of the surface at (ξ, σ),

$$\Delta\bar{p}(\xi, \sigma) = \bar{p}_u(\xi, \sigma) - \bar{p}_\ell(\xi, \sigma) \quad (43)$$

and $d\xi\,d\sigma$ is the incremental area of the doublet sheet.

The normal wash at $(x, s, 0)$ given by Equation 40 is that due to a point pressure doublet at $(\xi, \sigma, n = 0)$. The total normal wash is the integral over the surface of all the pressure doublets,

$$\frac{\bar{w}(x, s, 0)}{U} = \frac{-1}{4\pi\rho U^2} \oint\!\!\oint \Delta\bar{p}(\xi, \sigma)\, K(x - \xi, s, \sigma, \omega, M)\, d\xi\, d\sigma \quad (44)$$

where $\oint$ denotes the Mangler formula for evaluating infinite integrals (Reference 7), and the kernel of the integral equation is (omitting the arguments ω, M for brevity)

$$K(x_o, s, \sigma) = \lim_{\substack{n \to 0 \\ N \to 0}} \left\{ e^{-i\frac{\omega x_o}{U}} P \int_{-\infty}^{x_o} \frac{e^{i\omega(\lambda - MR')/U\beta^2}}{R'} d\lambda \right\} \quad (45)$$

where

$$R' = \sqrt{\lambda^2 + \beta^2 r_1^2}$$

$$r_1 = \sqrt{y_o^2 + z_o^2}$$

and

$$x_o = x - \xi,\ y_o = y - \eta, \text{ and } z_o = z - \zeta$$

Now, by putting $k_1 = \omega r_1/U$ and $v = \lambda/\beta r_1$; then, by putting $u = -\frac{v - M\sqrt{1 + v^2}}{\beta}$ the integral in Equation 45 may be written

$$I_o \equiv \int_{-\infty}^{x_o} \frac{e^{i\omega(\lambda - MR')/U\beta^2}}{R'} d\lambda = \int_{u_1}^{\infty} \frac{e^{-ik_1 u}}{\sqrt{1 + u^2}} du \tag{46}$$

where

$$u_1 = -\frac{x_o - MR}{\beta^2 r_1}$$

By breaking up the interval of integration into three subintervals and in the first two integrals letting $u = w/i$

$$I_o = -i \int_0^1 \frac{e^{-k_1 w}}{\sqrt{1 - w^2}} dw + \int_1^{\infty} \frac{e^{-k_1 w}}{\sqrt{w^2 - 1}} dw - \int_0^{u_1} \frac{e^{-ik_1 u}}{\sqrt{1 + u^2}} du$$

or,

$$I_o = K_o(k_1) - i \int_0^1 \frac{e^{-k_1 w}}{\sqrt{1 - w^2}} dw - \int_0^{u_1} \frac{e^{-ik_1 u}}{\sqrt{1 + u^2}} du \tag{47}$$

where $K_o(k_1)$ is the modified Bessel function of the second kind and zeroth order of the argument k_1.

To obtain the analytic form of the kernel, we write the operator P (Equation 41) in a more convenient form, taking advantage of the fact that I_o is a function of only r_1 when x_o is held constant

$$PI_o = \cos\left[\gamma(y) - \gamma(\eta)\right] \left(\frac{1}{r_1} \frac{\partial I_o}{\partial r_1}\right)$$

$$+ \left[z_o \cos\gamma(y) - y_o \sin\gamma(y)\right] \left[z_o \cos\gamma(\eta) - y_o \sin\gamma(\eta)\right] \left(\frac{1}{r_1} \frac{\partial}{\partial r_1}\right)\left(\frac{1}{r_1} \frac{\partial I_o}{\partial r_1}\right)$$

The resulting kernel is

$$\bar{\bar{K}}(x_o, s, \sigma) = s_{TIP}{}^2 \, K(x_o, s, \sigma)$$

$$= e^{-i\frac{\omega x_o}{U}} \left\{ \frac{T_1 K_1(x_o, s, \sigma) + T_2 K_2(x_o, s, \sigma)}{\underline{r}_1{}^2} \right\} \quad (48)$$

where

$$T_1 = \cos\,[\,\gamma(\underline{s}) - \gamma(\underline{\sigma})]$$

$$T_2 = \left[\frac{\underline{z}_o}{\underline{r}_1}\cos\gamma(s) - \frac{\underline{y}_o}{\underline{r}_1}\sin\gamma(\underline{s})\right]\left[\frac{\underline{z}_o}{\underline{r}_1}\cos\gamma(\underline{\sigma}) - \frac{\underline{y}_o}{\underline{r}_1}\sin\gamma(\underline{\sigma})\right] \quad (48a)$$

$$K_1(x_o, s, \sigma) = -k_1 K_1(k_1) - \frac{x_o}{R} e^{-k_1 u_1} + ik_1 \int_0^1 \frac{w\,e^{-k_1 w}}{\sqrt{1 - w^2}}\,dw$$

$$+ ik_1 \int_0^{u_1} \frac{u e^{-ik_1 u}}{\sqrt{1 + u^2}}\,du$$

$$K_2(x_o, s, \sigma) = k_1{}^2 K_2(k_1) + \left(\frac{2x_o}{R} + \frac{\beta^2 r_1{}^2 x_o}{R^3} + i\,\frac{k_1 x_o(Mr_1 + Ru_1)}{R^2}\right) e^{-ik_1 u_1}$$

$$- ik_1 \int_0^1 \frac{w e^{-k_1 w}}{\sqrt{1 - w^2}}\,dw - ik_1{}^2 \int_0^1 \frac{w^2 e^{-k_1 w}}{\sqrt{1 - w^2}}\,dw$$

$$- ik_1 \int_0^{u_1} \frac{u e^{-ik_1 u}}{\sqrt{1 + u^2}}\,du + k_1{}^2 \int_0^{u_1} \frac{u^2 e^{-ik_1 u}}{\sqrt{1 + u^2}}\,du \quad (48b)$$

$$r_1 = s_{TIP}\sqrt{\underline{y}_o^{\,2} + z_o^{\,2}}$$

$$R = \sqrt{x_o^{\,2} + \beta^2 r_1^{\,2}}$$

$$u_1 = -\frac{x_o - MR}{\beta^2 r_1}$$

$$k_1 = \frac{\omega r_1}{U} \qquad (48c)$$

and $K_1(k_1)$ and $K_2(k_1)$ are modified Bessel functions of the second kind and first and second orders. The sub-bar indicates division by s_{TIP}, e.g., $\underline{r}_1 = r_1/s_{TIP}$.

V. THE BOUNDARY CONDITIONS

The remainder of the problem is to match the boundary conditions; i.e., to find a pressure-loading function $\Delta\bar{p}(\xi, \sigma)$ which, when inserted into Equation 45 and integrated over the surface, yields the kinematic downwashes at selected points on the surface w(x, s, 0).

In subsonic flow, the behavior of the pressure distribution is known in the area of the wing edges from a few of the exact solutions in lifting surface theory. In the neighborhood of the leading edge, the pressure should behave as

$$\lim_{\delta \to 0} \sqrt{\frac{1}{\delta}}$$

In the neighborhood of the trailing edge and all edges parallel to the free-stream direction, the pressure should behave as

$$\lim_{\delta \to 0} \sqrt{\delta}$$

where δ is the distance to the wing edge. Both Hsu and Watkins employ a linear superposition of functions that satisfy these conditions. Hsu's function differs from Watkins only in that for any given number of terms in the series, Hsu's terms are linear combinations of Watkins terms. We use a normalized form of the function given by Watkins:

$$\Delta\tilde{p}(\underline{\xi}, \underline{\sigma}) \equiv \frac{\rho U^2/2}{b(\sigma)} \sqrt{1 - \underline{\sigma}^2} \sum_{n=0}^{N} \sum_{m=0}^{M} a_{nm}\, \underline{\sigma}^m f_n(\tilde{\xi}) \tag{49}$$

where

$$f_o(\tilde{\xi}) \equiv \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}$$

$$f_n(\tilde{\xi}) \equiv \sqrt{1 - \tilde{\xi}^2}\, U_n(\tilde{\xi});\ 1 \le n$$

$$U_1(\tilde{\xi}) = 1.0$$

$$U_2(\tilde{\xi}) = -2\tilde{\xi}$$

$$U_n(\tilde{\xi}) = -(2\tilde{\xi}\ U_{n-1} + U_{n-2});\ 3 \le n$$

and

$$\tilde{\xi} \equiv \left(\xi - \frac{\xi_{LE} + \xi_{TE}}{2}\right)/b(\sigma)$$

$$b(\underline{\sigma}) = \frac{\xi_{TE} - \xi_{LE}}{2}$$

The a_{nm}'s are unknown pressure coefficients to be determined by matching the kinematic downwashes at the selected points $(x_j,\ s_r)$ on the surface. Substitution of Equation 49 into Equation 44 leads to the matrix equation given by Rodden and Revell (Reference 2) Equation 39, for the point set $x_j,\ s_r$

$$\left\{\frac{\overline{w}_i}{U}\right\} = \left[D^i_{nm}\right] \left\{a_{nm}\right\} \tag{50}$$

where, in this case,

$$D^i_{nm} = \frac{1}{8\pi} \int_{-1}^{1} \sqrt{1 - \underline{\sigma}^2}\ \underline{\sigma}^m \int_{-1}^{1} f_n(\tilde{\xi})\overline{\overline{K}}(x_j - \xi, \underline{s}_r, \underline{\sigma})\, d\tilde{\xi}\, d\underline{\sigma} \tag{51}$$

We now reexamine the fundamentals of the problem before proceeding to evaluate the integrals in Equation 51 and thence to solve Equation 50.

One of the basic reasons for development of the kernel function method is that pressure distributions over a continuous lifting surface are smooth continuous functions that can be represented with reasonable accuracy by a series of analytic functions. We point out that $f_n(\xi)$ can be written

$$f_o(\tilde{\xi}) = \frac{1 - \tilde{\xi}}{\sqrt{1 - \tilde{\xi}^2}}$$

$$f_n(\tilde{\xi}) = \frac{1 - \tilde{\xi}^2}{\sqrt{1 - \tilde{\xi}^2}}\ U_n(\tilde{\xi});\ 1 \ge n$$

and, therefore, the inner integral in Equation 51 may be written

$$I_1 = \int_{-1}^{1} \frac{P_n(\tilde{\xi})}{\sqrt{1 - \tilde{\xi}^2}} d\tilde{\xi} \tag{52}$$

where

$$P_o(\tilde{\xi}) = (1 - \tilde{\xi})\overline{\overline{K}}(x_j - \xi, \underline{s}_r, \underline{\sigma})$$

$$P_n(\tilde{\xi}) = (1 - \tilde{\xi}^2)U_n(\tilde{\xi})\overline{\overline{K}}(x_j - \xi, \underline{s}_r, \underline{\sigma});\ 1 \leq n$$

Now, we assume for the moment that the kernel $\overline{\overline{K}}(X_j - \xi, s_r, \sigma)$ can be represented with reasonable accuracy by a polynomial in $\tilde{\xi}$. Then, $P_n(\tilde{\xi})$ is also a polynomial in ξ, and the Chebyshev-Gauss quadrature formula may be used to obtain the exact value of the integral expression (52); i.e.,

$$\int_{-1}^{1} \frac{P_n(\tilde{\xi})}{\sqrt{1 - \tilde{\xi}^2}} d\tilde{\xi} = \sum_{k=1}^{K} \frac{\pi}{K} P_n(\tilde{\xi}_k) + E \tag{52a}$$

where

$$E = \frac{2\pi}{2^{2K}(2K)!} P_n^{(2K)}(\lambda)$$

and,

$$|\lambda| < 1.0$$

The error term E is zero if P_n is a polynomial of degree $\leq 2K - 1$.

A more accurate formula which utilizes the fact that $P_n(1.0) = 0$ is used by Hsu (and by us)

$$f_o(\tilde{\xi}) = \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}$$

$$f_n(\tilde{\xi}) = \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}} (1 + \tilde{\xi}) U_n(\tilde{\xi})$$

Then, the inner integral in Equation 51 may be written

$$I_1 = \int_{-1}^{1} \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}\, F_n(\tilde{\xi})\, d\tilde{\xi} \tag{53}$$

where

$$F_o(\tilde{\xi}) = \bar{\bar{K}}(x_j - \xi, \underline{s}_r, \underline{\sigma})$$

$$F_n(\tilde{\xi}) = (1 + \tilde{\xi})\, U_n(\tilde{\xi})\, \bar{\bar{K}}(x_j - \xi, \underline{s}_r, \underline{\sigma}),\ 1 \le n$$

This is evaluated by the L-point Jacobi-Gauss quadrature with the weight function $\sqrt{(1 - \tilde{\xi})/(1 + \tilde{\xi})}$ (see Reference 8, Chapter 8). The resulting formula

$$I \cong \sum_{k=1}^{L} W_k F_n(\tilde{\xi}_k) \tag{54}$$

is exact if $F_n(\tilde{\xi})$ is a polynomial of degree $\le 2L-1$, which corresponds to degree 2L for $P_n(\tilde{\xi})$. Putting $\tilde{\xi} = -\cos\theta$, the polynomials

$$\phi_m(\tilde{\xi}) = \frac{\cos\left(m + \frac{1}{2}\right)\theta}{\cos\frac{1}{2}\theta}$$

are orthogonal with respect to the weight function $\sqrt{(1 - \tilde{\xi})/(1 + \tilde{\xi})}$

$$\int_{-1}^{1} \phi_m(\tilde{\xi})\phi_n(\tilde{\xi}) \sqrt{\frac{1 - \tilde{\xi}}{1 + \xi}}\, d\tilde{\xi} = \begin{cases} 0, & m \ne n \\ \pi, & m = n \end{cases}$$

This is easily verified by expressing the integral in terms of θ. Referring to formulas in Reference 8, Chapter 8, Section 8.4, it can be shown, using these polynomials, that Equation 54 takes the form

$$\int_{-1}^{1} \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}\, F_n(\tilde{\xi})d\tilde{\xi} = \frac{2\pi}{2L + 1} \sum_{k=1}^{L} H_k F_n(\tilde{\xi}_k) \tag{55}$$

where

$$H_k = (1 - \tilde{\xi}_k)$$

and

$$\tilde{\xi}_k = -\cos\left(\frac{2k - 1}{2L + 1}\ \pi\right)$$

In two-dimensional, steady, incompressible flow, there is an optimum set of chordwise collocation stations $(\tilde{x}_i)$ for the determination of sectional lift, depending upon the order of the polynomial required to adequately represent the downwash distribution

$$\tilde{x}_j = -\cos\left(\frac{2j}{2N + 1}\ \pi\right),\ j = 1, 2, \ldots . N. \tag{56}$$

Since the behavior of the integrand for the chordwise loading is apt to exhibit similar characteristics near the surface edges, it is inferred that this set should also yield the best approximation in three-dimensional, unsteady, compressible flow. Note that the number of collocation points is not required to be the same as the number of integration points. As will be seen later, it is only necessary that the total number of downwash collocation points be equal to or greater than the number of pressure coefficients a_{nm}. When N chordwise integration stations are used, the quadrature used to evaluate the inner integral of Equation 51 is exact for integrands represented by a polynomial of degree $\leq 2N - 1$.

Hsu shows that an optimum set of interdigitated spanwise collocation stations and integration stations exists for evaluation of the outer integral in Equation 51. By reasoning similar to that used to establish the chordwise collocation stations, it was established that the optimum spanwise collocation stations are

$$\underline{s}_r = -\cos\frac{r}{M + 1}\ \pi\ ,\ r = 1, 2, \ldots M. \tag{57}$$

It was observed that the quadrature for the integral of difference between the actual and polynomial approximation of the spanwise loading is zero when the actual loading is precisely represented by a polynomial of degree $\leq 2M - 1$, and the polynomial approximation is of degree $= N - 1$.

Then, by substitution of Equation 55 into Equation 51,

$$D^i_{nm} = \frac{1}{8\pi} \int_{-1}^{1} \frac{\sqrt{1 - \underline{\sigma}^2}}{(\underline{s}_r - \underline{\sigma})^2} G_{nm} (\tilde{x}_j, \underline{s}_r, \underline{\sigma}) \, d\underline{\sigma} \tag{58}$$

where

$$G_{om} = \frac{2\pi}{2L+1} \sum_{k=1}^{L} \underline{\sigma}^m (1 - \tilde{\xi}_k) (\underline{s}_r - \underline{\sigma})^2 \overline{\overline{K}} (x_j - \xi_k, \underline{s}_r, \underline{\sigma})$$

(59)

$$G_{nm} = \frac{2\pi}{2L+1} \sum_{k=1}^{L} \underline{\sigma}^m (1 - \tilde{\xi}_k^2) U_n (\tilde{\xi}_k) (\underline{s}_r - \underline{\sigma})^2 \overline{\overline{K}} (x_j - \xi_k, \underline{s}_r, \underline{\sigma}); \; 1 \le n$$

Hsu established the form of the Gaussian quadrature and the spanwise integration stations. The difficulties of the singularity of the kernel at $\underline{\sigma} = \underline{s}_r$ and the difficulty of differentiation with respect to $\underline{\sigma}$ (he uses the steady-state lifting line formula to derive the form of the quadrature) are avoided by removal of the singularity at $\underline{\sigma} = \underline{s}_r$ and then by an integration by parts.

We first integrate by parts to get

$$D^i_{nm} = \frac{1}{8\pi} \int_{-1}^{1} \frac{\frac{\partial}{\partial \underline{\sigma}} \left[\sqrt{1 - \underline{\sigma}^2} \; G_{nm} (\tilde{x}_j, \underline{s}_r - \underline{\sigma}, \omega, M) \right]}{(\underline{s}_r - \underline{\sigma})} \, d\underline{\sigma}$$

which corresponds to Equation 58 in Reference 9. The Gaussian quadrature formula is developed and shows that when the number of integration stations is one greater than the number of collocation stations, and if they are interdigitated in the prescribed way

$$D^i_{nm} = \frac{1}{8\pi} \left\{ \sum_{p=1}^{M+1} \frac{\pi}{M+1} \frac{(1 - \underline{\sigma}_p^2)\, G_{nm}(\tilde{x}_j, \underline{s}_r, \underline{\sigma}_p)}{(\underline{s}_r - \underline{\sigma}_p)^2} - \pi (M+1)\, G_{nm}(\tilde{x}_j, \underline{s}_r, \underline{s}_r) \right\} \quad (60)$$

where

$$\underline{s}_r = -\cos \frac{r\pi}{M+1}$$

and

$$\underline{\sigma}_p = -\cos \frac{2p-1}{2(M+1)} \pi$$

$$r = 1, 2, \ldots M$$

Evaluation of the second term in the brackets requires the observation that the multiplier of K_2 in Equation 48 goes to zero whenever the collocation point is in the plane of the doublet sheet located at the integration point. Therefore, the finite part of the integral of the K_2 term is zero, and the entire contribution comes from the K_1 term. In this case, $\gamma(\underline{s}) = \gamma(\underline{\sigma})$ and $\underline{r}_1^2 = (\underline{s}_r - \underline{\sigma})^2 = 0.$

It can be shown that

$$K_1(x - \xi, \underline{s}_r, \underline{s}_r) = \begin{cases} -2, & x > \xi \\ 0, & x < \xi . \end{cases}$$

Thus, the chordwise integral which defines $G_{nm}(x_j, \underline{s}_r, \underline{s}_r)$ is

$$G_{nm}(x_j, \underline{s}_r, \underline{s}_r) = -2 \int_{-1}^{\tilde{x}_j} \underline{\sigma}^m f_n(\tilde{\xi})\, e^{-i\frac{\omega}{U}(x_j - \xi)}\, d\tilde{\xi}$$

If the range of integration is extended to $\tilde{\xi} = 1$ by making the integrand zero for $\tilde{\xi} > \tilde{x}_j$, the integral cannot be well approximated by a polynomial because it has a jump discontinuity at $\tilde{\xi} = \tilde{x}_j$. To overcome this difficulty, we write

$$G_{nm}(x_j, \underline{s}_r, \underline{s}_r) = -2 \int_{-1}^{\tilde{x}_j} \underline{\sigma}^m f_n(\tilde{\xi}) \left[e^{-i \frac{\omega}{U}(x_j - \xi)} - 1 \right] d\tilde{\xi}$$

$$-2 \int_{-1}^{\tilde{x}_j} \underline{\sigma}^m f_n(\tilde{\xi})\, d\tilde{\xi}. \tag{61a}$$

The second term here depends on the integrals

$$h_n(\tilde{x}_j) = \int_{-1}^{\tilde{x}_j} f_n(\tilde{\xi})\, d\tilde{\xi}$$

which may be evaluated exactly. We have

$$h_0(\tilde{x}_j) = \int_{-1}^{\tilde{x}_j} \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}\, d\tilde{\xi} = \frac{\pi}{2} + \sin^{-1} \tilde{x}_j + \sqrt{1 - \tilde{x}_j^{\,2}}$$

$$h_1(\tilde{x}_j) = \int_{-1}^{\tilde{x}_j} \sqrt{1 - \tilde{\xi}^2}\, d\tilde{\xi} = \frac{1}{2}\left(\frac{\pi}{2} + \sin^{-1} \tilde{x}_j + \tilde{x}_j \sqrt{1 - \tilde{x}_j^{\,2}}\right)$$

This list may be extended as far as it is needed.

The first integral in Equation 61a is considered as an integral over $-1 < \tilde{\xi} < 1$ with zero integrand when $\tilde{\xi} > \tilde{x}_j$, and Equation 55 is applied. The result is

$$G_{om}(x_j, \underline{s}_r, \underline{s}_r) = -2 \cdot \frac{2\pi}{2L + 1} \sum_{\substack{k \geq 1, \\ \tilde{\xi}_k < \tilde{x}_j}} \underline{\sigma}^m (1 - \tilde{\xi}_k) \left[e^{-i \frac{\omega}{U}(x_j - \xi_k)} - 1 \right]$$

$$- 2\, \underline{\sigma}^m g_o(\tilde{x}_j) \tag{61b}$$

$$G_{nm}(x_j, \underline{s}_r, \underline{s}_r) = -2 \frac{2\pi}{2L+1} \sum_{\substack{k \geq 1, \\ \tilde{\xi}_k < \tilde{x}_j}} \underline{\sigma}^m (1 - \tilde{\xi}_k^2) U_n(\tilde{\xi}_k) \left[e^{-i \frac{\omega}{U}(x_j - \xi_k)} - 1 \right]$$

$$- 2\underline{\sigma}^m g_n(\tilde{x}_j), \quad n \geq 1.$$

Equations 59 and 61 are used to evaluate the D^i_{nm}'s given by Equation 60. Equation 50 is then used to determine the pressure coefficients a_{nm} to match the downwashes (i.e. the $\overline{w}_i/U$'s) at the collocation points (x_j, s_r). Once the pressure coefficients are determined, the generalized forces are computed. A polynomial expression for the i^{th} modal deflections normal to the surface,

$$n^{(i)}(\xi, \underline{\sigma}) = \sum_{\nu=0}^{N} \sum_{\mu=0}^{M} b_{\nu\mu}^{(i)} \xi^\nu \underline{\sigma}^\mu$$

and Equation 49 are substituted into the equation

$$Q_{ij} = \int_{-s_{TIP}}^{s_{TIP}} \int_{x_{LE}}^{x_{TE}} n^{(i)}(\xi, \underline{\sigma}) \, \Delta\overline{p}^{(j)}(\tilde{\xi}, \underline{\sigma}) \, d\xi \, d\sigma$$

to get

$$Q_{ij} = s^2_{TIP} (1/2 \, \rho U^2) \sum_{n=0}^{N} \sum_{m=0}^{M} \sum_{\nu=0}^{N} \sum_{\mu=0}^{M} a_{nm}^{(j)} \, b_{\nu\mu}^{(i)} \, \Delta Q_{nm\nu\mu} \qquad (62)$$

where

$$\Delta Q_{nm\nu\mu} = \int_{-1}^{1} \int_{-1}^{1} \sqrt{1 - \underline{\sigma}^2} \, \underline{\sigma}^{m+u} \, \xi^\nu \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}} \, F_n(\tilde{\xi}) \, d\tilde{\xi} \, d\underline{\sigma}$$

The quadrature formula given by Equation 55 with L set equal to N may be used to evaluate the inner integral. For evaluation of the outer integral, a quadrature formula with the weight function $\sqrt{1 - \underline{\sigma}^2}$, analogous to Equation 60 for a nonsingular integral, may be used. This is not practical for a wing with folded tip, for which a different representation should be used on each plane part of the surface. Here, the spanwise integral over one part of the wing may have a square root factor at one end of the interval of integration, or at neither end. Such integrals may best be evaluated by a suitable application of ordinary Gaussian quadrature with a weight function of 1.0. The points and weights for this quadrature method are not given by simple formulas such as Equation 55. They are listed in many places, (e.g. Reference 5).

VI. APPLICATION TO A WING WITH A FOLDED TIP

The planform may be any continuous surface. However, the computer program was developed to treat either a planar or nonplanar planform of the type shown in Figure 2. (Only one-half the planform is shown.) To facilitate modifications of the program, comment cards are placed throughout the program to indicate where changes may be made to handle other nonplanar surfaces like that of the Paraglider.

In the application of the kernel function method to the planform shown in Figure 2, the computer program calculates from the equations of the leading and trailing edges the collocation and integration points for which the integrands of Equation 51 must be evaluated. For demonstration purposes, we calculate the collocation and integration points for values of L = 4, N = 6, and M = 10 and show the results in Figure 3.

$$x_{LE} = s \tan\lambda_{LE}, \quad x_{LETIP} = s_{F_L} \tan\lambda_{LE} + (s - s_{F_L})\tan\lambda_{LETIP}$$

$$x_{TE} = 2b_o + s\tan\lambda_{TE}, \quad x_{TETIP} = 2b_o + s_{F_L} \tan\Lambda_{TE} + (s - s_{F_L})\tan\lambda_{TETIF}$$

$$b(s) = 1/2\,(x_{TE} - x_{LE})$$

$$x_m = 1/2\,(x_{LE} + x_{TE})$$

$$x = x_m + b(s)\,\tilde{x}$$

$$\xi = \xi_m + b(\sigma)\,\tilde{\xi}$$

$$\tilde{x}_j = -\cos\frac{2j}{2N+1}\pi, \quad j = 1, 2, \ldots N.$$

$$\tilde{\xi}_k = -\cos\frac{2k-1}{2L+1}\pi, \quad k = 1, 2, \ldots L.$$

$$s = \underline{s}\, s_{TIP}$$

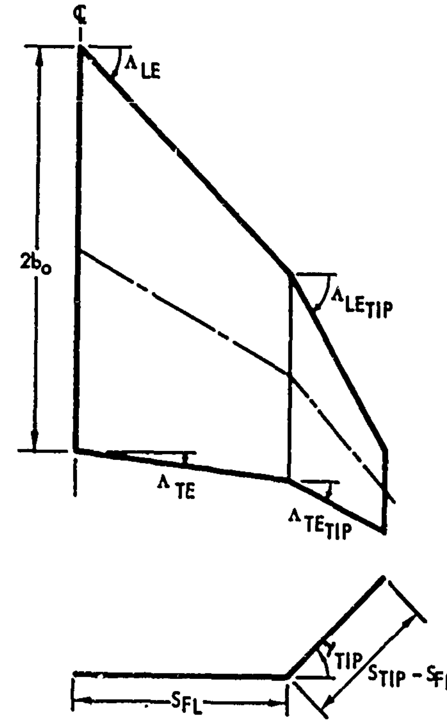


Figure 2. Planar Wing With Planar Symmetrically Folded Tips

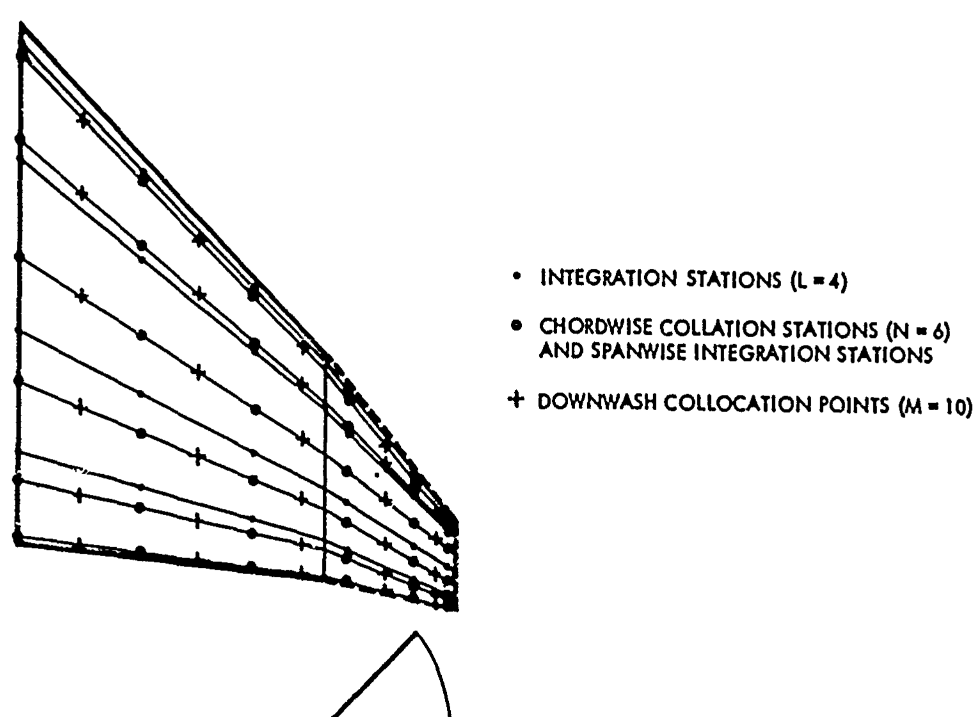


Figure 3. An Optimum Set of Collocation and Integration Points

$$\underline{s}_m = -\cos\frac{m\pi}{M+1}, \quad m = 1, 2, \ldots, M.$$

$$\underline{\sigma}_p = -\cos\frac{2p-1}{2(M+1)}\pi, \quad p = 1, 2, \ldots, (M+1).$$

We have also constructed a table of equations (Table 1), for the functions y_o, z_o, r_1, T_1, and T_2 for use in the expression for the kernel, Equations 48 through 48c. In Table 1, the subscript F_L indicates the fold line.

A difficulty is encountered in the spanwise integration because the kernel function has a finite discontinuity at the fold line. For example, note in Table 1 the change in T_1 and T_2 for receiving or collocation points on the wing as the sending or integration points shift from the port tip to the wing and from the wing to the starboard tip.

If we consider the kernel as a function of $\tilde{\xi}$ and $\underline{\sigma}$ for fixed values of x and $\underline{s}$,

$$q(\tilde{\xi}, \underline{\sigma}) = \bar{\bar{K}}(x - \xi, \underline{s}, \underline{\sigma}),$$

this function may be broken up into a simple discontinuous part g^{**} and a part g^* which is continuous across the fold lines:

$$g(\tilde{\xi}, \underline{\sigma}) = g^*(\tilde{\xi}, \underline{\sigma}) + g^{**}(\tilde{\xi}, \underline{\sigma}) \qquad (63)$$

To do this, define

$$g^{**}(\tilde{\xi}, \underline{\sigma}) = \begin{cases} g(\tilde{\xi}, \underline{\sigma}_{F_L}+) - g(\tilde{\xi}, \underline{\sigma}_{F_L}-), & \underline{\sigma} > \underline{\sigma}_{F_L} \\ 0, & -\underline{\sigma}_{F_L} < \underline{\sigma} < \underline{\sigma}_{F_L} \\ g(\tilde{\xi}, -\underline{\sigma}_{F_L}-) - g(\tilde{\xi}, -\underline{\sigma}_{F_L}+), & \underline{\sigma} < -\underline{\sigma}_{F_L} \end{cases} \qquad (64)$$

and then define g^* by Equation 63.

More explicitly, for $\underline{\sigma} > \underline{\sigma}_{F_L}$, define

$$\xi_{F_L} = b(\underline{\sigma}_{F_L})\tilde{\xi} + \frac{1}{2}\left[\xi_{LE}(\underline{\sigma}_{F_L}) + \xi_{TE}(\underline{\sigma}_{F_L})\right]$$

Table 1. Spanwise Parameter in the Kernel Function

Functions	Receiving Points (x, s)								
	Port Tip $S < -y_{F_L}$ and $\gamma(S) = -\gamma_{TIP}$			Wing $-y_{F_L} < S < y_{F_L}$ and $\gamma(S) = 0$			Starboard Tip $y_{F_L} < S$ and $\gamma(S) = \gamma_{TIP}$		
	Sending Points (ξ, σ)								
	PT $\gamma(\sigma) = -\gamma_{TIP}$	W $\gamma(\sigma) = 0$	ST $\gamma(\sigma) = \gamma_{TIP}$	PT $\gamma(\sigma) = \gamma_{TIP}$	W $\gamma(\sigma) = 0$	ST $\gamma(\sigma) = \gamma_{TIP}$	PT $\gamma(\sigma) = -\gamma_{TIP}$	W $\gamma(\sigma) = 0$	ST $\gamma(\sigma) = \gamma_{TIP}$
y_o	$s_o\not{c}$	$-\sigma^* + s^*\not{c}$	$(s^* - \sigma_*)\not{c} - 2y_{F_L}$	$s^* - \sigma^*\not{c}$	s_o	$s_* - \sigma_*\not{c}$	$(s_* - \sigma^*)\not{c} + 2y_{F_L}$	$-\sigma_* + s_*\not{c}$	$s_o\not{c}$
z_o	$-s_o\not{s}$	$-s^*\not{s}$	$-(s + \sigma)\not{s}$	$\sigma^*\not{s}$	0	$-\sigma_*\not{s}$	$(s_* + \sigma^*)\not{s}$	$s_*\not{s}$	$s_o\not{s}$
r_1	$\lvert s_o \rvert$	$\sqrt{s^{*2} + \sigma^{*2} - 2s^*\sigma^*\not{c}}$	$\sqrt{\left[(s^* - \sigma_*)\not{c} - 2y_{F_L}\right]^2 + \left[(s^* + \sigma_*)\not{s}\right]^2}$	$\sqrt{s^{*2} + \sigma^{*2} - 2s^*\sigma^*\not{c}}$	$\lvert s_o \rvert$	$\sqrt{s_*^2 + \sigma_*^2 - 2s_*\sigma_*\not{c}}$	$\sqrt{\left[(s_* - \sigma^*)\not{c} + 2y_{F_L}\right]^2 + \left[(s_* + \sigma^*)\not{s}\right]^2}$	$\sqrt{s_*^2 + \sigma_*^2 - 2s_*\sigma_*\not{c}}$	$\lvert s_o \rvert$
T_1	1.0	$\not{c}$	$\not{c}^2 - \not{s}^2$	$\not{c}$	1.0	$\not{c}$	$\not{c}^2 - \not{s}^2$	$\not{c}$	1 0
T_2	0	$s^*\sigma^*\not{s}^2$	$4(\sigma_*\not{c} + y_{F_L})(s^*\not{c} - y_{F_L})\not{s}^2$	$s^*\sigma^*\not{s}^2$	0	$s_o\sigma_*\not{s}^2$	$4(\sigma^*\not{c} - y_{F_L})(s_*\not{c} + y_{F_L})\not{s}^2$	$s_*\sigma_*\not{s}^2$	0

$s^* = s + y_{F_L}$, $\sigma^* = \sigma + y_{F_L}$, $s_* = s - y_{F_L}$, $\sigma_* = \sigma - y_{F_L}$, $s_o = s - \sigma$, $\not{s} = \sin\gamma_{TIP}$, $\not{c} = \cos\gamma_{TIP}$

Then

$$g^{**}(\tilde{\xi},\underline{\sigma}) = \overset{=}{K}^{ST}(x - \xi_{F_L}, \underline{s}, \underline{\sigma}_{F_L}) - \overset{=}{K}^{W}(x - \xi_{F_L}, \underline{s}, \underline{\sigma}_{F_L})$$

in which

$\gamma(\underline{\sigma}) = \gamma_{TIP}$ in $\overset{=}{K}^{ST}$ (ST ~ starboard tip)

$\gamma(\underline{\sigma}) = 0$ in $\overset{=}{K}^{W}$ (W ~ wing)

A similar formula applies when $\underline{\sigma} < -\underline{\sigma}_{F_L}$. g^{**} may be written in a form which indicates that it is independent of σ in each tip region:

$$g^{**}(\tilde{\xi},\underline{\sigma}) = \begin{cases} g^{**}{}_{ST}(\tilde{\xi}), & \underline{\sigma} > \underline{\sigma}_{F_L} \\ 0, & -\underline{\sigma}_{F_L} < \underline{\sigma} < \underline{\sigma}_{F_L} \\ g^{**}{}_{PT}(\tilde{\xi}), & \underline{\sigma} < -\underline{\sigma}_{F_L} \end{cases} \tag{65}$$

With the use of Equations 63 and 65, Equation 51 may be rewritten as

$$D^{i}_{mn} = \frac{1}{8\pi} \int_{-1}^{1} \sqrt{1 - \underline{\sigma}^2}\, \underline{\sigma}^m \int_{-1}^{1} f_n(\tilde{\xi})\, g^{*}(\tilde{\xi},\underline{\sigma})\, d\tilde{\xi}\, d\underline{\sigma}$$

$$+ \frac{1}{8\pi} \int_{-1}^{1} f_n(\tilde{\xi})\, g^{**}_{ST}(\tilde{\xi}) \int_{\underline{\sigma}_{F_L}}^{1} \sqrt{1 - \underline{\sigma}^2}\, \underline{\sigma}^m d\underline{\sigma}\, d\tilde{\xi}$$

$$+ \frac{1}{8\pi} \int_{-1}^{1} f_n(\tilde{\xi})\, g^{**}_{PT}(\tilde{\xi}) \int_{-1}^{-\underline{\sigma}_{F_L}} \sqrt{1 - \underline{\sigma}^2}\, \underline{\sigma}^m\, d\underline{\sigma}\, d\tilde{\xi}$$

The first of these three double integrals is calculated according to Equation 60. In the others, the inner integral may be evaluated exactly. The constants

$$u_m = \int_{\sigma_{F_L}}^{1} \sqrt{1 - \sigma^2}\, \sigma^m \, d\sigma$$

are given by formulas

$$u_0 = \frac{1}{2}\left[\frac{\pi}{2} - \sin^{-1}\sigma_{F_L} + \sigma_{F_L}\sqrt{1 - \sigma_{F_L}^2}\right]$$

$$u_1 = \frac{1}{3}\left(1 - \sigma_{F_L}^2\right)^{3/2}$$

etc.

In terms of these constants

$$D^i_{mn} = \frac{1}{8\pi}\int_{-1}^{1}\sqrt{1 - \sigma^2}\,\sigma^m \int_{-1}^{1} f_n(\tilde{\xi})\, g^*(\tilde{\xi},\sigma)\, d\xi\, d\sigma$$

$$+ \frac{1}{8\pi} u_m \int_{-1}^{1} f_n(\tilde{\xi})\left[g^{**}_{ST}(\tilde{\xi}) + (-1)^m g^{**}_{PT}(\tilde{\xi})\right] d\tilde{\xi}$$

The last integral in this formula is evaluated by Equation 55.

In the evaluation of O_{ij}, given by Equation 62 for the plane case, it is assumed that modes numbers i and j are either both symmetric in σ, or both antisymmetric. Then the contribution to the integral for $\sigma < 0$ is the same as the contribution for $\sigma > 0$. Let the deflection in the i^{th} mode be given by

$$n^{(i)}(\xi,\sigma) = \begin{cases} \sum_{\nu=0}^{N}\sum_{\mu=0}^{M} c^{(i)}_{\nu\mu}\, \xi^\nu \sigma^\mu, & 0 < \sigma < \sigma_{F_L} \\ \sum_{\nu=0}^{N}\sum_{\mu=0}^{M} d^{(i)}_{\nu\mu}\, \xi^\nu \sigma^\mu, & \sigma > \sigma_{F_L} \end{cases}$$

Then

$$O_{ij} = 2 \cdot y_{TIP}^2 \left(\frac{1}{2} \rho U^2\right) \sum_{n=0}^{N} \sum_{m=0}^{M} \sum_{\nu=0}^{N} \sum_{\mu=0}^{M}$$

$$a_{nm}^{(j)} \left\{ c_{\nu\mu}^{(i)} I_{nm\nu\mu}^{(1)} + d_{\nu\mu}^{(i)} I_{nm\nu\mu}^{(2)} \right\}$$

in which

$$I_{nm\nu\mu}^{(1)} = \int_0^{\sigma_{F_L}} \sqrt{1 - \underline{\sigma}^2}\, \underline{\sigma}^{m+\mu} \int_{-1}^{1} \xi^{\nu} \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}\, F_n(\tilde{\xi}) d\tilde{\xi} d\underline{\sigma}$$

$$I_{nm\nu\mu}^{(2)} = \int_{\sigma_{F_L}}^{1} \sqrt{1 - \underline{\sigma}^2}\, \underline{\sigma}^{m+\mu} \int_{-1}^{1} \xi^{\nu} \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}\, F_n(\tilde{\xi}) d\tilde{\xi} d\underline{\sigma}$$

Note that the inner integrals are the integrals of polynomials in $\tilde{\xi}$ multiplied by $\sqrt{(1 - \tilde{\xi})/(1 + \tilde{\xi})}$, by virtue of the relation

$$\xi = b(\underline{\sigma})\tilde{\xi} + \frac{1}{2}\left[\xi_{LE}(\underline{\sigma}) + \xi_{TE}(\underline{\sigma})\right]$$

Hence, the inner integral may be evaluated exactly by either Equation 52a or Equation 55 if enough points are used in the formulas. For the limits $\nu \leq 5$, $n \leq 4$ used in the computer program, six points are sufficient for Equation 52a, five points for Equation 55. Equation 52a was used with $K = 6$. (This choice was made arbitrarily; it would be just as good to use Equation 55.)

In the integrations over $\underline{\sigma}$, six-point Gaussian integration with weight function 1.0 was used. The basic formula is

$$\int_0^1 f(v)\,dv = \sum_{\ell=1}^{6} h_{\ell} f(v_{\ell}) \tag{66}$$

exact for f(v) a polynomial of degree at most 11. The constants occurring in this formula are given in the subroutine FORCE. They may be derived from a table given by Scarborough (Reference 6, p. 148).

In $I^{(1)}_{nm\nu\mu}$, Equation 66 is applied by putting

$$\sigma = \sigma_{F_L} v$$

The resulting expression is

$$I^{(1)}_{nm\nu\mu} = \frac{\pi}{6}\sigma_{F_L}\sum_{\ell=1}^{6} h_\ell \sqrt{1-\sigma_\ell^2}\,\sigma_\ell^{m+\mu}\sum_{k=1}^{6}\xi_k(\sigma_\ell)^\nu(1-\tilde{\xi}_k^2)F_n(\tilde{\xi}_k)$$

in which

$$\sigma_\ell = \sigma_{F_L} v_\ell$$

$$\xi_k(\sigma_\ell) = b(\sigma_\ell)\tilde{\xi}_k + \frac{1}{2}\left[\xi_{LE}(\sigma_\ell) + \xi_{TE}(\sigma_\ell)\right]$$

In $I^{(2)}_{nm\nu\mu}$, the transformation

$$\sigma = 1 - (1-\sigma_{F_L})v^2$$

was used. This makes the v-integrand behave like a polynomial at the ends of the interval. The resulting formula is

$$I^{(2)}_{nm\nu\mu} = \frac{\pi}{3}(1-\sigma_{F_L})\sum_{\ell=1}^{6} h_\ell v_\ell \sqrt{1-\sigma_\ell^2}\,\sigma_\ell^{m+\mu}\sum_{k=1}^{6}$$

$$\xi_k(\sigma_\ell)^\nu(1-\tilde{\xi}_k^2)F_n(\tilde{\xi}_k)$$

in which

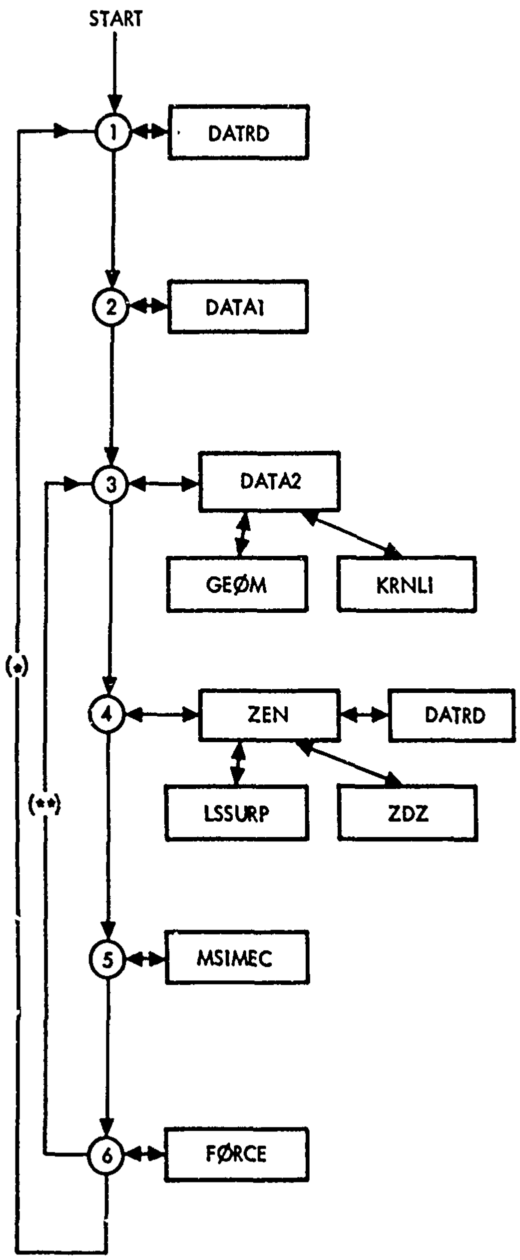
$$\sigma_\ell = 1 - (1-\sigma_{F_L})v_\ell^2$$

$$\xi_k(\sigma_\ell) = b(\sigma_\ell)\tilde{\xi}_k + \frac{1}{2}\left[\xi_{LE}(\sigma_\ell) + \xi_{TE}(\sigma_\ell)\right]$$

DESCRIPTION OF THE COMPUTER PROGRAM

A functional diagram of the computer program is given in Figure 4. With the exception of two subroutines named MSIMEC and MSIMER, all of the programs are written in Fortran IV. These two subroutines, written in machine language, are used for complex and real matrix inversion, respectively. There are certain limitations related to the various other subprograms, which are listed below.

Subprogram	Limitations
Subroutine Data	
NCC	The number of chordwise collocation stations must be ≤ 10.
NCS	The number of spanwise collocation stations must be ≤ 9.
NDATA	The number of sets of data must be ≤ 10.
N	The number of chordwise pressure modes ≤ 5.
M	The number of spanwise pressure modes ≤ 5.
Subroutine Zen	
MODES	This is the number of modes used in the calculation of generalized forces, and must be ≤ 10.
NPTS	For a planar wing, this is the number of points at which the deflection is given in the horizontal surface and must be ≤ 66. For a nonplanar wing, there must be ≤ 66 points for the deflections in the horizontal surface and ≤ 66 points for the deflections in the vertical surface.



1. The first set of data is read. This includes items 1-14 in the data array, the Mach numbers and frequencies, item 41, and perhaps some of the mode data that will be needed later.

2. Subroutine DATA1 performs most of the calculations that are independent of Mach number and frequency, such as setting up the arrays of collocation and integration stations.

3. This is the beginning of a loop that is gone through for each combination of Mach number and frequency specified. Subroutine DATA2 sets up the matrix (D^i_{nm}) of Equation 50.

4. In subroutine ZEN, any necessary data for the oscillation modes is read in. The left side of Equation 50 is set up for each mode.

5. Equation 50 is solved for the pressure coefficients for each mode by MSIMEC.

6. Subroutine FØRCE computes the generalized force matrix. The program returns to step 3 until the Mach numbers and frequencies are exhausted, then to step 1.

(*) RETURN TO BEGINNING FOR NEXT CASE

(**) LOOP ON NUMBER OF SETS OF MACH NUMBERS AND FREQUENCIES

Figure 4. Functional Flow Diagram—Main Program

USE OF THE COMPUTER PROGRAM

The following rules apply to optimum use of the computer program:

1. Before attempting to use the computer program compute the coefficients of polynomials of minimum order in x and y that will adequately represent the modal deflection distributions. Least-square fitted surfaces are very useful for this purpose, since weighting factors may be used to obtain a better fit in special regions.

2. Set the number of chordwise collocation points M equal to one plus the highest of the orders in x; and, unless there is special reason to reduce the number of chordwise integration points, set L equal to M.

3. Set the number of spanwise collocation points R equal to one plus the highest of the orders in y. This establishes the number of pressure coefficients a_{nm} at M x R. Their values are computed by matching exactly the downwashes at the M x R spanwise collocation points.

The input data is read by the subroutine DATRD. Use of this subroutine requires that, on each data card, the first 72 columns are six fields of width 12, as indicated on the sample date sheets (Figure 5). The first field contains an integer giving the location in the data array in which the number in the second field is to be stored. The numbers in the remaining fields are stored in consecutive locations. If a field is blank, the corresponding location in the data array is unchanged. DATRD reads any number of cards. A minus sign in column 1 indicates the last card to be read; if this minus sign is not present, DATRD continues with the next card. The storage locations of the data on a card are not affected by the sign in column 1. All floating point numbers must be written with decimal points. All integers must be at the right of their fields.

The data array is set up as follows:

1.	N	The number of chordwise pressure modes
2.	M	The number of spanwise pressure modes
3.	NCC	The number of chordwise collocation points
4.	NCS	The number of spanwise collocation points
5.	NDATA	The number of sets of values of Mach number and frequency to be used.
6.	NSYM	Indicator for symmetric (NSYM = +1) or antisymmetric (NSYM = -1) modes of oscillation.

7.	SFOLD	Distance spanwise from wing center line to fold line
8.	STIP	Semispan
9.	BO	One-half of the root chord
10.	ALFA1	The fold angle (in degrees)
11.	λ_{LE}	The sweep angle of the leading edge (in degrees)
12.	λ_{TE}	
13.	$\lambda_{LE_{TIP}}$	
14.	$\lambda_{TE_{TIP}}$	
21-30.	Values of Mach number.	NDATA of these must be entered.
31-40.	Values of reduced frequency, $\omega \cdot BO/U$.	NDATA of these must be entered.
41.	NMOD	The number of modes
42.	JD	Indicator for the type of input data for a mode. If JD=1, the deflections are given at a set of points on the wing (and tip). If JD=2, the coefficients of polynomials for the deflection of the wing and tip are given. If JD=0, the current mode and subsequent modes are not given by data. They are the same as the corresponding modes which were used for the previous frequency and Mach number.
43.	NPTSW	Number of points on the wing at which deflections are given (used if JD=1).
44.	NPTST	Number of points on tip at which deflections are given.
51-71.	Deflection coefficients on the wing.	The coefficients are stored as follows:

$$\overset{51}{a_{00}} + \overset{52}{a_{10}}\,x + \overset{53}{a_{20}}\,x^2 + \overset{54}{a_{30}}\,x^3 + \overset{55}{a_{40}}\,x^4 + \overset{56}{a_{50}}\,x^5 + y(\overset{57}{a_{01}} + \overset{58}{a_{11}}x + \overset{59}{a_{21}}x^2$$

$$+ \overset{60}{a_{31}}\,x^3 + \overset{61}{a_{41}}\,x^4) + y^2(\overset{62}{a_{02}} + \overset{63}{a_{12}}x + \overset{64}{a_{22}}x^2 + \overset{65}{a_{32}}x^3)$$

$$+ y^3(\overset{66}{a_{03}} + \overset{67}{a_{13}}x + \overset{68}{a_{23}}x^2) + y^4(\overset{69}{a_{04}} + \overset{70}{a_{14}}x) + \overset{71}{a_{05}}y^5$$

76-96.	Deflection coefficients on the tip.	Same storage rule as above, except all locations are increased by 25.
98.	Indicator that no more modes are to be read after the present one. (For current and subsequent frequencies and Mach numbers.)	
101-299.	Deflection data at points on the wing, in the order $x_1, s_1, n_1, x_2, s_2, n_2$, etc.	
301-499.	Deflection data at points on the tip.	

Items 1-41 must be read in the first set of data. There may be an additional set of data for each mode, for each Mach number and frequency case. After the indicator DAT(98) has been given a non-zero value, no more deflection data will be read. After JD has been given the value zero, no data will be read for the higher numbered modes.

The data in Figure 5 is for a 60° triangular wing folded at 75 percent semispan, at an angle of 30°. The root chord is 5.0 feet, making BO = 2.5. Three modes: plunge, pitch, and a third nonrigid mode are considered. The Mach number is 0.7, and six frequencies: 10, 20, 30, 40, 50, and 60 cps are used. The speed of sound is taken to be 1000 ft./sec. This gives reduced frequencies of 0.157, 0.314, 0.471, 0.628, 0.785, and 0.942.

Three spanwise and three chordwise pressure modes, six chordwise and eight spanwise collocation stations are specified.

In the set of cards numbered 15 - 30, which give deflection data, some of the cards have been omitted. Otherwise, this is a complete set of data for a computer run.

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE 1 of 5 JOB NO. ______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1	73 – 80	
13	3		N
25	3		M
37	6		NCC
49	8		NCS
61	6	1	NDATA
1	6	73 – 80	
13	1		NSYM
25	2.165		SFOLD
37	2.887		STIP
49	2.5		BO
61	30.0	2	Fold Angle
1	11	73 – 80	
13	60.0		Sweep Angles
25	0.0		
37	60.0		
49	0.0		
61		3	
1	21	73 – 80	
13	0.7		Mach Numbers
25	0.7		
37	0.7		
49	0.7		
61	0.7	4	

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Figure 5. Sample Data Sheets (Sheet 1 of 5)

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.______ PROGRAMMER______ DATE______ PAGE 2 of 5 JOB NO.______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	2 6		
13	0 . 7		Last Mach Number
25			
37			
49		73 80	
61		5	
1	3 1		
13	0 . 1 5 7		Reduced Frequencies
25	0 . 3 1 4		
37	0 . 4 7 1		
49	0 . 6 2 8	73 80	
61	0 . 7 8 5	6	
1	3 6		
13	0 . 9 4 2		
25			
37			
49		73 80	
61		7	
1	— 4 1		Minus sign indicates last card in first set of data
13	3		NMOD
25			
37			
49		73 80	
61		8	

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Figure 5. Sample Data Sheets (Sheet 2 of 5)

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.____ PROGRAMMER____ DATE____ PAGE 3 of 5 JOB NO.____

Col.	Sign	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1		4 2	73–80	
13		2		JD
25				
37				
49				
61			9	
1		5 1	73–80	
13		1 . 0		Coefficients of the deflection polynomial on the wing, which has the
25				constant value 1.0 in this mode.
37				
49				
61			1 0	
1	—	7 6	73–80	Minus sign indicates last card for first mode.
13		0 . 8 6 6		Coefficients of the deflection polynomial on the tip. A vertical
25				deflection of 1.0 causes a normal deflection of cos 30° = 0.866.
37				
49				
61			1 1	
1		5 1	73–80	
13		0 . 0		Coefficients on the wing for the second mode. The polynomial is 0.2X
25		0 . 2		
37				
49				
61			1 2	

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Figure 5. Sample Data Sheets (Sheet 3 of 5)

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE 4 of 5 JOB NO. ______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	76		
13	0.0		Coefficients on the tip for the second mode.
25	0.1732		
37			
49		73 … 80	
61		13	
1	42		
13	1		JD
25	22		NPTSW
37	6		NPTST
49		73 … 80	
61		14	
1	101		Deflection data on the wing begin.
13	0.9		X1
25	0.4		S1
37	0.314		N1
49	1.6	73 … 80	X2
61	0.4	15	S2
1	106		
13	0.323		N2
25	2.8		X3
37	0.4		S3
49	0.330	73 … 80	N3
61	3.6	16	X4

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Figure 5. Sample Data Sheets (Sheet 4 of 5)

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE 5 of 5 JOB NO. ______

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	166		End of deflection data on the wing
13	1.206		N22
25			
37			
49			
61		26	
1	301		Deflection data on the tip begin
13	4.25		X1
25	2.3		S1
37	1.38		N1
49	4.5		X2
61	2.3	27	S2
1	316		End of deflection data on the tip
13	4.8		X6
25	2.7		S6
37	1.44		N6
49			
61		30	
1	-98		
13	1		Indicator that modes are not to be read in hereafter
25			
37			
49			
61		31	

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Figure 5. Sample Data Sheets (Sheet 5 of 5)

VII. RESULTS

The computer program was applied to the rigid modes of two wings.

ASPECT RATIO 2.0 RECTANGULAR WING FOLDED AT 80 PERCENT SEMISPAN

C_{L_α} is plotted as a function of Mach number in Figure 6. The dashed curve is a plot of the approximation

$$C_{L_\alpha} = \frac{2\pi A.R.}{2 + \sqrt{4 + (A.R.)^2 (1 - M^2)}}$$

where A.R. is the aspect ratio. This is formula (6-31) of Reference 5, for the case of a rectangular wing.

TRIANGULAR WING WITH FOLDED TIPS

The configuration used was a triangular wing with a sweep angle of 65 degrees, folded at 60 percent semispan.

Figures 7 and 8 show C_{L_α} and C_{m_α} as functions of fold angle (at M = 0.8), and as functions of Mach number. Figure 9 is a plot of unsteady generalized forces for rigid oscillations of the wing in the pitching mode.

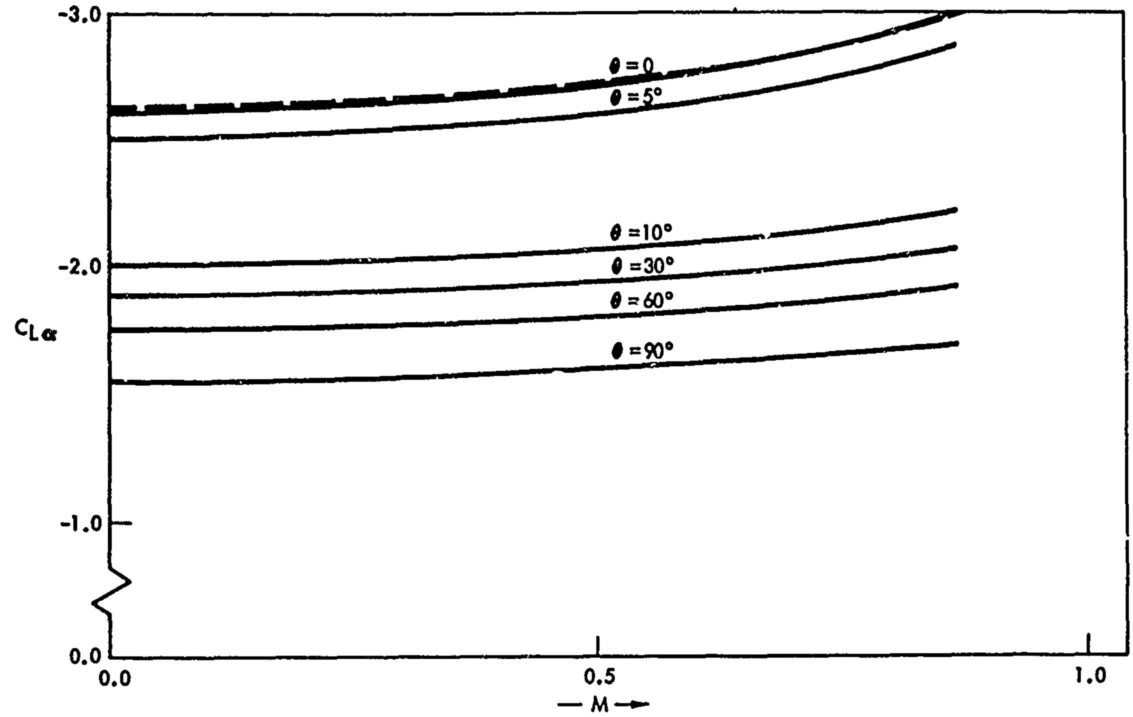


Figure 6. Lift and Moment Coefficients Vs Mach Number for Aspect Ratio 2.0 Rectangular Wing at Various Fold Angles

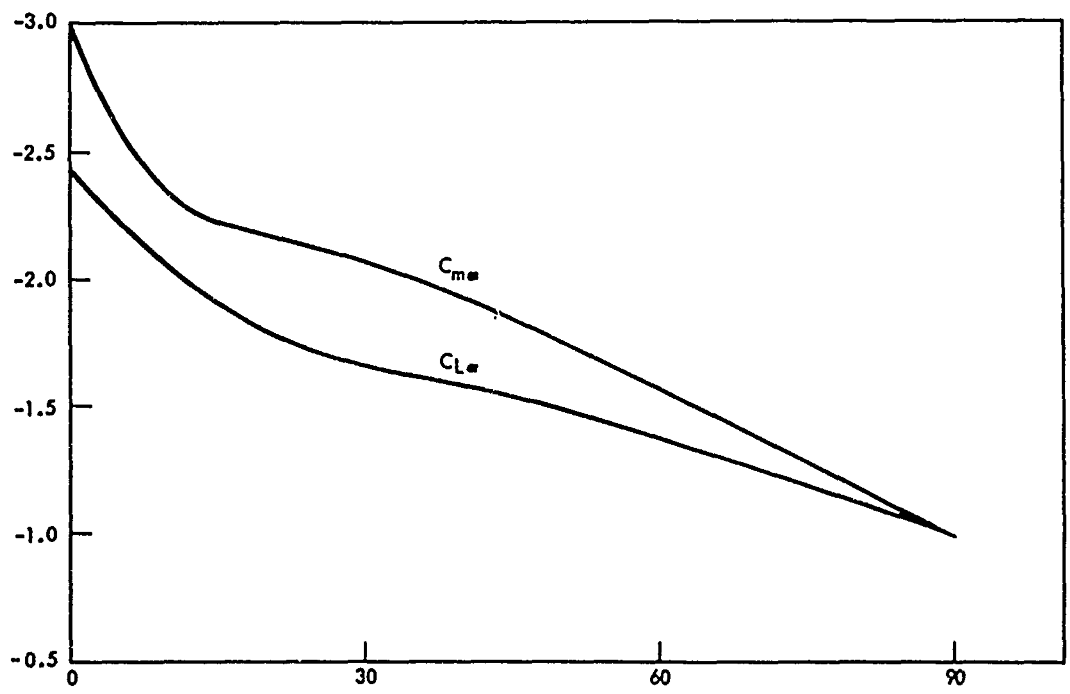


Figure 7. Lift and Moment Coefficients Vs Fold Angle for 65^{o} Triangular Wing at $M_{\infty} = 0.8$

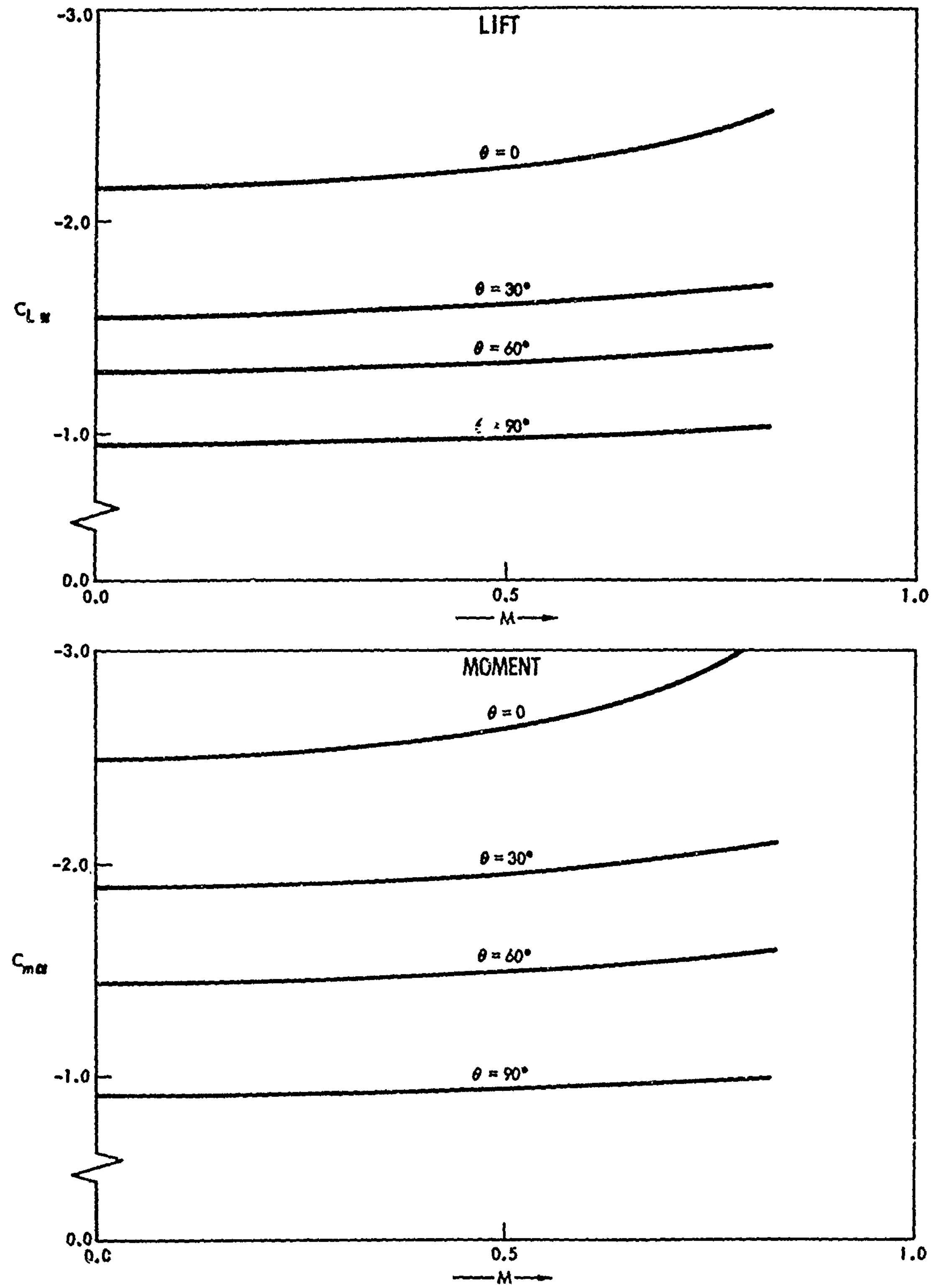
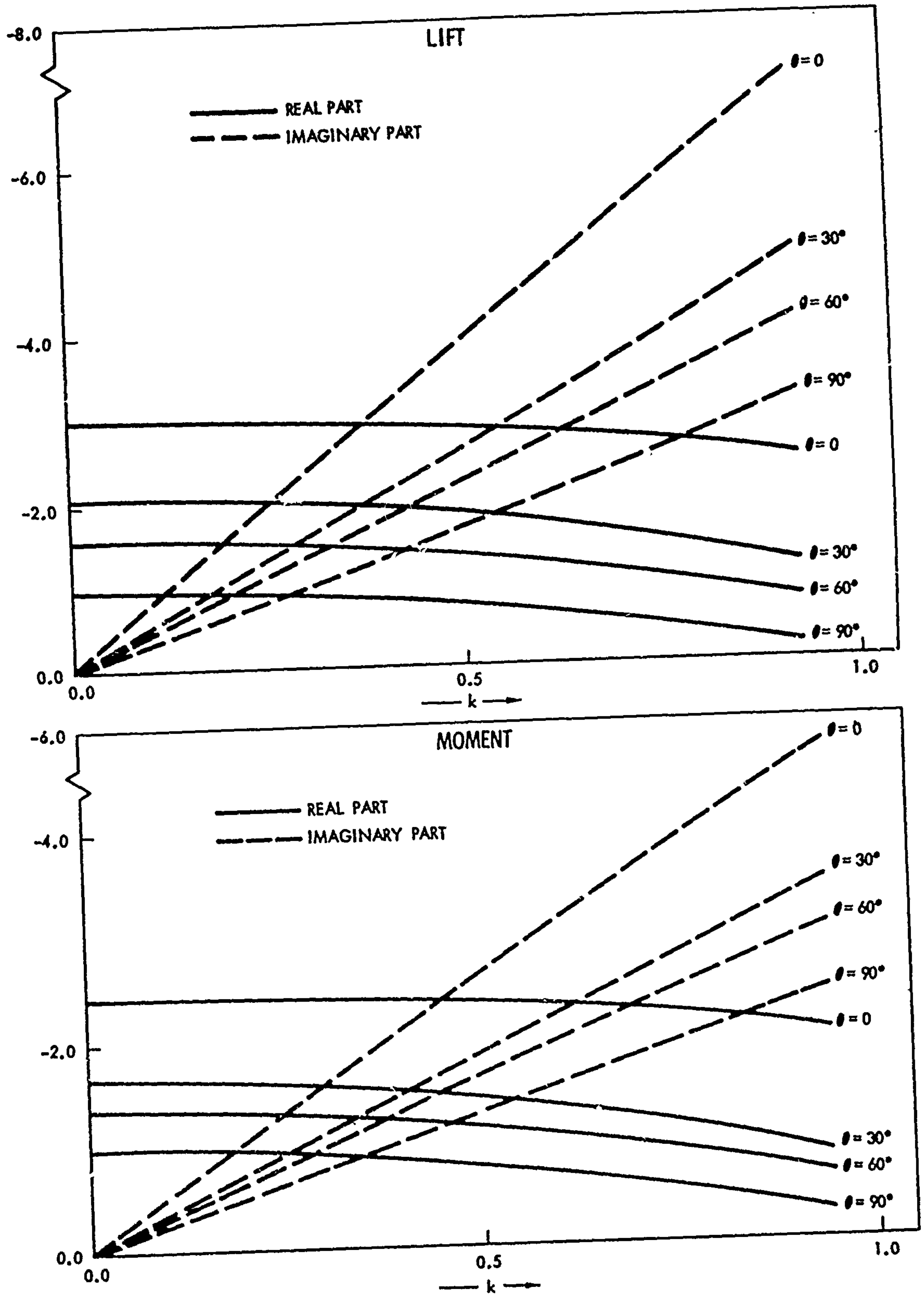


Figure 8. Lift and Moment Coefficients Vs Mach Number for 65° Triangular Wing at Various Fold Angles



Coefficients of Lift and Moment Due to Pitch Vs Reduced Frequency for a 65° Triangular Wing at M_∞ = 0.8 and Various Fold Angles

VIII. CONCLUSIONS AND RECOMMENDATIONS

The results obtained by the nonplanar kernal function method show the expected trend with increasing fold angle, which agrees with the observed experimental trend. For zero fold angle, the method reduces to that already used by Hsu (Reference 4).

Possible extensions of the method include the treatment of more general configurations, or of other specific configurations, such as the T-tail. Also, any generalizations proposed for the planar case should be considered here, such as the problem of a nonplanar wing with a control surface.

The formula that was used for the kernel function (page 19) should be useful in any future developments using the three-dimensional kernel function. The previously available formula was much longer. A special case of that formula is given in Reference 12. The use of the simplified kernel function, together with Hsu's method of integration, results in greatly reduced computer running times. This makes it practical to use the kernel function method as a tool in the preliminary analysis of new wing configurations.

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APPENDIX. COMPUTER PROGRAM LISTINGS

MP

```
      COMMON W(50,25),WI(50,25),A(2,25,25),B(2,25,10),XSV(10,5)
      COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)
      COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
      COMMON/COM1/FM,FR,BO,SF,ST
      COMMON /CDAT/DAT(500)
      DO 1 I=1,100
    1 DAT(I)=0.0
    2 CALL DATRD(DAT)
      CALL DATA1
      DO 200 ND=1,NDATA
      FR=DAT(ND+30)
      FM=DAT(ND+20)
      CALL DATA2
      CALL ZEN
      DO 35 I=1,NC
      DO 35 J=1,NC
      A(1,I,J)=0.0
      A(2,I,J)=0.0
      DO 35 K=1,NR
      A(1,I,J)=A(1,I,J)+W(K,I)*W(K,J)+WI(K,I)*WI(K,J)
      A(2,I,J)=A(2,I,J)+W(K,I)*WI(K,J)-WI(K,I)*W(K,J)
   35 CONTINUE
      K=MSIMEC(25,NC,NMOD,A,B)
      IF (K.NE.1) GO TO 99
      WRITE (6,650)
  650 FORMAT(1H18X,78HPRESSURE = STIP/B(S) * SUM OF A(N,M)*F(N,PSI)*SQRT
     1(1.0-(S/STIP)**2)*(S/STIP)**)
      IF (NSYM.LT.0) GO TO 10
      WRITE (6,651)
  651 FORMAT(1H+86X,5H(M-1))
      GO TO 15
   10 WRITE (6,652)
  652 FORMAT(1H+86X,1HM)
   15 LINES=1
      LINLIM=35-NC/2
      DO 40 I1=1,NMOD
      WRITE (6,700) I1
```

MP

```
700 FØRMAT(1H-20X,41HPRESSURE CØEFFICIENTS A(N,M) FØR MØDE NØ.I2/1H0
   1 2(9X,5HN    M6X,9HREAL PART7X,10HIMAG PART )/1H )
    J1=1
    JCARR=1
    DØ 19 NPR=1,N
    DØ 19 MPR=1,M
    IF (JCARR.EQ.0) GØ TØ 17
    JCARR=0
    WRITE (6,705) NPR,MPR,B(1,J1,I1),B(2,J1,I1)
    GØ TØ 19
 17 JCARR=1
    WRITE (6,706) NPR,MPR,B(1,J1,I1),B(2,J1,I1)
705 FØRMAT(I11,I4,1P2E16.5)
706 FØRMAT(1H+52X,2I4,1P2E16.5)
 19 J1=J1+1
    LINES=LINES+6+NC/2
    IF (LINES.LE.LINLIM) GØ TØ 40
    WRITE (6,708)
708 FØRMAT(1H1)
    LINES=0
 40 CØNTINUE
    IF (LINES.EQ.0) GØ TØ 24
    LINES=LINES+7+2*NMØD**2
    IF (LINES.LE.40) GØ TØ 23
    WRITE (6,708)
    GØ TØ 24
 23 WRITE (6,709)
709 FØRMAT(1H0)
 24 WRITE (6,45) FR,FM,DAT(10)
 45 FØRMAT(1H010X,26HGENERALIZED FØRCES - - K =F7.4,6H,  M =F7.4,
   1 15H,  FØLD ANGLE =F8.4,5H DEG.)
    WRITE (6,46)
 46 FØRMAT(1H05X,5HMØDES/4X,11HØSC.  DEFL.8X,9HREAL PART10X,9HIMAG PAR
   1T10X,10HABS. VALUE6X,11HPHASE ANGLE)
    CALL FØRCE
200 CØNTINUE
    GØ TØ 2
```

```
    MP

99 WRITE (6,98) ND
98 FØRMAT(57H1 PRESSURE CØEFFICIENTS CANNØT BE FØUND FØR DATA CASE NØ
  1.I2)
   GØ TØ 200
   END
```

DATA1

```
      SUBROUTINE DATA1
      COMMON W(50,25),WI(50,25),A(2,25,25),B(2,25,10),XSV(10,5)
      COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)
      COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
      COMMON/COM1/FM,FR,BO,SFOLD,STIP
      COMMON /COM2/ Q(5),DELP(5,10),NIC,NIS,ZIS,ZIP,ZCP,SFOST
      COMMON/COM2/BF,BMF,     COS1,SIN1,CT32P,CT32M,CT54P,CT54M
      COMMON /CDAT/DAT(500)
      COMMON /CONST/ PI,PIOVR2,RAD,ZI(10)
      N=DAT(1)
      M=DAT(2)
      NCC=DAT(3)
      NCS=DAT(4)
      NDATA=DAT(5)
      NSYM=DAT(6)
C     NCS = NO. OF COLLOCATION PTS. SPANWISE
C     NCC = NO. OF COLLOCATION PTS. CHORDWISE
C     NIS = NO. OF INTEGRATION PTS. SPANWISE
C     NIC = NO. OF INTEGRATION PTS. CHORDWISE
      NDEX = (1-NSYM)/2
      NIC = NCC
      NIS = NCS + 1
      ZIS = NIS
      BO=DAT(9)
      SFOLD=DAT(7)/BO
      STIP=DAT(8)/BO
      DO 15 I=1,10
      X(I)=DOTS
      Y(I)=DOTS
      PSI(I)=DOTS
   15 ETA(I)=DOTS
      SBO2=STIP**2
      WRITE (6,20) NCS,NCC,M,N,NDATA,NDATA
   20 FORMAT(1H1,54X,10HINPUT DATA/21X,16HCOLLOCATION PTS., 5X,14HPRESSU
     1RE MODES, 5X, 9HMACH NOS., 5X, 6HFREQS./21X,I2,5H SPAN,I3,6H CHORD
     2,4X,I2,5H SPAN,I3,6H CHORD,9X,I2,11X,I2)
   13 SFOST = SFOLD/STIP
```

DATA1

```
      ALFA1=DAT(10)/RAD
      ALFA2=(90.0-DAT(11))/RAD
      ALFA3=(90.0-DAT(12))/RAD
      ALFA4=(90.0-DAT(13))/RAD
      ALFA5=(90.0-DAT(14))/RAD
C     EVALUATE TIP INTEGRALS
      F= SQRT(1.0-SFØST**2)
      THE1=ATAN(SFØST/F)
      Q(1)=1.0-(THE1+F*SFØST)/PIØVR2
      F=F**3/PI
      Q(2)=F/3.0
      DØ 1607 IL=3,5
      F=F*SFØST
      Q(IL)=(F+ZI(IL-2)*Q(IL-2))/ZI(IL+1)
 1607 CØNTINUE
C     GENERATE CØLLØCATIØN AND INTEGRATIØN STATIØNS
      ZCP=2.0*PI/FLØAT(2*NCC+1)
      ZIR=SB02*ZCP/8.0
      F=0.5*ZCP
      DØ 27 J=1,NCC
      PSI(J)=-CØS(F)
      W=PSI(J)
      DØ 26 I=1,N
   26 DELP(I,J)=FXI(W,I)*ZIR
      I-NCC+1-J
      X(I)=-W
   27 F=F+ZCP
      ZS=PI/ZIS
      F=-ZS+PIØVR2
      DF=0.5*ZS
      DØ 29 J=1,NCS
      Y(J)=SIN(F)
      W=F+DF
      ETA(J)=SIN(W)
   29 F=F-ZS
      ETA(NIS)=-ETA(1)
      WRITE            ( 6,35)
```

```
      DATA1
 35   FORMAT(1H0,27X,47HCALCULATED COLLOCATION AND INTEGRATION STATIONS/
     11H ,24X,1HX,15X,3HPSI,15X,1HY,15X,3HETA)
      NLIN=MAX0(NIS,NCC)
      WRITE (6,36) (X(I),PSI(I),Y(I),ETA(I),I=1,NLIN)
   36 FORMAT(13X,4F17.4)
      WRITE (6,560)
      WRITE (6,565) (DAT(I+20),DAT(I+30),I=1,NDATA)
C     EVALUATE QUANTITIES FOR LATER USE
      COS1=COS(ALFA1)
      SIN1=SIN(ALFA1)
      NCS1=(NCS+1)/2
      IF (NSYM.LT.0) NCS1=NCS/2
      CTN2=0.5*COS(ALFA2)/SIN(ALFA2)
      CTN3=0.5*COS(ALFA3)/SIN(ALFA3)
      CTN4=0.5*COS(ALFA4)/SIN(ALFA4)
      CTN5=0.5*COS(ALFA5)/SIN(ALFA5)
      CT32P=CTN3+CTN2
      CT32M=CTN3-CTN2
      CT54P=CTN5+CTN4
      CT54M=CTN5-CTN4
      BF =1.0+SFOLD*CT32M
      BMF=1.0+SFOLD*CT32P
      NR=NCC*NCS1
      NC=M*N
      DO 82 J=1,10
      IF (Y(J).LE.SFOST) GO TO 84
   82 CONTINUE
   84 NCST=J-1
      ZIP=16.0/SB02
      RETURN
 560  FORMAT(1H0,35X, 9HMACH NOS. ,15X,6HFREQS.)
 565  FORMAT(1H ,36X,F8.4,14X,F8.4)
      END
```

DAT2

```
      SUBROUTINE DATA2
      COMMON WASH(50,25),WASHI(50,25),A(2,25,25),B(2,25,10),XSV(10,5)
      COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)
      COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
      COMMON/COM1/FM,FR,BO,SFOLD,STIP
      COMMON /COM2/ Q(5),DELP(5,10),NIC,NIS,ZIS,ZIP,ZCP,SFOST
      COMMON/COM2/BF,BMF,     COS1,SIN1,CT32P,CT32M,CT54P,CT54M
      COMMON /CDAT/DAT(500)
      COMMON /CONST/ PI,PIOVR2,RAD,ZI(10)
      DIMENSION RA(10),RB(10),RC(5),FS(5),FT(5),FU(5),FV(5),FW(5),FX(5)
      DIMENSION XU(2),XX(2),XY(2),XZ(2)
C     EVALUATION OF MATRIX OF INTEGRALS
      DO 38 J=1,1250
      WASH(J,1) = 0.0
 38   WASHI(J,1) = 0.0
      BETA2=1.0-FM**2
      IU =  0
      DO 1000 IQ=1,NCS1
      CALL GEOM(Y(IQ),XI1,XI2,CO2,SI2,YY,ZY)
      DO 1000 IP = 1,NCC
      XA=X(IP)*XI2+XI1
      XSV(IP,IQ) = XA
      IU = IU+1
      XARG=ZI(IP)*ZCP
      F=SQRT(1.0-X(IP)**2)
      RC(1)=2.0*(F+XARG)
      G=-2.0*X(IP)*F
      RC(2)=XARG-0.5*G
      DO 1604 IC=3,N
      H=-F-2.0*X(IP)*G
      RC(IC)=F/ZI(IC-2)-H/ZI(IC)
      F=G
 1604 G=H
 58   DO 1640 IN = 1,NIS
      YE=ETA(IN)
      CALL GEOM(YE,XI3,XI4,CO4,SI4,YETA,ZETA)
      SY=1.0
```

DAT2

```
      IF (YE.GE.0.0) GØ TØ 54
      SY=SIGN(1.0,NSYM)
      YE=SIGN(YE,NSYM)
   54 CØNTINUE
      YO=YY-YETA
      ZOO=ZY-ZETA
      DØ 1580 IJ=1,N
      FS(IJ) = 0.0
      FT(IJ) = 0.0
      FU(IJ) = 0.0
 1580 FV(IJ)=0.0
      DØ 1600 IL = 1,NIC
      XB=PSI(IL)*XI4+XI3
      XO = XA - XB
   51 CALL KRNLI(FR,XO,YO,ZOO,FM,BETA2,CØ2,SI2,CØ4,SI4,XX)
      IF (SI4.EQ.0.0) GØ TØ 93
C     ETA IS ØUTBØARD ØF FØLDLINE,CALCULATE KPRIME.
 96   IF(ETA(IN))101,101,102
  101 SA=YY+SFØLD
      GØ TØ 103
  102 SA=YY-SFØLD
  103 X1=XA-PSI(IL)*BF-BMF
      CALL KRNLI(FR,X1,SA,ZY,FM,BETA2,CØ2,SI2,CØ4,SI4,XY)
      CALL KRNLI(FR,X1,SA,ZY,FM,BETA2,CØ2,SI2,1.0,0.0,XZ)
C     KPRIME = XX - XY + XZ
      XU(1) = XY(1) - XZ(1)
      XU(2) = XY(2) - XZ(2)
      XX(1) = XX(1) - XU(1)
      XX(2) = XX(2) - XU(2)
      GØ TØ 94
   93 XU(1) = 0.0
      XU(2) = 0.0
   94 DØ 1590 IJ=1,N
      FS(IJ)=FS(IJ)+XX(1)*DELP(IJ,IL)
      FT(IJ)=FT(IJ)+XX(2)*DELP(IJ,IL)
      FU(IJ)=FU(IJ)+XU(1)*DELP(IJ,IL)
 1590 FV(IJ)=FV(IJ)+XU(2)*DELP(IJ,IL)
```

DAT2

```
 1600 CONTINUE
C     CHORDWISE INTEGRATION IS COMPLETE.
      IA = 0
      W=(1.0-YE**2)/ZIS
      IF (NDEX.NE.0) W=W*YE
      DO 1625 IL=1,M
      ILN=IL+NDEX
      DO 1620 IM = 1,N
      IA = 1 + IA
      WASH(IU,IA)=WASH(IU,IA)+FS(IM)*W
      WASHI(IU,IA)=WASHI(IU,IA)+FT(IM)*W
      IF (IN.NE.1.AND .IN.NE.NIS) GO TO 1620
      WASH(IU,IA)=WASH(IU,IA)+FU(IM)*Q(ILN)*SY
      WASHI(IU,IA)=WASHI(IU,IA)+FV(IM)*Q(ILN)*SY
 1620 CONTINUE
 1625 W=YE*W
 1640 CONTINUE
      DO 1601 LO=1,IP
      ARG=FR*(XA-PSI(LO)*XI2-XI1)
      RA(LO)=ZIP*(COS(ARG)-1.0)
      RB(LO)=-ZIP*SIN(ARG)
 1601 CONTINUE
C     RA = REAL, RB = IMAG
      DO 1646 IJ=1,N
      FW(IJ) = 0.0
      FX(IJ) = 0.0
      DO 1646 IK=1,IP
      FW(IJ) = FW(IJ) + DELP(IJ,IK)*RA(IK)
      FX(IJ) = FX(IJ) + DELP(IJ,IK)*RB(IK)
 1646 CONTINUE
      IA = 0
      YP=ZIS/8.0
      IF (NDEX.NE.0) YP=YP*Y(IQ)
      DO 1649 IL = 1,M
      DO 1648 IM = 1,N
      IA = 1+IA
      WASH(IU,IA) = WASH(IU,IA)-(FW(IM)+RC(IM))*YP
```

```
      DAT2
 1648 WASHI(IU,IA) = WASHI(IU,IA)-FX(IM)*YP
 1649 YP=YP*Y(IQ)
 1000 CONTINUE
 1010 WRITE (6,530) FR,FM
      DO 1015 K = 1,IU
      WRITE (6,555)
      WRITE          ( 6,535)(WASH(K,L),WASHI(K,L),L=1,IA)
 1015 CONTINUE
      RETURN
  530 FORMAT(1H121X,30HMATRIX FOR REDUCED FREQUENCY =F7.4,
     1  16H,  MACH NUMBER =F7.4)
 535  FORMAT(3(5X,1P2E15.4,1HI))
 555  FORMAT(1H )
      END
```

GEOM

```
      SUBROUTINE GEOM(Y,X1,X2,CO,SI,YY,ZY)
      COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
      COMMON/COM1/FM,FR,BO,SFOLD,STIP
      COMMON /COM2/ Q(5),DELP(5,10),NIC,NIS,ZIS,ZIP,ZCP,SFOST
      COMMON/COM2/BF,BMF,     COS1,SIN1,CT32P,CT32M,CT54P,CT54M
C     EVALUATION OF GEOMETRICAL QUANTITIES AT A GIVEN CHORD
      Y1=STIP*ABS(Y)
      F=Y1-SFOLD
      IF (F) 2,2,4
    2 CO=1.0
      SI=0.0
      YY=Y1
      ZY=0.0
      X1=1.0+Y1*CT32P
      X2=1.0+Y1*CT32M
      GO TO 6
    4 CO=COS1
      SI=SIN1
      YY=SFOLD+F*COS1
      ZY=F*SIN1
      X1=BMF+F*CT54P
      X2=BF+F*CT54M
    6 IF (Y.GE.0.0) RETURN
      SI=-SI
      YY=-YY
      RETURN
      END
```

KRNL

```
      SUBROUTINE KRNLI(CK,X,Y,ZO,CM,B2,CØ1,SI1,CØ2,SI2,AKERN)
      DIMENSIØN Z(6),H(6),AKERN(2)
      DATA H/0.08566225,0.18038079,2*0.23395697,0.18038079,0.08566225/
      DATA Z/0.96623476,0.83060469,0.61930959,0.38069041,0.16939531,
     10.03376524/
      R2 = Y*Y+ZO*ZO
      R =  SQRT(R2)
      CK1 = CK*R
      G1 = 0.0
      G2 = 0.0
      G3 = 0.0
      G4 = 0.0
      G5 = 0.0
      G6 = 0.0
    5 S2 = X*X + B2*R2
      S =  SQRT(S2)
      U1 = (CM*S-X)/(B2*R)
      UK = CK1*U1
      DØ 20 I = 1,6
      UZ = U1*Z(I)
      UZ2 = UZ**2
      F = UK*Z(I)
      CØ =  CØS(F)
      SI =  SIN(F)
      F = H(I)/ SQRT(1.0+UZ2)*UZ*U1
      G3 = G3 + F*CØ
      G4 = G4 - F*SI
      F = UZ*F
      G5 = G5 + F*CØ
      G6 = G6 - F*SI
      V = 1.0 - Z(I)**2
      F = H(I)*2.0*V* EXP(-CK1*V)/ SQRT(1.0+V)
      G1 = G1 + F
   20 G2 = G2 + V*F
      G7 = G1 + G3
      XS = X/S
      CØ =  CØS(UK)*XS
```

KRNL

```
      SI =  SIN(UK)*XS
      IF(CK.NE.0.0) GØ TØ 22
      F14 = 1.0
      F15 = 2.0
      GØ TØ 23
   22 F14 = CK1*BESEL(CK1,1,3)
      F15 = CK1*CK1*BESEL(CK1,2,3)
   23 F11 = -(CK1*G4+F14+CØ)
      F12 = CK1*G7+SI
      F = 2.0+B2*R2/S2
      FP = UK+CK*CM*R2/S
      F21 = CK1*G4+CK1*CK1*G5+F15+CØ*F +SI*FP
      F22 = CK1*(-G7+CK1*(G6-G2))+CØ*FP-SI*F
      XK = CK*X
      CØ =  CØS(XK)
      SI =  SIN(XK)
      F=(CØ1*CØ2+SI1*SI2)/R2
      FP=(Z0*CØ1-Y*SI1)*(Z0*CØ2-Y*SI2)/R2**2
      G1 = F*F11+FP*F21
      G2 = F*F12+FP*F22
      AKERN(1) = -CØ*G1-SI*G2
      AKERN(2) = SI*G1-CØ*G2
      RETURN
      END
```

ZEN

```
      SUBROUTINE ZEN
      COMMON W(50,25),WI(50,25),A(2,25,25),B(2,25,10),XSV(10,5)
      COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)
      COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
      COMMON/COM1/FM,FR,BO,SF,ST
      COMMON /CDAT/DAT(500)
      DIMENSION XH(66),SH(66),ZH(66),XCH(50),YCH(50),Z(50),DZ(50)
      DIMENSION WD(2,100)
      EQUIVALENCE (XH,A),(SH,A(67)),(ZH,A(133)),(XCH,A(200)),(Z,A(250))
      EQUIVALENCE (YCH,A(300)),(DZ,A(350)),(WD,A(400))
      LINES=50
      NMOD=DAT(41)
      IDEF=0
      DO 50 I=1,NMOD
      IF (DAT(98).NE.0.0) GO TO 20
      IF (IDEF.NE.0) GO TO 20
      CALL DATRD(DAT)
      JD=DAT(42)
      IF (JD-1) 2,4,16
    2 IDEF=1
      GO TO 20
C     READ DEFLECTION VALUES AT POINTS
    4 NPTS=DAT(43)
      L=101
      DO 8 J=1,NPTS
      XH(J)=DAT(L)
      SH(J)=DAT(L+1)
      ZH(J)=DAT(L+2)
    8 L=L+3
      NDH=5
    9 CALL LSSURP(XH,SH,ZH,NPTS,NDH,G,ZXY(1,I),IND)
      IF(IND.EQ.0) GO TO 10
      NDH=NDH-1
      GO TO 9
   10 NPTS=DAT(44)
      L=301
      DO 11 J=1,NPTS
```

```
      ZEN

      XH(J)=DAT(L)
      SH(J)=DAT(L+1)
      ZH(J)=DAT(L+2)
   11 L=L+3
      NDV=5
   12 CALL LSSURP(XH,SH,ZH,NPTS,NDV,G,ZZZ(1,I),IND)
      IF (IND.EQ.0) GØ TØ 20
      NDV=NDV-1
      GØ TØ 12
C     REA) PØLYNØMIAL CØEFFICIENTS
   16 DØ 17 J=1,21
      ZXY(J,I)=DAT(J+50)
   17 ZZZ(J,I)=DAT(J+75)
C     FIND UPWASHES
   20 K=0
      DØ 24 I2=1,NCST
      DØ 24 J=1,NCC
      K=K+1
      XCH(K)=BO*XSV(J,I2)
   24 YCH(K)=DAT(8)*Y(I2)
      LIND=8+K
      IF (LINES+LIND.LE.40) GØ TØ 25
      WRITE (6,41)
   41 FØRMAT(1H1)
      LINES=0
   25 LINES=LINES+LIND
      WRITE (6,43)
      WRITE(6,42) I,(ZZZ(J,I),J=1,21)
   42 FØRMAT(1H+5X,26HDEFLECTIØN CØEFFICIENTS ØN7X,8HMØDE NØ.I2/1H /
     1 (1P7E14.4))
   43 FØRMAT(1H032X,4HTIP,)
      CALL ZDZ(XCH,YCH,K,ZZZ(1,I),Z,DZ)
      DØ 26 I2=1,K
      WD(1,I2)=DZ(I2)*BO
   26 WD(2,I2)=FR*Z(I2)
      I3=NCST+1
      L=0
```

ZEN

```
      DO 27 I2=I3,NCS1
      DO 27 J=1,NCC
      L=L+1
      XCH(L)=BO*XSV(J,I2)
   27 YCH(L)=DAT(8)*Y(I2)
      LIND=8+L
      IF (LINES+LIND.LE.40) GO TO 28
      WRITE (6,41)
      LINES=0
   28 LINES=LINES+LIND
      WRITE (6,44)
   44 FORMAT(1H032X,5HWING,)
      WRITE (6,42) I,(ZXY(J,I),J=1,21)
      CALL ZDZ(XCH,YCH,L,ZXY(1,I),Z,DZ)
      J2=K+1
      DO 29 I2=1,L
      WD(1,J2)=DZ(I2)*BO
      WD(2,J2)=FR*Z(I2)
   29 J2=J2+1
      DO 33 J=1,NC
      B(1,J,I)=0.0
      B(2,J,I)=0.0
      DO 33 K=1,NR
      B(1,J,I)=B(1,J,I)+W(K,J)*WD(1,K)+WI(K,J)*WD(2,K)
   33 B(2,J,I)=B(2,J,I)+W(K,J)*WD(2,K)-WI(K,J)*WD(1,K)
   50 CONTINUE
      RETURN
      END
```

FØRC

```
      SUBRØUTINE FØRCE
      CØMMØN W(50,25),WI(50,25),A(2,25,25),AP(2,25,10),XSV(10,5)
      CØMMØN X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)
      CØMMØN /CØM1/ N,M,NR,NC,NDATA,NMØD,NSYM,NDEX,NCC,NCS1,NCST
      CØMMØN/CØM1/FM,FR,BO,SF,ST
      CØMMØN /CØM2/ Q(5),DELP(5,10),NIC,NIS,ZIS,ZIP,ZCP,SFØST
      CØMMØN/CØM2/BF,BMF,     CØS1,SIN1,CT32P,CT32M,CT54P,CT54M
      CØMMØN /CØNST/ PI,PIØVR2,RAD,ZI(10)
      DIMENSIØN AW(5,6,10),AT(5,6,10),AS(10)
      EQUIVALENCE (A,AW),(A(301),AT)
      DIMENSIØN FRC(2)
      DIMENSIØN U(6),V(6),H(6)
      DATA H/0.085662246,0.18038079,2*0.23395697,0.18038079,0.085662246/
      DATA V/0.96623476,0.83060459,0.61930959,0.38069041,0.16939531,
     10.03376524/
C     WEIGHTS AND PØINTS FØR 6 PØINT GAUSSIAN QUADRATURE ØN (0,1)
      DATA U/-0.965926,-0.707107,-0.258819,0.258819,0.707107,0.965926/
      DATA PI6/0.52359877/
C     PØINTS FØR 6 PØINT GAUSS QUADRATURE ØN (-1,1) WITH WEIGHT FUNCTIØN
C      1/SQRTF(1-U**2).  WEIGHTS ARE PI/6
      IDEX=NDEX-1
      DØ 2 I=1,300
      AT(I,1,1)=0.0
    2 AW(I,1,1)=0.0
    4 DS=ST-SF
      BT=BF+DS*CT54M
      DØ 12 L=1,2
    5 DØ 12 I=1,6
      GØ TØ (6,7),L
    6 SG=SF*V(I)
      XM=1.0+SG*CT32P
      B =1.0+SG*CT32M
      RS=SF
      GØ TØ 8
    7 SG=ST-DS*V(I)**2
      RS=2.0*DS*V(I)
      SG1=SG-SF
```

```
      FØRC

      XM=BMF+SG1*CT54P
      B = BF+SG1*CT54M
    8 RS = RS*PI6*H(I)*SQRT(ST**2-SG**2)
      DØ 9 K=1,10
      AS(K)=RS
    9 RS=SG*RS
      DØ 12 J=1,6
      Z=XM+B*U(J)
C       Z  IS  X
      DØ 12 L1=1,N
      FAC=FXI  (U(J),L1)
      GØ TØ (23,24),L
   23 DØ 11 L2=1,6
      DØ 10 K=1,10
   10 AW(L1,L2,K)=AW(L1,L2,K)+FAC*AS(K)
   11 FAC=Z*FAC
      GØ TØ 12
   24 DØ 21 L2=1,6
      DØ 20 K=1,10
   20 AT(L1,L2,K)=AT(L1,L2,K)+FAC*AS(K)
   21 FAC=Z*FAC
   12 CØNTINUE
      AREA=SF*(1.0+BF)+DS*(BF+BT)
      DØ 30 N1=1,NMØD
      DØ 30 N2=1,NMØD
      FRC(1)=0.0
      FRC(2)=0.0
      I2 = 1
      GO=1.0
      DØ 17 J=1,6
      G=GO
      GO=GO*BO
      JL=7-J
      DØ 17 I = 1,JL
      L3 = 1
      F1=ZXY(I2,N2)*G
      F2=ZZZ(I2,N2)*G
```

FØRC

```
      G=G*BO
      IF (F1.EQ.0.0 .AND. F2.EQ.0.0) GØ TØ 17
      Z=1.0
      DØ 19 K=1,M
      L2=K+J+IDEX
      DØ 18 L1=1,N
      F=F1*AW(L1,I,L2)+F2*AT(L1,I,L2)
      F=F*Z
      DØ 16 M1=1,2
   16 FRC(M1)=FRC(M1)+F*AP(M1,L3,N1)
   18 L3=L3+1
   19 Z=Z/ST
   17 I2 = 1 + I2
      F1=FRC(1)/AREA
      F2=FRC(2)/AREA
      F3=SQRT(F1**2+F2**2)
      F4=DØTS
      IF (F3.NE.0.0) F4=ATAN(F2,F1)*RAD
   30 WRITE (6,47) N1,N2,F1,F2,F3,F4
   47 FØRMAT(1H0216,1P3E19.5,0P1F16.4)
      RETURN
      END
```

LSSURP

```
      SUBROUTINE LSSURP(X,Y,Z,N,ND,NG,A,IND)
      DIMENSION X(1),Y(1),Z(1),A(21),C(21,21),B(21)
      DIMENSION G(21),F(21)
      DIMENSION JLIM(6)
C     A POLYNOMIAL WITH THE COEFFICIENTS (A) IS FITTED BY LEAST SQUARES
C      TO THE VALUES (Z) AT THE POINTS (X,Y)
C     DEGREE ND IS GIVEN, MUST BE AT MOST 5
C     NUMBER OF POINTS N IS UNRESTRICTED
C     IND=0 FOR SUCCESSFUL FIT
      IND=0
      ILIM=ND+1
      IF(ILIM.GT.6)ILIM=6
      IJLIM=ILIM+1
      DO 2 I=1,ILIM
    2 JLIM(I)=IJLIM-I
    3 NT=0
      DO 4 I=1,ILIM
    4 NT=NT+JLIM(I)
      DO 5 I=1,NT
      B(I)=0.0
      DO 5 J=1,NT
    5 C(J,I)=0.0
      DO 8 K=1,N
      YP=1.0
      L=1
      DO 7 I=1,ILIM
      XYP=YP
      JL=JLIM(I)
      DO 6 J=1,JL
      G(L)=XYP
      XYP=X(K)*XYP
    6 L=L+1
    7 YP=Y(K)*YP
      DO 8 I=1,NT
      B(I)=B(I)+G(I)*Z(K)
      DO 8 J=1,NT
    8 C(J,I)=C(J,I)+G(J)*G(I)
```

LSSURP

```
      DO 10 I=1,NT
      F(I)=0.0
      DO 9 J=1,NT
    9 F(I)=AMAX1(F(I),ABS(C(J,I)))
      DO 10 J=1,NT
   10 C(J,I)=C(J,I)/F(I)
      K=MSIMER(21,NT,1,C,B)
      IF (K.EQ.1) GO TO 15
   11 DO 12 I=1,ILIM
      IP=ILIM+1-I
      IF (JLIM(IP)+IP.EQ.IJLIM) GO TO 13
   12 CONTINUE
      IJLIM=IJLIM-1
      IF (IJLIM.GT.1) GO TO 11
      IND =1
      RETURN
   13 JLIM(IP)=JLIM(IP)-1
      IF (JLIM(IP).EQ.0) ILIM=IP-1
      GO TO 3
   15 NG=NT
      K=1
      L=1
      DO 18 I=1,ILIM
      JL=JLIM(I)
      JM=JL+1
      JN=7-I
      DO 16 J=1,JL
      A(L)=B(K)/F(K)
      K=K+1
   16 L=L+1
      IF (JN.LE.JL) GO TO 18
      DO 17 J=JM,JN
      A(L)=0.0
   17 L=L+1
   18 CONTINUE
      IF (L.GT.21) RETURN
      DO 19 K=L,21
```

```
 19 A(K)=0.0
    RETURN
    END
```

ZDZ

```
      SUBROUTINE ZDZ(X,Y,N,A,Z,DZ)
      DIMENSION C(7),X(300),Y(300),Z(300),DZ(300)
      DIMENSION A(21)
      DATA C/0.0,1.0,2.0,3.0,4.0,5.0,6.0/
C     FINDS VALUES OF Z,DZ/DX AT THE COLLOCATION POINTS
      DO 50 IN = 1,N
      Z(IN) = 0.0
      DZ(IN) = 0.0
      YP = 1.0
      NG = 1
      DO 20 I=1,6
      JL=7-I
      XYP = YP
      DO 10 J = 1,JL
      Z(IN) = Z(IN) + A(NG)*XYP
      XYP = X(IN)*XYP
      IF(J.EQ.1) GO TO 5
      DZ(IN) = DZ(IN) + C(J)*A(NG)*XYQ
      XYQ = X(IN)*XYQ
      GO TO 10
 5    XYQ = YP
 10   NG = NG + 1
 20   YP = Y(IN)*YP
 50   CONTINUE
      WRITE (6,101)
 101  FORMAT(1H0,17X,1HX,20X,1HY,20X,1HZ,20X,2HDZ)
      DO 100 I = 1,N
 100  WRITE (6,105) X(I),Y(I),Z(I),DZ(I)
 105  FORMAT(1H ,10X,1PE15.4,5X,1PE15.4,6X,1PE15.4,7X,1PE15.4)
      RETURN
      END
```

CONSTS

```
      BLOCK DATA
      COMMON /CONST/ PI,PIOVR2,RAD,ZI(10)
C     USEFUL CONSTANTS
      DATA PI/3.14159265/,PIOVR2/1.57079633/,RAD/57.2957795/
      DATA ZI/1.,2.,3.,4.,5.,6.,7.,8.,9.,10./
      END
```

FXI

```
      FUNCTION FXI(U1,K)
C     EVALUATES THE KTH CHORDWISE PRESSURE MODE AT U1
      U=U1
      IF (1-K) 2,4,8
    2 FXI=1.0-U*U
      IF (K-5) 3,5,8
    3 IF (K-3) 8,5,7
    4 FXI=1.0-U
      GO TO 8
    5 FXI=-2.0*U*FXI
      IF (3-K) 6,8,8
    6 FXI=FXI*(4.0*U*U-2.0)
      GO TO 8
    7 FXI=FXI*(3.0-4.0*FXI)
    8 RETURN
      END
```

```
        SIMR                   K=MSIMER(N,L,LB,A,B)
*               SIMULTANEOUS EQUATION SUBROUTINE
*       SOLVES THE SYSTEM OF EQUATIONS  A*X=B.
*       TO USE, SET K=MSIMER(N,L,LB,A,B)
*         WHERE N IS THE NUMBER OF ROWS FOR WHICH A IS
*         DIMENSIONED, AND L IS THE NUMBER OF EQUATIONS.
*         LB IS THE NUMBER OF COLUMNS IN B.
*       K=1 DENOTES SUCCESSFUL SOLUTION
*       K=2 FOR A SINGULAR OR ILL-CONDITIONED MATRIX
*       K=3 IF IMPROPER DATA IS GIVEN.
*         TO AVOID THIS SIGNAL, L MUST BE POSITIVE AND AT MOST 100,
*         N MUST NOT BE LESS THAN L, A MUST NOT INCLUDE A ROW
*         OF ZEROS.
*       A IS DESTROYED.
*       IF K=1, THE SOLUTION IS RETURNED IN B
*
*      ENTRY
*
MSIMER SAVE    1,2,3,4,5,6,7
*
*      PROLOGUE
*
        CLA*    3,4
        PAX     0,1
        TXL     E1,1,0
        TXH     E1,1,100
        PCD     0,1
        STD     A4-1
        STD     A6-1
        STD     2A6-1
        STD     A16
        STD     A16+1
        STD     2A21
        STD     A12+1
        STD     2A26
        STD     2A26+1
        STD     2A37
        STD     2A37+1
```

```
        SIMR                        K=MSIMER(N,L,LB,A,B)

        STD     A12+2
        STD     A20
        STD     A22
        STD     A25+1
        STD     A25+2
        STD     A33
        STD     A36+1
        STD     A36+2
        SXD     A52,1
        TXI     *+1,1,1
        SCD     A32,1
        TXI     *+1,1,-2
        SCD     A22+1,1
        CLA*    5,4
        PAX     0,7
        SXA     1A6-1,7
        SXA     A14,7
        SXA     1A21-1,7
        SXA     A26,7
        SXA     A37,7
        CLA*    4,4
        PAX     0,1
        TXL     E1,1,0
A52     TXH     E1,1,**
        SXA     A2,1
        SXA     A5-1,1
        SXD     A9,1
        SXD     A12,1
        TXI     *+1,1,-1
        SXD     A7,1
        SXD     A36,1
        SXD     A18,1
        SXD     A38-1,1
        SXD     A21-1,1
        SXD     A23,1
        SXD     A25,1
        SXD     A31,1
```

```
        SIMR                  K=MSIMER(N,L,LB,A,B)

        SCD     A51,1
        SCD     A51+1,1
        TXI     *+1,1,1
        CLA     6,4
        PAC     0,3
        CLA     7,4
        PAC     0,5
A51     TXI     *+1,3, -L+1
        TXI     *+1,5, -L+1
*
*       NORMALIZATION OF ROWS
*
A2      AXT     L,2
        PXA     0,3
        PAX     0,6
        PAX     0,7
        PXA     0,0
A3      LDQ     A,6
        LRS     0
        TLQ     *+2
        XCA
        TNX     A4,2,1
        TXI     A3,6, -N
A4      TZE     E1
        STO     T
        CLA     =1.0
        FDP     T
        STQ     T
        AXT     L,2
A5      FMP     A,7
        STO     A,7
        LDQ     T
        TNX     A6,2,1
        TXI     A5,7, -N
A6      PXA     0,5
        PAX     0,6
        AXT     LB,4
```

```
         SIMR                        K=MSIMER(N,L,LB,A,B)
1A6      FMP     B,6                                                00051120
         STØ     B,6                                                00051130
         LDQ     T                                                  00051140
         TNX     2A6,4,1                                            00051150
         TXI     1A6,6, -N                                          00051160
2A6      TNX     A7,1,1                                             00051170
         TXI     *+1,3,1                                            00051180
         TXI     A2,5,1                                             00051190
A7       TXH     B7,2,L -1                                          00051200
*                                                                   00051210
*       SEARCH FØR MAXIMUM PIVØT IN CØLUMN                          00051220
*                                                                   00051230
         PXA     0,2                                                00051240
         PAX     0,1                                                00051250
         PXA     0,3                                                00051260
         PAX     0,6                                                00051270
         PXA     0,0                                                00051280
A8       LDQ     A,6                                                00051290
         LRS     0                                                  00051300
         TLQ     *+3                                                00051310
         XCA                                                        00051320
         SXA     A10,1                                              00051330
         TXI     *+1,1,1                                            00051340
A9       TXH     *+2,1,L                                            00051350
         TXI     A8,6,-1                                            00051360
         LDQ     TØL                                                00051370
         TLQ     *+2                                                00051380
         TRA     E3                                                 00051390
A10      AXT     **,1                                               00051400
         SXD     *+1,2                                              00051410
         TNX     A17,1,**                                           00051420
*                                                                   00051430
*       RØW INTERCHANGE                                             00051440
*                                                                   00051450
         PXA     0,3                                                00051460
         PAX     0,6                                                00051470
         PAX     0,7                                                00051480
```

```
        SIMR                       K=MSIMER(N,L,LB,A,B)

        SCD     *+1,1
        TXI     *+1,7,**
        PXA     0,2
        PAX     0,4
A11     CLA     A,6
        LDQ     A,7
        STO     A,7
        STQ     A,6
        TXI     *+1,4,1
A12     TXH     A13,4,L
        TXI     *+1,6, -N
        TXI     A11,7, -N
A13     PXA     0,5
        PAX     0,6
        PAX     0,7
A14     AXT     LB,4
        SCD     *+1,1
        TXI     *+1,6,**
A15     CLA     B,7
        LDQ     B,6
        STO     B,6
        STQ     B,7
        TNX     A17,4,1
A16     TXI     *+1,6, -N
        TXI     A15,7, -N
*
*      DIVISION OF ROW BY PIVOT
*
A17     CLA     =1.0
        FDP     A,3
        STQ     AM
A18     TXH     A21,2,L -1
        PXA     0,2
        PAX     0,4
        PXA     0,3
        PAX     0,6
A20     TXI     *+1,6, -N
```

```
        SIMR              K=MSIMER(N,L,LB,A,B)

        LDQ     A,6
        FMP     AM
        STO     A,6
        TXI     *+1,4,1
        TXL     A20,4,L -1
A21     PXA     0,5
        PAX     0,6
        AXT     LB,4
1A21    LDQ     B,6
        FMP     AM
        STO     B,6
        TNX     *+2,4,1
2A21    TXI     1A21,6, -N
*
*       ROW REDUCTION
*
        PXA     0,2
        PAX     0,1
        PAX     0,4
        TNX     A31,1,1
        PXA     0,3
        PAX     0,6
        PAX     0,7
        STA     A29
        SXA     A26+1,5
        SXA     3A26,5
        TXI     *+1,6,1
A22     TXI     *+1,7, -N
        TXI     *+1,3, -N+1
        SXA     A28,7
A23     TXH     A26,2,L -1
        SXA     A27,3
A24     LDQ     A,6
        FMP     A,7
        CHS
        FAD     A,3
        STO     A,3
```

```
         SIMR          K=MSIMER(N,L,LB,A,B)
         TXI    *+1,4,1                                00052230
A25      TXH    A27,4,L -1                             00052240
         TXI    *+1,3, -N                              00052250
         TXI    A24,7, -N                              00052260
A27      AXT    **,3                                   00052270
A26      AXT    LB,4                                   00052280
         AXT    **,7                                   00052290
         TXI    *+1,7,1                                00052300
         SXA    *-2,7                                  00052310
1A26     LDQ    A,6                                    00052320
         FMP    B,5                                    00052330
         CHS                                           00052340
         FAD    B,7                                    00052350
         STO    B,7                                    00052360
         TNX    3A26,4,1                               00052370
2A26     TXI    *+1,5, -N                              00052380
         TXI    1A26,7, -N                             00052390
3A26     AXT    **,5                                   00052400
         TNX    A29,1,1                                00052410
A28      AXT    **,7                                   00052420
         PXA    0,2                                    00052430
         PAX    0,4                                    00052440
         TXI    *+1,3,1                                00052450
         TXI    A23,6,1                                00052460
A29      AXT    **,3                                   00052470
A31      TXH    A43,2,L -1                             00052480
         PXA    0,2                                    00052490
         PAX    0,1                                    00052500
         PAX    0,4                                    00052510
         PXA    0,3                                    00052520
         PAX    0,6                                    00052530
         PAX    0,7                                    00052540
A32      TXI    *+1,3, -N-1                            00052550
         TXI    *+1,6,-1                               00052560
A33      TXI    *+1,7, -N                              00052570
         SXA    A37+1,5                                00052580
         SXA    A41,5                                  00052590
```

```
        SIMR              K=MSIMER(N,L,LB,A,B)

        SXA     A40,3                                       00052600
        SXA     A39,7                                       00052610
A34     SXA     A38,3                                       00052620
A35     LDQ     A,6                                         00052630
        FMP     A,7                                         00052640
        CH                                                  00052650
        FAD     A,3                                         00052660
        STØ     A,3                                         00052670
        TXI     *+1,4,1                                     00052680
A36     TXH     A37,4,L -1                                  00052690
        TXI     *+1,3, -N                                   00052700
        TXI     A35,7, -N                                   00052710
A37     AXT     LB,4                                        00052720
        AXT     **,7                                        00052730
        TXI     *+1,7,-1                                    00052740
        SXA     *-2,7                                       00052750
1A37    LDQ     A,6                                         00052760
        FMP     B,5                                         00052770
        CHS                                                 00052780
        FAD     B,7                                         00052790
        STØ     B,7                                         00052800
        TNX     A41,4,1                                     00052810
2A37    TXI     *+1,5, -N                                   00052820
        TXI     1A37,7, -N                                  00052830
A41     AXT     **,5                                        00052840
        TXI     *+1,1,1                                     00052850
        TXH     A40,1,L -1                                  00052860
A38     AXT     **,3                                        00052870
A39     AXT     **,7                                        00052880
        PXA     0,2                                         00052890
        PAX     0,4                                         00052900
        TXI     *+1,3,-1                                    00052910
        TXI     A34,6,-1                                    00052920
A40     AXT     **,3                                        00052930
        TXI     *+1,5,-1                                    00052940
        TXI     A7,2,1                                      00052950
A45     CLA     =1                                          00052960
```

```
         SIMR                    K=MSIMER(N,L,LB,A,B)

         TRA      MSIMER+1                                            00052970
*                                                                     00052980
*       BRANCH FOR LAST ROW                                           00052990
*                                                                     00053000
B7       CLA      A,3                                                 00053010
         SSP                                                          00053020
         LDQ      TOL                                                 00053030
         TLQ      A17                                                 00053040
*                                                                     00053050
*       ERROR BRANCHES                                                00053060
*                                                                     00053070
E3       CLA      =2                                                  00053080
         TRA      MSIMER+1                                            00053090
E1       CLA      =3                                                  00053100
         TRA      MSIMER+1                                            00053110
*                                                                     00053120
*       STORAGE                                                       00053130
*                                                                     00053140
TOL      OCT      151400000000                                        00053150
AM       EQU      E.1                                                 00053160
T        EQU      E.2                                                 00053170
A        EQU      0                                                   00053180
B        EQU      0                                                   00053190
L        EQU      0                                                   00053200
N        EQU      0                                                   00053210
LB       EQU      0                                                   00053220
         END                                                          00053230
```

```
         SIMC                  K=MSIMEC(N,L,LB,A,B)

*        CØMPLEX SIMULTANEØUS EQUATIØN SUBRØUTINE
*        SØLVES THE SYSTEM ØF CØMPLEX EQUATIØNS A*X=B.
*        TØ USE, SET K=MSIMEC(N,L,LB,A,B)
*          WHERE N IS THE NUMBER ØF RØWS FØR WHICH A IS
*          DIMENSIØNED, AND L IS THE NUMBER ØF EQUATIØNS.
*          LB IS THE NUMBER ØF CØLUMNS IN B.
*        K=1 DENØTES SUCCESSFUL SØLUTIØN
*        K=2 FØR A SINGULAR ØR ILL-CØNDITIØNED MATRIX
*        K=3 IF IMPRØPER DATA IS GIVEN.
*          TØ AVØID THIS SIGNAL, L MUST BE PØSITIVE AND AT MØST 100,
*          N MUST NØT BE LESS THAN L, A MUST NØT INCLUDE A RØW
*          ØF ZERØS.
*        A IS DESTRØYED.
*        IF K=1, THE SØLUTIØN IS RETURNED IN B
*
*       ENTRY
*
MSIMEC SAVE     1,2,3,4,5,6,7
*
*       PRØLØGUE
*
        CLA*    3,4
        ALS     1
        PAX     0,1
        TXL     E1,1,1
        TXH     E1,1,200
        PCD     0,1
        STD     A4-1
        STD     A6-1
        STD     2A6-1
        STD     A16
        STD     A16+1
        STD     2A21
        STD     A12+1
        STD     2A26
        STD     2A26+1
        STD     2A37
```

```
         SIMC              K=MSIMEC(N,L,LB,A,B)

         STD      2A37+1                                00053620
         STD      A12+2                                 00053630
         STD      A20                                   00053640
         STD      A22                                   00053650
         STD      A25+1                                 00053660
         STD      A25+2                                 00053670
         STD      A33                                   00053680
         STD      A36+1                                 00053690
         STD      A36+2                                 00053700
         SXD      A52,1                                 00053710
         TXI      *+1,1,2                               00053720
         SCD      A32,1                                 00053730
         TXI      *+1,1,-4                              00053740
         SCD      A22+1,1                               00053750
         CLA*     5,4                                   00053760
         PAX      0,7                                   00053770
         SXA      1A6-1,7                               00053780
         SXA      A14,7                                 00053790
         SXA      1A21-1,7                              00053800
         SXA      A26,7                                 00053810
         SXA      A37,7                                 00053820
         CLA*     4,4                                   00053830
         ALS      1                                     00053840
         PAX      0,1                                   00053850
         TXL      E1,1,1                                00053860
A52      TXH      E1,1,**                               00053870
         SXA      A2,1                                  00053880
         SXA      A5-1,1                                00053890
         SXD      A9,1                                  00053900
         SXD      A12,1                                 00053910
         TXI      *+1,1,-2                              00053920
         SXD      A7,1                                  00053930
         SXD      A36,1                                 00053940
         SXD      A18,1                                 00053950
         SXD      A38-1,1                               00053960
         SXD      A21-1,1                               00053970
         SXD      A23,1                                 00053980
```

```
         SIMC                   K=MSIMEC(N,L,LB,A,B)

         SXD      A25,1                                                 00053990
         SXD      A31,1                                                 00054000
         SCD      A51,1                                                 00054010
         SCD      A51+1,1                                               00054020
         TXI      *+1,1,2                                               00054030
         CLA      6,4                                                   00054040
         PAC      0,3                                                   00054050
         CLA      7,4                                                   00054060
         PAC      0,5                                                   00054070
A51      TXI      *+1,3,2L -2                                           00054080
         TXI      *+1,5,2L -2                                           00054090
*                                                                       00054100
*       NØRMALIZATIØN ØF RØWS                                           00054110
*                                                                       00054120
A2       AXT      2L,2                                                  00054130
         PXA      0,3                                                   00054140
         PAX      0,6                                                   00054150
         PAX      0,7                                                   00054160
         PXA      0,0                                                   00054170
A3       LDQ      A,6                                                   00054180
         LRS      0                                                     00054190
         TLQ      *+2                                                   00054200
         XCA                                                            00054210
         LDQ      A+1,6                                                 00054220
         LRS      0                                                     00054230
         TLQ      *+2                                                   00054240
         XCA                                                            00054250
         TNX      A4,2,2                                                00054260
         TXI      A3,6,2N                                               00054270
A4       TZE      E1                                                    00054280
         STØ      T                                                     00054290
         CLA      =1.0                                                  00054300
         FDP      T                                                     00054310
         STQ      T                                                     00054320
         AXT      2L,2                                                  00054330
A5       FMP      A,7                                                   00054340
         STØ      A,7                                                   00054350
```

```
        SIMC               K=MSIMEC(N,L,LB,A,B)

        LDQ     T
        FMP     A+1,7
        STØ     A+1,7
        LDQ     T
        TNX     A6,2,2
        TXI     A5,7,2N
A6      PXA     0,5
        PAX     0,6
        AXT     LB,4
1A6     FMP     B,6
        STØ     B,6
        LDQ     T
        FMP     B+1,6
        STØ     B+1,6
        LDQ     T
        TNX     2A6,4,1
        TXI     1A6,6,2N
2A6     TNX     A7,1,2
        TXI     *+1,3,2
        TXI     A2,5,2
A7      TXH     B7,2,2L  -2
*
*       SEARCH FØR MAXIMUM PIVØT  IN CØLUMN
*
        PXA     0,2
        PAX     0,1
        PXA     0,3
        PAX     0,6
        PXA     0,0
A8      LDQ     A,6
        LRS     0
        TLQ     *+3
        XCA
        SXA     A10,1
        LDQ     A+1,6
        LRS     0
        TLQ     *+3
```

```
        SIMC              K=MSIMEC(N,L,LB,A,B)

        XCA                                                      00054730
        SXA     A10,1                                            00054740
        TXI     *+1,1,2                                          00054750
A9      TXH     *+2,1,2L                                         00054760
        TXI     A8,6,-2                                          00054770
        LDQ     TØL                                              00054780
        TLQ     *+2                                              00054790
        TRA     E3                                               00054800
A10     AXT     **,1                                             00054810
        SXD     *+1,2                                            00054820
        TNX     A17,1,**                                         00054830
*                                                                00054840
*     RØW INTERCHANGE                                            00054850
*                                                                00054860
        PXA     0,3                                              00054870
        PAX     0,6                                              00054880
        PAX     0,7                                              00054890
        SCD     *+1,1                                            00054900
        TXI     *+1,7,**                                         00054910
        PXA     0,2                                              00054920
        PAX     0,4                                              00054930
A11     CLA     A,6                                              00054940
        LDQ     A,7                                              00054950
        STØ     A,7                                              00054960
        STQ     A,6                                              00054970
        CLA     A+1,6                                            00054980
        LDQ     A+1,7                                            00054990
        STØ     A+1,7                                            00055000
        STQ     A+1,6                                            00055010
        TXI     *+1,4,2                                          00055020
A12     TXH     A13,4,2L                                         00055030
        TXI     *+1,6,2N                                         00055040
        TXI     A11,7,2N                                         00055050
A13     PXA     0,5                                              00055060
        PAX     0,6                                              00055070
        PAX     0,7                                              00055080
A14     AXT     LB,4                                             00055090
```

```
         SIMC             K=MSIMEC(N,L,LB,A,B)

         SCD      *+1,1                                           00055100
         TXI      *+1,6,**                                        00055110
A15      CLA      B,7                                             00055120
         LDQ      B,6                                             00055130
         STO      B,6                                             00055140
         STQ      B,7                                             00055150
         CLA      B+1,7                                           00055160
         LDQ      B+1,6                                           00055170
         STO      B+1,6                                           00055180
         STQ      B+1,7                                           00055190
         TNX      A17,4,1                                         00055200
A16      TXI      *+1,6,2N                                        00055210
         TXI      A15,7,2N                                        00055220
*                                                                 00055230
*       DIVISION OF ROW BY PIVOT                                  00055240
*                                                                 00055250
A17      NZT      A,3                                             00055260
         TRA      B1                                              00055270
         LDQ      A,3                                             00055280
         FMP      A,3                                             00055290
         STO      T                                               00055300
         LDQ      A+1,3                                           00055310
         FMP      A+1,3                                           00055320
         FAD      T                                               00055330
         STO      T                                               00055340
         CLA      A,3                                             00055350
         FDP      T                                               00055360
         STQ      AM                                              00055370
         CLA      A+1,3                                           00055380
         FDP      T                                               00055390
         STQ      AN                                              00055400
A18      TXH      A21,2,2L -2                                     00055410
         PXA      0,2                                             00055420
         PAX      0,4                                             00055430
         PXA      0,3                                             00055440
         PAX      0,6                                             00055450
A20      TXI      *+1,6,2N                                        00055460
```

```
        SIMC              K=MSIMEC(N,L,LB,A,B)
        LDQ     A,6                                     00055470
        FMP     AN                                      00055480
        STØ     T                                       00055490
        LDQ     A+1,6                                   00055500
        FMP     AM                                      00055510
        FSB     T                                       00055520
        LDQ     A+1,6                                   00055530
        STØ     A+1,6                                   00055540
        FMP     AN                                      00055550
        STØ     T                                       00055560
        LDQ     A,6                                     00055570
        FMP     AM                                      00055580
        FAD     T                                       00055590
        STØ     A,6                                     00055600
        TXI     *+1,4,2                                 00055610
        TXL     A20,4,2L -2                             00055620
A21     PXA     0,5                                     00055630
        PAX     0,6                                     00055640
        AXT     LB,4                                    00055650
1A21    LDQ     B,6                                     00055660
        FMP     AN                                      00055670
        STØ     T                                       00055680
        LDQ     B+1,6                                   00055690
        FMP     AM                                      00055700
        FSB     T                                       00055710
        LDQ     B+1,6                                   00055720
        STØ     B+1,6                                   00055730
        FMP     AN                                      00055740
        STØ     T                                       00055750
        LDQ     B,6                                     00055760
        FMP     AM                                      00055770
        FAD     T                                       00055780
        STØ     B,6                                     00055790
        TNX     *+2,4,1                                 00055800
2A21    TXI     1A21,6,2N                               00055810
*                                                       00055820
*      RØW REDUCTIØN                                    00055830
```

```
         SIMC          K=MSIMEC(N,L,LB,A,B)
*
         PXA     0,2
         PAX     0,1
         PAX     0,4
         TNX     A31,1,2
         PXA     0,3
         PAX     0,6
         PAX     0,7
         STA     A29
         SXA     A26+1,5
         SXA     3A26,5
         TXI     *+1,6,2
A22      TXI     *+1,7,2N
         TXI     *+1,3,2N -2
         SXA     A28,7
A23      TXH     A26,2,2L -2
         SXA     A27,3
A24      LDQ     A,6
         FMP     A,7
         STO     T
         LDQ     A+1,6
         FMP     A+1,7
         FSB     T
         FAD     A,3
         STO     A,3
         LDQ     A,6
         FMP     A+1,7
         STO     T
         LDQ     A+1,6
         FMP     A,7
         FAD     T
         CHS
         FAD     A+1,3
         STO     A+1,3
         TXI     *+1,4,2
A25      TXH     A27,4,2L -2
         TXI     *+1,3,2N
```

```
          SIMC                K=MSIMEC(N,L,LB,A,B)

          TXI     A24,7,2N                                        00056210
A27       AXT     **,3                                            00056220
A26       AXT     LB,4                                            00056230
          AXT     **,7                                            00056240
          TXI     *+1,7,2                                         00056250
          SXA     *-2,7                                           00056260
1A26      LDQ     A,6                                             00056270
          FMP     B,5                                             00056280
          STØ     T                                               00056290
          LDQ     A+1,6                                           00056300
          FMP     B+1,5                                           00056310
          FSB     T                                               0005632C
          FAD     B,7                                             00056330
          STØ     B,7                                             00056340
          LDQ     A,6                                             00056350
          FMP     B+1,5                                           00056360
          STØ     T                                               00056370
          LDQ     A+1,6                                           00056380
          FMP     B,5                                             00056390
          FAD     T                                               00056400
          CHS                                                     00056410
          FAD     B+1,7                                           00056420
          STØ     B+1,7                                           00056430
          TNX     3A26,4,1                                        00056440
2A26      TXI     *+1,5,2N                                        00056450
          TXI     1A26,7,2N                                       00056460
3A26      AXT     **,5                                            00056470
          TNX     A29,1,2                                         00056480
A28       AXT     **,7                                            00056490
          PXA     0,2                                             00056500
          PAX     0,4                                             00056510
          TXI     *+1,3,2                                         00056520
          TXI     A23,6,2                                         00056530
A29       AXT     **,3                                            00056540
A31       TXH     A43,2,2L -2                                     00056550
          PXA     0,2                                             00056560
          PAX     0,1                                             00056570
```

```
          SIMC                          K=MSIMEC(N,L,LB,A,B)
          PAX     0,4                                                    00056580
          PXA     0,3                                                    00056590
          PAX     0,6                                                    00056600
          PAX     0,7                                                    00056610
A32       TXI     *+1,3,2N +2                                            00056620
          TXI     *+1,6,-2                                               00056630
A33       TXI     *+1,7,2N                                               00056640
          SXA     A37+1,5                                                00056650
          SXA     A41,5                                                  00056660
          SXA     A40,3                                                  00056670
          SXA     A39,7                                                  00056680
A34       SXA     A38,3                                                  00056690
A35       LDQ     A,6                                                    00056700
          FMP     A,7                                                    00056710
          STO     T                                                      00056720
          LDQ     A+1,6                                                  00056730
          FMP     A+1,7                                                  00056740
          FSB     T                                                      00056750
          FAD     A,3                                                    00056760
          STO     A,3                                                    00056770
          LDQ     A,6                                                    00056780
          FMP     A+1,7                                                  00056790
          STO     T                                                      00056800
          LDQ     A+1,6                                                  00056810
          FMP     A,7                                                    00056820
          FAD     T                                                      00056830
          CHS                                                            00056840
          FAD     A+1,3                                                  00056850
          STO     A+1,3                                                  00056860
          TXI     *+1,4,2                                                00056870
A36       TXH     A37,4,2L -2                                            00056880
          TXI     *+1,3,2N                                               00056890
          TXI     A35,7,2N                                               00056900
A37       AXT     LB,4                                                   00056910
          AXT     **,7                                                   00056920
          TXI     *+1,7,-2                                               00056930
          SXA     *-2,7                                                  00056940
```

```
        SIMC                     K=MSIMEC(N,L,LB,A,B)
1A37    LDQ     A,6                                        00056950
        FMP     B,5                                        00056960
        STØ     T                                          00056970
        LDQ     A+1,6                                      00056980
        FMP     B+1,5                                      00056990
        FSB     T                                          00057000
        FAD     B,7                                        00057010
        STØ     B,7                                        00057020
        LDQ     A,6                                        00057030
        FMP     B+1,5                                      00057040
        STØ     T                                          00057050
        LDQ     A+1,6                                      00057060
        FMP     B,5                                        00057070
        FAD     T                                          00057080
        CHS                                                00057090
        FAD     B+1,7                                      00057100
        STØ     B+1,7                                      00057110
        TNX     A41,4,1                                    00057120
2A37    TXI     *+1,5,2N                                   00057130
        TXI     1A37,7,2N                                  00057140
A41     AXT     **,5                                       00057150
        TXI     *+1,1,2                                    00057160
        TXH     A40,1,2L -2                                00057170
A38     AXT     **,3                                       00057180
A39     AXT     **,7                                       00057190
        PXA     0,2                                        00057200
        PAX     0,4                                        00057210
        TXI     *+1,3,-2                                   00057220
        TXI     A34,6,-2                                   00057230
A40     AXT     **,3                                       00057240
        TXI     *+1,5,-2                                   00057250
        TXI     A7,2,2                                     00057260
A43     CLA     =1                                         00057270
        TRA     MSIMEC+1                                   00057280
B1      STZ     AM                                         00057290
        CLA     =1.0                                       00057300
        FDP     A+1,3                                      00057310
```

```
         SIMC                K=MSIMEC(N,L,LB,A,B)

         STQ      AN
         TRA      A18
*
*      BRANCH FOR LAST ROW
*
B7       CLA      A,3
         SSP
         LDQ      TOL
         TLQ      A17+2
         CLA      A+1,3
         SSP
         TLQ      A17
*
*      ERROR BRANCHES
*
E3       CLA      =2
         TRA      MSIMEC+1
E1       CLA      =3
         TRA      MSIMEC+1
*
*      STORAGE
*
TOL      OCT      151400000000
AM
AN
T
A        EQU      0
B        EQU      0
2L       EQU      0
2N       EQU      0
LB       EQU      0
         END
```

```
        BESEL   700                 N.A.A. SUBROUTINE LIBRARY

*               BESSEL FUNCTIONS - JNX, INX, YNX, KNX                        BESE0010
*               CALLING SEQUENCE IS BESEL (X,N,P)                            BESE0020
*               N IS ORDER                                                   BESE0030
*               P EQUALS 3 FOR KNX                                           BESE0060
 BESEL  SAVE                                                                 BESE0090
        CLA*    3,4                                                          BESE0100
        TMI     ERR                 ERROR ON-X                               BESE0110
        STO     FRE                 X TO FRE                                 BESE0120
        FDP     FL02                X/2                                      BESE0130
        STQ     FRE+24                                                       BESE0140
        XCA                                                                  BESE0150
        TZE     AAA                                                          BESE0160
        SXA     X4,4                                                         BESE0170
        CALL    ALOG(FRE+24)                                                 BESE0180
    X4  AXT     **,4                                                         BESE0190
   AAA  STO     FRE+6                                                        BESE0200
        CLA*    4,4                                                          BESE0210
        STO     FRE+14              N AT B35                                 BESE0220
        ACL     MAGNM                                                        BESE0230
        FAD     MAGNM                                                        BESE0240
        STO     FRE+1               N IN FL. PT.                             BESE0250
        TRA     K                                                            BESE0320
*               ERROR RETURN                                                 BESE0340
   ERR  CALL    .FXEM.(57)                                                   BESE0350
*               ESCAPE - NORMAL                                              BESE0370
   ESC  RETURN  BESEL                                                        BESE0380
*               TEST FOR SWITCHOVER, SERIES TO ASYMPTOTIC EXPANSION          BESE2580
 K      CLM                                                                  BESE2590
        SLW     FRE+2                                                        BESE2600
        SLW     FRE+16                                                       BESE2610
        SLW     FRE+17                                                       BESE2620
        CLA     FRE                                                          BESE2630
        TZE     ERR                                                          BESE2640
        FSB     KSWO                                                         BESE2650
        TPL     KA                                                           BESE2660
 K1     TSX     A,4                                                          BESE2670
        STO     FRE+18                                                       BESE2680
```

```
      TSX    B,4                                      BESE2690
      XCA                                             BESE2700
      FMP    FRE+18                                   BESE2710
      FAD    FRE+16                                   BESE2720
      STØ    FRE+16                                   BESE2730
      CAS    FRE+17                                   BESE2740
      TRA    K2                                       BESE2750
      TRA    K3                                       BESE2760
K2    STØ    FRE+17                                   BESE2770
      CLA    FRE+2                                    BESE2780
      FAD    FLØ1                                     BESE2790
      STØ    FRE+2                                    BESE2800
      TRA    K1                                       BESE2810
K3    STØ    FRE+18                                   BESE2820
      CLM                                             BESE2830
      SLW    FRE+2                                    BESE2840
      SLW    FRE+16                                   BESE2850
      CLA    FLØ1                                     BESE2860
      STØ    FRE+15                                   BESE2870
K4    TSX    C,4                                      BESE2880
      XCA                                             BESE2890
      FMP    FRE+15                                   BESE2900
      FAD    FRE+16                                   BESE2910
      STØ    FRE+16                                   BESE2920
      CLS    FRE+15                                   BESE2930
      STØ    FRE+15                                   BESE2940
      CLA    FRE+2                                    BESE2950
      FAD    FLØ1                                     BESE2960
      STØ    FRE+2                                    BESE2970
      FSB    FRE+1                                    BESE2980
      TMI    K4                                       BESE2990
K6    CLA    FRE+14                                   BESE3000
      LBT                                             BESE3010
      TRA    K7                                       BESE3020
      CLA    FLØ1                                     BESE3030
      TRA    K8                                       BESE3040
K7    CLS    FLØ1                                     BESE3050
```

```
K8   XCA                                     BESE3060
     FMP   FRE+18                            BESE3070
     FAD   FRE+16                            BESE3080
     FDP   FLØ2                              BESE3090
     STQ   FRE+21                            BESE3100
     CLA   FRE+21                            BESE3110
     TRA   ESC                               BESE3120
*          ASYMPTØTIC EXPANSIØN ØF KNX       BESE3130
KA   CLS   FRE                               BESE3140
     FAD   FLØ88                             BESE3150
     TMI   KA1                               BESE3160
     CLS   FRE                               BESE3170
KA1  STØ   FRE+24                            BESE3180
     CALL  EXP(FRE+24)                       BESE3190
     STØ   FRE+21                            BESE3200
*          CALCULATE SQUARE RT ØF PI/2X      BESE3210
     CLA   FRE                               BESE3220
     FAD   FRE                               BESE3230
     STØ   FRE+18                            BESE3240
KA3  CLA   PI                                BESE3250
     FDP   FRE+18                            BESE3260
     STQ   FRE+19                            BESE3270
     CALL  SQRT(FRE+19)                      BESE3280
     XCA                                     BESE3290
     FMP   FRE+21                            BESE3300
     STØ   FRE+21                            BESE3310
     CLA   FLØ1                              BESE3320
     STØ   FRE+3                             BESE3330
     STØ   FRE+9                             BESE3340
     LDQ   FRE                               BESE3350
     FMP   FLØ8                              BESE3360
     STØ   FRE+18                            BESE3370
     STØ   FRE+11                            BESE3380
     LDQ   FRE+1                             BESE3390
     FMP   FRE+1                             BESE3400
     XCA                                     BESE3410
     FMP   FLØ4                              BESE3420
```

```
       STØ    FRE+7                                  BESE3430
       FSB    FLØ1                                   BESE3440
       FDP    FRE+11                                 BESE3450
       STQ    FRE+8                                  BESE3460
       CLA    FRE+8                                  BESE3470
KA4    FAD    FRE+16                                 BESE3480
       STØ    FRE+16                                 BESE3490
       CAS    FRE+17                                 BESE3500
       TRA    KA5                                    BESE3510
       TRA    KA6                                    BESE3520
KA5    STØ    FRE+17                                 BESE3530
       CLA    FRE+3                                  BESE3540
       FAD    FLØ1                                   BESE3550
       STØ    FRE+3                                  BESE3560
       TSX    D,4                                    BESE3570
       TRA    KA4                                    BESE3580
KA6    FAD    FLØ1                                   BESE3590
       XCA                                           BESE3600
       FMP    FRE+21                                 BESE3610
       STØ    FRE+19                                 BESE3620
       CLS    FRE                                    BESE3630
       FAD    FLØ88                                  BESE3640
       TMI    KA7                                    BESE3650
       CLA    FRE+19                                 BESE3660
       TRA    ESC                                    BESE3670
KA7    LDQ    FRE+19                                 BESE3680
KA8    FMP    EM88                                   BESE3690
       TRA    ESC                                    BESE3700
*             FACTØRIAL                              BESE3710
*             M IN ACC                               BESE3720
*             LEAVE M/ IN ACC                        BESE3730
FACT   TNZ    FACT+3                                 BESE3740
       CLA    FLØ1                                   BESE3750
       TRA    1,4                                    BESE3760
       STØ    FRE+4                                  BESE3770
       STØ    FRE+5                                  BESE3780
       CLA    FRE+4                                  BESE3790
```

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        FSB     FLØ1                                      BESE3800
        TZE     FACT+12                                   BESE3810
        STØ     FRE+4                                     BESE3820
        LDQ     FRE+5                                     BESE3830
        FMP     FRE+4                                     BESE3840
        TRA     FACT+4                                    BESE3850
        CLA     FRE+5                                     BESE3860
        TRA     1,4                                       BESE3870
*               SUBR.                                     BESE3880
*               X/2,N+2S/S/N+S/                           BESE3890
*               X/2,N AND S IN STØRAGE                    BESE3900
*               RESULT LEFT IN ACC                        BESE3910
 A      SXD     FRE+10,4                                  BESE3920
        CLA     FRE+1                                     BESE3930
        FAD     FRE+2                                     BESE3940
        STØ     FRE+7                                     BESE3950
        FAD     FRE+2                                     BESE3960
        STØ     FRE+8                                     BESE3970
        CLA     FRE+2                                     BESE3980
        TSX     FACT,4          13.1.6                    BESE3990
        STØ     FRE+9           13.1.7                    BESE4000
        CLA     FRE+7           .8                        BESE4010
        TSX     FACT,4          .9                        BESE4020
        XCA                                               BESE4030
        FMP     FRE+9                                     BESE4040
        STØ     FRE+9                                     BESE4050
        CLA     FRE+8                                     BESE4060
        TNZ     A1                                        BESE4070
        CLA     FLØ1                                      BESE4080
        TRA     A2                                        BESE4090
 A1     LDQ     FRE+6                                     BESE4100
        FMP     FRE+8                                     BESE4110
        STØ     FRE+24                                    BESE4120
        CALL    EXP(FRE+24)                               BESE4130
 A2     FDP     FRE+9                                     BESE4140
        STQ     FRE+9                                     BESE4150
        CLA     FRE+9                                     BESE4160
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        LXD    FRE+10,4                                          BESE4170
        TRA    1,4                                               BESE4180
*              PARENTHETICAL SUBR                                BESE4190
*              X/2,N,AND S                                       BESE4200
*              LEAVE RESULT IN ACC.                              BESE4210
 B      SXD    FRE+10,4                                          BESE4220
        CLA    FRE+2                                             BESE4230
        TSX    B1,4                                              BESE4240
        CHS                                                      BESE4250
        STØ    FRE+8                                             BESE4260
        CLA    FRE+1                                             BESE4270
        FAD    FRE+2                                             BESE4280
        TSX    B1,4                                              BESE4290
        CHS                                                      BESE4300
        FAD    FRE+8                                             BESE4310
        FAD    2GAM                                              BESE4320
        STØ    FRE+8                                             BESE4330
        LDQ    FRE+6                                             BESE4340
        FMP    FLØ2                                              BESE4350
        FAD    FRE+8                                             BESE4360
        LXD    FRE+10,4                                          BESE4370
        TRA    1,4                                               BESE4380
*              SUMMATIØN ØF 1/Y FRØM R EQUALS 1 TØ LIMIT         BESE4390
 B1     STØ    FRE+3                                             BESE4400
        TZE    1,4                                               BESE4410
        CLM                                                      BESE4420
        SLW    FRE+9                                             BESE4430
        CLA    FLØ1                                              BESE4440
        FDP    FRE+3                                             BESE4450
        STQ    FRE+11                                            BESE4460
        CLA    FRE+11                                            BESE4470
        FAD    FRE+9                                             BESE4480
        STØ    FRE+9                                             BESE4490
        CLA    FRE+3                                             BESE4500
        FSB    FLØ1                                              BESE4510
        TZE    B1+15                                             BESE4520
        STØ    FRE+3                                             BESE4530
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         TRA     B1+4                                                  BESE4540
         CLA     FRE+9                                                 BESE4550
         TRA     1,4                                                   BESE4560
*                SUBROUTINE FOR LAST TERM OF SERIES OF YN AND KN       BESE4570
*                X/2,N AND S IN STORAGE                                BESE4580
*                LEAVE RESULT IN ACCUMULATOR                           BESE4590
 C       CLA     FRE+1                                                 BESE4600
         FSB     FL01                                                  BESE4610
         TPL     C3                                                    BESE4620
         ZAC                                                           BESE4630
         TRA     1,4                                                   BESE4640
 C3      SXD     FRE+10,4                                              BESE4650
         CLA     FRE+2                                                 BESE4660
         TSX     FACT,4                                                BESE4670
         STO     FRE+7                                                 BESE4680
         CLA     FRE+1                                                 BESE4690
         FSB     FRE+2                                                 BESE4700
         FSB     FL01                                                  BESE4710
         TSX     FACT,4                                                BESE4720
         FDP     FRE+7                                                 BESE4730
         STQ     FRE+7                                                 BESE4740
         CLA     FRE+2                                                 BESE4750
         FAD     FRE+2                                                 BESE4760
         FSB     FRE+1                                                 BESE4770
         TNZ     C1                                                    BESE4780
         LDQ     FL01                                                  BESE4790
         TRA     C2                                                    BESE4800
 C1      STO     FRE+8                                                 BESE4810
         LDQ     FRE+6                                                 BESE4820
         FMP     FRE+8                                                 BESE4830
         STO     FRE+24                                                BESE4840
         CALL    EXP(FRE+24)                                           BESE4850
         XCA                                                           BESE4860
 C2      FMP     FRE+7                                                 BESE4870
         LXD     FRE+10,4                                              BESE4880
         TRA     1,4                                                   BESE4890
*                LEAVE RESULT IN ACC.                                  BESE4900
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D       LDQ     FRE+3                                   BESE4910
        FMP     FRE+18                                  BESE4920
        STO     FRE+11                                  BESE4930
        CLA     FRE+9                                   BESE4940
        FAD     FL02                                    BESE4950
        STO     FRE+9                                   BESE4960
        LDQ     FRE+9                                   BESE4970
        FMP     FRE+9                                   BESE4980
        SSM                                             BESE4990
        FAD     FRE+7                                   BESE5000
        FDP     FRE+11                                  BESE5010
        STQ     FRE+2                                   BESE5020
        CLA     FRE+2                                   BESE5030
        ADD     FL01                                    BESE5040
        TMI     D1                                      BESE5050
D2      LDQ     FRE+2                                   BESE5060
        FMP     FRE+8                                   BESE5070
        STO     FRE+8                                   BESE5080
        TRA     1,4                                     BESE5090
D1      LDQ     FRE+2                                   BESE5100
        FMP     FRE+8                                   BESE5110
        FAD     FRE+16                                  BESE5120
        FAD     FRE+16                                  BESE5130
        FDP     FL02                                    BESE5140
        STQ     FRE+16                                  BESE5150
        CLA     FRE+16                                  BESE5160
        TRA     2,4                                     BESE5170
*               CONSTANTS FOR BESSEL FUNCTIONS          BESE6710
KSW0    DEC     3.7                                     BESE6740
2GAM    DEC     1.154431330                             BESE6760
PI      DEC     3.14159265                              BESE6765
FL01    DEC     1.                                      BESE6840
FL02    DEC     2.                                      BESE6850
FL04    DEC     4.                                      BESE6870
FL08    DEC     8.                                      BESE6890
FL088   DEC     88.                                     BESE6910
EM88    OCT     002407555014                            BESE6930
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MAGNM ØCT   +233000000000                    BESE6940
  FRE BSS   25                               BESE6960
      END                                    BESE6970
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14. KEY WORDS	LINK A		LINK B		LINK C	
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Downey, California

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Andrew, L. V.

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Wright-Patterson AFB, Ohio 45433

13. ABSTRACT

Equations for pressure distributions and generalized aerodynamic forces are derived for a thin nonplanar lifting surface in simple harmonic motion at subsonic speeds. A digital computer program, written in Fortran IV, is also presented. The computer program will generate up to a ten by ten matrix of generalized aerodynamic forces when given data for the geometry of a planar lifting surface with a folded planar tip, the flight Mach number, the reduced frequency of motion, and some control constants. Control surface deflections are not accounted for in this study. The kernel function method given by Watkins, Runyan, and Woolston (Reference 1), which relates the pressure distribution to the downwash on a planar lifting surface, has been extended and applied to a nonplanar lifting surface. Hsu's technique (Reference 4) of employing Gaussian quadrature formulas is used when integrating the product of the kernel function and the lift function over the planform area. Recommendations are made to extend the method to account for blunted leading edges and the accompanying airfoil thickness and to account for control surface deflections.